

# DISCUSSION PAPER SERIES

No. 7948

## ENDOGENOUS SPATIAL DIFFERENTIATION WITH VERTICAL CONTRACTING

Frago Kourandi and Nikolaos Vettas

*INDUSTRIAL ORGANIZATION*



**C**entre for **E**conomic **P**olicy **R**esearch

[www.cepr.org](http://www.cepr.org)

Available online at:

[www.cepr.org/pubs/dps/DP7948.asp](http://www.cepr.org/pubs/dps/DP7948.asp)

# ENDOGENOUS SPATIAL DIFFERENTIATION WITH VERTICAL CONTRACTING

Frago Kourandi, Athens University of Economics and Business  
Nikolaos Vettas, Athens University of Economics and Business and CEPR

Discussion Paper No. 7948  
August 2010

Centre for Economic Policy Research  
53–56 Gt Sutton St, London EC1V 0DG, UK  
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820  
Email: [cepr@cepr.org](mailto:cepr@cepr.org), Website: [www.cepr.org](http://www.cepr.org)

This Discussion Paper is issued under the auspices of the Centre's research programme in **INDUSTRIAL ORGANIZATION**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as an educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Frago Kourandi and Nikolaos Vettas

## **ABSTRACT**

### **Endogenous Spatial Differentiation with Vertical Contracting\***

We set-up a linear city model with duopoly upstream and downstream. Consumers have a transportation cost when buying from a retailer, and retailers have a transportation cost when buying from a wholesaler. We characterize the equilibria in a five-stage game where location and pricing decisions (wholesale and retail) by all four firms are endogenous. The usual demand and price competition effects are modified and an additional strategic effect emerges, since the retailers' marginal costs become endogenous. Firms tend to locate farther away from the market center relative to the vertically integration case. When the wholesalers choose locations before the retailers, each wholesaler locates closer to the market center relative to the retailer locations, and relative to when the wholesalers cannot move first. Each wholesaler does this to strengthen the strategic position of its retailer by credibly pulling him towards the market center. As a result, the intensity of competition is higher and industry profit is lower when upstream locations are chosen before downstream locations. Variations of the model and welfare analysis are provided.

JEL Classification: L13 and R32

Keywords: linear city, locations, spatial differentiation, strategic commitment and vertical contracting

Frago Kourandi  
Department of Economics  
Athens University of Economics and  
Business  
76 Patision Street  
Athens 10434  
GREECE

Email: [kourandi@aueb.gr](mailto:kourandi@aueb.gr)

For further Discussion Papers by this author see:  
[www.cepr.org/pubs/new-dps/dplist.asp?authorid=169527](http://www.cepr.org/pubs/new-dps/dplist.asp?authorid=169527)

Nikolaos Vettas  
Department of Economics  
Athens University of Economics and  
Business  
76 Patision Street  
Athens 10434  
GREECE

Email: [nvettas@aueb.gr](mailto:nvettas@aueb.gr)

For further Discussion Papers by this author see:  
[www.cepr.org/pubs/new-dps/dplist.asp?authorid=121328](http://www.cepr.org/pubs/new-dps/dplist.asp?authorid=121328)

\* For comments on previous versions, we thank participants in seminars and the 2009 Salento IO Workshop, the 2009 Maastricht Conference on Economic Design, the Ljubljana EARIE 2009 conference, and the CEPR 2010 IO School at Toulouse.

Submitted 24 July 2010

# 1 Introduction

A large number of studies has examined various important aspects of how oligopolistic firms choose their locations (or horizontal differentiation) and then compete in prices. Maximal, minimal or intermediate differentiation may emerge in the market depending on the balance between a direct demand effect that pulls firms towards the centre of the market and a strategic price effect that pushes them away from one another. This literature typically focuses on the direct market relation between the oligopolists and the final consumers. In contrast, the nature of vertical relations are important in the great majority of the oligopolistic markets, that is, how upstream firms (wholesalers) trade with downstream firms (retailers) is interrelated with how downstream firms trade with final consumers. In this paper, we study endogenously the horizontal product differentiation (or locations) and pricing decisions in vertically linked duopolies, or, more simply put, we endogenize the locations and pricing decisions of wholesalers and retailers on a Hotelling line.

We study a two-tier horizontally differentiated duopoly model. We assume exclusive vertical contracting with each downstream firm able to turn one unit it gets from its upstream firm into one unit in the final market. Upstream and downstream firms are differentiated with respect to their horizontal locations - as usual, these can be interpreted either geographically or in the space of product characteristics.<sup>1</sup> Choices are made sequentially. We study two versions of the model, first when any location on the real line can be chosen and, second, when location can only be chosen within the interval within which consumers are uniformly distributed. Allowing upstream firms and downstream firms to locate anywhere on the real line allows a clearer and richer characterization of the equilibrium location incentives.

The main questions that can be answered in a framework that combines vertical and product differentiation elements refer to the implications of vertical pricing rules for the location choices and the role that upstream and/or downstream differentiation plays for vertical trade and final prices. The strategic incentives are quite rich, as each firm's choice affects the subsequent choices of both firms and of the final consumers. For instance, upstream locations affect downstream locations, then wholesale prices and then the final prices. In addition, the quantity that each upstream firm will sell is equal to the quantity that the corresponding retailer will sell to the final consumers and this quantity depends (directly or indirectly) on all the choices made by the firms.

In models of horizontal differentiation without upstream firms, there are two opposite forces, one pushing firms close to each other, the "demand" effect, and one to the opposite direction, the "price competition" effect. In our model there is a third force affecting the wholesale prices and the transportation cost that the downstream firms pay when moving to the centre of the line.

---

<sup>1</sup>So, we may be thinking about retailers and wholesalers with their point of sale at different geographical locations or choosing different varieties of a horizontal product characteristic: milk sold by wholesalers may differ with respect to its fat content and retailers have a cost when using that to produce daily products, also differentiated with respect to their fat content.

The interaction of these forces give the equilibrium result. We obtain a number of results. In our basic model (with locations anywhere on the real line) we find that there is a unique subgame perfect equilibrium. In this, all firms locate outside the unit interval but wholesalers always locate closer to the city center than retailers. Relative to the vertical integration benchmark, all locations are farther away from the center (and therefore the social cost is also higher, including both the intermediate and the final transportation cost). Double marginalization emerges, with all firms making positive profits and final prices higher than under vertical integration. When locations are restricted within the unit interval, maximum differentiation is obtained, with each chain choosing an endpoint location.

Important insights are obtained when we modify the order of location choices. When wholesalers chose their locations first, they choose to locate closer to the center relative to when location choices are simultaneous or when retailers move first. They do this to credibly pull their retailers towards the center and offer them a stronger strategic position in the final market. In equilibrium, this strategic behavior leads to more intense competition and lower profits when wholesalers choose their locations first. The industry would, thus, prefer retailers to choose locations first (or at the same time as the wholesalers).

The remainder of the paper is as follows. Related literature is briefly discussed in Section 2. Section 3 sets up the basic model. Section 4 derives the equilibrium and studies its properties. Extensions of the basic model are examined in Section 5. Section 6 modifies the basic model with firms allowed to locate only in the unit interval. For fixed upstream locations in Section 7, we explore the possibility that downstream firms may "reverse" their locations on the line. Section 8 concludes.

## 2 Related literature

Our paper contributes to two distinct literatures, on product differentiation and on vertical contracting. Each of these contains a number of influential papers and is too vast to survey here.<sup>2</sup> We only refer to work that is more closely related to the specific setting of our model. The strand of the product differentiation literature we are building on, starts with the classic linear-city model of Hotelling (1929), modified by D'Aspremont *et al.* (1979) in a way that the strategic price incentives can also be more easily characterized.<sup>3</sup> Nevertheless, only very few papers have exam-

---

<sup>2</sup>For a review and some key results on product differentiation see e.g. Anderson *et al.* (1992), Gabszewicz and Thisse (1992) and for vertical relations Rey and Tirole (2007) and Rey and Verge (2008).

<sup>3</sup>Many variants of the linear city model has been studied. Among other, Anderson and Neven (1991) solve the location-pricing game when oligopolists compete in quantities. Ziss (1993) examines the D'Aspremont *et al.* model with heterogeneous production costs. If the marginal cost difference is sufficient small, a price and location equilibrium exists in which both firms enter and maximum differentiation emerges. Anderson and Engers (1994) study a price-taking equilibrium in the spatial setting. In Vettas (2003) and Vettas and Christou (2005), firms are horizontally as

ined the vertical chain interactions in a horizontal differentiation framework. This creates a gap in the literature since in reality many market structures have an important vertical element with upstream firms supplying the downstream firms. Marginal production costs of the downstream firms are therefore endogenously determined in the vertical chain. Thus, location choices by the upstream and/or downstream firms are affected by the transactions in a vertical environment.<sup>4</sup>

Gupta *et al.* (1994) assume that an upstream monopolist sets the wholesale price based on its observation of the locations chosen by the downstream firms and that the downstream firms can price discriminate. Beladi *et al.* (2008) study an upstream monopoly and a downstream duopoly where two-part tariffs are signed and the downstream firms cannot produce all varieties demanded. Aiura and Sato (2008) examine an upstream monopolist at the center of the city and supplying two retailers. Retailers do not pay wholesale prices but only a transportation cost and choose their locations and final prices. Here, we consider in contrast a model with two upstream and two downstream firms that pay linear wholesale prices. A location-price equilibrium is also analyzed by Brekke and Straume (2004) where each downstream firm has its own supplier, upstream firms bargain about the input prices with the downstream firms, but upstream firms are not product differentiated. Allain (2002) and Laussel (2006) examine the situation where two upstream firms are exogenously brand differentiated and two downstream firms are exogenously spatially differentiated, thus, consumers face four different products (and are distributed on a rectangle). In our paper, in contrast, consumers care directly only about the final spatial differentiation and we further endogenize the location choices in both levels assuming input prices are set by the wholesalers.

Previous work by Matsushima (2004) has also studied the two upstream and two downstream firm structure on the line and finds equilibrium locations that depend on the transportation cost parameters. Our model differs in that upstream firms do not restrict wholesale prices to be equal to the rival firm's transportation cost, the upstream firms supply their own downstream firms and firms can locate outside the unit interval. Further, we assume that it is the downstream firms that pay the transportation costs when supplied by the upstream firms. We also present a sequential location choice by the upstream firms first and the downstream firms after, as opposed to simultaneous symmetric location choices by Matsushima (2004).<sup>5</sup>

---

well as vertically differentiated. Tabuchi and Thisse (1995) and Lambertini (1997) allow firms to locate along the entire real line, while consumers are concentrated around the market center.

<sup>4</sup>Dobson and Waterson (1996) study the exclusivity in an exogenously non-horizontal differentiated successive duopoly in upstream and downstream level with consumers facing four varieties. Inderst and Shaffer (2007) analyse the impact of retail mergers on product variety in a non-Hotelling type differentiated model.

<sup>5</sup>Matsushima (2009) also studies the incentives for vertical mergers in a location model and Matsumura and Matsushima (2009) examines the mixed strategy equilibria under large cost differentials.

### 3 The basic model

We set up a model where upstream and downstream duopolies locate on a line as follows. Consumers are uniformly distributed on a  $[0,1]$  interval and have unit demands. There are two upstream firms, A and B, and two downstream firms, X and Y, each choosing a unique location on the entire real line. There is exclusive dealing and the downstream firms have a simple fixed-proportions technology: firm X turns each unit it purchases from firm A into one unit that it can sell to the final consumers; likewise, for firms Y and B.

The locations of firms A and B are denoted by  $a$  and  $1 - b$ , respectively. The locations of firms X and Y are denoted by  $x$  and  $1 - y$ , respectively. Thus, for the A and X chain,  $a$  and  $x$  measure how much to the right of endpoint 0 each firm is located, while for the B and Y chain,  $b$  and  $y$  measure how much to the left of endpoint 1 each firm is located. Without loss of generality, we consider  $1 - a - b > 0$ , so that A is located to the left of B. In the main body of the paper we will also focus on the case where  $1 - x - y > 0$  so that X is also to the left of Y; in Section 7 we also discuss the possibility that the downstream locations switch to the "wrong" side of the line relative to their upstream suppliers.

A transportation cost has to be paid both in the wholesale and retail market. So that our results are easily comparable to the literature, we follow the often used assumption that this cost is quadratic in distance. A consumer located at point  $z$  pays transportation cost  $t(x - z)^2$  when purchasing a product from firm X and  $t(1 - y - z)^2$  when purchasing a product from firm Y. In turn, firm X pays transportation cost  $\tau(x - a)^2$  when purchasing a unit from firm A and firm Y pays transportation cost  $\tau(y - b)^2$  when purchasing a unit from firm B. These costs,  $t > 0$  and  $\tau > 0$ , may be real transportation costs (for example depending on the weight or the volume of the product) or may be product characteristic transformation costs necessary to convert one unit of the upstream firm's input to one unit of final good. We consider linear pricing at the wholesale and retail level. A final consumer who purchases a unit from downstream firm X or Y pays the price set by this firm ( $p_X$  or  $p_Y$ ) plus the transportation cost between the chosen firm's location and his own location. We also assume that the basic reservation value of each consumer is high enough so that each consumer purchases one unit of the product. A downstream firm who purchases a unit from upstream firm A or B pays the price set by this firm ( $w_A$  or  $w_B$ ) plus the transportation cost between the two trading firms. Apart from locations, the products are homogeneous and production costs are assumed zero for simplicity. Note that the final consumers care about the upstream locations and prices indirectly, that is, only to the extent they affect the downstream locations and prices.

Each of the four firms seeks to maximize its own profit and each consumer his own net surplus. We assume there are no information asymmetries and proceed to analyze a sequential game where all locations and prices are endogenous. We view locations as longer-run (and more difficult to change) variables than prices. Therefore, the main model we analyze is a five stage game as



follows:

- (1) Upstream firms A and B simultaneously choose their locations,  $a, b$ .
- (2) Downstream firms X and Y simultaneously choose their locations,  $x, y$ .
- (3) Upstream firms simultaneously set linear wholesale prices,  $w_A, w_B$ : firm A charges  $w_A$  to firm X and firm B charges  $w_B$  to firm Y.
- (4) Having observed both wholesale prices, downstream firms simultaneously set their (final) product prices,  $p_X, p_Y$ .
- (5) Having observed the firms' locations and final prices, each consumer purchases one unit of the product from one of the downstream firms X and Y.

We proceed backwards, solving for the subgame perfect Nash equilibria of the game.

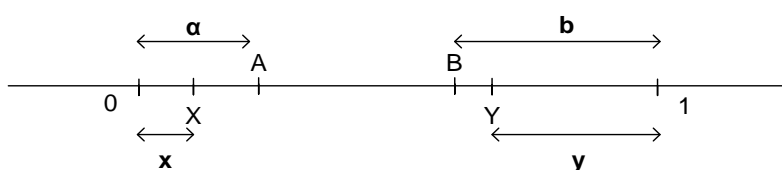


Figure 1: Upstream and downstream locations (example)

In subsequent Sections of the paper and following the analysis of our main case, we also consider alternative assumptions about the timing of location choices, when downstream locations can be chosen before the upstream locations and also when all locations (upstream and downstream) are chosen simultaneously. We also examine other extensions of the analysis: when firm locations cannot be on the entire real line but are restricted within the unit interval; and when downstream locations can be at the opposite side on the line relative to their upstream suppliers. But first we proceed to the analysis of our main case.

## 4 Equilibrium analysis

To obtain a subgame perfect equilibrium, we proceed backwards. First, we consider trade in the downstream market to find equilibrium retail prices and consumer choices given the wholesale prices and the locations of all firms. Second, we derive the equilibrium wholesale prices given all firms' locations and anticipating equilibrium behavior in the retail market. Third, we derive the downstream firms' equilibrium locations given the upstream locations and anticipating pricing, wholesale and then retail. Finally, we derive upstream equilibrium locations anticipating equilibrium in all the subsequent stages. Once we derive the equilibrium, we study its properties. As should be expected when dealing with such a five-stage game, the analysis is tedious at times and some formal details are in the Appendix, or are presented in brevity to facilitate the continuity of the arguments and the intuition.

## 4.1 Consumers choice and retail prices

This part of analysis corresponds to duopoly competition in the final (retail) market with fixed locations and potentially different unit costs.<sup>6</sup> For some locations and costs, both retailers sell in the market, while for locations and costs that significantly favor one of the retailers, we obtain a corner solution where the rival makes no sales. The costs faced by the retailers are equal to the wholesale price plus the transportation cost they have to pay per unit.

Given the firms' locations and the wholesale prices, we calculate the demand functions for the downstream firms X and Y. Let  $z$  be the demand of firm X and  $1 - z$  the demand of firm Y. The firms' profit functions are

$$\begin{aligned}\Pi_X &= (p_X - f_X)z, \\ \Pi_Y &= (p_Y - f_Y)(1 - z),\end{aligned}$$

where  $f_X$  (respectively  $f_Y$ ) is the *aggregate* marginal cost, that is, the wholesale price *plus* the transportation cost of firm X (respectively Y) with  $f_X \equiv w_A + \tau(x - a)^2$  and  $f_Y \equiv w_B + \tau(y - b)^2$ . When both firms sell positive quantities, demand for each firm is characterized by the presence of an indifferent consumer located at  $z$ , as follows:

$$p_X + t(x - z)^2 = p_Y + t(1 - y - z)^2. \quad (1)$$

Of course, the location of the indifferent consumer depends on the downstream firms' locations, product prices and the transportation cost parameter  $t$ , that is,  $z = z(p_X, p_Y, x, y, t)$ ; to simplify the exposition we suppress the arguments of this function and solve (1) to obtain

$$z = \frac{1 + x - y}{2} + \frac{p_Y - p_X}{2t(1 - x - y)}. \quad (2)$$

Further, taking into account the possibility that *all* consumers may choose to purchase from one firm, demand for firm X will be equal to

$$z = \begin{cases} 1 & \text{if } \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} \geq 1 \\ \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} & \text{if } 0 < \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} < 1 \\ 0 & \text{if } \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} \leq 0. \end{cases} \quad (3)$$

We note that the demand and profit functions for the downstream firms are continuous in both firms' prices. For intermediate values of the prices, the market is shared,  $z \in (0, 1)$ , and as firm X reduces  $p_X$ , the indifferent consumer moves to the right on the line (thus,  $z$  increases). When  $p_X$  decreases below the threshold stated in expression (3), firm X captures all the demand ( $z = 1$ ) and its profits is simply equal to its profit margin  $p_X - f_X$ . Likewise, for high values of  $p_X$ , firm X makes no sales and its profit is driven to zero. Figure 2 presents two examples of the demand and profit functions for different values of the parameters.

---

<sup>6</sup>Ziss (1993) studies the linear city model with different unit costs. The analysis in this part parallels his.

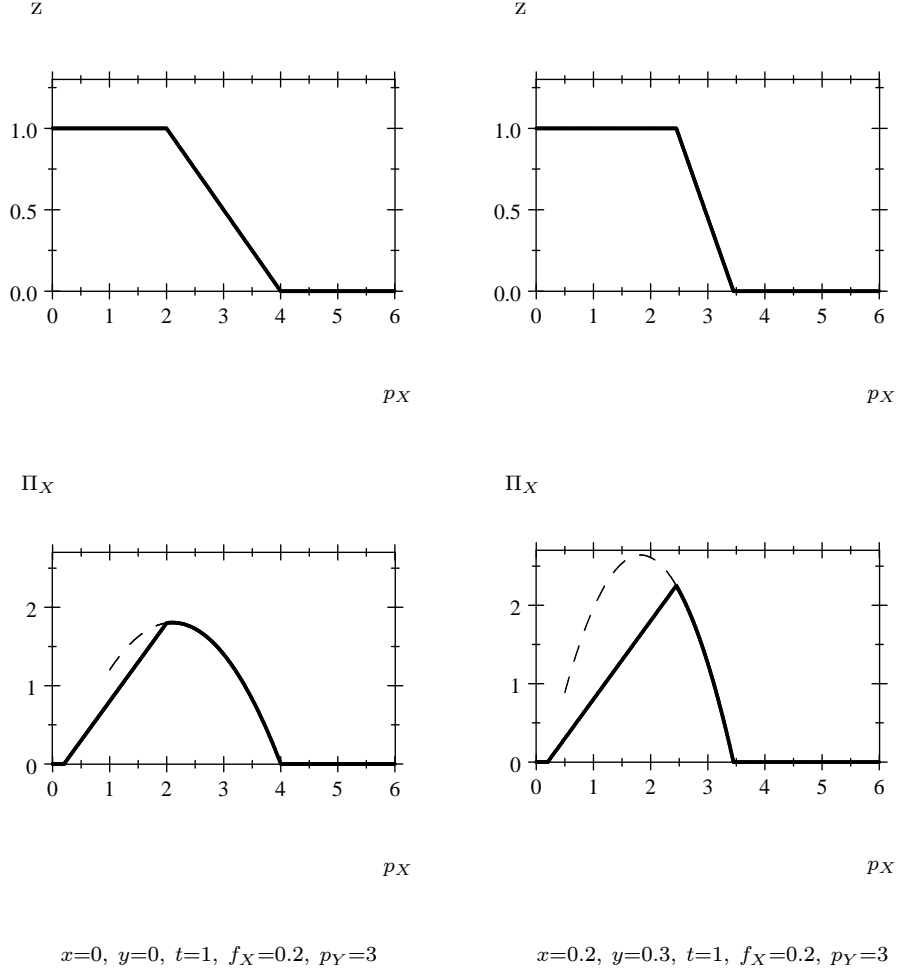


Figure 2: Downstream demand and profits

Given the profit functions for the downstream firms, we now proceed to the characterization of equilibrium retail prices. The analysis is standard but special care should be taken for the characterization of the corner cases:

**Lemma 1** *The equilibrium price and profit for firm X are:*

$$p_X = \begin{cases} f_Y - t(1-x-y)(1+y-x) & \text{if } f_X \leq \overline{f_X} \\ \frac{1}{3}(t(1-x-y)(3+x-y) + f_Y + 2f_X) & \text{if } \overline{f_X} < f_X < \widehat{f_X} \\ f_X & \text{if } f_X \geq \widehat{f_X} \end{cases} \quad (4)$$

and

$$\Pi_X = \begin{cases} f_Y - t(1-x-y)(1+y-x) - f_X & \text{if } f_X \leq \overline{f_X} \\ \frac{(t(1-x-y)(3+x-y) + f_Y - f_X)^2}{18t(1-x-y)} & \text{if } \overline{f_X} < f_X < \widehat{f_X} \\ 0 & \text{if } f_X \geq \widehat{f_X}, \end{cases} \quad (5)$$

where

$$\begin{aligned} \widehat{f_X} &\equiv f_Y + t(1-x-y)(3-y+x), \\ \overline{f_X} &\equiv f_Y - t(1-x-y)(3-x+y). \end{aligned}$$

Prices and profits for firm Y are derived in a symmetric way.

When firm X has low enough *aggregate* marginal cost (lower than the critical value  $\overline{f_X}$ ) it is the only one that sells in the market, while Y has zero demand. The opposite is true when X has a high *aggregate* marginal cost (higher than  $\widehat{f_X}$ ). When the difference in the marginal costs of the two firms is not too large,  $\overline{f_X} < f_X < \widehat{f_X}$ , then both downstream firms make sales in the final goods' market.

Two properties of the downstream market equilibrium should be noted. The first derivative of the final prices and the profits with respect to the transportation cost parameter  $t$  of the consumers is positive: the more differentiated the final goods are, the higher are the prices set by the retailers and the profits they obtain. Also, we observe that  $\frac{dp_i}{df_i} = \frac{2}{3} < 1$ . Thus, following an increase in his aggregate marginal cost, a retailer passes part of this to the final consumers but absorbs the rest.

## 4.2 Wholesale prices

In this stage, the two upstream firms compete by setting their wholesale prices. Each upstream firm exclusively supplies its retailer, firm A supplies firm X and firm B supplies firm Y. Even though each upstream firm cannot supply the rival retailer, competition takes place: the wholesale prices shape competition in the subsequent stage where retail prices and final demand is determined. If the wholesale price charged to retailer X by the upstream firm A is high, retailer X has a relative cost disadvantage compared with the rival retailer and, thus, the demand obtained by firm X (and, in turn, by firm A) is low.

Since a downstream firm turns one unit of the good it purchases in the wholesale market into one unit it sells in the retail market, the demand function of firms A and B are  $D_A = z$  and  $D_B = 1 - z$  and their profit functions are  $\Pi_A = w_A z$  and  $\Pi_B = w_B(1 - z)$ . Assuming equilibrium will follow when setting retail prices, as characterized in the previous Subsection, the upstream profit functions can be expressed as:

$$\begin{aligned}
\Pi_A &= w_A \\
&\text{if } w_A \leq w_B + \tau \left( (y - b)^2 - (x - a)^2 \right) + t(y - x + 3)(x + y - 1) \\
&= w_A \left( \frac{2t(1-x-y)(y-x) + w_B - w_A + \tau \left( (y-b)^2 - (x-a)^2 \right)}{6t(1-x-y)} + \frac{1}{2}(1 + x - y) \right) \\
&\text{if } \left\{ \begin{array}{l} w_A > w_B + \tau \left( (y - b)^2 - (x - a)^2 \right) + t(y - x + 3)(x + y - 1) \\ w_A < w_B + \tau \left( (y - b)^2 - (x - a)^2 \right) - t(x - y + 3)(x + y - 1) \end{array} \right\} \quad (6) \\
&= 0 \\
&\text{if } w_A \geq w_B + \tau \left( (y - b)^2 - (x - a)^2 \right) - t(x - y + 3)(x + y - 1)
\end{aligned}$$

and

$$\begin{aligned}
\Pi_B &= 0 \\
&\text{if } w_A \leq w_B + \tau((y-b)^2 - (x-a)^2) + t(y-x+3)(x+y-1) \\
&= w_B \left( 1 - \left( \frac{2t(1-x-y)(y-x) + w_B - w_A + \tau((y-b)^2 - (x-a)^2)}{6t(1-x-y)} + \frac{1}{2}(1+x-y) \right) \right) \\
&\text{if } \left\{ \begin{array}{l} w_A > w_B + \tau((y-b)^2 - (x-a)^2) + t(y-x+3)(x+y-1) \\ w_A < w_B + \tau((y-b)^2 - (x-a)^2) - t(x-y+3)(x+y-1) \end{array} \right\} \quad (7) \\
&= w_B \\
&\text{if } w_A \geq w_B + \tau((y-b)^2 - (x-a)^2) - t(x-y+3)(x+y-1).
\end{aligned}$$

Note that these profits are functions of the wholesale prices and all four locations. When the *aggregate* marginal cost  $w_A + \tau(a-x)^2$  faced by firm X is low enough compared to its rival, firm A captures the whole demand via its retailer X ( $z = 1$ ). When the *aggregate* marginal cost faced by firm X is high enough, its demand (and by implication also the demand of firm A) becomes zero ( $z = 0$ ). For intermediate values of  $w_A + \tau(a-x)^2$  we obtain  $z \in (0, 1)$ , both retailers make sales in the final market, and therefore, both wholesalers make sales in the upstream market. We should stress the role that upstream and downstream locations play in our model since they crucially affect the marginal costs of the retailers. For some locations, even if an upstream firm charges a zero wholesale price to its retailer, it cannot obtain positive demand and the *aggregate* marginal cost of one retailer is always lower than the rival's.<sup>7</sup> This happens when the distance of one retailer from its supplier is very large compared to the rival's distance from its own supplier. Clearly, these cases will tend to emerge when the transportation cost parameter  $\tau$  is high.

Maximization of the profit functions (6) and (7) with respect to the wholesale prices (note that the profit functions are quasi-concave) implies the reaction functions of the upstream firms:

$$\begin{aligned}
w_A = R_A(w_B) &= w_B + \tau((y-b)^2 - (x-a)^2) + t(y-x+3)(x+y-1) \\
&\text{if } w_B \geq \tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y) \\
&= \frac{1}{2}w_B + \frac{t(x-y+3)(1-x-y) - \tau((x-a)^2 - (y-b)^2)}{2} \\
&\text{if } \left\{ \begin{array}{l} w_B > \tau((x-a)^2 - (y-b)^2) - t(x-y+3)(1-x-y) \\ w_B < \tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y) \end{array} \right\} \\
&= 0 \\
&\text{if } w_B \leq \tau((x-a)^2 - (y-b)^2) - t(x-y+3)(1-x-y)
\end{aligned}$$

---

<sup>7</sup>To illustrate, take for example  $t = \tau = 1, a = b = 0.2, x = 0.06, y = 0.9$ . Then even if  $w_B = 0$  firm A takes the whole demand.

and

$$\begin{aligned}
w_B = R_B(w_A) &= 0 \\
&\text{if } w_A \leq \tau((y-b)^2 - (x-a)^2) - t(y-x+3)(1-x-y) \\
&= \frac{1}{2}w_A + \frac{t(y-x+3)(1-x-y) - \tau((y-b)^2 - (x-a)^2)}{2} \\
&\text{if } \left\{ \begin{array}{l} w_A > \tau((y-b)^2 - (x-a)^2) - t(y-x+3)(1-x-y) \\ w_A < \tau((y-b)^2 - (x-a)^2) + t(x-y+9)(1-x-y) \end{array} \right\} \\
&= w_A - \tau((y-b)^2 - (x-a)^2) + t(x-y+3)(x+y-1) \\
&\text{if } w_A \geq \tau((y-b)^2 - (x-a)^2) + t(x-y+9)(1-x-y).
\end{aligned}$$

In Figure 3 (I) we present these reaction functions for some parameters values  $t, \tau$  and locations  $a, b, x, y$  for which both firms sell. In Figure 3 (II) we show a case where firm B faces a strong cost disadvantage as it is located far away from its supplier, and this implies a high aggregate marginal cost  $w_B + \tau(y-b)^2$  for its retailer. Even if the wholesale price it sets is reduced to zero, firm B cannot obtain positive demand. In equilibrium, firm A serves the whole market via its retailer X. We also note that the wholesale prices are strategic complements, an increase in  $w_B$  leads to an increase to  $w_A$  and vice versa.

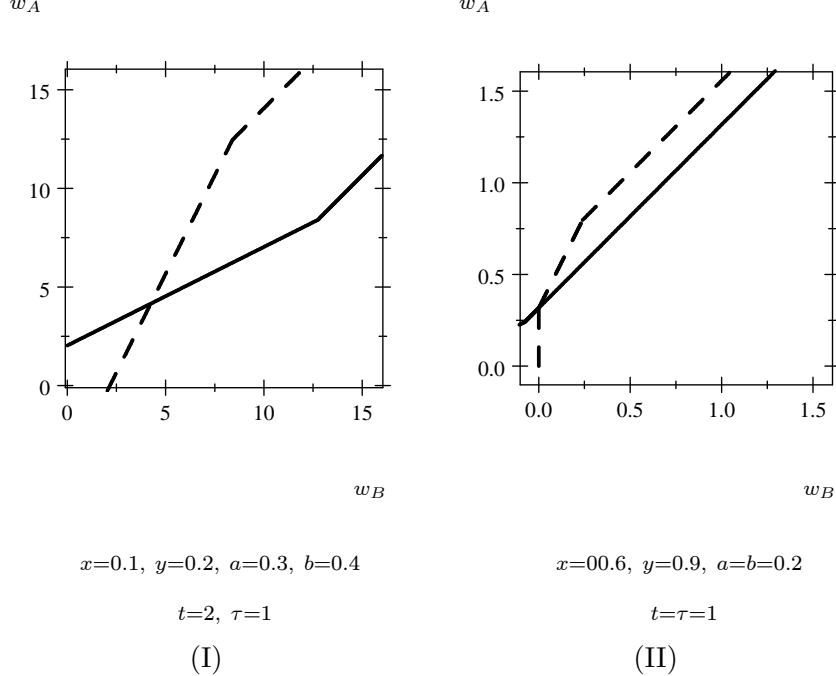


Figure 3:  $R_A(w_B)$ : solid line,  $R_B(w_A)$ : dash line

From the reaction functions we obtain:

**Lemma 2** *The equilibrium wholesale prices (as functions of the upstream and downstream locations) are:*

$$\begin{aligned}
w_A &= \tau \left( (y-b)^2 - (x-a)^2 \right) + t(y-x+3)(x+y-1) \\
&\quad \text{if } \tau \left( (x-a)^2 - (y-b)^2 \right) + t(y-x+9)(1-x-y) \leq 0 \\
&= \frac{\tau((y-b)^2 - (x-a)^2) + t(x-y+9)(1-x-y)}{3} \\
&\quad \text{if } \left\{ \begin{array}{l} \tau \left( (y-b)^2 - (x-a)^2 \right) + t(x-y+9)(1-x-y) > 0 \\ \tau \left( (x-a)^2 - (y-b)^2 \right) + t(y-x+9)(1-x-y) > 0 \end{array} \right\} \quad (8) \\
&= 0 \\
&\quad \text{if } \tau \left( (x-a)^2 - (y-b)^2 \right) - t(x-y+9)(1-x-y) \geq 0
\end{aligned}$$

and

$$\begin{aligned}
w_B &= 0 \\
&\quad \text{if } \tau \left( (x-a)^2 - (y-b)^2 \right) + t(y-x+9)(1-x-y) \leq 0 \\
&= \frac{\tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y)}{3} \\
&\quad \text{if } \left\{ \begin{array}{l} \tau \left( (y-b)^2 - (x-a)^2 \right) + t(x-y+9)(1-x-y) > 0 \\ \tau \left( (x-a)^2 - (y-b)^2 \right) + t(y-x+9)(1-x-y) > 0 \end{array} \right\} \quad (9) \\
&= \tau \left( (x-a)^2 - (y-b)^2 \right) + t(x-y+3)(x+y-1) \\
&\quad \text{if } \tau \left( (x-a)^2 - (y-b)^2 \right) - t(x-y+9)(1-x-y) \geq 0.
\end{aligned}$$

In equilibrium, when the upstream locations are asymmetric enough relative to the downstream locations, the disadvantaged upstream firm sets a zero price and there are no sales for itself (and its retailer) while the rival upstream firm sets the highest price that allows it to capture the whole demand. Otherwise, the market is shared (upstream and downstream). In either case, by substituting the equilibrium wholesale prices (8) and (9) into expressions (6) and (7), we obtain the equilibrium upstream profits in this stage.

### 4.3 Downstream locations

Taking as given the locations of the upstream firms and assuming that the retail and wholesale prices will be subsequently chosen in equilibrium, the downstream firms simultaneously choose their locations to maximize their profits,

$$\begin{aligned}
\Pi_X &= 2t(1-x-y) \\
&\text{if } \tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y) \leq 0 \\
&= \frac{(\tau((y-b)^2 - (x-a)^2) + t(x-y+9)(1-x-y))^2}{162t(1-x-y)} \\
&\text{if } \left\{ \begin{array}{l} \tau((y-b)^2 - (x-a)^2) + t(x-y+9)(1-x-y) > 0 \\ \tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y) > 0 \end{array} \right\} \quad (10) \\
&= 0 \\
&\text{if } \tau((x-a)^2 - (y-b)^2) - t(x-y+9)(1-x-y) \geq 0
\end{aligned}$$

and

$$\begin{aligned}
\Pi_Y &= 0 \\
&\text{if } \tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y) \leq 0 \\
&= \frac{(\tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y))^2}{162t(1-x-y)} \\
&\text{if } \left\{ \begin{array}{l} \tau((y-b)^2 - (x-a)^2) + t(x-y+9)(1-x-y) > 0 \\ \tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y) > 0 \end{array} \right\} \quad (11) \\
&= 2t(1-x-y) \\
&\text{if } \tau((x-a)^2 - (y-b)^2) - t(x-y+9)(1-x-y) \geq 0.
\end{aligned}$$

In our main model firms are allowed to locate anywhere on the real line, that is, also outside the unit interval where consumers live. Like in the previous steps of analysis, special care should be taken about the corner cases. Is it possible that in equilibrium one retailer captures the whole demand and the other gets nothing? In other words, is it possible that for certain wholesale locations, a retailer cannot avoid obtaining zero demand no matter what location it would choose? If so, for what values of the upstream locations this may occur? We find that, if the upstream locations are asymmetric enough, we may obtain a corner solution in the continuation of the game, that is, one retailer has no way to avoid making zero sales. Upstream firms are located asymmetrically enough and this leads a retailer to take the whole demand. Take, for example, firm B that supplies firm Y to be located far away from the unit interval and firm A that supplies firm X to be located in the centre of the unit interval. Then, the transportation cost that retailer Y pays when supplied by firm B is high compared to the rival's transportation cost. Based on its cost advantage, firm X maximizes its profits by choosing a location  $x$  that allows it to capture the whole market. The alternative, choosing a location far away from the unit interval would lead to zero demand or, on any event, to demand lower than unity and firm X's profits would not be maximized. Figure 4 (I) presents the profit function of firm X when it captures the whole demand.



In contrast, in Figure 4 (II) firm X maximizes its profit when it shares the market with firm Y, as its cost advantage is not high enough to allow it take all the demand.

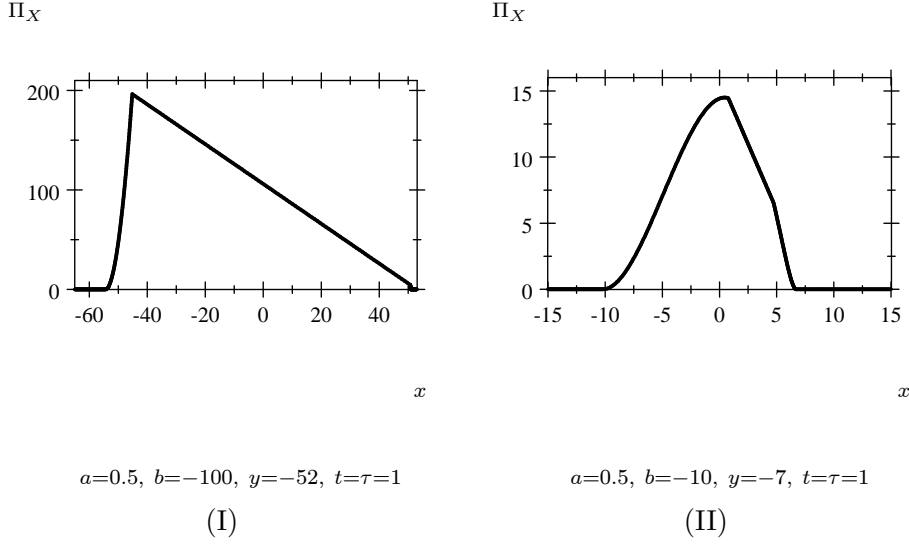


Figure 4: Downstream profits

We have to determine exactly when a corner solution will emerge. From the profit functions above, we calculate that firm X serves the whole demand if  $\tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y) \leq 0$ . When firm B is located far away from the unit interval, its retailer Y has to decide where to locate in order not to pay a high transportation cost but also get some positive demand. If Y locates at its supplier's location, it minimizes its own transportation cost but it is then located far away from the consumers. In contrast, when it locates in or near the unit interval it is close to the final consumers but faces a high transportation cost. In equilibrium, Y chooses a location  $y$  that maximizes the expression  $\tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y)$ , that is, it minimizes the area where its demand is zero. By direct calculations it follows that  $y = \frac{b\tau - 4t}{t + \tau}$ . Given this location  $y$ , firm X maximizes its own profits. This argument holds symmetrically when firm A is located far away from the unit interval and firm X has a cost disadvantage. Of course, when neither firm has a high cost disadvantage in equilibrium the firms share the demand. From this analysis we obtain:<sup>8</sup>

**Lemma 3** *The equilibrium downstream locations (as functions of the upstream locations) are:*

---

<sup>8</sup>The second order conditions of the downstream firms are satisfied at the equilibrium locations. Profit margins are positive at the equilibrium point  $x, y$  if  $a - b + 45 > \frac{9(-9t+16\tau(1-a-b))}{4\tau(1-a-b)}$  and  $a - b - 45 < \frac{9(9t-16\tau(1-a-b))}{4\tau(1-a-b)}$ . The second order conditions for firms X and Y are satisfied for  $a - b - 45 < \frac{243t^2 + 32\tau^2(a+b-1)^2}{4t\tau(1-a-b)}$  and  $a - b + 45 > \frac{-243t^2 - 32\tau^2(a+b-1)^2}{4t\tau(1-a-b)}$  respectively. This holds as  $\frac{9(9t+16\tau(a+b-1))}{4\tau(1-a-b)} < \frac{243t^2 + 32\tau^2(a+b-1)^2}{4t\tau(1-a-b)}$  and  $\frac{9(-9t+16\tau(1-a-b))}{4\tau(1-a-b)} > \frac{-243t^2 - 32\tau^2(a+b-1)^2}{4t\tau(1-a-b)}$ .

$$\begin{aligned}
(x, y) &= \left( \frac{(5t+a\tau-\sqrt{t\tau(a-b-9)(1-a-b)})}{t+\tau}, \frac{b\tau-4t}{t+\tau} \right) \\
&\text{if } 81t + 4\tau(a-b-9)(a+b-1) \leq 0 \\
&= \left( \frac{(16a\tau^2(1-a-b)+4t\tau(17a+6b-a^2+b^2-7)-63t^2)}{4(t+\tau)(4\tau(1-a-b)+9t)}, \frac{(16b\tau^2(1-a-b)+4t\tau(17b+6a-b^2+a^2-7)-63t^2)}{4(t+\tau)(4\tau(1-a-b)+9t)} \right) \\
&\text{if } \left\{ \begin{array}{l} 81t - 4\tau(a-b+9)(a+b-1) > 0 \\ 81t + 4\tau(a-b-9)(a+b-1) > 0 \end{array} \right\} \\
&= \left( \frac{a\tau-4t}{t+\tau}, \frac{(5t+b\tau-\sqrt{t\tau(a-b+9)(a+b-1)})}{t+\tau} \right) \\
&\text{if } 81t - 4\tau(a-b+9)(a+b-1) \leq 0.
\end{aligned} \tag{12}$$

The equilibrium downstream profits in this stage can be obtained by substituting the equilibrium downstream locations (12) into the profits (10) and (11).

#### 4.4 Upstream locations

In this stage, upstream firms choose their locations to maximize their profits,

$$\begin{aligned}
\Pi_A &= \frac{6t(\tau(1-a-b)+\sqrt{t\tau(a-b-9)(1-a-b)})}{t+\tau} \\
&\text{if } 81t + 4\tau(a-b-9)(a+b-1) \leq 0 \\
&= \frac{t(9t+2\tau(1-a-b))(81t+4\tau(a-b+9)(1-a-b))^2}{108(t+\tau)(9t+4\tau(1-a-b))^2} \\
&\text{if } \left\{ \begin{array}{l} 81t - 4\tau(a-b+9)(a+b-1) > 0 \\ 81t + 4\tau(a-b-9)(a+b-1) > 0 \end{array} \right\} \\
&= 0 \\
&\text{if } 81t - 4\tau(a-b+9)(a+b-1) \leq 0
\end{aligned}$$

and

$$\begin{aligned}
\Pi_B &= 0 \\
&\text{if } 81t + 4\tau(a - b - 9)(a + b - 1) \leq 0 \\
&= \frac{t(9t+2\tau(1-a-b))(81t+4\tau(b-a+9)(1-a-b))^2}{108(t+\tau)(9t+4\tau(1-a-b))^2} \\
&\text{if } \begin{cases} 81t - 4\tau(a - b + 9)(a + b - 1) > 0 \\ 81t + 4\tau(a - b - 9)(a + b - 1) > 0 \end{cases} \\
&= \frac{6t(\tau(1-a-b) + \sqrt{t\tau(b-a-9)(1-a-b)})}{t+\tau} \\
&\text{if } 81t - 4\tau(a - b + 9)(a + b - 1) \leq 0.
\end{aligned}$$

An upstream firm will not choose a location that will make its own retailer face so high aggregate marginal cost that would make it obtain zero demand in the subsequent stage. Obviously, this is because such an outcome would also lead to zero demand and profits for this upstream firm. Therefore in equilibrium, both upstream firms avoid receiving zero demand and they share the market. The relevant profit functions reduce to:

$$\begin{aligned}
\Pi_A &= \frac{t(9t + 2\tau(1 - a - b))(81t + 4\tau(a - b + 9)(1 - a - b))^2}{108(t + \tau)(9t + 4\tau(1 - a - b))^2}, \\
\Pi_B &= \frac{t(9t + 2\tau(1 - a - b))(81t + 4\tau(b - a + 9)(1 - a - b))^2}{108(t + \tau)(9t + 4\tau(1 - a - b))^2}.
\end{aligned}$$

From the first order conditions we obtain<sup>9</sup> the equilibrium upstream locations,  $a = \frac{9t-5\tau-9\sqrt{\tau^2+t^2}}{8\tau}$ ,  $b = \frac{9t-5\tau-9\sqrt{\tau^2+t^2}}{8\tau}$ . Notice that  $a < 0$  and  $b < 0$ , thus, upstream firms are located outside the unit interval for all values of the parameters  $t$  and  $\tau$ . Combining this result with Lemmas 1-3 we obtain:

**Proposition 1** *The unique subgame perfect equilibrium outcome in the five stage game is:*

$$\begin{aligned}
a^* &= b^* = \frac{9t - 5\tau - 9\sqrt{\tau^2 + t^2}}{8\tau}, \\
x^* &= y^* = \frac{-\left(5(t + \tau) + 9\sqrt{\tau^2 + t^2}\right)}{8(t + \tau)}, \\
w_A^* &= w_B^* = \frac{27t(t + \tau + \sqrt{\tau^2 + t^2})}{4(t + \tau)}, \\
p_X^* &= p_Y^* = \frac{9t\left(9t^3 + 41t^2\tau + 73t\tau^2 + 32\tau^3 + \sqrt{\tau^2 + t^2}(-9t + 32\tau)(t + \tau)\right)}{32\tau(t + \tau)^2}, \\
z^* &= 0.5, \\
\Pi_A^* &= \Pi_B^* = \frac{27t(t + \tau + \sqrt{\tau^2 + t^2})}{8(t + \tau)}, \\
\Pi_X^* &= \Pi_Y^* = \frac{9t(t + \tau + \sqrt{\tau^2 + t^2})}{8(t + \tau)}.
\end{aligned}$$

---

<sup>9</sup>The second order conditions are satisfied in equilibrium.

We note that, in equilibrium, the market is shared equally, all four firms obtain positive profits and all locations are outside the unit interval. Further, upstream firms locate closer to each other (and to the market center) relative to the downstream firms ( $a > x$ ). We discuss the equilibrium properties in detail in the next Subsection.

#### 4.5 Equilibrium properties

Here we discuss the equilibrium we have obtained just above, and compare its properties to standard D' Aspremont *et al.* (1979) model and to the social optimum. The vertical integration case and its comparison to our basic model is examined in Section 6. We start by noting that:

**Remark 1** *Profits for all firms are strictly positive for all parameter values. As the downstream transportation cost parameter  $t$  increases, all four prices and profit levels increase.*

We obtain this result by directly differentiating the relevant expressions in Proposition 1: wholesale and retail prices increase ( $\frac{dw_A^*}{dt} > 0, \frac{dp_X^*}{dt} > 0$ ) and this also implies higher profit levels ( $\frac{d\Pi_A^*}{dt} > 0, \frac{d\Pi_X^*}{dt} > 0$ ), with symmetric relations holding for firms B and Y. As should be expected, when final consumers view the products as less differentiated, competition becomes less intense. In the extreme case where  $t$  is zero, consumers buy from the cheapest retailer since there is no product differentiation at the downstream level, and this leads to zero wholesale and retail prices and profits for all firms.

**Remark 2** *As the upstream transportation cost parameter  $\tau$  increases, the wholesale prices and all four profit levels increase when  $\tau > t$ , and decrease otherwise. No general conclusion can be derived regarding the retail prices.*

By directly differentiating the relevant expressions in Proposition 1, we obtain:

$$\begin{aligned}\frac{dw_A^*}{d\tau} &= \frac{27t^2(\tau - t)}{4\sqrt{\tau^2 + t^2}(t + \tau)^2}, \\ \frac{dp_X^*}{d\tau} &= \frac{9t^2 \left( (t + \tau)(41\tau^3 - 32t\tau^2 + 18t^2\tau + 9t^3) - 9\sqrt{\tau^2 + t^2}(\tau(t\tau + \tau^2 + 3t^2) + t^3) \right)}{32\tau^2\sqrt{\tau^2 + t^2}(t + \tau)^3}, \\ \frac{d\Pi_A^*}{d\tau} &= \frac{27t^2(\tau - t)}{8\sqrt{\tau^2 + t^2}(t + \tau)^2}, \quad \frac{d\Pi_X^*}{d\tau} = \frac{9t^2(\tau - t)}{8\sqrt{\tau^2 + t^2}(t + \tau)^2}.\end{aligned}$$

To illustrate, take for example  $\tau = 2t$ . Then  $\frac{dw_A^*}{d\tau} = 0.3354$ ,  $\frac{dp_X^*}{d\tau} = 0.4107$ ,  $\frac{d\Pi_A^*}{d\tau} = 0.1677$ ,  $\frac{d\Pi_X^*}{d\tau} =$

0.0559. When upstream transportation cost parameter  $\tau$  is double the downstream transportation cost parameter  $t$ , an increase in the cost parameter  $\tau$ , can be partly passed to the final consumers and the upstream firms absorb the rest. In the limit case where  $\tau$  goes to zero, we obtain:  $\lim_{\tau \rightarrow 0^+} w_A^* = 13.5t$ ,  $\lim_{\tau \rightarrow 0^+} p_X^* = 18t$ ,  $\lim_{\tau \rightarrow 0^+} \Pi_A^* = 6.75t$ ,  $\lim_{\tau \rightarrow 0^+} \Pi_X^* = 2.25t$

Now we contrast our results to the D' Aspremont *et al.* (1979) model, that is, the case where there are no wholesalers and no vertical structure. We find that the introduction of the upstream firms affects both the final prices and the equilibrium locations.

**Remark 3** *At the D' Aspremont et al. (1979) model there is only one stage of competition and firms would locate at  $-\frac{1}{4}$  from each endpoint of the unit interval and equilibrium prices will be equal to  $\frac{3t}{2}$ .*

The above result follows from an application of the D' Aspremont *et al.* (1979) model when there is only one stage of competition and firms are allowed to locate anywhere on the real line (see e.g. Tabuchi and Thisse (1995) and Lambertini (1997)).

By direct comparison to Proposition 1, we find that:

**Remark 4** *In our model, final prices are higher than the D' Aspremont et al. (1979) model.*

The introduction of the wholesalers increase the final prices. The D' Aspremont *et al.* (1979) results are also obtained if we assume that firms are vertically integrated and each pair of vertically integrated firms (firm X with its supplier A, firm Y with its supplier B) act as one firm and choose a unique location and final price. In Section 6, we study the vertical integration case when keeping the timing of the game as in our basic model. The upstream firms in the vertical integrated chains charge zero wholesale prices to their retailers but are allowed to choose different locations compared to their own retailers.

Now we turn to the locations. Again by Proposition 1 and direct calculations, we obtain:

**Remark 5** *As the downstream transportation cost parameter  $t$  increases, upstream firms move closer to the unit interval for all values of  $\tau$ , however, downstream firms move closer to the unit interval when  $\tau > t$ . For  $\tau < t$ , the downstream firms move farther away from the unit interval.*

We obtain this result by directly differentiating the relevant expressions in Proposition 1: upstream location increases ( $\frac{da^*}{dt} > 0$ ) for all values of  $\tau$ , but downstream location increases ( $\frac{dx^*}{dt} > 0$ ) when  $\tau > t$ . When final consumers view the products as more differentiated, upstream firms move closer to the unit interval and retailers follow them when retail transportation costs are high. In the extreme case when  $t = 0$ , all four firms locate at  $a^*=b^*=x^*=y^* = -1.75$ .

**Remark 6** *As the upstream transportation cost parameter  $\tau$  increases, upstream firms move farther away from the unit interval for all values of  $t$ , however, downstream firms move to the same direction when  $\tau > t$ . For  $\tau < t$ , the downstream firms move closer to the unit interval.*

By directly calculations in Proposition 1, we obtain  $\frac{da^*}{d\tau} < 0$  for all  $t$  and  $\frac{dx^*}{d\tau} < 0$  for  $\tau > t$ . Otherwise, for  $\tau < t$ , we obtain  $\frac{dx^*}{d\tau} > 0$ . In the limit case where  $\tau$  goes to zero, we obtain:  $\lim_{\tau \rightarrow 0^+} a^* = -0.625$ ,  $\lim_{\tau \rightarrow 0^+} x^* = -1.75$ . The location incentives will be discussed in more detail later in this Subsection.

**Remark 7** *Downstream locations are farther away from the unit interval compared to the upstream locations and the latter are farther away compared to the D' Aspremont et al. (1979) model :  $x^* = y^* < a^* = b^* < -\frac{1}{4}$ .*

Subsequently in this Subsection, we will discuss in detail the forces that shape the equilibrium locations, but it is useful to first briefly turn to the welfare properties of the equilibrium. Final consumers face unit demand, thus, the social cost ( $SC$ ) simply equals the transportation cost paid

by the retailers plus the transportation cost paid by the final consumers:

$$SC = \int_0^z t(k-x)^2 dk + \int_z^1 t(1-y-k)^2 dk + \tau(x-a)^2 z + \tau(y-b)^2 (1-z). \quad (13)$$

By substituting the locations from Proposition 1, we calculate that in equilibrium total social cost is:

$$\begin{aligned} SC &= \int_0^{\frac{1}{2}} t(k-x)^2 dk + \int_{\frac{1}{2}}^1 t(1-y-k)^2 dk + \frac{(\tau(x-a)^2 + \tau(y-b)^2)}{2} \\ &= \frac{18t \left( \sqrt{\tau^2 + t^2} (-9t + 7\tau) + 7\tau(t + \tau) + 9t^2 \right)}{64\tau(t + \tau)} + \frac{t}{12} > 0. \end{aligned}$$

A social planner in contrast, would minimize the total transportation cost (13). Standard calculations imply:

**Remark 8** *The social optimum locations are  $a = b = x = y = \frac{1}{4}$ .*

Firm X should be located together with its supplier A, and firm Y together with its supplier B, so as to eliminate the transportation costs of the retailers. Also, they are both located inside the unit interval (and splitting the distances equally) to minimize the transportation cost paid by the consumers. The optimum social cost is  $SC = \frac{t}{48}$ . We find that:

**Proposition 2** *We have  $x^* = y^* < a^* = b^* < -\frac{1}{4} < \frac{1}{4}$ .*

In the equilibrium of our model, firms locate outside the unit interval in contrast to the socially optimum locations which are within the unit interval. Thus, the corresponding total social cost is strictly higher than the optimal cost and also strictly higher than the cost at the D'Aspremont *et al.* (1979) model.

Now let us discuss the equilibrium locations in more detail. In models of horizontal differentiation with duopolists selling directly to final consumers, there are two opposite forces, one pushing firms close to each other to directly obtain higher demand and one in the opposite direction to reduce the intensity of price competition. In our model, these forces are modified and a third force also emerges. Given the locations of the other three firms, if the downstream firm moves closer to the centre of the unit interval this affects its aggregate marginal cost, the wholesale price plus the transportation cost. This effect may be working to bring firms either closer to each other or apart, depending on the locations of the upstream firms. The marginal production cost of the retailers is location dependent.

In a standard linear city duopoly model as in D'Aspremont *et al.* (1979) the marginal production cost is exogenous. Denoting this cost by  $c$ , profits are  $\Pi_X = (p_X - c)z$  for firm X. By the envelope theorem, we obtain:

$$\frac{d\Pi_X}{dx} = (p_X - c) \left( \frac{dz}{dx} + \frac{dz}{dp_Y} \frac{dp_Y}{dx} \right),$$

evaluated at the equilibrium prices.<sup>10</sup> The direct demand effect that pushes firms close to each other corresponds to  $\frac{dz}{dx}$  while the indirect price competition effect that pushes firms to the opposite direction corresponds to  $\frac{dz}{dp_Y} \frac{dp_Y}{dx}$ . In our model the marginal production cost for the retailers is endogenous and equal to the wholesale price plus the transportation cost,  $f_X = w_A + \tau(a - x)^2$ . The profit function of firm X is  $\Pi_X = (p_X - f_X)z$  and by the envelope theorem we now obtain:

$$\frac{d\Pi_X}{dx} = (p_X - f_X) \left( \frac{dz}{dx} + \left( \frac{dz}{dp_Y} \frac{dp_Y}{dx} + \frac{dz}{dp_Y} \frac{dp_Y}{df_X} \frac{df_X}{dx} + \frac{dz}{dp_Y} \frac{dp_Y}{df_Y} \frac{df_Y}{dx} \right) \right) - \frac{df_X}{dx} z.$$

A change in the location  $x$  affects the aggregate marginal costs of both retailers and the final prices directly and indirectly via the change in the marginal costs. The direct demand effect again corresponds to  $\frac{dz}{dx}$  but the indirect price competition effect now corresponds to the expression  $\frac{dz}{dp_Y} \frac{dp_Y}{dx} + \frac{dz}{dp_Y} \frac{dp_Y}{df_X} \frac{df_X}{dx} + \frac{dz}{dp_Y} \frac{dp_Y}{df_Y} \frac{df_Y}{dx}$  as final prices are affected by the endogenous marginal production costs of the retailers. A change in  $x$ , leads to a change in the wholesale prices and the retailers' transportation cost, therefore, final prices change too. Finally, profits are also affected directly by the change in the marginal cost itself,  $\frac{df_X}{dx}$ .

**Remark 9** *In addition to the demand effect and the price competition effect, the aggregate marginal cost effect plays a key role in shaping the retailer location incentives. These three forces co-determine the equilibrium locations.*

As the profit margin  $(p_X - f_X)$  and the demand  $z$  are positive, the following derivatives determine the effect of  $x$  on the profit  $\Pi_X$  :

$$\frac{dz}{dx} = \frac{\tau((y-b)^2 - (x-a)^2) + t(9-y+x)(1-x-y)}{18t(1-x-y)^2} - \frac{x}{1-x-y},$$

$$\frac{dz}{dp_Y} \frac{dp_Y}{dx} + \frac{dz}{dp_Y} \frac{dp_Y}{df_X} \frac{df_X}{dx} + \frac{dz}{dp_Y} \frac{dp_Y}{df_Y} \frac{df_Y}{dx} = \frac{4(t(x-5) - \tau(a-x))}{9t(1-x-y)},$$

$$\text{and } \frac{df_X}{dx} = -\frac{2}{3}(t(x+4) + 2\tau(a-x)).$$

The price competition effect  $\frac{4(t(x-5) - \tau(a-x))}{9t(1-x-y)}$  is negative if firm A is located to the right of firm X ( $a > x$ ) and  $x < 5$ . Given  $a, b, y$ , if retailer X moves to right (that is, if  $x$  increases) this may either increase or decrease the aggregate marginal cost that it pays depending on the location of its supplier A:  $\frac{df_X}{dx} = \frac{d(w_A + \tau(a-x)^2)}{dx} = -\frac{2}{3}(t(x+4) + 2\tau(a-x))$  where  $w_A$  is given by (8). If firm A is located to the right of firm X ( $a > x$ ) with  $x > 0$  then  $\frac{df_X}{dx} < 0$  and as  $x$  increases the marginal cost of firm X decreases, which is a positive effect. So, by increasing  $x$ , the marginal cost is reduced, demand is increased (demand effect) and prices fall (price competition effect). But if firm A is located to the left of firm X ( $a < x$ ), the sign of  $\frac{df_X}{dx}$  depends on the values of the cost parameters. If  $\frac{df_X}{dx} > 0$ , this means that, as  $x$  increases, the aggregate marginal cost increases too, which may lead to an increase in the retail price and a reduction in demand. Therefore, as firm X moves to the

---

<sup>10</sup>See Tirole (1989), page 281.

centre, the sign of the demand effect and the price competition effect is ambiguous. In equilibrium, the upstream firms are located closer to the unit interval compared to the downstream firms and all four firms are outside the unit interval ( $x < a < 0, y < b < 0$ ). The forces pushing firms (X relative to Y and A relative to B) close to each other are dominated by the forces pushing firms to the opposite direction and this total effect is stronger for the downstream firms.

## 5 Modified order of location choices

We present now some extensions of the basic model which allows us to obtain additional insights into the problem, especially about the location incentives. We start, in this Section, by considering what happens when the wholesalers cannot choose their locations first. As we show, the equilibrium locations are affected in a systematic way.

### 5.1 Simultaneous location choices

Suppose that the four firms make their location choices simultaneously in the first stage, while the pricing stages of the game remains the same. Now, in the second stage of this modified game the wholesale prices are determined, followed by the final prices, whereas final consumers decide which retailer to buy from. By inserting the wholesale and final prices that we have obtained in our proceeding analysis into the profits of the upstream and downstream firms, we obtain the profit functions depending on the four location choices. We observe that  $\Pi_A = 3\Pi_X$  and  $\Pi_B = 3\Pi_Y$  for all location values and parameters  $t, \tau$ :

$$\begin{aligned}\Pi_X &= \frac{(\tau((y-b)^2 - (x-a)^2) + t(x-y+9)(1-x-y))^2}{162t(1-x-y)}, \\ \Pi_Y &= \frac{(\tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y))^2}{162t(1-x-y)}, \\ \Pi_A &= \frac{(\tau((y-b)^2 - (x-a)^2) + t(x-y+9)(1-x-y))^2}{54t(1-x-y)}, \\ \Pi_B &= \frac{(\tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y))^2}{54t(1-x-y)}.\end{aligned}$$

The four firms simultaneously seek to maximize their profits with respect to their locations. From the first order conditions of the upstream firms (and the fact that the corresponding profit margins are positive), we find that each wholesaler chooses to have the same location as the corresponding retailer:

$$a = x \text{ and } b = y.$$

Further, from the first order conditions of the retailers we obtain:<sup>11</sup>  $a = x = b = y = -1.75$ . Thus, we have:

---

<sup>11</sup>The second order conditions are satisfied.



**Proposition 3** *In the simultaneous locations choice model, the equilibrium outcome is:*

$$\begin{aligned} a^* &= b^* = x^* = y^* = -1.75, \\ w_A^* &= w_B^* = 13.5t, \quad p_X^* = p_Y^* = 18t, \\ z^* &= 0.5, \\ \Pi_A^* &= \Pi_B^* = 6.75t, \quad \Pi_X^* = \Pi_Y^* = 2.25t. \end{aligned}$$

Like in our basic model, all firms are located outside the unit interval, now at  $-1.75$ . We observe that the equilibrium locations are independent of the parameter  $\tau$ , as downstream firms are located at the same point of their suppliers, which means that they pay zero transportation costs in equilibrium. Equilibrium locations are also independent of the parameter  $t$ ; this is a similar result to the standard linear city model where firms are located in equilibrium at  $-0.25$  for all (quadratic) transportation cost parameters. Compared to that model, firms in our model are located farther away ( $-1.75 < -0.25$ ). The introduction of the wholesalers in the problem, which makes the production costs for the retailers endogenous, pushes the locations farther away. The forces that push the firms closer to each other are dominated by the forces that push them to the opposite direction and the latter are stronger at a model with endogenous production costs.<sup>12</sup>

## 5.2 Downstream locations first

We could also consider our sequential locations model but with reversed order of location choices. Downstream firms now choose their locations first and upstream location choices follow. This timing may be especially relevant if the retailers have a stronger position in the market than the wholesalers. The remaining of the game is the same. From the analysis of our simultaneous locations case just above, we know that, in the second stage, upstream firms locate exactly on top of their corresponding retailers ( $a = x$ ,  $b = y$ ). Given these upstream locations the equilibrium downstream locations in the first stage are calculated as  $x = y = -1.75$ . Therefore, the sequential locations outcome where downstream firms determine their locations first coincides with the simultaneous locations outcome (Proposition 3).

## 5.3 Comparison

It is important to compare the equilibrium locations in our simultaneous locations game (Proposition 3) with the sequential locations game (Proposition 1). In the previous Section, we found that

---

<sup>12</sup>Brekke and Straume (2004) allow wholesale prices to be determined through bargaining between the upstream and downstream firms but they do not study the location choices of the upstream firms. In contrast, we assume that wholesale prices are set by the upstream firms and that their locations are endogenous. When the bargaining power of the upstream firms equals one, and there is no product differentiation at the upstream level ( $\tau = 0$ ) the two models deliver the same results, firms locate at  $-1.75$ .

$x < a < 0$  and  $y < b < 0$ , which means that all firms are located outside the unit interval with the upstream firms to be closer to the center. We can further calculate that for all parameter values  $-1.75 < x < a < 0$  and  $-1.75 < y < b < 0$ . Thus, we find that in the simultaneous locations game firms move farther away from the city center than when upstream locations are chosen first. In addition, profits for both upstream and downstream firms and wholesale and final prices are *higher* compared to the sequential location choice game. Why then do not upstream firms locate at  $-1.75$  in our basic model, to obtain higher profits? Each upstream firm has a unilateral incentive to deviate from  $-1.75$  and move closer to the center given that its rival is located at  $-1.75$  and that downstream locations will follow equation (12). Each wholesaler does this in an effort to offer a strategic commitment incentive to its own retailer, to help it move credibly closer to the market center and, in this way, strengthen its position in the final market. However, both retailers become more competitive in this way and all firms obtain lower equilibrium profits. We can say that they are trapped in a Prisoners' Dilemma in locations when wholesalers choose locations first. Both upstream firms jointly prefer being located at  $-1.75$  (which also leads to  $x = y = -1.75$ ) however there is a unilateral incentive for each to move closer to the unit interval to (indirectly) increase its own demand and enjoy higher profits. For example, if firm B were located at  $-1.75$  then firm A would like to deviate from  $-1.75$  and move closer to the unit interval ( $a$  increases) and pull firm X also closer to the unit interval ( $x$  increases too). Demand would then increase for firms X and A, as well as profits.

**Remark 10** *In the sequential locations game, wholesalers move first and locate closer to the center in an effort to also pull their retailers closer to the center, but they end up with lower equilibrium profits than in the simultaneous locations game or when retailers choose their locations first.*

Thus, we find that firms would prefer to be in a game where they set their locations simultaneously compared to the sequential game where upstream firms move first. When upstream firms set their locations first, they can unilaterally affect the downstream locations and this leads to a stronger competition between them. Likewise, when the location choices are reversed compared to our basic model and downstream firms choose their locations first, the equilibrium locations coincide with the simultaneous equilibrium locations. If retailers are already located at  $-1.75$ , the suppliers have no other choice but to locate on their retailers' locations to maximize their profits. In summary, all four firms prefer the downstream firms to locate first (or that locations are simultaneous) so that upstream firms locate at  $-1.75$  and not move to a location closer to the center. Higher profits can be obtained either with simultaneous location choices or when the downstream firms locate first, compared to our basic model.

This result is reminiscent of other results in the sequential choices and strategic commitment literature. In particular, Fershtman and Judd (1987), Bonanno and Vickers (1988), Brander and Spencer (1985), and others have compared contracting and pricing incentives in structures with a

vertical dimension. Owners reward managers for revenue or market shares, upstream firms choose low wholesale prices, governments choose export subsidies, all in an effort to unilaterally strengthen the strategic position of their downstream agents (technically, to shift their reaction functions to a more aggressive position). Here, we identify how this incentive for strategic commitment to aggressive behavior affects upstream and downstream equilibrium locations (and prices) along the line.

## 6 Vertical integration

We study now the vertical integration benchmark for the various timing assumptions on the order of the location choices of the game. Under vertical integration (firm X with its supplier A and firm Y with its supplier B), the wholesalers charge zero wholesale prices to their own retailers in the vertical chain. However, as in the vertical separation model, upstream and downstream firms are allowed to choose different locations. We start, in this Section, by considering the case where upstream firms choose locations first. Then we modify the order of location choices as we did in the previous Section and compare the vertical integration results to the vertical separation. Do we observe double marginalization? That is, are the final prices under vertical separation higher than the vertical integration?

### 6.1 Upstream locations first

In the first stage of this vertical integration game, the upstream firms choose their locations and the downstream locations follow.<sup>13</sup> In the third stage, the two vertically integrated chains compete for the consumers by setting the final prices. We proceed backwards to solve for the subgame perfect equilibrium. The third stage of the game remains the same as in our basic model and the final prices for the chain A-X are given by equation (4). Each vertically integrated pair of firms maximizes the joint profits of its chain. Now the marginal production costs,  $f_X$  and  $f_Y$ , are simply the transportation costs of the retailers, since the wholesale prices are zero. The equilibrium profits of the vertically integrated chain A-X,  $\Pi_X^{VI}$ , are as follows:

$$\Pi_X^{VI} = \begin{cases} f_Y - t(1-x-y)(1+y-x) - f_X & \text{if } f_X \leq \overline{f}_X \\ \frac{(t(1-x-y)(3+x-y)+f_Y-f_X)^2}{18t(1-x-y)} & \text{if } \overline{f}_X < f_X < \widehat{f}_X \\ 0 & \text{if } f_X \geq \widehat{f}_X, \end{cases} \quad (14)$$

where

$$\begin{aligned} \widehat{f}_X &= f_Y + t(1-x-y)(3-y+x) \\ \overline{f}_X &= f_Y - t(1-x-y)(3-x+y), \end{aligned}$$

---

<sup>13</sup>We may be thinking about vertically integrated firms with the factory and the point of sale at different geographical locations.

and

$$\begin{aligned} f_X &= \tau(a-x)^2 \\ f_Y &= \tau(y-b)^2. \end{aligned}$$

Similarly, we calculate the joint profits for the chain B-Y.

In the second stage, taking as given the locations of the upstream firms and assuming that the retail prices will be subsequently chosen in equilibrium, the downstream firms simultaneously choose their locations to maximize their joint profits. Firms are allowed to locate anywhere on the real line, thus, again special care should be taken about the corner cases. If upstream firms are located asymmetrically enough, this can lead a retailer to take the whole demand. Following the same logic as in Subsection 4.3, the retailer that has cost disadvantage minimizes the area where its demand is zero. Of course, when neither retailer has a high cost disadvantage, firms share the demand. From this analysis we obtain the equilibrium downstream locations (as functions of the upstream locations):

$$\begin{aligned} (x, y) &= \left( \frac{t+a\tau}{t+\tau}, \frac{b\tau-t}{t+\tau} \right) \\ &\quad \text{if } 9t - 4\tau(a-b-3)(1-a-b) \leq 0 \\ &= \left( \frac{3t^2+4t\tau(-5a+a^2-b^2+1)+16\tau^2a(a+b-1)}{4(t+\tau)(-3t+4\tau(a+b-1))}, \frac{3t^2+4t\tau(-5b-a^2+b^2+1)+16\tau^2b(a+b-1)}{4(t+\tau)(-3t+4\tau(a+b-1))} \right) \\ &\quad \text{if } \begin{cases} 9t - 4\tau(a-b-3)(1-a-b) > 0 \\ 9t + 4\tau(a-b+3)(1-a-b) > 0 \end{cases} \\ &= \left( \frac{a\tau-t}{t+\tau}, \frac{t+b\tau}{t+\tau} \right) \\ &\quad \text{if } 9t + 4\tau(a-b+3)(1-a-b) \leq 0. \end{aligned} \tag{15}$$

In the first stage of the game, upstream firms choose locations to maximize their joint profits:

$$\begin{aligned} \Pi_X^{VI} &= \frac{t(t+\tau(a-b-1)(1-a-b))}{t+\tau} \\ &\quad \text{if } 9t - 4\tau(a-b-3)(1-a-b) \leq 0 \\ &= \frac{t(3t+2\tau(1-a-b))(9t+4\tau(a-b+3)(1-a-b))^2}{36(t+\tau)(3t+4\tau(1-a-b))^2} \\ &\quad \text{if } \begin{cases} 9t - 4\tau(a-b-3)(1-a-b) > 0 \\ 9t + 4\tau(a-b+3)(1-a-b) > 0 \end{cases} \\ &= 0 \\ &\quad \text{if } 9t + 4\tau(a-b+3)(1-a-b) \leq 0. \end{aligned}$$

Analogously, we calculate for firms B-Y.

An upstream firm will not choose a location that will make its own retailer obtain zero demand. Therefore, in equilibrium both vertical chains avoid receiving zero demand. The relevant joint

profit functions reduce to:

$$\begin{aligned}\Pi_X^{VI} &= \frac{t(3t + 2\tau(1 - a - b))(9t + 4\tau(a - b + 3)(1 - a - b))^2}{36(t + \tau)(3t + 4\tau(1 - a - b))^2} \\ \Pi_Y^{VI} &= \frac{t(3t + 2\tau(1 - a - b))(9t + 4\tau(b - a + 3)(1 - a - b))^2}{36(t + \tau)(3t + 4\tau(1 - a - b))^2}\end{aligned}$$

From the first order conditions we obtain:  $a = b = \frac{3t + \tau - 3\sqrt{\tau^2 + t^2}}{8\tau}$ .

**Proposition 4** *The unique subgame perfect equilibrium outcome under vertical integration and upstream locations first is:*

$$\begin{aligned}a^{VI} &= b^{VI} = \frac{3t + \tau - 3\sqrt{\tau^2 + t^2}}{8\tau} \\ x^{VI} &= y^{VI} = \frac{t + \tau - 3\sqrt{t^2 + \tau^2}}{8(t + \tau)} \\ p_X^{VI} &= p_Y^{VI} = \frac{3t(8\tau^3 + 19t\tau^2 + 11t^2\tau + 3t^3 + \sqrt{\tau^2 + t^2}(-3t + 8\tau)(t + \tau))}{32\tau(t + \tau)^2} \\ z^{VI} &= 0.5 \\ \Pi_X^{VI} &= \Pi_Y^{VI} = \frac{3t(t + \tau + \sqrt{t^2 + \tau^2})}{8(t + \tau)}\end{aligned}$$

We note that, in equilibrium, the market is equally shared between the two pairs of vertically integrated firms. Firms obtain positive profits for all parameter values and the retailers are located outside the unit interval ( $x^{VI} = y^{VI} < 0$ ). Further, upstream firms are located closer to the center relative to the downstream firms ( $a^{VI} > x^{VI}$ ) and upstream firms are located within the unit interval ( $a^{VI} > 0$ ) when  $t > \frac{4\tau}{3}$ .

By substituting the equilibrium locations under vertical integration in the social cost function (13), we calculate that, in equilibrium, the total social cost is:

$$\begin{aligned}SC^{VI} &= \int_0^{\frac{1}{2}} t(k - x)^2 dk + \int_{\frac{1}{2}}^1 t(1 - y - k)^2 dk + \frac{(\tau(x - a)^2 + \tau(y - b)^2)}{2} \\ &= \frac{t(17t\tau + 17\tau^2 + 27t^2 - 9\sqrt{\tau^2 + t^2}(3t - \tau))}{96\tau(t + \tau)}\end{aligned}$$

In a subsequent Subsection, we compare the vertical integration outcome to the equilibrium outcome in our basic model.

## 6.2 Simultaneous location choices

Suppose now that the vertical integrated firms choose their locations simultaneously in the first stage, while the pricing stage of the game remains the same. Each pair of vertically integrated firms

choose their upstream and downstream locations simultaneously to maximize their joint profits:

$$\begin{aligned}\Pi_X^{VI} &= \frac{(t(1-x-y)(3+x-y) + \tau(y-b)^2 - \tau(x-a)^2)^2}{18t(1-x-y)} \\ \Pi_Y^{VI} &= \frac{(t(1-x-y)(3+y-x) + \tau(x-a)^2 - \tau(y-b)^2)^2}{18t(1-x-y)}\end{aligned}$$

Profits  $\Pi_X^{VI}$  are maximized with respect to  $a$  and  $x$  and profits  $\Pi_Y^{VI}$  are maximized with respect to  $b$  and  $y$ . From the first order conditions of the upstream firms, we find that each wholesaler chooses to have the same location as the corresponding retailer:

$$a = x \text{ and } b = y.$$

Further, from the first order conditions of the downstream firms we obtain:<sup>14</sup>  $a = x = b = y = -\frac{1}{4}$ . Thus, we have:

**Proposition 5** *In the simultaneous locations choice model, the equilibrium outcome under vertical integration is:*

$$\begin{aligned}a^{VI} &= b^{VI} = x^{VI} = y^{VI} = -\frac{1}{4} \\ p_X^{VI} &= p_Y^{VI} = \frac{3t}{2} \\ z^{VI} &= 0.5 \\ \Pi_X^{VI} &= \Pi_Y^{VI} = \frac{3t}{4}\end{aligned}$$

All four firms locate outside the unit interval, now at  $-\frac{1}{4}$ . Note also that this result coincides with the result of the standard  $D'$  Aspremont et al. (1979) model where there are no wholesalers.

### 6.3 Downstream locations first

With reversed order of location choices, downstream firms choose their locations first and upstream location choices follow. The final stage of the game is the same. From the analysis of our simultaneous locations case just above, we know that, in the second stage, upstream firms locate exactly on top of their corresponding retailers ( $a = x$ ,  $b = y$ ). Given these upstream locations the equilibrium downstream locations in the first stage are calculated as  $x = y = -\frac{1}{4}$ . Therefore, the sequential locations outcome where downstream firms determine their locations first coincides with the simultaneous locations outcome.

### 6.4 Comparison

First, we compare the vertical integration case under the various timing assumptions on the order of the location choices and then we compare the vertical integration to the vertical separation

---

<sup>14</sup>The second order conditions are satisfied.

outcome. We calculate that, for all parameter values,  $-\frac{1}{4} < x^{VI} < a^{VI}$  and  $-\frac{1}{4} < y^{VI} < b^{VI}$ . In the simultaneous locations game the vertically integrated firms move farther away from the city center relative to the case where upstream locations are chosen first. In addition, joint profits are *higher* compared to the sequential locations choice game. Why then do not upstream firms locate at  $-\frac{1}{4}$  in the sequential locations choice game, to obtain higher profits? The argument parallels the argument in Subsection 5.3. Each upstream firm has a unilateral incentive to deviate from  $-\frac{1}{4}$  and move closer to the center given that its rival is located at  $-\frac{1}{4}$  and that downstream locations will follow equation (15). Each wholesaler does this in an effort to offer a strategic commitment incentive to its own retailer, to help it move credibly closer to the market center and, in this way, strengthen its position in the final market. However, both retailers become more competitive in this way and all firms obtain lower equilibrium profits. Thus, we conclude that

**Remark 11** *Vertically integrated firms would prefer to set their locations simultaneously (or the retailers to move first) compared to the sequential game where upstream firms move first.*

It is important now to compare the equilibrium outcome under vertical integration (Proposition 4) to the vertical separation (Proposition 1). We compare the equilibrium locations and obtain:

**Proposition 6** *We have  $-1.75 < x^* = y^* < a^* = b^* < -\frac{1}{4} < x^{VI} = y^{VI} < a^{VI} = b^{VI} < \frac{1}{4}$ .*

Under vertical separation downstream firms locate farther away relative to the upstream firms ( $x^* = y^* < a^* = b^*$ ) and the latter are located farther away relative to the simultaneous locations choices under vertical integration ( $a^* = b^* < -\frac{1}{4}$ ). Further, under vertical integration when upstream locations are chosen first, the downstream and upstream firms move closer to the center relative to the vertical separation ( $x^* = y^* < a^* = b^* < -\frac{1}{4} < x^{VI} = y^{VI} < a^{VI} = b^{VI}$ ) with the upstream firms to be located even closer to the center. However, product differentiation under vertical integration is greater than the social optimum ( $x^{VI} = y^{VI} < a^{VI} = b^{VI} < \frac{1}{4}$ ). Under vertical integration, the *aggregate* marginal cost effect (third effect) is modified, since the wholesale prices are zero. A change in the location of firm X, still affects the marginal cost ( $f_X$ ) that it pays since it affects its transportation cost but there is no effect through the wholesale price as in our basic model. The equilibrium locations under vertical integration are closer to the unit interval, thus, the price competition effect is less intense when there is no competition on the wholesale prices.

Now, let us turn to the final prices. We find that, in equilibrium, the final prices under vertical separation are higher than under the vertical integration ( $p_X^* = p_Y^* > p_X^{VI} = p_Y^{VI}$ ). This result did not have to necessarily hold since in our model we have duopoly competition in each stage. We find that each upstream firm raises its price to a level that makes its retailer charge a higher final price than the one would see in equilibrium under vertical integration. Only part of the negative effect to its retailer of an increased wholesale price is taken into account by the wholesaler, while the remainder of this effect emerges as an externality. We also compare for each symmetric pair of

upstream and downstream locations ( $a = b, x = y$ ) the final prices under vertical separation and under vertical integration and find that:

**Proposition 7** *Final prices under vertical separation are always higher than under vertical integration,  $p_X^*(a = b, x = y) = p_Y^*(a = b, x = y) > p_X^{VI}(a = b, x = y) = p_Y^{VI}(a = b, x = y)$ . Double marginalization emerges in equilibrium.*

We fix the upstream and downstream locations and solve the vertical separation and vertical integration case. We find that double marginalization emerges under vertical separation, with positive profit margins at both upstream and downstream levels. We also compare the social cost under vertical separation to the social cost under vertical integration and have:

**Remark 12** *The social cost under vertical separation is higher than under vertical integration.*

Under vertical integration, the equilibrium locations are closer to the social optimum relative to the equilibrium locations under vertical separation, thus, social cost is higher under vertical separation in equilibrium.

## 7 Other extensions

In this Section we present two more extensions of our basic model. First, we study the model where firms are not allowed to locate outside the unit interval. Then, we allow retailers to locate to the opposite side of their wholesalers on the line.

### 7.1 Locations in the unit interval

In this Subsection we modify our basic model so that firms are restricted to locate within the  $[0,1]$ , an assumption often made in the literature. Our analysis of the game is as in the basic model, however now  $a, b, x$  and  $y$  cannot take negative values. Thus, we have to solve for the restricted location choices in the first and second stage of the game with the other stages remaining the same.

In the second stage of the game, the downstream firms set their locations and their profit functions are given by expressions (10) and (11). Can one of the two retailers in equilibrium serve the whole market? Firm X takes the whole demand if  $\tau((x - a)^2 - (y - b)^2) + t(y - x + 9)(1 - x - y) \leq 0$ . Given that all firms are located in the unit interval, this expression can be negative only when  $\tau(y - b)^2$  is higher than  $\tau(x - a)^2 + t(y - x + 9)(1 - x - y)$ . Thus, firm Y can avoid receiving zero demand by setting  $y = b$ , that is, locate at the same point as its supplier. Firm Y has some local monopoly power and always serves some consumers located close to that firm. Likewise, firm X can avoid receiving zero demand by setting  $x = a$ . Therefore, in equilibrium no retailer can serve the whole demand, each retailer can assure at least some sales. The profit functions, thus, reduce to:

$$\Pi_X = \frac{(\tau((y - b)^2 - (x - a)^2) + t(x - y + 9)(1 - x - y))^2}{162t(1 - x - y)}, \quad (16)$$



and

$$\Pi_Y = \frac{(\tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y))^2}{162t(1-x-y)}.$$

From the first order conditions we obtain (note that the profit margin is positive<sup>15</sup>):

$$x = \frac{(16a\tau^2(1-a-b) + 4t\tau(17a+6b-a^2+b^2-7) - 63t^2)}{4(t+\tau)(4\tau(1-a-b) + 9t)},$$

and

$$y = \frac{(16b\tau^2(1-a-b) + 4t\tau(17b+6a-b^2+a^2-7) - 63t^2)}{4(t+\tau)(4\tau(1-a-b) + 9t)}.$$

However, the pair of locations  $(x, y)$  is positive only if the numerators of  $x$  and  $y$  are positive (also note that  $x, y < 1$ ). Otherwise, due to the concavity of the profit functions, the equilibrium locations become zero. We get four cases, depending on the values of  $a, b$  and the parameter values  $t, \tau$ . In the second stage of the game, both retailers may be located within  $[0,1]$ , both retailers may be located at the two opposite endpoints or one retailer within  $[0,1]$  and the rival at the endpoint.

Nevertheless, in the first stage of the game, the wholesalers choose symmetric locations ( $a = b$ ), thus, we cannot obtain, as an equilibrium outcome, asymmetric locations downstream. Further, from the analysis of the unrestricted locations model, we find that all four firms are located outside the unit interval. Thus, in the restricted locations model, we cannot obtain an equilibrium with all four firms located in  $[0,1]$ , since there does not exist such an equilibrium in the unrestricted locations model.

Also, in the unrestricted locations model, the forces that push retailers away from the centre of the unit line are stronger relative to the wholesalers. Thus, there are two possible equilibrium outcomes in the restricted locations model. Retailers locate at the two opposite endpoints and the wholesalers either locate within the unit interval or at the endpoints as their corresponding retailers. We prove that when retailers are located at the two opposite endpoints ( $x = y = 0$ ), the wholesalers maximize their profits, in the restricted locations model, by locating at the two opposite endpoints as their retailers ( $a = b = 0$ ).

The following results summarize the equilibrium outcome and the social cost in equilibrium.<sup>16</sup>

**Proposition 8** *The equilibrium outcome with locations restricted in the unit interval is:*

$$\begin{aligned} a^* &= b^* = x^* = y^* = 0, \\ w_A^* &= w_B^* = 3t, \quad p_X^* = p_Y^* = 4t, \\ z^* &= 0.5, \\ \Pi_A^* &= \Pi_B^* = \frac{3t}{2}, \quad \Pi_X^* = \Pi_Y^* = \frac{t}{2}. \end{aligned}$$

---

<sup>15</sup>The profit margin of firm X is positive when  $\tau((y-b)^2 - (x-a)^2) + t(x-y+9)(1-x-y) > 0$  and the profit margin of firm Y is positive when  $\tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y) > 0$ . Also, the second order conditions are satisfied in equilibrium.

<sup>16</sup>In equilibrium, the inequality  $\overline{f_X} < f_X < \widehat{f_X}$  holds true and corresponds to  $-3t < 0 < 3t$ .

**Remark 13** *The social cost in equilibrium with locations within the unit interval is equal to  $\frac{t}{12}$ . As the transportation cost parameter  $t$  increases, the social cost increases too.*

In equilibrium, the social cost is simply equal to the transportation cost of the consumers since this is a unit final demand model (prices do not affect welfare) and the downstream firms are located at the same point as their suppliers:

$$SC = \int_0^{\frac{1}{2}} t(z-x)^2 dz + \int_{\frac{1}{2}}^1 t(1-y-z)^2 dz = \int_0^{\frac{1}{2}} tz^2 dz + \int_{\frac{1}{2}}^1 t(1-z)^2 dz = \frac{t}{12}.$$

As the transportation cost parameter  $t$  of the consumers increases, products become more differentiated and the profits of the upstream and downstream firms increase. We observe that the cost parameter  $\tau$  that affects the transportation cost paid by the downstream firms when supplied by the upstream firms does not affect the equilibrium prices and profits as retailers are located at the same point as their suppliers. The wholesale and product prices in the restricted locations model are lower than in our basic model where firms locate anywhere on the real line. Again, there are three forces that affect the location choices. The demand effect, the price competition effect and the aggregate marginal cost effect. The forces that push firms farther away from each other dominate the forces that push them towards each other and this lead to maximum differentiation. This result was expected since in the unrestricted locations model we have obtained that all four firms locate outside the unit interval (and since the profit functions are quasi-concave).

We should also note that if we modify the restricted locations model to have all four firms simultaneously choose their locations, we obtain that again upstream firms locate at the same point as their retailers and that maximum differentiation occurs ( $a = x = 0$  and  $b = y = 0$ ) as opposed to the unrestricted locations model where all firms locate at  $-1.75$ . So, in the simultaneous locations choice model with locations within  $[0,1]$  we obtain the same equilibrium outcome compared to the sequential location choices with restricted locations.<sup>17</sup>

## 7.2 Retailers locating at the opposite side from the wholesalers

In our analysis we have assumed that upstream firm A locates to the left of B: this is simply a matter of labelling and without loss of generality. We have also proceeded to the equilibrium derivation assuming that each downstream firm locates at the same side of the line as the corresponding upstream (X with A and Y with B). Doing so has allowed us to simplify the exposition in the first two stages of the game and focus on the core arguments. Here, for completeness, we investigate the possibility that, while A is to the left of B, X locates to the right of Y. We find that for some upstream locations and parameter values it could also be an equilibrium in the second stage of

---

<sup>17</sup>Matsushima (2004) solves the simultaneous restricted locations choice model so that upstream firms price discriminate among downstream firms. He finds that for some parameters values downstream firms are located closer to the center of the unit interval compared to the upstream firms in an effort to reduce the wholesale prices they pay. In our model, there is no such incentive since each retailer has its own supplier and no price discrimination takes place.

the game that the retailers locate at the opposite side of the line relative to the corresponding wholesaler. We also show that second stage equilibrium profit is lower in this arrangement. Still, we do not find an equilibrium of the entire game when all locations (upstream and downstream) are endogenous and are chosen so that wholesalers' and retailers' locations are chosen at the opposite side of one another.

We proceed with the analysis. Let us fix the upstream locations (with  $1 - a - b > 0$ ). Thus far we have assumed that  $1 - x - y > 0$ . In Appendix A, we calculate the profit functions of the downstream firms when firm X is located at the same point of Y (with  $1 - x - y = 0$ ) or to the right of firm Y (with  $1 - x - y < 0$ ). The overall profit function of firm X is then:

$$\Pi_X = \begin{cases} \Pi_X^L & \text{if } 1 - x - y > 0 \\ 0 & \text{if } 1 - x - y = 0 \\ \Pi_X^R & \text{if } 1 - x - y < 0, \end{cases}$$

where  $\Pi_X^L$  is the profit for X when X is to the left (L) of Y and  $\Pi_X^R$  is the profit when X is to the right (R) of Y with:

$$\begin{aligned} \Pi_X^L &= 2t(1 - x - y) \\ &\text{if } \tau((x - a)^2 - (y - b)^2) + t(y - x + 9)(1 - x - y) \leq 0 \\ &= \frac{(\tau((y - b)^2 - (x - a)^2) + t(x - y + 9)(1 - x - y))^2}{162t(1 - x - y)} \\ &\text{if } \left\{ \begin{array}{l} \tau((y - b)^2 - (x - a)^2) + t(x - y + 9)(1 - x - y) > 0 \\ \tau((x - a)^2 - (y - b)^2) + t(y - x + 9)(1 - x - y) > 0 \end{array} \right\} \quad (17) \\ &= 0 \\ &\text{if } \tau((x - a)^2 - (y - b)^2) - t(x - y + 9)(1 - x - y) \geq 0 \end{aligned}$$

and

$$\begin{aligned} \Pi_X^R &= 2t(x + y - 1) \\ &\text{if } \tau((x - a)^2 - (y - b)^2) + t(x - y + 9)(x + y - 1) \leq 0 \\ &= \frac{(\tau((y - b)^2 - (x - a)^2) + t(y - x + 9)(x + y - 1))^2}{162t(x + y - 1)} \\ &\text{if } \left\{ \begin{array}{l} \tau((y - b)^2 - (x - a)^2) + t(y - x + 9)(x + y - 1) > 0 \\ \tau((x - a)^2 - (y - b)^2) + t(x - y + 9)(x + y - 1) > 0 \end{array} \right\} \quad (18) \\ &= 0 \\ &\text{if } \tau((x - a)^2 - (y - b)^2) - t(y - x + 9)(x + y - 1) \geq 0. \end{aligned}$$

We can also write the profit function of firm Y in an analogous way.

Our main result in this Subsection is that for symmetric upstream locations ( $a = b$ ) and sufficiently high transportation cost parameter  $t$  there is an additional equilibrium when the downstream locations are set. In the second stage of the game, given the location of firm Y (and the locations of the upstream firms and the cost parameters  $t, \tau$ ) firm X can choose to locate either to the left or to the right of firm Y. We have already characterized in Subsection (4.3) an equilibrium where firm X locates to the left of firm Y (see Appendix B for the proof). Now, we will prove that, for some parameter values, there is another equilibrium where firm X locates to the right of firm Y.

If there is an equilibrium where firm X is located to the right of firm Y, with locations say  $(\hat{x}, \hat{y})$ , the optimal location choices of the downstream firms will be given by the maximization of the downstream firms' profits  $\Pi_X^R$  (equation (18)) and  $\Pi_Y^R$  respectively. From the first order conditions we calculate that:

$$\begin{aligned}\hat{x} &= \frac{16a\tau^2(a+b-1) + 4t\tau(19a + 12b + a^2 - b^2 - 11) + 99t^2}{4(t+\tau)(4\tau(a+b-1) + 9t)}, \\ \hat{y} &= \frac{16b\tau^2(a+b-1) + 4t\tau(19b + 12a + b^2 - a^2 - 11) + 99t^2}{4(t+\tau)(4\tau(a+b-1) + 9t)}\end{aligned}\tag{19}$$

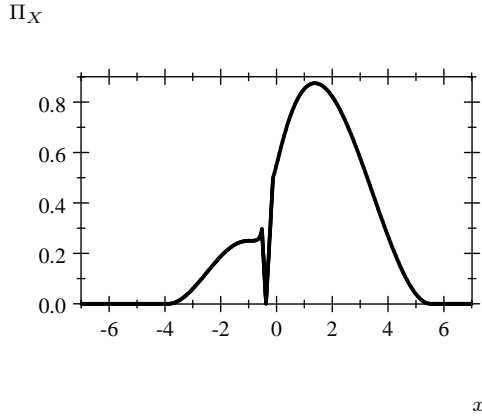
if

$$\begin{aligned}9t + 2\tau(a+b-1) &> 0, \\ \frac{(81t + 4\tau(a-b-9)(1-a-b))}{9t + 4\tau(a+b-1)} &> 0, \frac{(81t + 4\tau(a-b+9)(a+b-1))}{(9t + 4\tau(a+b-1))} > 0\end{aligned}$$

with

$$\begin{aligned}\Pi_X^R(\hat{x}, \hat{y}) &= \frac{t(9t + 2\tau(a+b-1))((81t + 4\tau(a-b-9)(1-a-b)))^2}{324(t+\tau)(9t + 4\tau(a+b-1))^2}, \\ \Pi_Y^R(\hat{x}, \hat{y}) &= \frac{t(9t + 2\tau(a+b-1))((81t + 4\tau(a-b+9)(a+b-1)))^2}{324(t+\tau)(9t + 4\tau(a+b-1))^2}.\end{aligned}$$

The first constraint is necessary to have positive equilibrium profits and the other two to have positive profit margins (to assure that the calculated quantities are not negative). Thus, if such an equilibrium  $(\hat{x}, \hat{y})$  exists, it will satisfy the conditions under (19).



$$a=b=0, y=1.375, t=\tau=1$$

Figure 5: Downstream profits with firm X located either to the left or right of firm Y

Figure 5 illustrates the nature of the problem. When the upstream firms are located at the two opposite endpoints and the transportation cost parameters are equal to one, we obtain  $\hat{y} = 1.375$ . In Figure 5, we plot the profit function of firm X allowing X to be located either to the left or to the right of firm Y. When firm X is located to the left of firm Y and far enough from the unit interval and its own supplier, it gets zero demand and profits. There is an interval where firm X is located to the left of firm Y and both firms sell to the final consumers. However, when X is located close to Y, either to the left or to the right, X captures the whole demand. Firm X has a cost advantage compared to firm Y and reduces its retail price to capture the whole demand. This cost advantage is greater when firm X is located to the right of firm Y, since firm X is closer to its supplier, firm A. Thus, the profits of firm X when it captures the whole demand are higher when it chooses the side that is closer to its supplier. There is again an interval where the two retailers share the market, but now firm X is to the right of firm Y. Finally, if firm X is located to the right of firm Y and away from the unit interval, it obtains zero demand since it has a significant cost disadvantage and is located away from  $[0,1]$  where consumers live.

We prove that for symmetric upstream locations and some parameter values  $t, \tau$ , firm X (given  $y = \hat{y} = \frac{11t+4a\tau}{4(t+\tau)}$ ) obtains higher profits when it locates to the right of firm Y and shares the market with Y compared to the profits obtained when it locates to the left of Y and shares the market or it locates to the left of Y and captures the whole demand or it locates to the right of Y and captures the whole demand. The total maximum of firm X's profits is to the right of firm Y when both firms sell to the final consumers. Figure 5 presents an example. Employing symmetry, we prove that, for certain parameter values and given  $\hat{x}$ , the best response of firm Y is  $\hat{y}$ .

**Proposition 9** *For fixed and symmetric upstream locations and  $t \geq \frac{4\tau(1-2a)}{9}$ , the pair of locations  $\hat{x} = \hat{y} = \frac{11t+4a\tau}{4(t+\tau)}$  constitutes an equilibrium in the second stage of the game.*

We find that: if the transportation cost parameter of the consumers,  $t$ , is high enough (compared to  $\tau$ ), it can also be an equilibrium in the second stage of the game that retailers locate at the opposite side of the corresponding wholesaler. In the extreme case where  $t$  is positive and  $\tau$  is zero, retailers pay zero transportation costs when supplied by their wholesalers. This is equivalent to locating at the optimum points on the real line either to the same or the opposite side of their wholesalers. In Figure 5, we assume  $a = b = 0$  and  $\tau = 1$ , therefore we need  $t \geq \frac{4}{9}$ , which is satisfied since  $t = 1$ .

Therefore, there can be two equilibria in the second stage of the game for fixed upstream locations,  $a = b$ , and for  $t \geq \frac{4\tau(1-2a)}{9}$ . Retailers either locate to the same side (see  $(x^*, y^*)$ , from equation (12) for  $a = b$ ) or to the opposite side of their wholesalers  $(\hat{x}, \hat{y})$  :

$$x^* = y^* = \frac{-7t + 4a\tau}{4(t + \tau)}, \quad \Pi_X^L(x^*, y^*) = \Pi_Y^L(x^*, y^*) = \frac{t(9t + 2\tau(1 - 2a))}{4(t + \tau)}$$

and

$$\hat{x} = \hat{y} = \frac{11t + 4a\tau}{4(t + \tau)}, \quad \Pi_X^R(\hat{x}, \hat{y}) = \Pi_Y^R(\hat{x}, \hat{y}) = \frac{t(9t - 2\tau(1 - 2a))}{4(t + \tau)}.$$

We now compare the profits in the two equilibria and find that equilibrium  $(x^*, y^*)$  Pareto dominates  $(\hat{x}, \hat{y})$  since  $\Pi_X^L(x^*, y^*) > \Pi_X^R(\hat{x}, \hat{y})$ .<sup>18</sup> Downstream equilibrium profits are higher when retailers are located closer to their suppliers.

Can this behavior be part of a subgame perfect equilibrium in the whole game? If we make the upstream firms locate at the points indicated in Proposition 1 ( $a^* = b^* = \frac{9t - 5\tau - 9\sqrt{\tau^2 + t^2}}{8\tau}$ ) we calculate that we can have this additional equilibrium  $\hat{x} = \hat{y} = \frac{31t - 5\tau - 9\sqrt{\tau^2 + t^2}}{8(t + \tau)}$  in the second stage of the game when  $t \geq \frac{4}{3}\tau$ . Retailers locate either at  $(x^*, y^*)$  or at  $(\hat{x}, \hat{y})$  in the second stage of the game for these fixed upstream locations. He have proved numerically that the pair  $(a^*, b^*)$  is not an equilibrium pair of locations in the first stage of the game, when it is anticipated that the retailers, in the equilibrium of the subsequent stage, locate at  $(\hat{x}, \hat{y})$ . Each wholesaler then has an incentive to deviate from  $(a^*, b^*)$ .

## 8 Conclusion

Our paper contributes to the horizontal differentiation literature and the vertical contracting literature. We have studied a linear city model with duopoly upstream and downstream. Wholesalers and then retailers choose their locations and then their prices, before consumers make their choices. We derive a unique subgame perfect equilibrium of this five stage game and examine its properties. We find that wholesalers choose to become less differentiated (that is, locate closer to the unit interval) than the retailers and that differentiation is greater compared to the vertical integration benchmark (which in turn is greater than in the social optimum). We also find positive profit margins both upstream and downstream and that final prices are higher than under vertical integration

---

<sup>18</sup>For symmetric upstream locations, we have  $a = b < \frac{1}{2}$ .

(“double marginalization”). Thus, vertical separation in our duopoly implies both a higher social cost (of transportation) and higher final prices for the consumers.

Considering a number of extensions offers additional insights into the problem. When firms’ locations are restricted to the unit interval, maximum differentiation is obtained. We also modify the order of location choices. When locations are chosen simultaneously by all four firms or the downstream locations are chosen before the upstream, the upstream and the downstream firms in each pair choose the same location. This location is also farther away from the city center relative to the case when upstream locations are chosen first: in that case, the wholesalers locate closer to the city center with the goal to also pull their retailers closer to the center, so that they can each strategically strengthen their retailer’s position in the downstream market. Still, since this happens for both vertical chains, the end result is a more competitive market (than when all locations are chosen simultaneously) and profits are lower.

The framework studied in this paper allows a number of additional extensions and modifications. While the results presented here take a relatively simple and clear form, the equilibrium calculations for this model or its variations are quite involved, as one should expect from a five stage game, especially when care has to be taken for possible corner solutions. Still, a number of extensions appear promising. In companion work we consider the role of price discrimination and non exclusive vertical relations. Modifications of the product differentiation structure, pricing and contracting and the timing of the game may offer additional important insights, and so would models where consumers may directly care also about the upstream choices (and not only indirectly, as in our model). Of course, empirical work that studies the interplay of product differentiation and vertical contracting will also be very important and hopefully our theoretical study of the topic also contributes in this direction.

## Appendix A

**Profit function of firm X for locations  $x$  to the left or the right of  $y$ .** In Section (4.3) we provide the profit functions of the downstream firms when  $1 - x - y > 0$ , that is, firm X is located to the left of firm Y. Now, we also study the case where  $1 - x - y = 0$  and  $1 - x - y < 0$  where firm X is located at the same point or to the right of firm Y. When  $1 - x - y < 0$  we cannot simply replace in the profit functions (10) and (11),  $x$  with  $1 - y$  and  $y$  with  $1 - x$  since there is exclusive dealing between firm X and its supplier A and between firm Y and its supplier B. If firm X is located to the right of firm Y, it is still supplied by firm A which is located to the left of firm B. Thus, when firm X is located to the right of firm Y is not completely symmetric to when firm Y is located to the right of firm X, since they have different suppliers.

Assume  $1 - x - y = 0$ , so that retailers are located at the same point on the line. There is no product differentiation downstream and consumers buy one unit of the product from the cheapest retailer. The demand of firm X,  $z$ , becomes:

$$z = D_X = D_A = \begin{cases} 1 & \text{if } p_X < p_Y \\ \frac{1}{2} & \text{if } p_X = p_Y \\ 0 & \text{if } p_X > p_Y. \end{cases}$$

Downstream competition is very intense (Bertrand competition) and pushes final prices to the marginal cost. Firm X faces aggregate marginal cost  $f_X = w_A + \tau(x - a)^2$  and firm Y  $f_Y = w_B + \tau(y - b)^2$ . Thus, final price is set equal to  $\max\{f_X, f_Y\}$  and the firm with the lowest aggregate marginal cost captures the whole demand and enjoys positive profits. The profit function of firm X becomes:

$$\Pi_X = \begin{cases} w_B - w_A + \tau \left( (y - b)^2 - (x - a)^2 \right) & \text{if } w_A + \tau(x - a)^2 < w_B + \tau(y - b)^2 \\ 0 & \text{if } w_A + \tau(x - a)^2 \geq w_B + \tau(y - b)^2. \end{cases}$$

Since firm X is supplied by firm A, the profit function of firm A is:

$$\Pi_A = \begin{cases} w_A & \text{if } w_A < w_B + \tau \left( (y - b)^2 - (x - a)^2 \right) \\ 0 & \text{if } w_A \geq w_B + \tau \left( (y - b)^2 - (x - a)^2 \right). \end{cases}$$

Downstream competition is transferred to the upstream level and the wholesale prices are reduced to the difference in the transportation costs of the two retailers (minus  $\varepsilon \simeq 0$ ). If firm B charges a wholesale price higher than that level, firm A can undercut  $w_B$  and capture the whole demand via its retailer X. Thus:

$$w_A = \begin{cases} \tau \left( (1 - x - b)^2 - (x - a)^2 \right) & \text{if } (1 - x - b)^2 - (x - a)^2 > 0 \\ 0 & \text{otherwise,} \end{cases}$$

$$w_B = \begin{cases} \tau \left( (x - a)^2 - (1 - x - b)^2 \right) & \text{if } (x - a)^2 - (1 - x - b)^2 > 0 \\ 0 & \text{otherwise.} \end{cases}$$

The profits of X reduce to zero,  $\Pi_X = 0$ , as  $\tau \left( (1 - x - b)^2 - (x - a)^2 \right) - \left( \tau \left( (1 - x - b)^2 - (x - a)^2 \right) \right) = 0$ . In an analogous manner  $\Pi_Y = 0$ .



Assume  $1 - x - y < 0$ , so that firm X is located to the right of firm Y. The indifferent consumer is given by:

$$p_Y + t(z - (1 - y))^2 = p_X + t(x - z)^2$$

and the demand of firm Y ( $z$ ) that is located on the left of firm X is:

$$z = D_Y = D_B = \begin{cases} 1 & \text{if } \frac{1+x-y}{2} + \frac{p_X - p_Y}{2t(x+y-1)} \geq 1 \\ \frac{1+x-y}{2} + \frac{p_X - p_Y}{2t(x+y-1)} & \text{if } 0 < \frac{1+x-y}{2} + \frac{p_X - p_Y}{2t(x+y-1)} < 1 \\ 0 & \text{if } \frac{1+x-y}{2} + \frac{p_X - p_Y}{2t(x+y-1)} \leq 0. \end{cases}$$

The profit functions of firms X and Y are:

$$\begin{aligned} \Pi_X^R &= (p_X - f_X)(1 - z), \\ \text{and } \Pi_Y^R &= (p_Y - f_Y)z. \end{aligned}$$

The superscript  $R$  refers to the case where firm X is to the right of firm Y. From the first order conditions, we obtain the equilibrium final price for firm X (likewise, for firm Y):

$$p_X^R = \begin{cases} f_Y - t(x + y - 1)(1 + x - y) & \text{if } f_X \leq f_Y - t(3 - y + x)(x + y - 1) \\ \frac{1}{3}(t(x + y - 1)(3 + y - x) + f_Y + 2f_X) & \text{if } \begin{cases} f_Y - t(3 - y + x)(x + y - 1) < f_X \\ f_X < f_Y + t(3 - x + y)(x + y - 1) \end{cases} \\ f_X & \text{if } f_X \geq f_Y + t(3 - x + y)(x + y - 1) \end{cases}$$

with the respective profits

$$\Pi_X^R = \begin{cases} f_Y - t(x - y + 1)(x + y - 1) - f_X & \text{if } f_X \leq f_Y - t(3 - y + x)(x + y - 1) \\ \frac{(t(x+y-1)(3+y-x)+f_Y-f_X)^2}{18t(x+y-1)} & \text{if } \begin{cases} f_Y - t(3 - y + x)(x + y - 1) < f_X \\ f_X < f_Y + t(3 - x + y)(x + y - 1) \end{cases} \\ 0 & \text{if } f_X \geq f_Y + t(3 - x + y)(x + y - 1). \end{cases}$$

In the third stage, upstream firms seek to maximize their profits  $\Pi_A^R = w_A(1 - z)$  and  $\Pi_B^R = w_B z$  with respect to their wholesale prices. Assuming equilibrium in the subsequent stage, the profit function of firm A becomes:

$$\begin{aligned} \Pi_A^R &= 0 \\ &\text{if } w_A \geq w_B + \tau \left( (y - b)^2 - (x - a)^2 \right) + t(y - x + 3)(x + y - 1) \\ &= w_A \left( 1 - \left( \frac{2t(y-x)(x+y-1) + w_A - w_B + \tau((x-a)^2 - (y-b)^2)}{6t(x+y-1)} + \frac{1}{2}(1 + x - y) \right) \right) \\ &\text{if } \begin{cases} w_A < w_B + \tau \left( (y - b)^2 - (x - a)^2 \right) + t(y - x + 3)(x + y - 1) \\ w_A > w_B + \tau \left( (y - b)^2 - (x - a)^2 \right) - t(x - y + 3)(x + y - 1) \end{cases} \\ &= w_A \\ &\text{if } w_A \leq w_B + \tau \left( (y - b)^2 - (x - a)^2 \right) - t(x - y + 3)(x + y - 1). \end{aligned}$$

When the aggregate marginal cost faced by firm X,  $w_A + \tau(x - a)^2$ , is high enough, the demand of firm X reduces to zero, thus, firm A gets zero demand too. For intermediate prices,  $w_A$ , the market is shared with firm B and for very low prices firm A captures the whole demand. As in section (4.2), we calculate the equilibrium wholesale prices for firm A (analogously for firm B):

$$\begin{aligned}
w_A^R &= \tau \left( (y - b)^2 - (x - a)^2 \right) - t(x - y + 3)(x + y - 1) \\
&\quad \text{if } \tau \left( (x - a)^2 - (y - b)^2 \right) + t(x - y + 9)(x + y - 1) \leq 0 \\
&= \frac{\tau((y-b)^2 - (x-a)^2) + t(y-x+9)(x+y-1)}{3} \\
&\quad \text{if } \left\{ \begin{array}{l} \tau \left( (y - b)^2 - (x - a)^2 \right) + t(y - x + 9)(x + y - 1) > 0 \\ \tau \left( (x - a)^2 - (y - b)^2 \right) + t(x - y + 9)(x + y - 1) > 0 \end{array} \right\} \\
&= 0 \\
&\quad \text{if } \tau \left( (x - a)^2 - (y - b)^2 \right) - t(y - x + 9)(x + y - 1) \geq 0.
\end{aligned}$$

The corresponding profits for firm X are:

$$\begin{aligned}
\Pi_X^R &= 2t(x + y - 1) \\
&\quad \text{if } \tau \left( (x - a)^2 - (y - b)^2 \right) + t(x - y + 9)(x + y - 1) \leq 0 \\
&= \frac{(\tau((y-b)^2 - (x-a)^2) + t(y-x+9)(x+y-1))^2}{162t(x+y-1)} \\
&\quad \text{if } \left\{ \begin{array}{l} \tau \left( (y - b)^2 - (x - a)^2 \right) + t(y - x + 9)(x + y - 1) > 0 \\ \tau \left( (x - a)^2 - (y - b)^2 \right) + t(x - y + 9)(x + y - 1) > 0 \end{array} \right\} \\
&= 0 \\
&\quad \text{if } \tau \left( (x - a)^2 - (y - b)^2 \right) - t(y - x + 9)(x + y - 1) \geq 0.
\end{aligned}$$

Section (4.3) provides the profits of firm X when located to the left of firm Y ( $\Pi_X^L$ ) and Appendix A presents the profits of firm X when X is located at the same point or to the right of firm Y ( $\Pi_X^R$ ).

Thus,

$$\Pi_X = \begin{cases} \Pi_X^L & \text{if } 1 - x - y > 0 \\ 0 & \text{if } 1 - x - y = 0 \\ \Pi_X^R & \text{if } 1 - x - y < 0. \end{cases}$$

■

## Appendix B

**Proof of the downstream equilibrium locations.** In Section (4.3) we prove that there is an equilibrium in the second stage of the game, when firm X is located to the left of firm Y, say  $(x^*, y^*)$  with:

$$\begin{aligned}
x^* &= \frac{(16a\tau^2(1-a-b) + 4t\tau(17a + 6b - a^2 + b^2 - 7) - 63t^2)}{4(t + \tau)(4\tau(1-a-b) + 9t)}, \\
y^* &= \frac{(16b\tau^2(1-a-b) + 4t\tau(17b + 6a - b^2 + a^2 - 7) - 63t^2)}{4(t + \tau)(4\tau(1-a-b) + 9t)}.
\end{aligned}$$

However, we do not compare the profits of firm X when located either to the left or the right of firm Y. We assume that firm X is located to the left. Now we complete the proof and show that given  $y = y^*$ , firm X prefers to locate at the left of firm Y at point  $x^*$ , compared to the right of firm Y. For simplicity, we provide the proof for symmetric upstream locations ( $a = b$ ) since this will be the equilibrium outcome.

Initially we prove that given  $y = y^* = \frac{-7t+4a\tau}{4(t+\tau)}$ , firm X cannot capture the whole demand either located to the left or to the right of firm Y. Firm X would serve the whole market located to the left of firm Y if:

$$\tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y) \leq 0$$

$$\begin{aligned} \text{or } x &\in (x_1, x_2) \text{ with} \\ x_1 &\equiv \frac{20t + 4a\tau - 3\sqrt{t(9t - 16\tau + 32a\tau)}}{4(t + \tau)}, \\ x_2 &\equiv \frac{20t + 4a\tau + 3\sqrt{t(9t - 16\tau + 32a\tau)}}{4(t + \tau)}. \end{aligned}$$

Since  $1-y^* < x_1 < x_2$ , we have that inequality  $\tau((x-a)^2 - (y-b)^2) + t(y-x+9)(1-x-y) \leq 0$  is not satisfied. Firm X would serve the whole market located to the right of firm Y if:

$$\tau((x-a)^2 - (y-b)^2) + t(x-y+9)(x+y-1) \leq 0$$

$$\begin{aligned} \text{or } x &\in (x_4, x_3) \text{ with} \\ x_3 &\equiv \frac{-16t + 4a\tau + 3\sqrt{t(81t + 16\tau - 32a\tau)}}{4(t + \tau)}, \\ x_4 &\equiv \frac{-16t + 4a\tau - 3\sqrt{t(81t + 16\tau - 32a\tau)}}{4(t + \tau)}. \end{aligned}$$

Since  $1-y^L > x_3 > x_4$ , we have that inequality  $\tau((x-a)^2 - (y-b)^2) + t(x-y+9)(x+y-1) \leq 0$  is not satisfied.

Thus, we have to prove that firm X prefers to share the market with firm Y and locate to the left of Y. Firm X maximizes its profits to the left at  $x^*$  with:

$$\begin{aligned} \Pi_X^L(x^*, y^*) &= \frac{t(9t - 2\tau(2a - 1))}{4(t + \tau)}, \\ x^* &= \frac{-7t + 4a\tau}{4(t + \tau)} \end{aligned}$$

and to the right at  $x^+$  with

$$\begin{aligned} \Pi_X^R(x^+, y^*) &= \frac{(9t - 2\tau(2a - 1))(9t - 16\tau + 32a\tau)^2}{8748t(t + \tau)}, \\ x^+ &= \frac{51t + 16\tau - 20a\tau}{12(t + \tau)} \end{aligned}$$

for

$$9t + 16\tau(2a - 1) > 0.$$

The constraint  $9t + 16\tau(2a - 1) > 0$  is necessary so that both firms sell, when X is located to the right of Y. If  $a$  is low enough, that is, upstream firms are located away from the unit interval, firm X can never sell when located to the right of firm Y since its supplier A is far away. We compare the profits  $\Pi_X^L(x^*, y^*)$  and  $\Pi_X^R(x^+, y^*)$  and obtain  $\Pi_X^L(x^*, y^*) > \Pi_X^R(x^+, y^*)$ . ■

### Appendix C

**Proof of the second equilibrium in the second stage of the game for fixed and symmetric upstream locations.** For symmetric upstream locations ( $a = b$ ) and given  $y = \hat{y} = \frac{11t+4a\tau}{4(t+\tau)}$ , the profits of firm X when it locates to the right of firm Y and shares the market are maximized at  $\hat{x}$  where:

$$\begin{aligned}\Pi_X^R(\hat{x}, \hat{y}) &= \frac{t(9t + 2\tau(2a - 1))}{4(t + \tau)}, \\ \hat{x} &= \frac{11t + 4a\tau}{4(t + \tau)}.\end{aligned}$$

Firm X maximizes its profits when it locates to the left of firm Y and shares the market at  $x^-$  with:

$$\begin{aligned}\Pi_X^L(x^-, \hat{y}) &= \frac{\left(4(-9t + 4\tau(2a - 1))\sqrt{(9t - 8\tau + 16a\tau)^2 + 4(-81t^2 + 288t\tau(2a - 1) + 32\tau^2(2a - 1)^2)}\right)^2}{279936t(t + \tau)\left(9t + 4\tau(1 - 2a) + \sqrt{(9t - 8\tau + 16a\tau)^2}\right)}, \\ x^- &= -\frac{\left(15t + 2\tau(a - 2) + \frac{1}{2}\sqrt{(9t - 8\tau + 16a\tau)^2}\right)}{6(t + \tau)}.\end{aligned}$$

When firm X locates to the left of firm Y and captures the whole demand, its profits are maximized at  $x_m^L$  with:

$$\begin{aligned}\Pi_X^L(x_m^L, \hat{y}) &= 2t(1 - x - y) = \frac{t\left(4\tau(1 - 2a) - 27t + 3\sqrt{t(81t + 16\tau(2a - 1))}\right)}{2(t + \tau)}, \\ x_m^L &= \frac{20t + 4a\tau - 3\sqrt{t(81t + 16\tau(2a - 1))}}{4(t + \tau)}.\end{aligned}$$

Finally, firm X captures the whole demand located to the right of firm Y and maximizes its profits at  $x_m^R$  where:

$$\begin{aligned}\Pi_X^R(x_m^R, \hat{y}) &= \frac{t\left(-9t - 4\tau + 8a\tau + 3\sqrt{t(9t - 16\tau(2a - 1))}\right)}{2(t + \tau)}, \\ x_m^R &= \frac{-16t + 4a\tau + 3\sqrt{t(9t + 16\tau(1 - 2a))}}{4(t + \tau)}.\end{aligned}$$

The  $(\hat{x}, \hat{y})$  pair of locations is an equilibrium in the second stage of the game when:

$$\begin{aligned}\Pi_X^R(\hat{x}, \hat{y}) &\geq \Pi_X^L(x^-, \hat{y}), \\ \Pi_X^R(\hat{x}, \hat{y}) &\geq \Pi_X^L(x_m^L, \hat{y}), \\ \text{and } \Pi_X^R(\hat{x}, \hat{y}) &\geq \Pi_X^R(x_m^R, \hat{y})\end{aligned}$$

are satisfied. By direct calculations we find that when  $\Pi_X^R(\hat{x}, \hat{y}) \geq \Pi_X^L(x^-, \hat{y})$  then  $\Pi_X^R(\hat{x}, \hat{y}) \geq \Pi_X^L(x_m^L, \hat{y})$  and  $\Pi_X^R(\hat{x}, \hat{y}) \geq \Pi_X^R(x_m^R, \hat{y})$ . Inequality  $\Pi_X^R(\hat{x}, \hat{y}) \geq \Pi_X^L(x^-, \hat{y})$  is satisfied when:

$$t \geq \frac{4\tau(1-2a)}{9}.$$

■

## References

- [1] Aiura H. and Sato Y. (2008), Welfare properties of spatial competition with location-dependent costs, *Regional Science and Urban Economics*, 38 (1), 32-48.
- [2] Allain M.L. (2002), The balance of power between manufacturers and retailers: a differentiation model, *Louvain Economic Review*, 68, 359-370.
- [3] Anderson S., de Palma A. and Thisse J.-F. (1992), *Discrete choice theory of product differentiation*, MIT Press.
- [4] Anderson S. and Engers M., (1994), Spatial competition with price-taking firms, *Economica*, 61, 125-136.
- [5] Anderson S. and Neven D., (1991), Cournot competition yields spatial agglomeration, *International Economic Review*, 32, 4, 793-808.
- [6] Beladi H., Chakrabarti A. and Marjit S. (2008), Vertical mergers and downstream spatial competition with different product varieties, *Economics Letters*.
- [7] Bonanno G. and Vickers J. (1988), Vertical Separation, *The Journal of Industrial Economics* 36, 3, 257-265.
- [8] Brander J. and Spencer B. (1985), Export subsidies and international market share rivalry, *Journal of International Economics* 18, 83-100.
- [9] Brekke K. and Straume O. R. (2004), Bilateral monopolies and location choice, *Regional Science & Urban Economics* 34, 275-288.
- [10] Christou C. and Vettas N. (2005), Location choices under quality uncertainty, *Mathematical Social Sciences* 50, 268-278.
- [11] D'Aspremont C., Gabszewicz J.J. and Thisse J.-F. (1979), On Hotelling's "Stability in competition", *Econometrica* 47, 1145-1150.
- [12] Dobson P. and Waterson M. (1996), Exclusive trading contracts in successive differentiated duopoly, *Southern Economic Journal*, 63 (2), 361-377.

- [13] Fershtman C. and Judd K. (1987), Equilibrium Incentives in Oligopoly, *The American Economic Review* 77, 5, 927-940.
- [14] Gabszewicz J.J. and Thisse J.-F. (1992), Location. In: Aumann R., Hart S. (eds.), *Handbook of Game Theory*, vol. I. North-Holland, Amsterdam, 281-304 (chapter 9).
- [15] Gupta B.H., Kats A. and Pal D. (1994), Upstream monopoly, downstream competition and spatial price discrimination, *Regional Science and Urban Economics* 24, 529–542.
- [16] Hotelling H. (1929), Stability in competition, *Economic Journal* 39, 41– 57.
- [17] Inderst R. and Shaffer G. (2007), Retail mergers, buyer power and product variety, *The Economic Journal* 117, 45-67.
- [18] Lambertini L. (1997), Unicity of the equilibrium in the unconstrained Hotelling model, *Regional Science and Urban Economics* 27, 785-798.
- [19] Laussel D. (2006), Are manufacturers competing with or through supermarkets? A theoretical investigation, *Contributions to Theoretical Economics*, 6, 1, 10.
- [20] Matsumura T. and Matsushima N. (2009), Cost differential and mixed strategy equilibria in a Hotelling model, *The Annals of Regional Science*, 43, 1, 215-234.
- [21] Matsushima N. (2004), Technology of upstream firms and equilibrium product differentiation, *International Journal of Industrial Organization*, 22, 1091– 1114.
- [22] Matsushima N. (2009), Vertical mergers and product differentiation, *The Journal of Industrial Economics*, LVII, 4, 812-834.
- [23] Rey P. and Tirole J. (2007), A Primer on Foreclosure, In: Armstrong M. and Rob Porter R. (eds.), *Handbook of Industrial Organization III*.
- [24] Rey P. and Verge T. (2008), Economics of Vertical Restraints, In: Buccirossi P. (ed.), *Handbook of Antitrust Economics*, MIT Press (chapter 9).
- [25] Tabuchi T. and Thisse J.-F. (1995), Asymmetric equilibria in spatial competition, *International Journal of Industrial Organization* 13, 213-227.
- [26] Tirole J. (1989), *The Theory of Industrial Organization*, MIT Press.
- [27] Vettas N. (1999), Location as signal of quality, CEPR discussion paper No. 2165.
- [28] Ziss S. (1993), Entry deterrence, cost advantage and horizontal product differentiation, *Regional Science and Urban Economics* 23, 523– 543.