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SAMUELSON HYPOTHESIS:  
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# **THE HARROD-BALASSA- SAMUELSON HYPOTHESIS: REAL EXCHANGE RATES AND THEIR LONG-RUN EQUILIBRIUM**

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## ABSTRACT

### The Harrod-Balassa-Samuelson Hypothesis: Real Exchange Rates and their Long-Run Equilibrium\*

Frictionless, perfectly competitive traded-goods markets justify thinking of purchasing power parity (PPP) as the main driver of exchange rates in the long-run. But differences in the traded/non-traded sectors of economies tend to be persistent and affect movements in local price levels in ways that upset the PPP balance (the underpinning of the Harrod-Balassa-Samuelson hypothesis, HBS). This paper uses panel-data techniques on a broad collection of countries to investigate the long-run properties of the PPP/HBS equilibrium using novel local projection methods for cointegrated systems. These semi-parametric methods isolate the long-run behavior of the data from contaminating factors such as frictions not explicitly modelled and thought to have effects only in the short-run. Absent the short-run effects, we find that the estimated speed of reversion to long-run equilibrium is much higher. In addition, the HBS effects means that the real exchange rate is converging not to a steady mean, but to a slowly to a moving target. The common failure to properly model this effect also biases the estimated speed of reversion downwards. Thus, the so-called

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# 1 Introduction

Parity conditions remain a key foundation of international macroeconomics. When exchange rates and the prices of goods are fully flexible and trade between countries is frictionless, equilibrium in perfectly competitive markets is characterized by goods selling at the same price everywhere – the purchasing power parity (PPP) condition is a manifestation of this ideal equilibrium. By analogy, in frictionless risk-neutral markets expected returns to different currency investments should equalize – the uncovered interest rate parity (UIP) condition being a manifestation of this observation. In the real world, the costs of moving goods across oceans, tariffs, regulations, capital controls, fear of floating, limits to arbitrage, and other frictions may undermine the basic premise underlying PPP and UIP. Yet these mechanisms are so basic and so powerful that one would expect them to hold to a first approximation and thus provide a basis to broadly characterize the behavior of exchange rates, interest rates, and prices.

Financial products can be seamlessly traded across the world at the speed of light but when it comes to real economies, there are many goods (often associated with services) that cannot be traded across borders. For this reason, comparisons of the economy-wide price levels in two countries with their bilateral exchange rate will reflect not only PPP-related forces but also the effects of differences in the relative prices of tradeable and non-tradeable goods. To the extent that these differences are attributable to differences in economic development and these differences persist over time, the Harrod-Balassa-Samuelson (HBS) (Harrod 1933; Balassa 1964; Samuelson 1964) hypothesis extends the PPP condition to account for differences in productivity across countries.

This paper investigates explanations of long-run secular movements in exchange rates described by the HBS effect. Several features of our work separate our contributions from an extensive ongoing literature on this subject (see, *inter alia*, Chinn and Johnston 1996; Ito et al. 1999; Halpern and Wyplosz 2001; Choudhri and Khan 2005; Lee and Tang 2007; Lothian and Taylor 2008).<sup>1</sup> Our core principle has been to let the data speak for itself and for this reason we use a relatively extensive collection of countries – 21 OECD countries to be precise. We then investigate bilateral relationships of these OECD countries using both the U.S. and the “world” as counterparties.

However, perhaps the most innovative features of our analysis come from the statistical methods that we introduce: local projections for cointegrated systems. This approach, which extends the local projections method introduced in Jordà (2005, 2009), allows for semi-parametric mea-

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<sup>1</sup> Earlier theoretical and empirical contributions are well summarized in the papers published the *Review of International Economics* 1994 special issue.

surement of the dynamics of long-run equilibrium adjustment. Moreover, local projections provide a natural and simple separation of these adjustment dynamics into the component explained purely by long-run forces (encapsulated primarily in the HBS hypothesis) from the component explained by frictions thought to have effects only in the short-run and for which no explicit economic justification is given nor sought in this paper. We hope that discerning readers will find the methods useful for applications other than the one offered in this paper.

Traditionally, cointegration is examined through the prism of a vector error-correction model (VECM) but VECMs are difficult to extend to panels because they are parametrically intensive. In addition, parametrically tractable specifications and the structure of VECMs restrict the range of dynamics and half-lives that can be estimated from the data. Finally, conventional estimates from these models offer half-live estimates that conflate the process of long-run adjustment (the focus of this paper) with the effect of short-run frictions on long-run adjustment (not the focus of this paper). To be sure, we are not advocating that forces other than those encapsulated by the HBS effect are unimportant in explaining exchange rate movements. Much like a feather blown by the wind returns to the ground by the force of gravity, so do exchange rates return to their long-run equilibrium levels and the strength of this pull is the object of our interest. But just as the flight of the feather follows the whimsy of the wind, so do exchange rates follow the whimsy of forces whose characterization we reserve for future research.

## 2 Statistical Design

### 2.1 Preliminaries

We begin by presenting the basic set-up in a more traditional linear context to better communicate the main ideas. Let  $e_{t+1}$  denote the logarithm of the nominal exchange rate in terms of currency units of domestic currency per units of foreign currency, let  $p_{t+1}$  and  $p_{t+1}^*$  denote the logarithm of the price indices home and abroad. In the strict version of PPP without frictions and where the price indices refer only to traded goods, then the real exchange rate defined as  $q_{t+1} = e_{t+1} + (p_{t+1}^* - p_{t+1})$  would only exhibit temporary fluctuations around a long-run equilibrium level, or in time series parlance, it would be a stationary variable and hence implicitly define a cointegrating vector. Similarly, let  $i_t$  and  $i_t^*$  denote the one-period, risk-free interest rate home and abroad. The UIP condition can be seen as a statement about the absence of arbitrage of a zero net investment position. Specifically, let the variable *momentum* denoted as  $m_t$  refer to the ex-ante nominal excess returns of such an investment, i.e.,  $m_{t+1} = (i_t^* - i_t) + \Delta e_{t+1}$ , then in the absence of arbitrage opportunities the ex-ante value of momentum should be zero, that is  $E_t m_{t+1} = (i_t^* - i_t) + E_t \Delta e_{t+1} = 0$ , which should make  $m_{t+1}$  a stationary and unpredictable variable. Momentum

can also be expressed in terms of real excess returns as

$$m_{t+1} = (r_t^* - r_t) + \Delta q_{t+1} \quad (1)$$

where  $r_t = i_t - \pi_{t+1}$  with  $\pi_{t+1} = \Delta p_{t+1}$ , and similarly for  $r_t^*$  and  $\pi_{t+1}^*$ ; and  $q_{t+1} = e_{t+1} + (p_{t+1}^* - p_{t+1})$  so that  $\Delta q_{t+1} = \Delta e_{t+1} + (\pi_{t+1}^* - \pi_{t+1})$ . This reformulation of momentum with real variables makes the interaction between PPP and UIP more readily apparent. Finally, we extend the formulation of the system with  $x_{t+1}$  and  $x_{t+1}^*$ , which denote the logarithm of home and abroad productivity proxies. If  $q_{t+1}$  is not a stationary variable, persistent deviations from long-run equilibrium could be explained by persistent productivity differentials (in the traded/non-traded composition) between trading partners according to the Harrod-Balassa-Samuelson hypothesis. The implication of this hypothesis can be stated in terms of the variable  $z_{t+1} = q_{t+1} - \beta(x_{t+1}^* - x_{t+1})$ , which should be stationary under the HBS hypothesis.

The dynamic interactions between nominal exchange rates, inflation, interest rate and productivity differentials are the constituent elements of a system whose linear combinations explain the dynamic behavior of HBS-adjusted PPP, and UIP jointly, specifically

$$\Delta \mathbf{y}_{t+1} = \begin{bmatrix} \Delta e_{t+1} \\ \pi_{t+1}^* - \pi_{t+1} \\ i_t^* - i_t \\ \Delta x_{t+1}^* - \Delta x_{t+1} \end{bmatrix}. \quad (2)$$

Several features of this system deserve mention. We will show momentarily that  $e_{t+1}$ ,  $(p_{t+1}^* - p_{t+1})$ , and  $(x_{t+1}^* - x_{t+1})$  are  $I(1)$  variables but that they are cointegrated (and hence  $\Delta \mathbf{y}_{t+1}$  is  $I(0)$ ), a property that we exploit to derive the HBS-adjusted PPP relation below. In addition, we will show formally (in Table 1 below) that each of the elements in  $\Delta \mathbf{y}_{t+1}$  is stationary with appropriate panel unit root tests. The timing of the variable  $(i_t^* - i_t)$  may seem odd but simply reflects the observation that at time  $t$ , the interest rate for delivery at time  $t+1$  is known. Finally, appropriate panel cointegration tests reported in Table 2 show that  $z_{t+1} = q_{t+1} - \beta(x_{t+1}^* - x_{t+1})$  is a proper cointegration vector.

Notice that expression (2) does not characterize the stochastic process we think  $\Delta \mathbf{y}_{t+1}$  may follow. This is by design since we will be interested in calculating certain sample statistics of interest without referencing to any one specific model. The tools required to estimate these sample statistics are discussed in the next section.

## 2.2 Local Projections for Cointegrated Systems

Long-run equilibrium in exchange rates under the HBS hypothesis takes on the form  $z_{t+1} = e_{t+1} + (p_{t+1}^* - p_{t+1}) - \beta(x_{t+1}^* - x_{t+1})$ . When  $\beta = 0$  this is clearly just the usual PPP condition. We are interested in learning how long-run equilibrium is restored in response to shocks and how this adjustment process manifests itself in the dynamics of nominal exchange rates and prices.

In order to present our approach, we find it convenient to discuss the basic features of a typical linear specification first. We do this with as general a notation as possible to accommodate applications other than the one we pursue in this paper for the benefit of readers with other applications in mind. Suppose  $\mathbf{y}_t$  is an  $n \times 1$  vector of non-stationary random variables, such as that in expression (2), that is first-difference stationary with Wold representation:

$$\Delta \mathbf{y}_t = \sum_{j=0}^{\infty} C_j \boldsymbol{\varepsilon}_{t-j} \quad (3)$$

where the constant and other deterministic components are omitted for convenience but without loss of generality. The  $n \times n$  matrices  $C_j$  are such that  $C_0 = I_n$ ;  $\sum_{j=0}^{\infty} j \|C_j\| < \infty$  where  $\|C_j\|^2 = \text{tr}(C_j' C_j)$ ; and  $\boldsymbol{\varepsilon}_t$  is an  $n \times 1$  vector white noise (relaxing these and subsequent assumptions is certainly possible but distracting from the exposition of the main results here).

Further, assume that  $\mathbf{y}_t$  has a finite vector autoregressive (VAR) representation of order  $p + 1$  in the levels and given by

$$\mathbf{y}_{t+1} = \Phi_1 \mathbf{y}_t + \dots + \Phi_{p+1} \mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_{t+1} \quad (4)$$

where the  $\Phi_j$   $j = 1, \dots, p + 1$  are  $n \times n$  coefficient matrices. Expression (4) can be equivalently written (see e.g. Hamilton, 1994) as

$$\mathbf{y}_{t+1} = \Psi_1 \Delta \mathbf{y}_t + \dots + \Psi_p \Delta \mathbf{y}_{t-p+1} + \Pi \mathbf{y}_t + \boldsymbol{\varepsilon}_{t+1} \quad (5)$$

with  $\Psi_j = -[\Phi_{j+1} + \dots + \Phi_{p+1}]$ ,  $j = 1, \dots, p$  and  $\Pi = \sum_{j=1}^{p+1} \Phi_j$ . Subtracting  $\mathbf{y}_t$  from both sides of expression (5) we obtain

$$\Delta \mathbf{y}_{t+1} = \Psi_1 \Delta \mathbf{y}_t + \dots + \Psi_p \Delta \mathbf{y}_{t-p+1} + \Psi_0 \mathbf{y}_t + \boldsymbol{\varepsilon}_{t+1} \quad (6)$$

where  $\Psi_0 = \Pi - I = -\Phi(1)$ .

When there is cointegration among the elements of  $\mathbf{y}_t$ , then there exists an  $n \times k$  matrix  $A$  such that  $\mathbf{z}_t = A' \mathbf{y}_t$  is  $I(0)$  and contains the  $k$  cointegrating vectors. In our specific application  $k = 1$ . Further, the  $n \times n$  matrix  $\Phi(1)$  is reduced-rank with rank  $k$  and thus can be expressed as  $\Phi(1) = BA'$  where  $B$  is an  $n \times k$  matrix so that expression (6) becomes the well-known error-correction representation for  $\mathbf{y}_t$ . Notice that (3) can be recast as

$$\mathbf{y}_t = \mathbf{y}_0 + C(1) \sum_{j=1}^t \boldsymbol{\varepsilon}_j + \sum_{j=0}^{\infty} C_j^* \boldsymbol{\varepsilon}_{t-j} - \sum_{j=0}^{\infty} C_j^* \boldsymbol{\varepsilon}_{-j}$$



and assuming for convenience that the initial conditions  $\mathbf{y}_0$  and  $\varepsilon_{-j}$  are zero, further simplified into

$$\mathbf{y}_t = C(1) \sum_{j=1}^t \varepsilon_j + \sum_{j=0}^{\infty} C_j^* \varepsilon_{t-j} \quad (7)$$

where the first term contains the stochastic trend components, the second term the cyclical components and  $C_j^* = -[C_{j+1} + \dots]$ ,  $j = 0, 1, \dots$ . Expression (7) is the well-known Beveridge-Nelson decomposition and when there is cointegration,  $A'\mathbf{y}_t$  is stationary which implies that  $A'C(1) = 0$  and hence an impulse response representation for  $\mathbf{z}_t$  can be easily derived to be from (7)

$$\mathbf{z}_t = \sum_{j=0}^{\infty} A'C_j^* \varepsilon_{t-j} = A' \sum_{j=0}^{\infty} \left( \sum_{i=0}^j C_i \varepsilon_{t-j} \right). \quad (8)$$

This is the essence of the Granger-representation theorem.

In order to investigate how the dynamics of the long-run equilibrium relations in  $\mathbf{z}_t$  affect the dynamic responses characterized by the terms  $C_j$ , we find it convenient to construct these terms using projection arguments in a state-space representation of expressions (5) and (6), specifically

$$\begin{bmatrix} \mathbf{z}_{t+1} \\ \Delta \mathbf{y}_{t+1} \\ \Delta \mathbf{y}_t \\ \vdots \\ \Delta \mathbf{y}_{t-p+1} \end{bmatrix} = \begin{bmatrix} A'\Pi & A'\Psi_1 & \dots & A'\Psi_{p-2} & A'\Psi_{p-1} \\ -B & \Psi_1 & \dots & \Psi_{p-2} & \Psi_{p-1} \\ \mathbf{0}_k & I_n & \dots & \mathbf{0}_n & \mathbf{0}_n \\ \vdots & \vdots & \dots & \vdots & \vdots \\ \mathbf{0}_k & \mathbf{0}_n & \dots & I_n & \mathbf{0}_n \end{bmatrix} \begin{bmatrix} \mathbf{z}_t \\ \Delta \mathbf{y}_t \\ \Delta \mathbf{y}_{t-1} \\ \vdots \\ \Delta \mathbf{y}_{t-p} \end{bmatrix} + \begin{bmatrix} A'\varepsilon_{t+1} \\ \varepsilon_{t+1} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} \quad (9)$$

or more compactly

$$\boldsymbol{\xi}_{t+1} = G\boldsymbol{\xi}_t + \boldsymbol{\eta}_{t+1}. \quad (10)$$

Expression (10) is convenient because it allows one to construct linear forecasts of  $\mathbf{z}_{t+h}$  and  $\Delta \mathbf{y}_{t+h}$  for  $h = 1, \dots, H$  given information up to time  $t$ .

Specifically, notice that for  $z_{t+h}$  and  $\Delta \mathbf{y}_{t+h}$

$$\begin{cases} z_{t+h} = G_{[1,1]}^h z_t + G_{[1,2]}^h \Delta \mathbf{y}_t + \sum_{j=3}^{p-2} G_{[1,j]}^h \Delta \mathbf{y}_{t-j+2} + u_{t+h} \\ u_{t+h} = A'\varepsilon_{t+h} + A'(I + C_1)\varepsilon_{t+h-1} + \dots + A'(I + C_1 + \dots + C_{h-1})\varepsilon_{t+1} \\ \Delta \mathbf{y}_{t+h} = G_{[2,1]}^h z_t + G_{[2,2]}^h \Delta \mathbf{y}_t + \sum_{j=3}^{p-2} G_{[2,j]}^h \Delta \mathbf{y}_{t-j+2} + \mathbf{v}_{t+h} \\ \mathbf{v}_{t+h} = \varepsilon_{t+h} + C_1\varepsilon_{t+h-1} + \dots + C_{h-1}\varepsilon_{t+1} \end{cases} \quad (11)$$

for  $h = 1, \dots, H$  and where  $G_{[i,j]}^h$  denotes the  $[i, j]$  block of the matrix  $G$  raised to the  $h^{th}$  power. Therefore, it is easy to see that the impulse responses associated to the system in (2) due to a shock to the long-run equilibrium are:

$$\begin{aligned} IR(z_{t+h}|A'\varepsilon_{t+1} = \mathbf{1}; \Upsilon_t) &= E_L(z_{t+h}|A'\varepsilon_{t+1} = \mathbf{1}; \Upsilon_t) - E_L(z_{t+h}|A'\varepsilon_{t+1} = \mathbf{0}; \Upsilon_t) \\ &= A'(I + C_1 + \dots + C_h)A = G_{[1,1]}^h + G_{[1,2]}^h A \end{aligned} \quad (12)$$

and

$$\begin{aligned} IR(\Delta \mathbf{y}_{t+h} | A' \varepsilon_{t+1} = \mathbf{1}; \Upsilon_t) &= E_L(\Delta \mathbf{y}_{t+h} | A' \varepsilon_{t+1} = \mathbf{1}; \Upsilon_t) - E_L(\Delta \mathbf{y}_{t+h} | A' \varepsilon_{t+1} = \mathbf{0}; \Upsilon_t) \\ &= C_h A = G_{[2,1]}^h + G_{[2,2]}^h A \end{aligned} \quad (13)$$

Several features of expressions (12) and (13) deserve comment. First, the definitions of the impulse responses are somewhat unusual because they have the flavor of a dynamic average treatment effect where the treatment is standardized to be  $A' \varepsilon_{t+1} = \mathbf{1}$  and the non-treatment  $A' \varepsilon_{t+1} = \mathbf{0}$ . That is, the impulse response focuses directly on disturbances to long-run equilibrium, but it is not explicit about the source of this disturbance (i.e. we do not examine shocks to any one particular element of  $\Delta \mathbf{y}_{t+1}$ ). In our application  $A$  is  $n \times 1$  so that  $z_{t+1}$  is scalar and  $A' \varepsilon_{t+1} = 1$  is also scalar and uniquely determined. For this reason, we are not required to make the type of identification assumption (such as the ubiquitous Cholesky assumptions) commonly found in the vector autoregressive (VAR) literature regarding the constituent elements in  $\Delta \mathbf{y}_{t+1}$ . Second, for reference to what has been traditional in the literature in the past we note that the usual (not-orthogonalized) impulse responses analyzed correspond to the elements in  $C_h$  in expression (13), which in our set up amounts to  $G_{[2,1]}^h A' + G_{[2,2]}^h$ . Third, notice that the impulse responses in expressions (12) and (13) depend on the sum of two components,  $G_{[i,1]}^h$  and  $G_{[i,2]}^h$ ,  $i \in \{1, 2\}$ . The terms  $G_{[i,1]}^h$  describe the effect that adjustment to the long-run equilibrium  $z_t$  has on the impulse response. The terms  $G_{[i,2]}^h$  describe instead the effect that the short-run dynamics in  $\Delta \mathbf{y}_{t+1}$  have on the impulse response. In our application, such demarcation is very useful because it orthogonalizes the dynamic response to the PPP-HBS term from the response to short-term frictions due to unmodelled factors.

It has been traditional to estimate vector error correction models (VECMs) and obtain estimates of the system's impulse responses ( $C_h$ ) with nonlinear transformations of the conditional mean parameter estimates. Instead, we find it more convenient and suitable to the economic question of interest to directly estimate the terms  $G_{[i,j]}^h$ ,  $i, j \in \{1, 2\}$  with local projections (Jordà 2005, 2009) from expressions (11). This is advantageous for several reasons: (1) direct estimates of expressions (11) can be done equation by equation with little loss of efficiency (see Jordà, 2005) and therefore can be adapted directly to panel estimation. A VECM specification is too parametrically intensive in our application to afford sufficiently rich dynamics; (2) direct estimation of (11) does not restrict the dynamics of the estimated impulse responses across periods and is therefore robust to misspecification and generalizable to allow for nonlinear effects (see Jordà 2005 for examples in the stationary context); (3) because the coefficients in  $G_{[i',j]}^h$ ,  $i, j \in \{1, 2\}$  are computed directly from regression, computation of standard errors is straight-forward. By contrast,

VECM estimates are based on nonlinear functions of estimated parameters with differing rates of asymptotic convergence which complicates calculation of appropriate inferential procedures; and (4) direct estimates provide the orthogonalization of the impulse responses that we seek in a natural and uncomplicated way whereas orthogonalization with VECM estimates would require complicated nonlinear transformations once again.

To get a better sense of the large sample properties of the local projection estimator, consider first the simpler non-panel case. Let  $Z_H$  be the  $(T - p - H) \times kH$  matrix that collects observations for  $(\mathbf{z}_{t+1}, \dots, \mathbf{z}_{t+H})'$ ; and let  $Y_H$  be the  $(T - p - H) \times nH$  matrix that collects observations for  $(\Delta \mathbf{y}_{t+1}, \dots, \Delta \mathbf{y}_{t+H})'$ . Next define the regressor matrices, with  $X$  a  $(T - p - H) \times (k + n)$  matrix that collects observations for  $(\mathbf{z}_t, \Delta \mathbf{y}_t)'$  and  $W$  a  $(T - p - H) \times (1 + np)$  matrix that collects observations for  $(1, \Delta \mathbf{y}_{t-1}, \dots, \Delta \mathbf{y}_{t-p})'$ . Notice that  $W$  collects all the regressors whose coefficients are of no direct interest and hence the matrix  $M = I - W(W'W)^{-1}W'$  projects their effect away.

A direct estimate of the  $G_{[i,j]}^h$  given a first stage estimate of  $A'$  by conventional methods (or based on economic restrictions) can be found easily with the local projection estimator

$$\begin{aligned} \widehat{G}_z &= \begin{bmatrix} \widehat{G}_{[1,1]}^1 & \dots & \widehat{G}_{[1,1]}^H \\ \widehat{G}_{[1,2]}^1 & \dots & \widehat{G}_{[1,2]}^H \end{bmatrix} = (X'MX)^{-1} (X'MZ_H) \\ \widehat{G}_y &= \begin{bmatrix} \widehat{G}_{[2,1]}^1 & \dots & \widehat{G}_{[2,1]}^H \\ \widehat{G}_{[2,2]}^1 & \dots & \widehat{G}_{[2,2]}^H \end{bmatrix} = (X'MX)^{-1} (X'MY_H) \end{aligned} \quad (14)$$

with covariance matrices for  $\widehat{g}_z = \text{vec}(\widehat{G}_z)$  and  $\widehat{g}_y = \text{vec}(\widehat{G}_y)$  respectively

$$\widehat{\Omega}_i = \left[ (X'MX)^{-1} \otimes \widehat{\Sigma}_i \right]; \widehat{\Sigma}_i = \frac{\widehat{V}_i' \widehat{V}_i}{T - p - H} \text{ for } i \in \{z, y\} \quad (15)$$

with  $\widehat{V}_z = MZ_H - MX\widehat{G}_z$  and  $\widehat{V}_y = MY_H - MX\widehat{G}_y$ . Thus, the impulse responses in expression (13) can be constructed given estimates  $\widehat{A}$ ,  $\widehat{G}_z$  and  $\widehat{G}_y$ . Further, since  $A'$  is either imposed from theory or superconsistently estimated from typical cointegration procedures, then the standard regularity conditions made in expression (3) and the results in proposition 2 in Jordà and Kozicki (2010) are all that is needed to show that

$$\sqrt{T - p - H} (\widehat{g}_i - g_i) \rightarrow N(0, \Omega_i); i \in \{z, y\}$$

so that standard inferential procedures are readily available using (14) and (15).

A convenient feature of the local projection approach discussed in Jordà (2005) is that to estimate impulse responses in practice, one can obtain consistent estimates of the elements of  $\widehat{G}_{[i,j]}^h$  using equation-by-equation methods with little loss of efficiency when standard errors are calculated with non-parametric heteroscedasticity and autocorrelation robust (HAC) variance-covariance matrix estimators. Since most econometric packages are well suited to estimate panel

regressions, and have pre-built routines for (HAC) variance-covariance estimators, we find useful to take this more convenient approach since it lowers entry barriers to other researchers wishing to use our methods.

Extensions of the local projection estimator to panels in (11) is therefore straight-forward. In our application, we estimate (11) equation by equation for the panel of countries we consider and allow for country-fixed effects. It is well-known that in a regression model for panel data containing lags of the endogenous variable, the within-groups estimator can be severely downward biased when the serial correlation in the endogenous variable is high and the time series dimension  $T$  is short, regardless of the cross-section dimension  $M$ . This is often called the Nickell (1981) bias and a standard solution is to apply the Arellano and Bond (1991) GMM estimator. However, in our setting  $T = 31$  and  $M = 21$  which perhaps is best characterized by  $T/M \rightarrow c > 0$ ,  $T, M \rightarrow \infty$  asymptotics. Alvarez and Arellano (2003) show that in this case, the within-group estimator has a vanishing downward bias. In the simple case with no exogenous regressors and first order serial correlation, this bias is  $(1/T)(1 + \rho)$ , where  $\rho$  is the autocorrelation coefficient. For example, in the extreme where  $\rho \approx 1$ , the bias is approximately 0.06 given our sample size. However, the crude GMM estimator in this case is inconsistent despite being consistent for fixed  $T$ . For this reason we proceed with the within-group estimator. We remark on these issues because readers may find the panel-cointegration impulse response estimator useful for other applications where the rates at which  $M$  and  $T$  grow will differ from ours and where different estimation procedures will be indicated. The reader is referred to Alvarez and Arellano (2003), who provide a very good discussion on these issues.

Summarizing, the half-life of the PPP/HBS relation can be estimated directly by regressing leads of the cointegrating vector on its current values and on current and lagged values of  $\Delta \mathbf{y}_t$ . Standard errors for the relevant coefficients can be obtained by standard methods, which facilitate construction of confidence bands. The approach has several advantages: although we use a linear set-up to explain the method, it should be clear that ours is a semi-parametric method that does not restrict the half-life to decay monotonically. More generally and as is discussed in Jordà (2005), there is no obligation to rely on linear projections and one is free to examine nonlinear adjustment to long-run equilibrium with nonlinear projections (an approach best left for another paper here). Nonlinearities are easily accommodated with our equation-by-equation approach because they do not require full blown specification of what the nonlinear stochastic process for the entire system might be.

### 3 Data Description

We begin by describing the data. Our analysis is based on quarterly data over the 1973Q2–2008Q4 period for 21 OECD countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States. Nominal exchange rates and CPI data come mainly from the International Financial Statistics (IFS) database maintained by the IMF, except for Germany’s CPI, available from the OECD’s Main Economic Indicators database. CPI data are not seasonally adjusted at the source and therefore were harmonized with the X11 procedure, which is the standard method of seasonal adjustment for many statistical agencies. Data on GDP and GDP deflators (denoted PGDP) come primarily from the OECD’s Outlook database (except for Germany, which comes from the IFS database). Interest rate data come from the Global Financial database. A data file with full descriptions is available upon request.<sup>2</sup>

Throughout our analysis, lowercase symbols will always denote logarithms. Bilateral comparisons are made with respect to a reference country (denoted with the superscript \* in expression 2). The U.S. is a natural counterparty and so is the “rest of the world.” We consider the latter as it is well known that the choice of base country can substantially affect the statistical properties of real exchange rate dynamics. In particular, induced cross-sectional dependence can be an issue (see, e.g. O’Connell, 1998) and bias may result from the idiosyncratic behavior of a particular base country (see, e.g. Taylor, 2002).

In the case of the U.S. base, we construct for all other countries  $i$  the logarithms of real exchange rates as  $q_{it}^{US} = e_{it}^{US} + p_t^{US} - p_{it}$ , where  $e_{it}^{US}$  is the log nominal exchange rate, denoted in units of home currency per USD, and  $p_{it}$  and  $p_t^{US}$  are the log CPI levels of the home country and the U.S., respectively. An increase in  $q_{it}^{US}$  means a real depreciation of home currency, i.e., home goods are becoming less expensive relative to the U.S. The relative productivity term, or *HBS term*, is naïvely measured by the log real GDP per capita ratio,  $x_t^{US} - x_{it}$ , where  $x_{it}$  and  $x_t^{US}$  denote the log real GDP per capita levels of the home country and the U.S., respectively. The inflation differential is defined as  $\pi_t^{US} - \pi_{it}$ , where  $\pi_{it}$  and  $\pi_t^{US}$  are the inflation rates in percent per quarter for each country, while the interest rate differential is defined as,  $i_t^{US} - i_{it}$ , where  $i_{it}$

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<sup>2</sup> Note that in the OECD databases, data for the Euro area countries are now expressed in euro, so pre-1999 data were converted from national currency using official euro conversion rates. Consequently, “national currency” in always refers to euro for the Euro area countries, and not the legacy currencies. Germany GDP data from the IFS in billions of deutsche mark at quarterly level were converted to millions of Euro at annual level. Regarding 3-month interest rate series, in cases that 3-month interbank rates (IB) are not available, changes of alternative rates were applied to the end IB observation to recover levels. The alternative rates we used as substitutes were (in order of preference) the 3-month commercial paper rate, the 3-month T-bill rate, Government note yields, and other fixed-income rates. For the 11 Euro area countries, the EuroLIBOR rates are spliced at the end.

and  $i_t^{US}$  are 3-month interest rates, also in percent per quarter. Finally, the variable *momentum*,  $m^{US}$ , is defined as the rate of return in percent per quarter for the carry trade strategy of going long in the U.S. dollar while going short in home currency,  $m_{t+1}^{US} = \Delta e_{i,t+1}^{US} + (i_t^{US} - i_{it})$ , where the units are commensurate because the difference is taken over quarterly observations and the interest rates are measured on a per quarter basis.

We also generate a complete set of series relative to the “World” base, i.e., relative to the average value of all other countries in the sample. The world-based series can be easily constructed using the U.S.-based series. Suppose that  $y_{i,t}^{US}$  denotes the collection of U.S.-based variables for country  $i$ . Then the vector of world-based variables is given by  $y_{i,t}^{World} = y_{i,t}^{US} - \frac{1}{M-1} \sum_{j \neq i} y_{j,t}^{US}$ ,  $i = 1, 2, \dots, 21$ . To summarize the key features of our raw data for the HBS hypothesis, charts showing the time series for  $q_{it}$  and  $x_t^R - x_{it}$  for each country are presented in the Appendix Figures A1 (U.S. base) and A2 (“World” base).

### 3.1 Panel Unit Root Tests

For each of the 21 countries in our sample and each of the relevant variables in the system of expression (2), Table 1 reports univariate unit root tests based on Elliott, Rothenberg and Stock’s (1996) DF-GLS procedure. In addition, we also conduct Pesaran’s (2003) CADF test for non-stationarity in heterogeneous panels with cross-section dependence. This test proposes as its null hypothesis that all cross-section units in the panel are non-stationary and is consistent for the alternative that all or only a fraction of the units are stationary. Under each variable-name heading, the first column (in plain text) shows the test statistics for the U.S.-base series, and the second column (in italics) refers to the “World”-base series.

The individual DF-GLS test statistics suggest quite clearly that log CPI differentials ( $p_t^* - p_{it}$ ), log nominal exchange rates ( $e_t$ ) and log real GDP per capita ratios ( $x_t^* - x_{it}$ ) are non-stationary, whereas all elements in  $\Delta y_{t+1}$  are stationary.<sup>3</sup> However, the DF-GLS tests on log real exchange rates ( $q_t$ ), which are intended to check whether  $e_{t+1}$ ,  $(p_{t+1}^* - p_{t+1})$ , and  $(x_{t+1}^* - x_{t+1})$  are cointegrated, yield mixed evidence, especially those relative to the U.S. This is not too surprising. Our data series have a short span: “only” four decades. A robust and powerful rejection of the non-stationarity null with slowly-reverting series like real exchange rates may require a span of data covering a century or more (Frankel 1986). Thus, we consider these short-span univariate tests inconclusive.

However, in the literature on real exchange rate dynamics, when long span data are not avail-

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<sup>3</sup> I.e. interest rate differentials ( $i_t^* - i_{it}$ ), inflation rate differentials ( $\pi_t^* - \pi_{it}$ ), first differences of log nominal exchange rates ( $\Delta e_{it}$ ), and first differences of log real GDP per capita ratio ( $\Delta x_t^* - \Delta x_{it}$ ).



able, one widely used method to enhance the power of unit root tests is to employ panel methods (see the discussion in Taylor and Taylor, 2004). The Pesaran CADF panel tests show that, at the conventional significance level, that all  $x_t^* - x_{it}$  series in the panel are non-stationary, whereas for any other variables, at least one series in the panel is stationary. As a result, we also test for stationarity based on the magnitude of the test statistic. As large negative values of the CADF statistic indicate rejections of null of universal non-stationarity,  $\Delta e_{it}$ , and  $\Delta x_t^* - \Delta x_{it}$  are shown to be the variables most likely to be stationary, followed by  $i_t^* - i_{it}$ ,  $\pi_t^* - \pi_{it}$ , then by  $p_t^* - p_{it}$ ,  $e_{it}$ , and  $q_{it}$ .

Based on the weak power of the univariate tests, and the supportive panel results, one might proceed under the assumption that the real exchange rate is stationary, in accord with the findings from longer-span studies (e.g., Taylor, 2002). However, according to the HBS theory there is another possibility: that the real exchange rate bears an equilibrium relationship to—and cointegrates with—a relative productivity proxy.

### 3.2 Panel Cointegration Tests

Thus, we next examine the cointegration properties between the real exchange rate,  $q_{it}$  and the productivity proxy differential,  $(x_t^* - x_{it})$ , for reference base U.S. or “World.” We begin by testing the null of no cointegration with the battery of seven residual-based test statistics presented in Pedroni (1999).

Each of these statistics is a panel version of the conventional univariate counterparts presented in Phillips and Perron (1988) and Phillips and Ouliaris (1990). The first four statistics are based on pooling the data across the within dimension of the panel, specifically, the panel- $v$  statistic is a type of nonparametric variance ratio statistic, the panel- $\rho$  statistic is analogous to the Phillips and Perron (1998)  $\rho$ -statistic. Similarly, the nonparametric panel- $t$  statistic is analogous to the Phillips and Perron (1998)  $t$ -statistic, and finally the parametric  $t$ -statistic is a panel version of the augmented Dickey Fuller  $t$ -statistic. The remaining three statistics are constructed by pooling the data along the between-dimension of the panel. These statistics in effect compute the group mean of the individual conventional time series statistics, and hence the name, group-mean statistics. The group- $\rho$  and group- $t$  statistics are analogous to the Phillips and Perron (1998)  $\rho$ -statistic and  $t$ -statistic, respectively; and the last parametric group- $t$  statistic is analogous again to the ADF  $t$ -statistic. For the panel- $v$  statistics, large positive values indicate rejection, whereas for all other statistics, large negative values indicate rejection of the null.

For our purposes, we implement Pedroni’s tests using estimated residuals from the equation

$$q_{it} = \alpha_i + \beta_i(x_t^* - x_{it}) + u_{it}, \quad (16)$$



where  $q_{it}$  is the logarithm of the real exchange rate for country  $i$ ,  $(x_t^* - x_{it})$  is the productivity differential for country  $i$ , and each is computed using both the U.S. or World as the reference country, the  $\alpha_i$  are country fixed-effects, and the  $\beta_i$  are potentially heterogenous slope parameters for each country. Allowing for heterogeneity in  $\beta_i$  is important: it ensures that we do not bias the results toward rejecting the null of no cointegration.

However, these tests are not unproblematic. We should remark that the critical values tabulated by Pedroni (1999, 2004) for these cointegration tests rely on the assumption of cross-sectional independence in the error term, a condition that is likely to be violated when the reference country is the U.S. but which is much alleviated when using the World as the reference country and one makes further allowance for fixed (common) time effects in expression (16).

Furthermore, Kremers, Ericsson, and Dolado (1992) suggest that the Pedroni-type residual-based cointegration tests require the long-run cointegrating vector for the variables in the levels to be equal to the short-run adjustment process for the variables in the differences. Failure of this common factor restriction causes significant loss of power in the Pedroni procedures.

For these reasons, we considered four additional cointegration tests proposed in Westerlund (2007).<sup>4</sup> These tests are based on residual tests that explicitly model short-run dynamics and do not rely on the common factor assumption. They can also be implemented in such a way as to allow for cross-sectional dependence.

The four Westerlund (2007) tests are panel data extensions of the cointegration tests proposed by Banerjee, Dolado, and Mestre (1998) and the underlying premise is to test the null of no cointegration with residual based tests from regressions given by

$$\begin{aligned} \Delta q_{it+1} = & \delta_i + \alpha_i(q_{it} - \beta_i(x_t^* - x_{it})) + \\ & \sum_{j=0}^{p_i} \phi_{ij} \Delta q_{it-j} + \sum_{j=0}^{p_i} \gamma_{ij} \Delta(x_{t-j}^* - x_{it-j}) + u_{it}. \end{aligned} \quad (17)$$

The panel statistics denoted  $P_\tau$  and  $P_\alpha$  (using the nomenclature in Westerlund, 2007) test the null of no cointegration against the simultaneous alternative that the panel is cointegrated, whereas the group mean statistics  $G_\tau$  and  $G_\alpha$  test the null of no cointegration against the alternative that at least one element in the panel is cointegrated. We use the Barlett kernel to estimate the long-run variances semi-parametrically, and obtain robust critical values with the bootstrap to account for cross-sectional correlation.

Table 2 summarizes the battery of Pedroni's and Westerlund's cointegration tests. The null hypothesis of no cointegration between  $q_{it}$  and  $x_t^* - x_{it}$  is rejected firmly by all Pedroni test

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<sup>4</sup> These tests are implemented using STATA code prepared by Persyn and Westerlund (2008).

**Table 2. Panel Cointegration Tests**

Pedroni's Tests	US base	"World" base	Westerlund's Tests	US base	"World" base
Panel-v	7.70***	<i>8.04***</i>	P $\tau$	-2.64	-4.79***
Panel- $\rho$	-5.41***	<i>-6.03***</i>	P $\alpha$	-3.92**	-6.53***
Panel-t	-3.74***	<i>-4.13***</i>	G $\tau$	-1.77	-4.13***
Panel-t (parametric)	-2.76***	<i>-3.82***</i>	G $\alpha$	-1.40	-4.52***
Group- $\rho$	-2.94***	<i>-4.81***</i>			
Group-t	-2.85***	<i>-3.79***</i>			
Group-t (parametric)	-1.77*	<i>-3.30***</i>			

1. The test results of US-based series are in regular fonts while those of world-based series are in *italics*.
2. The panel statistics test the null of no cointegration against the alternative that all units in the panel are cointegrated, whereas the group mean statistics test the null of no cointegration against the alternative that at least one element in the panel is cointegrated. The significance of the pedroni's test statistics is determined by standard normal critical values, while the significance of the Westerlund's (2007) test statistics is determined by the robust critical values generated by 1000 times bootstraps. \*, \*\* and \*\*\* indicate 10%, 5% and 1% rejection levels, respectively.
3. The optimum lag length in the ADF-type regression of Pedroni's tests is selected by Schwarz information criterion with a maximum 12 lags. The fixed number of lags  $p_i$  to be included in the error correction equations of Westerlund's tests is determined by Akaike information criterion from 0 to 2 lags for each separate series.

statistics, for both the US-base and "World"-base data. However, these rejections, especially when the reference country is the U.S. may not be as strong as they appear to be. This is due to the fact that we evaluate the significance of the Pedroni's standardized test statistics using  $N(0, 1)$  critical values, despite possible cross-sectional independence. In contrast, the significance of the Westerlund test statistics is determined by robust critical values generated by 1000-time bootstraps. The null of no cointegration is rejected by all four of the Westerlund test statistics at the 1% significance level for the "World"-base series, while only by the  $P_\alpha$  statistic at the 5% level for the US-base series. Overall, the panel cointegration tests indicate that  $q_{it}$  and  $x_t^* - x_{it}$  are cointegrated, especially those relative to the "World"-base.

## 4 Estimating the HBS Cointegrating Vector

In estimating the panel cointegrating vector, we employ the group-mean panel DOLS estimator suggested in Pedroni (2001). As he points out, the point estimates for these between-dimension estimators can be interpreted as the mean value for the cointegrating vectors in the event that the true cointegrating vectors are heterogeneous. Specifically, Pedroni (2000) shows that the between-dimension estimators appear to suffer from much lower small-sample size distortion than the within-dimension estimators.

To obtain the group mean panel DOLS estimator, we first estimate country-specific cointegrating vectors using dynamic OLS suggested by Phillips and Loretan (1991). The DOLS for an individual country is formulated as:

$$q_{i,t} = \alpha_i + \beta_i(x_{i,t} - x_t^*) + \sum_{s=-p_i}^{p_i} \theta_{i,s}(\Delta x_{i,t+s} - \Delta x_{t+s}^*) + \mu_{i,t}, \quad (18)$$

where asterisk denotes either the US or the “World” counterparts. The country-specific optimal numbers of lags and leads,  $p_i$  are selected by AIC. Next we construct the group-mean panel DOLS estimator according to Pedroni (2001),  $\hat{\beta}_{GM} = N^{-1} \sum_{i=1}^N \hat{\beta}_i$ , where  $\hat{\beta}_i$  is the DOLS estimator, applied to the  $i$ th country of the panel. The associated t-statistic for the between-dimension estimator above can be constructed as  $t_{\hat{\beta}_{GM}} = N^{-1/2} \sum_{i=1}^N t_{\hat{\beta}_i}$ . As can be seen clearly from the group mean formula, the panel estimate of the cointegrating vector is an average of the estimated individual cointegrating vectors.

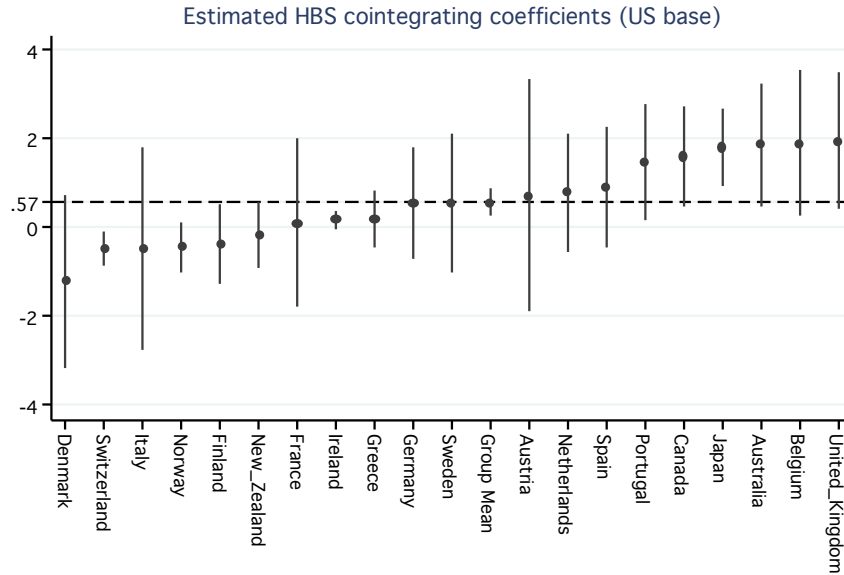
Table 3 reports the estimated individual slopes,  $\hat{\beta}_i$ , and group mean slope,  $\hat{\beta}_{GM}$  for our HBS term, for the U.S. and “World” base cases. A few results are worth noting. First, the point estimates are positive in most cases as predicted by the HBS theory, albeit they do vary greatly among different countries. Second, the slopes estimated with the “World”-base data are in general steeper and more significant than the corresponding ones estimated with the U.S.-base data. As a result, the “World”-base group mean slope estimate is about twice the size of the U.S.-base one, though both are positive and statistically significant at the 1% level.

The positive group mean estimate leads us to conclude that the HBS effect is present in the panel, even though there is substantial variation in point estimates (and wide confidence bands) as we look at individual countries. A slope of 0.57 for the U.S. base and 0.78 for the world base is suggestive of an economically meaningful and qualitatively significant elasticity of price levels with respect real income levels, and, within the range of estimation uncertainty, is consistent with reasonable values for the share of nontraded goods in typical theoretical models of the HBS effect.

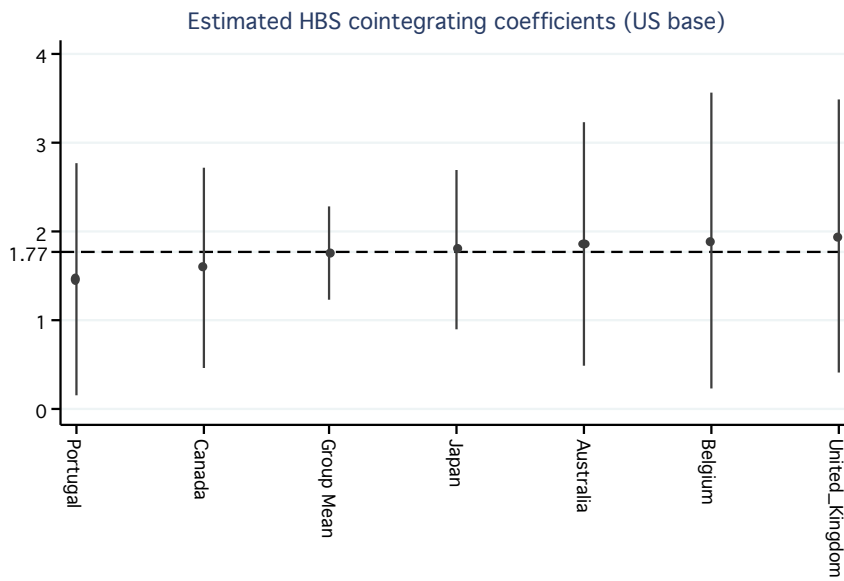
To clearly illustrate the magnitude of these effects, Figure 1(a) plots the HBS slope estimates (individual and group mean) for the full sample using the U.S. base, where the group mean is 0.57. Figure 1(b) repeats the exercise when the panel estimation is limited to the 6 countries (top tercile) for which the largest HBS slope coefficient is found; for this group all coefficients are significant and the group mean slope estimate rises to 1.77. Figure 2 repeats this exercise for the “World” base, where the group mean is 0.78 for the full sample and 1.72 in the case of the 6 top tercile countries. These results support the hypothesis that an HBS effect is present, and although possibly heterogeneous, it could be qualitatively large in some subset of the countries in our sample. It now remains to be seen whether the existence of such effects makes a material difference to the methods used to assess the dynamic speed of adjustment of the real exchange rate to its long run equilibrium value.

Figure 1: Estimated HBS cointegrating coefficients (U.S. base)

(a) Sample = all countries



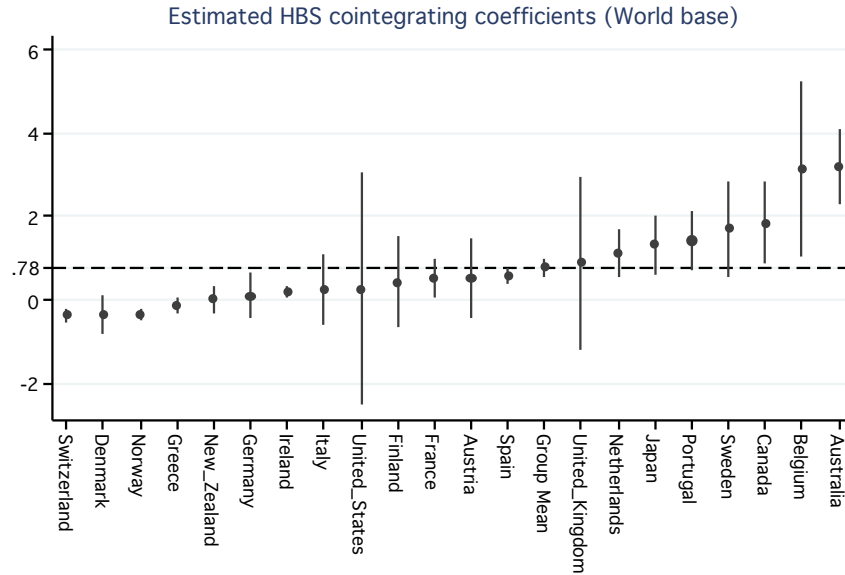
(b) Sample = countries with significant coefficients



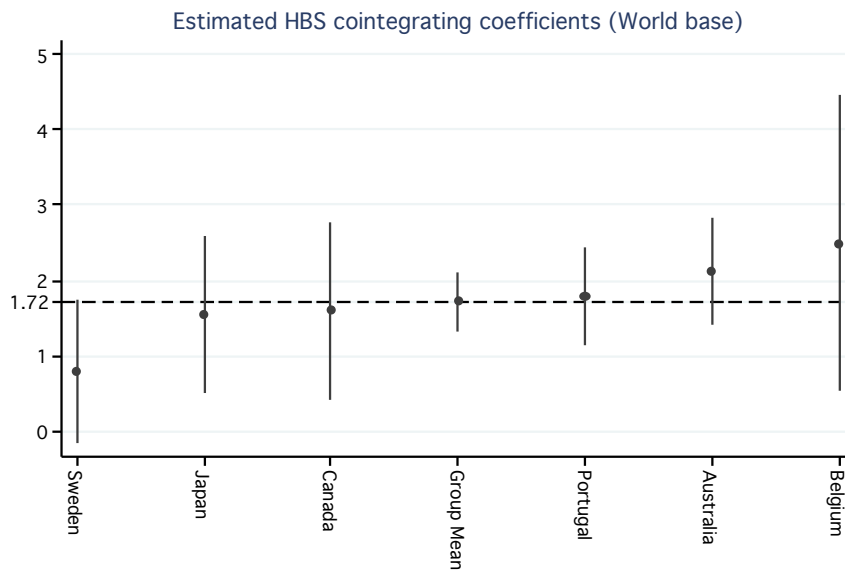
Notes: The sample period is 1973Q2 to 2008Q4.

Figure 2: Estimated HBS cointegrating coefficients (“world” base)

(a) Sample = all countries



(b) Sample = countries with significant coefficients



Notes: The sample period is 1973Q2 to 2008Q4.

**Table 3. Estimated HBS Cointegrating Coefficients**

Country	US base			World base		
	Leads/Lags	Slope	t-value	Leads/Lags	Slope	t-value
Australia	1	1.87***	2.67	1	3.20***	6.9
Austria	1	0.72	0.54	1	0.55	1.12
Belgium	1	1.90**	2.26	1	3.16***	2.96
Canada	2	1.61***	2.81	1	1.85***	3.75
Denmark	1	-1.2	-1.22	1	-0.34	-1.39
Finland	1	-0.35	-0.79	1	0.42	0.76
France	1	0.11	0.12	1	0.53**	2.31
Germany	1	0.54	0.84	1	0.11	0.43
Greece	1	0.19	0.58	2	-0.13	-1.21
Ireland	1	0.18*	1.82	1	0.18**	2.45
Italy	1	-0.46	-0.4	1	0.24	0.55
Japan	1	1.81***	3.98	2	1.33***	3.64
Netherlands	1	0.8	1.17	1	1.11***	3.82
New Zealand	1	-0.17	-0.45	1	0.03	0.2
Norway	1	-0.44	-1.48	1	-0.33***	-4.79
Portugal	1	1.47**	2.2	2	1.43***	3.99
Spain	1	0.91	1.31	1	0.57***	5.9
Sweden	1	0.54	0.67	2	1.72***	2.91
Switzerland	1	-0.48**	-2.39	1	-0.36***	-4.1
United Kingdom	1	1.96**	2.51	1	0.89	0.84
United States	-	-	-	1	0.27	0.19
<b>Group Mean</b>		<b>0.57***</b>	3.74		<b>0.78***</b>	6.81

\*, \*\* indicate 10% and 5% rejection levels, respectively

## 5 Estimating Half-lives of Deviations from Equilibrium

The previous sections provide supporting evidence that recommends analyzing the system in expression (2) using  $z_t = q_t - \beta(x_t^* - x_t)$  as the cointegrating vector. We proceed by defining the system's cointegrating vector for each country  $i$  as  $z_{it} = q_{it} - \hat{\beta}_{GM}(x_{it}^* - x_{it})$ , where  $\hat{\beta}_{GM}$  is the group mean slope estimate which is applied as a homogenous slope to all units in the panel.

The key results are shown in Figures 3 through 6. Each figure has an identical format for consistent presentation. The top panel of each of these figures (labeled “panel (a)”) displays two impulse responses to a unit shock to long-run equilibrium when the HBS term is excluded (so that we have pure PPP effects only). The left-hand graph displays the traditional response, which conflates long-run and short-run dynamics, as we explained previously (these are labeled “incorrect”). The right-hand graph displays the response that isolates the long-run dynamics only (and are therefore labeled “correct”). The bottom panels of each of Figures 3 to 6 (labeled “panel (b)”) are organized like those in panel (a) but they refer to the case when HBS effects are included instead.

While there is no ambiguity when constructing the long-run response to a shock in  $z$  (the

response labeled correct), the short-run responses will vary depending on the sources of the shock to  $z$  (and the responses labeled incorrect will also vary). In order to stack the odds against our procedures, we decided to construct a unit disturbance in  $z$  using weights inverse to the variability (measured by the mean absolute deviation) of each component in the cointegrating vector (i.e., nominal exchange rates, price levels and productivity differentials) in the differences. We also experimented with weights determined by the cointegrating relation itself, but these resulted in incorrect responses with even longer total half-lives and wider confidence bands than the ones reported in Figures 3 to 6.

Next, Figures 3 and 4 use the U.S. as the reference country whereas Figures 5 and 6 use the “World” as the base. Finally, the differences between figures for a given country (3 versus 4 for the U.S. base and 5 versus 6 for the World base) refer to whether we look at all countries in our sample (Figures 3 and 5), or else the top tercile of countries by the size of the HBS effect in the cointegrating term (Figures 4 and 6). It is this last subset of graphs where one would expect to find that HBS effects are strongest and this will prove to be correct. Simultaneous confidence regions with 95% coverage are reported using Scheffé bands (see Jordà 2009).

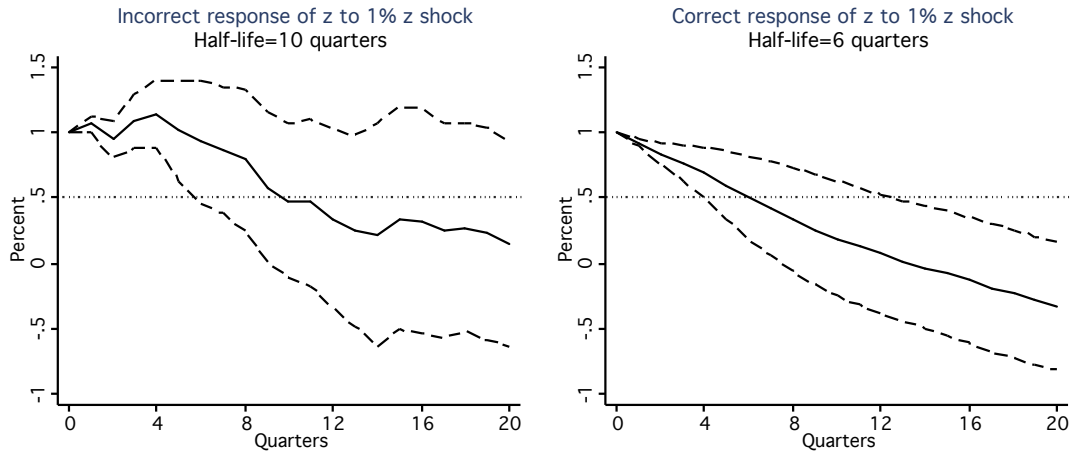
Several results deserve comment. First, there are no significant differences arising from whether we use the US or the World as the base country – the conclusions are qualitatively and quantitatively similar. Second, half-lives computed from *incorrect* responses are uniformly longer (by as much as five quarters) than the *correct* responses. Moreover, the contaminating effect of short-run dynamics is clearly visible in the lack of smoothness and imprecision that the *incorrect* responses display. In contrast, *correct* responses are smoother and much more precisely estimated. Third, Figures 4 and 6, which correspond to the experiments based on the top tercile of countries by the strength of the HBS effect in the cointegration term, show that the inclusion of the HBS term (seen by comparing panels (a) and (b) in these figures) results in a considerable reduction in estimated half-lives in the *correct* responses.

This is one of our principal findings: omitting the HBS effect, as is common, leads to an upwardly biased estimate of the half-lives of deviations from real exchange rate equilibrium. However, this depends on the correct modeling of the dynamics.

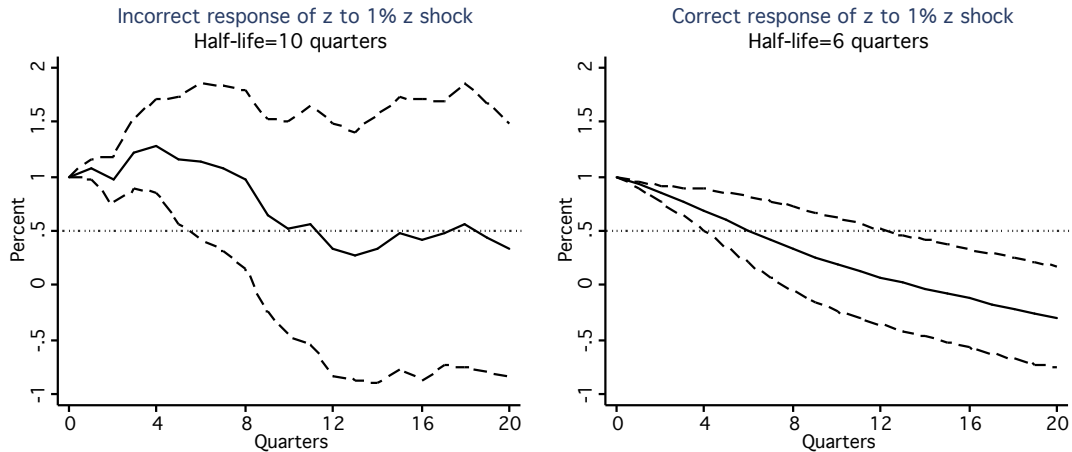
Interestingly, the effect on the *incorrect* responses is ambiguous: for example, in Figure 4 (US base) the incorrect response actually results in a longer half-life when the HBS term is included, whereas in Figure 6 the effect is the opposite. Half-life estimates from *incorrect* responses corresponds with what has been found in the previous “PPP puzzle” literature. For example, Glen (1992), using nine countries over the period 1900-1987 found half-lives of 3.3 years. Our estimates using the World as the base result in a half-life estimate of 3 years, as can be seen in Figure

Figure 3: Estimated impulse response functions (U.S. base)

(a) Excluding the HBS term, sample = all countries



(b) Including the HBS term, sample = all countries

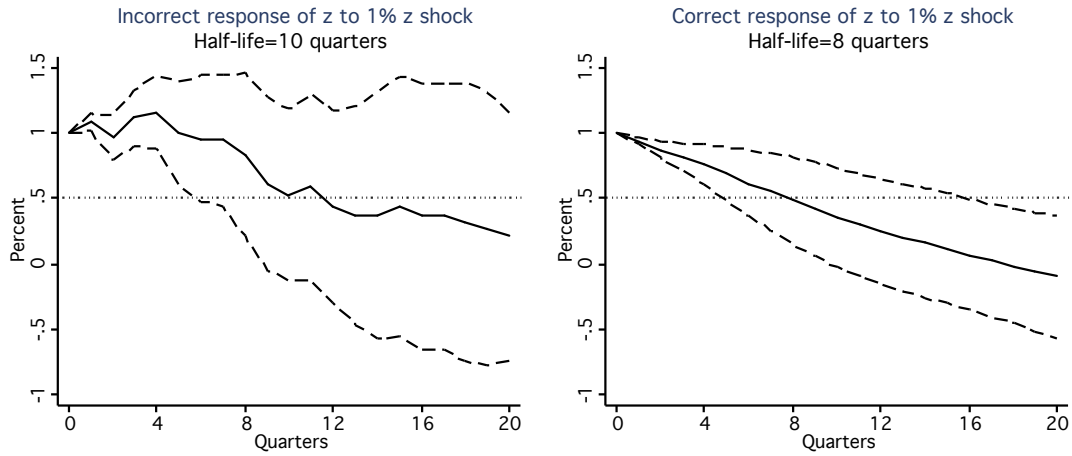


Notes: In panel (a), a one unit shock to  $q$  consists of .85 unit shock to  $e$  and .15 unit shock to  $(p^* - p)$ . In panel (b), a one unit shock to  $z$  consists of .71 unit shock to  $e$ , .12 unit shock to  $(p^* - p)$  and .17 unit shock to  $(x^* - x)$ . The 95% simultaneous confidence bands are shown. Sample countries in these charts: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Spain, Sweden, Switzerland, United Kingdom.

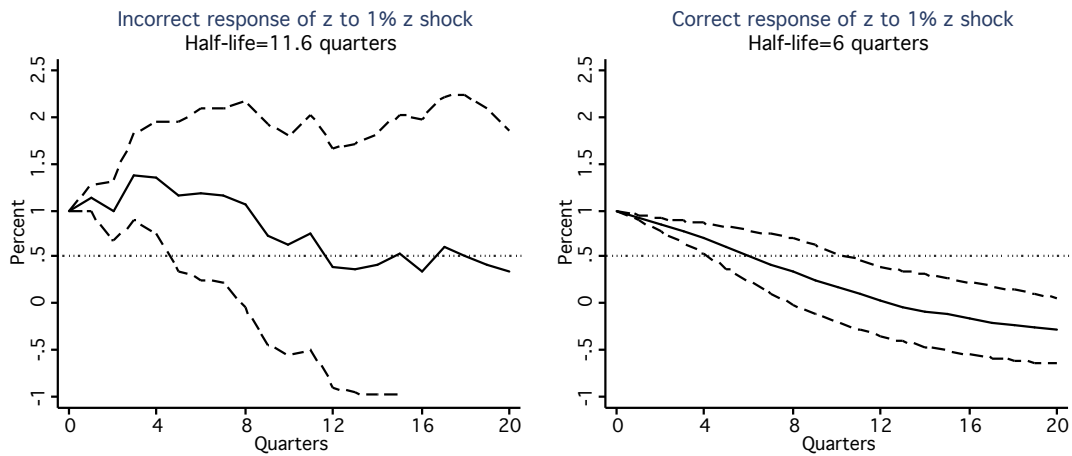


Figure 4: Estimated impulse response functions (U.S. base)

(a) Excluding the HBS term, sample = countries with significant coefficients



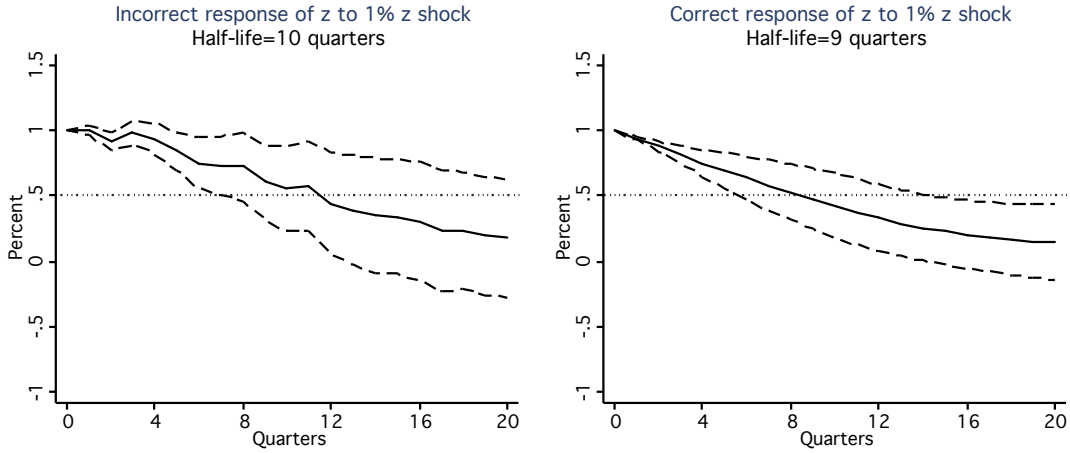
(b) Including the HBS term, sample = countries with significant coefficients



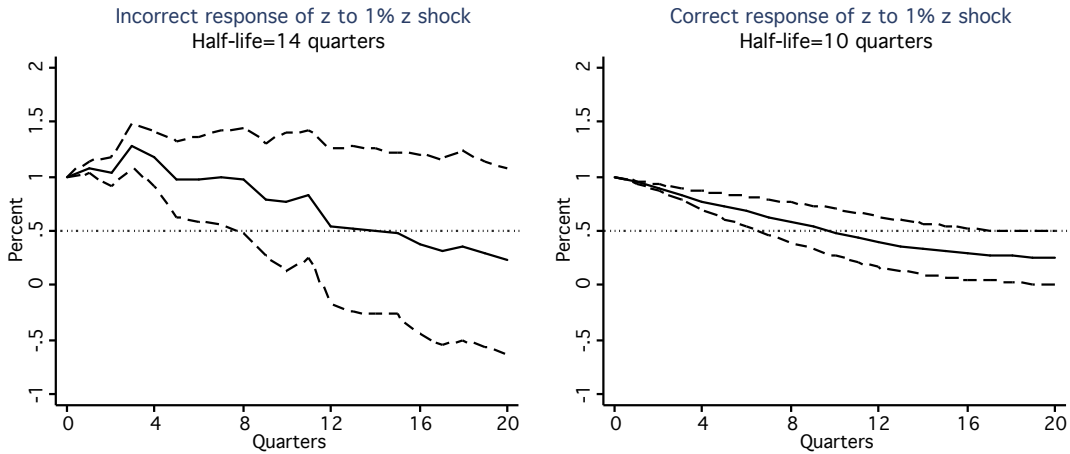
Notes: In panel (a), a one unit shock to  $q$  consists of .84 unit shock to  $e$  and .16 unit shock to  $(p^* - p)$ . In panel (b), a one unit shock to  $z$  consists of .71 unit shock to  $e$ , .14 unit shock to  $(p^* - p)$  and .15 unit shock to  $(x^* - x)$ . The 95% simultaneous confidence bands are shown. Sample countries in these charts: Australia, Belgium, Canada, Japan, United Kingdom, Portugal.

Figure 5: Estimated impulse response functions (“world” base)

(a) Excluding the HBS term, sample = all countries



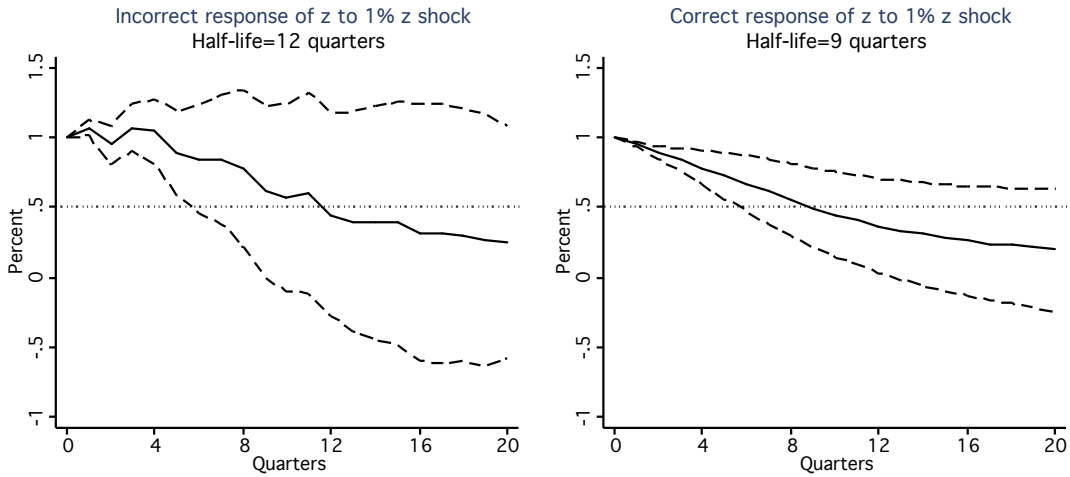
(b) Including the HBS term, sample = all countries



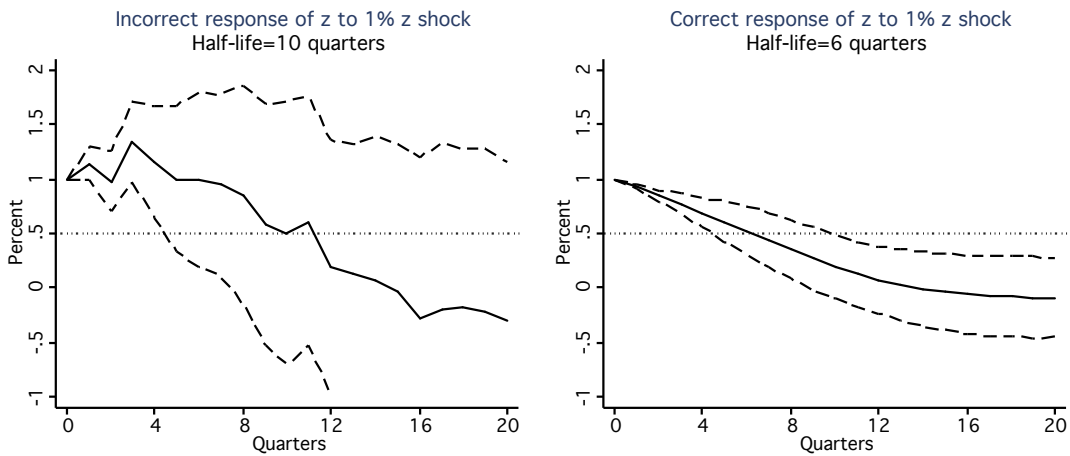
Notes: In panel (a), a one unit shock to  $q$  consists of .78 unit shock to  $e$  and .22 unit shock to  $(p * -p)$ . In panel (b), a one unit shock to  $z$  consists of .61 unit shock to  $e$ , .17 unit shock to  $(p * -p)$  and .22 unit shock to  $(x * -x)$ . The 95% simultaneous confidence bands are shown. Sample countries in these charts: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Spain, Sweden, Switzerland, United Kingdom.

Figure 6: Estimated impulse response functions (“world” base)

(a) Excluding the HBS term, sample = countries with significant coefficients



(b) Including the HBS term, sample = countries with significant coefficients



Notes: In panel (a), a one unit shock to  $q$  consists of .81 unit shock to  $e$  and .19 unit shock to  $(p^* - p)$ . In panel (b), a one unit shock to  $z$  consists of .68 unit shock to  $e$ , .16 unit shock to  $(p^* - p)$  and .16 unit shock to  $(x^* - x)$ . The 95% simultaneous confidence bands are shown. Sample countries in these charts: Australia, Belgium, Canada, Sweden, Portugal, Japan.

5(b). Taylor (2002), using 20 countries (mostly in the OECD) for the period 1850-1996 reports mean and median half-lives of 2-3 years. Using more complex nonlinear specifications based on an exponential smooth transition model, Peel, Sarno and Taylor (2001) found half-lives in the 3-5year range using four bilateral dollar exchange rates for the recent free floating period.

These results are reassuring because they suggest that neither the use of a local projection estimator; the specific cross-section of countries considered; the time period used; the frequency of the data; the linearity of the specification; nor the country used as base explain the differences between the literature and our findings. Rather, the main result that half-lives are about three to five quarters shorter than previously estimated is driven primarily by correctly allowing for the contribution of the HBS effect to long-run equilibrium adjustment. And to this end, it is important to isolate the contaminating effects of short-run frictions, which our local projection estimator for cointegrated systems clearly shows how to do.

## 6 Conclusion

Models of open economies describe the behavior of aggregate variables over medium- and long-run frequencies meant to reflect the time-scale of relevant policy questions. A central ingredient of such models is an assumptions about how secular movements in exchange rates are determined. In this respect, it has been traditional to focus on equilibrium conditions in fully flexible and frictionless markets, such as the well-known PPP and UIP conditions. While basic and powerful, these mechanisms have found precarious support in the data.

Harrod (1933), Balassa (1964) and Samuelson (1964) extended the notion of the PPP conditions to account for differences in the traded/non-traded sectors across economies that may persist over time due to differences in productivity. This paper investigates whether this mechanism provides sufficient texture to explain movements of exchange rates in the long-run by introducing new empirical methods whose applicability transcends this paper.

We take the view that there are many factors that influence exchange rates but we are interested in those whose effects are felt over the long-run rather than those whose effects are short-lived. The methods that we introduce are not simply another way of answering this question: they provide a decomposition of the data that is central to obtaining the correct answer.

The local projection approach serves to formulate how one can measure adjustment to long-run equilibrium in terms of the intrinsic long-run dynamics that the PPP/HBS hypothesis generates from all other factors whose role is limited to explain short-run movements. Previous studies do not make this distinction and hence provide contaminated measures of the HBS hypothesis. Not surprisingly, we find that empirical estimates of half-lives are considerably shorter than what has

been previously reported. Such findings provide support not only for the HBS hypothesis, but also for the view that equilibrium adjustment speeds are not so puzzling.

The local projection approach for cointegrated systems is an important econometric contribution in its own right. The methods not only proved useful in our application but also open the door for more sophisticated analysis of non-linear error correction adjustments that have been hitherto complicated by the need to specify nonlinear stochastic processes for the entire system of variables considered. It is our hope that this paper will also help to illustrate the advantages of this approach and inspire further applications.

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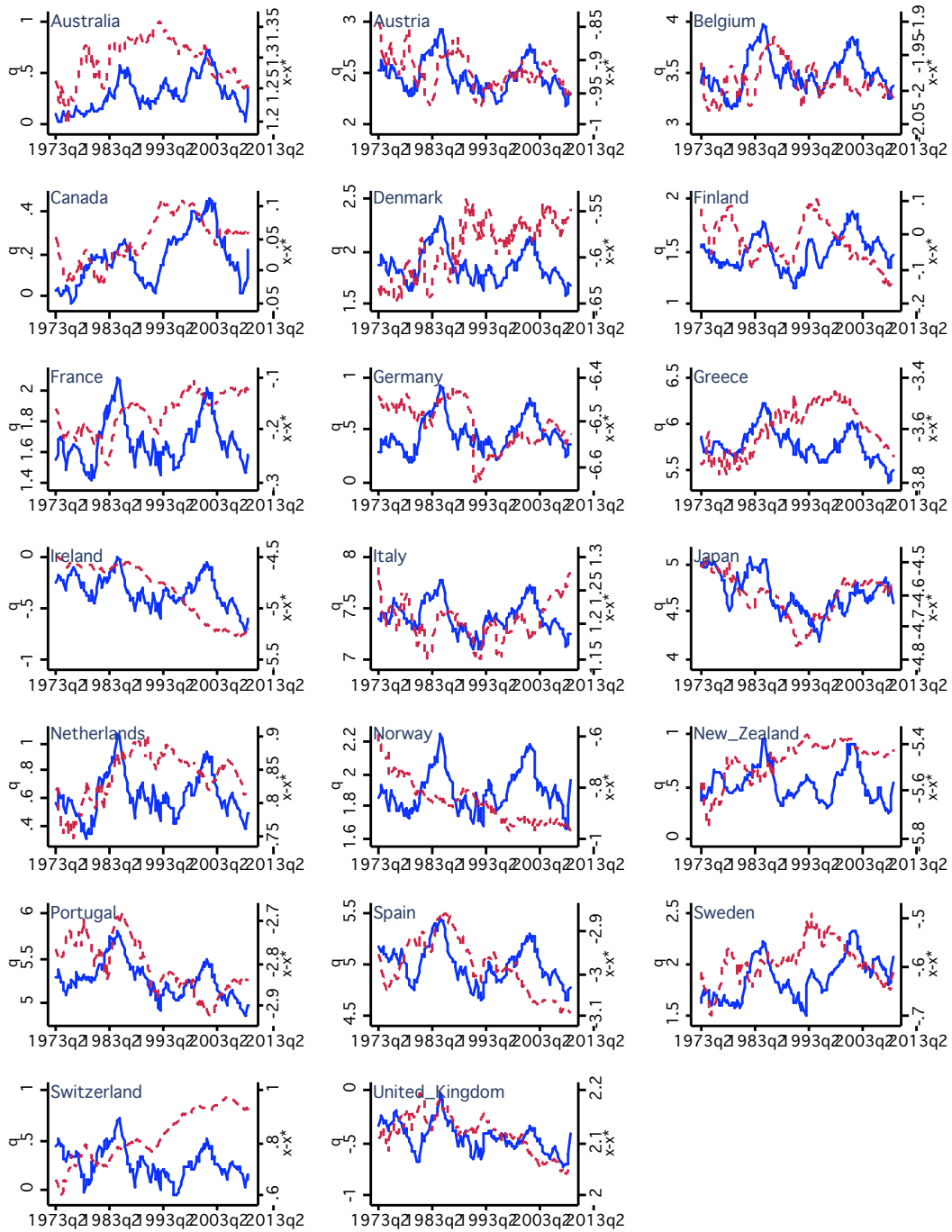
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## Appendix A: Raw Data

Figures A1 and A2 show the raw data for log real exchange rates and log relative GDP per capita for the U.S. and world bases, respectively.

Figure A1

IFS data:US-Based Log Real Exchange Rates and Log Per Capita Output Ratios

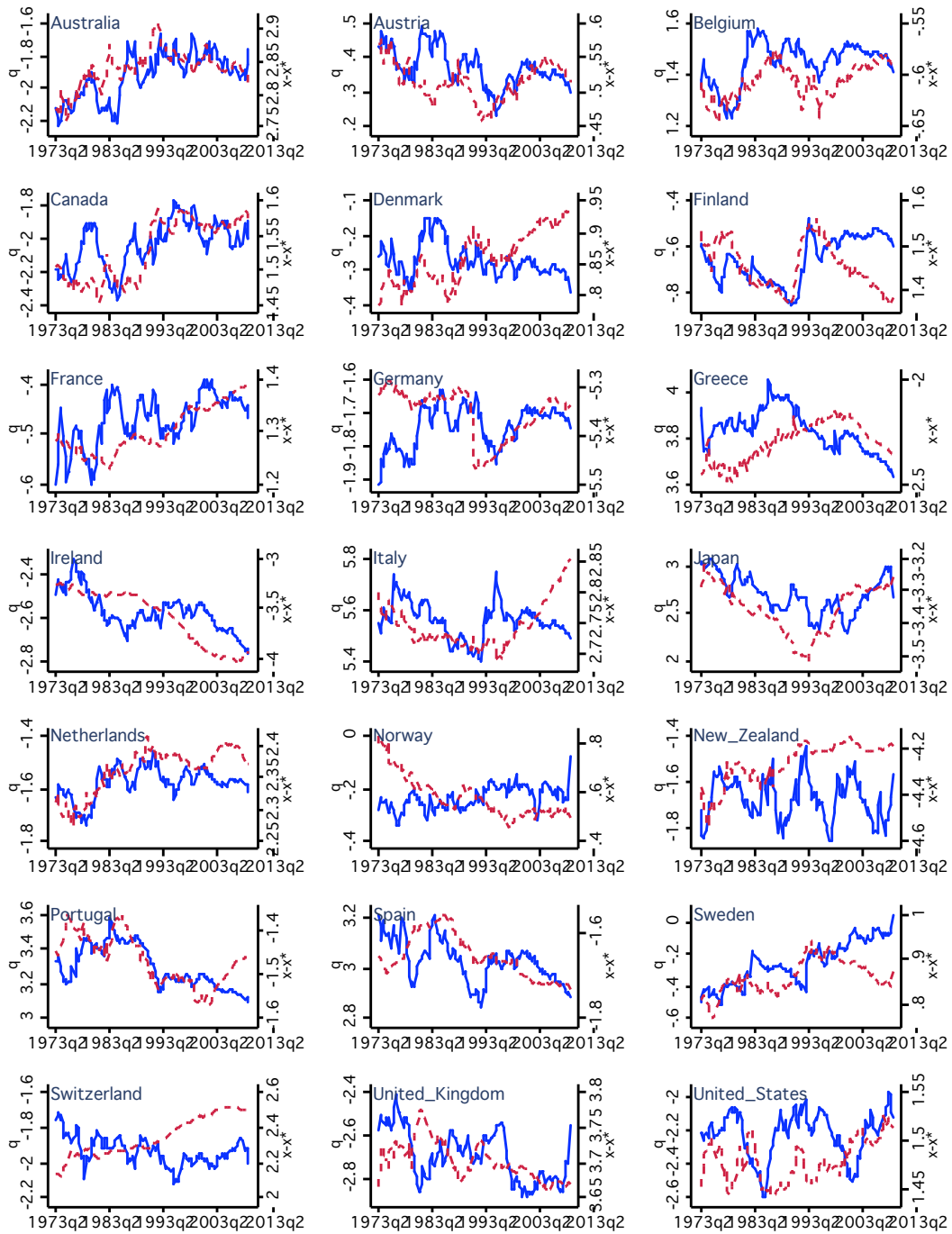


Note: Solid blue lines represent log real exchange rates, and Dashed red colored lines represent log real GDP per capita



Figure A2

IFS data:World-Based Log Real Exchange Rates and Log Per Capita Output Ratios



Note: Solid blue lines represent log real exchange rates, and Dashed red colored lines represent log real GDP per capita