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ABSTRACT

Optimal price setting with observation and menu costs*

We study the price setting problem of a firm in the presence of both observation and menu costs. In this problem the firm optimally decides when to collect costly information on the adequacy of its price, an activity which we refer to as a price “review”. Upon each review, the firm chooses whether to adjust its price, subject to a menu cost, and when to conduct the next price review. This behavior is consistent with recent survey evidence documenting that firms revise prices infrequently and that only a few price revisions yield a price adjustment. The goal of the paper is to study how the firm's choices map into several observable statistics, depending on the level and relative magnitude of the observation vs the menu cost. The observable statistics are: the frequency of price reviews, the frequency of price adjustments, the size-distribution of price adjustments, and the shape of the hazard rate of price adjustments. We provide an analytical characterization of the firm's decisions and a mapping from the structural parameters to the observable statistics. We compare these statistics with the ones obtained for the models with only one type of cost. The predictions of the model can, with suitable data, be used to quantify the importance of the menu cost vs. the information cost. We also consider a version of the model where several price adjustment are allowed between observations, a form of *price plans* or indexation. We find that no indexation is optimal for small inflation rates.

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1 Introduction

Consumers in the modern market place need to sort through an overwhelming number of available options, and hence, may not be able to pay serious attention to each and every feasible alternative. Consequently, some options may receive more attention than others. This may be due to the fact that some options are better than others along some salient dimension. For example, when searching for a laptop computer, a very low price or a very light weight will most likely draw one’s attention; when flipping through TV channels in search of a program to view, one may pay greater attention to a sensational news report, or to a special guest appearance by a celebrity on a sit-com. Alternatively, a consumer may pay more serious attention to items that are *similar* to options he is already familiar with.

Thus, the mere offering of a particular item can have an indirect effect on a firm’s market share by drawing attention to the firm and other items it offers. For instance, the items that stores display on their shop front and web retailers put on their home-pages can exert a positive externality on other items, by persuading consumers to enter the store/website and browse its selection. Similarly, the shows and news items that a TV network chooses to broadcast may persuade viewers to stay tuned to that channel and therefore become exposed to other programs. As a result, consumers whose attention is initially attracted to a firm because of a particular item may end up consuming another item that it offers. Firms may take this indirect marketing effect into account when designing a “product line”. Specifically, they may introduce an item even when the direct demand for this item fails to cover its cost.

We explore this motive by proposing a stylized model of market competition over consumers with limited attention. In our model, firms offer menus of “items” in response to consumer preferences over such menus. Consumers’ limited attention gives firms an incentive to expand their menu and include “pure attention grabbers” - namely, items that do not add to the consumer’s utility from the menu, and whose sole function is therefore to attract consumers’ attention to other items the firm offers. We analyze the firms’ trade-off between the cost of adding pure attention grabbers and the benefit of the extra market share they may generate.

The following examples illustrate a variety of contexts in which certain items may be offered even if they are rarely consumed, because they attract consumers to the firm and persuade them to consider other items that are offered.

Example 0.1. Think of a consumer who wants to buy a new laptop computer. He initially considers a particular model x , possibly because it is his current machine. The

consumer may then notice that a computer store offers a model y that is significantly lighter than x . This gives the consumer a sufficient reason to consider y in addition to x . Upon closer inspection, the consumer realizes that he does not like y as much as he does x . However, since he is already inside the store, he may browse the other laptop computers on offer and find a model z that he ranks above both x and y . Thus, although few consumers may actually buy y , this model functions as a “door opener” that attracts consumers’ serious attention to the other products offered by the store.¹

Example 0.2. Consider the recent strategy of fast-food chains (notably McDonald’s in 2004) of enriching their menus with “healthy” options such as salads and fresh fruit, in an attempt to appeal to health-conscious customers. One may argue (see Warner (2006) for a journalistic account) that the motive behind this marketing move is not so much to generate large direct revenues from the healthy options, but to create a more health-conscious image that will induce a segment of the consumer population to consider McDonald’s restaurants. Once at the restaurant, these consumers will not necessarily choose the healthiest items in the menu, and their consumption decision at the restaurant will involve other motives (such as price, or how filling the meal is).

Example 0.3. The use of attention-grabbing items is often associated with competition among media platforms, such as broadcast television, newspapers or internet portals. Consider the case of broadcast TV. Viewers have a tendency to adopt a default channel that serves as a “home base”. For the competing channel, the challenge is first to draw the viewer’s attention, and then to convince him to stay with it. The channel’s programming strategy takes this motive into account. For instance, the channel may wish to introduce sensational shows, or sensational news flashes, because of their attention-grabbing value.² Alternatively, it may wish to air programs that are identical or similar to the viewer’s favorite shows on his default channel, so that he can recognize familiar genres while on a channel-flipping cruise.

We propose a theoretical framework that incorporates the strategic use of attention grabbers into models of market competition. In this paper, we take only a first step in

¹One recent example is the launch of Apple’s Macbook Air, the thinnest available laptop, measuring 0.76 inches at its thickest point and tapering to just 0.16 inches. These extreme features will most likely attract the attention of consumers contemplating a switch from Windows-based laptops. However, such consumers may decide not to switch upon learning that the Macbook air requires an external DVD drive, or that it only has a single USB port.

²A recent study by the Project of Excellence in Journalism (Rosenstiel et al. (2007)) argues that “In reporting their priorities, TV producers and journalists said things like, ‘People are always drawn to yellow tape and flashing lights’ or ‘urgent stories are the attention grabbers’. Others repeated the familiar mantra, ‘if it bleeds, it leads’.”

this direction, by analyzing a model that focuses exclusively on the above-mentioned trade-off between the cost of offering pure attention grabbers and the indirect gain in market share that they may generate. In the model, two firms, interpreted as media platforms as in Example 0.3, face a continuum of identical consumers having well-defined preferences over menus. The firms simultaneously choose a menu of “items” (in the TV example, an item is a program). It is costly for a firm to add items to its menu. Each firm aims to maximize (the value of) its market share minus the fixed costs associated with its menu.

Each consumer is initially assigned to one firm i (each firm initially gets half the consumers), which is interpreted as his default media provider. The consumer’s decision whether to switch to the competing firm j follows a two-stage procedure. In the first stage, it is determined whether the consumer will pay attention to j ’s menu. Conditional on the consumer’s attention being drawn to j ’s menu, the consumer will switch if and only if he finds j ’s menu strictly superior to i ’s menu, according to his preferences over menus. Thus, the consumer’s choice procedure is biased in favor of his “home base”: he switches to another firm only if his attention is drawn to its menu and he strictly prefers it to his default menu.

We assume that no two menus are perfect substitutes: the consumer is never indifferent between any two menus that do not contain one another. If consumers are indifferent between a menu M and a larger menu $M' \supset M$, we say that the items in $M' \setminus M$ are “pure attention grabbers”. Our interpretation of this indifference is that when consumers are endowed with the menu M' , they do not consume the items in $M' \setminus M$ on a regular basis. We refer to the smallest subset of M that does not contain pure attention grabbers as the set of “content items” in M . Our no-perfect-substitutes assumption guarantees that this subset, denoted $L(M)$, is unique.

The novel element of the model is the attention generation process in the first stage of the consumers’ choice procedure. Here we extend a modeling approach presented in Eliaz and Spiegler (in press). The consumer is endowed with a primitive called an “attention function” f , which determines whether the consumer will pay attention to the new menu M_j given the set of content items in the default menu $L(M_i)$. Thus, whether or not the consumer’s attention will be drawn to the new media provider depends on the subset of regularly consumed items in the default menu as well as the entire menu offered by the new provider. We view the attention function as an unobservable personal characteristic of the consumer which can be elicited (at least partially) from observed choices. The attention function captures the ease of attracting the consumer’s attention under various circumstances. The case of a rational consumer

is subsumed into the model as a special case, in which the consumer always considers all available menus and thus always chooses according to his preferences over menus.

We wish to emphasize that our main objective in this paper is to propose a theoretical approach for incorporating competition over consumers' *attention* into I.O. models. We interpret the model in media-market terms for expositional purposes, as it adds to the concreteness of the presentation. The model itself is very stylized and should not be mistaken for a descriptively faithful account of real-life media industries. We hope to demonstrate the kind of questions and answers one can obtain with this modeling approach, which we believe can serve as a platform for more descriptively faithful applications to media markets and other industries. The following key elements of the market model do seem to fit the media scenario.

(i) *The firms' objective function.* For media platforms such as commercial broadcasting networks, newspapers, content websites or search engines, prices do not play a strategic role. Because their profit is mostly generated by advertisements, it is directly related to the amount of traffic they attract.

(ii) *Each consumer has a "default" provider.* Consumers of newspapers, broadcast television and online content tend to exhibit some degree of loyalty to a particular newspaper, TV network or an internet portal. For example, in a study based on minute-by-minute television viewing for 1,067 individuals (Meyer and Muthaly (2008)), the authors conclude that "people who watch a lot of television are less likely to switch frequently between channels." As to internet browsing, Bucklin and Sismeiro (2003) and Zauberaman (2003) present evidence that users develop "within-site-lock-in".

(iii) *The scarcity of consumer attention and the role of content in allocating it.* The need to attract a viewer/reader's attention is best captured by the editorial choices of headlines and newsflashes, as well as the level of sensationalism (e.g., the degree of violence or obscenity) of television programs (e.g., the escalating level of extremity adopted by reality shows such as "Fear factor" or talk shows such as "Jerry Springer"). Of course, these content strategies are partly a response to changing viewers' tastes, but we believe it may be insightful to think of them also as a response to changes in viewers' attention span.³

The consumer's choice procedure determines the market share that each firm receives under any profile of menus they offer. This completes the specification of a sym-

³One arena where sensationalism is intensely used for attention-grabbing purposes is local television news. According to the Boston Globe (Bennet (2007)), "The past two decades have seen a marked shift in local television news across the country, away from in-depth coverage and towards speed and spectacle."

metric, complete-information, simultaneous-move game played between the two firms. Our assumptions on the firms’ cost structure imply that if consumers were rational, then in Nash equilibrium both firms would offer the smallest menu that maximizes consumers’ utility, and hence, contains no pure attention grabbers. We show that under a few mild assumptions on the model’s primitives, symmetric Nash equilibrium departs from this rational-consumer benchmark: the probability that firms offer menus that maximize consumer utility is strictly between zero and one. Moreover, firms employ pure attention grabbers with positive probability.

The analytic heart of the paper focuses on two classes of attention functions. It is for these classes that we provide a characterization of symmetric Nash equilibria. We begin in Section 3 with the case in which items can be ordered according to their degree of *salience* or according to how well they attract attention. For a menu to attract a consumer’s attention, it must contain an item that is at least as “sensational” as all content items in the consumer’s default menu. We show that in this case of “salience-based” attention, symmetric Nash equilibria have several strong properties. First, while the equilibrium outcome departs from the rational-consumer benchmark, firms earn the same profits as if consumers had unlimited attention. Second, the only menus that contain pure attention grabbers in equilibrium are those that maximize consumer utility. Third, the probability that firms offer such utility-maximizing menus is a decreasing function of the cost of the item with the highest “sensation value”. Finally, this item is employed with positive probability as a pure attention grabber.

In Section 4 we turn to another class of attention functions, which we refer to as “similarity-based”. Here we assume for simplicity that every menu has only one content item, e.g. the favorite show on a TV channel. The consumer considers a new media provider if and only if it offers an item that is *similar* to the content item on the consumer’s default menu.⁴ Items are ordered along the real line, such that similarity means proximity. We show that as in the case of salience-based attention, firms’ profits in symmetric Nash equilibrium are the same as in the rational-consumer benchmark. In the extreme case in which one item resembles another if and only if the two are identical, we provide a complete characterization of symmetric equilibria, including

⁴Kennedy (2002) analyzes programme introductions by television networks and compares the payoff to imitative and differentiated introductions. His analysis indicates that the networks imitate one another when introducing new programs, and that on average, imitative introductions *underperform* in terms of rating relative to differentiated introductions. The author concludes that this finding “suggests that non-payoff-maximizing imitation is common in at least one industry”. We propose to interpret the author’s finding as evidence suggesting that a television programme that imitates a programme aired by another network serves as an attention grabber and therefore its overall value to the network is not generated purely by its direct demand.

the probability that each item is offered as a real content item and as a pure attention grabber, and the rate at which consumers switch suppliers in equilibrium.

In both cases of salience-based and similarity-based attention, we see that industry profits are as if attention was not scarce. Although low-cost, low-quality menus are offered in equilibrium, the equilibrium cost of pure attention grabbers turns out to dissipate whatever excess profits such menus might enable. In Section 5, we show that whenever firms earn rational-consumer equilibrium profits, the equilibrium has an important property that relates two aspects of a firm’s strategy: the quality of its menu and whether or not it contains pure attention grabbers. Specifically, for every pair of menus M and M' that are offered in equilibrium, if M is not the best menu, and if consumer attention is drawn from M to M' only as a result of pure attention grabbers in M' , then it must be the case that the consumer prefers M' to M . This result, referred to as the “*effective marketing property*”, extends a similar finding in Eliaz and Spiegler (in press).

Our assumption that all consumers are identical is clearly unrealistic, and its role in the present paper is to sharpen our understanding of the role of attention grabbing in a competitive environment. In Section 6 we introduce preference heterogeneity into a model with salience-based attention. We assume that for every consumer type, each menu has a single content item. We also assume that the best attention grabber is not the favorite item for any consumer type. We show that if menu costs are sufficiently small, there is a symmetric Nash equilibrium that mimics a particular specification of the homogenous-consumers case analyzed in Section 3. Thus, many of the properties derived for the homogenous-consumers case carry over to the heterogeneous-consumers case.

Related literature

This paper extends Eliaz and Spiegler (in press), where we originally introduced the idea of a two-stage choice procedure in which consumers first form a “consideration set”, which is a subset of the objectively feasible set of market alternatives, and then apply preferences to the consideration set.⁵ In both papers, only the first stage of the choice procedure is sensitive to the firms’ marketing strategies. Both papers study market models in which firms choose which product to offer and how to market it, aiming to maximize the value of their market share minus the fixed costs associated with their strategies. Finally, the two papers have a few themes in common: the ques-

⁵The notion of *consideration sets* originates from the marketing literature, which has long recognized that the consumption decision follows a two-step decision process. For extensive surveys of this literature, see Alba, Hutchinson and Lynch (1991) and Roberts and Lattin (1997).

tion of whether competitive marketing brings industry profits to the rational-consumer benchmark level, and the question of how the firms' product design and marketing strategies are correlated, as captured by the effective marketing property.

However, there are several substantial differences between the two papers. First and foremost, the formalism used here is quite different than the one used in Eliaz and Spiegler (in press). In particular, there are important contrasts in how each paper models firms' strategies and the consumers' choice process. While in Eliaz and Spiegler (in press) there is an a-priori distinction between product design and marketing strategies, in the present paper the marketing strategies in question, pure attention grabbers, are themselves products. Thus, two consumers with different preferences would have a different partition of a given menu into content items and pure attention grabbers. This not only adds a technical complication to the model, but also changes the analysis when the consumer population is heterogeneous (an extension Eliaz and Spiegler (in press) do not address). Second, there is the obvious difference in the marketing strategies under examination: the use of attention-grabbing products by multi-product firms in the present paper, as opposed to the use of advertising and product display by single-product firms in Eliaz and Spiegler (in press). Finally, the classes of attention functions analyzed in the two papers are different and lead to very different analysis.

Piccione and Spiegler (2009) study the two-stage procedure in a market model that incorporates price setting while abstracting from fixed costs. In that model, single-product firms choose the price of their product as well as its "price format". Whether or not the consumer makes a price comparison between the two firms is purely a function of the firms' price formats, which captures the complexity of comparing them. The Piccione-Spiegler specification of the two-stage procedure and the firms' objective function leads to a market model that differs substantially from this paper.

A choice-theoretic analysis of decision processes that involve consideration set formation is explored in Masatlioglu and Nakajima (2009) and in Masatlioglu, Nakajima and Ozbay (2009). The first paper axiomatizes a more general choice procedure than ours, in which the consumer iteratively constructs consideration sets starting from some exogenously given default option. The second paper axiomatizes a two-stage choice procedure in which first, the decision-maker employs an "attention filter" to shrink the objectively feasible set to a consideration set, and second, he applies his preferences to the consideration set. Both papers are concerned with eliciting the parameters of the choice procedures (e.g., the preference orderings and the attention filter) from observed behavior. As such, these papers complement our own, which deals with strategic manipulation of consumers' consideration sets.

Another related strand in the decision-theoretic literature concerns preferences over menus (e.g., Kreps (1979), Dekel, Lipman and Rustichini (2001), Gul and Pesendorfer (2001)). Indeed, in the concluding section we show that a special case of our model with salience-based attention can be re-interpreted as an instance of a “naïve” multi-selves model, a re-interpretation with interesting welfare implications.

The pure attention grabbers in our paper constitute a particular form of “loss leaders”: since they are not regularly consumed, they superficially fail to cover their cost. By comparison, the notion of loss leaders in the literature typically refers to products that *are* consumed on a regular basis yet generate a direct loss because they are priced below marginal cost (e.g., see Lal and Matutes (1994)).⁶

The indirect effect that a television show can have on viewers is documented in Anand and Shachar (2004). That paper provides empirical evidence that the introduction of a new television show to a network increases the extent to which viewers watch *other* shows on that network. The authors, however, do not interpret their finding as evidence of the use of attention grabbers. Instead, they offer an explanation based on the idea that a consumer who observes some product of a firm infers information about the entire product line of that firm.

Finally, this paper joins the theoretical literature on market interactions between profit-maximizing firms and boundedly rational consumers. Ellison (2006), Armstrong (2008) and Spiegel (forthcoming) provide general treatments of this growing research field.

2 A Model

We analyze an idealized model of competition between media platforms. Let X be a finite set of “items”, where $|X| \geq 2$. A menu is a non-empty subset of X . Let $P(X)$ be the set of all menus. Two firms play a symmetric complete-information game in which they simultaneously choose menus. A mixed strategy for a firm is a probability distribution $\sigma \in \Delta(P(X))$. We denote the support of σ by $S(\sigma)$. Each menu carries a fixed cost, defined as $c(M) = \sum_{x \in M} c_x$, where $c_x > 0$ is the fixed cost associated with the item x . Each firm aims to maximize the value of its market share minus its costs. Henceforth, we shall normalize the costs to be expressed in terms of market share.

The two firms face a continuum of identical consumers, who are characterized by

⁶One notable exception is Kamenica (2008), which illustrates a signalling equilibrium in which there is positive probability that a monopolist produces a high quality product even in a state of nature where all consumer types strictly prefer other products in the firm’s product line. Kamenica refers to such a product as a “premium loss leader”.

two primitives: a *preference relation* \succsim over the set of menus $P(X)$, and an “*attention function*” $f : X \times P(X) \rightarrow \{0, 1\}$ that governs the attention grabbing process. The preference relation \succsim satisfies two properties:

Monotonicity: For every $M, M' \in P(X)$, if $M \subset M'$ then $M' \succsim M$.

No perfect substitutes: For every $M, M' \in P(X)$, if $M \sim M'$, then $M \subseteq M'$ or $M' \subseteq M$.

We interpret monotonicity as a free disposal property: the consumer is free to watch any subset of a given menu of programs. The no-perfect-substitutes assumption says that the consumer is never indifferent between menus that do not contain one another. It has the following immediate implication, which will play an important role in the sequel: for every menu M , there is a *unique* subset $L(M) \subseteq M$ satisfying the following property: if $M' \subseteq M$ and $M' \sim M$, then $M' \supseteq L(M)$. We interpret $L(M)$ as the set of items the consumer actually consumes *regularly* from the menu M . The items in $L(M)$ are referred to as “*content items*” in M , while the items in $M \setminus L(M)$ are referred to as the “*pure attention grabbers*” in M . Thus, an item functions as a “pure attention grabber” in a menu if its inclusion is not necessary for satisfying consumer tastes; and $L(M)$ is the unique smallest subset of M that does not contain pure attention grabbers.

Definition 1 (beating relation) *We say that M beats M' if the following two conditions hold: (i) there exists $x \in M$ such that $f(x, L(M')) = 1$; (ii) $M \succ M'$.*

Armed with this definition, we are finally ready to describe consumer choice. Given a profile of menus (M_1, M_2) , consumers choose according to the following two-stage procedure. Each consumer is initially assigned (with equal probability) to a random firm $i = 1, 2$. This initial assignment represents the consumer’s default. The consumer switches to firm $j \neq i$ if and only if M_j *beats* M_i .

The interpretation is as follows. The consumer has a tendency to stick to his default media provider, and not even consider alternative providers, due to lack of attention or sheer inertia. The consumer considers a new firm only if its menu includes an item that satisfies a certain criterion (captured by f) in relation to the items he *regularly* consumes from the default provider. The existence of such an item draws the consumer’s attention to the new firm. Having considered its menu, the consumer will switch to it only if he finds it strictly superior (according to his true underlying preferences \succsim) to his default menu.⁷

⁷Note that although the second argument of f can be any $M \in P(X)$, for the choice procedure it only matters how f acts on the set of *content items* in the default menu.

The beating relation is nothing but the strict *revealed* preference relation over menus induced by the consumer's choice procedure. To an outside observer, M_j is revealed to be preferred to M_i if a consumer for whom M_i is the default menu switches to M_j . According to our model, this revealed preference relation typically fails to coincide with the consumer's true preference relation \succsim , because it also reflects his limited attention. And as we shall see below, the beating relation may be intransitive.

The following example illustrates how consumer choice may be sensitive to pure attention grabbers.

Example 1 *Let $X = \{a, b\}$, and assume $\{a, b\} \sim \{a\} \succ \{b\}$, $f(b, \{b\}) = 1$, $f(a, \{b\}) = 0$. Then, if a consumer is initially assigned to a firm that offers the menu $\{b\}$ and the rival firm offers the menu $\{a\}$, the consumer will stick to his default firm. However, if the rival firm offers $\{a, b\}$, the consumer will switch to the new firm.*

The tuple $\langle X, c, \succsim, f \rangle$ fully defines the simultaneous-move game played between the firms, where $P(X)$ is the strategy space and firm i 's payoff function is as follows:

$$\pi_i(M_1, M_2) = \begin{cases} \frac{1}{2}[1 + \max_{x \in M_i} f(x, L(M_j))] - c(M_i) & \text{if } M_i \succ M_j \\ \frac{1}{2}[1 - \max_{x \in M_j} f(x, L(M_i))] - c(M_i) & \text{if } M_j \succ M_i \\ \frac{1}{2} - c(M_i) & \text{if } M_i \sim M_j \end{cases} \quad (1)$$

We impose the following assumptions on the primitives \succsim, c, f .

(A1) For every $M, M' \in P(X)$, if $M \succ M'$ then $c(L(M)) > c(L(M'))$.

(A2) For every $M \in P(X)$, there exists $x \in X$ such that $f(x, M) = 1$.

(A3) $c(X) < \frac{1}{2}$

Assumption (A1) links the costs of providing a menu with consumer preferences. When neither M nor M' contain pure attention grabbers, if consumers prefer M to M' , then it must be more costly to provide the more desirable menu M . This assumption enables us to interpret \succsim as a quality ranking: higher-quality menus are more costly to provide. The concluding section discusses the extent to which our results are robust to a weakening of this assumption. Assumption (A2) means that for any set of content items in the menu of the consumer's default firm, there is an item that the competing firm can include in its menu, that will attract consumer attention. Put differently, a firm cannot prevent consumer attention from being drawn to its rival. The interpretation of (A3) is that costs are not too high, in the sense that when firms share the market

equally, each has an incentive to do “whatever it takes” to win the entire market. Thus, (A2) implies that it is feasible for a firm to attract the attention of its rival’s consumers, while (A3) implies that it will have an incentive to do so if this leads to a 50% increase in its market share.

To simplify the exposition, we introduce the following two pieces of notation. First, denote $M^* = L(X)$. By monotonicity of \succsim , M^* is the smallest menu among those that consumers find most desirable. Henceforth, we shall refer to M^* as the best menu for consumers. Second, given a mixed strategy σ , define

$$\beta_\sigma(M) = \sum_{M' \sim M} \sigma(M')$$

This is the probability that σ assigns to menus that consumers find exactly as good as M (including, of course, M itself).

The case of a consumer who is rational in the sense of always choosing according to his true underlying preferences \succsim is captured by an attention function f satisfying $f(x, M) = 1$ for all $(x, M) \in X \times P(X)$. We refer to this case as the “rational benchmark”. In this case, both firms offer the menu M^* and earn a payoff of $\frac{1}{2} - c(M^*)$ in Nash equilibrium. This is also the max-min payoff under (A2) and (A3). The reason is as follows. The worst-case scenario for a firm, regardless of its strategy, is that its rival chooses the universal set X . But the best-reply against X is M^* because this is the least costly menu that generates a market share of $\frac{1}{2}$ against X .

Consumer rationality is not a necessary condition for the rational-consumer outcome to emerge in equilibrium, as the following remark observes.

Remark 1 *Suppose that M^* beats every menu $M \in P(X)$ for which $M^* \succ M$. Then, both firms offer M^* with probability one in Nash equilibrium.*

We omit the proof, as it is quite conventional. For the rest of the paper, we assume that the condition for the rational-consumer outcome fails:

(A4) There exists $M \in P(X)$ such that $M^* \succ M$ and yet M^* does not beat M .

This assumption, combined with (A1), implies that when one firm offers M^* , its opponent is able to offer a lower-cost, lower-quality menu M such that consumers’ attention will not be drawn from M to M^* . We adopt Assumptions (A1)-(A4) throughout the rest of the paper. They turn out to imply that symmetric Nash equilibria are necessarily mixed. The following result provides a preliminary characterization.

Proposition 1 *Let σ be a symmetric Nash equilibrium strategy. Then: (i) $\beta_\sigma(M^*) \in (0, 1)$; (ii) there exists $M \in S(\sigma)$ such that $M^* \subset M$.*

Proof. (i) Suppose that $\beta_\sigma(M^*) = 0$. Consider a menu $M \in S(\sigma)$ such that $M' \succsim M$ for all $M' \in S(\sigma)$. Then, M beats no menu in $S(\sigma)$. Therefore, M generates a market share of at most $\frac{1}{2}$. If a firm deviates from M to X , the deviation is profitable. By (A2), it raises the firm's market share from $\frac{1}{2}$ to 1, whereas by (A3), it changes its cost by $c(X) - c(M) < \frac{1}{2}$. Now suppose that $\beta_\sigma(M^*) = 1$. Since M^* is the (unique) least costly menu M such that $M \sim M^*$, each firm must offer M^* with probability one. By (A1) and (A4), there exists a menu M' such that M' is less costly than M^* and M^* does not beat M' , it is profitable for a firm to deviate to M' . It follows that $\beta_\sigma(M^*) \in (0, 1)$.

(ii) Assume the contrary. By (i), $\beta_\sigma(M^*) > 0$, hence $\beta_\sigma(M^*) = \sigma(M^*)$. Let \mathcal{M}_1 denote the set of menus in $S(\sigma)$ that M^* beats, and let \mathcal{M}_0 denote the set of menus $M \in S(\sigma)$ for which $M^* \succ M$ yet M^* does not beat M . Recall that all menus are weakly worse than M^* , hence, the set $\mathcal{M}_0 \cup \mathcal{M}_1$ includes all the menus other than M^* .

Suppose \mathcal{M}_1 is empty. Then M^* generates a payoff of $\frac{1}{2} - c(M^*)$. Let $\tilde{M} \in S(\sigma)$ be a \succsim -maximal menu in \mathcal{M}_0 . By (A1), $c(L(\tilde{M})) < c(M^*)$. Moreover, by the definition of the beating relation, no menu in $S(\sigma)$ beats $L(\tilde{M})$. Therefore, if a firm deviated to $L(\tilde{M})$, it would generate a market share of at least $\frac{1}{2}$ while costing less than $c(M^*)$, hence the deviation would be profitable. It follows that \mathcal{M}_1 is non-empty.

Let M_* denote some \succsim -minimal menu in \mathcal{M}_1 . Thus, M_* does not beat any menu in \mathcal{M}_1 . Suppose that a firm deviates from M_* to M^* . This deviation is unprofitable only if the following inequality holds:

$$\frac{1}{2}\sigma(M^*) + \frac{1}{2} \sum_{M \in \mathcal{M}_1} \sigma(M) - c(M^*) + c(M_*) \leq 0 \quad (2)$$

Now suppose that a firm deviates from M^* to X . This deviation is unprofitable only if the following inequality holds:

$$\frac{1}{2} \sum_{M \in \mathcal{M}_0} \sigma(M) - c(X) + c(M^*) \leq 0 \quad (3)$$

Note that $S(\sigma) = \{M^*\} \cup \mathcal{M}_0 \cup \mathcal{M}_1$. Therefore, adding up the two inequalities yields the inequality

$$\frac{1}{2} \leq c(X) - c(M_*) < c(X)$$

in contradiction to (A3). ■

Thus, when the outcome of symmetric Nash equilibrium departs from the rational-consumer benchmark (in the sense that sub-optimal menus are offered with positive probability), the probability that utility maximizing menus are offered is positive, and pure attention grabbers are offered with positive probability. Since a pure attention grabber is costly to offer and makes no difference for consumer welfare, the equilibrium use of pure attention grabbers is socially wasteful. The rationale for the use of pure attention grabbers is that they exert a positive externality on other items in the firm's menu - they attract consumers' attention to these other items, thus increasing the firm's market share.

Comment: The interpretation of $L(M)$

Recall that the subset $L(M)$ is defined in terms of the preference relation \succsim over menus: $L(M)$ is the smallest \succsim -equivalent subset of M . At the same time, we interpreted $L(M)$ as the set of items that the consumer *regularly consumes from* M . This interpretation justifies our assumption that the attention grabbing process does not depend on pure attention grabbers in the default menu: whether or not consumer attention is drawn away from M should not depend on items in M that are rarely consumed. Also recall that the pure attention grabbers in any menu are by definition irrelevant for the preference ranking. It follows that in our model, pure attention grabbers in the consumer's default menu are entirely irrelevant for his choice.

Our interpretation of $L(M)$ does not rule out the possibility that consumers occasionally watch pure attention grabbers. However, consumers would not demand any compensation if these items were removed from the menu. For example, a sensational reality show will constitute a pure attention grabber if a consumer refuses to pay a premium to have access to this program, even though he might occasionally watch the program when it is freely available.

The irrelevance of pure attention grabbers in the consumer's default menu will play an important role in our analysis. It implies that when firm j considers whether to add a pure attention grabber to its menu, it weighs the extra menu cost only against the benefit of attracting the attention of consumers who are initially assigned to the rival firm i . In particular, firm j need not worry that adding the attention grabber might affect the choice of those consumers for whom it is the default provider.

We should point out that as far as the next section is concerned, none of our results would change if we adopted an alternative definition of the beating relation, in which M' replaces $L(M')$. Sections 4 and 5, however, rely on our original definition of the beating relation.

3 Saliency-Based Attention

In this section we analyze in detail a special case of our model. We say that f is a *saliency-based attention function* if there is a complete, anti-symmetric and transitive binary relation R on X , such that $f(x, M) = 1$ if and only if xRy for all $y \in M$. A saliency-based attention function captures the idea that items can be ordered according to their attention grabbing powers. For instance, R can represent the sensation value of different types of news items. In order to attract attention, a competing channel should broadcast news items that are at least as sensational as anything the consumer regularly watches on his default channel. Note that the attention relation R is reflexive - i.e., xRx for all $x \in X$. For every menu M , let $r(M)$ denote the R -maximal item in M . Denote $x^* = r(X)$. By (A1), $c_{x^*} < c(M^*)$. By (A4), $x^* \notin M^*$.

The following example illustrates that although the attention function is based on a complete and transitive binary relation, the consumer's choice between menus is typically inconsistent with maximization of a utility function over menus, since the beating relation may be intransitive.

Example 2 *Suppose $xRyRz$ and that \succsim satisfies $\{z, y\} \succ \{x, y\} \sim \{y\} \succ \{x\}$. The menu $\{z, y\}$ beats the menu $\{x, y\}$ because $L(\{x, y\}) = \{y\}$ and yRy . The menu $\{x, y\}$ beats $\{x\}$ because xRx . However, the menu $\{z, y\}$ does not beat $\{x\}$ since $y \not R x$ and $z \not R x$. The revealed “indifference” relation over menus is also intransitive. To see why, note that $\{x\}$ does not beat $\{z, y\}$ because $\{z, y\} \succ \{x\}$. We have already seen that $\{z, y\}$ does not beat $\{x\}$. Thus, consumer choices “reveal” that he is indifferent between $\{z, y\}$ and $\{x\}$. Similarly, $\{x\}$ does not beat $\{y\}$ (because $\{y\} \succ \{x\}$) and $\{y\}$ does not beat $\{x\}$ (because $y \not R x$). Thus, consumer choices “reveal” that he is indifferent between $\{x\}$ and $\{y\}$. However, $\{z, y\}$ beats $\{y\}$, because yRy and $\{z, y\} \succ \{y\}$.*

If consumers behaved as if they were maximizing some utility function over menus (which need not coincide with \succsim), then by the assumption that $c(X) < \frac{1}{2}$, competitive forces would push firms to offer the cheapest menu among those that are optimal according to this revealed preference relation. The fact that consumers choose between menus in a way that cannot be rationalized is what makes this model non-trivial to analyze.

3.1 An Example: Low-Quality Sensations

We illustrate the structure of symmetric Nash equilibria in this model with the following simple example. Assume that $c_{x^*} < c_x$ for all $x \neq x^*$. That is, the item with the

highest “sensation value” is also the cheapest to produce. By (A1) and our no-perfect-substitutes assumption, this means that $\{x\} \succ \{x^*\}$ for every $x \neq x^*$. In other words, the best attention grabber is also the worst item in terms of consumer preferences. Thus, there is an extreme tension between the items that maximize consumer welfare and the items that attract the most attention.

There is a symmetric Nash equilibrium in this case, where the mixed equilibrium strategy σ is as follows:

$$\sigma\{x^*\} = 2c_{x^*} \quad (4)$$

$$\sigma(M^*) = 1 - 2c(M^*) \quad (5)$$

$$\sigma(M^* \cup \{x^*\}) = 2c(M^*) - 2c_{x^*} \quad (6)$$

To see why this is an equilibrium, let us write down the payoff that each of the three pure strategies generates against σ . The menu M^* generates a market share of $\frac{1}{2}$ because it does not beat any other menu. The menu $\{x^*\}$ generates a market share of $\frac{1}{2} - \frac{1}{2}\sigma(M^* \cup \{x^*\})$ because it is only beaten by $M^* \cup \{x^*\}$. The latter menu generates a market share of $\frac{1}{2} + \frac{1}{2}\sigma(\{x^*\})$ because $\{x^*\}$ is the only menu that $M^* \cup \{x^*\}$ beats. It is easy to see that all three menus generate a payoff of $\frac{1}{2} - c(M^*)$ against σ .

Suppose there exists some menu M outside the support of σ , which yields a higher payoff against σ . Among all the menus that are \succsim -equivalent to M^* , the menu $M^* \cup \{x^*\}$ is the cheapest except M^* , and in addition, it attracts attention away from every possible default menu. Therefore, it must be the case that $M^* \succ M$, in which case it follows that M is necessarily beaten by $M^* \cup \{x^*\}$. Suppose M beats $\{x^*\}$. Since x^* is the best attention grabber in X , it must be that $x^* \in M$. Therefore,

$$c(M) = c(M \setminus \{x^*\}) + c_{x^*} > 2c_{x^*}$$

The market share that M generates is at most (in the best-case scenario where M^* does not beat M),

$$\frac{1}{2}[1 - \sigma(M^* \cup \{x^*\})] + \frac{1}{2}\sigma(\{x^*\}) = \frac{1}{2} - c(M^*) + 2c_{x^*}$$

It follows that the expected payoff from M is strictly lower than $\frac{1}{2} - c(M^*)$, the expected payoff from each pure strategy in σ .

If M does not beat x^* , then the highest market share it can generate is $\frac{1}{2} - \frac{1}{2}\sigma(M^* \cup \{x^*\})$. But since $c(M) > c_{x^*}$, this same market share can be achieved with lower cost by

offering $\{x^*\}$. Hence, M cannot generate a higher expected payoff against σ compared with the payoff generated by each menu in σ , a contradiction. It follows that σ is a symmetric equilibrium strategy. In fact, it is the only symmetric equilibrium, as we will show later.

Observe that in this equilibrium, the total probability that x^* is offered is $2c(M^*)$. However, as c_{x^*} goes down, x^* is offered more frequently as a pure attention grabber and less frequently as a content item.

3.2 Equilibrium Characterization

We now turn to characterize the symmetric Nash equilibria. All the results in this sub-section are based on the assumption that f is a salience-based attention function.

Proposition 2 *Let σ be a symmetric Nash equilibrium strategy. Then:*

- (i) *Firms earn the max-min payoff $\frac{1}{2} - c(M^*)$.*
- (ii) *If $M \in S(\sigma)$ contains a pure attention grabber, then $M \sim M^*$.*
- (iii) $\beta_\sigma(M^*) = 1 - 2c_{x^*}$.
- (iv) $\sigma(M^* \cup \{x^*\}) > 0$.

The proof relies on two lemmas. The first lemma establishes that equilibrium menus never contain more than one pure attention grabber. The second lemma shows that the rational-consumer menu M^* is offered with positive probability in any symmetric equilibrium. Moreover, this menu fails to attract attention from any inferior menu that is offered in equilibrium.

Lemma 1 *Let σ be a symmetric Nash equilibrium strategy. Then, every $M \in S(\sigma)$ contains at most one pure attention grabber.*

Proof. Assume that $M \in S(\sigma)$ contains at least two pure attention grabbers x, y , where xRy . If a firm deviates from M to $M \setminus \{y\}$, it reduces its cost without changing its market share, for the following reasons. First, $M \sim M \setminus \{y\}$ by the assumption that y is a pure attention grabber in M . Second, $M \setminus \{y\}$ beats exactly the same menus as M , because $r(M \setminus \{y\}) = r(M)$. Third, $M \setminus \{y\}$ is beaten by exactly the same menus as M , because $L(M \setminus \{y\}) = L(M)$. ■

Lemma 2 *Let σ be a symmetric Nash equilibrium strategy. Then, $M^* \in S(\sigma)$ and there exists no menu $M \in S(\sigma)$ that is beaten by M^* .*

Proof. Assume the contrary. Define $\mathcal{M}_\sigma = \{M \in S(\sigma) \mid M \sim M^*\}$. By Proposition 1, $\sum_{M \in \mathcal{M}_\sigma} \sigma(M) = \beta_\sigma(M^*) \in (0, 1)$. Suppose that \mathcal{M}_σ includes a menu $M \neq M^*$ that beats no menu in $S(\sigma)$. Therefore, M generates a market share of $\frac{1}{2}$. By the definition of M^* , $c(M) > c(M^*)$. It follows that M yields a payoff strictly below the max-min level $\frac{1}{2} - c(M^*)$, a contradiction. The remaining possibility is that for every $M \in \mathcal{M}_\sigma$, there exists $\tilde{M} \in S(\sigma)$ such that M beats \tilde{M} . Our task in this proof is to rule out this possibility.

List the menus in \mathcal{M}_σ as follows: M_1, \dots, M_K , $K \geq 1$, such that

$$r(M_K)Rr(M_{K-1})R \cdots Rr(M_1)$$

For every $M_k \in \mathcal{M}_\sigma$, let \tilde{M}_k be one of the \succsim -minimal menus among those that are members of $S(\sigma)$ and beaten by M_k . By definition, $r(M_1)Rx$ for all $x \in L(\tilde{M}_1)$. By transitivity of R , it follows that for every $k = 2, \dots, K$, $r(M_k)Rx$ for all $x \in L(\tilde{M}_1)$ - i.e., \tilde{M}_1 is beaten by every menu in \mathcal{M}_σ .

Assume that \tilde{M}_1 beats some $M \in S(\sigma)$. That is, $r(\tilde{M}_1)Rx$ for every $x \in L(M)$. Let us distinguish between two cases. First, suppose that $r(\tilde{M}_1) \in L(\tilde{M}_1)$. Then, $r(M_1)Rr(\tilde{M}_1)$, and by the transitivity of R , $r(M_1)Rx$ for every $x \in L(M)$, contradicting the definition of \tilde{M}_1 as a \succsim -minimal menu in $S(\sigma)$ that is beaten by M_1 . Second, suppose that $r(\tilde{M}_1) \in \tilde{M}_1 \setminus L(\tilde{M}_1)$ - i.e., that $r(\tilde{M}_1)$ is a pure attention grabber in \tilde{M}_1 . By Lemma 1, \tilde{M}_1 contains no other pure attention grabbers except $r(\tilde{M}_1)$. Note that it must be the case that $r(\tilde{M}_1)Rr(M_1)$ and $r(M_1) \not R r(\tilde{M}_1)$ - otherwise, M_1 would beat all the menus that \tilde{M}_1 beats, thus contradicting the definition of \tilde{M}_1 . Let \mathcal{B} denote the set of menus in $S(\sigma)$ that are beaten by \tilde{M}_1 and not by $L(\tilde{M}_1)$. From the firms' decision not to deviate from \tilde{M}_1 to $L(\tilde{M}_1)$, we conclude that

$$\frac{1}{2} \sum_{M \in \mathcal{B}} \sigma(M) - c_{r(\tilde{M}_1)} \geq 0$$

At the same time, from the firms' decision not to deviate from M_1 to a menu that replaces $r(M_1)$ with $r(\tilde{M}_1)$, we conclude that

$$\frac{1}{2} \sum_{M \in \mathcal{B}} \sigma(M) - c_{r(\tilde{M}_1)} + c_{r(M_1)} \leq 0$$

The two inequalities contradict each other.

We have thus established that \tilde{M}_1 beats no menu in $S(\sigma)$, as well as beaten by every menu in \mathcal{M}_σ . Suppose that a firm deviates from \tilde{M}_1 to $M^* \cup \{x^*\}$. Then, the

firm increases its market share by at least $\frac{1}{2}\beta_\sigma(M^*) + \frac{1}{2}(1 - \beta_\sigma(M^*)) = \frac{1}{2}$, which by assumption is strictly higher than the change in the cost. Therefore, the deviation is profitable, a contradiction. ■

We are now ready to prove the proposition.

Proof of Proposition 2. (i) This follows immediately from Lemma 2. Since M^* belongs to $S(\sigma)$ and beats no menu in $S(\sigma)$, it generates a market share of $\frac{1}{2}$ and therefore yields a payoff of $\frac{1}{2} - c(M^*)$.

(ii) Assume that there exists a menu $M \in S(\sigma)$ such that: (i) $M^* \succ M$; and (ii) $L(M) \subset M$. If $r(M) \in L(M)$, then every menu beats M if and only if it beats $L(M)$, and every menu is beaten by M if and only if it is beaten by $L(M)$. Since $c(L(M)) < c(M)$, it is profitable for a firm to deviate from M to $L(M)$. It follows that $r(M) \notin L(M)$, hence $M = L(M) \cup \{r(M)\}$. Now consider the menu $M^* \cup \{r(M)\}$. This menu beats every menu $M'' \in S(\sigma)$ that is beaten by M and not by $L(M)$. In addition, by construction, the menu $M^* \cup \{r(M)\}$ beats M . By Lemma 2, M^* beats no menu in $S(\sigma)$. It follows that the benefit from adding $r(M)$ to M^* in terms of added market share is strictly higher than the cost of this addition. Therefore, the deviation is profitable, a contradiction.

(iii) Assume that $\beta_\sigma(M^*) < 1 - 2c_{x^*}$. By Lemma 2, M^* beats no menu in $S(\sigma)$. Therefore, in order for a deviation to $M^* \cup \{x^*\}$ to be unprofitable, it must be that $c_{x^*} \geq \frac{1}{2}[1 - \beta_\sigma(M^*)]$, a contradiction. Now assume that $\beta_\sigma(M^*) > 1 - 2c_{x^*}$. By part (ii) of Proposition 1, there exists $M \in S(\sigma)$ such that $M \supset M^*$. Let \mathcal{M}^* denote the set of menus $M' \in S(\sigma)$ that M beats. The set \mathcal{M}^* must be non-empty - otherwise, M generates a payoff below $\frac{1}{2} - c(M^*)$, a contradiction. By part (i), M generates a payoff of $\frac{1}{2} - c(M^*)$ against σ . Therefore:

$$\frac{1}{2} - c(M^*) = \frac{1}{2} - c(M) + \frac{1}{2} \sum_{M' \in \mathcal{M}^*} \sigma(M')$$

By definition, $\sum_{M' \in \mathcal{M}^*} \sigma(M') \leq 1 - \beta_\sigma(M^*)$. Therefore,

$$c(M) - c(M^*) \leq \frac{1}{2}[1 - \beta_\sigma(M^*)] < c_{x^*}$$

Hence, none of these menus M includes x^* . Let M^{**} be the \succsim -maximal menu among all menus M for which $M^* \succ M$ and $x^* \in L(M)$. Thus, M^{**} is not beaten by any menu in $S(\sigma)$. Hence, it achieves a market share of at least $\frac{1}{2}$. By (A1), $c[L(M^{**})] < c[L(M^*)]$. But this means that M^{**} generates a payoff higher than $\frac{1}{2} - c(M^*)$, in contradiction

to part (i) of the proposition.

(iv) Assume $M^* \cup \{x^*\}$ does not belong to $S(\sigma)$. Then, a firm that deviates to M^{**} , as defined in the proof of (iii), would earn more than $\frac{1}{2} - c(M^*)$, in contradiction to part (i) of the proposition. ■

Thus, symmetric Nash equilibria in this model have several strong properties. First, although the equilibrium outcome departs from the rational-consumer benchmark, firms' profits are equal to the max-min level, which, as we saw, coincides with the rational-consumer benchmark. In other words, industry profits are in some sense “competitive”. The use of pure attention grabbers is restricted to menus that consumers find optimal. In particular, the R -maximal item x^* is employed with positive probability as a pure attention grabber to attract attention to M^* . In contrast, when firms offer sub-optimal menus, they do not adorn them with pure attention grabbers. Finally, the probability that sub-optimal menus are offered is entirely determined by the cost of the best attention grabber. As this cost goes up, the probability that consumers are offered menus that maximize their utility goes down.

On a somewhat speculative note, this result provides a perspective into the ongoing debate over the sensationalism of broadcast television, particularly news (see Bennet (2007)). Critics in this debate attack popular channels for engaging in empty rating-driven sensationalism. Broadcasters typically retort that they “give the public what it wants”. Viewed through the prism of Proposition 2, both parties to this debate are right to some extent. Indeed, media providers use sensationalism as a pure attention grabbing device that does not directly increase consumer welfare. However, as the cost of the sensational items declines, it is more likely that these items help give viewers what they want, because they help to draw their attention to a package that maximizes their utility.

Recall that in our discussion of the example in the previous sub-section, we claimed that there exist no symmetric equilibria apart from the one given there. We can now apply Lemma 1 and Proposition 2 to prove this claim.

Proposition 3 *If $c_{x^*} < c_x$ for all $x \neq x^*$, then (4)-(6) is the unique symmetric equilibrium strategy.*

Proof. Let σ be some symmetric equilibrium. By Proposition 2, firms earn the max-min payoff and both M^* and $M^* \cup \{x^*\}$ are in $S(\sigma)$. Suppose $S(\sigma)$ also contains some $M \notin \{M^*, \{x^*\}, (M^* \cup \{x^*\})\}$. If $M \in S(\sigma)$ contains a pure attention grabber, then by part (ii) of Proposition 2, $M \sim M^*$. By Lemma 1, $M = M^* \cup \{y\}$ for some $y \in X$.

If $y \neq x^*$, then $M^* \cup \{x^*\}$ achieves at least as high a market share as M but with lower costs. Hence, M does not contain a pure attention grabber.

Denote by \mathcal{A} the set of menus $M \in S(\sigma) \setminus \{M^*, \{x^*\}\}$ for which $L(M) = M$. Let \tilde{M} be the \succsim -minimal menu in \mathcal{A} . Suppose $x^* \notin \tilde{M}$. Then \tilde{M} does not beat any menu in $S(\sigma)$. Let $\mathcal{B} \subset \mathcal{A}$ denote the subset of menus in \mathcal{A} that beat $\{x^*\}$. If \mathcal{B} is non-empty, then every menu in this set must include x^* . By the definition of \tilde{M} , every menu in \mathcal{B} must also beat \tilde{M} . It follows that both \tilde{M} and $\{x^*\}$ achieve exactly the same market share, but $\{x^*\}$ is cheaper. Suppose $x^* \in \tilde{M}$. Then \tilde{M} necessarily beats x^* , but every menu in $S(\sigma)$ that beats $\{x^*\}$ also beats \tilde{M} . Hence, the gain in market share from playing \tilde{M} instead of $\{x^*\}$ is $\frac{1}{2}\sigma(\{x^*\})$. Since $\tilde{M} \in S(\sigma)$, it must be that $c(\tilde{M} \setminus \{x^*\}) \leq \frac{1}{2}\sigma(\{x^*\})$. Since by assumption, x^* is the cheapest item, it must be true that $c_{x^*} < \frac{1}{2}\sigma(\{x^*\})$. But by part (iii) of Proposition 2, $c_{x^*} = \frac{1}{2}[1 - \beta_\sigma(M^*)]$. Since we've assumed that $S(\sigma)$ includes \tilde{M} , in addition to M^* , $M^* \cup \{x^*\}$ and $\{x^*\}$, we conclude that $1 - \beta_\sigma(M^*) > \sigma(\{x^*\})$, hence $c_{x^*} > \frac{1}{2}\sigma(\{x^*\})$, a contradiction. It follows that $\tilde{M} \notin S(\sigma)$, which implies that $S(\sigma)$ can only include M^* , $M^* \cup \{x^*\}$ or $\{x^*\}$. It is straightforward to show that $S(\sigma)$ must include all of these menus. Therefore, the unique symmetric equilibrium is given by (4)-(6). ■

Thus, when the tension between the things that maximize consumers' utility and the things that maximize their attention is the strongest, the structure of equilibrium is extremely simple: each firm offers either the attention grabber only, or the best menu only, or the two combined. When x^* is not the cheapest alternative, one can construct equilibria with a more complicated structure.

4 Similarity-Based Attention

In the previous section, we assumed that items can be ordered according to how well they attract attention, independently of what they attract attention from. In many cases, however, an item attracts attention if it is *similar* to what the consumer regularly consumes. For instance, think of a TV viewer on a channel-flipping cruise. If he stumbles upon a familiar show, he may pause and pay more attention to the channel on which the show is aired.

Likewise, when a channel programs shows that contain features that are familiar to viewers from their TV habits, viewers are more likely to *recall* the channel and thus consider it as an option when thinking about what to watch on TV. Several studies in psychology and marketing confirm this intuition. For example, subjects in Markman

and Gentner (1997) were asked to make similarity comparisons between pairs of pictures and were then probed for recall. The recall probes were figures taken from the pictures and were either alignable (related to the commonalities) or nonalignable differences between the pairs. The authors show that the alignable differences were better memory probes than the nonalignable differences. Following up on these results, Zhang and Markman (1998) showed that attributes that differentiate later entrants from the first entrant are better remembered and listed more often in judgment formation protocols if the attributes are comparable along some common aspect (i.e., they are alignable differences) than if they do not correspond to any attributes of the first entrant (i.e., they are nonalignable differences).

Our model can capture this idea, provided that we interpret the attention function f as an object that captures the role of recall in the attention-generation process. We envision the consumer as trying to recall from memory those menus that are available to him before making his media consumption decision. The default menu is easily recalled since the consumer is used to it. However, a new menu may or may not be recalled, and the consumer will find it easier to recall it if it contains items that are similar to what the consumer is already familiar with.

For simplicity, we assume in this section that consumers have max-max preferences over menus. Formally, we assume that there is a linear order \succ^* on X such that $M \succ M'$ if and only if there exists $x \in M$ such that $x \succ^* y$ for all $y \in M'$. The interpretation is that every menu contains a single item which the consumer regularly consumes. By (A1), $x \succ^* y$ if and only if $c_x > c_y$. For every menu M , let $b(M)$ denote the \succ^* -maximal item in M . Thus, for every menu M , $L(M) = \{b(M)\}$. Denote $y^* = b(X)$. Given a mixed strategy σ , define $\beta_\sigma(x) = \sum_{b(M)=x} \sigma(M)$ to be the probability that x is offered as a \succ^* -maximal item in a menu.

To incorporate similarity considerations, we impose some structure on the set of items. Assume that $X \subset \mathbb{R}$. For every $x \in X$, let $I(x)$ be a neighborhood of x . Assume that the attention function f satisfies the following: for every $x, y \in X$, $f(x, \{y\}) = 1$ if $y \in I(x)$. Our assumptions on \succ and f imply the following definition of the beating relation. For every $M, M' \in P(X)$, M beats M' if the following two conditions hold: (i) there exists $x \in M$ such that $b(M') \in I(x)$; (ii) $b(M) \succ^* b(M')$.

Note that the attention function induces a reflexive binary relation R on X , defined as follows: xRy if $y \in I(x)$. This is the similarity relation that underlies the attention function. The interpretation is that consideration sets are constructed according to similarity judgments. For each product y , there is a set of products that resemble it. The consumer is willing to consider substitutes to his default if the competing

firm offers some item it resembles. Note that by (A4), there exists $x \in X$ such that $x \notin I(y^*)$. Since $M^* = \{y^*\}$, the max-min payoff is $\frac{1}{2} - c_{y^*}$.⁸

We now investigate symmetric Nash equilibria under this class of attention functions. We begin with an important lemma that relates the probability that an inferior item is offered as a content item (i.e., as the \succ^* -maximal item on a menu) to its cost.

Lemma 3 *Let σ be a symmetric Nash equilibrium strategy. Then, $\beta_\sigma(x) \leq 2c_x$ for all $x \neq y^*$.*

Proof. Assume the contrary. Let x be the \succ^* -minimal product for which $\frac{1}{2}\beta_\sigma(x) > c_x$. Suppose that there exists a menu $M \in S(\sigma)$ such that $b(M) \succ^* x$ and $y \not R x$ for all $y \in M$. Then, M does not beat any menu M' with $b(M') = x$. If a firm deviates from M to $M \cup \{x\}$, then since $b(M) \succ^* x$, the probability that some menu M'' with $b(M'') \succ^* b(M)$ beats M does not change. Therefore, by reflexivity of R , the deviation increases the firm's payoff by at least $\frac{1}{2}\beta_\sigma(x) - c_x > 0$, hence it is profitable. It follows that for every $M \in S(\sigma)$ for which $b(M) \succ^* x$, there exists some $y \in M$ such that $y R x$, so that M beats any M' with $b(M') = x$.

Now consider a menu $M \in S(\sigma)$ with $b(M) = x$ (there must be such a menu, since by assumption, $\frac{1}{2}\beta_\sigma(x) > c_x > 0$), and suppose that a firm deviates to $M \cup \{y^*\}$. The cost of this deviation is c_{y^*} , whereas the gained market share is at least $\frac{1}{2} \sum_{y \succ^* x} \beta(y)$. The reason is that first, $M \cup \{y^*\}$ beats any menu M' with $b(M') = x$; and second, whereas prior to the deviation every menu $M' \in S(\sigma)$ with $b(M') \succ^* x$ had beaten M (as we showed in the previous paragraph), after the deviation no menu beats $M \cup \{y^*\}$. In order for this deviation to be unprofitable, we must have $\frac{1}{2} \sum_{y \succ^* x} \beta(y) \leq c_{y^*}$. By the definition of x , $\frac{1}{2}\beta_\sigma(z) \leq c_z$ whenever $x \succ^* z$. Adding up these inequalities, we obtain $\frac{1}{2} \sum_{y \in X} \beta(y) \leq c_{y^*} + \sum_{y|x \succ^* y} c_y < c(X)$. Since the L.H.S of this inequality is by definition $\frac{1}{2}$, we obtain $\frac{1}{2} - c(X) \leq 0$, contradicting condition (A3). ■

Lemma 3 implies that $\beta_\sigma(y^*) \geq 1 - 2 \sum_{x \neq y^*} c_x$. That is, the probability that firms offer the best item has a lower bound that decreases with the cost of inferior products. This result relies only on the reflexivity of R , and thus does not rest on the additional topological structure we imposed.

⁸The relation R is not necessarily symmetric. That is, it is possible that $x \in I(y)$ and $y \notin I(x)$. For evidence that similarity judgments are not always symmetric, see Tversky (1977). In addition, our assumptions do not rule out the possibility that R is complete, transitive and anti-symmetric. Therefore, the case of order-based consideration and max-max preferences is subsumed as a special case of the following analysis.

Lemma 4 *Let σ be a symmetric Nash equilibrium strategy. For every $M \in S(\sigma)$ with $b(M) \neq y^*$ there exists $M' \in S(\sigma)$ with $b(M') = y^*$ such that M' does not beat M .*

Proof. Assume the contrary and let $M \in S(\sigma)$ be a menu which is beaten by all $M' \in S(\sigma)$ with $b(M') = y^*$. If a firm deviates from M to $M \cup \{y^*\}$, it increases its market share by more than $\frac{1}{2}\beta_\sigma(y^*)$. In order for this deviation to be unprofitable, we must have $\beta_\sigma(y^*) \leq 2c_{y^*}$. Combined with Lemma 3, we obtain $\sum_x \beta_\sigma(x) \leq 2c(X)$. Since the L.H.S is equal to one, we obtain a contradiction. ■

Using this lemma, we can now show that in equilibrium, firms cannot sustain a payoff above the rational-consumer benchmark level.

Proposition 4 *Firms earn the max-min payoff $\frac{1}{2} - c_{y^*}$ in any symmetric Nash equilibrium.*

Proof. We begin the proof with some preliminaries. Define $\mathcal{M} = \{M \subseteq X \setminus \{y^*\} \mid M \cup \{y^*\} \in S(\sigma)\}$. Denote $B_\sigma(M) = \{z \in X \setminus \{y^*\} \mid \beta_\sigma(z) > 0 \text{ and } z \in I(x) \text{ for some } x \in M\}$. Let

$$\Delta(M) = \frac{1}{2} \left[\sum_{z \in B_\sigma(M)} \beta_\sigma(z) - c(M) \right]$$

be the net payoff gain from adding the subset M to $\{y^*\}$, given that the rival firm plays σ . Note that in the menu $M \cup \{y^*\}$, the items in M are all pure attention grabbers. The function Δ is sub-additive: for every M, M' , $\Delta(M \cup M') \leq \Delta(M) + \Delta(M')$. For every M and M' such that $M' \subset M$, denote $\delta(M', M) = \Delta(M) - \Delta(M \setminus M')$. Thus, $\delta(M', M)$ is the marginal contribution of M' to the profit generated by M (when these sets are combined with y^*). Finally, for every $x \in X$, let $y^*(x)$ and $y_*(x)$ be the largest and smallest elements in X that belong to $I(x)$.

Assume that firms earn a payoff strictly above $\frac{1}{2} - c_{y^*}$ under σ . By Lemma 4, $\beta_\sigma(x) = 0$ for all x satisfying y^*Rx and $x \neq y^*$. This means that the menu $\{y^*\}$ generates a payoff of $\frac{1}{2} - c_{y^*}$ against σ , hence it does not belong to $S(\sigma)$. By Proposition 1, $\beta_\sigma(y^*) > 0$. Therefore, in order for a menu $M \cup \{y^*\} \in S(\sigma)$, $M \in \mathcal{M}$, to generate a payoff above $\frac{1}{2} - c_{y^*}$, it must be the case that $\Delta(M) > 0$. We will show that this leads to a contradiction with Lemma 4.

Since by Proposition 1, $\beta_\sigma(y^*) < 1$, \mathcal{M} must contain at least two menus - otherwise, Lemma 4 is trivially violated. For every $M \in \mathcal{M}$, let $m \in M$ be the item with the maximal $y^*(x)$ among the elements $x \in M$ with $\Delta(\{m\}) > 0$. Because Δ is sub-additive, $\Delta(M) > 0$ implies that there exists $x' \in M$ such that $\Delta(\{x'\}) > 0$. If x' is the item with the highest $y^*(x)$ among all $x \in M$, then $x' = m$. Otherwise, every

$x \in M$ with $y^*(x) > y^*(m)$ satisfies $\Delta(x) = 0$. Order the elements of each $M \in \mathcal{M}$ such that $M = \{x_1^M, \dots, x_{n(M)}^M, \dots, x_{|M|}^M\}$, where $x_{n(M)}^M = m$. By sub-additivity, $\Delta[\{x_{n(M)+1}^M, \dots, x_{|M|}^M\}] = 0$.

Order the menus $M \in \mathcal{M}$ according to $y^*(m)$, such that $\mathcal{M} = \{M_1, \dots, M_K\}$, $y^*(m_1) \geq \dots \geq y^*(m_K)$. We already saw that $K \geq 2$. Suppose that $y_*(m_j) > y^*(m_K)$ for some $j = 1, \dots, K - 1$. Then, M_K cannot be a best-reply to σ . The reason is that a firm can deviate to the menu $\{x_1^{M_K}, \dots, m_K, m_j\}$, and this deviation will be profitable. The reason for this is that the removal of $\{x_{n(M_K)+1}^{M_K}, \dots, x_{|M_K|}^{M_K}\}$ from M_K does not affect profits, whereas by construction, $\Delta(m_j) > 0$, and $B_\sigma\{m_j\}$ and $B_\sigma(\{x_1^{M_K}, \dots, m_K\})$ are mutually disjoint, and therefore adding m_j strictly raises profits. It follows that $y_*(m_j) \leq y^*(m_K)$ for every $j = 1, \dots, K - 1$. By construction, $y^*(m_K) \leq y^*(m_j)$ for every $j = 1, \dots, K$. Since $I(m_j)$ is a real interval for every $j = 1, \dots, K$, it follows that $y^*(m_K) \in B_\sigma(M)$ for every $M \in \mathcal{M}$, contradicting Lemma 4. To see why we obtain a contradiction, note that for $M_K \cup \{y^*\}$ to be played in an equilibrium σ , there must be some menu \hat{M} in $S(\sigma)$, which is beaten by $M_K \cup \{y^*\}$. But then \hat{M} will be beaten by any menu that contains y^* , in contradiction to Lemma 4. ■

Identity-based attention

An extreme case of similarity-based attention is when $I(x) = \{x\}$ for all $x \in X$, such that the similarity relation R is in fact the identity relation: xRy if and only if $x = y$. Define

$$\alpha_\sigma(x) = \sum_{M|x \in M; x \neq b(M)} \sigma(M)$$

to be the probability that an item x is offered as a pure attention grabber under σ .

Proposition 5 *Suppose that $I(x) = \{x\}$ for all $x \in X$. Then, in any symmetric Nash equilibrium σ , $\beta_\sigma(x) = 2c_x$ and $\alpha_\sigma(x) = 2(c_{y^*} - c_x)$ for all $x \neq y^*$.*

Proof. By Proposition 4, firms earn a payoff of $\frac{1}{2} - c_{y^*}$ in symmetric Nash equilibrium. Observe that under the identity attention relation, M beats M' if and only if $b(M) \succ^* b(M')$ and $b(M') \in M$. Suppose that $\alpha_\sigma(x) = 0$ for some $x \neq y^*$. Then, if a firm plays $\{x\}$, it earns $\frac{1}{2} - c_x > \frac{1}{2} - c_{y^*}$, a contradiction. Therefore, $\alpha_\sigma(x) > 0$ for all $x \neq y^*$. Let $M \in S(\sigma)$ be a menu that includes some $x \neq y^*$ as a pure attention grabber. By Lemma 3, $\beta_\sigma(x) \leq 2c_x$. If the inequality is strict, it is profitable for a firm to deviate from M to $M \setminus \{x\}$. It follows that $\beta_\sigma(x) = 2c_x$. But this means that any menu $M \in S(\sigma)$ with $b(M) = x$, $x \neq y^*$, yields the same payoff against σ as the singleton $\{x\}$. Therefore, $\frac{1}{2}[1 - \alpha_\sigma(x)] - c_x = \frac{1}{2} - c_{y^*}$, i.e. $\alpha_\sigma(x) = 2c_{y^*} - 2c_x$. ■

Thus, as an inferior product becomes more costly, it is offered more often as a content item and less often as a pure attention grabber. The total probability that any inferior product is offered is $2c_{y^*}$.

5 The Effective Marketing Property

One of the features of symmetric equilibria under salience-based attention functions was that pure attention grabbers were offered only in conjunction with the menu M^* , which is optimal from the consumers' point of view. This property does not hold for general attention functions. For example, under identity-based attention (see the previous section), it is easy to construct equilibria in which menus offered with positive probability that are inferior to M^* contain pure attention grabbers.

In this section we will see that equilibria in which firms earn rational-consumer profits satisfy a weaker property that links the inclusion of pure attention grabbers in a menu to its quality. This property extends and adapts a similar result (which goes by the same name) derived in Eliaz and Spiegler (in press) in a different market environment (see our discussion in the Introduction). Its proof does not rely on assumptions (A1)-(A4).

Consider an arbitrary tuple (\succsim, f, c) , such that \succsim satisfies monotonicity and no-perfect-substitutes. Suppose that a consumer is initially assigned to a firm that offers a menu M' , which is strictly worse than M^* . Suppose also that the consumer's attention is drawn to the competing firm's menu M only because of a pure attention grabber in M . We show that if M and M' are drawn from an equilibrium strategy that induces rational-consumer profits, it must be the case that $M \succ M'$, hence the consumer will switch away from M' to M . A priori, the fact that a pure attention grabber attracts the consumer to consider a menu does not guarantee that he will *choose* that menu over his default option. The connection between the two emerges in equilibrium, as a result of competitive forces.

Proposition 6 (Effective Marketing Property) *Suppose that a symmetric Nash equilibrium strategy σ induces the max-min payoff $\frac{1}{2} - c(M^*)$. Let M and M' be two menus in $S(\sigma)$ which satisfy the following properties: (1) $M^* \succ M'$; (2) $f(x, L(M')) = 1$ for some $x \in M \setminus L(M)$; (3) $f(x, L(M)) = 0$ for all $x \in L(M)$. Then, $M \succ M'$.*

Proof. Assume the contrary - i.e., there exist menus $M, M' \in S(\sigma)$ that satisfy properties 1-3 above, and yet $M \not\succeq M'$. Let \mathcal{B} denote the set of menus in $S(\sigma)$ that

are beaten by M and not by $L(M)$. Note that $M' \notin \mathcal{B}$. From the firms' decision not to deviate from M to $L(M)$, we conclude that

$$\frac{1}{2} \sum_{\tilde{M} \in \mathcal{B}} \sigma(\tilde{M}) - c(M \setminus L(M)) \geq 0$$

The reason is that when a firm adds a pure attention grabber to a menu it offers, it can change only the set of menus that the firm's menu beats, but not the set of menus that the firm's menu is beaten by.

Now suppose that a firm deviates to the menu $M^* \cup (M \setminus L(M))$. By assumption, firms earn rational-consumer profits in equilibrium. Therefore, M^* does not beat any menu in $S(\sigma)$. In order for the deviation to be unprofitable, the following inequality must hold:

$$\frac{1}{2} \sum_{\tilde{M} \in \mathcal{B}} \sigma(\tilde{M}) + \frac{1}{2} \sigma(M') - c(M \setminus L(M)) \leq 0$$

The reason is that adding $M \setminus L(M)$ to M^* allows a firm to beat not only all the menus in \mathcal{B} , but also the menu M' . However, the two inequalities we derived contradict each other. ■

As we saw in Sections 3 and 4, Proposition 6 is not vacuous, because there exist large classes of attention functions for which all symmetric Nash equilibria induce rational-consumer profits. In Section 7 we comment on the generality of rational-consumer equilibrium profits.

We conclude this section with a demonstration that the effective marketing property can be useful in characterizing the rate at which consumers switch firms in equilibrium. Recall the case of identity-based attention analyzed in the previous section. Given the equilibrium characterization of $\beta_\sigma(\cdot)$ and $\alpha_\sigma(\cdot)$ in Proposition 5, we can calculate the fraction of consumers who switch a supplier given a symmetric equilibrium strategy σ . We denote this fraction by $\lambda(\sigma)$. By the effective marketing property, a consumer switches from one firm to the other if and only if the highest-quality item in the former's menu is offered as a pure attention grabber by the latter. This leads to the following expression:

$$\lambda(\sigma) = \sum_{x \neq y^*} \beta_\sigma(x) \alpha_\sigma(x) = \sum_{x \neq y^*} 4c_x(c_{y^*} - c_x)$$

Our assumptions on menu costs ensure that $\lambda(\sigma) \in (0, 1)$. Thus, consumers switch suppliers in equilibrium. By comparison, no switching occurs in the rational-consumer benchmark. Note that $\lambda(\sigma)$ behaves non-monotonically in menu costs, and approaches an upper bound of $(n - 1) \cdot c_{y^*}^2$ as the costs of all items $x \neq y^*$ cluster near $c_{y^*}/2$. The

reason for this non-monotonicity is that as an inferior item becomes more costly to add, it is offered less frequently as a pure attention grabber and more frequently as a content item.

Observe that the switching rate is exactly equal to the equilibrium expected cost of pure attention grabbers: for each $x \neq y^*$, the probability x is offered as a pure attention grabber by each firm is by definition $\alpha_\sigma(x)$, while by Proposition 5, $\beta_\sigma(x)$ is equal to twice the cost of x . Thus, the general relation between the social cost of pure attention grabbers and their role in attracting consumers' attention is especially transparent in the case of identity-based attention: the “deadweight loss” associated with pure attention grabbers is equal to consumers' switching rate.

6 Heterogeneous Consumer Preferences

In our analysis thus far we have maintained the simplifying assumption that consumers have identical tastes. This section explores the implications of relaxing this assumption in the context of salience-based attention. In particular, we wish to provide a partial “representative agent” justification for the model analyzed in Section 3. In the original model, we assumed consumer homogeneity but did not force $L(M)$ to be a singleton for all M . In contrast, in the present section we allow for taste heterogeneity but force $L(M)$ to be a singleton for all M (as we did in Section 4). Thus, we may interpret consumer choices in Section 3 as the behavior of a “representative agent” relative to a consumer population with a particular distribution of preferences.

Partition the grand set X into two subsets, $A = \{a_1, \dots, a_m\}$ and $B = \{b_1, \dots, b_n\}$. There are m consumer types, where type i is fully characterized by a preference relation \succ_i^* , which is a linear ordering on X that ranks a_i at the top. The fraction of each type in the consumer population is $\frac{1}{m}$. With respect to menu costs, assume $c_x = c \leq \frac{1}{2(m+1)}$ for all $x \in X$. The upper bound on costs plays the same role as the 50% bound we imposed in Section 2, namely it provides a clear “rational-consumer benchmark” and ensures a certain minimal level of competitiveness.

We begin by characterizing the rational-consumer benchmark for this environment. We omit the proof for brevity.

Remark 2 *Suppose all consumer types are endowed with the perfect-attention attention function: $f(x, M) = 1$ for all $x \in X$, $M \in P(X)$. Then, there exists a unique Nash equilibrium, in which both firms offer A .*

In contrast, assume now that all consumer types share a salience-based attention function as in Section 3. That is, let R be a complete, transitive and anti-symmetric binary relation on X . For all consumers, the attention function is as follows: $f(x, M) = 1$ if and only if xRy for all $y \in M$. Thus, while we assume preference heterogeneity among consumers, we retain the assumption that they are all identical as far as the attention grabbing process is concerned. For any $S \subseteq X$, let $r(S)$ denote the R -maximal element in S . Let $a^* \equiv r(A)$ and $b^* \equiv r(B)$. Assume $r(X) = b^*$. That is, the item with the highest “sensation value” is not a most preferred item for any consumer type.

It turns out that in this case, there exists a symmetric equilibrium which has similar features to the symmetric equilibrium when all consumers have identical tastes and the least-preferred item is also the best attention grabber. In this equilibrium, firms’ expected payoff is the same as in the rational-consumer benchmark, and the effective marketing property continues to hold for all consumer types.

Proposition 7 *Under the above specification of R , the following is a symmetric Nash equilibrium:*

$$\sigma(\{b^*\}) = 2c \tag{7}$$

$$\sigma(A \cup \{b^*\}) = 2(m-1)c \tag{8}$$

$$\sigma(A) = 1 - 2mc \tag{9}$$

Proof. First, note that by our assumption on the size of costs, the expressions in (7)-(9) are probabilities. Second, note that each of the menus in the support generates an expected payoff of $\frac{1}{2} - mc$. Suppose a firm, say firm 1, deviates to playing $A' \cup B'$, where $A' \subseteq A$ and $B' \subseteq B$. If $A' \cup B'$ does no better than $A \cup \{b^*\}$ against the proposed equilibrium σ , then it cannot do better than any of the other pure strategies in σ , and so it is not a profitable deviation.

Notice that $A' \cup B'$ is potentially a profitable deviation only if it contains fewer elements than $A \cup \{b^*\}$ does. Let k be the difference between the cardinality of $A \cup \{b^*\}$ and the cardinality of $A' \cup B'$. Then $k \leq m$. Let $k' \equiv |A - A'|$. Then $k \leq k'$. We consider two cases.

Assume $b^* \notin B'$. Then $A' \cup B'$ does not steal consumers from a firm offering $\{b^*\}$, while $A \cup \{b^*\}$ does. In addition, any consumer whose favorite item is in $A - A'$ will switch from $A' \cup B'$ to $A \cup \{b^*\}$. The best scenario that can happen when a firm deviates to $A' \cup B'$ is that no consumer leaves the firm when the other firm offers A . Suppose

this is true. This gives us an upper bound on the expected market share $A' \cup B'$ can generate. So the expected gain from this deviation is at most $k'c$, which is the savings in costs. The expected loss is $\frac{1}{2} \cdot 2c$, the probability that the consumer starts with the other firm and the other firm offers b^* , plus $\frac{1}{2} \cdot \frac{k'}{m} \cdot 2(m-1)c$, the probability that the consumer starts with the deviant firm, the consumer's favorite item is in $A - A'$, and the other firm offers $A \cup \{b^*\}$. Thus, the total expected loss is $k'c + (1 - \frac{k'}{m})c$, while the expected gain is only $k'c$. So on net, the deviation leads to an expected loss of at least $(1 - \frac{k'}{m})c > 0$.

Assume next that $b^* \in B'$. Then $A' \cup B'$ steals consumers from the other firm, when that firm offers $\{b^*\}$: it steals all consumers whose top item is in A' and may steal other consumers who rank at least one element in $A' \cup B'$ above b^* . So at most, $A' \cup B'$ steals *all* consumers who start with b^* . But because B' does contain b^* the deviation saves at most $(k' - 1)c$. The expected loss is now at least $k'c - (\frac{k'}{m})c$. So on net, the deviation leads to an expected loss of $(1 - \frac{k'}{m})c > 0$. ■

7 Concluding Remarks

This paper analyzed a stylized model of market competition that emphasized consumers' limited attention and the role of the firms' "product line" decisions in manipulating consumers' attention. Equilibrium behavior departs from the benchmark of rational consumers with unlimited attention. Firms offer menus that are inferior to the consumers' first-best and employ costly pure attention grabbers in equilibrium. For two natural special cases of our model, industry profits are exactly the same as if consumers had unlimited attention: the costly use of pure attention grabbers wears off any collusive payoff firms might earn as a result of consumers' bounded rationality. This result has an important corollary regarding consumer conversion: whenever consumers' attention is drawn to a menu thanks to a pure attention grabber it contains, they end up switching to this menu.

How general are rational-consumer equilibrium profits?

The following is an example of an attention function that satisfies assumptions (A1)-(A4), and yet gives rise to equilibria that sustain profits above the rational-consumer level (this is a variant on an example given in Eliaz and Spiegler (in press)). Let $X = \{0, 1\}^3 \setminus \{(0, 0, 0)\}$. Define a linear ordering \succ^* over X which satisfies the following property: if $\sum_{k=1}^3 x_k > \sum_{k=1}^3 y_k$, then $x \succ^* y$. Assume that the consumers' preferences over menus are as follows: $M \succ M'$ if and only if there exists $x \in M$ such that $x \succ^* y$ for all $y \in M'$. Therefore, $M^* = \{(1, 1, 1)\}$. Assume further that $f(x, \{y\}) = 1$ if and

only if $x_k = y_k$ for at least two components $k \in \{1, 2, 3\}$. This is a similarity-based attention function in the same spirit of Section 4, except that the topology over X that defines the similarity relation is different.

One can show that for an appropriately specified cost function, there is a continuum of symmetric equilibria with the following properties: (i) the support of the equilibrium strategy consists of $\{(1, 1, 1), (1, 1, 0)\}$, $\{(1, 1, 1), (1, 0, 1)\}$, $\{(1, 1, 1), (0, 0, 1)\}$, $\{(1, 0, 0)\}$, $\{(0, 1, 0)\}$ and $\{(0, 0, 1)\}$; (ii) the equilibrium payoff is strictly above the rational-consumer (max-min) level of $\frac{1}{2} - c_{(1,1,1)}$. There is also a symmetric equilibrium that induces rational-consumer payoffs.

How typical is this counter-example? We conjecture that for generic cost functions, any attention function that satisfies (A1)-(A3) induces rational-consumer payoffs in symmetric equilibrium. When (A3) is significantly strengthened - i.e., when menu costs are sufficiently small - the result holds with no need for a genericity requirement. The proof of this result is simple and close to a parallel result in Eliaz and Spiegler (in press), and therefore omitted.

The relation between costs and preferences

Our primary motivation for assuming that consumer preferences over menus are positively related to the cost of providing them is to introduce an anti-competitive force due to consumer inattention. In order to depart from the unlimited-attention benchmark, a firm should have an incentive to exploit a consumer who fails to consider its rival. Assumption (A1) implies this incentive. However, we should point out that this motive to degrade quality can be achieved with a weaker assumption than (A1) - namely, that there exists some menu which is not beaten by M^* and costs strictly less than M^* . In fact, many of our results would continue to hold under this weaker assumption. (specifically, part (i) of Proposition 1, parts (i) – (ii) of Proposition 2, Lemmas (3)-(4), Proposition (4) and Proposition 6). Also our analysis in Section 6 does not depend on (A1), as it is not well-defined with heterogeneous preferences.

A comment on welfare analysis

Recall that consumer choice in our model is in general inconsistent with the maximization of a utility function over menus. Therefore, welfare analysis in our model cannot be given a conventional revealed preference justification. Throughout this paper, we interpreted \succsim as the consumers' true preferences over menus, and used it to analyze consumer welfare. However, there are alternative interpretations of our choice model that might suggest different welfare criteria.

Recall the case of salience-based attention studied in Section 3. Assume that consumers have max-max preferences over menus (i.e., there is a linear ordering \succ^* over X

such that $M \succ M'$ if and only if there exists $x \in M$ for which $x \succ^* y$ for all $y \in M'$). This specification admits an alternative interpretation in the spirit of the literature on *dynamically inconsistent preferences*, whereby the rationale that consumers use to rank menus differs from the rationale they use when ranking items *within* a given menu. According to this interpretation, the binary relation R represents the preferences over items of the consumer's "first-period self", whereas \succ^* represents the preference over items of his "second-period self". The consumer is naïve in the sense of O'Donoghue and Rabin (1999): when he chooses between menus, he erroneously believes that he will use his first-period self's preference relation R to choose an item from menus, whereas in actuality he uses his second-period self's preference relation \succ^* .

When economists study such two-stage, multi-self choice models with naïve decision makers, they often use the first-period self's preference relation as the normative welfare criterion, because it tends to represent cool deliberation, whereas the second-period self's preference relation captures visceral urges that are inconsistent with long-run well-being. It follows that if we adopted this alternative interpretation of the model, we would be led to conduct a welfare analysis that replaces \succ with R as a welfare criterion. Note, however, that this ambiguity arises in a very special specification of our model. At any rate, this discussion demonstrates the subtlety of welfare analysis in market models with boundedly rational consumers.

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