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# ABSTRACT

## Debt-sensitive Majority Rules*

We examine debt-sensitive majority rules. According to such a rule, the higher a planned public debt, the higher the parliamentary majority required to approve it. In a two-period model we compare debt-sensitive majority rules with the simple majority rule when individuals differ regarding their benefits from public-good provision. We establish the existence of Condorcet winners under debt-sensitive majority rules and derive their properties. We find that equilibrium debt-levels are lower under the debt-sensitive majority rule if preferences regarding public goods are sufficiently heterogeneous and if the impact of debt on future public-good provision is sufficiently strong. We illustrate how debt-sensitive majority rules act as political stabilizers in the event of negative macroeconomic shocks.

JEL Classification: D72, E61, H41 and H63
Keywords: debt restriction, debt-sensitive majority rule, fiscal policy, public debt, public goods, simple majority rule and voting

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# 1. Introduction

Most western countries have experienced rising debt levels over the last few decades. This will be accelerated by the recent economic and financial crisis, as indicated by Figure 1. The issue of debt restriction has become a major concern for policy-makers and economists.

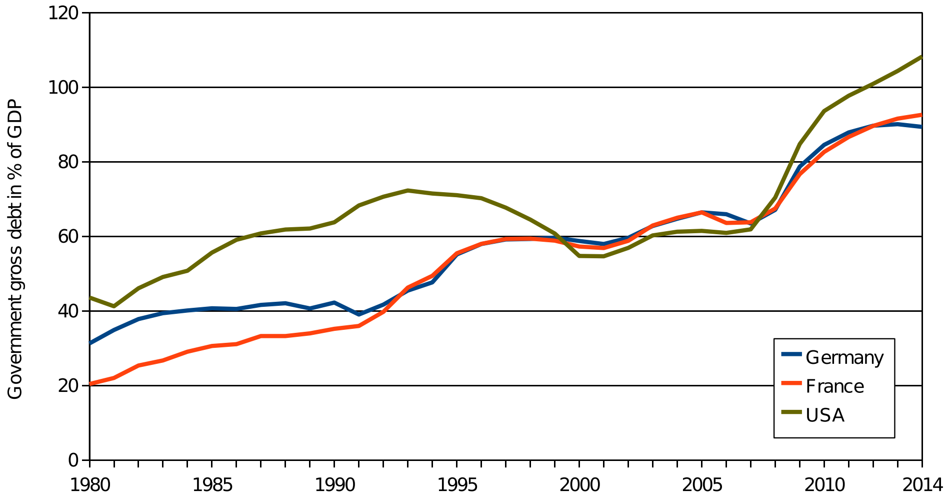


Figure 1: Government debt for selected countries (years 2009–2014 estimated).
Source: International Monetary Fund (2009)

In this paper, we examine whether and how new voting rules called debt-sensitive majority rules can provide tighter debt brakes than standard voting procedures. Under debt-sensitive majority rules, the required majority to pass a budget increases with the size of the budget deficit.

We consider a two-period model with private and public goods. Individuals differ in their preferences for public goods. Debt has a negative impact on the government's ability to provide public goods in the second period. In the first period, the society votes on a policy package including taxes, a level of public-good provision, and the amount of debt.

We establish the existence of Condorcet winners under debt-sensitive majority rules and derive their properties. Then we compare the outcome of a standard simple majority voting rule to that of a debt-sensitive majority rule. We show that a suitable debt-sensitive majority rule can restrict debt accumulation (a) if the individuals are sufficiently heterogeneous, (b) if the society comprises individuals with a high valuation for public goods, and (c) if the impact of debt on future public-good provision is sufficiently strong. In contrast to fixed debt limits, debt-sensitive majority rules act as stabilizers when negative macroeconomic shocks occur. Debt-sensitive voting rules create fiscal space by permitting fiscal deficits in downturns and hence lead to economic stabilization. As stabilization is the result of the voting procedure, we call debt-sensitive majority rules *political* stabilizers.

While government debt can be justified on normative grounds – for instance, by the famous tax-smoothing argument proposed by Barro (1979) –, the literature has emphasized that political and economic forces tend to push public debt beyond socially desirable levels. Fragmented governments are prone to excessive spending when few groups benefit from public goods but the costs are distributed over the society as a whole (Weingast et al., 1981; Baron and Ferejohn, 1989; Baron, 1991). In a dynamic framework, Battaglini and Coate (2008) show that a government's incentive for pork-barrel spending accounts for an underprovision of public goods and an accumulation of debt over time. Yared (2010) shows that a rent-seeking government might raise debts and taxes above a society's optimal level. The accumulation of debts enables governments to shift fiscal burdens to future generations, as these generations are underrepresented or not represented at all in today's elections (Song et al., 2009). Within a monetary union, public debt might even be raised further due to free-riding behavior: the debt burden of a single country has to be borne by all member countries (Beetsma and Bovenberg, 1999; Neck and Sturm, 2008).

Several proposals have been made to limit excessive public-debt accumulation. For instance, fixed budget-limits, debt-brakes, rainy-day funds, and balanced-budget rules have been implemented. These rules, however, face credibility and flexibility problems. On the one hand, tight budget rules and debt-brakes have proved not to be very credible, as they have either been repealed temporarily, or violations have not been sanctioned. On the other hand, where rules have allowed for room to maneuver, policy-makers have exploited their flexibility in quieter periods as well, so debt accumulation has not been curbed. Canova (2006) and Auerbach (2008) have assessed the success of fiscal rules in most U.S. states, and their findings show that these rules have had little impact on debt accumulation.

Debt-sensitive majority rules are a new concept.[1] They belong to the class of proposal-dependent majority rules. Other types of proposal-dependent majority rules in the

[1]For a policy discussion see Gersbach (2007).

context of cake division and public-good provision have been discussed by Gersbach (2004) and Gersbach (2009).

Our paper is organized as follows. In Section 2 we outline the basic model and introduce the voting rules. In Section 3 we determine optimal policies. In Section 4 we prove the existence of Condorcet winners and derive the unique equilibrium under the simple majority rule. In Section 5 we prove the existence of a unique equilibrium under a debt-sensitive majority rule. Section 6 contains a number of examples. We develop conditions under which the debt-sensitive rule will restrict excessive debt accumulation. In Section 7 we discuss how debt-sensitive majority rules can act as political stabilizers. In Section 8 we outline procedures for implementing debt-sensitive majority rules and discuss extensions of the model. Section 9 concludes.

## 2. Model

We consider an economy populated by a finite or infinite number of agents who live for two periods. Agents work when they are young but do not work when they are old. We consider a competitive labor market and assume that each agent inelastically supplies one unit of labor when he/she is young. All workers receive the same wage $w$. We focus on the collective decision of the agents when they are young.

### 2.1. Utility function

We follow Song et al. (2009) in assuming that the utility of an agent in period 1 is given by the Cobb-Douglas utility function

$$U_\theta = \log c + \theta \log g + \beta\theta \log h, \tag{1}$$

where $c$ is the life-time consumption level of the private good and $g$ and $h$ denote public-good consumption in periods 1 and 2, respectively. The discount factor is denoted by $\beta$. We note that the expression $\log c$ can be justified as follows: If the intertemporal utility from private consumption is given by

$$\log c_1 + \beta \log c_2,$$

with $c_1$ and $c_2$ being the consumption levels in period 1 and period 2, then the budget available for private consumption is $c$ with

$$c = c_1 + (1 + r)^{-1} c_2.$$

Here $r$ is the real interest rate. Maximizing utility, the agent will choose

$$c_1 = \frac{1}{1 + \beta(1 + r)} \cdot c \quad \text{and} \quad c_2 = \frac{\beta(1 + r)}{1 + \beta(1 + r)} \cdot c.$$

Hence, the utility from private consumption equals

$$\log c_1 + \beta \log c_2 = (1 + \beta) \log c + \beta \log\bigl(\beta(1 + r)\bigr) - (1 + \beta) \log\bigl(1 + \beta(1 + r)\bigr).$$

Dividing this expression by $(1 + \beta)$ and dropping constant terms, we obtain the term $\log c$ in the utility function (1).

The agents differ in the relative weight $\theta > 0$ of utility from public goods. The distribution of types $\theta$ over the society is given by a probability measure $\mu$, which means that the mass of agents is normalized to unity. We assume that the median type is unique, i. e. that there is a unique type $\theta^*$ such that the intervals $(0; \theta^*]$ and $[\theta^*; \infty)$ both comprise a mass of at least 1/2 of agents.

The amount of private consumption by an agent is given by his budget constraint

$$c = w(1 - \tau), \tag{2}$$

with $\tau$ being the income tax rate.

### 2.2. Fiscal policy

Fiscal policy is determined by a parliament that represents the electorate. The parliament decides upon the tax rate $\tau$ and the amount $g$ of public good provided in the first period. It faces an inherited debt level $a$. The debt level at the beginning of the second period is denoted by $b$. The evolution of debt is given by

$$b = Ra + g - \tau w. \tag{3}$$

We assume that debt is issued on the international capital market and that government bonds are held by foreigners. $R$ denotes the interest rate factor for public debt. $R - 1$

is the interest rate.

There exists a maximum level $\bar{b}$ of government debt that can be issued. This upper limit is reached when interest payment exhausts the maximum tax revenue. Thus,

$$\bar{b} = \frac{\bar{\tau} w}{R - 1}, \tag{4}$$

where $\bar{\tau}$ is the revenue-maximizing tax rate. For ease of presentation, we assume that labor is supplied inelastically and the entire income can be taxed away, i. e. $\bar{\tau} = 1$.[2] We exclude subsidies and thus only consider tax rates $\tau \geq 0$. We assume that initial debt is below the level above which state bankruptcy would be inevitable, i. e. $Ra - w < \bar{b}$.

In the second period, a future parliament decides upon the amount $h$ of the public good for that period. Thus, provision of the public good in the second period depends on the inherited debt $b$. We do not model the parliament's decision explicitly, but describe the result of this decision by a function $h(b)$. We assume that for $b < \bar{b}$, $\log h$ is twice continuously differentiable with

$$(\log h)'(b) < 0 \quad \text{and} \quad (\log h)''(b) < 0, \tag{5}$$

which means that $\log h(b)$ is strictly decreasing and strictly concave. If debt inherited from period 1 reaches $\bar{b}$, the total tax income $\bar{\tau} w$ has to be used to finance interest payments on government debt, and no public goods can be provided, which implies $h(\bar{b}) = 0$.

For the general analysis in this paper we do not specify the function $h$ exactly. For the theoretical investigations in the coming sections we only require $h$ to fulfill the above assumptions. Specific examples describing specific reactions of future parliaments to higher debt levels will be discussed in Section 6.

It is useful to make the following definitions:

**Definition 1** *We denote the set of policies by $\mathcal{P} := [0; 1] \times \mathbb{R}_{\geq 0}$. A* policy *is a pair $(\tau, g) \in \mathcal{P}$, consisting of a tax rate $\tau$ and a level of government goods $g$.*

[2] There are various ways to relax this assumption. For instance, one might assume that households supply inelastically one unit of labor as long as the tax rate does not exceed a threshold $\bar{\bar{\tau}} < \bar{\tau} = 1$, and do not work if the tax level exceeds this threshold. Our results are fully applicable to this case. A more sophisticated extension is to consider $w$ as a smooth function of the tax rate $\tau$, with tax revenue reaching a maximum at a tax rate $\bar{\bar{\tau}} < 1$.

*A policy determines the associated debt level*

$$B(\tau, g) := Ra + g - \tau w. \tag{6}$$

*For $b \leq \bar{b}$ define*

$$\mathcal{P}_{=b} := \big\{(\tau, g) \in \mathcal{P} \bigm| B(\tau, g) = b\big\},$$

*and*

$$\mathcal{P}_{\leq b} := \big\{(\tau, g) \in \mathcal{P} \bigm| B(\tau, g) \leq b\big\}.$$

In words, $\mathcal{P}_{=b}$ is the set of all policies that imply a debt level of exactly $b$ left to the next period, and $\mathcal{P}_{\leq b}$ is the set of all policies that imply a debt level of $b$ at most.

By Equations (1), (2), and (3), the utility of an agent of type $\theta$ from a policy $(\tau, g) \in \mathcal{P}$ is

$$V_\theta(\tau, g) = \begin{cases} \log(1-\tau) + \theta \log g + \beta\theta \log h\big(B(\tau, g)\big) & \text{if } \tau < 1,\ g > 0, \text{ and } B(\tau, g) < \bar{b}, \\ -\infty & \text{otherwise.} \end{cases} \tag{7}$$

Here we have made a monotonic transformation and dropped the constant term $\log w$, to simplify the exposition. $V_\theta$ can be viewed as the indirect utility function of an agent of type $\theta$.

The parliament chooses a policy according to some voting scheme to be specified below. We summarize the sequence of events in Figure 2.

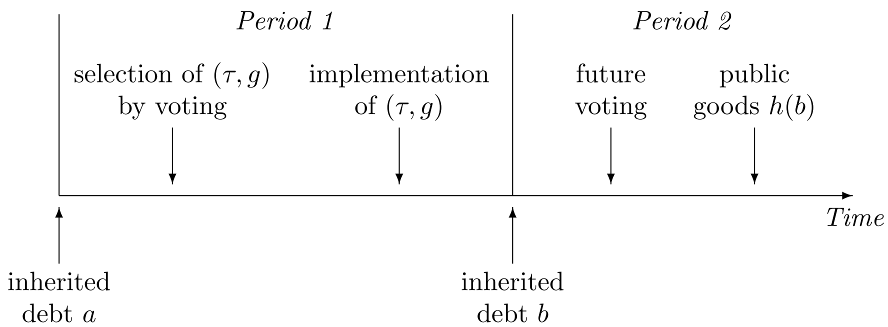


Figure 2: Sequence of events.

### 2.3. Voting schemes

We now compare the outcome under the simple majority rule and debt-sensitive majority rules. The rules are defined in the following.

#### 2.3.1. Simple majority rule

We assume that each member of parliament can make a proposal $(\tau, g) \in \mathcal{P}$ on the tax rate and the level of public-good provision. The legislature decides between the proposals by pairwise voting.

**Definition 2 (simple majority rule)** *The voting process under the simple majority rule is described as follows:*

- *The proposals are sorted arbitrarily and pairwise voting occurs sequentially. First, the parliament decides between the first two proposals. The winning proposal is pitted against the third proposal, etc. The proposal that survives this process is implemented.*
- *In pairwise voting, a proposal wins against another proposal if it receives a majority of strictly more than* $0.5$*. If both proposals receive an equal vote share of 0.5, the winning proposal is chosen by fair randomization.*

In general, the outcome of such a voting game may depend on the order in which the proposals appear in the voting process. In Section 4, however, we prove the existence of a unique Condorcet winner. As the Condorcet winner beats all other proposals in the voting process, the order in which the proposals enter the process is immaterial. Furthermore, the voting process does not necessarily have to be sequential in the sense that the winner of the previous vote then competes against the next proposal. We could use any form of a tournament with pairwise votes to determine the winning proposal.

We assume there is no commitment problem, i. e. the winning proposal will always be implemented.

### 2.3.2. Debt-sensitive majority rule

An exogenously given debt-sensitive majority rule is a rule that is sensitive as to the planned government debt level $b$. The rule is established by constitution and requires a majority of parliamentarians that grows with the level of planned public debt.

We define a status quo policy as any policy following a *no-deficit rule*. That is, such a policy does not increase the debt level. If a status quo policy has to be implemented, tax revenues have to cover the expenditures for public goods and the interest payments on the inherited debt.

**Definition 3** *A* status quo policy *is a tax / public good plan $(\tau, g) \in \mathcal{P}$ that does not increase the government debt level, i. e. for which $B(\tau, g) \leq a$.*

We next introduce the concept of a debt-sensitive majority rule:

**Definition 4 (debt-sensitive majority rule)** *A* debt-sensitive majority rule *is described by a left-continuous, weakly increasing function $\phi\colon \mathbb{R}_{\geq 0} \to [0; 1]$ with the following consequences in the voting process:*

*Stage 1: Proposal-making*

- *Each legislator makes*[3] *a proposal $(\tau, g)$. The winning proposal $(\tau^F, g^F)$ is determined by the simple majority rule.*

*Stage 2: Winning proposal versus status quo*

*The legislators decide between the winning proposal $(\tau^F, g^F)$ in stage 1 and implementing a status-quo policy. The winning proposal $(\tau^F, g^F)$ needs a majority of $\phi(b)$ to be passed, with $b = B(\tau^F, g^F)$. Specifically,*

- *if $(\tau^F, g^F)$ receives a vote share greater than or equal to $\phi(b)$, the winning proposal of stage 1 has been passed and will be implemented;*
- *if $(\tau^F, g^F)$ receives a vote share smaller than $\phi(b)$, a status-quo policy has to be implemented. This status-quo policy is chosen by simple majority voting, where only proposals from the set $\mathcal{P}_{\leq a}$ are allowed.*

[3] Of course, a legislator can abstain from proposal-making. It is however easy to see that such a deviation can never be profitable.

As we shall see in Section 5, the left-continuity of $\phi$ will ensure uniqueness of the voting outcome. To illustrate our definition, we provide a simple example. Let $\delta_1 = 0.5\,\%$ and $\delta_2 = 2\,\%$. Consider the rule

$$\phi(b) := \begin{cases} 0.50 & \text{for } b \leq a + \delta_1 \cdot \text{GDP}, \\ 0.55 & \text{for } a + \delta_1 \cdot \text{GDP} < b \leq a + \delta_2 \cdot \text{GDP}, \\ 0.67 & \text{for } b > a + \delta_2 \cdot \text{GDP}, \end{cases} \tag{8}$$

where $a$ denotes debt inherited from the previous period. According to this rule, government budgets with new debts below $\delta_1 = 0.5\,\%$ of GDP can be passed with a simple majority. New debts between $\delta_1 = 0.5\,\%$ and $\delta_2 = 2\,\%$ of GDP require the support of 55 % of legislators, while higher new debts require a 2/3 majority in parliament.

### 2.4. Equilibrium concept

In Sections 4 and 5 we determine the equilibrium outcomes under the two voting regimes. We will employ the concept of subgame-perfect strong Nash equilibrium for the voting games of Definitions 2 and 4. Strong Nash equilibrium was proposed and introduced by Aumann (1959). A strong equilibrium is defined to be a strategy profile where no subset of players can jointly deviate in a way that is beneficial for all of them. Consequently, in a strong Nash equilibrium players cannot do better, even if they are allowed to communicate and collaborate before the game. This property is especially desirable for the analysis of a political decision process.[4]

## 3. Optimal Policies

To prepare our analysis of voting outcomes in Sections 4 and 5, we now examine the policies that agents consider to be optimal. For this purpose, we show that the indirect utility function is strictly concave. The concavity guarantees the existence and the uniqueness of a maximum. We characterize this maximum explicitly.

[4]An alternative solution concept would be coalition-proof Nash equilibrium (Bernheim et al., 1987). Coalition-proof Nash equilibrium requires robustness against deviation only by those coalitions that are internally stable. Any strong Nash equilibrium is coalition-proof.

### 3.1. Properties of the indirect utility function

Under debt-sensitive majority rules, policies that imply a high debt level will not survive the second stage of the voting process and thus will be excluded from the set of policies that legislators will consider in the first stage. Moreover, the status-quo policy is chosen from the set $\mathcal{P}_{\le a}$, which is a proper subset of $\mathcal{P}$. In the following, we prove that in $\mathcal{P}$, as well as in proper subsets of $\mathcal{P}$, utility-maximizing policies for each type $\theta$ exist, and that these policies are unique.

Let

$$\mathcal{P}^{\mathrm{f}} := \big\{(\tau, g) \in \mathcal{P} \;\big|\; V_\theta(\tau, g) > -\infty\big\}.$$

This definition does not depend on the choice of $\theta$. We now state

**Proposition 1**

*(i) The set $\mathcal{P}^{\mathrm{f}}$ is convex, and for each $\theta > 0$ the restriction of $V_\theta$ to $\mathcal{P}^{\mathrm{f}}$ is strictly concave.*

*(ii) For any closed and convex set $A \subseteq \mathcal{P}$, $A \neq \emptyset$, the restriction of $V_\theta$ to $A$ attains a maximum; the point of maximum is unique if $\mathcal{P}^{\mathrm{f}} \cap A \neq \emptyset$.*

The proposition can be proved by examining the partial derivatives of $V_\theta$. For $(\tau, g) \in \mathcal{P}^{\mathrm{f}}$, the first-order partial derivatives are given by

$$\partial_\tau V_\theta(\tau, g) = -\frac{1}{1-\tau} - \beta\theta w (\log h)'(Ra + g - \tau w) \tag{9}$$

and

$$\partial_g V_\theta(\tau, g) = \frac{\theta}{g} + \beta\theta (\log h)'(Ra + g - \tau w). \tag{10}$$

The second-order derivatives are

$$\partial_\tau\partial_\tau V_\theta(\tau, g) = -\frac{1}{(1-\tau)^2} + \beta\theta w^2 (\log h)''(Ra + g - \tau w), \tag{11}$$

$$\partial_g\partial_\tau V_\theta(\tau, g) = -\beta\theta w (\log h)''(Ra + g - \tau w), \tag{12}$$

$$\partial_g\partial_g V_\theta(\tau, g) = -\frac{\theta}{g^2} + \beta\theta (\log h)''(Ra + g - \tau w). \tag{13}$$

By inspecting the Hessian, we find that $V_\theta$ is strictly concave. For details, we refer the reader to the proof of Proposition 1 in the Appendix.

An immediate consequence of Proposition 1 is the following corollary:

**Corollary 1** *$V_\theta$ possesses a unique point of maximum $\big(\tau^0(\theta), g^0(\theta)\big)$ on $\mathcal{P}$. Furthermore, for each debt level $b \in (Ra - w; \bar{b})$ there is a unique optimal policy $\big(\tau^0_{=b}(\theta), g^0_{=b}(\theta)\big)$ among all policies in $\mathcal{P}_{=b}$, as well as a unique optimal policy $\big(\tau^0_{\leq b}(\theta), g^0_{\leq b}(\theta)\big)$ in $\mathcal{P}_{\leq b}$.*

The optimal policy $(\tau^0(\theta), g^0(\theta))$ within $\mathcal{P}$ is characterized by the first-order conditions

$$\partial_\tau V_\theta\big(\tau^0(\theta), g^0(\theta)\big) \leq 0 \text{ and } \partial_g V_\theta\big(\tau^0(\theta), g^0(\theta)\big) = 0,$$

where strict inequality is only permitted if $\tau^0(\theta) = 0$. Note that the $\leq$ sign results from the strict concavity of $V_\theta$, which implies that a corner solution at $\tau = 0$ will occur if and only if $\partial_\tau V_\theta(0, g) \leq 0$. We do not need to consider the other possible corner solution $\tau = 1$, as $\lim_{\tau \to 1} V_\theta(\tau, g) = -\infty$ and thus a tax rate of $\tau = 1$ cannot be optimal. For the same reason, we can neglect boundary solutions with $g = 0$. Due to (9) and (10), the first-order conditions read

$$\frac{1}{1-\tau} + \beta\theta w \cdot (\log h)'(Ra + g - \tau w) \geq 0, \tag{14}$$

$$\frac{\theta}{g} + \beta\theta \cdot (\log h)'(Ra + g - \tau w) = 0. \tag{15}$$

For the case of an interior solution with $\tau > 0$, Equation (14) holds with equality, and the combination of both equations yields

$$\tau = 1 - \frac{g}{\theta w}\,. \tag{16}$$

Inserting this expression into the budget restriction leads to

$$g = \frac{b - Ra + w}{1 + \theta^{-1}}\,. \tag{17}$$

Inserting this result into (15) yields

$$\frac{1 + \theta^{-1}}{b - Ra + w} + \beta(\log h)'(b) = 0.$$

Thus, in the case of an interior solution, the optimal debt level $b^0(\theta) := B\big(\tau^0(\theta), g^0(\theta)\big)$ for type $\theta$ satisfies the equation

$$b^0(\theta) = Ra - w + \left(1 + \frac{1}{\theta}\right) \frac{-1}{\beta(\log h)'(b^0(\theta))}\,. \tag{18}$$

From $b^0(\theta)$, the optimal values $g^0(\theta)$ and $\tau^0(\theta)$ are determined by the Equations (16) and (17). A corner solution will occur if and only if this procedure leads to a negative value of $\tau$. Then $\tau^0(\theta) = 0$, and $g^0(\theta)$ and $b^0(\theta)$ are determined by Equation (15) and the budget constraint (3). In the Appendix we prove

**Proposition 2** *The optimal debt level $b^0(\theta)$ depends negatively on the taste parameter $\theta$ for public goods.*

## 3.2. Optimal policy with an upper debt limit

In the following, we determine the optimal policy for an agent of type $\theta$ among all policies associated with a debt level below some upper limit $\tilde{b}$, i. e. the optimal policy in the set $\mathcal{P}_{\leq \tilde{b}}$. Recall that we denote this optimal policy by $\big(g^0_{\leq \tilde{b}}(\theta), \tau^0_{\leq \tilde{b}}(\theta)\big)$; we define

$$b^0_{\leq \tilde{b}}(\theta) := B\big(\tau^0_{\leq \tilde{b}}(\theta), g^0_{\leq \tilde{b}}(\theta)\big)$$

to be the corresponding debt level.

We first derive the optimal policy $\big(\tau^0_{=b}(\theta), g^0_{=b}(\theta)\big)$ within $\mathcal{P}_{=b}$ for a given $b$. Since $\tau \leq 1$ and $g \geq 0$, and due to Equation (4), we can restrict ourselves to $b$ satisfying

$$Ra - w < b < \bar{b}. \tag{19}$$

**Lemma 1** *For $b$ satisfying (19), we have*

$$\tau^0_{=b}(\theta) = \max\left\{0; \frac{\theta w - b + Ra}{w(1+\theta)}\right\}, \tag{20}$$

$$g^0_{=b}(\theta) = b - Ra + \tau^0_{=b}(\theta) w. \tag{21}$$

The proof of this lemma is given in the Appendix. The expressions from the lemma can be rewritten as

$$1 - \tau^0_{=b}(\theta) = \min\left\{1; \frac{w + b - Ra}{w(1+\theta)}\right\}, \tag{22}$$

$$g^0_{=b}(\theta) = \max\left\{b - Ra; \frac{\theta(w + b - Ra)}{1+\theta}\right\}. \tag{23}$$

By these equations, the indirect utility of type $\theta$ as a function of the debt level $b$ amounts

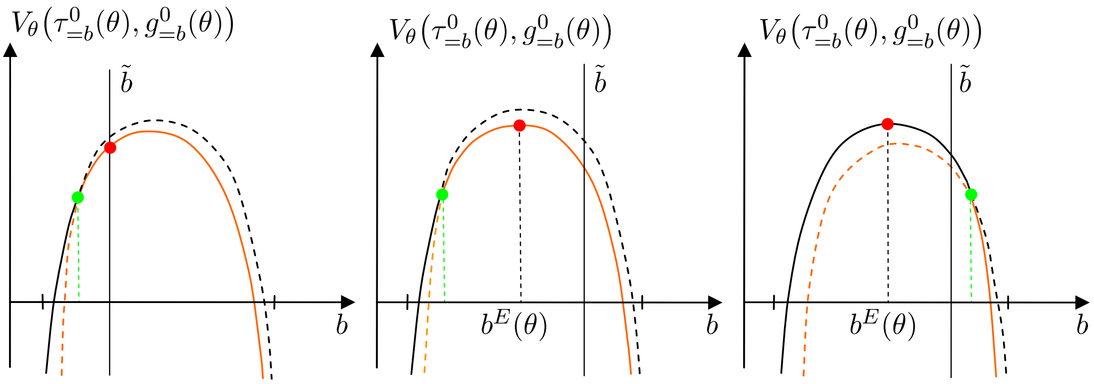


Figure 3: The three candidates for the optimal debt level from Equation (27). The solid curve represents the function $b \mapsto V_\theta\big(\tau^0_{=b}(\theta), g^0_{=b}(\theta)\big)$ from Equation (24) on the interval $\big(Ra - w; \bar b\big)$. As we explain in the text, the black curve corresponds to the first case and the orange curve corresponds to the second case of Equation (27). The green vertical line indicates the debt level $b = Ra + \theta w$, right of which $\tau^0_{=b}(\theta) = 0$.

to

$$
\begin{aligned}
&V_\theta\big(\tau^0_{=b}(\theta), g^0_{=b}(\theta)\big)\\
&= \log \min\left\{1; \frac{w+b-Ra}{w(1+\theta)}\right\} + \theta \log \max\left\{b - Ra; \frac{\theta(w+b-Ra)}{1+\theta}\right\} + \theta\beta \log h(b)\\
&= \begin{cases} C + (1+\theta)\log(w+b-Ra) + \theta\beta\log h(b) & \text{if } b < Ra + \theta w, \\ \theta \log(b - Ra) + \theta\beta \log h(b) & \text{if } b \geq Ra + \theta w, \end{cases}
\end{aligned} \tag{24}
$$

with the constant $C$ reading

$$C := -\log w - (1+\theta)\log(1+\theta) + \theta \log \theta.$$

For any $\tilde b \in (Ra - w; \bar b)$, under the constraint that debt is restricted to $\tilde b$, the optimal debt level is

$$b^0_{\leq \tilde b}(\theta) = \operatorname*{arg\,max}_{b \leq \tilde b} V_\theta\big(\tau^0_{=b}(\theta), g^0_{=b}(\theta)\big).$$

We use $b^E(\theta)$ to denote the maximum of the function $b \mapsto V_\theta\big(\tau^0_{=b}(\theta), g^0_{=b}(\theta)\big)$. This function is depicted in Figure 3. The first case of Equation (24) corresponds to the black curve, the second case to the orange curve. The graph of the function is given by the black curve for debt levels below $Ra + \theta w$, which marks the osculation point of the two curves (the green point in the figure), and by the orange curve for higher debt levels. The point of maximum $b^E(\theta)$ is given by the point of maximum of the orange curve if the slope is positive at the osculation point; it is given by the point of maximum of the black curve if the slope is negative.

The figure illustrates three cases for $b^0_{\leq \tilde{b}}(\theta)$. If the debt constraint $\tilde{b}$ is binding, the utility-maximizing debt level $b^0_{\leq \tilde{b}}(\theta)$ is given by $\tilde{b}$ (left diagram). If the debt constraint is not binding, $b^0_{\leq \tilde{b}}(\theta)$ is given by the maximum $b^E(\theta)$, which is either the maximum of the orange curve (middle diagram) or the black curve (right diagram). The following proposition gives a formal characterization:

**Proposition 3** *Suppose* $\tilde{b} \in \left(Ra - w; \bar{b}\right]$. *If* $-(\log h)'(Ra + \theta w) > 1/(\beta\theta w)$, *let* $b^E(\theta)$ *be the unique solution of*

$$\frac{1+\theta}{w+b-Ra} = -\theta\beta(\log h)'(b). \tag{25}$$

*If* $-(\log h)'(Ra + \theta w) \leq 1/(\beta\theta w)$, *let* $b^E(\theta)$ *be the unique solution of*

$$\frac{\theta}{b-Ra} = -\theta\beta(\log h)'(b). \tag{26}$$

*With this definition,* $b^E(\theta)$ *is the minimum of the solution of* (25) *and the solution of* (26). *The optimal debt level* $b$ *under the constraint* $b \leq \tilde{b}$ *is given by*

$$b^0_{\leq \tilde{b}}(\theta) = \min\left\{\tilde{b}; b^E(\theta)\right\}. \tag{27}$$

*The optimal tax rate and the optimal level of government expenditure are given by*

$$\begin{aligned}
\tau^0_{\leq \tilde{b}}(\theta) &= \max\left\{\frac{\theta w - \tilde{b} + Ra}{w(1+\theta)}; \frac{\theta w - b^E(\theta) + Ra}{w(1+\theta)}; 0\right\}, &(28)\\
g^0_{\leq \tilde{b}}(\theta) &= b^0_{\leq \tilde{b}}(\theta) - Ra + \tau^0_{\leq \tilde{b}}(\theta) w. &(29)
\end{aligned}$$

We prove this proposition in the Appendix. For future use, we emphasize

**Proposition 4** *For* $\tilde{b} \leq Ra + \theta w$, *we have*

$$1 - \tau^0_{\leq \tilde{b}}(\theta) = \frac{1}{\theta w} \cdot g^0_{\leq \tilde{b}}(\theta) \tag{30}$$

*and*

$$b^0_{\leq \tilde{b}}(\theta) = Ra - w + \frac{1+\theta}{\theta} \cdot g^0_{\leq \tilde{b}}(\theta). \tag{31}$$

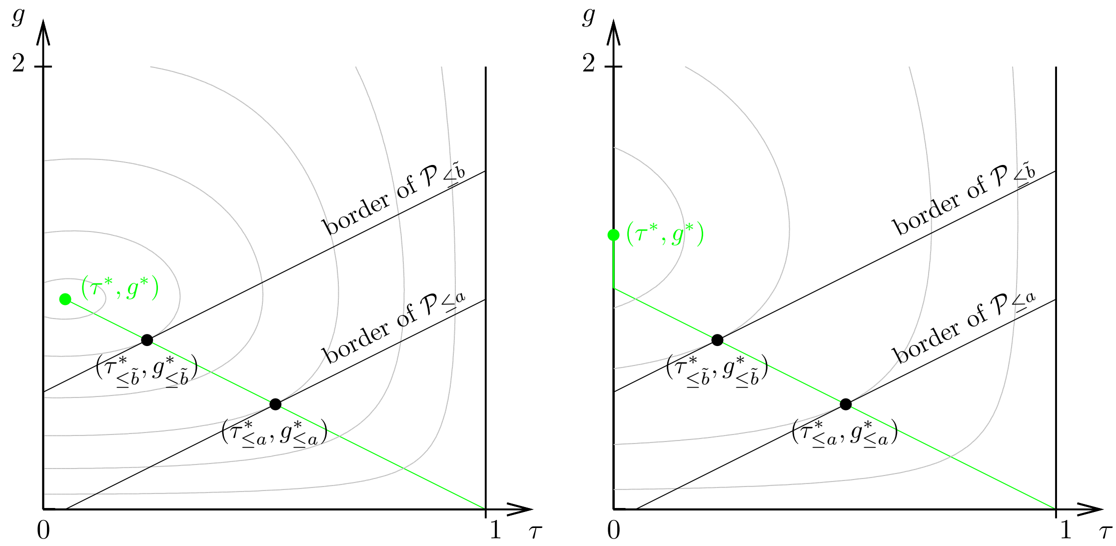


Figure 4: Indifference curves. The parameter values for both diagrams are taken from Example 3 (see page 26). The only difference between the diagrams: For the left diagram, we have used the value $\zeta = 20$ from the example, whereas for the right diagram we have changed it to $\zeta = 15$ in order to obtain a corner solution for $\tau^*$.

For the proof we again refer to the Appendix.

In the subsequent analysis the median type $\theta^*$ is particularly important. To simplify the notation, we will henceforth use the abbreviations $\tau^* := \tau^0(\theta^*)$, $g^* := g^0(\theta^*)$, $b^* := b^0(\theta^*)$, $\tau^*_{\le b} := \tau^0_{\le b}(\theta^*)$ etc. to describe the optimal values for the median type.

We provide a graphical illustration of the agents' optimization problem that we have analyzed in this section. The left-hand diagram in Figure 4 exhibits the indifference curves of the median voter $\theta^*$ and his desired policy $(\tau^*, g^*)$, with the parameters that will also be used in Example 3 on page 26. If debt is restricted to some $\tilde{b}$, his best policy within $\mathcal{P}_{\le \tilde{b}}$ is given by $(\tau^*_{\le \tilde{b}}, g^*_{\le \tilde{b}})$; it is located on the highest indifference curve that touches $\mathcal{P}_{\le \tilde{b}}$. The status quo $(\tau^*_{\le a}, g^*_{\le a})$ is determined in the same way, with debt being restricted to $a$. The points $(\tau^*_{\le \tilde{b}}, g^*_{\le \tilde{b}})$ for varying values of $\tilde{b}$ are depicted in green. If $\tau^* > 0$, as is the case in the left-hand diagram, they lie on the straight line that connects the points $(\tau^*, g^*)$ and $(1, 0)$; this line is characterized by Equation (30).

The right-hand diagram illustrates the case of a corner solution $\tau^* = 0$. Here the green line hits the vertical axis, and the point $(\tau^*, g^*)$ is located on the vertical axis. The set of points $(\tau^*_{\leq\tilde{b}}, g^*_{\leq\tilde{b}})$ (green) has a kink; it contains the complete downward-sloping straight line that is defined by Equation (30), as well as the segment on the vertical axis between the intersection point and the optimum $(\tau^*, g^*)$.

# 4. Condorcet Winners and the Outcomes Under the Simple Majority Rule

In this section, we turn our attention to the equilibrium of the voting process under the simple majority rule, which we specified in Section 2.3.1. We will establish the existence of a unique Condorcet winner, i. e. the winner against all other policy proposals under pairwise voting. We show that the Condorcet winner is determined by the median voter's preferences. We further show that under the simple majority rule, the Condorcet winner is the equilibrium policy.

We use the notation of McKelvey (1974). For any two proposals $(\tau, g)$ and $(\tau', g')$, we write $(\tau, g) \succ_\theta (\tau', g')$ if and only if a type-$\theta$ agent strictly prefers $(\tau, g)$ to $(\tau', g')$, i. e.

$$(\tau, g) \succ_\theta (\tau', g') \;:\Leftrightarrow\; V_\theta(\tau, g) > V_\theta(\tau', g').$$

We introduce the abbreviation $\big\{(\tau, g) \succ_\theta (\tau', g')\big\} := \big\{\theta > 0 \;\big|\; (\tau, g) \succ_\theta (\tau', g')\big\}$.

For any measurable subset $M \subseteq (0; \infty)$, we use $|M|$ to denote the measure of $M$, which means that a fraction $|M| := \mu(M)$ of all agents belongs to $M$. We use the shorthand version

$$\big|(\tau, g) \succ_\theta (\tau', g')\big| := \Big|\big\{(\tau, g) \succ_\theta (\tau', g')\big\}\Big|.$$

We shall use a similar notation for $\succsim_\theta$, $\precsim_\theta$, and $\prec_\theta$.

**Definition 5** *A proposal $(\tau, g)$ is called a* Condorcet winner *within a set $A \subseteq \mathcal{P}$ of policies, if no alternative $(\tau', g')$ is strictly preferred to $(\tau, g)$ by a simple majority of voters. Formally, $(\tau, g)$ is called a Condorcet winner if*

$$\big|(\tau', g') \succ_\theta (\tau, g)\big| \leq \frac{1}{2} \quad \textit{for all } (\tau', g') \in A.$$

The concept of a Condorcet winner within a proper subset $A$ of $\mathcal{P}$ will be particularly relevant for the analysis of the debt-sensitive majority rules, while only the case $A = \mathcal{P}$ will matter for simple majority rule.

In the following proposition, we describe the set of agents preferring one particular policy to another:

**Proposition 5** *Consider any two policies $(\tau, g), (\tau', g') \in \mathcal{P}$. Let $M := \big\{(\tau, g) \succ_\theta (\tau', g')\big\}$. Then, if $\tau = \tau'$, either $M = \emptyset$ or $M = \big(0; \infty\big)$. If $\tau < \tau'$, either $M = \emptyset$ or $M = \big(0; \infty\big)$, or $\tilde{\theta} \in \big(0; \infty\big)$ exists such that $M = \big(0; \tilde{\theta}\big)$. If $\tau > \tau'$, either $M = \emptyset$ or $M = \big(0; \infty\big)$, or $\tilde{\theta} \in \big(0; \infty\big)$ exists such that $M = \big(\tilde{\theta}; \infty\big)$. In particular, $\big\{(\tau, g) \succ_\theta (\tau', g')\big\}$ and $\big\{(\tau, g) \precsim (\tau', g')\big\}$ are (possibly empty) intervals.*

**Proof.** If $V_\theta(\tau, g) = -\infty$ or $V_\theta(\tau', g') = -\infty$, the statement is obvious. Otherwise, define the function $d \colon (0; \infty) \to \mathbb{R}$ by

$$d(\theta) := \frac{1}{\theta} \cdot \big(V_\theta(\tau, g) - V_\theta(\tau', g')\big).$$

By Equation (7) we have

$$d(\theta) = \frac{1}{\theta}\big(\log(1 - \tau) - \log(1 - \tau')\big) + \tilde{C},$$

with $\tilde{C}$ being a term that does not depend on $\theta$. We now observe that $d$ is constant if $\tau = \tau'$, strictly decreasing if $\tau < \tau'$, and strictly increasing if $\tau > \tau'$. Furthermore, $d$ is continuous. Since $M = \big\{\theta \bigm| d(\theta) > 0\big\}$, the assertion follows. $\square$

Inspecting the proof, we observe

**Remark 1** If $\tilde{\theta}$ in Proposition 5 exists, an individual of type $\tilde{\theta}$ will be indifferent as to the proposals $(\tau, g)$ and $(\tau', g')$.

Together with the uniqueness of the median, Proposition 5 yields the following technical result, which we prove in the Appendix:

**Corollary 2** *For any two policies $(\tau, g), (\tau', g') \in \mathcal{P}$, let $M := \big\{(\tau, g) \succ_\theta (\tau', g')\big\}$. If the median type $\theta^*$ is contained in $M$, then $|M| > 1/2$.*

As a consequence of Proposition 5 and Corollary 2, we obtain a characterization of Condorcet winners:

**Corollary 3** *Consider any set $A \subseteq \mathcal{P}$. A policy proposal is a Condorcet winner in $A$ if and only if it is the median voter's optimal policy in $A$.*

**Proof.** Consider any proposal $(\tau, g)$ that is not a Condorcet winner. Then a proposal $(\tau', g')$ exists with

$$\big|(\tau', g') \succ_\theta (\tau, g)\big| > \frac{1}{2}\,. \tag{32}$$

According to Proposition 5, the set $\big\{(\tau', g') \succ_\theta (\tau, g)\big\}$ is an interval; thus inequality (32) implies that the set contains the median voter's type. Hence the median voter strictly prefers $(\tau', g')$ to $(\tau, g)$, so that he does not consider $(\tau, g)$ to be optimal in $A$.

To prove the reverse, take any proposal $(\tau, g)$ that is not considered to be optimal by the median voter. Let $(\tau', g')$ be any proposal that he strictly prefers to $(\tau, g)$. From Corollary 2 we know that $\big|(\tau', g') \succ_\theta (\tau, g)\big| > 1/2$. Hence $(\tau, g)$ is not a Condorcet winner. $\square$

Corollary 3 says that Condorcet winners coincide with optimal policies of the median voter. This property enables us to characterize voting outcomes below. Recall that $\theta^*$ denotes the median voter's type and that we use the abbreviations $\tau^* := \tau^0(\theta^*)$, $g^* := g^0(\theta^*)$, $b^* := b^0(\theta^*)$, $\tau^*_{\leq b} := \tau^0_{\leq b}(\theta^*)$ etc.

**Corollary 4** *For each $b$ a unique Condorcet winner in $\mathcal{P}_{\leq b}$ exists. It is given by $(\tau^*_{\leq b}, g^*_{\leq b})$. In particular, $(\tau^*, g^*)$ is the unique Condorcet winner in $\mathcal{P}$.*

**Proof.** The corollary follows from Proposition 1, applied to $V_{\theta^*}$, and Corollary 3. $\square$

**Corollary 5** *Consider any set $A \subseteq \mathcal{P}$. If $(\tau, g)$ is a unique Condorcet winner within $A$, then*

$$\big|(\tau', g') \succ_\theta (\tau, g)\big| < \frac{1}{2} \quad \textit{for any proposal } (\tau', g') \in A.$$

The proof can be found in the Appendix.

As the following proposition states, the Condorcet winner is the equilibrium outcome when a simple majority rule is applied:

**Proposition 6** *Let $A \subseteq \mathcal{P}$ be a set of policies, and suppose a unique Condorcet winner $(\tau, g)$ within $A$ exists. If the agents decide among the policies of $A$ using sequential pair-wise voting with a simple majority rule, then a subgame-perfect strong Nash equilibrium exists, and $(\tau, g)$ will be the equilibrium outcome in any subgame-perfect strong Nash equilibrium.*

**Proof.** Let $(\tau, g)$ be a Condorcet winner within $A$. Consider the strategy profile in which everybody proposes $(\tau, g)$ and the proposal is accepted unanimously. This is a subgame-perfect strong Nash equilibrium, which we clarify as follows: Since $(\tau, g)$ is the unique Condorcet winner within $A$, Corollary 5 yields

$$\left|(\tau', g') \succ_\theta (\tau, g)\right| < \frac{1}{2} \quad \text{for any proposal } (\tau', g') \in A.$$

Hence, only a minority of agents would contemplate pursuing a strategy leading to an outcome different from $(\tau, g)$, but such a minority would not have the power to alter the outcome. Consequently, no coalition has an incentive to deviate from the before-mentioned strategy profile.

It remains to be shown that the outcome of any subgame-perfect strong Nash equilibrium is a Condorcet winner. Suppose there were a subgame-perfect strong Nash equilibrium with an outcome $(\tau', g')$ that is not a Condorcet winner. Then a proposal $(\tau'', g'')$ would exist such that

$$\left|(\tau'', g'') \succ_\theta (\tau', g')\right| > \frac{1}{2}.$$

Thus, a majority of agents would deviate by proposing and voting for $(\tau'', g'')$. This would contradict the fact that $(\tau', g')$ is the outcome of a subgame-perfect strong Nash equilibrium. □

An important remark is in order here. A unanimous voting result, as presented in the proof, is unlikely to be observed in actual parliamentary voting. Minorities will fight for their ideas, even if they know that their proposals will not be successful. As we have just shown, no coalition of parliamentarians has the power to secure an outcome that is better than $(\tau, g)$. Hence, in our situation, parliamentarians would have other motives for deviating than influencing the voting outcome. For the purpose of the proof, it is sufficient to consider the simplest possible strategy profile, where all agents vote for the winning proposal.

We conclude this section and combine our findings in the first main result:

**Theorem 1** *The median voter's desired policy $(\tau^*, g^*)$ is the unique voting outcome under the simple majority rule.*

## 5. Debt-sensitive Majority Rule

In this section, we analyze the outcome under a debt-sensitive majority rule. Again, we employ the concept of subgame-perfect strong Nash equilibrium. We show that the equilibrium policy is unique and characterize the equilibrium.

### 5.1. Equilibrium: definition and existence

Recall that by Definition 4, a debt-sensitive majority rule is described by a function $\phi$, which is assumed to be left-continuous and weakly increasing.

For any policy $(\tau, g) \in \mathcal{P}$, we use

$$\alpha(\tau, g) := \left|(\tau, g) \succsim_\theta \left(\tau^*_{\leq a}, g^*_{\leq a}\right)\right| \tag{33}$$

to denote the fraction of voters who in a direct comparison at least weakly prefer $(\tau, g)$ to the status quo optimal for the median type.

The following equilibrium definition will turn out to be appropriate:

**Definition 6** *Let $\phi$ be a debt-sensitive majority rule. A policy $(\tau, g)$ is called a* semi-equilibrium under $\phi$ *if it satisfies the following requirements:*

*(i) $(\tau, g)$ is a Condorcet winner in $\mathcal{P}_{\leq B(\tau,g)}$,*

*(ii) $\alpha(\tau, g) \geq \phi\big(B(\tau, g)\big)$.*

*A semi-equilibrium $(\tau, g)$ under $\phi$ is called an* equilibrium under $\phi$ *if no semi-equilibrium $(\tau', g')$ with $B(\tau', g') > B(\tau, g)$ exists.*

To illustrate this definition, we refer once again to Figure 4. The semi-equilibria are exactly those points between $(\tau^*, g^*)$ and the status quo $(\tau^*_{\leq a}, g^*_{\leq a})$ for which condition (ii) is fulfilled. The semi-equilibrium with the highest associated debt-level, that is, the highest of these points, is the equilibrium.

In the subsequent proposition we show that the equilibria in the sense of Definition 6 are the equilibrium outcomes of the voting game described in Section 2.3.2.

**Proposition 7** *A policy $(\tau, g)$ is an equilibrium (in the sense of Definition 6) if and only if it is the outcome of a subgame-perfect strong Nash equilibrium of the voting process under the debt-sensitive majority rule (in the sense of Definition 4).*

The proof is very similar to that of Proposition 6 and is provided in the Appendix.

We next state our second main result.

**Theorem 2** *For each debt-sensitive majority rule $\phi$, a unique equilibrium under $\phi$ exists.*

The proof of the theorem can be found in the Appendix.

## 5.2. Characterization of the equilibrium policy

We now characterize the equilibrium policy in more detail. The fraction of people who prefer the proposal $(\tau^*_{\le b}, g^*_{\le b})$ to the status quo $(\tau^*_{\le a}, g^*_{\le a})$ is given by $\alpha(\tau^*_{\le b}, g^*_{\le b})$, as defined in Equation (33). As shown in the last section, the equilibrium debt level under the debt-sensitive majority rule $\phi(\cdot)$ is the largest $b$ such that $\alpha(\tau^*_{\le b}, g^*_{\le b}) \ge \phi(b)$. The following proposition enables us to determine $\alpha(\tau^*_{\le b}, g^*_{\le b})$.

**Proposition 8** *Define the function $T\colon [a;\bar{b}) \to \mathbb{R}$ by*

$$T(b) := \begin{cases} \dfrac{\log g^*_{=b} - \log g^*_{=a} + \beta\big(\log h(b) - \log h(a)\big)}{\log(1-\tau^*_{=a}) - \log(1-\tau^*_{=b})} & \text{for } b \in (a;\bar{b}), \\ -\infty & \text{for } b = a. \end{cases} \tag{34}$$

*(i) For $b$ with $a < b \le Ra + \theta^* w$, we have*

$$T(b) = -1 + \beta \cdot \frac{\log h(a) - \log h(b)}{\log(w+b-Ra) - \log(w+a-Ra)}. \tag{35}$$

*(ii) $T(b)$ is strictly increasing for $a \le b \le Ra + \theta^* w$.*

*(iii) Consider $b$ with $a \le b \le b^*$. An agent of type $\theta$ will prefer the policy proposal $(\tau^*_{\le b}, g^*_{\le b})$ to the status quo $(\tau^*_{\le a}, g^*_{\le a})$ if and only if*

$$\frac{1}{\theta} \ge T(b). \tag{36}$$

The proof of this proposition can be found in the Appendix. The proposition permits a direct characterization of the equilibrium proposal under a particular debt-sensitive majority rule.

**Corollary 6** *If $b^* \geq a$, the equilibrium debt level under a debt-sensitive majority rule $\phi$ is the largest $b \in [a; b^*]$ satisfying*

$$\alpha(\tau^*_{\leq b}, g^*_{\leq b}) \equiv \mu\big\{\theta \bigm| 1/\theta \geq T(b)\big\} \geq \phi(b).$$

The equilibrium under the debt-sensitive majority rule is the Condorcet winner within a set of policies that honor some upper debt limit. This upper debt limit is chosen so that the supermajority required by the rule will prefer the Condorcet winner within the restricted set to the Condorcet winner when no new debt is allowed (status quo).

We further note that for $b^* < a$ even the median voter will favor a policy that implies debt reduction. Hence, any debt-sensitive majority rule will yield the same outcome as a simple majority rule in such circumstances.

# 6. Examples

## 6.1. The setup

In the following, we compare the simple and the debt-sensitive majority rule in several examples. In all these examples, the relationship between debt and government expenditure is given by

$$h(b) = \gamma(\bar{b} - b)^\zeta \quad \text{for } b < \bar{b}, \tag{37}$$

with some $\gamma, \zeta > 0$.

Equation (25) then reads

$$\frac{1+\theta}{w + b - Ra} = \frac{\beta\theta\zeta}{\bar{b} - b}, \tag{38}$$

and Equation (26) reads

$$\frac{\theta}{b - Ra} = \frac{\beta\theta\zeta}{\bar{b} - b}. \tag{39}$$

$b^E(\theta)$ is the solution of Equation (38) or (39), depending on whether $-(\log h)'(Ra + \theta w)$ is above or below $1/(\beta\theta w)$; hence,

$$b^E(\theta) = \begin{cases} \dfrac{\beta\theta\zeta(Ra - w) + (1+\theta)\bar{b}}{1 + \theta + \beta\zeta\theta} & \text{for } \dfrac{\beta\theta\zeta w}{\bar{b} - Ra - \theta w} \leq 1, \\ \dfrac{\beta\zeta Ra + \bar{b}}{1 + \beta\zeta} & \text{for } \dfrac{\beta\theta\zeta w}{\bar{b} - Ra - \theta w} > 1. \end{cases} \tag{40}$$

We obtain the optimal debt level from Equation (27). It is given by

$$b^0_{\leq\tilde{b}}(\theta) = \min\left\{\tilde{b},\ \frac{\beta\theta\zeta(Ra-w)+(1+\theta)\bar{b}}{1+\theta+\beta\zeta\theta},\ \frac{\beta\zeta Ra+\bar{b}}{1+\beta\zeta}\right\}. \tag{41}$$

Using Equations (28) and (29), we arrive at

$$1-\tau^0_{\leq\tilde{b}}(\theta) = \min\left\{\frac{w+\tilde{b}-Ra}{w(1+\theta)},\ \frac{w+\bar{b}-Ra}{w(1+\theta+\beta\zeta\theta)},\ 1\right\}, \tag{42}$$

$$g^0_{\leq\tilde{b}}(\theta) = b^0_{\leq\tilde{b}}(\theta) - Ra + \tau^0_{\leq\tilde{b}}(\theta)w. \tag{43}$$

In Section 4 we showed that under the simple majority rule policy $(\tau^*, g^*)$ will be chosen. This policy corresponds to the median voter optimum if there is no restriction on the debt level allowed, which means $(\tau^*, g^*) = \left(\tau^0_{\leq\bar{b}}(\theta^*), g^0_{\leq\bar{b}}(\theta^*)\right)$ and $b^* = b^0_{\leq\bar{b}}$. With the Equations (41), (42), and (43), and taking into account that $1+\theta^*+\beta\zeta\theta^* > 1+\theta^*$, we obtain the outcome under the simple majority rule:

$$b^* = \min\left\{\frac{\beta\zeta\theta^*(Ra-w)+(1+\theta^*)\bar{b}}{1+\theta^*+\beta\zeta\theta^*},\ \frac{\beta\zeta Ra+\bar{b}}{1+\beta\zeta}\right\}, \tag{44}$$

$$\tau^* = \max\left\{1-\frac{w+\bar{b}-Ra}{w(1+\theta^*+\beta\zeta\theta^*)},\ 0\right\}, \tag{45}$$

$$g^* = b^* - Ra + \tau^* w\,. \tag{46}$$

To analyze debt-sensitive majority rules, we use the results from Section 5.2. For $h$ given by (37), we determine the fraction of agents who prefer the policy proposal $\left(\tau^*_{\leq b}, g^*_{\leq b}\right)$ to the status quo $\left(\tau^*_{\leq a}, g^*_{\leq a}\right)$, i.e. the types for which Condition (36) is fulfilled. The function $T$ is given by Equation (34). For $a < b \leq Ra + \theta^* w$, we can use Equation (35) and obtain

$$T(b) = -1 + \beta\zeta \cdot \frac{\log\left(\bar{b}-a\right) - \log\left(\bar{b}-b\right)}{\log(w+b-Ra) - \log(w+a-Ra)}\ . \tag{47}$$

## 6.2. Parametrized examples

Here we illustrate the impact of debt-sensitive majority rules for specific parameter values.

**Example 1 (linear $h(b)$ with low initial debt)** We first consider the parameter values $a = 1$, $w = 1$, $\bar{b} = 20$, $R = 1.05$, $\beta = 1/R$, $\zeta = 1$. Let $1/\theta$, which is the relative

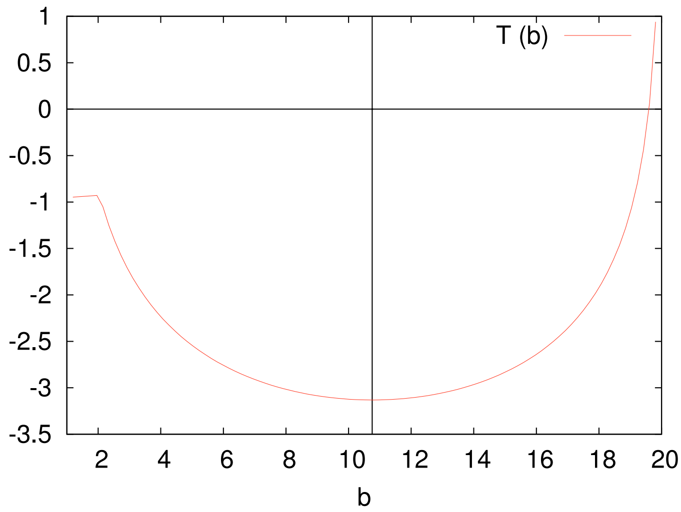


Figure 5: Function $T$ for Example 1.

weight of consumption in the utility function, be uniformly distributed on the interval $(0; 2]$. Then the median type is $\theta^* = 1$.

For the simple majority rule, Equations (44), (45), and (46) yield

$$b^* = 10.76, \qquad \tau^* = 0, \qquad g^* = 9.71,$$

which is a corner solution – a majority of agents prefers to finance government expenditure exclusively via debt. This result is mainly driven by the very low initial debt level, which leads to a relatively low utility loss for debt-financing compared to tax-financing. The status quo is given by the Equations (41) to (43)

$$b^*_{\leq a} = a = 1, \qquad \tau^*_{\leq a} = 0.525, \qquad g^*_{\leq a} = 0.475.$$

Figure 5 shows the graph of the function $T$, as defined in Equation (34). The curve has a kink at a debt level of $b = Ra + \theta^* w = 2.05$, above which the median type's desired tax rate is zero. The vertical line depicts the equilibrium debt level $b^*$ under the simple majority rule. Function $T$ is negative on the interval $(a; b^*)$. This means that all agents prefer the optimal policy $(\tau^*, g^*)$ of the median voter to the status quo. Hence, a debt-sensitive majority rule as introduced in Section 5 will have no effect. $\diamondsuit$

**Example 2 (linear $h(b)$ with high initial debt)** Now we consider a higher initial debt level given by $a = 19$, leaving the other parameters unchanged.

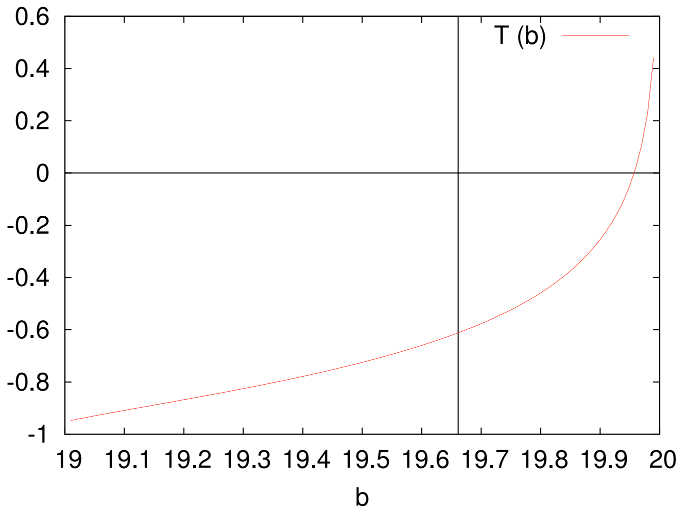


Figure 6: Function $T$ for Example 2.

Here we obtain the outcome

$$b^* = 19.66, \qquad \tau^* = 0.64, \qquad g^* = 0.36.$$

In this example, the status quo is given by

$$b^*_{\leq a} = a = 19, \qquad \tau^*_{\leq a} = 0.975, \qquad g^*_{\leq a} = 0.025.$$

Function $T$ is depicted in Figure 6. Again, a debt-sensitive majority rule will have no effect, as $T$ is negative on the relevant interval $(a, b^*)$. $\diamondsuit$

In Example 1 we observed a corner optimum involving a tax rate of zero, whereas we obtained an inner optimum in Example 2. In both examples debt accumulation cannot be prevented by a debt-sensitive majority rule. The reason is as follows. First, the marginal disutility of debt is small, as future government expenditures depend linearly on debt. Second, the status quo is extremely bad compared to the median voter's optimal policy, even for large values of $\theta$. This is still true if initial debt is very high, because severe taxation is necessary to maintain the status quo in such circumstances, which results in a sharp cut on private consumption and thus high marginal disutility. This effect outweighs the negative impact of debt-making on future public-good provision.

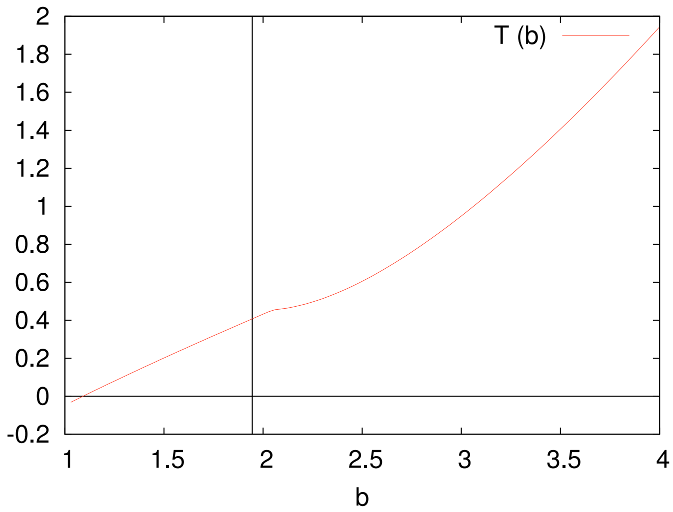


Figure 7: Function $T$ for Example 3.

We consider a third example, where higher debt causes a disproportionately high decrease in $h$. This dependence may be due to the fact that higher indebtedness causes higher interest rates in future, as lenders will demand higher risk premiums.

**Example 3 (stronger impact of debt on future utility)** In this example, we calibrate the model with $a = 1$, $w = 1$, $\bar{b} = 20$, $R = 1.05$, $\beta = 1/R$, $\zeta = 20$. These parameter values yield

$$b^* = 1.95, \qquad \tau^* = 0.05, \qquad g^* = 0.95.$$

The status quo is the same as in Example 1. Function $T$ is depicted in Figure 7. We observe that $T(b)$ is no longer negative on the whole interval $(a, b^*)$, but positive for $b > 1.1$. Thus, proposals that imply a debt level near $b^*$ are not accepted unanimously. The fraction $\alpha(\tau^*_{\leq b}, g^*_{\leq b}) = \mu\{\theta \mid 1/\theta \geq T(b)\}$ of parliamentarians who accept policy proposal $(\tau^*_{\leq b}, g^*_{\leq b})$ is decreasing in $b$ and is strictly below one if $b$ is larger than 1.1. Hence, we can prevent debt accumulation beyond this level by employing a suitable debt-sensitive majority rule. As we have demonstrated in Section 5, the resulting debt level is the highest $b$ such that $\mu\{\theta \mid 1/\theta \geq T(b)\} \leq \phi(b)$.

Consider, for instance, the debt-sensitive majority rule $\phi(b) = \frac{1}{2} + \frac{2}{3}(b - a)$ depicted in Figure 8. The intersection of the graphs of $\mu\{\theta \mid 1/\theta \geq T(b)\}$ and $\phi(b)$ gives the equilibrium value of $b$. In this example, future debt level is reduced from 1.95 to around 1.55 by the introduction of the debt-sensitive majority rule $\phi(b)$. $\diamondsuit$

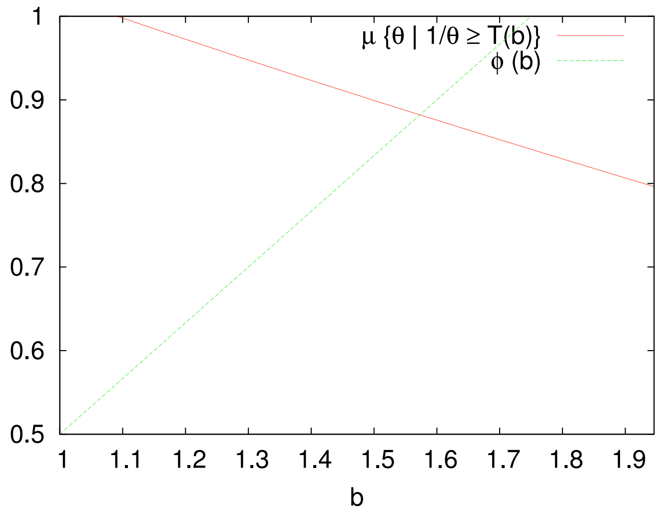


Figure 8: Function $\mu\{\theta \mid 1/\theta \geq T(b)\}$ (solid line) and $\phi(b)$ (dotted line) for Example 3.

## 6.3. A sufficient condition for the effectiveness of debt-sensitive majority rules

Examples 1–3 indicate that a debt-sensitive majority rule will only have an effect if the graph of $T$ hits the horizontal axis at a point left of $b^*$. We now provide a sufficient condition for debt-sensitive majority rules to be effective.

A situation similar to the ones shown in Examples 1 and 2 will not arise if $T$ starts above the horizontal axis, i.e. if $\lim_{b\downarrow a} T(b) \geq 0$. By L'Hospital's rule,

$$\lim_{b\downarrow a} T(b) = -1 + \beta \lim_{b\downarrow a} \frac{-(\log h)'(b)}{(w+b-Ra)^{-1}} = -1 - \beta(w+a-Ra)(\log h)'(a).$$

Thus we obtain

**Proposition 9** *Suppose that $b^* > a$ and agents with arbitrarily large $\theta$ exist, i.e. 0 is a boundary point for the support of the distribution of $1/\theta$. Then a sufficient condition for a debt-restricting debt-sensitive majority rule to exist is*

$$-\beta(w+a-Ra)(\log h)'(a) \geq 1.$$

Agents with high values of $\theta$ reflect those people in society who strongly rely on public services and public goods provided by the government. Examples are older people who rely heavily on public transportation or public health services, or people particularly concerned about security.

### 6.4. Risky public-good provision

As a further example, we consider risky public-good provision. There are many different reasons for such risks. For instance, a high level of future debt increases the risk of a substantial cut in future public-good provision, since a future downturn might reduce the ability to pay interest, interest rates might rise, or the country might even default. Future electorates might also re-allocate government funds to activities that do not benefit the current electorate. In order to capture such uncertainties in our model, let $\log h(b)$ be a mixture of a first term reflecting a normal economic situation and a second term reflecting a situation in which public-good provision is severely restricted. The weight of the second term increases with the debt level. We use the following specification:

$$\log h(b) = \frac{b}{\bar{b}} \log\bigl(\gamma_1 (\bar{b} - b)^{\zeta}\bigr) + \left(1 - \frac{b}{\bar{b}}\right) \log\bigl(\gamma_2 (\bar{b} - b)^{\zeta}\bigr), \tag{48}$$

with $\gamma_2$ considerably larger than $\gamma_1$. The value $1 - b/\bar{b}$ describes the probability of normal economic circumstances. The fraction $b/\bar{b}$ reflects the probability of a drastic reduction of public-good provision. This reduction is modeled by a low value of $\gamma_1$. The probability $b/\bar{b}$ that the amount of public goods will be severely restricted increases with the level of public debt.

Again, the median voter's desired debt level follows from Equation (27). It is given by $b^* = b^E(\theta^*)$, with $b^E(\theta^*)$ being the solution of Equation (25), which now reads

$$\frac{1+\theta}{w + b - Ra} = -\beta\theta \left( \frac{\log \gamma_1 - \log \gamma_2}{\bar{b}} - \frac{\zeta}{\bar{b} - b} \right),$$

or the solution of Equation (26), which reads

$$\frac{\theta}{b - Ra} = -\beta\theta \left( \frac{\log \gamma_1 - \log \gamma_2}{\bar{b}} - \frac{\zeta}{\bar{b} - b} \right).$$

From Equation (35) we obtain

$$T(b) = \frac{\log g^*_{=b} - \log g^*_{=a} + \beta\bigl[(b-a)\bar{b}^{-1}(\log \gamma_2 - \log \gamma_1) + \zeta\bigl(\log(\bar{b} - a) - \log(\bar{b} - b)\bigr)\bigr]}{\log(1 - \tau^*_{=a}) - \log(1 - \tau^*_{=b})}.$$

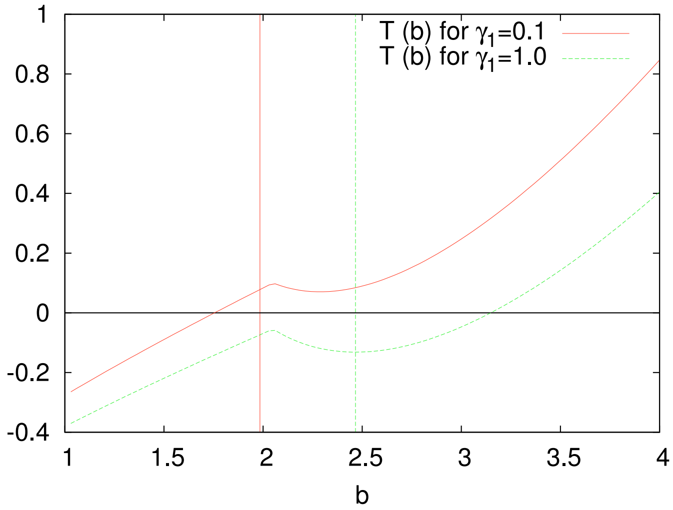


Figure 9: Functions $T$ for Section 6.4.

We use the distribution of $\theta$ from Example 1 and the parameter values $a = 1$, $w = 1$, $\bar{b} = 20$, $R = 1.05$, $\beta = 1/R$, $\zeta = 13$, $\gamma_1 = 0.1$, and $\gamma_2 = 1$. These yield

$$b^* = 1.98, \qquad \tau^* = 0.03, \qquad g^* = 0.97.$$

Again, the status quo is that of Example 1. In Figure 9, function $T$ is plotted for this example and for a scenario without the risk of a severe decline in public-good provision (i. e. $\gamma_1 = \gamma_2 = 1$). In the latter scenario, the optimal values for the median type are

$$b^* = 2.47, \qquad \tau^* = 0, \qquad g^* = 1.42.$$

The median voter's optimal debt $b^*$ is lower in the presence of risk, which results in lower debt-making under the simple majority rule as well. This is illustrated by the left-hand shift of the vertical line. The figure illustrates that a debt-sensitive majority rule can be effective, as the solid $T$ curve is partly above the horizontal axis. In the benchmark scenario without risky public-good provision, the $T$ curve (dashed) hits the horizontal line at a point to the right of $b^*$, and so the introduction of a debt-sensitive majority rule would have no effect.

The upward shift of the $T$ curve can be explained as follows: In the scenario with risky public-good provision, the expected utility $\log h(b)$ from future public-good consumption is lower relative to the status-quo utility $\log h(a)$ for any debt level $b$. So agents with a high value of $\theta$ will now tend to favor the status quo over the median voter's desired policy, if this policy is connected with a high debt level. This leads to a larger fraction of agents opposing debt accumulation compared to the case where $\gamma_1 = \gamma_2 = 1$.

## 7. Debt-sensitive Majority Rules as Political Stabilizers

Debt-sensitive majority rules can limit debt accumulation. The same goal, however, could be achieved by fixed debt limits. Indeed, the following proposition holds as a direct consequence of Proposition 7:

**Proposition 10** *Consider some debt-sensitive majority rule with an associated policy outcome $(\tau^{\phi}, g^{\phi})$ and an associated debt level $b^{\phi} = B(\tau^{\phi}, g^{\phi})$. If a debt limit of $b^{\mathrm{fix}} = b^{\phi}$ were fixed by constitution, the simple majority rule would yield the same outcome $(\tau^{\phi}, g^{\phi})$.*

We now illustrate that in contrast to fixed debt limits, a debt-sensitive majority rule can stabilize macroeconomic shocks. We demonstrate this property for a negative income shock.

Consider a temporary shock that causes a decline of $w$ to $w_{\mathrm{r}} < w$. Since the shock is not persistent, $\bar{b}$ is not affected. From Equation (35) we observe that with the reduction in wages, function $T$ increases more slowly than before, as long as we do not have a corner solution at $\tau = 0$. Due to the recession the median voter's desired debt level also increases from $b^*$ to $b^*_{\mathrm{r}}$ (see Equation 25). According to Corollary 6, the wage shock will increase the debt level chosen under the debt-sensitive majority rule, i. e. $b^{\phi}_{\mathrm{r}} > b^{\phi}$.

With a fixed debt limit, however, the debt level would remain unchanged at $b^{\mathrm{fix}} < b^{\phi}_{\mathrm{r}}$. When negative shocks occur, the debt-sensitive majority rule thus guarantees more flexibility in policy-making by allowing for, and resulting in, a higher debt level compared to a fixed debt limit. As indicated by Equations (31) and (30), the higher debt level under the debt-sensitive majority rule is accompanied by higher public-good provision and a lower tax rate. Thus debt-sensitive majority rules act as political stabilizers.

## 8. Implementation and Extensions

Our analysis has indicated that debt-sensitive majority rules may act as a debt-brake. Nevertheless, their introduction may turn out to be difficult, as such rules may not be in the interest of the current electorate. In Section 8.1 we assess ways of overcoming these difficulties. Several extensions of our model are discussed in Section 8.2.

### 8.1. Implementation

A debt-sensitive majority rule allows a minority to block debt accumulation. In those cases in which the rule is effective, it leads to an outcome that makes a majority of agents worse off than under simple majority voting. Hence, the introduction of a debt-sensitive majority rule will not be supported by a majority in parliament. In this section, we outline two ways in which debt-sensitive majority rules could be introduced despite this fact.

*Prompt introduction, future effectiveness.* Consider a situation in which the present debt is moderate, so that the proposed debt-sensitive majority rule would not bind immediately. If, in such a situation, agents expect a future political environment to be less disciplined with respect to debt accumulation, then a debt-restricting policy rule could obtain the support of a majority.

*Delayed application plus altruism.* A promising way of introducing debt-sensitive majority rules is immediate introduction, but delayed application. Suppose agents exhibit some degree of altruism, which may be small. An agent who cares for his descendants to some extent will fear excessive debt accumulation, as it limits the government's ability to provide public goods. An altruistic agent may consider the introduction of a debt-sensitive rule that will only come into effect in the future. In the first generation for which the debt-sensitive majority rule is effective, a majority of agents will experience a utility loss – these agents will inherit a relatively high debt level from their parents, and at the same time they will be limited in their debt-financing potential by the debt-sensitive majority rule. The second and all subsequent generations, however, may benefit. If their gains outweigh the aggregate utility loss of the first generation, a majority of the current generation may be willing to adopt a debt-sensitive majority rule today if it is applied from the next generation onwards. The introduction has to be coupled with the rule that abolition requires a supermajority. Otherwise, the next generation would immediately abolish the debt-sensitive majority rule.[5]

Another reason favoring delayed application is common in parliamentary democracies, where politicians are elected only for a limited period of time. Politicians may be willing

[5]The concept of delayed application of constitutional rules in the context of a long-term investment project is discussed in Gersbach and Kleinschmidt (2009).

to introduce unpopular policy measures for some future point in time, as they will be no longer in office when negative side-effects appear. Then they need not fear being voted out.

A recent example of delayed application of a long-term, welfare-improving policy is the recent debt-brake in Germany *(Schuldenbremse)*. It states that the government's budget has to be nearly balanced every year – "nearly" meaning that the budget deficit is restricted to 0.35 % of GDP. Debt accumulated in downturns has to be repaid as soon as the economy recovers. The rule was introduced in 2009 and will be fully enforced by the year 2020 (see Mody and Stehn, 2009, and Article 115 of the German Constitution).

## 8.2. Further extensions

So far, we have assumed that agents differ in their taste for public goods. In practice, there are other sources of heterogeneities among citizens that may increase the strength of debt-sensitive majority rules in considerably limiting government debt accumulation. In this section we sketch the likely consequences of three sources of heterogeneity.

*Initial old generation.* We could add an old generation in the first period. Such a generation would be in favor of high debt. If the old generation were the majority, debt-sensitive majority rules would be particularly effective, as in such circumstances the simple majority rule would lead to very high debt levels.

*Different degrees of altruism.* Suppose that agents are altruistic and differ in the weight they attach to future utility. In our model, the simplest way to analyze the effects of altruism is to interpret $h(b)$ as the overall utility of future periods, which comprises the agent's own utility in old age and the discounted utility stream of future generations. Then different levels of $\beta$ would reflect different degrees of altruism. Since the utility $h(b)$ of future periods is strictly decreasing in the debt level $b$ left to the next generation, a higher value of $\beta$ exhibits higher altruistic motives and implies a higher distaste for public debt. If there is heterogeneity regarding $\beta$, highly altruistic agents will block the tendency toward higher public debt under a debt-sensitive majority rule. This is impossible under a simple majority rule.

*Income heterogeneity.* Assume that agents differ in labor income. The way in which income heterogeneities affect our results depends on the tax scheme. If the income tax rate is constant (flat tax scheme), our results will remain unaltered, the reason being that the individual wage enters the indirect utility function as a constant summand and thus does not influence the agents' policy preferences.

We next consider a two-level tax system where individuals with low incomes are exempted from taxation. Such a tax system affects our model and findings in two directions.

First, *non-taxed* individuals desire high tax rates to finance a high level of public goods, as their utility is given by

$$\begin{aligned} V_\theta^{\text{non-taxed}} &= \log(1-\tau)\big|_{\tau=0} + \theta \log g + \theta\beta \log h(b) \\ &= \theta \log g + \theta\beta \log h(b). \end{aligned}$$

Since monotonic transformations of the utility function do not affect the agents' preferences, one can describe the preferences of non-taxed agents by the utility function

$$\tilde{V}_\theta^{\text{non-taxed}} = \log g + \beta \log h(b).$$

Analogously, the utility of a *taxed* agent, given by Equation (7), can be described by

$$\tilde{V}_\theta = \frac{1}{\theta} V_\theta.$$

Since

$$\tilde{V}_\theta^{\text{non-taxed}} = \lim_{\theta' \to \infty} \frac{1}{\theta'} \tilde{V}_{\theta'},$$

individuals exempted from taxation will behave exactly like taxed individuals with an infinite weight $\theta$ on public-good consumption. Hence, to study the impact of a group of non-taxed individuals, we can treat the non-taxed individuals as individuals with an extremely high value of $\theta$. Thus the share of individuals with high $\theta$ increases.

Second, for any given tax rate, the per-capita tax revenue will be smaller, since only a fraction of the society can be taxed. As the wage rate only enters the optimization problem via the government budget constraint, a two-level tax system formally corresponds to a decrease in the wage rate $w$ in a system with a single tax rate.

To illustrate these two changes induced by a two-level tax system, we modify Example 3 and assume that 15 % of the individuals are exempted from taxation. We further assume that they earn 5 % of the total wage. Assume that the decision not to tax low incomes can be revoked if debt cannot be serviced otherwise. This means that the maximum debt level $\bar{b}$ remains unaffected. We consider two scenarios. In the *first scenario*, we

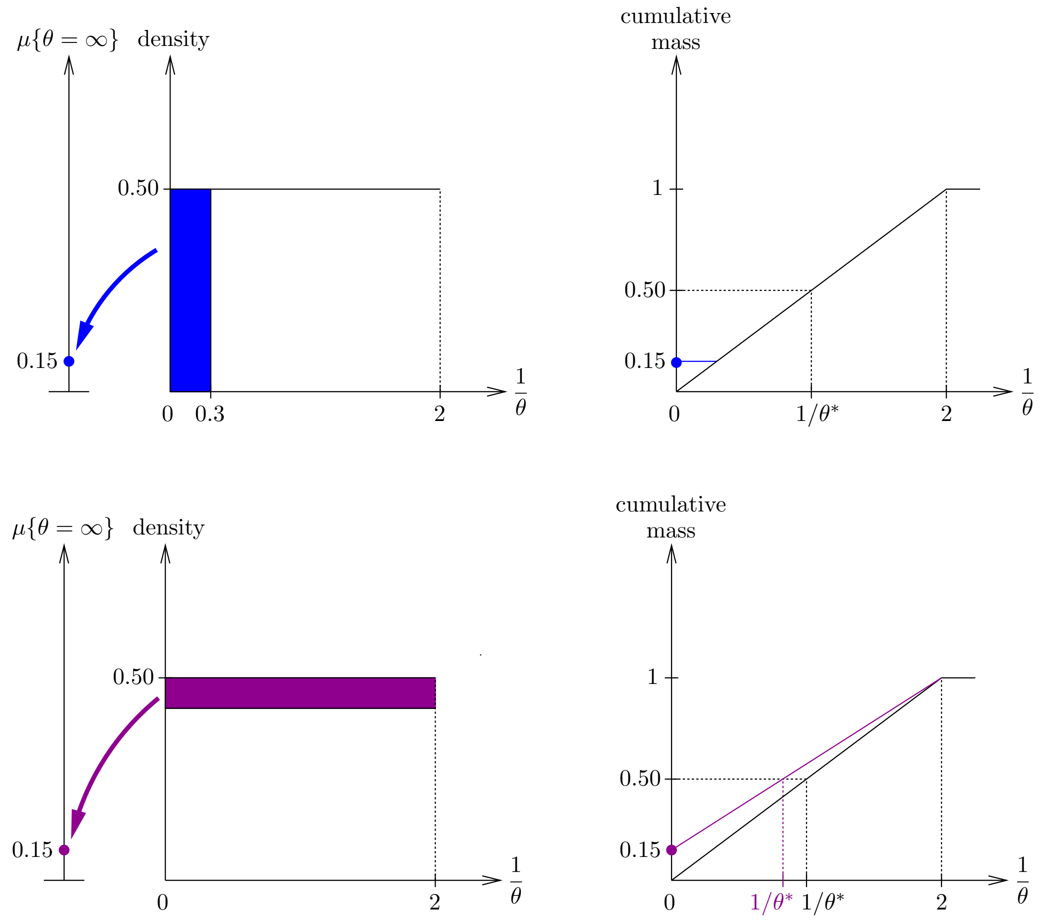


Figure 10: Exempting agents from taxation corresponds to assigning them $\theta = \infty$. In the upper two diagrams, agents with $0 < 1/\theta < 0.3$ are exempted from taxation; the median type $\theta^*$ does not change. This corresponds to the first scenario in the main text. In the lower two diagrams, the non-taxed agents are distributed uniformly over all types; the median type $\theta^*$ shifts upwards. This is the second scenario.

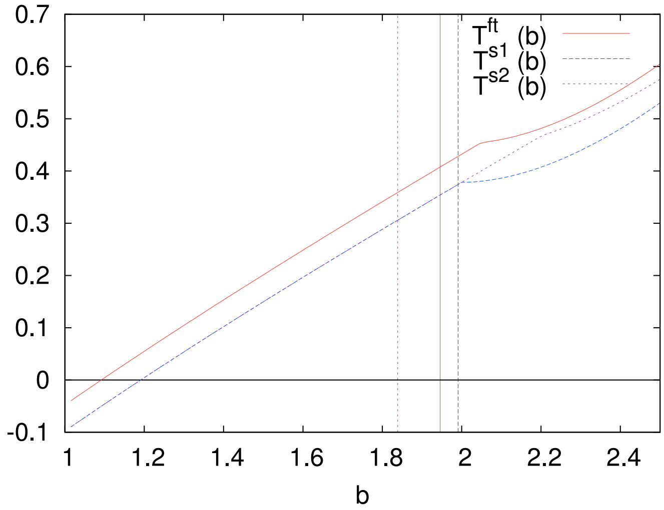


Figure 11: Function $T$ for a flat tax system ($T^{\mathrm{ft}}$), and for a two-level tax system in the first scenario ($T^{\mathrm{s1}}$) and in the second scenario ($T^{\mathrm{s2}}$). The parameter values are those that are given in the text. The vertical lines indicate the resulting debt levels under the simple majority rule in a flat tax system (solid red line) and in a two-level system (dotted blue and violet lines).

assume that the non-taxed individuals are those with the highest valuation of public-good consumption anyway (i. e. individuals with $0 < 1/\theta < 0.3$ are exempted from taxation), so that the median type remains unchanged. We illustrate this in the upper two diagrams of Figure 10. As a *second scenario*, at the end of this section, we will discuss the effects of a changing median type. This, for instance, occurs if the agents exempted from taxation are spread uniformly over all types, as depicted in the two lower diagrams of Figure 10.

*First scenario:* Our assumptions on the parameter values imply that due to the introduction of the two-level tax system the taxed wage decreases from $w = 1.0$ to $w = 0.95$, while all other parameter values, including $\theta^* = 1$, remain unaltered. The decrease of $w$ leads to a downward shift of the function $T$, which can easily be seen from the derivative

$$\frac{\mathrm{d}}{\mathrm{d}w}T(b) = -\beta \cdot \underbrace{\frac{\log h(a) - \log h(b)}{\big(\log(w+b-Ra) - \log(w+a-Ra)\big)^2}}_{>0} \cdot \underbrace{\left(\frac{1}{w+b-Ra} - \frac{1}{w+a-Ra}\right)}_{<0}$$
$$> 0.$$

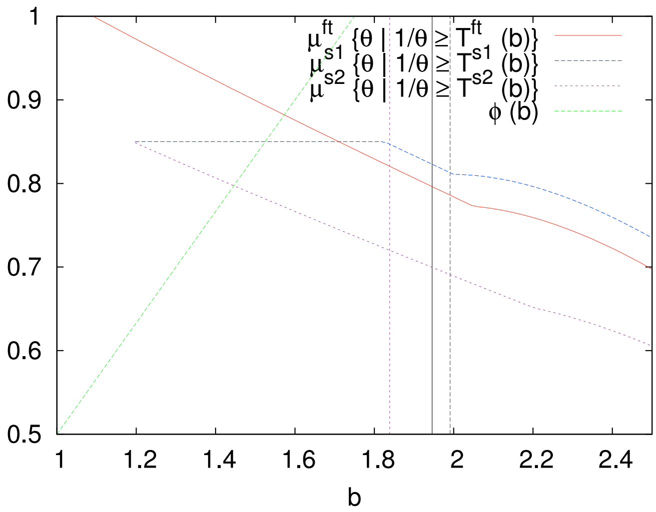


Figure 12: Shares of agents accepting a debt level of $b$, given by $\mu\{\theta \mid 1/\theta \geq T(b)\}$ for a flat tax system and for the two scenarios of a two-level tax system, as well as a debt-sensitive majority rule $\phi(b)$. As in Figure 11, the vertical lines indicate the resulting debt levels under the simple majority rule.

Intuitively, if the tax base is lower, the tax revenue will be smaller for any given tax level; hence *taxed* agents will consider taxation a less attractive instrument and favor debt-making. We see from Equation (18) that, as long as we have an interior solution involving a strictly positive tax rate, $b^*$ is shifted to the right. For our example, the downward shift of the $T$ curve and the change of $b^*$ are illustrated by Figure 11 (shift from the red curve to the blue curve). Compared to Example 3, a two-level tax system makes the equilibrium variables change to

$$b^* = 1.99, \qquad \tau^* = 0.005, \qquad g^* = 0.945.$$

Figure 12 shows how the shares of agents accepting a debt level of $b$, represented by $\mu\{\theta \mid T(b) \geq 1/\theta\}$, changes under a two-level tax system. The non-taxed agents, who comprise 15 % of the society, are the ones most opposed to debt-making. This is reflected by the fact that the blue curve is at the value of 0.85 for debt levels between 1.2 and about 1.8. The first kink of the blue line indicates the debt level at which further agents begin to oppose. In our example the green curve hits the blue curve before the red curve. Hence the debt-sensitive majority rule is more effective in limiting debt accumulation under a two-level tax scheme than under a flat tax system. One can also construct numerical examples with the opposite conclusion, in which taxation designed to achieve a particular level of revenues becomes sufficiently unfavorable for taxed individuals. Graphically, this would mean that the downward shift of the $T$ curve would dominate.

*Second scenario:* So far, we have assumed that the median type remains unaltered by the exemption of agents from taxation. In general, the median type $\theta^*$ tends to shift upwards. For an illustration, see the lower two diagrams of Figure 10 and the violet curves in Figures 11 and 12. Given an interior optimum with $\tau^* > 0$, an upward shift of the median type will cause $b^*$ to decrease, as we saw at the end of Section 3.1. Hence, by the upward shift of the median type, the policy will be more disciplined in a two-level tax system than in a flat tax system even if a simple majority rule is applied. This may reduce the *relative* advantage of a debt-sensitive majority rule over a simple majority rule.

Overall, we can say that the strength of a debt-sensitive rule in moderating debt accumulation tends to become more pronounced, the larger the untaxed fraction of a society is, because then high debt levels are opposed by strong minorities.

# 9. Conclusion

Using a simple model of public-good provision, we have examined the effectiveness of debt-sensitive majority rules in restricting the excessive accumulation of public debts. The analysis has left out many issues that deserve further scrutiny in future research. In Section 8, we referred to a variety of useful extensions of our model that strengthen the effectiveness of debt-sensitive majority rules. Moreover, it will be useful to calibrate debt-sensitive majority rules to business-cycle movements and to develop debt-sensitive majority rules for practical applications. Such rules may involve three or four critical thresholds at which the required majority increases. An extension of our model to an infinite (or longer-term) horizon will be worth pursuing, as this will allow comparison of the long-term debt paths under the two voting rules.

# A. Proofs

**Proof of Proposition 1.** Consider any $\theta > 0$. The convexity of $\mathcal{P}^{\mathrm{f}}$ is obvious. On $\mathcal{P}^{\mathrm{f}}$, the function $V_\theta$ is twice continuously differentiable. The first- and second-order derivatives are given by the Equations (9) to (13) in the main text.

The assumption $(\log h)'' < 0$ from Equation (5) implies

$$\partial_\tau \partial_\tau V_\theta < 0$$

and

$$\det \operatorname{Hess} V_\theta = \frac{\theta}{(1-\tau)^2 g^2} - \beta\theta\Big(\frac{w^2\theta}{g^2} + \frac{1}{(1-\tau)^2}\Big) \cdot (\log h)''(Ra + g - \tau w) > 0;$$

hence the Hessian

$$\operatorname{Hess} V_\theta = \begin{pmatrix} \partial_\tau \partial_\tau V_\theta & \partial_g \partial_\tau V_\theta \\ \partial_g \partial_\tau V_\theta & \partial_g \partial_g V_\theta \end{pmatrix}$$

is negative definite. Therefore, function $V_\theta$ is strictly concave on $\mathcal{P}^{\mathrm{f}}$.

Consider any closed and convex set $A \subseteq \mathcal{P}$ with $\mathcal{P}^{\mathrm{f}} \cap A \neq \emptyset$. For $c \in \mathbb{R}$, define $G_c := \big\{(\tau, g) \in A \bigm| V_\theta(\tau, g) \geq c\big\}$. Some $c$ exists such that the set $G_c$ is non-empty. Any point of maximum of $V_\theta$ on $A$ lies within $G_c$. Since $V_\theta$ is continuous and $A$ is closed, $G_c$ is closed. In addition, since $\max_{\tau \in [0;1]} V_\theta(\tau, g) \to -\infty$ for both $g \to 0$ and $g \to \infty$, $G_c$ is bounded, hence it is compact. Therefore the restriction of $V_\theta$ to $A$ attains a maximum on $G_c$, which by the definition of $G_c$, is a global maximum of the restriction of $V_\theta$ to $A$.

Any point of maximum of the restriction of $V_\theta$ to $A$ must lie in $\mathcal{P}^{\mathrm{f}} \cap A$. The convexity of $\mathcal{P}^{\mathrm{f}} \cap A$ and the strict concavity of $V_\theta$ on $\mathcal{P}^{\mathrm{f}} \cap A$ imply that the point of maximum is unique. $\square$

**Proof of Proposition 2.** By differentiating Equation (18) totally, we obtain

$$\mathrm{d}b^0 = \frac{1}{\theta^2 \beta (\log h)'(b^0)} \mathrm{d}\theta + \Big(1 + \frac{1}{\theta}\Big) \frac{(\log h)''(b^0)}{\beta\big((\log h)'(b^0)\big)^2} \mathrm{d}b^0.$$

As $(\log h)'$ is negative and monotonically decreasing, we obtain

$$\frac{\mathrm{d}b^0}{\mathrm{d}\theta} < 0.$$

□

**Proof of Lemma 1.** Solving the budget constraint (3) with respect to $g$ and inserting into (7) yields

$$V_\theta(\tau, b - Ra + \tau w) = \log(1-\tau) + \theta \log(b - Ra + \tau w) + \beta\theta \log h(b). \tag{49}$$

The derivative with respect to $\tau$ reads

$$\frac{\mathrm{d}}{\mathrm{d}\tau} V_\theta(\tau, b - Ra + \tau w) = -\frac{1}{1-\tau} + \theta \frac{w}{b - Ra + \tau w}\ . \tag{50}$$

This expression is decreasing in $\tau$. Hence the expression in Equation (49), as a function of $\tau$, reaches its maximum in $\tau = 0$ if and only if

$$\frac{\mathrm{d}}{\mathrm{d}\tau} V_\theta(\tau, b - Ra + \tau w)\Big|_{\tau=0} \leq 0.$$

An interior optimum is obtained when (50) equals zero for some $\tau > 0$. Solving for $\tau$ yields

$$\tau = \frac{\theta w - b + Ra}{w(1+\theta)}. \tag{51}$$

A corner solution at $\tau = 0$ occurs if and only if

$$\begin{aligned} & \frac{\mathrm{d}}{\mathrm{d}\tau} V_\theta(\tau, b - Ra + \tau w)\Big|_{\tau=0} = -\frac{1}{1-\tau} + \theta \frac{w}{b - Ra + \tau w}\Big|_{\tau=0} \leq 0, \\ \Leftrightarrow \quad & \theta \frac{w}{b - Ra} \leq 1, \\ \Leftrightarrow \quad & \theta w \leq b - Ra. \end{aligned}$$

Therefore we obtain a corner solution at $\tau = 0$ if and only if the right-hand side of Equation (51) takes a non-positive value. □

**Proof of Proposition 3.** Due to the first-order condition, a local optimum for $b$ for the function

$$f_1\colon (Ra - w; \bar{b}) \to \mathbb{R}, \quad b \mapsto C + (1+\theta)\log(w + b - Ra) + \theta\beta \log h(b), \tag{52}$$

which corresponds to the first case of (24), is given by the solution of Equation (25). As the left-hand side of this equation is monotonically decreasing in $b$ with $(1+\theta)/(w+b-Ra) \to +\infty$ for $b \downarrow Ra - w$, and the right-hand side is monotonically increasing in $b$ and converging to $+\infty$ for $b \uparrow \bar{b}$, a unique solution exists. Since (52) is strictly concave, the solution is a maximizer. A similar argument holds for the second case of (24), i. e. the function

$$f_2\colon (Ra;\bar{b}) \to \mathbb{R}, \quad b \mapsto \theta \log(b - Ra) + \theta\beta \log h(b),$$

and the solution of Equation (26).

Inspecting (24), we observe that $b \mapsto V_\theta\big(\tau^0_{=b}(\theta), g^0_{=b}(\theta)\big)$ is continuously differentiable for $b \in (Ra - w; \bar{b})$ and strictly concave. It attains its maximum at the solution of Equation (25) if and only if the derivative of (52) is negative (or zero) at $b = Ra + \theta w$, i. e. if and only if

$$\begin{aligned}
& \frac{\mathrm{d}}{\mathrm{d}b}\big((1+\theta)\log(w+b-Ra) + \theta\beta \log h(b)\big)\Big|_{b=Ra+\theta w} \leq 0 \\
\Leftrightarrow \quad & \frac{1+\theta}{w+b-Ra} + \theta\beta(\log h)'(b)\Big|_{b=Ra+\theta w} \leq 0 \\
\Leftrightarrow \quad & -(\log h)'(Ra+\theta w) \geq \frac{1}{\beta\theta w}.
\end{aligned}$$

Similarly, the function attains its maximum at the solution of Equation (26) if and only if the derivative is positive (or zero). Therefore the point of maximum is given by $b^E(\theta)$.

We have to show that $b^E(\theta)$ is the minimum of the maximizer of $f_1$ and the maximizer of $f_2$. The derivatives $f_1'$ and $f_2'$ are continuous and strictly decreasing. As we have seen above, there is exactly one point of intersection, at $Ra + \theta w$. Calculation shows that $f_1''(Ra + \theta w) < f_2''(Ra + \theta w)$. Hence, the root of $f_2'$ is left of the root of $f_1'$ if $f_1'(Ra + \theta w) < 0$ and right of the root of $f_1'$ if $f_1'(Ra + \theta w) > 0$.

By concavity, the function $b \mapsto V_\theta\big(\tau^0_{=b}(\theta), g^0_{=b}(\theta)\big)$ is increasing left of the maximum; hence

$$\begin{aligned}
b^0_{\leq\tilde{b}}(\theta) &= \operatorname*{arg\,max}_{b \leq \tilde{b}} V_\theta\big(\tau^0_{=b}(\theta), g^0_{=b}(\theta)\big) \\
&= \begin{cases} \tilde{b} & \text{if } \tilde{b} \leq b^E(\theta), \\ b^E(\theta) & \text{if } \tilde{b} > b^E(\theta) \end{cases} \\
&= \min\big\{\tilde{b}, b^E(\theta)\big\},
\end{aligned}$$

which is (27). By inserting this into (20) and (21), we obtain (28) and (29). $\square$

**Proof of Proposition 4.** For $\tilde{b} \le Ra + \theta w$, the first term on the right-hand side of Equation (28) is non-negative. Hence, we can neglect the third term (the zero) in this case, and we obtain

$$\tau^0_{\le \tilde{b}}(\theta) = \frac{1}{w(1+\theta)} \cdot \big(\theta w - b^0_{\le \tilde{b}}(\theta) + Ra\big).$$

Rearranging yields

$$b^0_{\le \tilde{b}}(\theta) = \theta w + Ra - (1+\theta) w \tau^0_{\le \tilde{b}}(\theta).$$

Using (29), we obtain

$$g^0_{\le \tilde{b}}(\theta) = \theta w - \tau^0_{\le \tilde{b}}(\theta)\theta w.$$

From this equation we obtain (30). The budget constraint (3) then yields Equation (31). $\square$

**Proof of Corollary 2.** If $\tau = \tau'$, $M = \emptyset$ or $M = (0;\infty)$ by Proposition 5; hence, there is nothing to prove. We consider the case $\tau > \tau'$; the case $\tau < \tau'$ is similar.

Since $\theta^* \in M$, we have $|M| \ge 1/2$. Assume that $|M| = 1/2$. By Proposition 5, $\tilde{\theta}$ exists such that $M = (0;\tilde{\theta})$. Since $\mu(M) = 1/2$, $\mu\big([\tilde{\theta};\infty)\big) = 1/2$. Hence $\tilde{\theta}$ is a median. Since the median is unique, we have $\theta^* = \tilde{\theta}$. It follows that $\theta^* \notin M$. Hence $\theta^* \in M$ implies $|M| > 1/2$. $\square$

**Proof of Corollary 5.** Consider any $(\tau', g') \in A$ with $(\tau', g') \ne (\tau, g)$. Since $(\tau', g')$ is a Condorcet winner within $A$, we have

$$\big|(\tau', g') \succ_\theta (\tau, g)\big| \le \frac{1}{2}\,.$$

To obtain a contradiction, we assume that

$$\big|(\tau', g') \succ_\theta (\tau, g)\big| = \frac{1}{2}\,. \tag{53}$$

By Proposition 5, we know that $\big\{(\tau', g') \succ_\theta (\tau, g)\big\}$ takes the form $(0;\tilde{\theta})$ or $(\tilde{\theta};\infty)$. By Equation (53), $\tilde{\theta}$ is a median, and by the uniqueness of the median, $\tilde{\theta} = \theta^*$. Hence, by Remark 1, the median type $\theta^*$ is indifferent between the two proposals $(\tau, g)$ and $(\tau', g')$. By Corollary 3, optimal policies of the median type and Condorcet winners are the same. Since the Condorcet winner was assumed to be unique, we obtain a contradiction. $\square$

**Proof of Proposition 7.** We proceed as in the proof of Proposition 6. Since voting is now two-staged, the arguments are more involved.

Let $(\tau^\phi, g^\phi)$ be an equilibrium in the sense of Definition 6, and let $b^\phi = B(\tau^\phi, g^\phi)$ be the associated debt level. We show that a subgame-perfect strong Nash equilibrium with the outcome $(\tau^\phi, g^\phi)$ exists (i. e. the "only if" part of Proposition 7).

Consider the following strategy profile: Everybody proposes $(\tau^\phi, g^\phi)$; the proposal is accepted unanimously in both stages. This is a subgame-perfect strong Nash equilibrium, which we clarify as follows: No proposal $(\tau, g)$ with $B(\tau, g) > b^\phi$ will survive stage 2. Since $(\tau^\phi, g^\phi)$ is the unique Condorcet winner within the set of policies $\mathcal{P}_{\leq b^\phi}$, Corollary 5 yields

$$\big|(\tau, g) \succ_\theta (\tau^\phi, g^\phi)\big| < \frac{1}{2} \quad \text{for any proposal } (\tau, g) \text{ with } B(\tau, g) \leq b^\phi.$$

Hence, only a minority of agents would contemplate pursuing a strategy leading to an outcome different from $(\tau^\phi, g^\phi)$, but such a minority would not have the power to alter the outcome. Consequently, no coalition has an incentive to deviate from the above-mentioned strategy profile.

It remains to be shown that the outcome of any subgame-perfect strong Nash equilibrium is an equilibrium in the sense of Definition 6 (i. e. the "if" part of Proposition 7).

Let $(\tau, g)$ be the outcome of a subgame-perfect strong Nash equilibrium. Then $(\tau, g)$ is a status quo policy or a policy with $B(\tau, g) > a$ that survives the two stages of the voting game. In all cases, $(\tau, g)$ is the unique Condorcet winner within $\mathcal{P}_{\leq B(\tau,g)}$, and $\alpha(\tau, g) \geq \phi\big(B(\tau, g)\big)$. Hence $(\tau, g)$ is a semi-equilibrium.

Suppose that another semi-equilibrium $(\tau', g')$ with $B(\tau', g') > B(\tau, g)$ exists. As $(\tau', g')$ is the unique Condorcet winner within $\mathcal{P}_{\leq B(\tau',g')}$ and $(\tau, g)$ is contained in the set, $(\tau', g')$ will be strictly preferred to $(\tau, g)$ by the median voter, due to Corollary 3. Hence, by Corollary 2,

$$\big|(\tau', g') \succ_\theta (\tau, g)\big| > \frac{1}{2}.$$

It follows that a majority of agents has an incentive to deviate by proposing and voting for $(\tau', g')$, so $(\tau, g)$ would not be the outcome of a subgame-perfect strong Nash equilibrium. This is a contradiction, therefore $(\tau, g)$ is an equilibrium.

Again, as we argued after the proof of Proposition 6, a unanimous voting outcome is unlikely to occur in reality, but it is sufficient for the purpose of the proof. $\square$

**Proof of Theorem 2.** We first prove uniqueness. Consider two equilibria $(\tau, g)$ and $(\tau', g')$. By definition of equilibrium, $B(\tau, g) = B(\tau', g')$. Since by Corollary 4, the Condorcet winner in $\mathcal{P}_{\leq B(\tau,g)} = \mathcal{P}_{\leq B(\tau',g')}$ is unique, $(\tau, g) = (\tau', g')$.

Now we turn to existence. From Corollary 4 we know that for any $b$, a unique Condorcet winner in $\mathcal{P}_{\leq b}$ exists. It is given by the proposal $\left(\tau^*_{\leq b}, g^*_{\leq b}\right)$, which the median voter considers to be optimal among all proposals in the set $\mathcal{P}_{\leq b}$. Let

$$\mathcal{P}^* := \left\{ \left(\tau^*_{\leq b}, g^*_{\leq b}\right) \,\middle|\, Ra - w < b \leq \bar{b} \right\}$$

be the set of all such proposals, and let

$$\mathcal{S} := \Big\{ (\tau, g) \in \mathcal{P}^* : \alpha(\tau, g) \geq \phi\big(B(\tau, g)\big) \Big\}.$$

By construction, $\mathcal{S}$ is the set of all semi-equilibria. Since by Equation (33) $\alpha\left(\tau^*_{\leq a}, g^*_{\leq a}\right) = 1$, the policy $\left(\tau^*_{\leq a}, g^*_{\leq a}\right)$ is contained in $\mathcal{S}$; hence $\mathcal{S} \neq \emptyset$. Let

$$\hat{b} := \sup \left\{ B(\tau, g) \,\middle|\, (\tau, g) \in \mathcal{S} \right\},$$

and define $\hat{\tau} := \tau^*_{\leq \hat{b}}$, $\hat{g} := g^*_{\leq \hat{b}}$.

We claim that $\left(\hat{\tau}, \hat{g}\right)$ is an equilibrium under $\phi$. To verify this, we only need show that $\left(\hat{\tau}, \hat{g}\right) \in \mathcal{S}$, i.e. that it is a semi-equilibrium. Assertion (i) is clearly fulfilled. In order to prove (ii), consider a sequence of policies $(\tau_n, g_n) \in \mathcal{S}$ with $B(\tau_n, g_n) \to \hat{b}$. Since $\phi$ is assumed to be left-continuous and weakly increasing, it is lower semi-continuous[6]. As $B(\cdot)$, $b \mapsto \tau^*_{\leq b}$ and $b \mapsto g^*_{\leq b}$ are continuous functions, the function

$$b \mapsto \phi\Big(B\big(\tau^*_{\leq b}, g^*_{\leq b}\big)\Big)$$

were lower semi-continuous as well. If we already knew that the function

$$b \mapsto \alpha\big(\tau^*_{\leq b}, g^*_{\leq b}\big) \tag{54}$$

[6] A function $f$ is called lower semi-continuous if for all $x$ and all sequences $(x_n)$ with $x_n \to x$, $\liminf_{n\to\infty} f(x_n) \geq f(x)$. A function $f$ is called upper semi-continuous if for all $x$ and all sequences $(x_n)$ with $x_n \to x$, $\limsup_{n\to\infty} f(x_n) \leq f(x)$.

is upper semi-continuous, we could conclude that

$$\begin{aligned}\alpha\big(\hat{\tau},\hat{g}\big) &= \alpha\big(\tau^*_{\le\hat{b}}, g^*_{\le\hat{b}}\big)\\ &\ge \lim_{n\to\infty}\alpha\big(\tau^*_{\le b_n}, g^*_{\le b_n}\big)\\ &\ge \lim_{n\to\infty}\phi\Big(B\big(\tau^*_{\le b_n}, g^*_{\le b_n}\big)\Big)\\ &\ge \phi\Big(B\big(\tau^*_{\le\hat{b}}, g^*_{\le\hat{b}}\big)\Big)\\ &= \phi\big(B(\hat{\tau},\hat{g})\big),\end{aligned}$$

and the proof of the theorem would be complete.

We are left with the task of proving that the function given in (54) is upper semi-continuous. For any $b$, let

$$M(b) := \Big\{\theta \;\Big|\; V_\theta\big(\tau^*_{\le b}, g^*_{\le b}\big) - V_\theta\big(\tau^*_{\le a}, g^*_{\le a}\big) < 0\Big\}$$

be the set of types that strictly prefer the status quo over policy $(\tau^*_{\le b}, g^*_{\le b})$. Then

$$\alpha\big(\tau^*_{\le b}, g^*_{\le b}\big) = 1 - \big|M(b)\big|. \tag{55}$$

Consider any $b_0$ and a sequence $(b_n)$ with $b_n \to b_0$. As for each $\theta$ the function

$$b \mapsto V_\theta\big(\tau^*_{\le b}, g^*_{\le b}\big) - V_\theta\big(\tau^*_{\le a}, g^*_{\le a}\big)$$

is continuous, we have

$$\begin{aligned}M(b_0) &= \Big\{\theta \;\Big|\; V_\theta\big(\tau^*_{\le b_0}, g^*_{\le b_0}\big) - V_\theta\big(\tau^*_{\le a}, g^*_{\le a}\big) < 0\Big\}\\ &\subseteq \Big\{\theta \;\Big|\; \exists n\ \forall m \ge n \;\Big|\; V_\theta\big(\tau^*_{\le b_m}, g^*_{\le b_m}\big) - V_\theta\big(\tau^*_{\le a}, g^*_{\le a}\big) < 0\Big\}\\ &= \bigcup_{n=1}^{\infty}\bigcap_{m=n}^{\infty} M(b_m).\end{aligned}\tag{56}$$

This means that if a type $\theta$ prefers the status quo $\big(\tau^*_{\le a}, g^*_{\le a}\big)$ to $\big(\tau^*_{\le b_0}, g^*_{\le b_0}\big)$, he will prefer the status quo to the approximating proposals $\big(\tau^*_{\le b_m}, g^*_{\le b_m}\big)$ for all $m$ that are sufficiently large.

Since $\mu$ is a probability measure, it is $\sigma$-additive and hence $\sigma$-continuous, which means the following: If the $A_n$ form a sequence of sets with $A_1 \subseteq A_2 \subseteq A_3 \subseteq \dots$ and $A :=$

$\bigcup_{n=1}^{\infty} A_n$, then

$$\mu(A) = \lim_{n\to\infty} \mu(A_n).$$

Equation (56) shows that by choosing

$$A_n := \bigcap_{m=n}^{\infty} M(b_m),$$

we obtain

$$\big|M(b_0)\big| \leq \left|\bigcup_{n=1}^{\infty}\bigcap_{m=n}^{\infty} M(b_m)\right| = \lim_{n\to\infty}\left|\bigcap_{m=n}^{\infty} M(b_m)\right|.$$

Using the fact that $\bigcap_{m=n}^{\infty} M(b_m) \subseteq M(b_n)$ for all $n$, we arrive at

$$\big|M(b_0)\big| \leq \lim_{n\to\infty}\big|M(b_n)\big|.$$

With (55) this yields

$$\alpha\big(\tau^*_{\leq b_0}, g^*_{\leq b_0}\big) \geq \lim_{n\to\infty} \alpha\big(\tau^*_{\leq b_n}, g^*_{\leq b_n}\big).$$

Since $b_0$ and the sequence $b_n$ are arbitrary, this shows the upper semi-continuity of $b \mapsto \alpha\big(\tau^*_{\leq b}, g^*_{\leq b}\big)$. □

**Proof of Proposition 8.** Part (i) follows from the definition of $T$ and Equations (20) and (21).

Part (ii): To prove that $T(b)$ is strictly increasing in $b$ for $a < b \leq Ra + \theta^* w$, observe that its derivative with respect to $b$ is given by

$$T'(b) = \beta \cdot \big(\log(w + b - Ra) - \log(w + a - Ra)\big)^{-2} \cdot z(b),$$

where

$$z(b) := -(\log h)'(b) \cdot \big(\log(w + b - Ra) - \log(w + a - Ra)\big) - \frac{\log h(a) - \log h(b)}{w + b - Ra}.$$

It is sufficient to show that $z(b) > 0$ for $b > a$. This is immediate. Since

$$\begin{aligned} z'(b) = &- (\log h)''(b) \cdot \big(\log(w + b - Ra) - \log(w + a - Ra)\big) \\ &- (\log h)'(b) \cdot \frac{1}{w + b - Ra} + (\log h)'(b) \cdot \frac{1}{w + b - Ra} \\ &+ \big(\log h(a) - \log h(b)\big) \cdot \frac{1}{(w + b - Ra)^2} \\ &> 0, \end{aligned}$$

we obtain $z(b) > z(a) = 0$ for $b > a$.

Part (iii): An agent of type $\theta$ will prefer $(\tau^*_{\le b}, g^*_{\le b})$ to $(\tau^*_{\le a}, g^*_{\le a})$ if and only if the inequality

$$V_\theta(\tau^*_{\le b}, g^*_{\le b}) - V_\theta(\tau^*_{\le a}, g^*_{\le a}) \ge 0 \tag{57}$$

is satisfied. By Equation (7),

$$V_\theta(\tau^*_{\le b}, g^*_{\le b}) = \log(1 - \tau^*_{\le b}) + \theta \log g^*_{\le b} + \beta\theta \log h(b^*_{\le b}).$$

Thus (57) is equivalent to

$$0 \le \log(1 - \tau^*_{\le b}) - \log(1 - \tau^*_{\le a}) + \theta\big(\log g^*_{\le b} - \log g^*_{\le a}\big) + \beta\theta\Big(\log h\big(b^*_{\le b}\big) - \log h\big(b^*_{\le a}\big)\Big). \tag{58}$$

For $b \le b^*$, we have

$$b^*_{\le b} = b, \quad \tau^*_{\le b} = \tau^*_{=b}, \quad g^*_{\le b} = g^*_{=b}$$

and similarly for $a \le b^*$

$$b^*_{\le a} = a, \quad \tau^*_{\le a} = \tau^*_{=a}, \quad g^*_{\le a} = g^*_{=a}.$$

From this, we observe that for $a < b \le b^*$, Condition (58) is equivalent to

$$0 \le \log(1 - \tau^*_{=b}) - \log(1 - \tau^*_{=a}) + \theta\big(\log g^*_{=b} - \log g^*_{=a}\big) + \beta\theta\Big(\log h\big(b^*_{=b}\big) - \log h\big(b^*_{=a}\big)\Big).$$

Solving for $1/\theta$ yields

$$\frac{1}{\theta} \ge T(b).$$

□

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