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No. 7874

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ISSN 0265-8003

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Discussion Paper No. 7874
June 2010

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ABSTRACT

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JEL Classification: D44, D82, J31 and J41
Keywords: all-pay auctions, bid caps and multi-stage contests

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Submitted 23 May 2010

Caps in Sequential Contests

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Abstract

We study a sequential two-stage all-pay auction with two identical prizes. In each stage, the players compete for one prize and each player may win either one or two prizes. The designer may impose a cap on the players' bids in each of the stages. We analyze the equilibrium in this sequential all-pay auction with bid caps and show that capping the players' bids is profitable for a designer who wishes to maximize the players' expected total bid.

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KEYWORDS: Multi-stage contests, All-pay auctions, Bid caps

1 Introduction

In many competitions, we can often observe situations where severe constraints are imposed on contestants. For example, in the US electoral campaign, there is a specific maximum campaign contribution that a single agent can make to a candidate.[1] Also several sports leagues (e.g., the NBA) implement a salary cap, which is a limit to the total amount of money a team can spend on players' salaries. The actual amount of the cap varies on a year-to-year basis, and is calculated as a percentage of the league's revenue from the previous season.[2] Professional NBA players also face a variable salary cap where the maximum amount of money a

[1] Political Action Committees (PACs) can contribute at most \$5,000 per election to a candidate, while individuals can contribute at most \$1,000. About caps in political lobbying see Che and Gale (1998).

[2] For instance, in 2007-08, the NBA salary cap was approximately US \$55.63 million per team, and for the 2008-09 season it was \$58.68 million. The 2009-10 salary cap has been set at \$57.7 million.

player can sign for is contingent on the number of years that he has played and on the total of the salary cap.[3]

The caps imposed on players have an ambiguous effect on their bids. On the one hand, relatively weak players believe that they will have a higher chance to win and therefore will make more effort than in a contest without a bid cap. On the other hand, relatively strong players will make less effort than in a contest without a bid cap. The effect of the bid cap on the players' total bid, therefore, depends on the trade-off between the increase of the weak players' effort and the decrease of the strong players' effort. The all-pay auction would seem to be the natural model to examine the effect of bid caps because of its simple structure and its advantages over other contest forms.[4] Laffont and Robert (1996), for example, showed that an all-pay auction with a reserve price is a revenue-maximizing mechanism for selling one object to bidders who face linear costs and a common and common-knowledge fixed budget constraint. Maskin (2000) showed that an all-pay auction is constrained efficient, namely, it maximizes expected welfare subject to incentive-compatibility and budget constraints. These results subsequently motivated several researchers to study the effect of bid caps in all-pay auctions. Che and Gale (1998) calculated the bidding equilibrium of a complete information all-pay auction with two bidders having different valuations for a prize and linear cost functions, and demonstrated that a bid cap can increase the players' total bid. Gavious, Moldovanu and Sela (2003) studied symmetric all-pay auctions under incomplete information and showed that, regardless of the number of bidders, if agents have linear or concave cost functions then setting a bid cap is not profitable for a designer who wishes to maximize the average bid. On the other hand, if agents have convex cost functions (i.e. an increasing marginal cost) then effectively capping the bids is profitable for a designer facing a sufficiently large number of bidders. Sahuguet's (2006) findings indicate that in asymmetric all-pay auctions under incomplete information and linear cost functions, capping the bids may be profitable for the

[3] The maximum salary of a player with 6 or fewer years of experience is \$9,000,000 or 25% of the total salary cap (2009-10: \$14,472,500). For a player with 7–9 years of experience, the maximum is \$11,000,000 or 30% of the cap (2009-10: \$17,310,000), and for a player with 10+ years of experience, the maximum is \$14,000,000 or 35% of the cap (2008-2009: \$20,195,000).

[4] The economic literature on all-pay auctions is quite extensive. All-pay auction models with complete information about the prize's value to different players have been studied, among others, by Hillman and Riley (1989), Baye Kovenock and de Vries (1993, 1996) and Siegel (2009). All-pay auctions with incomplete information about the prize's values, have been studied, among others, by Krishna and Morgan (1997) and Moldovanu and Sela (2001, 2006).

designer who wishes to maximize the average bid. Kirkegaard (2009) studied asymmetric all-pay auctions under incomplete information in which a strong and a weak contestant compete and where a contestant may suffer from a handicap or benefit from a head start. His results show that it is generally profitable to give the weak contestant a head start but it may or may not be profitable to handicap the strong contestant. He also found that the weak contestant may have a head start as well as a handicap.[5]

All the above mentioned papers focus on bid caps in one-stage all pay auctions. Works on bid caps in sequential multi-stage contests, particularly sequential all-pay auctions, are relatively sparse.[6] The reason for this gap is that the effect of bid caps in sequential multi-stage contests on players' equilibrium strategies is much more complex to analyze than in one-stage contests, as a bid cap in any stage of a sequential contest affects not only the players' strategies in that stage but also the players' strategies in all the other stages. Therefore a bid cap may increase the players' effort in some stages but decrease it in others. In this paper, we extend the model of the sequential two-stage all-pay auction under complete information studied by Sela (2010) by allow the designer to impose bid caps in each of the two stages. Our model has two players and two identical prizes and each player may win more than one prize. The players' marginal values are non-increasing such that the marginal value of the second prize is not larger than the marginal value of the first one. In our sequential model like in Che and Gale (1998), a bid cap in the second stage will increase the players' total bid in this stage. But, a bid cap in the second stage may decrease the players' bid in the first stage such that it is not clear at all that a bid cap in the second stage is as effective as in the one-stage all-pay auction. Furthermore, a bid cap in the first stage does not change only the players' bids in the first stage but also the players' probabilities of winning in that stage. Thus a bid cap in the first stage changes the players' expected bid in the second stage as well. However, despite this seemingly complex effect of bid caps on players' strategies in our sequential all-pay auction, we show that by choosing the right bid caps the designer can always increase the players' total bid. In particular, we show that to increase the total bid it is

[5] Konrad (2002) examined a two-bidder model under complete information with head starts and handicaps.

[6] The literature presents only a few sequential auctions with constrained bidders. For example, Pitchik and Schotter (1988) studied complete information sequential auctions with two financially constrained bidders and two independent objects. Benoit and Krishna (2001) extended this model to more than two bidders, assuming synergies among the objects and that budgets chosen by the bidders. They note that the seller may benefit from budget constraints, but that this feature cannot occur in their model if only one object is auctioned.

sufficient to cap the players' bids in the first stage of the sequential all-pay auction. However, this last result does not imply that the bid cap in the second stage is not profitable for a designer who wishes to maximize the players' total bid. Indeed, we show that if there is no dominant player such that both of his marginal values are larger than those of his opponent, the combination of bid caps in both stages of the sequential all-pay auction is the optimal setting.

The paper is organized as follows: In Section 2, we introduce our sequential two-stage all-pay auction with bid caps. In Sections 3 and 4, we analyze the equilibrium behavior in both stages of this model. In Section 5 we analyze the effect of the bid caps on the expected total bid. Section 6 concludes.

2 The model

We consider a sequential all-pay auction with two players (denoted by $i = a, b$) and two stages (denoted by $t = 1, 2$). In each of the stages, a single (identical) prize is awarded. Let v_j^i denotes player i's marginal value for winning his j-th prize. That is, if player i wins only one prize his value is v_1^i and if he wins two prizes his value is $v_1^i + v_2^i$. We assume that the marginal values are non-increasing, namely, $v_1^i \geq v_2^i$ and that they are common knowledge.

Each player i submits a bid (effort) in the first stage $x_1^i \leq d_1$ where $d_1 \in [0, 1]$ is a commonly known bid cap. The player with the highest bid wins the first prize and all the players pay their bids. The players know the identity of the winner in the first stage before the beginning of the second stage, such that the players' values in the second stage are common knowledge. Then, each player i submits a bid in the second stage $x_2^i \leq d_2$ where $d_2 \in [0, 1]$ is a commonly known bid cap. The player with the highest bid in the second stage wins the second prize and all the players pay their bids. We assume that the bid caps can be controlled by the contest designer who wishes to maximize the players' total (average) bid in both stages.

3 Equilibrium - second stage

In order to analyze a subgame-perfect equilibrium of a sequential two-stage all-pay auction with two players we begin by analyzing the second stage and go backwards to the first stage. We assume first that there is no bid cap in the second stage. Then, if player i's value in that stage is v^i, $i = a, b$, where $v^a \geq v^b$, according to Baye, Kovenock and de Vries (1996), there is always a unique mixed-strategy equilibrium in which players a and b randomize on the interval $[0, v^b]$ according to their bid cumulative distribution functions, which are given by

$$\begin{aligned} v^a F^b(x) - x &= v^a - v^b \\ v^b F^a(x) - x &= 0 \end{aligned}$$

Thus, player a's equilibrium bid is uniformly distributed; that is

$$F^a(x) = \frac{x}{v^b}$$

while player b's equilibrium bid is distributed according to the cumulative distribution function

$$F^b(x) = \frac{v^a - v^b + x}{v^a}$$

Player a's probability to win is $1 - \frac{v^b}{2v^a}$ and his expected payoff is $v^a - v^b$, while player b's probability to win is $\frac{v^b}{2v^a}$ and his expected payoff is zero.

Now assume that there is a bid cap $d_2 \in \left[0, v^b\right]$ in the second stage. Note that if $d_2 > v^b$ the bid cap is not effective since it is not binding to the players. According to Che and Gale (1998), if $d_2 \in \left[0, \frac{v^b}{2}\right]$ there is an equilibrium with pure strategies in which each player submits a bid equal to the bid cap $\frac{v^b}{2}$, each of the players wins with probability of 0.5 and the expected payoff of player $i = a, b$ is $\frac{v^i}{2} - d_2$. If $d_2 \in (\frac{v^b}{2}, v^b]$ there is a mixed-strategy equilibrium in which players a and b randomize on the interval $[0, 2d_2 - v^b] \cup \{d_2\}$ according to their bid cumulative distribution functions which are given by

$$\begin{aligned} v^a F^b(x) - x &= v^a \left[F^b(2d_2 - v^b) + \frac{1 - F^b(2d_2 - v^b)}{2} \right] - d_2 \\ v^b F^a(x) - x &= v^b \left[F^a(2d_2 - v^b) + \frac{1 - F^a(2d_2 - v^b)}{2} \right] - d_2 \end{aligned}$$

Here the LHS of the above equations are the expected payoffs of the players if they submit a bid $x \in [0, 2d_2 - v^b]$ and the RHS are the expected payoffs if they submit a bid equal to d_2. Thus, player a's equilibrium bid is distributed according to the cumulative distribution function

$$F^a(x)=\begin{cases} \frac{x}{v^b} \text{ if } x\in[0,2d_2-v^b] \\ \frac{2d_2-v^b}{v^b} \text{ if } x\in(2d_2-v^b,d_2) \\ 1 \quad \text{if} \quad x=d_2 \end{cases}$$

while player b's equilibrium bid is distributed according to the cumulative distribution function

$$F^b(x)=\begin{cases} 1-\frac{v^b-x}{v^a} \text{ if } x\in[0,2d_2-v^b] \\ 1-\frac{2v^b-2d_2}{v^a} \text{ if } x\in(2d_2-v^b,d_2) \\ 1 \quad \text{if} \quad x=d_2 \end{cases}$$

Then, player a's probability of winning in the second stage is $1-\frac{v^b}{2v^a}$ and his expected payoff is v^a-v^b, while player b's probability of winning is $\frac{v^b}{2v^a}$ and his expected payoff is zero. Note that in the case where $d_2\in(\frac{v^b}{2},v^b]$ the players' probabilities of winning and their expected payoffs are the same as in the case without a bid cap.

4 Equilibrium - first stage

We divide the analysis of the equilibrium in the first stage into three cases: 1. We have a bid cap d_1 in the first stage only. 2. We have a bid cap d_2 in the second stage only. 3. We have bid caps d_1 and d_2 in both stages. Suppose first that there is a bid cap d_1 in the first stage only, and assume that player i's marginal value in stage $t=1,2$ is v_t^i. If player a wins in the first stage his payoff is v_1^a and then the players' values in the second stage are v_2^a and v_1^b. If $v_2^a>v_1^b$, the expected payoff of player a will be $v_2^a-v_1^b$ in the second stage. Otherwise, if $v_2^a\leq v_1^b$, the expected payoff of player a in the second stage will be zero. Thus, if player a wins in the first stage, his expected payoff in both stages is $v_1^a+\max\{v_2^a-v_1^b,0\}$.

If player a doesn't win in the first stage then the players' values in the second stage are v_1^a and v_2^b. If $v_1^a>v_2^b$, the expected payoff of player a will be $v_1^a-v_2^b$ in the second stage. Otherwise, if $v_1^a\leq v_2^b$, the expected payoff of player a in the second stage will be zero. Thus, if player a does not win in the first stage his expected payoff is $\max\{v_1^a-v_2^b,0\}$. A similar argument holds for player b. The induced value of each player in the first stage (denoted by $\widehat{v}_1^i, i\in\{a,b\}$) is the difference between his expected payoff in the contest when he wins or not in the first stage. Thus, the induced values of the players in the first stage are:

$$\begin{aligned} \widehat{v}_1^a &= v_1^a + \max\{v_2^a - v_1^b, 0\} - \max\{v_1^a - v_2^b, 0\} \\ \widehat{v}_1^b &= v_1^b + \max\{v_2^b - v_1^a, 0\} - \max\{v_1^b - v_2^a, 0\} \end{aligned} \tag{1}$$

Note that since the players' marginal values are positive, the induced values are positive as well. Then, using the induced values, the players' equilibrium strategies in the first stage of the sequential two-stage all-pay auction can be stated as follows.

Proposition 1 *Assume that $\widehat{v}_1^j \geq \widehat{v}_1^k$, $j,k \in \{a,b\}$. Then, in the unique subgame-perfect equilibrium of the sequential all-pay auction with a bid cap d_1, the players' strategies in the first stage are as follows: If $d_1 \in \left[0, \frac{\widehat{v}_1^k}{2}\right]$ both players use the pure strategies*

$$x^a = x^b = d_1$$

If $d_1 \in \left(\frac{\widehat{v}_1^k}{2}, \widehat{v}_1^k\right]$, player j's equilibrium bid is distributed according to

$$F_1^j(x) = \begin{cases} \frac{x}{\widehat{v}_1^k} \text{ if } x \in [0, 2d_1 - \widehat{v}_1^k] \\ \frac{2d_1 - \widehat{v}_1^k}{\widehat{v}_1^k} \text{ if } x \in (2d_1 - \widehat{v}_1^k, d_1) \\ 1 \quad \text{if} \quad x = d_1 \end{cases} \tag{2}$$

while player k's equilibrium bid is distributed according to

$$F_1^k(x) = \begin{cases} 1 - \frac{\widehat{v}_1^k - x}{\widehat{v}_1^j} \text{ if } x \in [0, 2d_1 - \widehat{v}_1^k] \\ 1 - \frac{2\widehat{v}_1^k - 2d_1}{\widehat{v}_1^j} \text{ if } x \in (2d_1 - \widehat{v}_1^k, d_1) \\ 1 \quad \text{if} \quad x = d_1 \end{cases} \tag{3}$$

Proof. See Appendix. ■

Suppose now that there is a bid cap d_2 in the second stage only. We assume that $d_2 \in \left[0, \min(\frac{v_2^a}{2}, \frac{v_2^b}{2})\right]$ since otherwise the bid cap in the second stage is not necessarily binding to both players. We omit the equilibrium analysis for when $d_2 > \min(\frac{v_2^a}{2}, \frac{v_2^b}{2})$, namely, for when a bid cap is binding to only one of the players or it is not binding to both players, since these cases are not relevant to the intent of our study. Then, given that the bid cap d_2 is binding to both players, if player a wins in the first stage his payoff in this stage is v_1^a and in the second stage the expected payoff of player a will be $\frac{v_2^a}{2} - d_2$. Thus, if player a wins in the first stage, his expected payoff in both stages is $v_1^a + \frac{v_2^a}{2} - d_2$. If player a doesn't win in the first stage his expected payoff in the second stage is $\frac{v_1^a}{2} - d_2$. A similar argument holds for player b. The induced values of both players in the first stage, which are the differences between their expected payoffs in the contest if they win or not in the first stage, are given by

$$\widetilde{v}_1^a = \frac{v_1^a + v_2^a}{2}, \widetilde{v}_1^b = \frac{v_1^b + v_2^b}{2}$$

Using the induced values, the players' equilibrium strategies in the first stage of the sequential two-stage all-pay auction with a bid cap in the second stage can be stated as follows.

Proposition 2 *Suppose that $\widetilde{v}_1^j \geq \widetilde{v}_1^k$, $j, k \in \{a, b\}$. Then in the unique subgame-perfect equilibrium of the sequential two-stage all-pay auction with a bid cap d_2, the players' strategies in the first stage are as follows: If $d_2 \in \left[0, \min(\frac{v_2^a}{2}, \frac{v_2^b}{2})\right]$, player j's equilibrium bid is distributed according to*

$$F_1^j(x) = \frac{x}{\widetilde{v}_1^k} \tag{4}$$

while player k's equilibrium bid is distributed according to

$$F_1^k(x) = \frac{x + \widetilde{v}_1^j - \widetilde{v}_1^k}{\widetilde{v}_1^j} \tag{5}$$

Proof. See Appendix. ■

We assume now that the bid caps in both stages are binding. As was shown above, the bid cap in the second stage is binding to both players iff $d_2 \in \left[0, \min(\frac{v_2^a}{2}, \frac{v_2^b}{2})\right]$. We now find the values of d_1 that are binding to both players in the first stage. If player a wins in the first stage, his payoff is $v_1^a - d_1$ and in the second stage the expected payoff of player a will be $\frac{v_2^a}{2} - d_2$. Thus, if player a wins in the first stage, his expected payoff in the both stages is $v_1^a + \frac{v_2^a}{2} - d_1 - d_2$. If player a doesn't win in the first stage his expected payoff in both stages is $\frac{v_1^a}{2} - d_1 - d_2$. A similar argument holds for player b. Since both players have the same probability to win in the first stage and both choose to participate in the first stage we have for all $i \in \{a, b\}$,

$$\frac{1}{2}(v_1^i + \frac{v_2^i}{2} - d_1 - d_2) + \frac{1}{2}(\frac{v_1^i}{2} - d_1 - d_2) \geq \frac{v_1^i}{2} - d_2 \tag{6}$$

where the LHS is the expected payoff of player i in the sequential all-pay auction and the RHS is the expected payoff of player i if he would participate in the second stage only. Thus, by (6) we obtain that

Proposition 3 *In the sequential two-stage all-pay auction with bid caps d_1 and d_2, if $d_2 \in \left[0, \min(\frac{v_2^a}{2}, \frac{v_2^b}{2})\right]$ and $d_1 \in \left[0, \min(\frac{v_1^a+v_2^a}{4}, \frac{v_1^b+v_2^b}{4})\right]$ there is a unique subgame-perfect equilibrium in which both players choose d_1 in the first stage and d_2 in the second stage.*

Using the above equilibrium analysis we now examine the efficiency of capping the players' bids for a designer who wishes to maximize the players' expected total bid.

5 Total bid

Che and Gale (1998) showed that in the one-stage all-pay auction an endogenous bid cap is profitable for a designer who wishes to maximize the expected total bid. In this section, we generalize this result and show that in the sequential two-stage all-pay auction endogenous bid caps increase the players' expected total bid. Moreover, we show that a bid cap in the first stage only is sufficient to increase the expected total bid.

If there is no bid cap in the first stage, the players' equilibrium strategies are given by Proposition 1 where d_1 is equal to the lower induced value, namely, $d_1 = \widehat{v}_1^k$. Then the expected total bid in the first stage of the sequential all-pay auction is given by $\frac{\widehat{v}_1^k}{2}(1+\frac{\widehat{v}_1^k}{\widehat{v}_1^j})$. If player j wins in the first stage and that happens with probability $1-\frac{\widehat{v}_1^k}{2\widehat{v}_1^j}$, then the expected total bid in the second stage is $\frac{\min\{v_2^j,v_1^k\}}{2}(1+\frac{\min\{v_2^j,v_1^k\}}{\max\{v_2^j,v_1^k\}})$. But if player k wins in the first stage and that happens with probability $\frac{\widehat{v}_1^k}{2\widehat{v}_1^j}$, then the expected total bid in the second stage is $\frac{\min\{v_1^j,v_2^k\}}{2}(1+\frac{\min\{v_1^j,v_2^k\}}{\max\{v_1^j,v_2^k\}})$. Thus, the expected total bid in the sequential all-pay auction without any bid cap is given by

$$TE_0 = \frac{\widehat{v}_1^k}{2}(1+\frac{\widehat{v}_1^k}{\widehat{v}_1^j}) + (1-\frac{\widehat{v}_1^k}{2\widehat{v}_1^j})\frac{\min\{v_2^j,v_1^k\}}{2}(1+\frac{\min\{v_2^j,v_1^k\}}{\max\{v_2^j,v_1^k\}}) + \frac{\widehat{v}_1^k}{2\widehat{v}_1^j}\frac{\min\{v_1^j,v_2^k\}}{2}(1+\frac{\min\{v_1^j,v_2^k\}}{\max\{v_1^j,v_2^k\}}) \quad (7)$$

On the other hand, if there is a bid cap in the first stage $d_1 = \frac{\widehat{v}_1^k}{2}$,[7] by Proposition 1 the expected total bid in the first stage is $2d_1 = \widehat{v}_1^k$, and the expected total bid in the second stage is either $\frac{\min\{v_2^j,v_1^k\}}{2}(1+\frac{\min\{v_2^j,v_1^k\}}{\max\{v_2^j,v_1^k\}})$ or $\frac{\min\{v_1^j,v_2^k\}}{2}(1+\frac{\min\{v_1^j,v_2^k\}}{\max\{v_1^j,v_2^k\}})$ with the same probability. Thus, the expected total bid in the sequential all-pay auction with a bid cap $d_1 = \frac{\widehat{v}_1^k}{2}$ in the first stage is given by

$$TE_1 = \widehat{v}_1^k + \frac{1}{2}\frac{\min\{v_2^j,v_1^k\}}{2}(1+\frac{\min\{v_2^j,v_1^k\}}{\max\{v_2^j,v_1^k\}}) + \frac{1}{2}\frac{\min\{v_1^j,v_2^k\}}{2}(1+\frac{\min\{v_1^j,v_2^k\}}{\max\{v_1^j,v_2^k\}}) \quad (8)$$

[7] This is the maximal value of the bid cap in the first stage that is binding to both players.

Note that if the bid cap in the first stage satisfies $d_1 \in \left(\frac{\widehat{v}_1^k}{2}, \widehat{v}_1^k\right]$, based on Che and Gale (1998), the expected total bid and the players' probabilities of winning in the first stage are exactly the same as without a bid cap. Therefore the expected total bid in both stages is the same as in the sequential all-pay auction without a bid cap. In the following, by comparing the expected total bid in the sequential all-pay auction without bid caps and with a bid cap in the first stage, we demonstrate that capping the bids in the sequential all-pay auction increases the players' expected total bid.

Proposition 4 *For every sequential all-pay auction there are bid caps $(d_1, d_2) > (0, 0)$ that increase the players' expected total bid. In particular, the total bid in the sequential all-pay auction with a bid cap of $d_1 = \frac{\widehat{v}_1^k}{2}$ in the first stage $(d_2 = 0)$ is larger than the expected total bid without bid caps.*

Proof. See Appendix. ∎

So far, we have shown that capping the players' bids in the first stage increases the players' expected total bid. The effect of the a bid cap in the second stage, however, is ambiguous since it affects the players' strategies in both stages. In other words, while we know that a bid cap increases the players' expected total bid in the second stage it might decrease the players' expected total bid in the first stage such that the total effect of the bid cap in the second stage is not clear and will depend on the exact marginal values of the players in both stages. However, below we show that a bid cap in the second stage can be an optimal complement to a bid cap in the first stage such that bid caps in both stages might be the optimal setting. By Proposition 3, if there is a bid cap in each of the stages of the sequential all-pay auction and both caps have the maximal values which are binding to both players then the players' expected total bid is given by

$$TE_{12} = 2d_1 + 2d_2 = 2\min\left(\frac{v_1^j + v_2^j}{4}, \frac{v_1^k + v_2^k}{4}\right) + 2\min\left(\frac{v_2^j}{2}, \frac{v_2^k}{2}\right) \tag{9}$$

A comparison of (9) and (8) yields

Proposition 5 *If the players' values satisfy $v_1^i \geq v_2^j$ $i, j \in \{a, b\}$ then the sequential all-pay auction with bid caps in both stages yields a higher expected total effort than the sequential all-pay auction with a bid cap in the first stage only.*

Proof. See Appendix. ∎

It can be easily verified that if the condition of Proposition 5, according to which $v_1^i \geq v_2^j$ $i, j \in \{a, b\}$ does not hold, that is, there exists a dominant player such that his both values are larger than both values of his opponent, then the expected total bid in the sequential all-pay auction with bid caps in both stages is not necessarily larger than the expected total bid in the sequential all-pay auction with a bid cap in the first stage only. Hence, the optimal setting for a designer who wishes to maximize the players' expected total bid could be either with a bid cap in one stage only or in the second stage only or in both stages. In any case, bid caps are effective for increasing the expected total bids in the sequential all-pay auction.

6 Concluding remarks

We studied a sequential two-stage all-pay auction where the players' marginal values for the prizes are non-increasing. We showed that a designer who wishes to maximize the expected total bid should impose bid caps in either one of the stages or in both of them. In particular, we showed that the expected total bid in the sequential all-pay auction with the optimal bid cap in the first stage is larger than in the sequential all-pay auction without bid caps. We also found that the bid cap in the second stage of a sequential all-pay auction may have a positive effect on the players' expected total bid. It is most likely that these results according to which bid caps have a positive effect on players' total effort can be generalized for any relation between the players' marginal values. However, such a generalization is somewhat complex since neither the bid cap in the first stage nor in the second stage has a significant and clear effect as it does in the first stage of our present model.

7 Appendix

7.1 Proof of Proposition 1

Without loss of generality, assume that $\widehat{v}_1^a > \widehat{v}_1^b$. If $d_1 \in \left[0, \frac{\widehat{v}_1^b}{2}\right]$ then both players use the pure strategies $x^a = x^b = d_1$ and their expected payoffs in the contest are

$$u^a = \frac{v_1^a}{2} + \frac{\max\{v_2^a - v_1^b, 0\}}{2} + \frac{\max\{v_1^a - v_2^b, 0\}}{2} - d_1, u^b = \frac{v_1^b}{2} + \frac{\max\{v_2^b - v_1^a, 0\}}{2} + \frac{\max\{v_1^b - v_2^a, 0\}}{2} - d_1$$

If $d_1 \in \left(\frac{\widehat{v}_1^b}{2}, \widehat{v}_1^b\right]$ there is no pure strategy equilibrium and similarly to the one-stage all-pay auction with a bid cap the players randomize on the interval $[0, \widehat{x}] \cup \{d_1\}$ according to their effort cumulative distribution functions, $F_1^a(x)$ and $F_1^b(x)$, which are given by the indifference conditions:

$$\begin{aligned}
& (v_1^a + \max\{v_2^a - v_1^b, 0\})F_1^b(x) + (\max\{v_1^a - v_2^b, 0\})(1 - F_1^b(x)) - x \\
= \; & (v_1^a + \max\{v_2^a - v_1^b, 0\})(F_1^b(\widehat{x}) + \frac{1 - F_1^b(\widehat{x})}{2}) + (\max\{v_1^a - v_2^b, 0\})(1 - F_1^b(\widehat{x}) - \frac{1 - F_1^b(\widehat{x})}{2}) - d_1 \\
& (v_1^b + \max\{v_2^b - v_1^a, 0\})F_1^a(x) + (\max\{v_1^b - v_2^a, 0\})(1 - F_1^a(x)) - x \\
= \; & (v_1^b + \max\{v_2^b - v_1^a, 0\})(F_1^a(\widehat{x}) + \frac{1 - F_1^a(\widehat{x})}{2}) + (\max\{v_1^b - v_2^a, 0\})(1 - F_1^a(\widehat{x}) - \frac{1 - F_1^a(\widehat{x})}{2}) - d_1
\end{aligned}$$

where the LHS of the above equations are the expected payoffs of the players if they submit a bid $x \in [0, \widehat{x}]$ and the RHS are the expected payoffs of the players if they submit a bid equal to d_1.

Using the induced values $\widehat{v}_1^a, \widehat{v}_1^b$, the above indifference conditions can be written as

$$\begin{aligned}
\widehat{v}_1^a F_1^b(x) - x &= \widehat{v}_1^a (F_1^b(\widehat{x}) + \frac{1 - F_1^b(\widehat{x})}{2}) - d_1 \\
\widehat{v}_1^b F_1^a(x) - x &= \widehat{v}_1^b (F_1^a(\widehat{x}) + \frac{1 - F_1^a(\widehat{x})}{2}) - d_1
\end{aligned} \tag{10}$$

The system of equations (10) describing the players' mixed strategies $F_1^a(x), F_1^b(x)$ in the first stage of the sequential two-prize all-pay auction with a bid cap d_1 is identical to that describing the equilibrium strategies of players in the standard one-stage all-pay auction with a bid cap d_1 where the players' values are the induced marginal values $\widehat{v}_1^a, \widehat{v}_1^b$. Hence, according to the analysis of the standard one-stage all-pay auction (Che and Gale 1998), there is a unique mixed strategy equilibrium in the first stage of the sequential two-prize all-pay auction which is given by (2) and (3). Then the players' expected payoffs in the contest are

$$\begin{aligned}
u^a &= \widehat{v}_1^a - \widehat{v}_1^b + \max\{v_1^a - v_2^b, 0\} \\
u^b &= \max\{v_1^b - v_2^a, 0\}
\end{aligned}$$

Q.E.D.

7.2 Proof of Proposition 2

Without loss of generality, assume that $\widetilde{v}_1^a > \widetilde{v}_1^b$. If $d_2 \in \left[0, \min(\frac{v_2^a}{2}, \frac{v_2^b}{2})\right]$ then the players randomize in the first stage according to their effort cumulative distribution functions, $F_1^a(x)$ and $F_1^b(x)$, which are given by the indifference conditions:

$$(v_1^a + \frac{v_2^a}{2} - d_2)F_1^b(x) + (\frac{v_1^a}{2} - d_2)(1 - F_1^b(x)) - x = \frac{v_1^a + v_2^a}{2} - \frac{v_1^b + v_2^b}{2} + \frac{v_1^a}{2} - d_2$$
$$(v_1^b + \frac{v_2^b}{2} - d_2)F_1^a(x) + (\frac{v_1^b}{2} - d_2)(1 - F_1^a(x)) - x = \frac{v_1^b}{2} - d_2$$

Using the induced values $\widetilde{v}_1^a, \widetilde{v}_1^b$, the above indifference conditions can be written as

$$\begin{aligned} \widetilde{v}_1^a F_1^b(x) - x &= \widetilde{v}_1^a - \widetilde{v}_1^b \\ \widetilde{v}_1^b F_1^a(x) - x &= 0 \end{aligned}$$

Hence, based on the analysis of the standard one-stage all-pay auction (Baye, Kovenock and de Vries 1996), there is a unique mixed strategy equilibrium in the first stage of the sequential two-prize all-pay auction which is given by (4) and (5). *Q.E.D.*

7.3 Proof of Proposition 4

Assume that the players' values in both stages are $v_1 \geq v_2 \geq v_3 \geq v_4$. Given that the players values are non-increasing we have three possible cases:

$$\begin{aligned} A &: v^a = (v_1, v_4), v^b = (v_2, v_3). \\ B &: v^a = (v_1, v_3), v^b = (v_2, v_4). \\ C &: v^a = (v_1, v_2), v^b = (v_3, v_4). \end{aligned}$$

We wish to show that for all these cases, independent of the values of v_i $i = 1, 2, 3, 4$, the expected total bid in the sequential contest with a bid cap in the first stage TE_1 (given by (8)) is larger than the expected total bid in the sequential contest without any cap TE_0 (given by (7)). In all of these cases, by (8) and (7) we obtain that if the players' induced values are identical, namely, $\widehat{v}^a = \widehat{v}^b$, then $TE_1 = TE_0$. We now show that in each of the three cases the point at which $\widehat{v}^a = \widehat{v}^b$ is the maximal point of the difference $TE_0 - TE_1$.

7.3.1 Case A: $v^a = (v_1, v_4), v^b = (v_2, v_3)$

In this case, by (1) the players' induced values are $\widehat{v}^a = v_3$ and $\widehat{v}^b = v_4$. By (8) and (7) we have

$$TE_0 - TE_1 = \frac{v_1 v_3 v_4^2 + v_1 v_2 v_4^2 + v_2 v_3^2 v_4 - v_1 v_2 v_3^2 - v_2 v_3^3 - v_1 v_4^3}{4 v_1 v_2 v_3} \tag{11}$$

We intend to show that the maximal value of (11) is obtained when $v_3 = v_4$ and that this value is equal to zero. Therefore, we solve the following maximization problem:

$$
\begin{aligned}
\max_{v_1,v_2,v_3,v_4} TE_0 - TE_1 &= \max_{v_1,v_2,v_3,v_4} v_1 v_3 v_4^2 + v_1 v_2 v_4^2 + v_2 v_3^2 v_4 - v_1 v_2 v_3^2 - v_2 v_3^3 - v_1 v_4^3 \\
& \text{s.t.} \\
v_2 - v_1 &\leq 0 \\
v_3 - v_2 &\leq 0 \\
v_4 - v_3 &\leq 0
\end{aligned}
$$

The Lagrangian is given by

$$
\begin{aligned}
L &= v_1 v_3 v_4^2 + v_1 v_2 v_4^2 + v_2 v_3^2 v_4 - v_1 v_2 v_3^2 - v_2 v_3^3 - v_1 v_4^3 \\
& -\alpha_1(v_2 - v_1) - \alpha_2(v_3 - v_2) - \alpha_3(v_4 - v_3)
\end{aligned}
$$

where $\alpha_j, j = 1,2,3$ are the Lagrangian multipliers. The first-order conditions are:

$$
\begin{aligned}
v_3 v_4^2 + v_2 v_4^2 - v_2 v_3^2 - v_4^3 &= -\alpha_1 \\
v_1 v_4^2 + v_3^2 v_4 - v_1 v_3^2 - v_3^3 &= \alpha_1 - \alpha_2 \\
v_1 v_4^2 + 2 v_2 v_3 v_4 - 2 v_1 v_2 v_3 - 3 v_2 v_3^2 &= \alpha_2 - \alpha_3 \\
2 v_1 v_3 v_4 + 2 v_1 v_2 v_4 + v_2 v_3^2 - 3 v_1 v_4^2 &= \alpha_3
\end{aligned}
$$

The solution of this system of equations is

$$
\begin{aligned}
v_1 &\geq v_2 \geq v_3 = v_4 \\
\alpha_1 &= \alpha_2 = 0 \\
\alpha_3 &= v_2 v_3^2 + v_1 v_3(2v_2 - v_3) > 0
\end{aligned}
$$

Therefore the maximal value of $TE_0 - TE_1$ is obtained when $v_3 = v_4$ and this value is equal to zero.

7.3.2 Case B: $v^a = (v_1, v_3), v^b = (v_2, v_4)$

In this case, by (1) the players' induced values are $\widehat{v}^a = v_4$ and $\widehat{v}^b = v_3$. By (8) and (7) we have

$$
TE_0 - TE_1 = \frac{v_2 v_3 v_4^2 + v_1 v_2 v_4^2 + v_1 v_3^2 v_4 - v_1 v_2 v_3^2 - v_1 v_3^3 - v_2 v_4^3}{4 v_1 v_2 v_3} \tag{12}
$$

We intend to show that the maximal value of (12) is obtained when $v_3 = v_4$ and that this value is equal to zero. Therefore, we solve the following maximization problem:

$$\begin{aligned}
\max_{v_1,v_2,v_3,v_4} TE_0 - TE_1 &= \max_{v_1,v_2,v_3,v_4} v_2v_3v_4^2 + v_1v_2v_4^2 + v_1v_3^2v_4 - v_1v_2v_3^2 - v_1v_3^3 - v_2v_4^3 \\
& \text{s.t.} \\
v_2 - v_1 &\leq 0 \\
v_3 - v_2 &\leq 0 \\
v_4 - v_3 &\leq 0
\end{aligned}$$

The Lagrangian is given by

$$\begin{aligned}
L &= v_2v_3v_4^2 + v_1v_2v_4^2 + v_1v_3^2v_4 - v_1v_2v_3^2 - v_1v_3^3 - v_2v_4^3 \\
&\quad -\alpha_1(v_2 - v_1) - \alpha_2(v_3 - v_2) - \alpha_3(v_4 - v_3)
\end{aligned}$$

where $\alpha_j, j = 1,2,3$ are the Lagrangian multipliers. The first-order conditions are:

$$\begin{aligned}
v_2v_4^2 + v_3^2v_4 - v_2v_3^2 - v_3^3 &= -\alpha_1 \\
v_1v_4^2 + v_3v_4^2 - v_1v_3^2 - v_4^3 &= \alpha_1 - \alpha_2 \\
v_2v_4^2 + 2v_1v_3v_4 - 2v_1v_2v_3 - 3v_1v_3^2 &= \alpha_2 - \alpha_3 \\
2v_2v_3v_4 + 2v_1v_2v_4 + v_1v_3^2 - 3v_2v_4^2 &= \alpha_3
\end{aligned}$$

The solution of this system of equations is

$$\begin{aligned}
v_1 &\geq v_2 \geq v_3 = v_4 \\
\alpha_1 &= \alpha_2 = 0 \\
\alpha_3 &= 2v_1v_2v_3 + v_3^2(v_1 - v_2) > 0
\end{aligned}$$

Therefore the maximal value of $TE_0 - TE_1$ is obtained when $v_3 = v_4$ and this value is equal to zero.

7.3.3 Case C: $v^a = (v_1, v_2), v^b = (v_3, v_4)$

In this case, by (1) the players' induced values are $\widehat{v}^a = v_2 + v_4 - v_3$ and $\widehat{v}^b = v_3$. We divide the analysis into two sub-cases.

1. If $v_2 + v_4 \geq 2v_3$, then by (8) and (7) we have

$$TE_0 - TE_1 = \frac{v_1v_2v_3v_4 + 3v_1v_2v_3^2 + 2v_2v_3v_4^2 + v_1v_3^2v_4}{4v_1v_2(v_2+v_4-v_3)} \qquad (13)$$
$$\frac{-v_1v_2^2v_3 - v_1v_2^2v_4 - v_1v_2v_4^2 - v_2^2v_4^2 - v_2v_4^3 - 2v_1v_3^3}{4v_1v_2(v_2+v_4-v_3)}$$

We intend to show that the maximal value of (13) is obtained when $v_2 + v_4 = 2v_3$ and that this value is equal to zero. Therefore, we solve the following maximization problem:

$$\begin{aligned}
\max_{v_1,v_2,v_3,v_4} TE_0 - TE_1 &= \max_{v_1,v_2,v_3,v_4} v_1v_2v_3v_4 + 3v_1v_2v_3^2 + 2v_2v_3v_4^2 + v_1v_3^2v_4 \\
&\quad -v_1v_2^2v_3 - v_1v_2^2v_4 - v_1v_2v_4^2 - v_2^2v_4^2 - v_2v_4^3 - 2v_1v_3^3 \\
&\quad \text{s.t.} \\
v_2 - v_1 &\leq 0 \\
v_3 - v_2 &\leq 0 \\
v_4 - v_3 &\leq 0 \\
2v_3 - v_2 - v_4 &\leq 0
\end{aligned}$$

The Lagrangian is given by

$$\begin{aligned}
L &= v_1v_2v_3v_4 + 3v_1v_2v_3^2 + 2v_2v_3v_4^2 + v_1v_3^2v_4 \\
&\quad -v_1v_2^2v_3 - v_1v_2^2v_4 - v_1v_2v_4^2 - v_2^2v_4^2 - v_2v_4^3 - 2v_1v_3^3 \\
&\quad -\alpha_1(v_2-v_1) - \alpha_2(v_3-v_2) - \alpha_3(v_4-v_3) - \alpha_4(2v_3-v_2-v_4)
\end{aligned}$$

where $\alpha_j, j = 1,2,3,4$ are the Lagrangian multipliers. The first-order conditions are:

$$\begin{aligned}
v_2v_3v_4 + 3v_2v_3^2 + v_3^2v_4 - v_2^2v_3 - v_2^2v_4 - v_2v_4^2 - 2v_3^3 &= -\alpha_1 \\
v_1v_3v_4 + 3v_1v_3^2 + 2v_3v_4^2 - 2v_1v_2v_3 - 2v_1v_2v_4 - v_1v_4^2 - 2v_2v_4^2 - v_4^3 &= \alpha_1 - \alpha_2 - \alpha_4 \\
v_1v_2v_4 + 6v_1v_2v_3 + 2v_2v_4^2 + 2v_1v_3v_4 - v_1v_2^2 - 6v_1v_3^2 &= \alpha_2 - \alpha_3 + 2\alpha_4 \\
v_1v_2v_3 + 4v_2v_3v_4 + v_1v_3^2 - v_1v_2^2 - 2v_1v_2v_4 - 2v_2^2v_4 - 3v_2v_4^2 &= \alpha_3 - \alpha_4
\end{aligned}$$

The solution of this system of equations is

$$\begin{aligned} v_2 + v_4 &= 2v_3 \\ \alpha_1 &= \alpha_2 = \alpha_3 = 0 \\ \alpha_4 &= \frac{4v_1 v_2 v_4 + 4v_2 v_4^2 + v_1(v_2^2 - v_4^2)}{4} > 0 \end{aligned}$$

Therefore the maximal value of $TE_0 - TE_1$ is obtained when $v_2 + v_4 = 2v_3$ and this value is equal to zero.

2. If $v_2 + v_4 \leq 2v_3$, then by (8) and (7) we have

$$\begin{aligned} TE_0 - TE_1 &= \frac{3v_1 v_2 v_3^2 + 2v_2 v_3 v_4^2 + 3v_1 v_2^2 v_4 + v_1 v_2 v_4^2 + v_1 v_3^2 v_4 + 2v_1 v_2^3}{4v_1 v_2 v_3} \\ &\quad \frac{-3v_1 v_2 v_3 v_4 - 5v_1 v_2^2 v_3 - 2v_1 v_3^3 - v_2^2 v_4^2 - v_2 v_4^3}{4v_1 v_2 v_3} \end{aligned} \tag{14}$$

We intend to show that the maximal value of (14) is obtained when $v_2 + v_4 = 2v_3$ and that this value is equal to zero. Therefore, we solve the following maximization problem:

$$\begin{aligned} \max_{v_1,v_2,v_3,v_4} TE_0 - TE_1 &= \max_{v_1,v_2,v_3,v_4} 3v_1 v_2 v_3^2 + 2v_2 v_3 v_4^2 + 3v_1 v_2^2 v_4 + v_1 v_2 v_4^2 + v_1 v_3^2 v_4 + 2v_1 v_2^3 \\ & -3v_1 v_2 v_3 v_4 - 5v_1 v_2^2 v_3 - 2v_1 v_3^3 - v_2^2 v_4^2 - v_2 v_4^3 \\ & \text{s.t.} \\ v_2 - v_1 &\leq 0 \\ v_3 - v_2 &\leq 0 \\ v_4 - v_3 &\leq 0 \\ v_2 + v_4 - 2v_3 &\leq 0 \end{aligned}$$

The Lagrangian is given by

$$\begin{aligned} L &= 3v_1 v_2 v_3^2 + 2v_2 v_3 v_4^2 + 3v_1 v_2^2 v_4 + v_1 v_2 v_4^2 + v_1 v_3^2 v_4 + 2v_1 v_2^3 \\ & -3v_1 v_2 v_3 v_4 - 5v_1 v_2^2 v_3 - 2v_1 v_3^3 - v_2^2 v_4^2 - v_2 v_4^3 \\ & -\alpha_1(v_2 - v_1) - \alpha_2(v_3 - v_2) - \alpha_3(v_4 - v_3) - \alpha_4(v_2 + v_4 - 2v_3) \end{aligned}$$

where $\alpha_j, j = 1,2,3,4$ are the Lagrangian multipliers. The first-order conditions are:

$$\begin{aligned}
3v_2v_3^2 + 2v_2^3 + 3v_2^2v_4 + v_2v_4^2 + v_3^2v_4 - 3v_2v_3v_4 - 5v_2^2v_3 - 2v_3^3 &= -\alpha_1 \\
3v_1v_3^2 + 2v_3v_4^2 + 6v_1v_2^2 + 6v_1v_2v_4 + v_1v_4^2 - 3v_1v_3v_4 - 10v_1v_2v_3 - 2v_2v_4^2 - v_4^3 &= \alpha_1 - \alpha_2 + \alpha_4 \\
6v_1v_2v_3 + 2v_2v_4^2 + 2v_1v_3v_4 - 3v_1v_2v_4 - 5v_1v_2^2 - 6v_1v_3^2 &= \alpha_2 - \alpha_3 - 2\alpha_4 \\
4v_2v_3v_4 + 3v_1v_2^2 + 2v_1v_2v_4 + v_1v_3^2 - 3v_1v_2v_3 - 2v_2^2v_4 - 3v_2v_4^2 &= \alpha_3 + \alpha_4
\end{aligned}$$

The solution of this system of equations is

$$\begin{aligned}
v_2 + v_4 &= 2v_3 \\
\alpha_1 &= \alpha_2 = \alpha_3 = 0 \\
\alpha_4 &= \frac{7v_1v_2^2 + v_1v_4^2 + 4v_2v_4(v_1 - v_4)}{4} > 0
\end{aligned}$$

Therefore the maximal value of $TE_0 - TE_1$ is obtained when $v_2 + v_4 = 2v_3$ and this value is equal to zero. *Q.E.D.*

7.4 Proof of Proposition 5

Assume that the players' values satisfy $v_1^a \geq v_2^b$ and $v_1^b \geq v_2^a$. Given that the players values are non-increasing we have two possible cases:

$$\begin{aligned}
A &: v^a = (v_1, v_4), v^b = (v_2, v_3) \\
B &: v^a = (v_1, v_3), v^b = (v_2, v_4)
\end{aligned}$$

where $v_1 \geq v_2 \geq v_3 \geq v_4$.

In case A by (8) and (9), if $v_1 + v_4 \geq v_2 + v_3$ we have

$$TE_{12} - TE_1 = \frac{v_1v_2(v_3 - v_4) + v_2(v_1v_2 - v_3^2) + v_1(v_2^2 - v_4^2)}{4v_1v_2} \geq 0$$

and if $v_1 + v_4 < v_2 + v_3$ we have

$$TE_{12} - TE_1 = \frac{v_1v_4(v_2 - v_4) + v_1v_2(v_1 - v_3) + v_2(v_1^2 - v_3^2)}{4v_1v_2} \geq 0$$

In case B by (8) and (9) we have

$$TE_{12} - TE_1 = \frac{v_2 v_4 (v_1 - v_4) + v_1 v_2 (v_2 - v_3) + v_1 (v_2^2 - v_3^2)}{4 v_1 v_2} \geq 0$$

Thus, we obtained that the players' expected total bid in the sequential all-pay auction with bid caps in both stages is larger than in the sequential all-pay auction with a bid cap in the first stage only. $Q.E.D.$

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