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## ABSTRACT

### Incentives, resources and the organization of the school system

We study a model where student effort and talent interact with parental and teachers' investments, as well as with school system resources. The model is rich, yet sufficiently stylized to provide novel implications. We can show, for example, that an improvement in parental outside options will reduce parental and school effort, which are partially compensated through school resources. In this way we provide a rationale for the ambiguous existing empirical evidence on the effect of school resources. We also provide a novel microfoundation for peer effects, with empirical implications on welfare and on preferences for sorting across schools.

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# 1 Introduction

Education policy is at the forefront of the social and political debate. The belief that education is a catalyst for a better and more equitable society gives it a central role in the political agenda in both developed and developing countries. As a consequence, a variety of policies and reforms are continuously being proposed with the objective of improving the outcomes of the education system. Surprisingly, the implementation and evaluation of these policies often overlooks the changes in behavior they can induce in the actors involved in the education process. For example, the debate about the role of education resources on student learning does not usually take into account behavioral responses from parents and school administrators. Similarly, proposals of educational vouchers generally disregard how different ways to sort students into schools would affect the determination of school policies, or their influence on parental involvement in education and, crucially, the political support for such schemes. In this paper, we study a model of education where student learning effort and outcomes, parental and school behavior, and public resources devoted to education are endogenously determined in an integrated and tractable framework.

In our model, the determination of educational outcomes is a process involving four participants: children, parents, headmasters and the policymaker. Each child chooses a certain level of effort devoted to learning. More able children obtain a higher learning outcome from a unit of effort. Altruistic parents and schools affect the effort decision through incentive schemes. However, inducing effort is costly for parents as well as for schools. Both for parents and teachers, there is the opportunity cost of time involved in setting up and executing the incentive scheme, which may also include monitoring or helping children with their learning tasks (such as homework). How costly it is for schools depends on their resources (class sizes, for example), which are determined by the policymaker. This integrated framework provides an accurate description of the workings of the educational process: parents, students and the education system interact in the determination of school resources, education quality, school and parent education methods and, through all these, on students results. An advantage of our framework is its tractability, which allows us to analyze many important dimensions of the education process.

We start with a case where children are homogeneous in terms of innate ability and parental opportunity cost of time. We find that the power of parental and school incentives and resources devoted to education increase with student innate ability. The results are less clear cut when we analyze the impact of an increase in the opportunity cost of time associated with parental involvement in their children learning process. This introduces, in a natural

way, the connection between labor market conditions and parental direct involvement in their children education. This link goes beyond the hourly wage. For example, it can also capture changes in opportunities and incentive for female participation in the labor market. In either case, the power of parental incentives (i.e., the reward for every effective unit of student effort) is decreasing in the opportunity cost of time.

The interaction between the school system and parental inputs is the reason why political considerations are important. As the parental opportunity cost of time increases, they would like to rely more heavily on school rewards, which triggers actions by those responsible for the education system. The policymaker anticipates participant choices and internalizes parental desires by increasing the resources devoted to education. Interestingly, the increase in school resources may not be accompanied by an overall increase in educational attainment. A result far too familiar for those in the educational policy arena.<sup>1</sup> Our model can predict “disappointingly” weak effects of school resources on student results even in situations where school resources do in fact affect learning *ceteris paribus*. The weak effect can be rationalized because *ceteris paribus* does not hold when resources increase. Parental involvement decreases because of a change in their opportunity costs. School resources increase to compensate for this reduction. These resources have in fact an effect, but this is not apparent because of concomitant changes in parental involvement in the education process. This process could also explain why the increase in expenditures per student observed in many countries during the last decades has not been followed by better test scores or improvements in other measures of student performance.<sup>2</sup>

We then allow for children to differ in terms of ability and parental opportunity costs of time, which leads to a number of insights. First, as the school determines the power of incentives for the average individual in the classroom, school rewards are positively affected by the mean ability of a student in a school. In equilibrium, this affects the intensity of

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<sup>1</sup>The empirical findings of the class-size literature are ambiguous. For example, Angrist and Lavy (1999); Krueger (1999); Urquiola (2006) report positive results of class-size on student attainment while others (Hanushek (2003); Hoxby (2000); Leuven, Oosterbeek, and RØonning (2008); Anghel and Cabrales (2010)) find no gains.

<sup>2</sup>See Hanushek (1998) for the case of the US. This result also provides a possible reason for a low cross-country correlation between education expenditures and school attainment levels results in standardized tests. See for example, Hanushek (2006). And this “anomaly” has been recognized for a long time. For example, in words of The Economist, “Glance at any league table of educational performance and you will find several Asian countries bunched near the top. The achievements of the region are a puzzle to people who think that educational success is all a matter of expenditures. Even in Japan most of the schools are ill-equipped by comparison with their western equivalents [...] The children are driven on by intense family pressure. Parents badger their children to succeed, but they also make big financial and personal sacrifices to help them do so. Mothers help their children with their homework [...] Fathers promise fancy toys and activities in return to examination success...” Quote from The Economist, November 21st 1992.

rewards that parents decide. Thus, peer effects arise endogenously, as the choice of effort and student rewards depend in equilibrium on the average ability of the student in the classroom. This effect is reinforced by the determination of school resources. The policy maker decides the level of resources optimally given the characteristics of the school attended by the median voter's child. Thus, the decision on school resources will be based on the average ability of this school and on the median child's ability. As a consequence, student effort depends on the mean abilities of peers at her/his school, plus the ability of the median child and his peers. Our model generates in this way a microfoundation for peer effects, rather than assuming them to come from some exogenous "contagion" process, as it is more common in the literature.

In this context, an increase of the opportunity cost of the median parent raises similar issues to those identified for the homogeneous case. However, the link between median child characteristics and individual effort generates a channel through which changes in the distribution of income (or talent) can affect the educational choices of households and schools. For example, an increase in the income of the median child's household will generate an increase of resources in the system. This will induce a positive effect on households in lower parts of the income distribution even if their incomes do not change. And the other way around, if the income of the median does not change (or it changes very little) in an environment where mean income is increasing markedly, there will be few changes in school outcomes (or even a regression) at a time when income appears to be fast increasing.

In a setting with heterogeneous children, we study the motivation for school sorting and its effects on educational achievement. We find that whether total student achievement is maximized by segregating students in public schools according to their ability depends crucially on how resources are allocated in segregated and not segregated settings. Our analysis identifies two channels driving this result: schools incentives and resources. Sorting according to ability implies that some schools' headmasters end up with higher average student ability and others with lower average student ability, as compared with an environment where children are randomly allocated to schools. Through the convexity of incentives, this translates in an increase in the power of incentives at schools with better students that more than compensates the decrease in incentives at schools with worse students. In our environment, however, resources to the school system may increase (decrease) if with sorting the average ability of the school attended by the median voter child increases (decreases).

Consider a situation where parents differ in culture (or other values such as religion) or the emphasis the schools put in different subjects (e.g., arts or sports). If school values

are chosen by the policy maker according to the demands of the median parent. We show that segregation according to these traits enhances student effort for children whose parents have values far from the median. The reason is as follows: parental incentives to motivate their children are greater when their values coincide with those generated at the school. Interestingly, those close to median are harmed by segregation in values because they benefit from the availability of higher resources in an equilibrium without segregation by values.

There are situations where ability sorting can give more ambiguous results. If sorting reduces the variance of talent within a classroom, teaching can be targeted better to individual needs and *cæteris paribus* improve learning. Duflo, Dupas, and Kremer (2008) argue that this is a possible explanation for their observation of a positive influence of ability sorting in all type of students. But one cannot reduce the variance of talent for all classrooms, as happens with ability sorting, without shifting the means in them. Thus, assuming that variance reduction in the classroom decreases the cost of teaching effort, we find that ability sorting increases the mean performance in the system, as well as that of the better able students. For students in lower parts of the distribution, the result is more ambiguous. This finding is important because it reconciles positive results of ability sorting into schools for children in all parts of the ability distribution (e.g., Ding and Lehrer (2007) and Duflo, Dupas, and Kremer (2008)), the fact that some studies (e.g., Ding and Lehrer (2007)) find a stronger effect in the upper parts of the distribution, and the ambiguous effects of ability sorting in earlier papers (e.g., Betts and Shkolnik (2000)).

We finally incorporate private schools in our setting. We show first that a mixed education system with public and private schools satisfies the condition for generating endogenous sorting by either parental opportunity cost of time or student talent. This allows us to analyze the effect of policies increasing school choice for parents, like a voucher scheme. We show that, even if the median voter is favored (and hence the voucher policy approved), the reaction of schools to the changes in classroom composition, will increase inequality in student scholar achievement. This is so because the worsening of peer effects in the schools where the less able students stay is magnified by the responses of other actors. The school principals will decrease the power of incentives at those schools, and policymakers will decrease the resources devoted to them. Hence, our framework allows us to understand in a simple way the effects of students' quality, and the reaction of other actors to this quality, on the incentives for school sorting.<sup>3</sup> Furthermore, we can then show how sorting feeds back

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<sup>3</sup>Previous papers focus on one of these elements. While Urquiola and Verhoogen (2009) and Epple, Romano, and Sieg (2006) focus on educational quality, Epple and Romano (1998, 2008) and Epple, Figlio, and Romano (2004) concentrate on purely peer effects. In Epple, Figlio, and Romano (2004), both ingredients



on school quality and classroom peer-effects.

Given that school resources are endogenously determined in our model, we also make a contribution to the literature on the determinants of class size. In Lazear (2001) class size is decided by the schools according to student behavior. For example, when students have a shorter attention span (i.e., they can be distracted more easily) students should be sorted in smaller classroom as they require closer attention. In Urquiola and Verhoogen (2009), schools differ in productivity and offer different quality levels (school size). As parents differ in earnings, sorting between schools with different class sizes arises naturally. Our model offers a complementary mechanism behind the determination of class size, which relies upon the interaction of parental and school motivation, which is partly determined by the government through the (strategic) choice of school resources.

We organize the paper as follow: In section 2, we set up the model. We characterize the equilibrium in section 3, where we discuss the interdependence between parental and school motivation systems, school resources and student performance. Section 4 contains a number of implications of our model for different school policies: tracking, faith schools, the effect of a voucher scheme (for which we first study endogenous school sorting into private schools), as well as policies inducing parents to participate in the school organization of activities.

## 2 The model

The model has four types of participants: children, parents, headmasters and the policymaker<sup>4</sup>. Each child chooses a certain level of effort to be devoted to learning. Her ability affects how much human capital she extracts from every unit of effort. Parents and schools affect the effort decision through incentive schemes. Inducing effort is costly for parents as well as for schools. Both for parents and teachers, the main cost is the opportunity cost of the time involved in setting up and executing the incentive scheme. The cost for schools depends as well on the level of resources (for example, class size), which are determined by a benevolent policymaker.

**Student performance and Children's short-term utility** School performance for child  $i$ ,  $H_i$ , is a linear function of her effort,  $e_i$ . In particular, we assume,

$$H_i = v_i e_i, \tag{1}$$

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are present in a model of higher education although no analytical solution is offered.

<sup>4</sup>For simplicity, we refer to students as she and to teachers as he.

where  $v_i$  is a measure of the ease at which she can learn by putting effort, a sort of total factor productivity in the child's production function. Furthermore, we assume that there is a cost of exerting effort that takes a quadratic form.

Children do not internalize directly the effect of their effort in human capital. Instead, they react to a short-term utility determined by the incentive schemes that the school and the family put in place (e.g., praise and acceptance from parents and teachers for achieving a goal). This assumption is certainly reasonable for primary school education, where children are learning about the consequences of postponing immediate gratification in exchange for greater future rewards. Denote by  $c_{1i}$  a summary of family  $i$ 's reward for every unit of effort and  $c_{2j}$  the school reward  $j$  for every unit of effort, so that

$$c_{ij}e_i = (c_{1i} + c_{2j})e_i.$$

This specification captures the idea that parents and school incentives are substitutes. Little is changed qualitatively once the reward systems of different principals are assumed to be complements. It also assumes away the issue of unconditional love (i.e., there is no intercept in the reward schemes) as we assume that children can't choose families or leave the school chosen by their parents.

These considerations imply that her short-term utility is given by:

$$U_{S_i} = (c_{1i} + c_{2j})e_i - \frac{1}{2}e_i^2. \quad (2)$$

Our assumption about the child's utility is that parent and school incentives enter positively in the utility so they can be interpreted as rewards. Of course, inducing effort may involve punishments as well. In this case, we could have written the utility in the alternative way:

$$U_{S_i} = -c_{ij}(1 - e_i) - \frac{1}{2}e_i^2 = c_{ij}e_i - c_{ij} - \frac{1}{2}e_i^2.$$

As will be clear below, this utility induces the same optimal action from her as the one we examine. Thus, provided the costs of the two incentive systems can be written in the same way, there will be no difference in any equilibrium value<sup>5</sup>.

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<sup>5</sup>In our setting, this is the case because we assume that parents care about  $H$  but not about her short term utility.

**Parents' utility** We assume that every parent has one child and that their utility is influenced by the sum of her performance and their own welfare, denoted by  $W_i$ .<sup>6</sup> Hence,

$$U_{P_i} = H_i + W_i.$$

Parental welfare depends on the time spent at work or pursuing leisure activities. This is the total time  $T$  available minus the time spent generating the reward which we assume is a linear function of the reward  $c_{1i}e_i$ . Thus, letting  $\psi_i$  be the opportunity cost of parent  $i$  yields,

$$W_i = \left(T - \frac{c_{1i}e_i}{2}\right) \psi_i,$$

and, therefore,

$$U_{P_i} = v_i e_i + \left(T - \frac{c_{1i}e_i}{2}\right) \psi_i. \quad (3)$$

Since  $\psi_i$  is an opportunity cost of time for parents, it can be interpreted as income or wages, although it can also be value of leisure or something else. Hence, in the remainder we often refer to this parameter as parental income.

**School system utilities** In our model, every school determines its own school ethos (learning incentives). We consider first the case of public (state) schools. In this case, we assume that headmasters chooses learning incentives (summarized in the parameter  $c_{2j}$ ) in order to maximize the sum of the average student performance and the welfare of the average teacher in the school. The average welfare of a teacher is determined by the difference between the total time available to him and the average time he devotes to his students, which we assume is a linear function of the reward. Thus, letting  $\gamma$  be the opportunity cost of teachers' time,

$$U_{HM_j} = \frac{1}{N_j} \sum_{i \in j} v_i e_i + \left(T - \frac{n_j}{N_j} \sum_{i \in j} \frac{c_{2j}e_i}{2}\right) \gamma, \quad (4)$$

where  $N_j$  is the total number of students in school  $j$  and  $n_j$  is the number of students per classroom.

The policymaker internalizes the utility of the (median-voter) parent (denoted by  $\bar{P}_i$ ), but

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<sup>6</sup>The utility function below does not internalize the child's cost of effort, so it is not purely "sympathetic". On the one hand this is reasonable, since this cost of effort is not observable. But we have also done the computations with strictly sympathetic parents' utility and there are no significant changes.

adds the cost of the system's actions, which being common to the system constitute a public good that neither parents nor headteachers internalize. This cost depends on the number of classes to be manned, which is the ratio of total number of students in the system,  $N$ , to the number of students per class,  $n$ . We assume that all public schools have the same class size so that  $n_j = n$  for all  $j$ . Manning costs are assumed to be quadratic in the number of classrooms. This can be justified by taking into consideration that the state has monopsony power in the market for teachers and faces a wage function that increases in the number of teachers hired. If the relationship between wages and the number of teachers hired is assumed linear, the cost function must be quadratic. Thus, the policymaker's preferences are represented as,

$$U_{PM} = U_{\bar{P}_i} - \frac{\omega'}{2} \left( \frac{N}{n} \right)^2, \quad (5)$$

where  $\omega'$  is a constant parameter summarizing the cost of reducing class size<sup>7</sup>. For ease of notation, in the remainder we denote  $\omega = \omega'N^2$ , so that

$$U_{PM} = U_{\bar{P}_i} - \frac{\omega}{2} \frac{1}{n^2}, \quad (6)$$

**The structure of the game** The policymaker announces first the policy variable ( $n$ ). After this announcement, parents and headmasters simultaneously decide their optimal levels of rewards per unit of effort  $c_{1i}$  and  $c_{2j}$ , respectively. After observing parents' and schools' announcements, the children decide their optimal level of effort,  $e_i$ .

### 3 Equilibrium

We solve the game by backward induction.

#### 3.1 Students', parents', and school choices

From equation (2), it follows that the optimal student action is

$$e_i = (c_{1i} + c_{2j}). \quad (7)$$

Substituting this expression into the parents' utility, equation (3), we obtain

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<sup>7</sup>The model abstracts from the issue of taxation. Thus, our formulation assumes that schools are financed out of lump sum taxation and the government keeps a balanced budget.

$$U_{P_i} = (c_{1i} + c_{2j}) v_i + \left( T - \frac{1}{2} c_{1i} (c_{1i} + c_{2j}) \right) \psi_i.$$

The first-order condition for the parents's problem is then

$$v_i - \left( c_{1i} + \frac{c_{2j}}{2} \right) \psi_i = 0.$$

Given that this condition is sufficient, the optimal choice of the parent is

$$c_{1i} = \max \left\{ \frac{v_i}{\psi_i} - \frac{c_{2j}}{2}, 0 \right\}, \quad (8)$$

which is always non-negative given that producing the rewards requires investing parental time.

It is clear from the expression for  $c_{1i}$  that the power of parental incentives are increasing in ability ( $v_i$ ) and decreasing in the opportunity cost of time ( $\psi_i$ ). Also, equation (8) shows the negative relationship between  $c_{1i}$  and  $c_{2j}$ .<sup>8</sup> When the stakes in the school for good performance are high, the gains from additional effort induced by parental rewards are smaller. We shall discuss below how both incentive schemes may compensate each other in responding to changes in  $\psi_i$ .

By substituting the optimal choice of children's effort into the utility function of the headmaster (4) we obtain:

$$U_{HMj} = \frac{1}{N_j} \sum_{i \in j} (c_{1i} + c_{2j}) v_i + \left( T - \frac{n_j}{N_j} \sum_{i \in j} \frac{c_{2j} (c_{1i} + c_{2j})}{2} \right) \gamma.$$

It follows that an interior solution for the headmaster's optimization problem implies

$$c_{2j} = \frac{\bar{v}_j}{\gamma n_j} - \frac{\bar{c}_{1j}}{2}, \quad (9)$$

where  $\bar{v}_j$  is the mean student ability and  $\bar{c}_{1j}$  is the mean parental reward for the students attending school  $j$ .

Following Epple and Romano (1998), we can interpret  $\bar{v}_j$  as the school's quality. On the hand, higher school quality is associated with greater classroom motivation. This association, which will tend to amplify school differences in student performance, will be important in the

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<sup>8</sup>The assumption of substitute rewards is not essential for this result. A similar result is obtained with other specifications where rewards are complements. An example of this is to assume  $H_i = v_i e_i^\alpha$ , with  $\alpha < 1$  and  $c_{ij} = c_{1i} c_{2j}$ . The driving force in this result is that greater school incentives will reduce the marginal benefit of parental effort.

emergence of peer effects and will provide the motivation for a segregated school system, as we shall discuss in the following section. Also, notice that a bigger class size has a negative effect on school rewards. Finally, a high level of motivation by the parents is associated with lower school incentives.

## 3.2 Equilibrium values for $c_1$ , $c_2$ and $n$

### 3.2.1 Homogeneous children

In order to solve for the first stage of the game, let us first assume that children are identical, so that  $v_i = v$  and  $\psi_i = \psi$  for all  $i \in \{1, \dots, N\}$ . In this case we have that  $c_{1i} = c_1$  for all  $i$  and  $c_{2j} = c_2$  for all  $j$ . Therefore, from (8) and (9), an interior solution for  $c_1$  and  $c_2$  implies

$$c_1 = \frac{v}{\psi} - \frac{c_2}{2} \quad (10)$$

and

$$c_2 = \frac{v}{\gamma n} - \frac{c_1}{2}. \quad (11)$$

In the first stage, the policymaker considers the optimal levels of rewards to be offered by parents and schools. After substituting (11) in (10) and plugging the resulting expression, together with (7), into (6) we obtain

$$U_{PM} = \frac{2v^2}{3} \left( \frac{1}{\psi} + \frac{1}{\gamma n} \right) + \left( T - \frac{2v^2}{9} \left( \frac{2}{\psi^2} + \frac{1}{\psi\gamma n} - \frac{1}{(\gamma n)^2} \right) \right) \psi - \frac{\omega}{2} \frac{1}{n^2}.$$

The interior solution that results from maximizing the above expression with respect to  $n$  is (assuming  $\omega > \psi (2v/3\gamma)^2$ )

$$n = \frac{\omega - \left( \frac{2v}{3\gamma} \right)^2 \psi}{\frac{1}{\gamma} \left( \frac{2v}{3} \right)^2}. \quad (12)$$

Therefore, class size is increasing in the cost of manning classes,  $\omega$ , and the opportunity cost of time of teachers,  $\gamma$ , and decreasing on student ability,  $v$ , and parental opportunity cost of time,  $\psi$ .

Finally, we substitute the optimal level of  $n$  in equation (11) and then into equations (7) and (10) to obtain the equilibrium values of  $c_1$ ,  $c_2$  and  $e$ . These are:

$$c_1 = \frac{2v}{3\psi} \left( \frac{2\omega - 3 \left( \frac{2v}{3\gamma} \right)^2 \psi}{\omega - \left( \frac{2v}{3\gamma} \right)^2 \psi} \right),$$

$$c_2 = \frac{2v}{3\psi} \left( \frac{3 \left( \frac{2v}{3\gamma} \right)^2 \psi - \omega}{\omega - \left( \frac{2v}{3\gamma} \right)^2 \psi} \right),$$

and

$$e = \frac{\frac{2v\omega}{3\psi}}{\omega - \left( \frac{2v}{3\gamma} \right)^2 \psi}. \quad (13)$$

From inspecting the above expressions it becomes clear that a necessary condition for a positive  $c_1$  is

$$\omega > \frac{3}{2} \left( \frac{2v}{3\gamma} \right)^2 \psi. \quad (14)$$

This condition is sufficient for  $n$  and  $e$  to be positive as well. A necessary condition for a positive  $c_2$  is

$$\omega < 3 \left( \frac{2v}{3\gamma} \right)^2 \psi. \quad (15)$$

The comparative static results of our model with respect to  $v$  are simple. An increase in student ability ( $v$ , which one can think of as innate or the result of early parental stimulation) reduces class size and leads to stronger school and parental incentives. These factors in turn induce higher student effort.

The effect on effort of an increase in  $\psi$  is less obvious. First, while  $c_1$  decreases when  $n$  falls (through the effect on  $c_2$ ), both  $e$  and  $c_2$  are positively associated with falls in  $n$ . A higher  $\psi$  imposes a higher opportunity cost for parents to engage in motivational activities. Hence,  $c_1$  is decreasing in  $\psi$ . The school system reacts to this by reducing  $n$  and therefore  $c_2$  is increasing in  $\psi$ . The driving force for this result is that the policymaker devotes more resources to classroom education, which lowers the cost of inducing effort by the school. Conversely, lower class size and the consequent stronger school ethos, reduces the gain from staying at home inducing children's effort.

In our model, the effect of reducing parents involvement is not always fully compensated by the school system so the net effect of an increase of  $\psi$  on student performance may be

negative. To see this:

$$\frac{\partial e}{\partial \psi} = -\frac{2\nu\omega}{3} \frac{\omega - 2\left(\frac{2\nu}{3\gamma}\right)^2 \psi}{\left[\omega\psi - \left(\frac{2\nu}{3\gamma}\right)^2 \psi^2\right]^2}$$

which implies that effort is decreasing in  $\psi$  when

$$\omega > 2\left(\frac{2\nu}{3\gamma}\right)^2 \psi.$$

Notice that from (14) and (15)

$$3\left(\frac{2\nu}{3\gamma}\right)^2 \psi > \omega > \frac{3}{2}\left(\frac{2\nu}{3\gamma}\right)^2 \psi$$

so that effort and performance can be both increasing or decreasing within our parametric range.

A key contribution of our model is that it provides a tractable structural framework where one can see how the impact of higher school resources may be mitigated by parents reactions and vice versa. For example, our model has identified two sources of variation for class size,  $\omega$  and  $\psi$ , that may lead to different policy estimates of the impact of class size on student performance. As we pointed out above, an increase in the opportunity cost of parents and a fall in the cost of manning classes both lead to lower class sizes. However, a fall in the cost of manning classes leads to a unambiguous increase in effort (as can be seen from (13)) and an improvement in student performance. The fact that an increase in class size may lead to different effects depending on the source of variability that generates these changes provides an interesting lens through which we can interpret the findings in the empirical literature. When the source of variability for class size comes from exogenous changes in the costs of manning classes, like in most randomized experiments, we should expect positive impacts on student performance. However, in cross-country or cross-state panel studies, where differences in class-size may result from increases in opportunity costs for the median parents, we may find more difficult to observe improvement in educational performance.



### 3.2.2 Heterogeneous children

We relax now the assumption of identical children and assume, as before, that they differ in their abilities ( $v_i$ ) and parent's income ( $\psi_i$ ). To gain in clarity, we define the following piece of notation:

$$\bar{\Omega}_j \equiv \frac{1}{N_j} \sum_{i \in j} \frac{v_i}{\psi_i}.$$

In words,  $\bar{\Omega}_j$  is the average at the school level of the ratio of student talent to parental opportunity cost. Thus, each school  $j$  is associated with a particular  $\bar{\Omega}_j$ .<sup>9</sup>

To obtain the utility of the policymaker, we substitute (9) in (8) and plug the resulting expression into (7). This yields

$$e_i^* = \frac{v_i}{\psi_i} + \frac{1}{3} \left( \frac{2}{\gamma n} \bar{v}_j - \bar{\Omega}_j \right), \quad (16)$$

where  $\bar{v}_j$  is the average talent in school  $j$ . Thus, equation (6) becomes:

$$\begin{aligned} U_{PM} = & \left( \frac{v_{i_M}}{\psi_{i_M}} + \frac{1}{3} \left( \frac{2}{\gamma n} \bar{v}_{j_M} - \bar{\Omega}_{j_M} \right) \right) v_{i_M} + \\ & + \frac{1}{2} \left( T - \left( \left( \frac{v_{i_M}}{\psi_{i_M}} \right)^2 - \frac{1}{9} \left( \frac{2}{\gamma n} \bar{v}_{j_M} - \bar{\Omega}_{j_M} \right)^2 \right) \right) \psi_{i_M} - \frac{\omega}{2} \frac{1}{n^2}, \end{aligned}$$

where  $M$  stands for the median voter and therefore  $\bar{v}_{j_M}$  and  $\bar{\Omega}_{j_M}$  express the characteristics in the school the median voter attends. Importantly, notice that the preferences of parents with respect to  $n$  is unimodal and therefore we can use the median voter theorem to find the optimal  $n$ .

The first-order condition for the policymaker's maximization problem if there is an interior solution is:

$$-\frac{2}{3} \frac{v_{i_M} \bar{v}_{j_M}}{\gamma n^2} - \frac{2}{9} \frac{\psi_{i_M} \bar{v}_{j_M}}{\gamma n^2} \left( \frac{2}{\gamma n} \bar{v}_{j_M} - \bar{\Omega}_{j_M} \right) + \frac{\omega}{n^3} = 0.$$

At this point, we make the following assumption which significantly reduces notational complexity:

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<sup>9</sup>Notice that in the plausible cases where  $v_i$  and  $\psi_i$  are correlated the ranking of schools would be indifferent to whether the ranking is based on  $v$ ,  $\psi$  or  $\bar{\Omega}$ .

ASSUMPTION 1

$$\frac{v_{i_M}}{\psi_{i_M}} = \bar{\Omega}_{j_M}.$$

This means that the median child talent to parental opportunity cost ratio is equal to the average talent to parental opportunity ratio in her school. Thus,

$$n = \frac{\omega - \left(\frac{2\bar{v}_{j_M}}{3\gamma}\right)^2 \psi_{i_M}}{\frac{1}{\gamma} \frac{4\bar{v}_{j_M} v_{i_M}}{9}} \quad (17)$$

if  $\omega - \frac{4}{9\gamma^2} (\bar{v}_{j_M})^2 \psi_{i_M} > 0$ . School resources increase in the opportunity cost of the median parent and the ability of the median child as well as the quality of the school she attends.

From the derivation of (17) it is clear that parents of children with  $v_i$  above  $v_{i_M}$  would like the level of school resources to be higher (e.g., smaller class sizes). So it would make sense for them to supply the school with extra resources, in the form of their own time and material resources. As we explore in the next section, this has strong implications for segregation. But even within public schools, they can choose, if allowed, to do so. This could explain why parents choose to organize activities in schools, which as Anghel and Cabrales (2010) document for the case of Spain have a sizable effect on student achievement.

For expositional ease, it is convenient to define:

$$\theta_j \equiv \frac{\bar{\Omega}_j}{\bar{\Omega}_{j_M}}$$

which under assumption (1) implies that

$$\bar{\Omega}_j = \theta_j \bar{\Omega}_{j_M} = \theta_j \frac{v_{i_M}}{\psi_{i_M}}.$$

Using equation (17), the equilibrium values for  $c_{1i}$ ,  $c_{2j}$  and  $e_i$  follow:

$$c_{1i} = \frac{v_i}{\psi_i} - \frac{1}{3} \frac{v_{i_M}}{\psi_{i_M}} \left( \frac{\left(\frac{2\bar{v}_j}{\bar{v}_{j_M}} + \theta_j\right) \left(\frac{2\bar{v}_{j_M}}{3\gamma}\right)^2 \psi_{i_M} - \theta_j \omega}{\omega - \left(\frac{2\bar{v}_{j_M}}{3\gamma}\right)^2 \psi_{i_M}} \right) \quad (18)$$

$$c_{2j} = \frac{2}{3} \frac{v_{i_M}}{\psi_{i_M}} \left( \frac{\left(\frac{2\bar{v}_j}{\bar{v}_{j_M}} + \theta_j\right) \left(\frac{2\bar{v}_{j_M}}{3\gamma}\right)^2 \psi_{i_M} - \theta_j \omega}{\omega - \left(\frac{2\bar{v}_{j_M}}{3\gamma}\right)^2 \psi_{i_M}} \right) \quad (19)$$

$$e_i = \frac{v_i}{\psi_i} + \frac{1}{3} \frac{v_{i_M}}{\psi_{i_M}} \left( \frac{\left(\frac{2\bar{v}_j}{\bar{v}_{j_M}} + \theta_j\right) \left(\frac{2\bar{v}_{j_M}}{3\gamma}\right)^2 \psi_{i_M} - \theta_j \omega}{\omega - \left(\frac{2\bar{v}_{j_M}}{3\gamma}\right)^2 \psi_{i_M}} \right) \quad (20)$$

**REMARK 1** *The interaction between parents, schools and education policy generates peer-effects.*

The expression for (20) reveals that, in equilibrium, the performance of student  $i$  depends on the ability of her peers at different levels. Therefore, the model provides a micro-foundation for the emergence of peer effects in the classroom without technological assumptions. First, we obtain peer-group effects in the sense that the student's own performance increases in  $\bar{v}_j$ . The driving force is the reward scheme at the school level, which depends on the mean ability of her peers. To our knowledge, this is new in the literature. Second, performance is affected by the cohort's median ability and the peer-group ability of the median student. This sort of peer-cohort effect result from the determination of the school resources that affects classroom motivation in public schools. We view these results as a cautionary note regarding empirical analyzes which aim at measuring the effect of different education policies as if they were exogenous to the political process<sup>10</sup>.

As in the case of homogeneous parents and children, an economy wide increase in parental opportunity costs induces a reduction of effort by the parent (the effect of  $\psi_i$  in (20) is negative) that is partially compensated by the increased effort of the school system (the effect of  $\psi_{i_M}$  in (19) is positive). But because of the link between median child and individual effort, changes in the distribution of income (or talent) can affect the outcomes. For example, an increase in  $\psi_{i_M}$  will generate an increase of resources (a decrease in  $n$ ) which will have a positive effect on lower income household even if their incomes do not change. Thus, a rising tide lift all votes in this case.<sup>11</sup> And the other way around, if  $\psi_{i_M}$  is unchanged (or almost)

<sup>10</sup>A related point appears in Besley and Case (2000).

<sup>11</sup>Corcoran and Evans (2010) find that 12 to 22 percent of the increase in local school spending in the U.S. over the period 1970-2000 is attributable to rising inequality.

in an environment where mean income is increasing markedly, there will be few changes in school outcomes (or even a regression, because of the negative reaction in  $c_{1i}$  of very high income households) at a time when GDP is increasing.

## 4 Implications

Our model emphasizes the interdependencies established between (parents and school) incentive systems, class resources and children's effort. As a consequence, student performance depends on group and cohort peer effects. The implications of this finding run deeply into the different variables of the system. We first analyze the effects of segregation within the public school system. Later, we introduce private schools and investigate sorting in school market and the effect of policies inducing segregation, like school vouchers.

### 4.1 School segregation in public education

Suppose it is costless to group students according to some characteristics. Would students be willing to be grouped by ability? Would they be willing to be grouped by their parents' values?

#### 4.1.1 Vertical segregation and efficiency

In a situation where there is no segregation, all the schools share the same distribution of students and therefore they should be identical in terms of their quality. Assuming for simplicity that the opportunity cost of time for parents is homogeneous (i.e.,  $\psi_i = \psi$ ), aggregate student performance becomes (as stated in equation (16)):

$$\bar{H}_{NS} = \int \left( \frac{v_i}{\psi} + \frac{1}{3} \left( \frac{2\bar{v}}{\gamma n_{NS}} - \frac{\bar{v}}{\psi} \right) \right) v_i df(v_i).$$

where  $n_{NS}$  is the optimal class size defined by (17). The above expression can be written as,

$$\bar{H}_{NS} = \int \frac{v_i^2}{\psi} df(v_i) + \frac{\bar{v}^2}{3} \left( \frac{2}{\gamma n_{NS}} - \frac{1}{\psi} \right). \quad (21)$$

On the other hand, consider  $l$  the number of public school and let the students be assigned to school according to their ability. In this case, aggregate student performance is:

$$\bar{H}_S = \int \frac{v_i^2}{\psi} df(v_i) + \sum_l \frac{\bar{v}_l^2}{3} \left( \frac{2}{\gamma n_S} - \frac{1}{\psi} \right) \quad (22)$$

where  $n_S$  is the optimal class size. Comparing (21) with (22) establishes the following result:

**PROPOSITION 1** *Suppose students are completely sorted at school according to their ability. Then,*

1. *For an equal class size  $n = n_{NS} = n_S$ , the average human capital increases with respect to a situation where all schools share the same distribution of students.*
2. *If class size,  $n$ , is determined as in (17), and average talent at the school of the median voter ( $\bar{v}_{j_M}$ ) increases with sorting, then the average human capital increases with respect to a situation where all schools share the same distribution of students.*

**Proof 1** *Part 1 follows from (21) and (22) by applying Jensen’s inequality. Part 2 then follows by noting that in (22)  $\bar{H}_S$  decreases in  $n_S$ , itself decreasing in  $\bar{v}_{j_M}$  by (17).*

**COROLLARY 1** *If class size,  $n$ , is determined as in (17) and average talent at the school of the median voter ( $\bar{v}_{j_M}$ ) decreases with sorting, then the average human capital may increase or decrease with respect to a situation where all schools share the same distribution of students.*

This result, is driven by the reactions of the educational authorities to changes in class composition. By definition, tracking by ability increases the mean ability of classmates of relatively talented students but has the opposite effect on peers of relatively less talented ones. For a fixed level of school resources, this provokes headmasters to increase (reduce) incentives in schools with greater (lower) mean student ability. This has a negative effect on the performance of less talented students and a positive effect on the more talented ones. The convexity of performance with respect to ability implies that the gains by high-ability students offset the losses incurred by the low-ability ones. On the other hand, whether tracking increases school resources will depend on whether the median child gets into a better school or stays in a deteriorated one. This effect is a novelty of our model, as most of the literature takes the level of school resources as exogenous with respect to sorting.<sup>12</sup>

#### 4.1.2 Horizontal segregation

Suppose now that parents differ in their preferences about the type of knowledge (and/or cultural/religious values) they would like their children to receive in the school. Clearly, schools build “values” beyond labor market ability, and parents do not always agree on the

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<sup>12</sup>See, for example, Urquiola and Verhoogen (2009), Epple and Romano (1998), Epple, Romano, and Sieg (2006) and Epple, Figlio, and Romano (2004).

best set of values. This includes differences in the predominant cultural trait or faith in the school, or the emphasis the school assigns to a particular subset of knowledge or skills, such as arts, sciences or sports, for which abilities may be imperfectly correlated. In this case, the relevant policy question is whether to allow for the creation of public schools differing in their “horizontal” characteristics.

To investigate the effects of horizontal segregation, we assume that parents’ differences in values are embedded in a parameter  $\tau_i$ . To summarize parents’ concerns, we assume that

$$U_{P_i} = F_i H_i + \left( T - \frac{1}{2} c_{1i} e_i \right) \psi_i \quad (23)$$

where

$$F_i = 1 - (\Phi - \tau_i)^2,$$

$\Phi$  is a policy parameter chosen by the school system and  $\tau_i$  is the parameter value that parent  $i$  thinks is best for the education of his children. The way  $F$  enters into the utility of parents implies that they will have more incentives to induce schooling effort in their children if the school offers values that match better with their preferences.

Substituting optimal effort into the parents’ utility, we obtain

$$U_{P_i} = F_i (c_{1i} + c_{2j}) v_i + \left( T - \frac{1}{2} c_{1i} (c_{1i} + c_{2j}) v_i \right) \psi_i.$$

Therefore, if there is an interior solution, the optimal choice for the parent is

$$c_{1i} = \frac{F_i v_i}{\psi_i} - \frac{c_{2j}}{2}. \quad (24)$$

To describe more easily the school, by analogy to the previous section, we define:

$$\widehat{\Omega}_j = \frac{1}{N_j} \sum_{i \in j} \frac{F_i v_i}{\psi_i}.$$

As the headmaster utility is unchanged with respect to (4),  $c_{2j}$  is determined by

$$c_{2j} = \frac{4\bar{v}_j}{3\gamma n_j} - \frac{2}{3} \widehat{\Omega}_j. \quad (25)$$

Given  $c_{1i}$  and  $c_{2j}$ , the objective for the policymaker is to maximize

$$\begin{aligned}
U_{PM} &= \left( \frac{F_{i_M} v_{i_M}}{\psi_{i_M}} + \frac{1}{3} \left( \frac{2}{\gamma n} \bar{v}_{j_M} - \widehat{\Omega}_M \right) \right) v_{i_M} + \\
&+ \frac{1}{2} \left( T - \left( \left( \frac{F_{i_M} v_{i_M}}{\psi_{i_M}} \right)^2 - \frac{1}{9} \left( \frac{2}{\gamma n} \bar{v}_{j_M} - \widehat{\Omega}_M \right)^2 \right) \right) \psi_{i_M} - \frac{\omega}{2} \frac{1}{n^2},
\end{aligned}$$

with respect to  $\Phi$  and  $n$ . If there is no cost associated to the choice of parameter  $\Phi$ , then the optimal choice for the school authority is to make  $\Phi = \tau_M$  (i.e., the value of  $\Phi$  preferred by the median voter). Therefore,  $F_{i_M} = 1$  and  $n$  is defined by:

$$n = \frac{\omega - \left( \frac{2\bar{v}_{j_M}}{3\gamma} \right)^2 \psi_{i_M}}{\frac{2v_{i_M}\bar{v}_{j_M}}{3\gamma} - \frac{2}{9\gamma} \widehat{\Omega}_M \bar{v}_{j_M} \psi_{i_M}}.$$

If, in addition to assumption 1, we further assume that the distribution of  $v_i / \psi_i$  is orthogonal to the distribution of  $\tau_i$ , we obtain that  $\widehat{\Omega}_M = \bar{F}_{j_M} \frac{v_{i_M}}{\psi_{i_M}}$ , so that:

$$n = \frac{\omega - \left( \frac{2\bar{v}_{j_M}}{3\gamma} \right)^2 \psi_{i_M}}{\frac{2(3-\bar{F}_{j_M})v_{i_M}\bar{v}_{j_M}}{9\gamma}}. \tag{26}$$

Notice that the optimal response from the policy maker to school heterogeneity (i.e., an  $\bar{F}_{j_M}$  further away from 1) is to reduce class size.

Using the above expressions and (24) in (7), we obtain an expression for the level of student effort that takes into account parents' cultural views:

$$e_i = \frac{F_i v_i}{\psi_i} + \frac{1}{3} \left( \frac{2(3-\bar{F}_{j_M})\bar{v}_j 4\bar{v}_{j_M} v_{i_M}}{9\gamma^2} - \widehat{\Omega}_j \right). \tag{27}$$

From the first term of (27), we can see that for a child that is educated in an environment with values very different from those of her family effort tends to fall. However, in a given school the teachers and school authorities tend to compensate for a lower average effort with a higher one of their own. This, in turn, means that while for many students their effort increases when they are segregated, it can actually decrease for some of them. The easiest way to see this is for the median voter's child. She is having her parents preferred  $\tau_i$  and in addition some extra effort from her teachers. Thus, when there is heterogeneity in  $\tau_i$  individuals far away from  $\tau_M$  would clearly benefit from moving out of the school system which has a  $\Phi$  different from their own tastes, if there was another one offering a more

conformable  $\Phi'$ .

To state these result somewhat more formally, assume there is a finite number of types  $\Psi = \{\tau_1, \tau_2, \dots, \tau_\psi\}$  such that for all  $i \in \{1, \dots, N\}$ ,  $\exists \Phi_j \in \Psi$  such that  $\Phi_j = \tau_i$ . Assume also that the distribution of  $v_i / \psi_i$  is orthogonal to the distribution of  $\tau_i$ . Then,

**PROPOSITION 2** *Suppose students are completely sorted at school according to their type  $\tau_i$*

1. *For an equal level of school resources  $n = n_{NS} = n_S$ , student performance increases with respect to a situation where they are all schooled together for those students for whom*

$$\frac{v_i (\Phi_{i_M} - \tau_i)^2}{\psi_i} - \frac{1}{3n} \sum_{i \in j} \frac{v_j (\Phi_{i_M} - \tau_j)^2}{\psi_j} > 0 \quad (28)$$

2. *When resources are determined as in (26) total school resources decrease in a situation where students are sorted according to their to their type  $\tau_i$ , that is  $n_{NS} < n_S$ .*

**Proof 2** *When all students are schooled with classmates sharing the same type,  $F_i = 1$  for all  $i \in \{1, \dots, N\}$ . Then by (27) the difference in effort levels between the segregated and the comprehensive school systems is given by (28). This proves part 1. Part 2 follows from (27) by noticing that  $\bar{F}_{j_M} = 1$  when students are sorted according to their type  $\tau_i$  but in comprehensive schools  $\bar{F}_{j_M} < 1$ .*

A crucial element behind our result is that parents increase their involvement in education if the school provided values, or approaches, that coincide more with those they want their children to acquire, or receive. This adds a new dimension to the process of socialization studied by Bisin and Verdier (2001). In their theory, (direct) socialization at home is a substitute to socialization in the school (assimilation). Thus, if children socialize in a school with similar cultural values, parents reduce their investment in transferring their values. In our model, however, acquiring values in the school requires effort and therefore requires parents incentives, which are larger if schools provide values that are more in line with those of the parents'.

Of course, it is possible that a disparity of educational models could be associated with a higher probability of social conflict. Hence a policymaker would probably need to balance the potential benefits of horizontal segregation which we highlight with the need for maintaining social cohesion on which our model is silent.



### 4.1.3 Combining horizontal and vertical segregation

Even more nuances are possible in this picture about horizontal and vertical segregation. Suppose, for example, that teaching is made easier if students in a class have more homogeneous  $v_i$ , which Dufló, Dupas, and Kremer (2008) conjecture is behind their observation of positive effects of tracking for all students. To see this formally, let

$$F_j = 1 + \frac{1}{N_j} \sum_{i \in j} (v_i - \bar{v}_j)^2,$$

and assume that the cost of incentives for the headmaster  $j$  are:

$$\left( T - \frac{n_j}{N_j} \sum_{i \in j} \frac{F_j c_{2j} e_i}{2} \right) \gamma,$$

then it is easy to see, following the same steps and assumption as in section (4.1.2), that:

$$\begin{aligned} c_{2j} &= \frac{2}{3} \left( \frac{2}{\gamma n F_j} \bar{v}_j - \bar{\Omega}_j \right), \\ n &= \frac{\omega - \left( \frac{2\bar{v}_{jM}}{3F_{jM}\gamma} \right)^2 \bar{\psi}_{jM}}{\frac{4v_{iM}\bar{v}_{jM}}{9\gamma F_{jM}}}, \end{aligned} \quad (29)$$

and therefore,

$$e_i = \left( \frac{v_i}{\psi_i} + \frac{1}{3} \left( \frac{\frac{4v_{iM}\bar{v}_{jM}}{9\gamma^2 F_{jM} F_j}}{\omega - \left( \frac{2\bar{v}_{jM}}{3F_{jM}\gamma} \right)^2 \bar{\psi}_{jM}} \bar{v}_j - \bar{\Omega}_j \right) \right) \quad (30)$$

The analysis of (30) yields the following result:

**PROPOSITION 3** *Suppose that the opportunity cost of time for parents is homogeneous (i.e.,  $\psi_i = \psi$ ). Then, comparing schooling children with type  $v_i$  exclusively with children of the same type to a situation where all schools share the same distribution of students:*

1. *For a fixed class size,  $n$ , the average human capital increases with respect to a situation where all schools share the same distribution of students.*
2. *If class size,  $n$ , is determined as in (29), and average talent at the school of the median voter,  $\bar{v}_{jM}$ , increases with sorting, then the average human capital increases with respect to a situation where all schools share the same distribution of students.*

3. For a fixed class size,  $n$ , human capital increases for students with abilities above the median.
4. The effect on students below the median is ambiguous.

**Proof 3** Notice first that separating students by ability level decreases  $F_j$  for all schools. This, plus an application of Jensen's inequality, shows parts 1 and 2, as in proposition 1. Part 3 follows from the fact that  $F_j$  decreases for all schools plus the fact that, above the median,  $\bar{v}_j$  increases. Part 4 follows because even though  $F_j$  decreases, below the median,  $\bar{v}_j$  decreases.

This result is important because it makes it easy to understand the positive results of ability sorting into schools for children in all parts of the ability distribution found by Ding and Lehrer (2007) and Duflo, Dupas, and Kremer (2008), the fact that Ding and Lehrer (2007) finds a stronger effect in the upper parts of the distribution, as well as the ambiguous effects of ability sorting found in earlier papers (see e.g. Betts and Shkolnik (2000)).

## 4.2 Endogenous segregation

The differential sensitivity of different types of parents to the composition of classrooms has segregation-inducing effects that we now analyze. The analysis will gain in clarity if we identify first a generic condition for an assignment equilibrium with sorting. Once a condition for this type of equilibrium is identified, we can then verify if it is satisfied for specific attributes, like talent or income, and a mixed system with public and private schools.

Formally, consider a generic attribute  $\xi$ . Take two schools  $\bar{\xi}_1$  and  $\bar{\xi}_2$ , where  $\bar{\xi}$  is an average of that attribute in the school. Parents are allowed to send their children to a private school paying a fee. The differential willingness to pay of two parents, with types  $\xi_i, \xi_{i'}$  and  $\xi_i > \xi_{i'}$  is:

$$\int_{\bar{\xi}_1}^{\bar{\xi}_2} \frac{\partial U_{P_i}}{\partial \bar{\xi}_j} d\bar{\xi}_j - \int_{\bar{\xi}_1}^{\bar{\xi}_2} \frac{\partial U_{P_{i'}}}{\partial \bar{\xi}_j} d\bar{\xi}_j = \int_{\bar{\xi}_1}^{\bar{\xi}_2} \left( \frac{\partial U_{P_i}}{\partial \bar{\xi}_j} - \frac{\partial U_{P_{i'}}}{\partial \bar{\xi}_j} \right) d\bar{\xi}_j > 0.$$

This type-monotonicity in relative gains is what leads to an equilibrium with school segregation by types.

More precisely, consider a finite set of schools  $l \in \{1, \dots, L\}$ , each with  $n_l$  slots. Assume as well that each  $n_l$  is high enough so that the compositional impact of changing one child's type on the  $\bar{\xi}_l$  of school  $l$  is small. Order arbitrarily the available schools.

We denote by *top-down sorting* the following assignment of children into schools according to their type. School 1 gets assigned the  $n_1$ –highest type children, school 2 the  $n_2$ –highest type children among the remaining ones, and so on until all children are assigned to one (and only one) school.

The top-down sorting leads to a segregated school structure with types stratified from higher to lower. Namely, given two schools  $l > l'$  and two children  $i, i'$  that are assigned to either school by top-down sorting, then,  $\xi_i > \xi_{i'}$  and  $\bar{\xi}_l \geq \bar{\xi}_{l'}$ . To ensure that this inequality is strict for at least one pair of players in two different communities, we assume that two successive communities cannot be fully occupied by players of the same type.

To join a school  $l$ , parents must pay a fee  $p_l$  to the owner of the school  $l$ . The last school (or set of schools) in the list is public, free and has enough capacity for  $N$  students (the full group). We say that an assignment of children to schools and a vector of school prices forms an equilibrium when, given the prices, no individual prefers to change communities and either a community is full or its associated fee is zero.

**PROPOSITION 4** *There exists an assignment equilibrium with top-down sorting if whenever  $P_i$  and  $P_{i'}$  are such that  $\xi_i > \xi_{i'}$  we have that*

$$\frac{\partial U_{P_i}}{\partial \bar{\xi}_j} - \frac{\partial U_{P_{i'}}}{\partial \bar{\xi}_j} > 0 \quad (31)$$

Letting  $\xi_{i^*(l)}$  be the type of the lowest type parent in school  $l$ , the fee for a full school  $l$  is defined recursively as:

$$p_l = \int_{\bar{\xi}_{l+1}}^{\bar{\xi}_l} \frac{\partial U_{P_{i^*(l)}}}{\partial \bar{\xi}_j} d\bar{\xi}_j + p_{l+1}, l = 1, \dots, L - 1, \quad (32)$$

and  $p_L = 0$ .

**Proof 4** *A parent of a child in school  $l$  with type  $\xi_i$  does not want to move the child to school  $l + 1$  provided that:*

$$\begin{aligned} U_{P_i}(\bar{\xi}_l) - p_l &\geq U_{P_i}(\bar{\xi}_{l+1}) - p_{l+1} \\ U_{P_i}(\bar{\xi}_l) - U_{P_i}(\bar{\xi}_{l+1}) &\geq p_l - p_{l+1}. \end{aligned}$$

*Such parent will have a type such that  $\xi_{i^*(l-1)} \geq \xi_i \geq \xi_{i^*(l)}$ . Then we have that:*

$$\begin{aligned}
U_{P_i}(\bar{\xi}_l) - U_{P_i}(\bar{\xi}_{l+1}) &= \int_{\bar{\xi}_{l+1}}^{\bar{\xi}_l} \frac{\partial U_{P_i}}{\partial \bar{\xi}_j} d\bar{\xi}_j \\
&\geq \int_{\bar{\xi}_{l+1}}^{\bar{\xi}_l} \frac{\partial U_{P_{i^*(l)}}}{\partial \bar{\xi}_j} d\bar{\xi}_j \\
&= p_l - p_{l+1},
\end{aligned}$$

where the inequality is true by (31). Similarly a parent of a child in school  $l$  with type  $\xi_i$  does not want to move the child to school  $l - 1$  provided that:

$$\begin{aligned}
U_{P_i}(\bar{\xi}_l) - p_l &\geq U_{P_i}(\bar{\xi}_{l-1}) - p_{l-1} \\
p_{l-1} - p_l &\geq U_{P_i}(\bar{\xi}_{l-1}) - U_{P_i}(\bar{\xi}_l)
\end{aligned}$$

Remember that  $\xi_{i^*(l-1)} \geq \xi_i \geq \xi_{i^*(l)}$ . Thus:

$$\begin{aligned}
p_{l-1} - p_l &= \int_{\bar{\xi}_l}^{\bar{\xi}_{l-1}} \frac{\partial U_{P_{i^*(l-1)}}}{\partial \bar{\xi}_j} d\bar{\xi}_j \\
&\geq \int_{\bar{\xi}_l}^{\bar{\xi}_{l-1}} \frac{\partial U_{P_i}}{\partial \bar{\xi}_j} d\bar{\xi}_j \\
&= U_{P_i}(\bar{\xi}_{l-1}) - U_{P_i}(\bar{\xi}_l),
\end{aligned}$$

where, again, the inequality is true by (31).

This condition provides a test for the existence of endogenous segregation in different settings and considering different attributes like income or student talent.

#### 4.2.1 Segregation with private schools

We explore the emergence of sorting in a framework with private schools. To this end, we need first to describe the governance structure of these schools. Once private schools' behavior is discussed, it has to be shown that this structure satisfies the condition for segregation given by equation (31).

**School behavior.** Private schools announce fees and allow parents to run schools as clubs.<sup>13</sup>

More precisely, once school  $l$  is formed, the headmaster chooses education policies (in our case,  $c_{2l}$  and  $n_j$ ) to maximize the utility of the median parent. Parents cover the running costs of the school in addition to the entry fee  $p_l$ . Hence, after school is formed the headmaster maximizes:

$$U_{P_{i_M}} = \left( \frac{v_{i_M}}{\psi_{i_M}} + \frac{c_{2l}}{2} \right) v_{i_M} + \left( T - \frac{1}{2} \left( \left( \frac{v_{i_M}}{\psi_{i_M}} \right)^2 - \left( \frac{c_{2l}}{2} \right)^2 \right) \right) \psi_{i_M} - \frac{\omega}{2n_l^2} - \frac{\gamma}{2} n_l c_{2l} \left( \bar{\Omega}_{l_M} + \frac{c_{2l}}{2} \right), \quad (33)$$

which represents the utility of the median parent in the school  $l$ . Notice that the optimal education policy depends on the characteristics of the median student. This feature differs from the case of a public school where both the resources determined by the policymaker and the incentives decided by the headmaster are based on the mean student abilities.

The utility of any parent in school  $l$  is:

$$U_{P_i} = \left( \frac{v_i}{\psi_i} + \frac{c_{2l}}{2} \right) v_i + \left( T - \frac{1}{2} \left( \left( \frac{v_i}{\psi_i} \right)^2 - \left( \frac{c_{2l}}{2} \right)^2 \right) \right) \psi_i - \frac{\omega}{2n_l^2} - \frac{\gamma}{2} n_l c_{2l} \left( \bar{\Omega}_{l_M} + \frac{c_{2l}}{2} \right),$$

which can be expressed in terms of  $U_{P_{i_M}}$  as

$$U_{P_i} = U_{P_{i_M}} + \frac{1}{2} \left( \frac{v_i^2}{\psi_i} - \frac{v_{i_M}^2}{\psi_{i_M}} \right) + (v_i - v_{i_M}) \frac{c_{2l}}{2} + \left( T + \frac{1}{2} \left( \frac{c_{2l}}{2} \right)^2 \right) (\psi_i - \psi_{i_M}). \quad (34)$$

At this point, we impose the following assumption for ease of computations:

**ASSUMPTION 2** *The distributions of  $v_i$  and  $\psi_i$  are such that:*

$$\bar{\Omega}_{l_M} = \frac{v_{i_M}}{\psi_{i_M}};$$

**COMMENT 1** *Assumptions (2) is equivalent to assumption (1) but now at the level of each school rather than at the level of the whole school system. It says that the median student within each school has a ratio of talent to parental opportunity cost of time that is equal to the average of that ratio at the school level.*

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<sup>13</sup>A similar school governance is assumed, for example, by Ferreyra (2007).

PROPOSITION 5 Let  $P_i$  and  $P_{i'}$  be such that  $v_i > v_{i'}$  and  $\psi_i > \psi_{i'}$ , then:

$$1. \quad \frac{\partial U_{P_i}}{\partial \bar{v}_l} - \frac{\partial U_{P_{i'}}}{\partial \bar{v}_l} > 0 \quad (35)$$

$$2. \quad \frac{\partial U_{P_i}}{\partial \bar{\psi}_l} - \frac{\partial U_{P_{i'}}}{\partial \bar{\psi}_l} > 0 \quad (36)$$

**Proof 5** Please see Appendix A.

REMARK 2 This result establishes (31) and hence, by proposition (4), it demonstrates the existence of an assignment equilibrium with top-down sorting.<sup>14</sup>

In our model, a private school attracting students from the public system affects the policy variables in a predictable way. Both higher parental income or talent induce an increase in school system resources and the power of school incentives. Students enrolling in a private school (and hence leaving the public school) tend to come from the upper parts of the talent and income distributions. From equation (17) one can see that these children leaving the public schools would entail automatically an increase in  $n$ . Similarly from equation (9) one can see that  $c_{2j}$  (school incentives) are directly reduced through the effect of the increase in  $n$ .<sup>15</sup> This effect is relevant for evaluating the effect of vouchers, to which we now turn.<sup>16</sup>

#### 4.2.2 Discussion

The presence of private schools leads to sorting. In line with the literature, we can discuss the effect of increasing the school choice through vouchers (see e.g. Epple and Romano (1998), Urquiola and Verhoogen (2009)). Our distinctive feature is our focus on the endogenous determination of peer-effects, and hence school quality and policy choices, as a result of the interaction of parents and the school system.<sup>17</sup>

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<sup>14</sup>The discussion in this section and the following abstracts from horizontal differentiation. Evidently, horizontal differences also produce an incentive for segregation. But as we saw in the previous section, there is no conflict of interest between groups about this issue when differences are purely horizontal, and hence (when costless) even a public system would make it possible.

<sup>15</sup>McMillan (2004) in an otherwise quite different model, also finds that public schools can decrease their performance in the presence of vouchers.

<sup>16</sup>In our model, the authorities invest more in public education the higher its marginal productivity, and this is why a lower median ability level in the class would decrease public funds. It is conceivable, though, that in the short run, public funds may be fixed. In that case, the fact that some children moved to a private school would increase *per capita* resources in the public one.

<sup>17</sup>Integrating in a analytically tractable framework peer-effects, school quality and education policies, both in terms of education incentives and resources, is, to the best of our knowledge, novel in the literature.

Taking the interaction between parents and the school system seriously also has ramifications for estimating the effect of vouchers on sorting outcomes.<sup>18</sup> Consider the implementation of a voucher scheme subsidizing private schools. The voucher will induce peers with higher talent or parents with higher opportunity costs of time to leave the public school.<sup>19</sup> As discussed above, this implies a decline in incentives and resources received by the students who stay in the public school.<sup>20</sup>

This result would mechanically follow from assuming the existence of peer effects as in most of the literature (e.g., Benabou (1993)). In our model, where peer-effects are endogenously generated by the interaction of parents and the school system, there is an amplification effect via the reaction of school resources and incentives to changes in the average ability of the peers. So, the negative effect of sorting on students left behind is greater than the estimates of model with exogenous peer-effects would suggest.<sup>21</sup> Also, when students can afford moving to better schools after receiving a voucher, the positive effect might also be smaller than the pre-voucher situation would suggest. This is because the receiving school would enroll on average less talented students, and its best students would like to move to a better private school. The new composition of the private school would entail a new median student and thus affect the level of resources and school incentives. Our effect hinges critically on the reactions of the actors involved in the educational process and the consequence of neglecting the implied feedback effects would overstate both the gains by those favored by the voucher policy and understate the losses suffered by those who stay in the public school.

The political feasibility of a voucher scheme would depend on whether the value of the voucher would make it possible for the median voter to move her/his child to a better (private) school. If not, the parents of the median voter child are made worse-off by the voucher-induced segregation, and thus they will not give its support if it comes to a vote. This would go a long way toward explaining the absence of such schemes in most developed economies.

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<sup>18</sup>In particular, the estimation of computational/structural general equilibrium models have become a common tool for policymakers to understand the impacts of various educational policies.

<sup>19</sup>This effect has been empirically uncovered by many studies. See for example, Howell and Peterson (2002) for the case of the US, Hsieh and Urquiola (2006), for Chile or Ladd (2002) for New Zealand.

<sup>20</sup>Altonji, Huang, and Taber (2004) provides evidence of this effect for the case of the U.S.

<sup>21</sup>Ferreira (2007) finds that a generalized voucher scheme even if positive in terms of welfare generates a negative effect on poor students.

## 5 Concluding remarks

We study a model where educational outcomes depend on student effort and talent. Student effort can be affected by parental and teachers' investments, as well as by school system resources. The model is rich, yet simple enough to deliver analytical predictions on a number of important problems. For example, when parental opportunity costs increase, both parental and teacher effort decrease, while school resources partially substitute for both. This can explain the lack of robust evidence for a class size effect in longitudinal (in contrast with experimental) data. The model also provides a microfoundation for "peer effects". Groups of children with higher average ability are more "profitable" to manage by teachers, who as a consequence exert more effort in them. Then, any child will benefit from their presence in the school. "Peer effects", as in other models, produce an incentive for sorting. We show that in some circumstances (e.g., when teaching technology favors low variance classrooms) sorting can be Pareto improving. Even in this case, the welfare gain from sorting is not evenly distributed, which can explain the ambiguous empirical evidence on sorting.

It is clear that there are circumstances when higher ability children are not necessarily those in which parents want to invest more effort. For example, if the objective of a parent is to have her child get into an Ivy league school and she is so talented that even without effort the goal would be achieved, there is no point in making the investment in incentives. On the other hand, a slightly less talented child may be on the verge of achieving the goal and some investment in incentives could be indeed profitable. Thus, in reality the relationship between talent and parental investment may be nonmonotonic. Introducing explicitly these nonmonotonicities would not change the relationship between parental opportunity costs and investments, as well as the reaction to this by the education system. Our specific results on segregation would indeed change. But the most important message is that heterogeneity in talent or opportunity cost create incentives for segregation through the responses of both politicians and educators to the school composition. And this message would remain unaffected by a potential nonmonotonic relationship between talent and parental investment.

The richness of the model allows it to be used in further research. The political aspects of school choice, for example, are barely scratched in this paper. Since the political authorities have a single instrument, school resources, and preferences over this instrument are single-peaked, we can resort to the median voter theorem in discussing the policymaker's choice. If there were more instruments (say, the level of funding of charter schools) more challenging (and more interesting) political interactions involving education could be studied (as in, for example, Boldrin and Montes (2005) or Levy (2005)). Another aspect we have not explored



is that of teacher heterogeneity. Once that is allowed, the issue of teacher sorting and teacher peer effects become important (something that Jackson (2009), and Jackson and Bruegmann (2009) have documented). We leave this sort of work for future research.

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## Appendix A: Proof of proposition 5

The first order conditions associated with (33) are:

$$\begin{aligned} \frac{1}{2}v_{i_M} + \frac{c_{2l}}{4}\psi_{i_M} - \frac{\gamma}{2}n_l(\bar{\Omega}_{l_M} + c_{2l})c_{2l} &= 0, \\ \frac{2\omega}{n_l^3} - \gamma\left(\bar{\Omega}_{l_M} + \frac{c_{2l}}{2}\right)c_{2l} &= 0. \end{aligned}$$

These conditions imply that:

$$\begin{aligned} n_l &= \frac{2v_{i_M} + c_{2l}\psi_{i_M}}{2\gamma(\bar{\Omega}_{l_M} + c_{2l})c_{2l}}, \\ 2\omega\left(\frac{2\gamma(\bar{\Omega}_{l_M} + c_{2l})c_{2l}}{2v_{i_M} + c_{2l}\psi_{i_M}}\right)^3 - \gamma\left(\bar{\Omega}_{l_M} + \frac{c_{2l}}{2}\right)c_{2l} &= 0. \end{aligned}$$

Using these conditions, assumption (2) and by equation (34), we can calculate the following derivatives:

$$\frac{\partial U_{Pi}}{\partial v_{iM}} = \frac{\partial U_{PiM}}{\partial v_{iM}} - \frac{v_{iM}}{\psi_{iM}} - \frac{c_{2l}}{2} + \frac{(v_i - v_{iM})}{2} \frac{\partial c_{2l}}{\partial v_{iM}} + \frac{c_{2l}}{4} (\psi_i - \psi_{iM}) \frac{\partial c_{2l}}{\partial v_{iM}} \quad (37)$$

$$\frac{\partial U_{Pi}}{\partial \psi_{iM}} = \frac{\partial U_{PiM}}{\partial \psi_{iM}} + \frac{1}{2} \frac{v_{iM}^2}{\psi_{iM}^2} - \frac{1}{2} \left( \frac{c_{2l}}{2} \right)^2 + \frac{(v_i - v_{iM})}{2} \frac{\partial c_{2l}}{\partial \psi_{iM}} + \frac{c_{2l}}{4} (\psi_i - \psi_{iM}) \quad (38)$$

$$\frac{\partial c_{2l}}{\partial v_{iM}} = - \frac{6\omega \left( \frac{2\gamma \left( \frac{v_{iM}}{\psi_{iM}} + c_{2l} \right) c_{2l}}{2v_{iM} + c_{2l}\psi_{iM}} \right)^2 \left( \frac{-2\gamma c_{2l}^2}{(2v_{iM} + c_{2l}\psi_{iM})^2} \right)}{6\omega \left( \frac{2\gamma \left( \frac{v_{iM}}{\psi_{iM}} + c_{2l} \right) c_{2l}}{2v_{iM} + c_{2l}\psi_{iM}} \right)^2 \left( \frac{4\gamma \left( \frac{v_{iM}^2}{\psi_{iM}^2} + 2c_{2l}v_{iM} \right) + 2\gamma c_{2l}^2 \psi_{iM}}{(2v_{iM} + c_{2l}\psi_{iM})^2} \right) - \gamma \left( \frac{v_i}{\psi_i} + c_{2l} \right)} > 0 \quad (39)$$

$$\frac{\partial c_{2l}}{\partial \psi_{iM}} = - \frac{6\omega \left( \frac{2\gamma \left( \frac{v_{iM}}{\psi_{iM}} + c_{2l} \right) c_{2l}}{2v_{iM} + c_{2l}\psi_{iM}} \right)^2 \left( \frac{-2\gamma \frac{v_{iM}}{\psi_{iM}^2} c_{2l} (2v_{iM} + c_{2l}\psi_{iM}) - 2\gamma \left( \frac{v_{iM}}{\psi_{iM}} + c_{2l} \right) c_{2l}^2}{(2v_{iM} + c_{2l}\psi_{iM})^2} \right)}{6\omega \left( \frac{2\gamma \left( \frac{\rho v_{iM}}{\psi_{iM}} + c_{2l} \right) c_{2l}}{2v_{iM} + c_{2l}\psi_{iM}} \right)^2 \left( \frac{4\gamma \left( \frac{\rho v_{iM}^2}{\psi_{iM}^2} + 2c_{2l}v_{iM} \right) + 2\gamma c_{2l}^2 \psi_{iM}}{(2v_{iM} + c_{2l}\psi_{iM})^2} \right) - \gamma \left( \frac{\rho v_i}{\psi_i} + c_{2l} \right)} > 0 \quad (40)$$

Let  $P_i$  and  $P_{i'}$  be such that  $v_i > v_{i'}$ , and  $\psi_i > \psi_{i'}$  then, from (37), we obtain:

$$\frac{\partial U_{Pi}}{\partial v_{iM}} - \frac{\partial U_{Pi'}}{\partial v_{iM}} = \frac{(v_i - v_{i'})}{2} \frac{\partial c_{2lM}}{\partial v_{iM}} + (\psi_i - \psi_{i'}) \frac{c_{2lM}}{4} \frac{\partial c_{2lM}}{\partial v_{iM}}$$

The first result then follows from (39) and (40).

Let now  $P_i$  and  $P_{i'}$  be such that  $v_i > v_{i'}$ , and  $\psi_i > \psi_{i'}$  then, from (38), we obtain:

$$\frac{\partial U_{Pi}}{\partial \psi_{iM}} - \frac{\partial U_{Pi'}}{\partial \psi_{iM}} = \frac{(v_i - v_{i'})}{2} \frac{\partial c_{2l}}{\partial \psi_{iM}} + (\psi_i - \psi_{i'}) \frac{c_{2l}}{4} \frac{\partial c_{2l}}{\partial \psi_{iM}}$$

And the second result then follows from from (39) and (40).