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Evgeny Zhelobodko, Sergey Kokovin, and Jacques-François Thisse

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Evgeny Zhelobodko, Novosibirsk State University
Sergey Kokovin, Novosibirsk State University
and Sobolev Institute of Mathematics
Jacques-François Thisse, Université Catholique de Louvain,
Université du Luxembourg and CEPR

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Centre for Economic Policy Research
53–56 Gt Sutton St, London EC1V 0DG, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Monopolistic Competition: Beyond the CES*

We propose a general model of monopolistic competition and derive a complete characterization of the market equilibrium based on an Arrow-Pratt measure of concavity of the utility, interpreted as the relative love for variety. When the relative love for variety increases with the consumption level, the market displays standard competitive effects. On the contrary, when it decreases, the equilibrium price increases with the number of firms and the market size, while the CES is the borderline case. Finally, we apply our setting to trade theory and uncover several new properties hindered by the CES, such as dumping and reverse dumping.

JEL Classification: D43, F12 and L13
Keywords: entry, monopolistic competition, relative love for variety and trade

Evgeny Zhelobodko
Department of Economics
Novosibirsk State University
ul.Pirogova 2
Novosibirsk, 630090
RUSSIA

Email: ezhel@ieie.nsc.ru

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Sergey Kokovin
Sobolev Institute of Mathematics
ul.Koptyuga 4
Novosibirsk, 630090
RUSSIA

Email: skokovin@math.nsc.ru

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Jacques-François Thisse
Université Catholique de Louvain
34 Voie du Roman Pays
B-1348 Louvain-la-Neuve
BELGIUM

Email: jacques.thisse@uclouvain.be

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=104425

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1 Introduction

Monopolistic competition has been used successfully in a wide range of fields, including economic growth and development, international trade, and economic geography. Although the CES utility model has been the workhorse of the vast majority of contributions based on monopolistic competition, it is fair to say that this model suffers from several major drawbacks. First, individual preferences lack flexibility since the elasticity of substitution is constant and the same across varieties. Second, the market outcome is not directly affected by the entry of new firms. In particular, markups and prices are independent of the number of competitors. This runs against empirical evidence, which shows that firms operating in bigger markets have lower markups (Syverson, 2007). Third, there is no scale effect, that is, the size of firms is independent of the number of consumers, which contradicts the fact that firms tend to be larger in larger markets (Campbell and Hopenhayn, 2005). Fourth, due to its very specific nature, the CES model yields fairly particular results, the robustness of which is not often checked. Last, in many applications, the CES utility is nested into an upper-tier Cobb-Douglas utility. This implies that expenditure shares and demands for different types of goods are independent from each other. To a large extent, these simplifying assumptions explain the success of the Dixit-Stiglitz model: the CES provides a convenient analytical tool that can be used as building-blocks in various settings.

Thus, we find it both meaningful and important to develop a more general model of monopolistic competition. The CES must be a special case of our setting to assess how our results depart from those obtained under the CES. To provide a better description of real world markets, we must also cope with the main issues highlighted in oligopoly theory. Developing such a general model and studying the properties of the market equilibrium is the main objective of this paper. To achieve our goal, we assume that preferences over the differentiated product are *additively separable* across varieties. However, unlike Dixit and Stiglitz (1977) who work mainly with a power function, we derive the properties of the market outcome for a general and unspecified utility function. Though still restrictive, we will see that additive preferences are rich enough to describe a range of market outcomes much richer than the CES. In particular, this setting will allow us to deal with various patterns of substitution through the *relative love for variety*, which is the counterpart of the measure of relative risk aversion à la Arrow-Pratt. We will show that the elasticity of substitution across varieties is the inverse of the relative love for variety, which implies that our model, unlike the CES, allows for the degree of product differentiation to be endogenous.

By ignoring strategic interactions, our setting remains tractable but sufficiently rich (i) to display a wide range of effects highlighted in industrial organization and (ii) to uncover new and unsuspected results under well-behaved utility functions. Specifically, we will show that *the market outcome depends on how the relative love of variety*, hence the elasticity of substitution, *varies with the consumption level.* To be more precise, the market outcome may obey two opposite patterns. On the one hand, when the relative love for variety increases with consumption, the equilibrium displays the standard *pro-competitive* effects generated by the entry of new firms and a larger size of the market, two effects that the CES does not apprehend: more firms, a larger market size, or both lead to *lower* market prices. On the other hand, when the relative love for variety decreases, the equilibrium displays an *anti-competitive* behavior, meaning that the entry of new firms, a larger market, or both lead to *higher* prices. Although at odds with the standard paradigm of entry, this result agrees with several contributions in product differentiation theory (Amir and Lambson, 2000; Chen and Riordan, 2007, 2008) as well as with empirical studies showing that entry or economic integration may lead to higher markups (Ward *et al.*, 2002; Badinger, 2007). It should not be viewed, therefore, as an exotica. What our paper adds to the literature is the idea that anti-competitive behavior need not be driven by defence strategies: it may result from the nature of preferences with utility functions which are otherwise well-behaved. We also want to stress that the CES is the dividing line between those two classes of utility functions since it does not display any of these effects. In addition, though the mass of firms always increases with the size of the market, it does so less than proportionally in the pro-competitive case and more than proportionally in the anti-competitive one. Last, whereas most existing papers focus on a single monopolistically competitive sector, we show that our main results can be extended to a multi-sector economy under fairly mild assumptions on the upper-tier utility. Therefore, our analysis is consistent with the idea that, though most sectors of the economy are probably pro-competitive, a few ones may be anti-competitive.

Our research strategy has also empirical appeal because it provides theoretical predictions that are sufficiently simple to be tested, sufficiently general to make sense on an empirical level, and precise enough to allow one to discriminate between different explanations. Furthermore, our approach also sheds new light on models that are commonly used in the empirics of trade. In particular, our analysis shows that a single market equilibrium, which leads to a specific value of the elasticity of substitution, can be rationalized by a CES model yielding this equilibrium. However, this does not mean that the CES can be used without questioning its relevance in studies comparing several markets and/or periods. Indeed, even when the CES provides a good approximation of preferences for a particular dataset, one may expect very different estimates of the elasticity of substitution to be obtained with different datasets. We need not assume changing preferences to rationalize this difference. It is sufficient to assume that elasticity of substitution across varieties varies with the consumption level.

Having this in mind, we apply our setting to international trade with the aim to uncover new results. Unlike what is known under the CES, we show that, depending on preferences, firms' pricing exhibits richer behaviors such as dumping (Brander and Krugman, 1983), reverse dumping (Greenhut *et al.*, 1987), or both. Recent empirical studies support the idea that, in several manufacturing sectors, French firms would adopt reverse dumping (Martin, 2009).

The idea of additive preferences is not really new since it goes back at least to Houtakker (1960), who introduced this specification precisely because it provides new impetus to empirical analysis. Using the same preference structure, Spence (1976) and Vives (1999, ch.6) have derived equilibrium conditions similar to ours. However, their main purpose is different from what we accomplish in this paper since their aim is to compare the free entry equilibrium and the social optimum. Our model also share several similarities with Krugman (1979) who shows how decreasing demand-elasticity yields what we call pro-competitive markets. Yet, Krugman did not explore the market implications of his model, perhaps because his purpose was different from ours. His approach has been ignored in subsequent works by trade theorists. As observed by Neary (2004, p.177), this is probably because Krugman's specification of preferences "has not proved tractable, and from Dixit and Norman (1980) and Krugman (1980) onwards, most writers have used the CES specification." Instead, we show that Krugman's approach is *tractable*. To be precise, by using the elasticity of the marginal utility, we can provide a complete characterization of the market outcome and of all the comparative statics implications in terms prices, consumption level, outputs, and mass of firms/varieties.[1]

The paper is organized as follows. The next section presents the model in the case of a one-sector economy. We characterize the short-run equilibrium in which the mass of firms is exogenous and the long-run equilibrium in which the mass of firms is determined by free entry and exit. In Section 3, we extend our results to the case of a multi-sector economy with a general upper-tier utility, as suggested by Dixit and Stiglitz (1977). In Section 4, we apply our approach to a new trade theory setting and derive the properties of firms' pricing behavior. Section 5 concludes.

[1]After completion of this paper, K. Behrens brought to our attention the paper by Bertoletti et al. (2008). Their analysis starts from the same primitives but does not go as far as us.

2 The one-sector economy

2.1 The basic model

The economy involves one differentiated good and one production factor - labor. There are L workers and each supplies E units of labor. Labor is chosen as the numéraire so that E is both a worker's income and expenditure. The differentiated good is made available as a continuum N of horizontally differentiated varieties indexed by $i \in [0, N]$ (endowed with the Lebesgue measure). They are provided by monopolistically competitive (hereafter, MC) firms. Each firm produces a single variety and no two firms sell the same variety. To operate every firm needs a fixed requirement $f > 0$ and a marginal requirement $c > 0$ of labor, so that the production cost of a firm supplying the quantity q is equal to $f + cq$.

Preferences. Consumers' preferences are additively separable (we discuss in the concluding section the case of non-separable preferences). Given a (measurable) price function $\mathbf{p} = p_{i \leq N}$ and an expenditure value E, every consumer chooses a (measurable) consumption function $\mathbf{x} = x_{i \leq N}$ to maximize her utility subject to the budget constraint:

$$\max_{x(\cdot)} \mathcal{U} \equiv \int_0^N u(x_i)\mathrm{d}i \qquad \text{s.t.} \quad \int_0^N p_i x_i \mathrm{d}i = E$$

where $u(\cdot)$ is a thrice continuously differentiable, strictly increasing and strictly concave function.[2] Note that preferences are not homothetic when u is not a power function.

The assumptions made on the utility u imply that a consumer displays a love for variety. Let indeed $Q > 0$ be any given quantity of the differentiated good. If she consumes the same number Q/n units of each variety $i \in [0, Q/n]$ with $n < N$, the consumer enjoys the utility level given by $\mathcal{U}(n; Q) = nu(Q/n) + (N - n)u(0)$. Note that $u(0) \neq 0$ implies that increasing the number of varieties affects the consumer's well-being even when she does not change her consumption pattern. This does not strike us as being plausible. For this reason, we assume from now on that $u(0) = 0$. That said, it is readily verified that $nu(Q/n)$ is a strictly increasing function of n under the assumptions made on u. Consequently, rather than concentrating her consumption over a small mass of varieties, the consumer prefers to spread it over the whole range of available varieties ($n = N$). This implies that our setting does not impose any additional restriction on u for consumers to exhibit a love for variety.

All of this has the following implication: individual consumption in the theory of monopolistic compe-

[2] We do not include firms' profits into the budget constraint for the following two reasons. First, when the mass of firms is fixed, each firm accurately treats the total profits parametrically because it is negligible. Second, when the mass of firms is determined by free entry, total profits are zero.

tition with love for variety is formally equivalent to individual decision-making in the Arrow-Pratt theory of risk aversion, the mix of risky assets being replaced with the mix of differentiated varieties. This will allow us to derive properties of firms' demands that are both intuitive and simple. More precisely, *the* key-concept for our study of monopolistic competition is what we call the *relative love for variety* (hereafter, RLV)

$$r_u(x_i) \equiv -\frac{x_i u''(x_i)}{u'(x_i)} > 0. \tag{1}$$

Under the CES, we have

$$u(x_i) = x_i^\rho$$

where ρ is a constant such that $0 < \rho \leq 1$, thus implying a constant RLV:

$$r_u(x_i) = 1 - \rho.$$

Behrens and Murata (2007) retain the CARA utility $u(x) = 1 - \exp(-\alpha x)$ where $\alpha > 0$ is the absolute love for variety, so that the corresponding RLV, i.e. $r_u = \alpha x$, increases with the consumption level.

To better understand the economic meaning of r_u, it turns out to be useful to evaluate it along the diagonal in the quantity space ($x_i = x$). Using the definition of the elasticity of substitution σ (see, e.g. Nadiri, 1982, p.442), we obtain

$$r_u(x) = \frac{1}{\sigma(x)}. \tag{2}$$

Thus, at a symmetric consumption pattern, *the RLV is the inverse of the elasticity of substitution across varieties*. However, unlike the CES where the elasticity of substitution is exogenous and constant, the value of σ varies here with the consumption level x or, equivalently, with the price level and the mass of varieties, as in the translog utility (Feenstra, 2003). In other words, a higher consumption of the differentiated product makes consumers' love for variety stronger when the RLV is increasing, while the love for variety gets weaker when the RLV is decreasing. This is because consumers' preferences for more balanced bundles of varieties become, respectively, stronger or weaker. Both schemes seem a priori plausible, which means that it is hard to make predictions about the behavior of the RLV without appealing to empirical studies.[3]

[3] The expression of the elasticity of substitution is more involved when the consumption pattern is asymmetric. Therefore, when firms are heterogeneous, the equilibrium value of σ varies with the two varieties under consideration when their consumption levels are different.

Demand. To determine the equilibrium consumption, we differentiate the Lagrangian

$$\mathcal{U} + \lambda \left[E - \int_0^N p_i x_i \mathrm{d}i \right]$$

with respect to x_i and get

$$u'(x_i) = \lambda p_i \tag{3}$$

where the Lagrange multiplier λ is determined by the price function $p(\cdot)$, the mass of varieties N, and the expenditure E. In other words, *the marginal utility of income captures all the market ingredients that matter to consumers* (and firms), very much as the price index does in the Dixit-Stiglitz model.

Clearly, the consumer's inverse demand

$$p_i(x_i) = u'(x_i)/\lambda \tag{4}$$

is univocally determined. Because the Lagrange multiplier acts only as a scaling factor, this expression implies that the inverse demand and the marginal utility display the same properties. In particular, $p_i(x_i)$ is strictly decreasing in x_i. Similarly, setting $\varphi \equiv (u')^{-1}$, we obtain the consumer's direct demand for variety i:

$$x_i(p_i) = \varphi(\lambda p_i).$$

Unlike Dixit and Stiglitz (1977), we do not assume that u, hence φ, takes a specific functional form. Instead, we will keep u unspecified. Since $\varphi' = 1/u'' < 0$, x_i is strictly decreasing in p_i.

Since there is a continuum of varieties, the consumption level of a single variety j has negligible impact on a consumer's utility. Thus, changing the price of variety j does not affect the i-s demand for $i \neq j$. Consequently, the optimal consumptions before and after the price change are (almost everywhere) the same. Therefore, the price choice made by firm j has no impact on firm i's demand function:

$$\frac{\partial x_i}{\partial p_j} = 0 \qquad \text{for } j \neq i$$

which means that *the individual demand for a variety depends only upon its price and the marginal utility of income*. It is readily verified that the elasticity of the inverse demand is equal to the RLV. Consequently, the price-elasticity of demand is equal to the inverse of the RLV:

$$\varepsilon_i(p_i) \equiv -\frac{p_i}{x_i}\frac{\partial x_i}{\partial p_i} = \frac{1}{r_u[x_i(p_i)]} \tag{5}$$

which is independent of the value of the multiplier λ though preferences are not quasi-linear (see also Appendix A). Therefore, *a stronger* (resp., weaker) *love for variety generates less* (resp., more) *elastic demands.* This is because a stronger love for variety induces consumers to focus more on a balanced mix of varieties, which in turn makes the demands for these varieties less sensitive to changes in their relative prices. This relationship builds the link with Krugman (1979) who assumes the price-elasticity to be decreasing, i.e. the RLV is upward sloping. Furthermore, (2) and (5) imply that, along the diagonal in the quantity space, the elasticity of substitution among varieties is equal to the price-elasticity of a variety's demand, as in the CES case (Vives, 1999). However, unlike the CES, this relationship ceases to hold off-diagonal, while both σ and ε_i vary with the common consumption level x.

Producers. Since all consumers face the same multiplier, the functional form of the demand for variety i is the same across consumers, which implies that the market demand is given by Lx_i. Because each firm accurately treats $\lambda(\cdot)$ as a parameter, it behaves like a monopolist on its market. Hence, maximizing profits with respect to price or quantity yields the same equilibrium outcome. Using (4), the profit function may be rewritten as follows:

$$\pi(x_i; x(\cdot), E) = \left[\frac{u'(x_i)}{\lambda(\cdot)} - c\right] Lx_i - f.$$

For any given value of $\lambda(\cdot)$, there *exists* a maximizer if the following two conditions hold:

$$\lim_{z\to 0}[u'(z) + zu''(z)] = \infty \qquad \lim_{z\to\infty}[u'(z) + zu''(z)] \leq 0. \tag{6}$$

Indeed, the marginal revenue curve intersects at least once the horizontal line $\lambda(\cdot)c$. Moreover, this maximizer is *unique* if $\pi(x_i; x(\cdot), E)$ is strictly concave with respect to x_i. Expressed in terms of the utility u, it is readily verified that this condition is equivalent to

$$r_{u'}(x_i) = -\frac{x_i u'''(x_i)}{u''(x_i)} < 2. \tag{7}$$

In words, this implies that inverse demands cannot be too convex. Throughout the rest of this paper, we assume that the conditions (6)-(7) hold.

As will be seen below, we need an additional condition on $r_u(0)$. It is well known that the strict concavity of profits means that the marginal revenue is strictly decreasing. Consequently, for a given utility u, two cases may arise. First, the equation $u^{'}(z) + zu''(z) = 0$ has no solution. Using (6), it must be that $r_u(z)$ is smaller than 1 for all $z > 0$. Second, the equation $u^{'}(z) + zu''(z) = 0$ has a solution z_0. Then, we have $r_u(z) < 1$ for $z < z_0$ and $r_u(z) > 1$ for $z > z_0$. In this case, we may restrict ourselves to the interval $(0, z_0)$ in which $r_u(z) < 1$ since all equilibria belong to $(0, z_0)$. Without loss of generality, we may then assume that $r_u(z) < 1$ for all relevant values of $z > 0$. In what follows, we will assume a somewhat stronger condition:

$$r_u(z) < 1 \qquad \text{for all } z \geq 0. \tag{8}$$

We are now ready to characterize the equilibria. In order to disentangle the various effects at work, it is both relevant and convenient to distinguish between what we call a *short-run equilibrium*, in which the mass N of firms is fixed, and a *long-run equilibrium* in the which the mass of firms is endogenously determined through free entry and exit.

2.2 The short-run equilibrium

Let $q_i \equiv Lx_i$ be firm i's output. Given the mass N of firms, $\bar{\mathbf{q}} = \bar{q}_{i \leq N}$ is a *short-run equilibrium* if no firm finds it profitable to change unilaterally its output while anticipating accurately the value of $\lambda(\cdot)$. For reasons that will become clear below, we focus on a symmetric equilibrium in which $\bar{x}_i = \bar{x}(N)$ for (almost) all $i \in [0, N]$. The resulting equilibrium price $\bar{p}$ then follows from (4).

It is readily verified that the first-order condition for firm i's profit maximization may be written as follows:

$$\bar{M} \equiv \frac{\bar{p} - c}{\bar{p}} = r_u(\bar{x}). \tag{9}$$

Hence, in equilibrium, *the mark-up of a firm is equal to the RLV.* Since $r_u(\bar{x}) < 1$, the elasticity of substitution must exceed 1 at any symmetric equilibrium. Furthermore, since all firms face the same Lagrange multiplier $\lambda(\cdot)$, the solutions to the first-order condition are the same across firms, which implies that all solutions (if any) are such that all prices are equal. Hence, if a short-run equilibrium exists, it is unique and symmetric.

The condition (9) and the budget condition imply the following two equilibrium equations:

$$1 - \frac{c}{\bar{p}} = r_u\left(\frac{E}{N\bar{p}}\right) \qquad \bar{x} = \frac{E}{N\bar{p}}. \tag{10}$$

Both sides of the left hand expression are continuous. Furthermore, since $l_1(p) \equiv 1 - c/p$ increases from 0 to 1 on $[c, \infty)$, it intersects $l_2(p) \equiv r_u(E/Np)$ at some $p > c$ if the two conditions $l_2(c) > 0$ and $l_2(\infty) < 1$ hold. The latter condition is equivalent to (8). Under this condition together with (6) and (7), the equations (10) have a unique and positive solution, which is the short-run equilibrium.

The impact of N. Differentiating the equilibrium condition (10) with respect to N leads to the expression

$$\left(\frac{c}{\bar{p}} + \frac{E}{N\bar{p}}r_u'\right)\frac{N}{\bar{p}}\frac{d\bar{p}}{dN} = -\frac{E}{N\bar{p}}r_u'.$$

Plugging the solution of (10) with respect to $c/\bar{p}$ and $\bar{x} = E/N\bar{p}$ into this expression yields

$$(1 - r_u + \bar{x}r_u')\frac{N}{\bar{p}}\frac{d\bar{p}}{dN} = -\bar{x}r_u'. \tag{11}$$

Differentiating (1) with respect to x_i, we obtain the identity:

$$\bar{x}r_u' = (1 + r_u - r_{u'})r_u. \tag{12}$$

Substituting for $\bar{x}r_u'$ into the left-hand side of (11), we get

$$1 - r_u + \bar{x}r_u' = 1 + r_u^2 - r_u r_{u'} > (1 - r_u)^2 \geq 0$$

where we have used (7). Therefore, (11) implies that $d\bar{p}/dN$ and $r_u'(\bar{x})$ have opposite signs. Consequently, we have:

Proposition 1 *If (6)-(8) hold, then there exists a unique and symmetric short-run equilibrium. Furthermore, when the relative love for variety increases (resp., decreases) with the consumption level, then the equilibrium price decreases (resp., increases) with the mass of firms. The equilibrium price is independent of the mass of firms if and only if the utility is given by a CES.*

When the RLV increases with the consumption level, the entry of new firms leads to a lower equilibrium price. This is the standard *pro-competitive* effect generated by entry, which works here as follows. Since $x = E/Np$, the consumption x evaluated at $\bar{p}$ decreases with N. Hence, $r_u' > 0$ and (2) implies a lower elasticity of substitution, hence a higher mark-up. The market is, therefore, more competitive and the equilibrium price lower. In contrast, when the RLV decreases with the consumption level, the entry of new firms leads to a higher equilibrium price. In this case, entry gives rise to an *anti-competitive* effect.

This unexpected result can be explained by reversing the above argument. The individual consumption being decreasing with N, varieties become more differentiated since $r'_u < 0$. This makes the market less competitive, which yields a higher price. The CES is the only function that has a constant RLV. It is, therefore, the only utility for which entry does not impact on the equilibrium price. Hence, we may safely conclude that the CES is the borderline between two very different, classes of utility functions.

This difference in results may be understood as follows. We have seen that the multiplier λ increases with N. Using (4) it is then readily verified that the entry of firms shifts down proportionally the incumbents' inverse demands. Figure 1 shows how the inverse demands change when λ increases from 1 to 2. In the left-handed panel, the marginal revenue curve is sufficiently flat for the equilibrium prices to go down. In contrast, in the right-handed panel, the stronger curvature of the inverse demand implies that the equilibrium price increases with N. Thus, whether the market is pro- or anti-competitive depends on the curvature of the inverse demands relative to the one of the CES-demand.

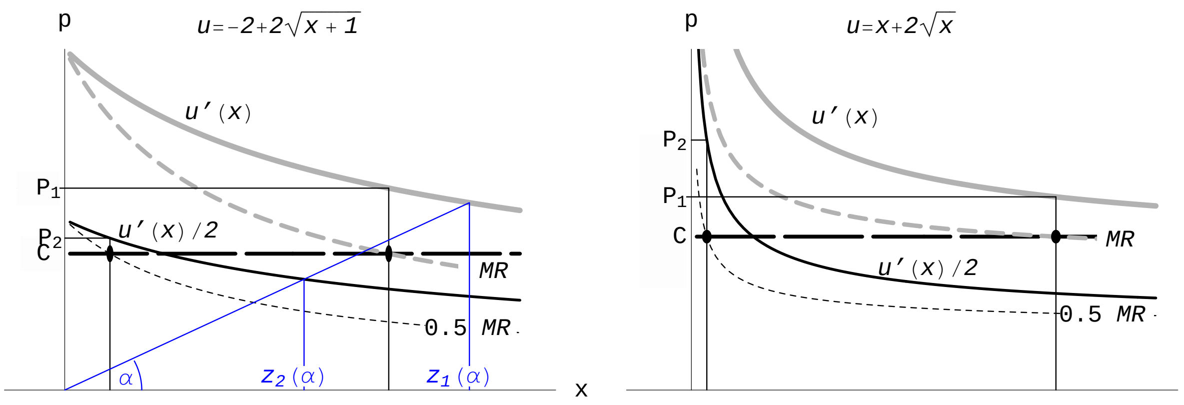

Figure 1: The impact of entry on pro- and anti-competitive demands.

Formally, curvature is expressed by the concept of radial convexity. Consider two positive and decreasing functions $f_1(x)$ and $f_2(x)$ and let $x_i(\alpha)$ be the solution to $f_i(x_i) = \alpha x_i$ for $\alpha \in (0, \pi/2)$ so that α and x_i move in opposite directions. We say that f_1 is *more radially convex* that f_2 if $d[f'_1(x_1(\alpha))/f'_2(x_2(\alpha))]/d\alpha > 0$ holds for all $\alpha \in (0, \pi/2)$. In words, the curvature of f_2 is stronger than that of f_1 when the ratio of the slopes of the two functions increases with the slope of the radius. Let $p_1(z)$ and $p_2(z)$ be two inverse demand functions (in Figure 1 $p_1(z) = u\prime(z)$ and $p_2(z) = u\prime(z)/2$ have same radial convexity). For any $\alpha \in (0, \pi/2)$, the corresponding radius intersects these two demands curves at $z_1(\alpha)$ and $z_2(\alpha)$ which are such that

$$\frac{p_1[z_1(\alpha)]}{z_1(\alpha)} = \frac{p_2[z_2(\alpha)]}{z_2(\alpha)} = \alpha.$$

Multiplying this expression by $p_1'[z_1(\alpha)]/p_2'[z_2(\alpha)]$ and using (5), we obtain

$$r_1[z_1(\alpha)]/r_2[z_2(\alpha)] = e_1[z_1(\alpha)]/e_2[z_2(\alpha)] = p_1'[z_1(\alpha)]/p_2'[z_2(\alpha)].$$

When $r_2(z)$ is constant, $r_1(z)$ is increasing (resp., decreasing) if and only if p_1 is less (resp., more) radially convex than the inverse isoelastic demand. Thus, whether the market is pro- or anti-competitive depends on the curvature of varieties' demand relative to the curvature of the CES-demand.

It remains to show that both the pro- and anti-competitive outcomes may be generated by well-behaved utility functions. To this end, we consider the following class of parametrized utility functions, called the "augmented-HARA," which enables us to derive both the pro- and anti-competitive outcomes:

$$u(x) = \frac{1}{\rho}\left[(a + hx)^\rho - a^\rho\right] + bx \tag{13}$$

where $a \geq 0$, $h > 0$, $b \geq 0$, and $0 < \rho < 1$. This expression boils down to the CES for $a = b = 0$, while it is equivalent to a standard HARA utility when $b = 0$. Setting (i) $a = 1$, $h = 1$, $b = 0$, $\rho = 1/2$, and (ii) $a = 0$, $h = 1$, $b = 1$, $\rho = 1/2$ in (13), we get

$$u_1(x) = 2\sqrt{x+1} - 2 \qquad u_2(x) = 2\sqrt{x} + x$$

which imply

$$r_{u_1}(x) = \frac{1}{2(1 + 1/x)} \qquad r_{u_2}(x) = \frac{1}{2(1+\sqrt{x})}.$$

Clearly, the former increases with x whereas the latter decreases, which means that the market outcome is pro-competitive under u_1 and anti-competitive under u_2. For simplicity, we have chosen to express the equilibrium prices through their inverse $N(\bar{p}_i)$ for $i = 1, 2$ in which E is normalized to 1. Under u_1, we have

$$N(\bar{p}_1) = \frac{2c - \bar{p}_1}{2\bar{p}_1(\bar{p}_1 - c)}$$

which is decreasing in $\bar{p}_1$ over the interval $(c, 2c)$, so that $\bar{p}_1(N)$ also decreases with N. Note that $\bar{p}_1$ is always smaller than $2c$ for a positive mass of firms to be active and tends to c when N tends to infinity.

Under u_2, we obtain

$$N(\bar{p}_2) = \frac{4(\bar{p}_2 - c)^2}{\bar{p}_2(\bar{p}_2 - 2c)^2}$$

which is also defined over $(c, 2c)$. Differentiating this expression shows that $N(\bar{p}_2)$ is increasing on this interval. Consequently, the equilibrium price decreases with N under u_1, whereas it increases under u_2.

2.3 The long-run equilibrium

A symmetric *long-run equilibrium* is defined by a mass of firm $\bar{N}$ and a symmetric equilibrium $\bar{x}$ such that firms earn zero profits:

$$(\bar{p} - c)L\bar{x} = f.$$

This shows that the equilibrium outcome depends on the "relative" market size $\tilde{L} \equiv L/f$, so that comparative statics in terms of $\tilde{L}$ allows one to capture shocks in both population and technology. Using the zero-profit condition and (9), we find the equilibrium consumption of each supplied variety:

$$\bar{x} = \frac{1}{c\tilde{L}}\left(\frac{1}{\bar{M}} - 1\right). \tag{14}$$

Furthermore, using the zero-profit condition and the budget constraint, we obtain

$$\frac{E}{\bar{N}} = \frac{f}{L\bar{M}} = \frac{1}{\tilde{L}\bar{M}}.$$

The conditions for a long-run equilibrium may then be written as follows.

Proposition 2 *Every symmetric long-run equilibrium must satisfy the following two conditions:*

$$\bar{M} = r_u\left[\frac{1}{c\tilde{L}}\left(\frac{1}{\bar{M}} - 1\right)\right] \tag{15}$$

$$\bar{N} = E\,\tilde{L}\bar{M}. \tag{16}$$

To illustrate, we go back to the above examples of pro-competitive utility u_1 and anti-competitive utility u_2 and determine the corresponding markups:

$$\bar{M}_1 = \frac{2}{\sqrt{8c\tilde{L}+1}+3} \qquad \bar{M}_2 = \frac{1}{2}\left(1 - \frac{1}{\sqrt{c\tilde{L}+1}}\right).$$

It is readily verified that the equilibrium prices are, respectively, decreasing and increasing with $\tilde{L}$.

The following remarks are in order. First, since (15) depends only upon the profit margin $\bar{M}$, the equilibrium conditions do not form a system of simultaneous equations. Solving (15) for $\bar{M}$ and plugging the solution into (16) yields the unique equilibrium value $\bar{N}$. Second, whatever the functional form of the utility u, the equilibrium price does not depend on the expenditure level E. This in turn implies that the equilibrium price prevailing in the MC-sector is independent of how the other sectors (if any) behave. This is because the price-elasticity of a firm's demand, hence this firm's profit-maximizing price, depends only upon the degree of love for variety within the MC-sector. We will explore in Section 3 some implications of this property. Last, the two parameters describing the size of the MC-sector, i.e. E and L, do not play the same role in determining the market outcome because $\bar{M}$ is independent of E, whereas $\bar{M}$ varies with L.

In the two sub-sections below, we study the impact of a change in the relative market size on the symmetric long-run equilibrium.

2.3.1 The impact of cost and market size

Price. Taking the total differential of (15) with respect to $\tilde{L}$, solving for $d\bar{M}/d\tilde{L}$ and multiplying both sides by $\tilde{L}/\bar{M}$, we obtain:

$$\frac{\tilde{L}}{\bar{M}}\frac{d\bar{M}}{d\tilde{L}} = -\frac{\bar{x}}{r_u + \frac{r'_u}{\tilde{L}r_u}}r'_u = -\frac{\bar{x}\,(1-r_u)}{r_u(1-r_u)+r'_u\bar{x}}r'_u = -\frac{\bar{x}\,(1-r_u)}{(2-r_{u'})\,r_u}r'_u \tag{17}$$

$$= \frac{\bar{x}(\bar{M}-1)}{(2-r_{u'})\,r_u}r'_u = (\bar{M}-1)\frac{1+r_u-r_{u'}}{2-r_{u'}} > \bar{M}-1 \tag{18}$$

where we have used successively (14), (15), (12), (11), and (8).

It follows from (15) that

$$\frac{\tilde{L}}{\bar{M}}\frac{d\bar{M}}{d\tilde{L}} = \frac{L}{\bar{M}}\frac{d\bar{M}}{dL} = -\frac{f}{\bar{M}}\frac{d\bar{M}}{df} = \frac{c}{\bar{M}}\frac{d\bar{M}}{dc}. \tag{19}$$

Since $r_u < 1$ and $r_{u'} < 2$, it follows from (17) that $d\bar{M}/d\tilde{L}$ and r'_u have opposite signs. Therefore, as in the foregoing, three cases may arise according to the sign of r'_u. For example, when $r'_u > 0$, the equilibrium mark-up decreases with $\tilde{L}$. As a result, the equilibrium price falls when the population size L increases, the level of fixed cost f decreases, or both. These effects are expected because a larger L or a smaller f fosters entry, which here leads to a lower market price. This corresponds to the standard pro-competitive effect generated by a bigger market. In contrast, when $r'_u < 0$, we fall back on the anti-competitive case uncovered in the above section.

Last, we show in Appendix B that a higher marginal cost always leads to a higher price. In addition, $r'_u > 0$ (resp., $r'_u < 0$) implies that a higher marginal cost leads to a less (resp., more) than proportional increase in market price. In other words, the pass-on varies with the RLV.

Industry size. To study how the size of the MC-sector changes with the structural parameters, we differentiate the equilibrium condition (16). Using (18), we then obtain

$$\frac{\tilde{L}}{\bar{N}}\frac{d\bar{N}}{d\tilde{L}} = 1 + \frac{\tilde{L}}{\bar{M}}\frac{d\bar{M}}{d\tilde{L}} > \bar{M} > 0.$$

Hence, regardless of the sign of r'_u, the equilibrium mass of firms is always an increasing function of the size of the economy. Thus, the RLV does not affect the pro-entry effect generated by a larger market. However, it affects the way this pro-entry effect reacts to market size: the above elasticity is smaller than 1 if and only if $r'_u > 0$, , which means that $\bar{N}(L)$ grows at a decreasing rate. In other words, when consumers display an increasing (resp., decreasing) RLV, a growing population enjoy a larger but less (resp., more) than proportionate mass of varieties. As it should now be expected, the mass of varieties grows linearly with L if and only if the utility is given by the CES:

$$\bar{N} = (1-\rho)\frac{L}{f}. \tag{20}$$

How does $\bar{N}$ react to the cost parameters? Since increasing f is tantamount to decreasing $\tilde{L}$, $\bar{N}$ must decreases with f. Furthermore, as shown in Appendix, a higher marginal cost leads to a larger mass of firms if and only if $r'_u > 0$.

Consumption and output. The impact of the above parameters can be obtained in a similar way by differentiating the corresponding equilibrium conditions (see Appendix B for more details). It is worth to single out two results. First, *the consumption of each variety always falls when the size of the economy rises*, the reason being that consumers prefer to spread their consumption over the wider range of varieties that results from the entry of new firms. Second, despite the larger mass of competitors, *a growing population induces each firm to produce more if and only if the RLV is increasing.* Again, this is because the entry of new firms leads to a lower market price.

2.3.2 Synthesis

Our results are summarized in the following two propositions.

Proposition 3 *The impact of the relative market size $\widetilde{L} \equiv L/f$ on the symmetric long-run equilibrium is as follows:*

	$r_u'(x) > 0$	$r_u'(x) = 0$	$r_u'(x) < 0$
$\mathcal{E}_{\bar{p}/\tilde{L}} \equiv \frac{\tilde{L}}{\bar{p}} \frac{d\bar{p}}{d\tilde{L}}$	$\downarrow : -\bar{M} < \mathcal{E}_{\bar{p}/\tilde{L}} < 0$	$o\ : \mathcal{E}_{\bar{p}/\tilde{L}} = 0$	$\uparrow : 0 < \mathcal{E}_{\bar{p}/\tilde{L}} \lesseqqgtr 1$
$\mathcal{E}_{\bar{N}/\tilde{L}} \equiv \frac{\tilde{L}}{\bar{N}} \frac{d\bar{N}}{d\tilde{L}}$	$\uparrow : 0 < \mathcal{E}_{\bar{N}/\tilde{L}} < 1$	$\uparrow : \mathcal{E}_{\bar{N}/\tilde{L}} = 1$	$\uparrow : 1 < \mathcal{E}_{\bar{N}/\tilde{L}}$
$\mathcal{E}_{\bar{x}/\tilde{L}} \equiv \frac{\tilde{L}}{\bar{x}} \frac{d\bar{x}}{d\tilde{L}}$	$\downarrow : -1 < \mathcal{E}_{\bar{x}/\tilde{L}} < 0$	$\downarrow : \mathcal{E}_{\bar{x}/\tilde{L}} = -1$	$\downarrow : \mathcal{E}_{\bar{x}/c} < -1$
$\mathcal{E}_{\bar{q}/\tilde{L}} \equiv \frac{\tilde{L}}{\bar{q}} \frac{d\bar{q}}{d\tilde{L}}$	$\uparrow : 0 < \mathcal{E}_{\bar{q}/\tilde{L}} < 1$	$o\ : \mathcal{E}_{\bar{q}/\tilde{L}} = 0$	$\downarrow : -1 \lesseqqgtr \mathcal{E}_{\bar{q}/\tilde{L}} < 0$

Again, we see that *what determines the properties of the market outcome is the variety-loving attitude of consumers.* That said, the following comments are in order. First, regarding the impact of market size on the equilibrium price, it is worth noting that *the long-run equilibrium inherits the pro- and anti-competitive properties of the short-run equilibrium.* To understand why, we observe that the size $\bar{N}$ of the industry always grows with L. Indeed, when there are more consumers, the profits of incumbents increase, thus attracting new firms. According to the sign of r_u', such an entry leads to a lower or higher market price. When $r_u' > 0$, the decrease in market price slows down the entry of new firms. However, since the elasticity of N with respect to L is smaller than 1, this negative feedback effect cannot outweigh the initial increase in N. Consequently, the market price is established at a level lower than the initial one. On the contrary, when $r_u' < 0$, the feedback effect is positive. This further pushes N upward, making the elasticity of N bigger than 1, which in turn yields a higher market price. Yet, this price does not become arbitrarily large because the individual consumption of each variety decreases at a rate exceeding 1. Under the CES, there is no feedback effect because the market price is unaffected by the entry of new firms. Thus, using the CES as a benchmark, when the market size grows, consumers face a smaller range of varieties and lower prices when they display an increasing RLV. In contrast, the range of varieties is wider and prices are higher under a decreasing RLV than under the CES.

To shed further light on the role of market size, it is worth investigating how the equilibrium value $\bar{\lambda}$ of the Lagrange multiplier changes with L. It follows immediately from (4) and (9) that

$$\bar{\lambda} = \frac{u'(\bar{x})[1 - r_u(\bar{x})]}{c} = \frac{u'(\bar{x}) + \bar{x}u''(\bar{x})}{c}.$$

Differentiating the numerator of this expression and using (7) shows that $\bar{\lambda}$ strictly decreases with $\bar{x}$. Proposition 4 then implies that, regardless of u, *a larger economy generates a higher marginal utility of income*. This a priori unsuspected result may be explained as follows. While consumers buy a large amount of each variety in a small economy, they buy a smaller amount in a large economy because they face a wider range varieties. This lower consumption makes income more valuable in the large economy than in the small one.

Another peculiar feature of the CES is that the equilibrium size of firms ($\bar{q}$) is independent of the market size. Our results show that *firms' size increases in the pro-competitive case.* This is because the industry size grows at a lower pace than the market size while prices go down. On the contrary, firms' size decreases in the anti-competitive case because the mass of firms increases at a more than proportionate rate and charge a higher price. These effects combine to yield a lower output. As expected, lowering the fixed cost is equivalent to raising market size.

Determining the impact of marginal cost is less straightforward.

Proposition 4 *The impact of marginal cost c on the long-run symmetric equilibrium is as follows:*

	$r'_u(x) > 0$	$r'_u(x) = 0$	$r'_u(x) < 0$
$\mathcal{E}_{\bar{p}/c} \equiv \frac{c}{\bar{p}} \frac{d\bar{p}}{dc}$	$\uparrow : 0 < 1 - \bar{M} < \mathcal{E}_{\bar{p}/c} < 1$	$\uparrow : \mathcal{E}_{\bar{p}/c} = 1$	$\uparrow : 1 < \mathcal{E}_{\bar{p}/c}$
$\mathcal{E}_{\bar{N}/c} \equiv \frac{c}{\bar{N}} \frac{d\bar{N}}{dc}$	$\downarrow : -(1 - \bar{M}) < \mathcal{E}_{\bar{N}/c} < 0$	$o\ : \mathcal{E}_{\bar{N}/c} = 0$	$\uparrow : 0 < \mathcal{E}_{\bar{N}/c}$
$\mathcal{E}_{\bar{x}/c} \equiv \frac{c}{\bar{x}} \frac{d\bar{x}}{dc}$	$\downarrow : -1 < \mathcal{E}_{\bar{x}/c} < 0$	$\downarrow : \mathcal{E}_{\bar{x}/c} = -1$	$\downarrow : \mathcal{E}_{\bar{x}/c} < -1$
$\mathcal{E}_{\bar{q}/c} \equiv \frac{c}{\bar{q}} \frac{d\bar{q}}{dc}$	$\downarrow : -1 < \mathcal{E}_{\bar{q}/c} < 0$	$\downarrow : \mathcal{E}_{\bar{q}/c} = -1$	$\downarrow : \mathcal{E}_{\bar{q}/c} < -1$

In the CES case, increasing the marginal cost leaves the markup unchanged. In contrast, in the pro-competitive (resp., anti-competitive) case, the markup decreases (resp., increases) because the market price increases less (resp., more) than proportionally. This in turn fosters the exit (resp., entry) of firms. Consequently, a technological change affects the degree of diversity in opposite directions. This discrepancy in results should be useful in empirical studies to distinguish between the two competition regimes.

In addition, under CES preferences, consumers always benefit from a larger market because prices remain constant while more varieties are available. This positive size effect is reinforced in the pro-competitive case because prices go down while the market supplies more varieties. Hence consumers are better-off. In the anti-competitive case, the impact of market size on welfare is not so clear. Indeed, although more varieties are still available, they are priced at a higher level. Therefore, if the equilibrium price increases at a much higher rate than the mass of varieties, one may expect the welfare level to decrease with the size of the market. Indeed, there exist well-behaved utility functions such that a growing market size is detrimental to consumers.

Before proceeding, we want to make a pause and discuss further the relevance of the CES in empirical works using the monopolistic competitive setting. In equilibrium, the RLV is equal to the inverse of the elasticity of substitution. Consequently, *one can rationalize the use of the CES once the value of* $\sigma(\bar{x}) = 1/r_u(\bar{x})$ *evaluated at the market outcome is known.* In other words, for any symmetric long-run equilibrium obtained within our framework, there exists a CES model that yields the same market outcome. Yet, this does not mean that the CES can be used doubtlessly in empirical analyses. In order to estimate a model with cross-section or panel dataset, we need data heterogeneity stemming from variations in the underlying structural parameters such as the relative market size L/f. Once we allow for such variations, Proposition 3 tells us that the corresponding elasticity of substitution also changes, except in the special case in which the real world would be described by the CES. All of this has the following major implication: it is likely to be meaningless to assume that the elasticity of substitution is the same across space and/or time (see Broda and Weinstein, 2006, and Head and Ries, 2001, among many others). This should not be interpreted as a negative message, however. Instead, it is our contention that *richer functional forms*, which encompasses both pro-and anti-competitive effects, *should be used in empirical analyses.* The results in Proposition 3 provide some guidelines that should help the empirical economist in detecting whether or not the market is pro-competitive, thereby helping her to choose a particular specification. To the very least, we find it fair to say that our analysis gives credence to such an alternative modeling strategy.

3 The multi-sector economy

Following Dixit and Stiglitz (1977), we now turn our attention to the case of a two-sector economy involving a differentiated good supplied under increasing returns and monopolistic competition, and a homogeneous good supplied under constant returns and perfect competition. Labor is the only production factor; it is perfectly mobile between sectors.

Each individual supplies inelastically one unit of labor and is endowed with preferences defined by

$$\max \mathcal{U} \equiv U(X, A) = U\left[\int_0^N u(x_i)\mathrm{d}i, A\right]$$

where U is increasing and strictly concave, while A denotes the consumption of the homogeneous good. To make sure that both goods X and A are produced at the market outcome, we assume that the marginal utility of each good tends to infinity when its consumption tends to zero.

Because there is perfect competition and constant returns in the agricultural sector, the price of the homogeneous good is equal to the equilibrium wage times a constant that measures the marginal productivity of labor. We then choose the unit of the homogeneous good for this constant to be equal to 1. Last, choosing the homogeneous good as the numéraire implies that the equilibrium wage is equal to 1 since the output of the agricultural sector is always positive. Since profits are zero, the budget constraint is given by

$$\int_0^N p_i x_i \mathrm{d}i + A = 1.$$

The consumer optimization problem may be decomposed in two subproblems, which in general do not correspond to a two-stage budgeting procedure. First, for any given expenditure $E < 1$, the consumer's program over the differentiated good is

$$\max \int_0^N u(x_i)\mathrm{d}i \qquad \text{s.t.} \int_0^N p_i x_i \mathrm{d}i = E.$$

As in the previous section, under the assumption of concave profits, we may focus on a symmetric outcome (p, N), so that the optimal value of the foregoing program is

$$v(p, N, E) \equiv Nu\left(\frac{E}{Np}\right).$$

The function v is the indirect utility level derived from consuming the differentiated good at the symmetric outcome. It follows from the properties of u that v is decreasing and convex in p, increasing in N, increasing and concave in E, while the cross-derivatives satisfy $v''_{pN} < 0$ and $v''_{EN} > 0$.

Second, the upper-tier maximization problem may be written as follows:

$$\max_E U\left(v(p, E, N), 1 - E\right)$$

in which $v(p, E, N)$ is the index of the differentiated good consumption. Let $E(p, N)$ be the unique solution to the first-order condition

$$U'_1(\cdot)v'_E(\cdot) = U'_2(\cdot). \tag{21}$$

evaluated at a symmetric outcome (p, N). Hence, the lower-tier optimization problem becomes

$$\max \int_0^N u(x_i)\mathrm{d}i \qquad \text{s.t.} \int_0^N p_i x_i \mathrm{d}i = E(p, N).$$

Hence, when consumers preferences are described by a general two-tier utility $U(\cdot)$, the properties of U are immaterial for the value of the equilibrium price. To illustrate, consider a Cobb-Douglas utility, i.e. $\mathcal{U}(X, A) = \alpha \log X + (1 - \alpha) \log A$. The corresponding expenditure function $E(p, N)$ may be obtained as follows. Let $e_u = xu'/u$ denote the elasticity of the lower-tier utility u with respect to consumption. After some manipulations the first-order condition (21) yields

$$\frac{1-\alpha}{\alpha} = \frac{1-E}{v} v'_E = \frac{1-E}{E} e_u(\bar{x})$$

where $\bar{x}$ denotes the long-run equilibrium consumption of every variety. Using (14) shows that $e_u(\bar{x})$ depends only upon $\bar{p}$, which is itself the unique solution to (15). Therefore, $e_u(\bar{x})$ is independent of N, so that the equilibrium expenditure on the differentiated good is defined by

$$\bar{E}(\bar{p}) = \frac{e_u[x(\bar{p})]}{(1-\alpha)/\alpha + e_u[x(\bar{p})]}.$$

This expression is tractable enough to be used in comparative static analyses for many specifications of the lower-tier utility u embodied in a Cobb-Douglas upper-tier utility.

On the other hand, when U is unspecified, it is not possible to derive a closed-form expression for E. However, we are able to derive the main properties of the long-run equilibrium under some mild assumptions on U and u. First, since the equilibrium price $\bar{p}$, individual consumption $\bar{x}$ and firm's output $\bar{q}$ are independent of the value of E (see (15)), their properties still hold within this general setting.

In contrast, the characterization of the equilibrium mass of varieties is more involved because it depends on E, which now also depends on N and p. In order to determine the properties of $\bar{N}$, we need some additional assumptions. Working with a specific expenditure function $E(p, N)$ may appear as the relevant empirical strategy. However, we prefer to identify sufficient conditions (see Appendix C) for the utilities U and u to yield an expenditure function $E(p, N)$ that satisfies intuitive properties, such as the following ones:

$$0 \leq \frac{p}{E} \cdot \frac{\partial E}{\partial p} < 1 \qquad \frac{N}{E} \cdot \frac{\partial E}{\partial N} < 1. \tag{22}$$

The interpretation of these conditions has some appeal. First, the assumption that X and A are complements in preferences ($U''_{12} \geq 0$) implies that the second and third inequalities hold. Such an assumption on U is fairly natural in a context in which both X and A refer to composite goods. Furthermore, the first inequality also implies some form of complementarity, which states that a higher price for the differentiated good leads consumers to spend more on this good. This agrees with the idea that the consumption of both X and A decreases because they are bad substitutes. Last, it is worth stressing that the conditions (22) can be checked once specific utilities are used in empirical analyses.

The following proposition, proven in Appendix D, extends our previous analysis to the case of two sectors.

Proposition 5 *In the two-sector economy, the long-run equilibrium prices, consumption and production vary with market size and cost parameters as in Proposition 3. Furthermore, if (22) holds, the equilibrium mass of varieties increases with the market size.*

It should be clear from the proof that, within a similar modeling structure, the argument developed above also applies to multi-sector economies with *several* differentiated and homogeneous goods.

4 An application to international pricing

The Dixit-Stiglitz model of monopolistic competition has proven to be very useful in the study of trade flows and firms' behavior on the international marketplace (Feenstra, 2004). Yet, this model delivers some predictions that do not fit the data (Bernard *et al.*, 2007). In particular, one of the most problematic results is related to firms' pricing behavior: the pass-on is equal to the trade costs. This is not a very plausible outcome because the existence of spatially separated markets allow firms to discriminate among consumers on the basis of their country of residence, as suggested by the empirical evidence (Martin, 2009; Manova and Zhang, 2009).[4] The purpose of this section is to show how firms price their varieties when the utilities U and u are unspecified, instead of being given, respectively, by a Cobb-Douglas and a CES.

As in the preceding section, we consider an economy endowed with two sectors. Two countries, H(ome) and F(oreign), of sizes L_H and L_F, where $L_H \geq L/2$ and $L_H + L_F = L$, are populated with identical consumers. As in new trade theories, manufacturing firms supplying the foreign country incur an iceberg-type trade cost given by $\tau > 1$, which implies that their marginal delivery cost is equal to $\tau c > c$.[5] Our

[4]Manova and Zhang argue that reverse dumping observed in Chinese exports is due to a quality premium. Although this explanation is likely to be part of the story, it is less clear why such a quality premium would increase with the distance from China.

[5]Observe that our analysis may be extended to the case in which the marginal delivery cost is given by $c+t$ (where $t > 0$) instead of τc.

purpose being to investigate how trade costs affect firms' pricing behavior in the MC-sector, we follow the literature and isolate this effect by working with a setting in which workers' wage is equalized between countries. This is guaranteed by assuming that the homogeneous good is costlessly traded. Under this assumption, the price of the homogeneous good is equal across countries. When this good is chosen as the numéraire, workers' wage is equal to 1 in both countries as long as the agricultural sector operates in both countries. The numbers of firms N^H and N^F in countries H and F are determined by the zero-profit conditions. However, as in the preceding section, the equilibrium prices can be determined without knowing the values of N^H and N^F.

The profit function of firm i located in country $k = H, F$ is given by

$$\pi(p_i^{kk}, p_i^{kl}; p^k(\cdot), p^l(\cdot), E) = (p_i^{kk} - c)L_k x_i^{kk} + (p_i^{kl} - \tau c)L_l x_i^{kl} - f \tag{23}$$

where p_i^{kk} (resp., p_i^{kl}) denotes the domestic price of variety i charged in country k (resp., the foreign price of this variety in country $l \neq k$), while x_i^{kk} (resp., x_i^{kl}) is the individual consumption of variety i in country k (resp., country l). Unless explicitly mentioned, we assume that intra-industry trade prevails.

The equilibrium conditions may be summarized as follows. Applying the profit-maximizing conditions (15) to (23), we obtain

$$p^{kk} = \frac{c}{1 - r_u(x^{kk})} \qquad p^{kl} = \frac{\tau c}{1 - r_u(x^{kl})} \tag{24}$$

where p^{kk} (resp., p^{kl}) denotes the symmetric domestic price in country k (resp., the symmetric foreign price in country $l \neq k$), while x^{kk} (resp., x^{kl}) is the individual consumption in country k (resp., country l) of a variety produced in k. Substituting the equilibrium prices (24) into the zero-profit conditions, we get:

$$\frac{x^{kk} r_u(x^{kk})}{1 - r_u(x^{kk})} L_k + \tau \frac{x^{kl} r_u(x^{kl})}{1 - r_u(x^{kl})} L_l = \frac{f}{c}. \tag{25}$$

Furthermore, consumers' budget in country k is:

$$N^k p^{kk} x^{kk} + N^l p^{lk} x^{lk} = E^k.$$

Using (24), it follows from the first-order condition for utility maximization that

$$\tau \frac{u'(x^{kk})}{u'(x^{lk})} = \tau \frac{p^{kk}}{p^{lk}} = \frac{1 - r_u(x^{lk})}{1 - r_u(x^{kk})}. \tag{26}$$

To determine the domestic and foreign prices set by firms in each country, we must rank the consumption levels of domestic and foreign varieties. Specifically, we show below that the inequalities $x^{FH} \leq x^{HF} < x^{HH} \leq x^{FF}$ always hold. The proof involves three steps.

(i) Setting

$$\phi(x) \equiv u'(x)\,[1 - r_u(x)]$$

the equation (26) may be rewritten as follows:

$$\tau\phi(x^{kk}) = \phi(x^{lk}). \tag{27}$$

Note that ϕ is strictly decreasing since

$$\phi' = 2u^{''} + xu^{'''} = (2 - r_{u'})u^{''} < 0$$

when $2 > r_{u'}$, that is, when profits are strictly concave. Therefore, since $\tau > 1$, it must be that

$$x^{kk} > x^{lk} \qquad \text{for } k, l = H, F \quad \text{and} \quad k \neq l. \tag{28}$$

In other words, *the individual consumption of a domestic variety always exceeds the individual consumption of a foreign variety.* Regardless of the difference in country sizes, the existence of trade costs suffices to bias consumers' purchases toward locally produced varieties.

(ii) We show in Appendix E.1 that

$$x^{HH} \leq x^{FF}. \tag{29}$$

In other words, *the individual consumption of a domestic variety is lower in the larger country than in the smaller one.* Furthermore, using (26) and (29), we obtain

$$\phi(x^{HF}) = \tau\phi(x^{FF}) \leq \tau\phi(x^{HH}) = \phi(x^{FH}).$$

Since $\phi(x)$ is strictly decreasing, it must be that

$$x^{HF} \geq x^{FH} \tag{30}$$

that is, *the consumption of a foreign variety is lower in the larger country than in the smaller one.* The equality holds if and only if $L_H = L_F$.

Note that $x^{HH} \leq x^{FF}$ and $x^{FH} \leq x^{HF}$ imply together that a consumer located in the larger country always buys less of every variety than a consumer in the smaller one. This a priori surprising result stems from the fact that the H-consumers spread their consumption over a wider range of domestic varieties,[6] which in turn makes the foreign varieties relatively less attractive to the domestic consumers.

(iii) Last, we show in Appendix E.2 that

$$x^{HH} > x^{HF}$$

always holds. Hence, *the local consumption always exceeds the foreign consumption.*

Consequently, combining this inequality with (28)-(30), it must be that

$$x^{FH} \leq x^{HF} < x^{HH} \leq x^{FF} \tag{31}$$

where all inequalities are strict if and only if $L_H > L_F$. It then follows from (26) that, in the pro-competitive case, prices are ranked as follows: $p^{FH} \leq p^{HF} < \tau p^{HH} \leq \tau p^{FF}$. On the other hand, in the anti-competitive case, the price ranking is reversed: $p^{FH} \geq p^{HF} > \tau p^{HH} \geq \tau p^{FF}$. Since all inequalities are strict when countries are asymmetric, we have:

Proposition 6 *Assume that U and u are such that (22) holds and that $L_H > L/2$. In the long-run equilibrium with asymmetric countries and two-way trade, the mass of firms in the larger country exceeds the one in the smaller country, while consumers' behavior and firms' pricing are described as follows:*

Pro-competitive utility	CES	Anti-competitive utility
Reciprocal dumping		Reciprocal reverse dumping
$x^{FH} < x^{HF} < x^{HH} < x^{FF}$	$x^{FH} < x^{HF} < x^{HH} < x^{FF}$	$x^{FH} < x^{HF} < x^{HH} < x^{FF}$
$p^{FH} < p^{HF} < \tau p^{HH} < \tau p^{FF}$	$p^{FH} = p^{HF} = \tau p^{HH} = \tau p^{FF}$	$p^{FH} > p^{HF} > \tau p^{HH} > \tau p^{FF}$

Hence, in the pro-competitive case firms' pricing involves *freight-absorption.* Since competition is tougher than in the CES case, foreign firms absorb some fraction of the trade costs. In contrast, in the anti-competitive case, the pass-through exceeds τ: foreign firms charge *phantom freights.* Since competition is now softer than in the CES case, foreign firms pass onto consumers a more than proportionate share of the trade costs.

[6] We show in Appendix E.3 that $\bar{N}^H > \bar{N}^F$.

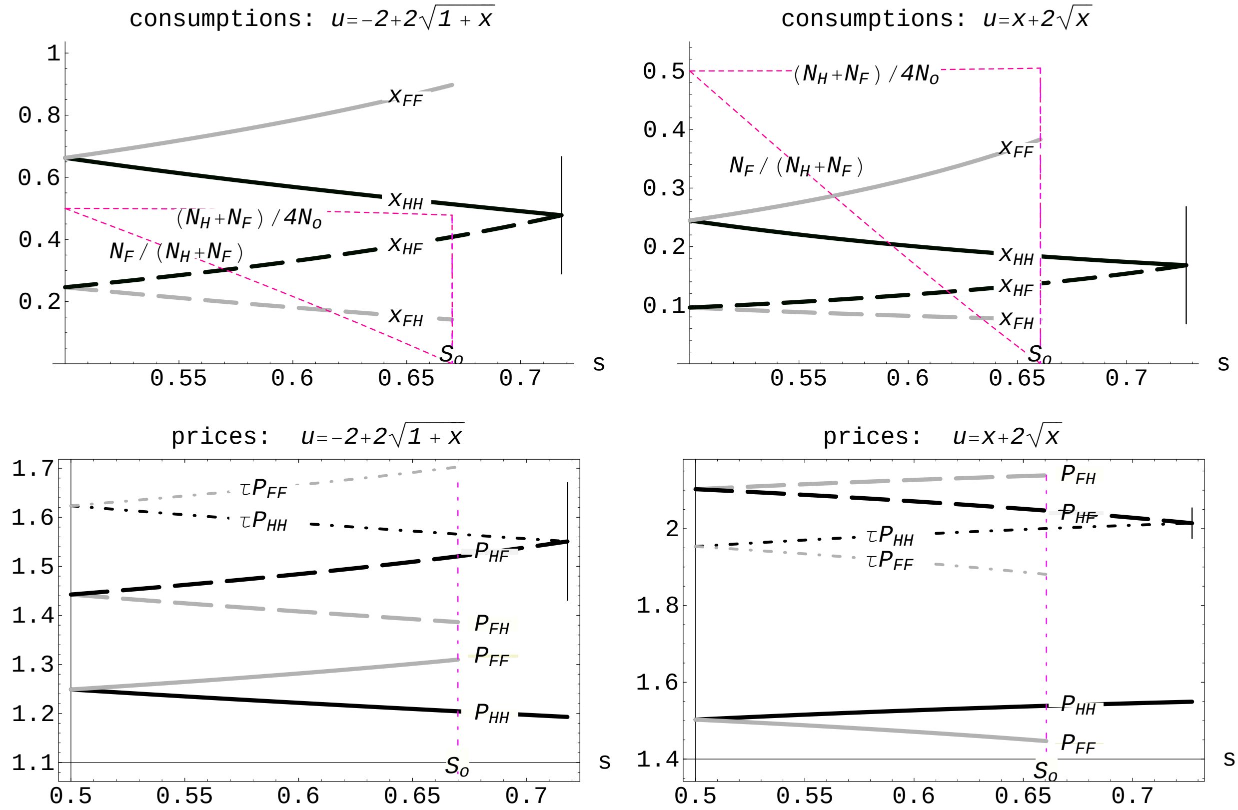

Figure 2: The impact of countries' asymmetry on the market outcome.

In Figure 2, we describe the evolution of the equilibrium outcome when the asymmetry between countries $s \equiv L_H/L_F$ rises from 1/2, the world population remaining constant. The upper-tier utility is a Cobb-Douglas in which the expenditure share on the differentiated good is 0.8, while $u_1(x)$ (the pro-competitive case described in the left panels) and $u_2(x)$ (the anti-competitive case described in the right panels) are the lower-tier utilities introduced in Section 2.2. The values of the parameters are $\tau = 1.3$, $c = 1$, $f = 1$, and $L = 10$ The solid lines describe the individual consumption and price of every domestic variety, whereas the dashed lines represent the individual consumption and price of every imported variety; the black lines refer to country H and the grey ones to F. Assume that $s = 1/2$ so that the two countries have the same mass of firms that behave in the same way. When a small number of consumers move from F to H, firms in H earn positive profits whereas firms in F make negative profits. As in Krugman (1980), this spurs entry in H and exit in F. Since domestic firms have a bigger effect on the local market than foreign firms, the foregoing results suggest that the now larger (resp., smaller) mass of firms in H (resp., F) negatively (resp., positively) impacts on the individual consumption of each domestic variety. As a result, the consumption of a variety produced in H (resp., F) decreases (resp., increases), which in turn implies $x^{HH} < x^{FF}$. Likewise, because local inverse demands are shifted downward (resp., upward) in country H (resp., F), it becomes harder for the foreign firms to export to H than to F. More generally, as the gap between the two countries widens, the mass of firms in H rises while the mass of firms in F falls. In country H, consumers face a wider range of domestic varieties, which makes the foreign varieties less attractive. Therefore, in country H the individual consumption of each variety decreases. For exactly the opposite reason, in country F the individual consumption of each variety increases.[7] When countries become sufficiently dissimilar, no firm operates in country F. In other words, when s exceeds some threshold s_0, *the smaller country no longer produces the differentiated good.* In this case, the local market in F is too small and the market in H too competitive for the operating profits of F-firms to cover their fixed costs. For some firms to operate in F, they should pay a wage lower than the prevailing equilibrium wage 1, but workers in F may guarantee to themselves this unit wage in the agricultural sector.

As shown in the two bottom panels when $s < s_0$, in the pro-competitive case, the dumping rate practiced by the small country firms ($p^{FH} - \tau p^{FF}$) increases monotonically with s, whereas the rate of dumping implemented by the large country firms ($p^{HF} - \tau p^{HH}$) decreases. In contrast, when markets are anti-competitive, the reverse dumping policy followed by the F-firms is exacerbated while the reverse dumping policy implemented by the H-firms is weakened.[8]

The foregoing discussion does not capture the entire richness of firms' pricing behavior on the international marketplace. Indeed, unlike what we have supposed so far, the RLV need not be monotone. Let us assume, for example, that the RLV first increases, and then decreases. Assume also that both markets are pro-competitive when the two countries have similar sizes. In this case, both firms choose dumping. However, as H gets bigger and F smaller, country F becomes anti-competitive, thus inducing firms located in country H to shift to reverse dumping. In other words, the choice between dumping or reverse dumping depends on the nature of preferences, while the difference in size may explain why firms may adopt similar or different pricing behaviors. In any case, all of this suggests that the pricing pattern chosen by firms vastly differs from what we know from the CES case where the foreign price is equal to the domestic price times τ.

5 Concluding remarks

Our main purpose was to develop a general, but tractable, model of monopolistic competition. Without having the explicit solution for the equilibrium outcome, we have been able to provide a full characterization

[7] The proof of monotonicity for any utility U and u is long and tedious. It may be obtained from the authors upon request.

[8] Using the monotonicity property mentioned in the preceding footnote, it is readily verified that those trends hold for general utilities.

of the market equilibrium and to derive necessary and sufficient conditions for the market to be pro- or anti-competitive. Interestingly, relatively minor changes in the specification of utility may result in opposite predictions, thus highlighting the need to be careful in the use of particular specifications. By showing how peculiar are the results obtained under a CES utility, we have seen that resorting to such a modeling strategy is at best problematic. This in turn suggests that the use of alternative and richer specifications should rank high on the research agenda.

As shown in Appendix F, our approach may comply with non-additive preferences, thus showing that the research strategy proposed here is relevant for the study of broader classes of preferences. This line of research that should rank high on the research agenda. Regarding other extensions, note that the assumption of a continuum of firms may be rationalized when the corresponding equilibrium is the limit of a sequence of economies involving a finite and growing number of firms. One appealing feature of our setting is that the monopolistically competitive equilibrium obtained in Section 2 can be shown to be the limit of oligopolistic economies in which firms compete either in quantity (Cournot) or prices (Bertrand). This provides a reconciliation of the two alternative approaches used in oligopoly theory in the case of large economies. Another interesting feature of our setting is that it allows one to work with several MC-sectors. More work is called for to understand better the nature of interactions across such sectors.

References

[1] Amir, R. and V.L. Lambson (2000) On the effects of entry in Cournot markets. *Review of Economic Studies* 67, 235-254.

[2] Behrens, K and Y. Murata (2007) General equilibrium models of monopolistic competition: a new approach. *Journal of Economic Theory* 136, 776-787.

[3] Bertoletti, P.E. Fumagalli and C. Poletti (2008) On price-increasing monopolistic competition. Memo, Università di Pavia.

[4] Bernard, A., J. Jensen, S. Redding, and P. Schott (2007) Firms in international trade. *Journal of Economic Perspectives* 21(3), 105-130.

[5] Badinger, H. (2007) Has the EU's Single Market Programme fostered competition? Testing for a decrease in mark-up ratios in EU industries. *Oxford Bulletin of Economics and Statistics* 69, 497-519.

[6] Broda, C. and D.E. Weinstein (2006) Globalization and the gains from variety. *Quarterly Journal of Economics* 121, 541-585.

[7] Brander, J. and P.R. Krugman (1983) A 'reciprocal dumping' model of international trade. *Journal of International Economics* 15, 313-321.

[8] Campbell, J.R. and H.A. Hopenhayn (2005) Market size matters. *Journal of Industrial Economics* LIII, 1-25.

[9] Chen, Y. and M.H. Riordan (2007) Price and variety in the spokes model. *Economic Journal* 117, 897-921.

[10] Chen, Y. and M.H. Riordan (2008) Price-increasing competition. *Rand Journal of Economics* 39, 1042-1058.

[11] Dixit, A.K. and V. Norman (1980) *Theory of International Trade.* Cambridge: Cambridge University Press.

[12] Dixit, A.K. and J.E. Stiglitz (1977) Monopolistic competition and optimum product diversity. *American Economic Review* 67, 297-308.

[13] Feenstra, R.C. (2003) A homothetic utility function for monopolistic competition models, without constant price elasticity. *Economics Letters* 78, 79-86.

[14] Feenstra, R.C. (2004) *Advanced International Trade: Theory and Evidence.* Princeton, NJ: Princeton University Press.

[15] Greenhut, M.L., H. Ohta and J. Sailors (1987) Reverse dumping: a form of spatial price discrimination. *Journal of Industrial Economics* XXXIV, 167-181.

[16] Head, K. and J. Ries (2001) Increasing returns versus national product differentiation as an explanation for the pattern of US-Canada trade. *American Economic Review* 91, 858-876.

[17] Houthakker, H.S. (1960) Additive preferences. *Econometrica* 28, 244-257.

[18] Krugman, P.R. (1979) Increasing returns, monopolistic competition, and international trade. *Journal of International Economics* 9, 469-479.

[19] Krugman, P.R. (1980) Scale economies, product differentiation, and the pattern of trade. *American Economic Review* 70, 950-959.

[20] Martin, J. (2009) Spatial price discrimination in international markets. CEPII Working Paper N°2009-21.

[21] Manova, K. and Z. Zhang (2009) Quality heterogeneity across firms and export destinations. NBER Working Paper No. 15342.

[22] Nadiri, M.I. (1982) Producers theory. In Arrow, K.J. and M.D. Intriligator (eds.) *Handbook of Mathematical Economics. Volume II.* Amsterdam: North-Holland, pp. 431-490.

[23] Neary, J.P. (2004) Monopolistic competition and international trade theory. In Brakman, S. and B.J. Heijdra (eds.) *The Monopolistic Competition Revolution in Retrospect.* Cambridge: Cambridge University Press, pp. 159-184.

[24] Ottaviano, G.I.P., T. Tabuchi, and J.-F. Thisse (2002) Agglomeration and trade revisited. *International Economic Review* 43, 409-436.

[25] Spence, M. (1976) Product selection, fixed costs, and monopolistic competition. Review of Economic Studies 43, 217-235.

[26] Syverson, C. (2007) Prices, spatial competition, and heterogeneous producers: an empirical test. *Journal of Industrial Economics* LV, 197-222.

[27] Vives, X. (1999) *Oligopoly Pricing. Old Ideas and New Tools.* Cambridge, MA: The MIT Press.

[28] Ward, M.B., J.P. Shimshack, J.M. Perloff, and J.M. Harris (2002) Effects of the private-label invasion in food industries. *American Journal of Agricultural Economics* 84, 961-973.

Appendix

A. The price-elasticity

We show how the price-elasticity of demand is related to the RLV. The first-order condition (4) implies that

$$u'(x_i) = u'(x_j)\frac{p_i}{p_j}.$$

Since $\varphi \equiv (u')^{-1}$, we get:

$$x_i = \varphi\left[\varphi^{-1}(x_j)\frac{p_i}{p_j}\right].$$

Differentiating this expression with respect to p_i and using the identity $\varphi' = 1/u''$ yields

$$\begin{aligned}\frac{\partial x_i}{\partial p_i} &= \frac{\partial}{\partial p_i}\varphi\left[\varphi^{-1}(x_j)\frac{p_i}{p_j}\right] = \varphi'\left[\varphi^{-1}(x_j)\frac{p_i}{p_j}\right]\cdot\left[\left(\varphi^{-1}\right)'(x_j)\frac{p_i}{p_j}\frac{\partial x_j}{\partial p_i} + \frac{\varphi^{-1}(x_j)}{p_j}\right] \\ &= \varphi'(\varphi^{-1}(x_i))\frac{\varphi^{-1}(x_i)}{p_i} = \frac{u'(x_i)}{p_i u''(x_i)} = -\frac{x_i}{p_i r_u(x_i)} \Leftrightarrow \varepsilon_i(p_i) = \frac{1}{r_u[x_i(p_i)]}.\end{aligned}$$

B. The impact of market size and cost on consumption and production

This appendix provides the missing results for Proposition 3 to hold.

The impact of marginal cost. When the marginal cost increases, everything else being equal, operating profits are lower so that the economy accommodates fewer firms. However, because $r'_u > 0$, a smaller mass of firms leads to a higher market price. Therefore, the impact of an increase in c generates two opposite effects. In order to determine the global impact, we rewrite the equilibrium markup as follows

$$\bar{p}(c) = \frac{c}{1 - \bar{M}(c)}$$

which leads to

$$\frac{c}{\bar{p}}\frac{d\bar{p}}{dc} = 1 + \frac{\bar{M}}{1-\bar{M}}\frac{c}{\bar{M}}\frac{d\bar{M}}{dc}.$$

Using (18), this implies

$$\frac{c}{\bar{p}}\frac{d\bar{p}}{dc} = 1 + \frac{\bar{M}}{1-\bar{M}}\frac{\tilde{L}}{\bar{M}}\frac{d\bar{M}}{d\tilde{L}} > 1 - \bar{M} > 0.$$

Consequently, the market price always increases with the marginal cost, regardless of the sign of r'_u. However, the elasticity of $\bar{p}$ with respect to c depends on the sign of r'_u. Indeed, if $r'_u > 0$, we have $d\bar{M}/d\tilde{L} < 0$. In this case, the above expression implies that $1 > (c/\bar{p})(d\bar{p}/dc) > 1 - \bar{M}$. Thus, each firm absorbs some fraction of the cost increase. On the other hand, if $r'_u < 0$, we have $(c/\bar{p})(d\bar{p}/dc) > 1$, meaning that a higher marginal cost leads to a more proportional increase in market price.

To determine the impact of c on the mass of firms, we differentiate $\bar{N}(c) = E\,\tilde{L}\bar{M}(c)$ with respect to c, use (19), and get

$$\frac{c}{\bar{N}}\frac{d\bar{N}}{dc} = \frac{\tilde{L}}{\bar{M}}\frac{d\bar{M}}{d\tilde{L}}.$$

Thus, when c rises, the equilibrium mass of firms may go up or down. Specifically, when $r'_u > 0$, the equilibrium price decreases with $\tilde{L}$, which together with (18) imply

$$-(1-\bar{M}) < \frac{c}{\bar{N}}\frac{d\bar{N}}{dc} < 0$$

whereas we have

$$\frac{c}{\bar{N}}\frac{d\bar{N}}{dc} > 0$$

when $r'_u < 0$. Note that, for $r'_u = 0$, an increase or a decrease in the marginal cost has no impact on the equilibrium mass of firms.

Consumption. Differentiating (14) and using the counterpart of (19), we get

$$\frac{\tilde{L}}{\bar{x}}\frac{d\bar{x}}{d\tilde{L}} = \frac{L}{\bar{x}}\frac{d\bar{x}}{dL} = \frac{c}{\bar{x}}\frac{d\bar{x}}{dc} = -1 - \frac{1}{1-\bar{M}}\frac{\tilde{L}}{\bar{M}}\frac{d\bar{M}}{d\tilde{L}} < 0.$$

Hence, the equilibrium consumption of each variety always decreases with $\tilde{L}$, regardless of the sign of $r_{u'}$. Clearly, the same holds when the marginal cost increases, whereas $(f/\bar{x})(d\bar{x}/df)$ is the same but has the opposite sign. These effects are similar to the one obtained under the CES.

Production. Recall that a firm's production is given by $\bar{q} = L\bar{x} = f\tilde{L}\bar{x}$. Thus, the elasticities of $\bar{q}$ with respect to c and f are the same as for $\bar{x}$. As for the impact of L on $\bar{q}$, the sign is a priori undetermined since $\bar{x}$ decreases with L. However, since we have

$$\frac{L}{\bar{q}}\frac{d\bar{q}}{dL} = -\frac{1}{1-\bar{M}}\frac{L}{\bar{M}}\frac{d\bar{M}}{dL}$$

it must be that $d\bar{q}/dL$ and $d\bar{M}/dL$ have opposite signs. Using the counterpart of (19), the impact of f and c is obtained in a similar way through $d\bar{q}/d\tilde{L}$.

C. Properties of the expenditure function

The purpose of this appendix is to prove the following two lemmas used to rationalize the assumption (22) made in Proposition 4.

Set

$$D \equiv U''_{11} \cdot \left(v'_E\right)^2 - 2U''_{12}v'_E + U''_{22} + U'_1 v''_{EE}.$$

Lemma 1 *If $U''_{21} \geq 0$, then the elasticity of E w.r.t. N is such that*

$$\frac{\partial E}{\partial N} \cdot \frac{N}{E} - 1 = \frac{-U''_{11}v'_E v + U''_{21}\left(v + v'_E E\right) - U''_{22}E}{DE} \leq 0.$$

Recall that $e_u = xu'/u$ denotes the elasticity of the lower-tier utility u.

Lemma 2 *If $U''_{21} \geq 0$ and the inequality*

$$\frac{1 - r_u(x)}{e_u(x)} \leq \frac{U''_{21}(X,Y)X}{U'_2(X,Y)} - \frac{U''_{11}(X,Y)X}{U'_1(X,Y)} \tag{C.1}$$

hold at a symmetric outcome, then the elasticity of E w.r.t. p is such that

$$-1 \leq \frac{\partial E}{\partial p} \cdot \frac{p}{E} - 1 = \frac{U'_1 v'_E + U''_{21}Ev'_E - EU''_{22}}{DE} \leq 0. \tag{C.2}$$

Remark 1. In the special case of a Cobb-Douglas upper utility, the right-hand side of (C.1) is 1 so that this condition boils down to

$$1 \leq r_u(x) + e_u(x)$$

which holds for many functions u, including the CES where $r_u(x) = 1 - \rho$ and $e_u(x) = \rho$.

Remark 2. Under $u(0) = 0$, the indirect utility function

$$v(p, E, N) = Nu\left(\frac{E}{pN}\right)$$

is homogeneous of degree 0 w.r.t. (p, E) and of degree 1 w.r.t. (E, N). Therefore, v'_E and v'_p are homogeneous of degree -1 w.r.t. (p, E) and of degree 0 w.r.t. (E, N). Finally, we have $v''_{EE} < 0$.

Before proceeding, recall that the first-order condition for the upper-tier utility maximization (21) is given by

$$U'_1(v(p, E, N), 1 - E)v'_E(p, E, N) - U'_2(v(p, E, N), 1 - E) = 0 \tag{C.3}$$

while the second-order condition is given by

$$D < 0.$$

Note that $U(v(p,\, E,\, N), 1-E)$ is concave w.r.t. E because U is concave while the concavity of u implies that of v.

Proof of Lemma 1. Differentiating (C.3) w.r.t. N and solving for $\partial E/\partial N$, we get

$$\frac{\partial E}{\partial N} = -\frac{U_{11}'' v_E' v_N' + U_1' v_{EN}'' - U_{21}'' v_N'}{D} = -\frac{(U_{11}'' v_E' - U_{21}'') v_N' + U_1' v_{EN}''}{D}.$$

Consequently,

$$\frac{\partial E}{\partial N} \cdot \frac{N}{E} - 1 = -N \frac{(U_{11}'' v_E' - U_{21}'') v_N' + U_1' v_{EN}''}{DE} - 1 =$$

$$\frac{-U_{11}'' \left[v_E' N v_N' + E\,(v_E')^2 \right] + U_{21}''\,(N v_N' + 2 v_E' E) - U_1'\,(N v_{EN}'' + E v_{EE}'') - E U_{22}''}{DE}.$$

Applying the Euler theorem to v and v', we obtain the following equalities:

$$-U_{11}'' \left[v_E' N v_N' + E \left(v_E' \right)^2 \right] = -U_{11}'' v_E' \left(N v_N' + E v_E' \right) = -U_{11}'' v_E' v$$

$$U_{21}'' \left(N v_N' + 2 E v_E' \right) = U_{21}'' \left(v + E v_E' \right)$$

$$-U_1' \left(N v_{EN}'' + E v_{EE}'' \right) = 0.$$

As a result, we have:

$$\frac{\partial E}{\partial N} \cdot \frac{N}{E} - 1 = \frac{-U_{11}'' v_E' v + U_{21}''\,(v + E v_E') - E U_{22}''}{DE}.$$

Since $U_{21}'' \geq 0$, the numerator of this expression is positive. Since $D < 0$, we have

$$\frac{\partial E}{\partial N} \cdot \frac{N}{E} - 1 \leq 0.$$

■

Proof of Lemma 2.

Step 1. Differentiating (C.3) w.r.t. p and solving for $\partial E/\partial p$, we get

$$\frac{\partial E}{\partial p} = \frac{-U_{11}'' v_p' v_E' - U_1' v_{Ep}'' + U_{21}'' v_p'}{D}. \tag{C.4}$$

which implies

$$\begin{aligned}
\frac{\partial E}{\partial p}\cdot\frac{p}{E}-1 &= p\frac{-U_{11}''v_p'v_E'-U_1'v_{Ep}''+U_{21}''v_p'}{DE}-1 \\
&= \frac{-U_{11}''\left[pv_p'v_E'+E\left(v_E'\right)^2\right]-U_1'\left(pv_{Ep}''+Ev_{EE}''\right)+U_{21}''\left(pv_p'+2Ev_E'\right)-EU_{22}''}{DE}.
\end{aligned}$$

Applying the Euler theorem to v and v' yields

$$-U_{11}''\left[pv_p'v_E'+E\left(v_E'\right)^2\right]=-U_{11}''v_E'\left(pv_p'+Ev_E'\right)=0$$

and

$$-U_1'\left(pv_{Ep}''+Ev_{EE}''\right)=U_1'v_E'>0.$$

Therefore,

$$\frac{\partial E}{\partial p}\cdot\frac{p}{E}-1=\frac{U_1'v_E'+U_{21}''Ev_E'-EU_{22}''}{DE}\leq 0$$

since $U_{21}''\geq 0$. Consequently, the right inequality of (C.2) is proven.

Step 2. To show that $\partial E/\partial p>0$, we rewrite (C.4) as follows:

$$\frac{\partial E}{\partial p}=\frac{v_p'}{D}\left(-U_{11}''v_E'-U_1'\frac{v_{Ep}''}{v_p'}+U_{21}''\right). \tag{C.5}$$

By definition of v, we have

$$v_p'=-\frac{Eu'}{p^2}<0 \qquad v_E'=\frac{u'}{p} \qquad v_{Ep}''=-\frac{u'}{p^2}-\frac{Eu''}{Np^3}.$$

Since $v_p'/D>0$, the sign of $\partial E/\partial p$ is the same as that of the bracketed term of (C.5). Substituting these expressions into (C.5) leads to

$$\begin{aligned}
& -U_{11}''v_E'-U_1'\frac{v_{Ep}''}{v_p'}+U_{21}'' \\
= & -U_{11}''\frac{u'}{p}-U_1'\frac{-\frac{u'}{p^2}-\frac{Eu''}{Np^3}}{-\frac{Eu'}{p^2}}+U_{21}'' \\
= & -\frac{U_1'}{E}\left[\left(\frac{U_{11}''Nu}{U_1'}-\frac{U_{21}''Nu}{U_2'}\right)\frac{Eu'}{Npu}+1+\frac{Eu''}{Npu'}\right].
\end{aligned}$$

Using $-U_1'/E < 0$ and $U_1' v_E'(p, E, N) = pU_2'/u'$, it follows from (C.1) that

$$\left(\frac{U_{11}''Nu}{U_1'} - \frac{U_{21}''Nu}{U_2'}\right)\frac{Eu'}{Npu} + 1 + \frac{Eu''}{Npu'} < 0 \Longrightarrow \frac{\partial E}{\partial p} > 0$$

which implies the left inequality of (C.2).■

D. The impact of market size on the mass of firms

We show that the equilibrium mass of firms decreases with the market size L (see Proposition 4). To this end, we exploit the following implicit relationship between $\bar{M}$ and L:

$$Nf = L\bar{M}(L)E(\bar{p}(L), N).$$

Differentiating this expression w.r.t. L, we get

$$\frac{\partial N}{\partial L} \cdot \frac{L}{N} = 1 + \frac{\partial \bar{M}}{\partial L}\frac{L}{\bar{M}} + \frac{\partial E}{\partial p}\frac{\bar{p}}{E}\frac{\partial \bar{p}}{\partial L}\frac{L}{\bar{p}} + \frac{\partial E}{\partial N}\frac{N}{E}\frac{\partial N}{\partial L}\frac{L}{N}$$

which implies

$$\frac{\partial N}{\partial L} \cdot \frac{L}{N}\left(1 - \frac{\partial E}{\partial N}\frac{N}{E}\right) = 1 + \left(1 + \frac{\partial E}{\partial p}\frac{\bar{p}}{E}\frac{\bar{M}}{1-\bar{M}}\right)\frac{\partial \bar{M}}{\partial L}\frac{L}{\bar{M}}.$$

Using Lemmas 1 and 2 of Appendix C, we obtain

$$\begin{aligned} \frac{\partial N}{\partial L} \cdot \frac{L}{N}\left(1 - \frac{\partial E}{\partial N}\frac{N}{E}\right) &\geq 1 + \left(1 + \frac{\partial E}{\partial p}\frac{\bar{p}}{E}\frac{\bar{M}}{1-\bar{M}}\right)(\bar{M} - 1) \\ &= \bar{M}\left(1 - \frac{\partial E}{\partial p}\frac{\bar{p}}{E}\right) > 0 \end{aligned}$$

which implies

$$\frac{\partial N}{\partial L} \cdot \frac{L}{N} > 0.$$

■

E. Consumption in international trade

Consumption of domestic and imported varieties. Let $y(x)$ be the unique solution to the consumer equilibrium condition (27) rewritten as follows:

$$\tau\phi(x) = \phi(y)$$

where $\phi(x) \equiv u'(x)\,[1 - r_u(x)]$. We have seen in Section 3.2 that x^{HH} and x^{FH} (resp., x^{FF} and , x^{HF}) satisfy the above equation.

Differentiating

$$\tau u'(x)[1 - r_u(x)] = u'(y)\,[1 - r_u(y)]$$

w.r.t. x, we obtain

$$\tau[2 - r_{u'}(x)]u''(x) = y'(x)[2 - r_{u'}(y)]u''(y)$$

which implies

$$y'(x) = \tau\frac{[2 - r_{u'}(x)]u''(x)}{u''(y)[2 - r_{u'}(y)]} > 0 \tag{E.1}$$

since profits are strictly concave.

Setting

$$\varphi(x) \equiv \frac{x r_u(x)}{1 - r_u(x)} - \tau\frac{y(x) r_u(y(x))}{1 - r_u(y(x))}$$

the zero-profit condition (25) and $L_H \geq L_F$ implies

$$\varphi(x^{HH}) \leq \varphi(x^{FF}). \tag{E.2}$$

We are show that $\varphi(x)$ is strictly increasing. Differentiating the first term of $\varphi(x)$ and using (12) yields

$$\left[\frac{x r_u(x)}{1 - r_u(x)}\right]' = \frac{[1 - r_u(x)]\,r_u(x) + x r_u'(x)}{[1 - r_u(x)]^2} = \frac{[2 - r_{u'}(x)]r_u(x)}{[1 - r_u(x)]^2}.$$

Repeating the same operation with the second term of φ and replacing y' by (E.1), we obtain:

$$\begin{aligned} \varphi'(x) &= \frac{[2 - r_{u'}(x)]r_u(x)}{[1 - r_u(x)]^2} - \tau y'\frac{[2 - r_{u'}(y)]r_u(y)}{[1 - r_u(y)]^2} \\ &= \frac{[2 - r_{u'}(x)]r_u(x)}{[1 - r_u(x)]^2} - \tau^2\frac{u''(x)[2 - r_{u'}(x)]r_u(y)}{u''(y)[1 - r_u(y)]^2}. \end{aligned}$$

Solving consumer's optimization leads to

$$\tau = \frac{u'(y)\,[1 - r_u(y)]}{u'(x)\,[1 - r_u(x)]}.$$

Replacing τ^2 into $\varphi'(x)$ yields

$$\begin{aligned}
\varphi'(x) &= [2 - r_{u'}(x)] \left\{ \frac{r_u(x)}{[1 - r_u(x)]^2} - \frac{u''(x)[u'(y)]^2 r_u(y)}{\{u'(x)[1 - r_u(x)]\}^2\, u''(y)} \right\} \\
&= \frac{2 - r_{u'}(x)}{[1 - r_u(x)]^2} \left\{ r_u(x) - \frac{u''(x)[u'(y)]^2 r_u(y)}{[u'(x)]^2 u''(y)} \right\} \\
&= \frac{2 - r_{u'}(x)}{[1 - r_u(x)]^2} \frac{r_u(x)}{x u'(x)} \left[x u'(x) - y u'(y) \right].
\end{aligned}$$

Observe that $xu'(x)$ is increasing w.r.t. x when $r_u < 1$ because $u' + xu'' = u'(1 - r_u)$. Therefore, since $x = x^{HH} > y = x^{HF}$ when $\tau > 1$, it must be that $xu'(x) - yu'(y) > 0$. This in turn implies $\varphi' > 0$. Therefore, it follows from (E.2) that

$$x^{HH} \leq x^{FF}$$

where the equality holds if and only if $L_H = L_F$.■

Consumption of domestic and exported varieties. We show that, under two-way trade, $x^{HH} > x^{HF}$. The proof involves three steps.

Step 1. Consumers' budgets in H and F are:

$$N^H p^{HH} x^{HH} + N^F p^{FH} x^{FH} = E^H$$

$$N^H p^{HF} x^{HF} + N^F p^{FF} x^{FF} = E^F.$$

Combining these two expressions, we get

$$\frac{N^H}{N^F} p^{HH} x^{HH} + p^{FH} x^{FH} = \frac{E^H}{E^F} \left(\frac{N^H}{N^F} p^{HF} x^{HF} + p^{FF} x^{FF} \right)$$

or

$$\frac{N^H}{N^F} = \frac{\frac{E^H}{E^F} p^{FF} x^{FF} - p^{FH} x^{FH}}{p^{HH} x^{HH} - \frac{E^H}{E^F} p^{HF} x^{HF}}.$$

Since there is two-way trade, the LHS of this expression is positive and finite. Consequently, we have

$$\frac{p^{FH}x^{FH}}{p^{FF}x^{FF}} < \frac{E^H}{E^F} < \frac{p^{HH}x^{HH}}{p^{HF}x^{HF}}. \tag{E.3}$$

We show that this interval is non-empty. (i) $zu'(z)$ is strictly increasing. Indeed, $u'(z) + zu''(z) = u'(z)[1 - r_u(z)]$ is positive because $1 > r_u(z)$ by (8) over the interval of all relevant values of z. (ii) Using the monotonicity of $zu'(z)$ on this interval together with the inequality $x^{HH} > x^{FH}$ (see (28)), we have

$$\frac{p^{HH}x^{HH}}{p^{FH}x^{FH}} = \frac{\frac{x^{HH}}{1-r_u^{HH}}}{\frac{\tau x^{FH}}{1-r_u^{FH}}} = \frac{x^{HH}\left(1 - r_u^{FH}\right)}{\tau x^{FH}\left(1 - r_u^{HH}\right)} = \frac{x^{HH}u'(x^{HH})}{x^{FH}u'(x^{FH})} > 1$$

which means that $p^{HH}x^{HH} > p^{FH}x^{FH}$. Similarly, it can be shown that $p^{FF}x^{FF} > p^{HF}x^{HF}$. Combining these two inequalities leads to

$$\frac{p^{FH}x^{FH}}{p^{FF}x^{FF}} < \frac{p^{HH}x^{HH}}{p^{HF}x^{HF}}.$$

Step 2. In what follows, we need the following inequality:

$$\frac{U_1'(v^H,\, A^H)}{U_2'(v^H,\, A^H)} < \frac{U_1'(v^F,\, A^F)}{U_2'(v^F,\, A^F)} \tag{E.4}$$

where v^i is the indirect utility of the differentiated good in country i and $A^i = 1 - E^i$ the consumption of the homogeneous good in this country. To prove (E.3), we first rewrite (C.3) for each country:

$$U_1'(v^H, 1 - E^H)\left(v^H\right)'_E = U_2'(v^H, 1 - E^H) \Leftrightarrow \frac{U_1'(v^H, 1 - E^H)}{U_2'(v^H, 1 - E^H)} = \frac{1}{\left(v^H\right)'_E}$$

$$U_1'(v^F, 1 - E^F)\left(v^F\right)'_E = U_2'(v^F, 1 - E^F) \Leftrightarrow \frac{U_1'(v^F, 1 - E^F)}{U_2'(v^F, 1 - E^F)} = \frac{1}{\left(v^F\right)'_E} \tag{E.5}$$

where $(v)'_E$ is the Lagrange multiplier in the consumers' lower-tier optimization under a given E. Using (3) and (24) yields

$$\left(v^i\right)'_E = \frac{u'(x^{ii})}{p^i} = \frac{u'(x^{ii})(1 - r_u(x^{ii}))}{c}.$$

Since $u'(x^{ii})[1 - r_u(x^{ii})]$ is decreasing and $x^{FF} > x^{HH}$ by (29), we have $\left(v^H\right)'_E > \left(v^F\right)'_E$. This together with (E.5) implies (E.4).

Step 3. The remaining of the proof is by contradiction. If $x^{HH} \leq x^{HF}$, then (E.3) implies

$$\frac{E^H}{E^F} < \frac{p^{HH}x^{HH}}{p^{HF}x^{HF}} \leq \frac{p^{HH}}{p^{HF}} = \frac{1}{\tau} < 1 \Longrightarrow E^H < E^F.$$

Hence, $1 - E^H > 1 - E^F$. Observe that the marginal rate of substitution between the differentiated and homogeneous goods, $U_1'(X,\, A)/U_2'(X,\, A)$, is decreasing in X and increasing in A when (22) holds. Indeed, (22) implies

$$\frac{U_{11}''U_2' - U_1'U_{21}''}{(U_2')^2} < 0 \qquad \frac{U_{12}''U_2' - U_1'U_{22}''}{(U_2')^2} > 0.$$

Therefore,

$$\frac{U_1'(v^H, 1 - E^H)}{U_2'(v^H, 1 - E^H)} > \frac{U_1'(v^H, 1 - E^F)}{U_2'(v^H, 1 - E^F)}.$$

Because $x^{HH} \leq x^{HF}$ and $x^{FH} \leq x^{FF}$ (see (29)), we find

$$v^H = N^H u(x^{HH}) + N^F u(x^{FH}) \leq N^H u(x^{HF}) + N^F u(x^{FF}) = v^F.$$

Since the marginal rate of substitution is decreasing in v, the above inequality implies

$$\frac{U_1'(v^H, 1 - E^H)}{U_2'(v^H, 1 - E^H)} \geq \frac{U_1'(v^H, 1 - E^F)}{U_2'(v^H, 1 - E^F)} \geq \frac{U_1'(v^F, 1 - E^F)}{U_2'(v^F, 1 - E^F)}$$

which contradicts (E.4).

Industry size We now show that $\bar{N}^H \geq \bar{N}^F$. It follows from (E.2) and $E^H \geq E^F$ that

$$\frac{N^H}{N^F} \geq \frac{p^{FF}x^{FF} - p^{FH}x^{FH}}{p^{HH}x^{HH} - p^{HF}x^{HF}}.$$

Since $p^{FF}x^{FF} \geq p^{HH}x^{HH}$ and $p^{FH}x^{FH} \leq p^{HF}x^{HF}$, the RHS of the above expression is larger than or equal to 1.■

F. The case of non-additive preferences

We show here that our approach may be applied to the following non-separable preferences:

$$\mathcal{U} \equiv U(N, \mathbf{x})$$

where U is assumed to be symmetric with respect to any non-zero interval of varieties (see below for an example). Applying the first-order condition for utility maximization yields the consumer's inverse demand for any variety i:

$$p_i(x_i) = \frac{1}{\lambda} U_i'(N, \mathbf{x})$$

where U_i' is the partial derivative of U with respect to x_i and λ the Lagrange multiplier. As in Section 3.2, the first-order condition for profit maximization gives

$$\bar{M} = \frac{\bar{p} - c}{\bar{p}} = r_U(N, \bar{x}) \tag{F.1}$$

where

$$r_U(N, x_i) \equiv -\frac{x_i U_i''(N, x_i)}{U_i'(N, x_i)}.$$

Therefore, at the symmetric equilibrium, the mark-up of a firm is equal to the RLV of U. Furthermore, the condition (9) and the budget constraint imply the following two equilibrium equations:

$$1 - \frac{c}{\bar{p}} = r_U\left(N, \frac{E}{N\bar{p}}\right) \qquad \bar{x} = \frac{E}{N\bar{p}}. \tag{F.2}$$

The only difference between the above conditions and their counterparts in the additive case is that the RLV now depends on N.

Using (F.1) and (F.2), we can repeat mutatis mutandis the arguments of Section 3.2. and 3.3 to derive results similar to Propositions 1 and 2 where u is replaced by U. Even though Proposition 3 slightly changes because r_U depends on N, pro- and anti-competitive behavior also occur under non-separable preferences.

To illustrate how our approach may be used to deal with non-separable preferences, consider the quadratic utility which is non-separable:

$$U = \alpha \int_0^N x_i \mathrm{d}i - \frac{\beta}{2} \int_0^N x_i^2 \mathrm{d}i - \frac{\gamma}{2} \left(\int_0^N x_i \mathrm{d}i \right)^2.$$

Thus,

$$U_i' = \alpha - \beta x_i - \gamma \int x_j \mathrm{d}j \qquad U_i'' = -\beta$$

The resulting RLV is given by

$$r_U(N, x_i) \equiv -\frac{x_i U_i''(N, x_i)}{U_i'(N, x_i)} = \frac{\beta x_i}{\alpha - \beta x_i - \gamma N x_i} \tag{F.3}$$

which can be used in the equilibrium equations without knowing explicitly the demand functions.

Can (F.3) be generated by an additive utility function such as

$$U(N, \mathbf{x}) = \int_0^N u(N, x_i)\mathrm{d}i \tag{F.4}$$

which would replace the quadratic utility while giving the same market outcome? To answer this question, we solve the following differential equation:

$$\log u'_{x_i}(N, x_i) = -\int \frac{\beta}{\alpha - \beta x_i - \gamma N x_i}\mathrm{d}x_i.$$

Integrating and choosing constants for $u(0) = 0$, we obtain

$$u(N, x_i) = a^{\frac{\beta-\gamma}{\beta-\gamma+N\gamma}+1} - [\alpha - (\beta - \gamma + N\gamma)x_i]^{\frac{\beta-\gamma}{\beta-\gamma+N\gamma}+1}$$

which is strictly increasing and concave in x_i. Substituting this expression into (F.4) yields an additive utility that gives the same market outcome as Ottaviano *et al.* (2002) with non-additive utility. This example suggests that the assumption of additive preferences might not be as restrictive as it looks like at first glance.