ISSN 0265-8003

DETRENDING AND BUSINESS CYCLE FACTS

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Discussion Paper No. 782
June 1993

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ABSTRACT

Detrending and Business Cycle Facts*

This paper examines the business cycle properties of a small set of real US macroeconomic time series using a variety of detrending methods. It is shown: (i) that both quantitatively and qualitatively 'stylized facts' of US business cycles vary widely across detrending methods; (ii) that alternative detrending filters extract different types of information from the original series; and (iii) the qualitative response of consumption, investment, hours and productivity to a typical shock in GNP have, depending on the method used, two types of patterns – one consistent with a Real Business Cycle model and one consistent with a Neo-Keynesian labour-hoarding story. Implications and suggestions for current macroeconomic practice are also presented.

JEL classification: B41, E32
Keywords: business cycles, stylized facts, filters, labour hoarding, technology shocks, detrending

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*I would like to thank Manuel Aurellano, Keith Blackburn, Tony Braun, Javier Diaz, Frank Diebold, Jordi Galí, Roger Kormendi, Eric Ghysels, Albert Jaeger, Albert Marcet, Jane Marrinan, Rick Mishkin, Franco Peracchi, Glen Rudebush, Guido Tabellini, Arnold Zellner and the participants at seminars at Brown, Columbia, NYU, Banco de España, Universitat Pompeu Fabra, University of Southampton, IGIER, WIFO, the 1990 Winter Meeting of the Econometric Society, Washington, the 1991 Meetings of the Society for Economic Dynamics and Control, Capri, and the 1991 European Meetings of the Econometric Society, Cambridge, for useful comments and suggestions.

Submitted 5 March 1993

NON-TECHNICAL SUMMARY

It has become increasingly popular in the applied macroeconomic literature to characterize the behaviour of macroeconomic variables over the business cycle using a set of uncontroversial summary statistics. The compilation of the so-called 'stylized facts' of the business cycle is important for two reasons. First, it gives a broad summary of the complex comovements existing among aggregates in the economy, allows a rough calculation of the magnitude of the fluctuations in economic variables and may guide researchers in choosing leading indicators for economic activity. Second, it provides a set of 'regularities' which can be used to examine the quantitative validity of theoretical models.

Any empirical examination of the business cycle, however, involves the delicate and controversial issue of detrending. This raises two problems. The first concerns the lack of a professional consensus on what constitutes business cycle fluctuations. The second concerns the use of an approach based on statistics versus one based on economics in analysing business cycles.

Consider first the issue of defining business cycles. Cyclical fluctuations are typically identified with deviations from the long-run (secular) path of the process. Within the empirical literature, however, there is fundamental disagreement on the properties of this secular component of macro variables and on its relationship with the cyclical component. Different detrending methods therefore embed different concepts of business cycle fluctuations.

In the past the representation and extraction of the secular component of macroeconomic variables was treated in a very straightforward fashion. The secular trend was represented with deterministic polynomial functions of time, assumed to be independent of the cyclical component and extracted using simple regression methods. More recently, macroeconomists have tended to represent long-run components using stochastic trends (unit root processes) which are either uncorrelated with the cyclical component of the series or perfectly correlated. In the last few years, however, this last idea has been seriously questioned and many now believe that the secular component of many macroeconomic time series is better represented by a deterministic function of time having infrequent but large breaks.

Since the issue of what is an 'appropriate' representation of the long-run features of macroeconomic variables cannot be solved with the samples of data which are currently available (typically 40–50 years), and since the choice of the relationship between the cyclical and secular components is arbitrary, statistically-based approaches to detrending raise questions about the robustness of certain reported 'stylized facts'.

The second problem connected with detrending – the question of a statistical versus an economic approach – arises from a well known concern about 'measurement without theory'. It is often argued that before variables can be selected and facts reported, a theory explaining the mechanism generating economic fluctuations is needed. This point of view has been advocated by researchers of two different camps: (i) those who use economic theory to pin down a decomposition of the actual time series into a long-run component and cyclical variations (i.e. economic models explicitly tell the researcher how to represent the trend and how to compute the cyclical component); and (ii) those who employ economic theory as an organizing principle for time-series analysis but use arbitrary procedures, which reflect the preferences of the researcher and the question to be investigated, to extract long-run components and to establish business cycle facts.

Dynamic economic theory, however, very rarely indicates the type of economic trend that aggregate macro variables may display and the relationship between secular and cyclical fluctuations. In other words, without a set of statistical facts which determine the properties of the secular component of the data, the precise economic relationship between the cyclical and secular component of variables is unknown, and the choice among different economic-based decompositions is arbitrary. Because of this circularity, all decompositions based on economic theory are, at best, attempts to approximate unknown features of macroeconomic time series and therefore subject to specification errors. This issue is relevant because there has been surprisingly little discussion in the literature on whether certain economic assumptions provide an appropriate characterization of actual business cycles or whether instead, they ignore important sources of fluctuations.

This paper focuses on two aspects of the problems just outlined: the robustness of business cycle features to different detrending methods, and the implication of the results for four questions which have received much attention in the literature – the behaviour of productivity over the business cycle, the presence of labour hoarding, the Dunlop-Tarshis puzzle, and the sources of cyclical fluctuations.

The approach of this paper is agnostic. Dynamic theory of economic fluctuations is used only to select the variables of interest for this study. None of the detrending procedures employed is believed to be the correct one, and all procedures are regarded as approximations which isolate aspects of the secular and cyclical components of the series. In this sense, different detrending methods are alternative windows which look at macroeconomic variables from different perspectives. The idea is to organize the information on business cycle fluctuations in a systematic manner in an attempt to identify whether there exists a set of relationships which is independent of the exact definition of cyclical fluctuations, and shed light on certain 'data anomalies' which have motivated recent developments in the theoretical literature.

I concentrate on a small set of real variables to maintain comparability with existing real business cycle (RBC) literature, but the type of problems encountered here are not unique to this literature and concern all approaches which use 'stylized facts' as qualitative or quantitative benchmarks to compare the properties of theoretical models.

The paper documents that the use of different detrending procedures makes an important difference to the variability, the serial- and cross-correlation properties of the estimated cyclical components of the seven series used (GNP, Consumption, Investment, Hours, Real Wage, Productivity and Capital Stock) vary widely across detrending procedures, but that only minor differences emerge in higher moments. I show that different detrending methods extract different types of information and that, even among the class of methods which extract cycles of similar length, significant differences may emerge. In addition, I argue that the qualitative responses of consumption, investment, hours and real wage to a shock in GNP exhibit two typical patterns; one that is consistent with the idea that business cycles are driven by technology shocks and one consistent with demand-driven cycles.

The first pattern fits an RBC tale: a temporary shock to output increases labour demand, so that hours and the real wage go up within a year's time. As the real wage increases, consumption increases and investment follows. Since the average productivity increases more that the real wage, profits increase and payments to holders of capital rise as well. Therefore the real return per unit of capital invested increases. This increase is correlated with the increase in hours. Hence hours move together with this measure of the real rate of return, a result which is consistent with the RBC emphasis on intertemporal substitution of labour. In addition, the responses of productivity are approximately coincident with the responses of GNP, a result which goes against the labour-hoarding explanation of business cycle fluctuations.

The second pattern of responses, on the other hand, fits the neo-Keynesian perspective better. A one standard error shock in GNP instantaneously increases consumption by about 1.2 times that amount, and because of wealth effects, decreases the amount of hours worked. To achieve this consumption increase, the economy depletes the capital stock and invests negative amounts. At least in the first phase of the cycle, the response of the average productivity of labour is negatively related to (and lags) output responses, a pattern which fits the labour-hoarding story. The demand driven expansion caused by the increase in consumption induces a further increase in output in the short run, possibly through the use of idle capacity or overtime and this drives hours and real wages up. When the consumption boom is exhausted, previous decisions are reverted: agents enjoy increasing amounts of leisure pushing hours below their long-run path in the medium run, investments decrease and the deterioration of the capital stock is reversed. The reconstruction of the capital stock is completed in about eight quarters and convergence to its steady-state path occurs after about fifteen quarters. Finally, because the capital stock is countercyclical, the real interest rate is large and positive in the first few quarters of the cycle. Despite large interest rates and real wage movements, hours move, relatively speaking, by only a small amount, a result which agrees with recent neo-Keynesian descriptions of the business cycle. While one need not agree with the exact details of the stories provided here, it is clear that there are two charaterizations of the transmission of GNP shocks which are consistent with contrasting theories of business cycles fluctuations.

Finally, I demonstrate that in some situations, e.g. in examining whether labour hoarding occurs, economic theory does suggest a class of methods which should be used to uncover the correct relationship. A few conclusions can be drawn from the exercise. First, the practice of solely employing a particular version of the 'Hodrick Prescott' filter in compiling business cycle statistics is dangerous. This filter produces results which are similar to those obtained with conventional band-pass filters (e.g. frequency domain masking the low frequency components of the data) and concentrates the attention of the researcher on cycles of average length of four to six years. There are some instances, however, *viz.* the productivity series, where a high proportion of the variability appears to be connected with cycles of slightly longer length and this high portion of variability is simply neglected in analyses which exclusively use this filter. The choice of the researcher to concentrate on cycles of four to six years length may also induce either extreme second-order properties in the detrended data and misdirect theoretical research trying to cope with them, or inappropriately characterize certain phenomena (e.g. labour hoarding). A more interactive relationship between theory and practice could be very useful here. Theory may indicate which cycles it wants to explain and therefore implicitly select a class of procedures which extract this type of cycles in the data, and empirical practice should indicate whether this theoretical choice leaves out important features of the data or produces distortions of various kind.

Second, since there are no quantitative stylized facts and very few qualitative features of the data which are robust across detrending methods and frequencies, the practice of building theoretical models whose numerical versions quantitatively match business cycle facts warrants a reconsideration. Because the major differences occur around business cycle frequencies, numerical exercises should at least be enlarged to provide results obtained with filters which emphasize different business cycle periodicities and theoretical work should try to explain why certain variables display a differential behaviour within business cycle frequencies, e.g. why productivity is procyclical when cycles of five years are considered, but is almost acyclical for cycles of three-year period.

Third, the results obtained with multivariate detrending methods which have their basis in dynamic economic theory are different from those obtained with statistically-based univariate procedures. Because, at least with the data set used here, there is very weak evidence of common (deterministic or stochastic) trends among the variables considered, caution should be exercised in imposing theoretical restrictions which are far from being satisfied in the data.

1 Introduction

Since the influential work of Hodrick and Prescott (1980) it has become increasingly popular to characterize the behavior of macroeconomic variables over the business cycle using a set of uncontroversial summary statistics (recent examples include Baxter and Stockman (1989), Kydland and Prescott (1990), Stock and Watson (1990) and Backus and Kehoe (1992)). The compilation of stylized facts of the business cycle is important for two reasons. First, it gives a coarse summary of the complex comovements existing among aggregates in the economy, allows a rough calculation of the magnitude of the fluctuations in economic variables and may guide researchers in choosing leading indicators for economic activity. Second, it provides a set of "regularities" which macroeconomists use as a benchmark to examine the validity of numerical versions of theoretical models.

Any empirical examination of the business cycle, however, involves the delicate and controversial issue of detrending. There are two problems connected with detrending. The first concerns the lack of a professional consensus on of what constitutes business fluctuations. The second concerns the use of a statistically–based approach vs. an economic–based approach to detrending.

Consider first the issue of what business cycles are. Business cycle fluctuations are typically identified with deviations from the trend of the process. However, within the empirical literature, there is fundamental disagreement on the properties of the trend and on its relationship with the cyclical component of a series. Therefore different detrending methods embed different concepts of business cycle fluctuations.

In the past the representation and extraction of the secular component was handled in a very simple way. The trend was represented with deterministic polynomial functions of time, assumed to be independent of the cyclical component and extracted using simple regression methods. Nelson and Plosser's (1982) findings turned around the traditional idea that the variance of the secular component is small relative to the variance of the cyclical component. Based on these results, Beveridge and Nelson (1981) produce a decomposition of a series where the secular component is highly variable (exhibits a unit root) and the secular and the cyclical components are perfectly correlated. Perron (1989), however, questions the evidence reported by Nelson and Plosser suggesting that the secular component is better represented by a deterministic function of time having structural breaks (see Christiano (1992) for a criticism of Perron's methodology). Recent work by Harvey

(1985), Watson (1986), Hamilton (1989) and Maravall (1992) propose alternative representations for the components of a time series and suggest different statistical methods for extracting trends.

Since the issue of what is an "appropriate" representation of the trend cannot be solved in small samples and since the choice of the relationship between the cyclical and secular components is arbitrary, statistical based approaches to detrending raise questions about the robustness of certain reported " facts". As Singleton (1988, p.372) observes "The stylized facts motivating recent specifications of the business cycle models may have been distorted by prefiltering procedures".

The second problem connected with detrending - the question of a statistical versus an economic based decomposition - arises from a standard "measurement without theory" concern. It is often argued that before variables can be selected and facts reported, a theory explaining the mechanism generating economic fluctuations is needed. This point of view has been advocated by those who use economic theory to choose an economic-based decomposition of the actual time series in deriving business cycle regularities (see e.g Singleton (1988), King, Plosser and Rebelo (1989) or King, Plosser, Stock and Watson (1991)) and also by those who employ economic theory as an organizing principle for time series analysis but use arbitrary filtering procedures, which reflect the preferences of the researcher and the question to be investigated, to establish business cycle facts (see e.g. Kydland and Prescott (1990) or Stock and Watson (1990)).

Dynamic economic theory, however, does not indicate the type of economic trend that series may display nor the relationship between secular and cyclical fluctuations. In other words, without a set of statistical facts pinning down the properties of the secular component of a time series, the precise economic relationship between the cyclical and secular components is unknown and the choice among economic-based decomposition arbitrary. This issue is particularly relevant because there has been surprisingly little discussion in the literature on whether economic assumptions provide an appropriate characterization of actual business cycles or whether they, instead, leave out important sources of fluctuations and on what they imply for business cycle regularities. Because of this circularity, all economic-based decompositions are, at best, attempts to approximate unknown features of a series and therefore subject to specification errors.

Compared to the vastness of the problems raised in this introduction, the focus of the paper is modest. I report the cyclical properties of a small set of real series using a number of different detrending methods. The approach of the paper is agnostic. Modern dynamic theory of real economic fluctuations is used only to select the variables of interest for this study. None of the detrending filters employed is believed to be the correct one. Instead, I assume that all procedures are approximations which isolate aspects of the secular and cyclical components of the series. In this sense, different detrending methods are alternative windows which look at series from different perspectives. The idea is to organize the information on business cycle fluctuations in a systematic manner in an attempt (i) to identify whether there exists a set of relationships which is independent of the exact definition of cyclical fluctuations and can be used as a benchmark to guide theoretical macroeconomic research, (ii) explain differences when they arise, (iii) provide new evidence on certain "data anomalies" which have motivated recent developments in the theoretical literature and propose new "puzzles" which may guide future developments.

I choose to concentrate on a small set of real variables to maintain comparability with existing real business cycle (RBC) literature but it should be clear that the type of problems outlined in this introduction are not unique to this literature and concern all approaches which use "stylized facts" as qualitative or quantitative benchmarks to compare the properties of theoretical models. Also, although neglecting monetary and financial variables may, in some situations, lead to spurious inference, the lack of these series from the list of variables examined here does not affect the substance of the arguments made.

I compare the properties of the cyclical components of seven real series (GNP, Consumption, Investment, Hours, Real Wage, Productivity and Capital Stock) obtained using seven univariate (Hodrick-Prescott, Beveridge-Nelson, Linear, Segmented, First Order Differencing, Unobservable Components, Frequency Domain Masking) and three multivariate (Cointegration, Common Linear and Multivariate Frequency Domain) detrending techniques. For each method I report sample moments, a few terms of the cross correlation function and the impulse response functions when GNP is shocked.

Antecedents of the type of research carried out here are King and Rebelo (1989), Baxter and Stockman (1989), Cogley and Nason (1991), Baxter (1991) and Harvey and Jaeger (1991). They demonstrated how the mechanical application of the Hodrick and Prescott filter to series which are either integrated or driven by deterministic trends may induce spurious results and how certain quantitative features of the business cycles are not robust to the choice of detrending.

The paper documents that the second order properties of the estimated cyclical components of the seven series vary widely across detrending procedures but that minor differential effects emerge in higher moments. I show that different detrending methods extract different types of information from the original series and that, even among the class of filters which extract cycles of similar length, significant differences may emerge. In addition, I argue that the qualitative responses of consumption, investment, hours and real wage to a typical shock in GNP exhibit two typical patterns which are consistent with either a technology driven or with a demand driven idea of business cycles. Finally, I demonstrate that in some situations, e.g. in examining whether labor hoarding occurs, economic theory does suggest which class of detrending methods should be used. In general, the unpleasant impression that there are no stylized facts can be avoided by clearly stating the purpose of the analysis, both in terms of the type of fluctuations one wants to examine and of the implications that different detrending filters have for business cycle characteristics.

The analysis of the paper completely ignores the possibility that measurement errors are present in the raw data. This is potentially a serious problem since the data collected by statistical agencies is massaged in so many ways that spurious results may obtain (see e.g. Wilcox (1992)). The crucial issue for this paper is whether these filtering procedures (which include sectoral and temporal aggregations, various adjustments and the use of proxies) induce differing amounts of measurement errors at different frequencies. Given the lack of information on the construction of various aggregates, I reluctantly sweep the problem under the rug and assume that measurement errors are negligible and constant across frequencies.

The paper is organized as follows: the next section describes various detrending procedures. Section 3 presents summary statistics. Section 4 analyzes certain "stylized facts" in light of the results of section 3, attempts to interpret the differences and discusses the implications for macroeconomic practice. Section 5 presents some conclusions.

2 Alternative Detrending Methods

This section briefly reviews the procedures used to extract trends from the observable time series. I divide the methods into two broad categories: "statistical" methods, which assume that the trend and the cycle are unobservable but use different assumptions to identify the two components, and "economic" methods, where the choice of trend is dictated by an economic model, by the preferences of the researcher or by the question being asked. Since only trend and cycle are assumed to exist, all the procedures implicitly assume that either the data has previously been seasonally adjusted or that the seasonal and the cyclical component of the series are lumped together and that irregular (high frequency) fluctuations play little role. Although these assumptions are not without consequences (see e.g. Canova (1991)), the implication of these restrictions will be neglected as a first approximation. Throughout this section I denote the natural logarithm of the observable time series by y_t, its trend (secular) component by x_t and its cyclical component by c_t.

2.1 Statistical Procedures

2.1.1 Polynomial Functions of Time

This procedure is the simplest and the oldest one. It assumes that the trend and the cyclical component of the (log) of each series are uncorrelated and that the secular component is a deterministic process which can be approximated by simple polynomial functions of time. These assumptions imply a model for y_t of the form

$$y_t = x_t + c_t \tag{1}$$

$$x_t = a + \sum_j^q b_{1j} f_j(t - t_0) \quad \text{if } t \leq \bar{t}$$

$$x_t = a + \sum_j^q b_{2j} f_j(t - t_1) \quad \text{if } \bar{t} + 1 \leq t \leq T \tag{2}$$

where q is typically chosen to be small, t_0 and t_1 are given points in time, scaling the origin of the trend. In (2), I allow for the possibility of a structural break in the secular component of the series at a known time $\bar{t}$. The trend can be estimated by fitting y_t to a constant and to scaled polynomial functions of time using least squares and by taking the predicted value of the regression (see e.g. Anderson (1971)) An estimate of the cyclical component is $\hat{c}_t = y_t - \hat{x}_t$. I present results obtained when $f_j(t - t_0) = t - t_0$ and $\bar{t} = T$ (LT in the tables), and when $f_j(t - t_0) = t - t_0$, $f_j(t - t_1) = t - t_1$ and $\bar{t} = 1973, 3$ (SEGM in the tables) [1].

[1]Experiments conducted using more elaborate deterministic functions of time (polynomials, exponentials and logarithms) produce results which are similar to those obtained with a linear function only and are not presented.

2.1.2 First Order Differences

The basic assumptions of the first order differencing procedure (FOD) are that the secular component of the series is a random walk with no drift, the cyclical component is stationary and that the two components are uncorrelated. Under these assumptions y_t has a unit root which is entirely due to the secular component of the series. Therefore y_t can be represented as:

$$y_t = y_{t-1} + \epsilon_t \tag{3}$$

and an estimate of c_t is obtained as $\hat{c}_t = y_t - y_{t-1}$.

2.1.3 Beveridge and Nelson's Procedure

Beveridge and Nelson's (1981) detrending procedure finds its roots in the seminal work of Nelson and Plosser (1982). The key identifying assumption of this procedure is that the cyclical component of the series is stationary while the secular component accounts for its nonstationary behavior. Let $w_t = (1-\ell)y_t$ and assume that w_t is a stationary ARMA process of the form

$$\phi(\ell)w_t = \mu' + \theta(\ell)\varepsilon_t \quad \varepsilon_t \sim i.i.d.(0, \sigma_\varepsilon^2) \tag{4}$$

where $\phi(\ell)$ and $\theta(\ell)$ are polynomials in the lag operator of order p and q respectively and where the roots of $\phi(z) = 0$ lie outside the unit circle. The moving average representation for w_t is $w_t = \mu + \gamma(\ell)\varepsilon_t$, where $\mu = \frac{\mu'}{\phi(1)}$ and $\gamma(\ell) = \phi^{-1}(\ell)\theta(\ell)$ and $\phi(1) = 1 - \sum_{j=1}^{p-1}\phi_j$.

Beveridge and Nelson show that, under the assumptions made, the secular component of a series can be defined as the long run forecast of y_t adjusted for its mean rate of change $k\mu$; i.e

$$x_t \equiv \hat{y}_t(k) - k\mu, \tag{5}$$

$$\hat{y}_t(k) = y_t + \hat{w}_t(1) + \cdots + \hat{w}_t(k) \tag{6}$$

with $\hat{w}_t(i) = E_t(w_{t+i}|y_t, y_{t-1}, \cdots) = (\sum_{i=1}^{k}\gamma_i)\varepsilon_t + (\sum_{i=2}^{k+1}\gamma_i)\varepsilon_{t-1} + \cdots, \qquad i = 1, \ldots k.$

For k sufficiently large, the first term of the right hand side of (5) is approximately constant and the secular component x_t is the value the series would have taken if it were on the long-run path. Therefore, letting k go to infinity (5)-(6) collapse to:

$$x_t - x_{t-1} = \mu + (\sum_{i=1}^{\infty}\gamma_i)\varepsilon_t, \qquad \lambda_0 \equiv 1 \tag{7}$$

The cyclical component of the series is defined by

$$\begin{aligned} c_t &= \hat{w}_t(1) + \cdots + \hat{w}_t(k) - k\mu \\ &= \sum_{j=1}^{\infty}\Big(\sum_{i=j+1}^{\infty} \gamma_i\Big)\varepsilon_{t-j} = \chi(\ell)\varepsilon_t \end{aligned} \tag{8}$$

Two characteristics of this decomposition should be noted. First, since the two components are driven by the same shock, this decomposition has the remarkable property that the secular and the cyclical components are perfectly correlated. Second, since this procedure relies on estimates of the γ's and on forecasts $w_t(k)$ obtained from an ARIMA model, the problems inherent to ARIMA specifications are carried over to this detrending method. For example, as Christiano and Eichenbaum (1990) and others have pointed out, there are several ARIMA models which fit the sample autocorrelations of a data set fairly well. However, since different ARIMA models having the same short run properties may have very different long run features, alternative specifications may lead to very different decompositions into trend and cycle. Also, as Maravall (1992) has argued, ARIMA models are designed to fit the short run properties of the data and they are very ill-suited to capture their long run features and, in particular, for long run forecasts.

Because the results vary considerably with the choice of $\theta(\ell)$ and $\phi(\ell)$, both in terms of the magnitude of the standard errors and of the relative size of the summary statistics, an appendix containing the results obtained using various ARIMA specifications is available on request. Here I present results obtained using $\theta(\ell) = 1\ \forall \ell$, five lags for $\phi(\ell)$, the value of 1955,2 as initial condition and the quick computational approach of Coddington and Winters (1987) (BN in the tables)

2.1.4 Unobserved Components Model

The unobserved components model, proposed by Harvey (1985) and Watson (1986) among others, is an alternative to ARIMA based procedures for decomposing aggregate series into secular and cyclical components. Unobserved components (UC) models are usually cast in a state space framework. The key identifying assumptions of this procedure are that the secular component follows a random walk with drift and that the cyclical component is a stationary finite order AR process. The most recent UC literature assumes that the drift term is itself a unit root process drifting over time (see e.g. Harvey and Jaeger (1991)). However, since the task here is to compare methodologies, not to find the best specification for each time series, I do not consider this nevertheless interesting possibility. The measurement equation is given by

$$y_t = x_t + c_t + \epsilon_t, \quad t = 1, \ldots T, \tag{9}$$

where $\epsilon_t \sim N(0, \sigma^2)$ $\forall t$ and $E(\epsilon_t \epsilon_{t-i}) = 0$ for $i \neq 0$. The transition equations for x_t and c_t are

$$\begin{aligned} x_t &= x_{t-1} + \delta + u_t, \\ c_t &= \phi(\ell) c_{t-1} + \nu_t \end{aligned} \tag{10}$$

where δ is a constant and the q roots of $\phi(z) = 0$ lie outside unit circle The stochastic properties of x_t and of c_t are fully characterized by the assumption that the distribution of u_t and ν_t are jointly normal with covariance matrix Σ and by the fact that ϵ_t is uncorrelated with u_t and ν_t. The parameters $\beta = (\sigma^2,\ \sigma_u^2,\ \sigma_\nu^2,\ \delta,\ \phi_j\ j = 1, \ldots, p)$ of the model are typically estimated using the prediction error decomposition of the likelihood and a smoothing algorithm which revises recursive estimates (see, e.g. Harvey (1985)). To simplify the procedure I use approximate maximum likelihood (method of moment) estimates for the β's using the autocovariances of $w_t = (1 - \ell) y_t$ (see Carvalho, Grether and Nerlove (1979)). Then, given initial conditions (typically a zero mean and a diagonal covariance matrix with large but finite elements) and an estimate of β, recursive estimates of the state vector $\alpha_t = [x_t,\ c_t, \ldots\ c_{t-q},\ 1]'$ are obtained with the Kalman filter. The first and second elements of α_t then provide recursive estimates of x_t and c_t. To maintain comparability with other procedures which are one sided, no smoothing of recursive estimates is undertaken. Also, because the results are not particularly sensitive to the choice of $\phi(\ell)$, I only report statistics obtained when $\phi(\ell)$ is a second order polynomial (UC in the tables).

2.1.5 Frequency Domain Methods

The frequency domain procedure employed here draws from Sims (1974). I assume that the cyclical and secular components of the series are independent, that the secular component has most of its power concentrated in a low frequency band of the spectrum and that away from zero the power of the secular component decays very fast. This identification procedure does not restrict the trend to be either deterministic or stochastic and allows for changes in the trend over time as long as the changes are not too frequent. The secular component can be recovered from y_t using

$$a(\omega) F_y(\omega) = F_x(\omega) \tag{11}$$

where $a(\omega)$ is a " low" pass filter and $F_y(\omega)$ and $F_x(\omega)$ are the Fourier transforms of y_t and x_t. In the time domain it can be shown (see e.g. Priestley, 1981, p.275) that the polynomial $a(\ell)$, the inverse Fourier transform of $a(\omega)$, has the form:

$$a(\ell) = \frac{sin(\omega_2 \ell) - sin(\omega_1 \ell)}{\pi \ell} \tag{12}$$

where ω_1 and ω_2 are the upper and lower limits of the frequency band where the secular component has all its power. The cyclical component of y_t can be estimated using $(1 - a(\ell))y_t$. The key to this procedure is the correct specification of the upper and lower limits of the filter. Following the NBER taxonomy, which describes business cycle fluctuations as those cycles with a 2-6 year periodicity, and the conventional wisdom that no complete cycle has exceeded 8 years in length, I chose $\omega_1 = 0$ and $\omega_2 = \frac{\pi}{15}$. Since the spectrum is symmetric around the origin, this choice of filter wipes out all the power in the band $(-\frac{\pi}{15}, \frac{\pi}{15})$. This identifying restriction implies that the power of the spectrum of y_t corresponding to cycles with length less that 30 quarters is entirely due to the cyclical component c_t (FREQ1 in the tables).

The above filter leaves a considerable amount of undesirable high frequency variability, which need not necessarily be identified with business cycle fluctuations. For this reason, I also consider a decomposition of y_t as in (9)) where ϵ_t is identified by the assumption that it has most of its power located in a high frequency band of the spectrum (as e.g. in Englund, Persson and Svensson (1991)). In this case the cyclical component of the series is obtained with a filter which, in addition to eliminating all cycles with period greater than thirty quarters, wipes out all cycles with period less than six quarters. This is achieved by choosing $a(\omega)$ to be:

$$\begin{aligned} a(\omega) &= 1 \qquad if\ \omega\ \varepsilon\ [0, \frac{\pi}{15}] \cup [\frac{\pi}{3}, \pi] \\ &= 0 \qquad \text{otherwise} \end{aligned}$$

The results are presented as FREQ2 in the tables. It is worthwhile noting that this filter has approximately the same properties as the "Batterworth" filter used by Stock and Watson (1990).

2.1.6 One Dimensional Index Model

The final procedure in the statistical group is multivariate and assumes that while each series is trending, either deterministically or stochastically or both, some linear combination of them does not have trends. The idea here is to extract the part of the secular component which is common to all series using an index model. The approach used is similar to the one employed by Stock and Watson (1989) and uses a multivariate frequency domain procedure to make the approach operational. The key assumption is that in the low frequencies of the spectrum there exists a one dimensional process (a secular component) which is common to all series (see Quah and Sargent (1991) for a two-index model). This process is characterized by the property that it has all its power at low frequencies and that away from zero it decays very fast. The model has the representation:

$$y_t = x_t + c_t \tag{13}$$

where y_t is a $n \times 1$ vector, $x_t = Az_t$ and z_t is a one dimensional process with $0 < S_z(\omega) < M, \ \forall\ \omega\ \varepsilon\ [\bar{\omega}, \pi]$ where $S_z(\omega)$ is the spectral density of z_t and M is a small number and x_t and c_t are independent. An estimate of x_t is obtained using a multivariate version of the procedure used for the univariate UC model and $\hat{c}_t$ is obtained from (13) (MFD in the tables).

2.2 Economic Procedures

2.2.1 A Model of Common Deterministic Trends

King, Plosser and Rebelo (1988) present a neoclassical model of capital accumulation with labor supply choices where there is deterministic labor augmenting technical progress. Their model implies that all endogenous variables have a common deterministic trend (the growth rate of labor augmenting technical progress) and that fluctuations around the common linear trend are all of a transitory nature. Each time series is therefore generated by a model like (1) where the secular and cyclical components are independent, where x_t is common to all series and given by

$$x_t = x_0 + \delta t \tag{14}$$

where δ is the growth rate of technological progress. Because total hours are bounded by the endowment of the economy, it must be the case that the trend in hours per-capita is zero. King, Plosser and Rebelo use a value of δ =.4% and detrend all the series using the resulting x_t. I follow their procedure and construct a deterministic trend which is common to all series. For the data set employed here the estimated value of δ is 0.7% [2]. x_0 is chosen to be an estimate of the unconditional

[2] The major reasons for the discrepancy between King, Plosser and Rebelo and my estimate are that they employ a different sample and they do not include the capital stock in the calculation of their common trend.

mean of each series. Since the hours series is measured in absolute terms, I detrend it using the growth rate of population (about 0.3% per quarter over the sample 55,3-86,3).

2.2.2 A Model of Common Stochastic Trends

King, Plosser, Stock and Watson (1991) propose a version of the theoretical model employed by King, Plosser and Rebelo (1988) where the long run properties of the endogenous variables are driven by the same nonstationary technological shock. The corresponding statistical common trend representation, developed in Stock and Watson (1988), implies that series have a common trend if there exists a cointegrating vector which makes all series simultaneously stationary. This approach produces as a by-product, a model driven decomposition into secular (nonstationary) and cyclical (stationary) components which is the multivariate counterpart of the method of Beveridge and Nelson. As in that framework, the two components are perfectly correlated because they are driven by the same shock. Let w_t be an $n \times 1$ vector of time series where $w_t = (1-\ell)y_t$ has moving average representation:

$$w_t = \delta + C(\ell)\epsilon_t + B(\ell)z_t \tag{15}$$

where $\alpha' C(1) = 0$, $\epsilon_t = G^{\frac{1}{2}} v_t$ with $v_t \sim iid\,(0, G)$ and z_t is a set of cointegrating vectors. Expanding (15) we have:

$$y_t = y_0 + \delta t + C(1)\zeta_t + D(\ell)\epsilon_t \tag{16}$$

where $D_j = -\sum_{i=1+j}^{\infty} C_i$ and $\zeta_t = \sum_{s=1}^{t} \epsilon_s$. Stock and Watson show that (16) implies that:

$$x_t = y_0 + A\tau_t = y_0 + \delta t + C(1)\zeta_t \tag{17}$$

$$c_t = D(\ell)\epsilon_t \tag{18}$$

where A is a $n \times k$ vector, $\tau_t = \mu + \tau_{t-1} + \eta_t$, η_t is a serially uncorrelated random noise and $dim(\tau_t) = k \leq n$. Rather than testing whether there is a cointegrating vector z_t, I work directly with the AR version of (15). That is, I estimate a vector error correction model (VECM) and use one lag of two cointegrating vectors (GNP/consumption, GNP/investment) to obtain estimates of $\delta, C(\ell)$ and ϵ_t. Then using (18) an estimate of the transitory component is obtained by taking $\hat{c}_t = y_t - y_0 - \hat{\delta} t - \hat{C}(1)\hat{\zeta}_t$.

As in the Beveridge-Nelson decomposition, estimates of the secular and transitory components differ for different specifications of the VECM model (both in terms of the number of variables and in terms of the choice of lag length). Here I present the results obtained using five lags for each variable and including all variables in the system (COIN in the tables)

2.2.3 The Hodrick and Prescott's Filter

The Hodrick and Prescott (HP) (1980) filter has two justifications: one economic and one statistical.

In the standard RBC literature the trend of a time series is not something intrinsic to the data but it is a representation of the preferences of the researcher and strongly depends on the economic question being investigated. For example, researchers interested in investigating 10-15 year cycles will extract different trends from those interested in investigating, say, 2-4 year cycles. The popularity of the HP filter among applied macroeconomists results from its flexibility to accommodate these needs since the implied trend line resembles what an analyst will draw by hand through the plot of the data (see e.g. Kydland and Prescott (1990)).

Like the two previous procedures, the selection mechanism that economic theory imposes on the data via the HP filter can be justified using the voluminous statistical literature on curve fitting (see e.g. Wabha (1980)) [3]. In this framework the HP filter is an optimal extractor of a trend which is stochastic but moves smoothly over time and is uncorrelated with the cyclical component. The assumption that the trend is smooth is imposed by assuming that the sum of squares of the second differences of x_t is small. An estimate of the secular component is obtained by minimizing:

$$\min_{[x_t]_{t=1}^{T}} \; [\sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=1}^{T}((x_t - x_{t-1}) - (x_{t-1} - x_{t-2}))]^2 \quad \lambda > 0 \tag{19}$$

where T is the sample size and λ is a parameter that penalizes the variability of trend. As the value of λ increases, the penalty imposed for large fluctuations in the secular component increases and the path for $\hat{x}_t$ becomes smoother. In this context, the "optimal" value of λ is series dependent, can be obtained by casting the problem into a signal extraction-prediction error decomposition framework, and is given by $\lambda = \frac{\sigma_1^2}{\sigma_2^2}$, where σ_1 and σ_2 are the standard errors of the trend and of the cyclical component of the series.

[3] Harvey and Jaeger (1991) offer also an unobservable component interpretation of this filter.

In the standard practice λ is not estimated from the data. In the RBC literature it is a-priori selected to isolate cyclical fluctuations which belong to the specific frequency band the researcher wants to investigate and the same value of λ is used for all series. For example, with quarterly data, λ is chosen on the assumption that the standard deviation of the cyclical component is forty times the standard deviation of the secular component, which results in a filter which extracts cycles of average amplitude of 4-6 years. While this approach is meaningful from the point of view of the RBC literature, the assumed magnitude of λ is debatable from a statistical point of view on two grounds. First, Nelson and Plosser (1982) estimated λ to be in the range $[\frac{1}{6}, 1]$ for most of the series they examine, a range far from the value of 40 assumed by Hodrick and Prescott. This implies that much of the variability that the Hodrick and Prescott filter attributes to the cyclical component is, in fact, part of the trend. Second, since economic aggregates possess different degrees of smoothness, the application of the Hodrick–Prescott filter with a uniformly standardized value of λ may distort features of summary correlations. Here I investigate the first of these possibilities by experimenting with two measures of λ: a standard one (HP1600 in the tables) and one obtained by assuming that the relative variability of the components is 2 (HP4 in the tables) [4].

In practical terms the procedure involves the computation of T first–order conditions with respect to $x_1, x_2, \cdots, x_T$ and the solution of a system of T linear simultaneous equations in T unknowns, of the form $\hat{x}_t = Ay_t$. An estimate of the cyclical component is obtained as $\hat{c}_t = y_t - \hat{x}_t$.

The ability of the HP filter to induce stationarity in trending series has been examined by King and Rebelo (1989). Some properties of the HP1600 filter when $T \to \infty$ and the penalty function is two-sided have been highlighted by Cogley and Nason (1991) and Harvey and Jaeger (1991).

3 The Results

This section presents results obtained using the logarithms of seasonally adjusted quarterly US time series for the period 1955,3-1986,3. GNP, Consumption, Investment, Hours and Real Wage Compensation are obtained from the Citibase data base. GNP measures Real Gross National Product in 1982 dollars (Citibase name: GNP82), consumption measures consumption expendi-

[4] A previous version of the paper also reported a decomposition where λ was separately estimated for each series by maximum likelihood. The results are not reported here since they represent an intermediate case between the two presented. In essence, because the degree of smoothness of the series examined do not differ very much, using the same λ does not induce large distortions in the properties of the six series examined.

ture by domestic residents on nondurables and services in 1982 dollars (Citibase names: GSC82 and GCN82), investment measures total fixed investment in plants and equipment plus consumer durables in 1982 dollars (Citibase names: GINPD82 and GCD82), hours measures the total number of hours of labor input as reported by establishment survey data (Citibase name: LPMHU) and the real wage is constructed using nominal total compensations of nonagricultural employees (Citibase name: GCOMP) and a measure of price (Citibase name: PUNEW). A quarterly series for the capital stock is reconstructed using the net capital stock (residential and nonresidential) for 1954, the quarterly series for investment and a depreciation rate of 2.5% per quarter. Finally, I also consider a productivity series constructed taking the difference between log(GNP) and log(Hours).

While this set of variables is pretty standard in aggregate analyses of the business cycle, different authors have used alternative measures of hours, real wage and productivity and capital. For example, Kydland and Prescott (1990) do not include residential capital in their capital stock series. To assess the sensitivity of the results to choice of series, I also examined other measures of consumption (only nondurables or total), of hours (household survey data), of real wage (output per man-hour in manufacturing) and productivity (Citibase name: LBOUT). Results obtained for these series are presented in an appendix.

Time plots for the log of the data, their estimated pseudo log spectrum and the estimated pseudo coherence of each series with GNP appear in figure 1 [5]. Plots of the estimates of the cyclical component of GNP obtained using different detrending methods appear in figure 2. Shaded areas in the time series plots indicate recessions according to NBER chronology. Shaded areas in the plots of the spectra and the coherence comprise cycles with periodicity of 2-6 years.

3.1 The Plots

Three features of the plots of the raw data deserve comment. First, all series exhibit upward movements after the removal of an exponential trend. Second, the spectral densities of the seven variables look alike. Third, the plots of the coherences with GNP show moderately high correlations at business cycle frequencies.

The plots of the estimated cyclical component of GNP indicate that detrending methods that impose a random walk on the secular component of the series (e.g. FOD, BN and UC) induce

[5] Pseudo spectra and pseudo coherences are computed knocking out frequency zero for each series. This is necessary because spectra and coherences do not exist for variables which may contain a unit root.

lower cyclical variability. At the opposite end of the spectrum LT, MLT and COIN leave the largest variability in the cyclical component. However, since some decompositions use (1) while others use (10) as the basic model, numerical differences may emerge simply because the irregular component of the series is handled in different ways.

The cyclical components of GNP obtained with linear (univariate and multivariate) and segmented filters look quite similar but have a slightly different mean value; those obtained with BN, FOD and HP4 filters resemble each other and the one obtained with FREQ1 is very similar to the cyclical component obtained with the HP1600 filter. Finally, the three multivariate methods produce cyclical components of GNP which are similar to each other, display long cycles and are different from those obtained using univariate methods (except, perhaps, LT). The cyclical components of the other six variables (not discussed for reason of space limitations) have essentially the same qualitative characteristics.

The cyclical components of all series might be classified into three general types of patterns. One, typically found with HP1600, SEGM, the frequency domain filters and, to a lesser extent, UC is where the cyclical components display cycles of 4-6 years and turning points for expansions and contractions correspond approximately to NBER dating. A second type, typically associated with linear detrending and the three multivariate procedure, is where cycles are generally long and turning points do not correspond to NBER chronology. Finally, a third type, which is typical of methods which impose a unit root on the trend, is where cyclical components are very erratic, display cycles of short length whose turning points have little agreement with NBER dating and which give many false alarms concerning the starting of an expansion or of a recession.

To obtain information on the type of cycles that each method extracts, it is instructive to examine the characterization of the 1979 and 1981-82 recessions given by each procedure. With all methods but FOD, MLT and COIN, the characterization of the 1979 recession was mild in terms of declines in GNP. However, with the last two filters GNP is below its long run trend from 1974 up to 1986, a result which casts doubts about the usefulness of these procedures to capture typically defined business cycle fluctuations. Finally, in three cases (UC, SEGM and MFD) the 1979 recession appears simply as a slowdown. For the 1981-82 recession all methods but BN and MFD locate the trough of the cycle sometime between 1981-82 but there is substantial disagreement regarding its magnitude. With MFD the 1981-82 recession appears as a strong slowdown, while with BN it shows up as an expansion and the trough of the cycle occurs only in late 1983, when NBER dating indicated that an expansion was well under way.

There are two conclusions that can be drawn from figure 2. First, different detrending methods emphasize cycles of different length in the data, some of which are too long to belong to the standard business cycle classification. Second, as a consequence, different detrending methods have different implications for the timing of turning points and the severity of recessions.

3.2 Summary Statistics

This section reports moments of the distribution of the cyclical components of the data, various short term cross correlations and the responses of the variables to a 1% standard error innovation in GNP. Table 1 presents the absolute standard deviations of the cyclical component of GNP and the relative standard deviations of the other six variables, in percentage of GNP standard errors. Table 2 presents cross correlations at lags (-1,0,1). In both tables a "*" indicates that the statistic in the cell differs at the 5% significance level from the statistic obtained with the HP1600 filter [6]. Table 3 displays the estimated coefficients of skewness and table 4 contains the estimated coefficients of excess kurtosis. A "*" in these two tables indicates that the statistic in the cell significantly differs from the statistic which would appear if the cyclical component of the series had a normal distribution (in which case, both are equal to 0) [7].

3.2.1 Standard Deviations

The magnitude of the standard deviations vary greatly across detrending methods. The absolute variability of the cyclical component of GNP is smallest for UC (0.38) and largest for MLT (6.01) while the HP1600 filter generates, approximately, the median value. Note that those methods which leave long swings in the cyclical component of the data typically generate high variability.

The range of relative variabilities is large as well. Consumption variability ranges between 34% and 98% of the variability of GNP, relative investment variability ranges from 216% to 672% and

[6] Under standard regularity conditions outlined e.g. in Newey and West (1987), the statistics $J_1 = (var_x(i) - var_x(HP1600))V_1^{-1}(var_x(i) - var_x(HP1600))$ and $J_2 = (cov_{x,GNP}(i) - cov_{x,GNP}(HP1600))V_2^{-1}(cov_{x,GNP}(i) - cov_{x,GNP}(HP1600))$ are distributed $\chi^2(1)$ where i stands for detrending method, x for the particular series examined and V_1 and V_2 are the asymptotic covariance matrices of the random variables $[var_x(i), var_x(HP1600)]$ and $[cov_{x,GNP}(i), cov_{x,GNP}(HP1600)]$ respectively.

[7] The tests for skewness and excess kurtosis used here are those of Kendall and Stuart (1958).

hours from 50% to 414%. The relative variability of real wage to GNP varies between 65% and 224% and the relative variability of productivity is between 49% and 409%, with the HP filters producing the lowest value in both cases. Qualitatively, the capital stock series displays an almost identical pattern to productivity although the range of relative variabilities is smaller (from 14% to 185%). Finally, hours can be either much less or much more volatile than productivity (ranging from 46% to 212%) (see also Baxter (1991)).

While it was relatively simple to group approaches when I considered the absolute variability of GNP, it is much harder to draw general conclusions regarding relative variabilities. For those methods which emphasize cycles of medium length in the data three features warrant mention. First, the magnitude of relative variabilities of the series filtered with HP are among the lowest, regardless of the value of the smoothing parameter employed. Second, although some switches appear for values close to 1, the relative variabilities generated with FOD are close to those obtained with the HP1600 and HP4, confirming some of the properties of the two filters described by King and Rebelo (1989). Third, the ordering of relative variabilities obtained with UC and FREQ filters differs substantially from those obtained with HP filters, with consumption, hours, real wage and productivity being the most affected.

Among methods which emphasize longer cycles (say 8-10 years), there is more agreement regarding the size and ordering of relative variability. For example, hours are less volatile then GNP and productivity while investment is about twice as volatile as GNP. For cycles of shorter length, regularities of this type are much harder to find.

3.2.2 Cross Correlations

The cross correlations of the cyclical components are also very sensitive to the detrending procedure used. For example, the contemporaneous cross correlation of consumption and GNP varies from 0.31 to 0.96 and that of hours and GNP varies from 0.17 to 0.88. Even more striking is the range of cross correlations between productivity and GNP which varies from -0.16 to about 0.75 and of hours and the real wage, from -0.05 to 0.85. Similarly, there is a wide range of cross correlations between productivity and past GNP (range -0.06 to 0.80) or real wage and past GNP (range 0.05 to 0.89). In general, the largest range in the lead and lag correlations occurs for hours and GNP while the smallest range occurs for consumption and GNP. In some cases, e.g.

the contemporaneous relationship between productivity and GNP, it is very hard even to sign the correlation with sufficient accuracy.

Among detrending methods, the HP1600 filter produces the highest contemporaneous correlation between hours and GNP and investment and GNP. In fact most of the contemporaneous correlations with GNP obtained with the HP1600 filter are significantly larger than those obtained with other methods (the exception are data detrended with frequency domain methods) and the hypothesis that the two sets of correlations are identical is frequently rejected. Hence, the magnitude or even the sign of various correlations substantially differs, even among those methods which extract cycles of similar length.

3.2.3 Higher Moments

Current work cataloging properties of business cycles typically reports only second moments [8]. Lingering in the background are one of two assumptions: either that the series are zero mean normal stochastic processes so that second moments summarize all that is contained in the data or that higher moments do not carry crucial information about the cyclical properties of the data. Recent work by Neftci (1984), Falk (1986), Delong and Summers (1986) and Pfann (1991) have considered higher moments in an attempt to detect asymmetries or fat tails in the distribution of the cyclical components of GNP and employment. Here I examine the properties of third and fourth moments to (i) indicate whether any detrending procedures induce significant distortions in the properties of the data, (ii) informally test the appropriateness of the normality assumption.

Perhaps surprisingly, given the vast range of results in tables 1 and 2, no major differences across detrending methods emerge for 5 of 7 series in the case of skewness and for 4 out of 7 series in the case of excess kurtosis. The major discrepancies are with the investment series, which is strongly left skewed with 5 methods (BN, LT, SEGM, MLT, COIN) and leptokurtic with 4 methods (HP1600, LT, SEGM, FREQ1) and with the capital series which is leptokurtic in 5 cases. To understand the source of divergences among methods for the investment series, note that all methods but FREQ2 generate both negative skewness and positive excess kurtosis, although their

[8] Examples include Kydland and Prescott (1990) or Stock and Watson (1990) for the US, Englund, Persson and Svensson (1991) for Sweden, Danthine and Girardin (1989) for Switzerland, Brandner and Neusser (1990) for Germany and Austria, Blackburn and Ravn (1990) for European countries, Fiorito and Kollintzas (1992) for the G-7, Backus and Kehoe (1992) for 10 countries.

magnitude is not always significant. Because FREQ2 is one of the two filters which eliminates high frequency variability (the other is UC), the skewed and leptokurtic behavior of investment probably has more to do with cycles of very short length rather than with business cycle fluctuations. For the capital stock series, on the other hand, leptokurtic behavior appears only with those methods which emphasize medium-long cycles in the data.

Comparing across procedures, excess skewness tend to appear primarily with LT and SEGM and excess kurtosis with LT, SEGM, HP1600 and FREQ1. For all these methods the assumption that the cyclical component is normal is highly inappropriate.

The size of the distortions in these higher moments induced by detrending can be evaluated by comparing the skewness and the excess kurtosis obtained before and after detrending [9]. For the original data all series are slightly left skewed but the coefficient of skewness is never different from zero. Investment, real wage and capital, on the other hand, have a marginally significant leptokurtosis. Hence, most detrending procedures do not severely distort the higher moments and do not seem to produce differential effects on their properties.

3.2.4 Impulse Responses

One final statistic which is typically examined is the impulse response function (IRF) when GNP is shocked by one standard deviation. Here I perform the exercise using a VAR system which includes the cyclical component of six variables (GNP, Hours, Real Wage, Consumption, Investment, and Capital). Because the IRF is a linear transformation of the data, the results for the average product of labor can be read off directly from the responses of GNP and Hours. The lag length of the system is method dependent and is chosen so that the residuals satisfy the white noise assumption. Since I am interested only in the responses to a GNP shock, I will not attempt a fully behavioral interpretation of the system. To identify GNP disturbances I assume that, within a quarter, no shocks others than its own affect GNP. Table 5 reports summary measures of the IRF. Figure 3 plots the IRF for HP1600 and COIN detrended data.

The properties of the IRF differ across detrending methods in several respects. First, the average length of an output cycle in response to an output shock varies with detrending procedure. For

[9] Since the tests for skewness and kurtosis are invalid in the presence of serial correlation, both the original and the filtered series are prewhitened with 12 lags before the statistics are computed.

example, the average cycle is about 3.5 years with the HP1600 filter, a little less than 2 years with the HP4 filter and about 1 year with the FOD filter. Second, the response of investment has varying degrees of persistence: it is essentially zero after 4 quarters when FOD is used while it is still sizable after 24 quarters with UC detrended data. Third, the size of the peak responses in consumption and investment is method dependent. For example, the peak response in consumption varies from 0.17 (with HP4) to 1.3 (with COIN) of the shock in GNP and peak investment response varies from 1.5 with HP4 and FOD to about 10.5 with MFD. Finally, the timing of the peak responses falls into two categories which appear to have little relationship with the length of the cycles various filters extract. In the first category, which includes most univariate filters (both HP filters, RW, BN, LT and FREQ1), a shock to output produces a peak response in output and real wage instantaneously, a 1-2 quarters lagged peak response in investment, a 2-4 quarter lagged peak response of consumption and hours, and a peak in capital with a 4-6 quarter delay. The exact timing of the peak response in hours constitutes the major difference among these methods, although the largest delay does not exceed 4 quarters. In addition, in all cases but UC, the size of the instantaneous response in productivity is always greater than the size of the instantaneous response in real wage.

In the second category which includes COIN (and somewhat MFD), output and the real wage display a peak response which lags the initial shock by 4-6 quarters, the peak in hours lags 2-3 quarters, the peak in consumption and investment lags 2-4 quarters and the peak in capital about 10 quarters. It should also be noticed that the magnitude of consumption responses exceeds the magnitude of output responses over the first 2-3 quarters of the cycle, the immediate response of investment and capital is negative and that the response of productivity is negative, at least in the first few quarters of the cycle [10]. Finally, for these filters the size of the peak response in all variables but capital exceeds the size of the disturbance in GNP, a results which does not hold true for the case of HP1600 detrended data.

Three conclusions can be drawn. First, qualitatively and quantitatively the second order properties of the data depend strongly on the detrending procedure used. Second, higher moments of the cyclical component of the data are more robust to the choice of detrending. Third, the transmission properties of a GNP shock are affected both quantitatively and qualitatively by the detrending

[10] The negative contemporaneous response of investment to a shock in GNP has been found also by Warne and Vredin (1991) using a COIN filter on Swedish data.

procedure employed. While quantitative differences are not surprising since different filters extract different types of information for each series and different amounts of variability across series, the lack of qualitative agreement among those methods which capture cycles of similar length casts doubts on the possibility to robustly characterize the interrelationships existing in the data.

4 Implications For Stylized Facts of the Business Cycle

4.1 Relative Variabilities

A number of stylized facts of the business cycle are stated in terms of the magnitude of the relative variability of one variable to GNP. For example, Kydland and Prescott (1990) or Backus and Kehoe (1992) suggest qualitative statements like: "consumption is less volatile than output" or quantitative statements like "investment is 2-3 times more volatile than output". The relative volatility of consumption to GNP is also crucial for tests of the permanent income hypothesis. Deaton (1987), for example, indicates that when GNP has a unit root consumption appears to be too smooth to be consistent with the permanent income hypothesis and this result has spurred substantial work in an attempt to rationalize this finding (see e.g. Quah (1989)) .

I find here that, qualitatively speaking, consumption is less volatile and investment more volatile than output. However, a quantitative statement on the size of these relative variabilities is somewhat difficult to make since values range from 0.34 to 0.98 in the first case and from 2.65 to 6.72 in the second. Among the methods which impose or allow for a unit root in GNP, Deaton's paradox holds, i.e. consumption tends to be less volatile than output. However, in at least three cases the relative variability exceeds 0.7 and in one case is 0.98. Hence, whether consumption is excessively smooth depends on the detrending procedure used and while the puzzle may be there, it is surely less dramatic than previously thought.

The relative variabilities of productivity to GNP and to hours are two commonly used statistics to gauge the behavior of labor markets over the cycle. In Prescott (1990, table 1) we find that the variability of productivity is less than the variability of GNP. Mankiw (1989, p.86)) claims that "Over the typical business cycle, employment varies substantially while determinants of the labor supply - the real wage and the real interest rate - vary only slightly"

Because the existing literature has measured productivity and real wage in different ways, I have experimented with two alternative measures of each. The results presented in the tables (and in the appendix) differ greatly across detrending methods and even qualitative statements are hazardous and represent a potentially misleading characterization of these relative variabilities. For 5 of the 12 methods both measures of productivity are more volatile than GNP but the relative ordering across detrending methods changes with the measure used. When the real wage is used in place of productivity (see Burnside, Eichenbaum and Rebelo (1992) and next subsection for some arguments which may justify this switch) the relative variability of real wage exceeds that of GNP for both measures in 9 out of 12 cases.

To try to account for the contrasting characterizations of the relative variability of the standard measure of productivity to GNP across methods, it is useful to examine the spectra of the two variables. It turns out that productivity has significantly more variability than GNP in the region corresponding to cycles of 8-10 years length. This variability is eliminated from the cyclical component extracted with methods like HP [11] and FOD, but it strongly appears with methods like LT. While one may wonder whether or not these cycles should belong to what analysts call the business cycle, two points need to be emphasized here. First, because some filters extract particular business cycle frequencies, they may neglect important sources of information included in cycles of slightly different length. Second, because productivity appears to have a substantial portion of variability outside the standard business cycles frequencies, theoretical work should provide some explanation for why this phenomena occurs.

The case of the relative variability of real wage to GNP is somewhat similar, although, quantitatively speaking, the real wage series does have a sizable portion of its variability in the region corresponding to 4-8 year cycles. Because the HP and FOD filters carve out only a portion of this region, they produce a smaller relative variability relative to other filters. As a consequence of these results, the relative variability of hours to productivity depends both on the measure of productivity and hours used and on the detrending method. For example, when the standard measure of productivity is used, productivity is more volatile then hours for those methods which leave long cycles in the cyclical component (LT, MLT or COIN methods). When the real wage is used, results do not appear to have much relationship with the type of cycles extracted by the filters and no general conclusions seem possible.

[11] Because of the shape of its theoretical gain function, the HP1600 tends to emphasize cycles with average periodicity of 4-6 years while reducing the importance of cycles of longer length (see e.g. Singleton (1988)).

4.2 Procyclical Productivity

Another typical set of stylized facts of the business cycle comes in the form of comovements across variables. Relationships which have attracted the attention of researchers include the correlations among productivity, hours and GNP. In this section I examine the question of the procyclicality of productivity. In the existing literature it is possible to find evidence of countercyclicality (Chirinko (1980)), of acyclicality (Geary and Kennan (1982)), and of procyclicality (Barski and Solon (1988) and Waldman and Delong (1991)). Whether productivity is procyclical, countercyclical or acyclical has important implications for the functioning of labor markets over the business cycle and the sources of disturbances in the economy. Procyclicality is, in fact, consistent with shifts in labor demand caused by shifts in the production function. Countercyclicality implies that shifts in the supply of labor are the primary source of disturbances.

In examining this relationship, it is common to interchange the real wage and productivity (see e.g Prescott (1986), McCullum (1989) or Bernanke and Parkinson (1991)). In a competitive world the real wage is chosen to be equal, in equilibrium, to the marginal product of labor (MPL). Because productivity here measures the average product of labor (APL) the equality need not hold. However, Christiano and Eichenbaum (1992) argue that substituting APL for the real wage is a reasonable approximation because in the real world one should expect the equality to hold on average, not on a period by period basis. In addition, since in many models MPL and APL are proportional, the result should not be badly distorted by this approximation.

In the current framework such a substitution appears to cause problems. When I use a measure of real wage, procyclicality appears with each method and the magnitude of the correlation is consistently above 0.5. When a measure of productivity is used I find that there are two cases where the correlation is negative, albeit small (BN and FREQ1), that the magnitude of the correlations is, in general, much smaller than that obtained with the real wage (the mean value around 0.10) and that the range of outcomes is very large. From the cross section of results one gets the impression that for cycles of 4-6 years productivity is probably acyclical [12].

The reason for these differences can be found by examining the coherence among pairs of series.

[12] The results presented in the appendix obtained with the alternative productivity series are more consistent across detrending methods. All correlations are in fact positive even though the range is large. With the alternative measure of wages, significant countercyclical behavior emerges in at least 3 cases (LT, MLT, COIN).

While the correlation between real wage and GNP is approximately constant over a large band of frequencies up to cycles of about 8 quarters, the magnitude of the correlation coefficients between productivity and GNP is very different by frequency: it is low in the region corresponding to 6-8 year cycles and to 4 years cycles. Within standard business cycles frequencies, the correlation goes from low to high to low to high again as the length of the cycle decreases, a result which strengthens the idea that productivity and GNP appear to have economic cycles of different length.

In sum, the identification of the average productivity with the real wage may lead to serious inconsistencies. The existence of noncompetitive aspects may be one reason for the divergence (see also Chirinko (1991), Sbordone (1992) and Bernanke and Parkinson (1991)). But, apart from the issue of which measure better represents the state of labor markets, it is clear that, whenever the productivity series is used, detrending methods which emphasize cycles of different length are bound to provide a very different characterization of the magnitude and even the sign of the correlation. As mentioned in section 3.2.2, the range of results is very large and even for methods which extract cycles of similar length the cyclical relationship among these variables varies. Hence, while it is safe to claim that, within a wide range of business cycle frequencies, the real wage is procyclical and highly correlated with GNP, the evidence for productivity is much weaker (see also McCullum (1989)) and the sign and the magnitude of the correlation depends on the exact business cycle frequencies examined.

4.3 The Dunlop–Tarshis Puzzle

A recurrent puzzle in the business cycle literature is the so called Dunlop–Tarshis paradox, i.e. the fact that the correlation between the return to working and the numbers of hours worked is very small. Kydland and Prescott (1988), for example, report that the contemporaneous correlation between a standard measure of hours and the real wage is approximately zero when HP1600 detrended data are used. Many models, both in the neoclassical and Keynesian tradition fail to account for this observation since they share the assumption that real wages and hours worked are on a fixed downward sloped marginal product of labour schedule. This implies that real wages and hours worked should be strongly negatively correlated. On the other hand, current RBC models which are driven by technology shocks, generate procyclical movements in hours and real wage via cyclical shifts in the production function.

To account for this discrepancy between theory and the data, Kydland and Prescott suggest that measurement errors may be important and attempt to reconstruct a real wage series which is free from these errors. Other authors like Christiano and Eichenbaum (1992) have instead modified existing RBC models so that the correlation between the two variables is approximately zero.

Here I find that when real wage is used the correlation is almost always positive and it is greater than 0.40 in half of the cases. When a standard measure of productivity is used in place of real wage the correlations are all negative and in 5 cases they are smaller than -0.50 [13]. With both measures, however, I find that the magnitude of the correlations varies substantially across methods and only in a few cases it is statistically equal to zero. Note also that the value of the correlation between hours and productivity obtained with HP1600 detrended data (-0.24) is very similar to existing estimates. For example, Christiano and Eichenbaum (1992), using a different hour series, obtain a point estimate of -0.16.

The change in sign occurring when APL is used rather than the real wage is easy to explain. In many cases, the productivity series is countercyclical up to the mid 1960's and procyclical afterwards but the negative relationship in the first part of the sample dominates. This appears to be due to the fact that hours were more volatile than GNP before the mid 1960's but less volatile afterwards. Over the subsamples 1955-1967 and 1968-1986, the relative variance of hours to GNP in fact switches from greater than 1 to less than 1.

The robustness of the correlation between real wage and hours is surprising. Except for the case of UC, the correlation is positive, a result which is entirely consistent with the idea that shifts in the production function account for this correlation. The association appears to be weaker for cycles of 8-10 year length and much stronger for fluctuations included in the standard definition of business cycle.

Hence the Dunlop-Tarshis paradox seem to be less of a puzzle than previously thought. Regardless of the measures of productivity and of real wage used, a small and insignificant association with hours occurs in only a few cases. The sign of the correlation strongly depends on whether real wage or productivity is used and the strength of the association depends on which cycles the analysis focuses on. However, there are very small discrepancies among methods which extract

[13] When the alternative measure of real wage is used all correlations exceed 0.50, while with a more direct measure of productivity the range of correlations is [-0.43, 0.82].

cycles of comparable length.

4.4 Labor Hoarding or No Labor Hoarding ?

The final stylized fact I examine is the relationship between productivity and lagged measures of economic activity. Some authors (e.g., Summers (1986) and McCallum (1989), Mankiw (1989)) have claimed that a negative correlation indicates the presence of labor hoarding, i.e. because of hiring and firing costs, firms adjust their workforce slowly and the cyclical behavior of productivity primarily reflects the cyclical behavior of output (see Rothemberg and Summers (1991)) [14]. In examining this relationship, a further complication to the choice between productivity and real wage measures arises because some authors (see e.g. Burnside, Eichenbaum and Rebelo (1992)) have used hours in place of GNP as an indicator of cyclical activity.

At first glance, it appears that the evidence concerning labor hoarding strongly depends on what measures of productivity and cyclical activity are used and on what detrending method is employed. For example, when the standard measure of productivity is used the sign of the correlation (APL_t, GNP_{t-1}) is almost equally split between positive and negative values, while when the real wage is used, it is mainly positive, except in one case, and significant. When hours are used as an indicator for cyclical activity, the correlation between productivity and lagged hours is primarily negative, although not much different from zero, while the correlation between real wage and lagged hours is positive [15]. The range of possible outcomes across detrending methods for each set of measures is also very wide, contributing to the impression that there is no clear pattern in the data.

In order to gain some intuition for why the descriptions of the phenomena contrast, it is useful to study the differences in one set of correlations across detrending methods. This exercise allows us to highlight particular features of various detrending filters and to stress the fact that a simple theoretical characterization of labor hoarding phenomena does suggest which class of detrending method should be used to extract this information from the data. If firms do hoard labor, it will be

[14] Although the intuition is simple, the mechanics of signing this coefficient is somewhat obscure. In particular, it seems necessary to assume that output is mean reverting to obtain a negative sign.

[15] The use of alternative measures of productivity, real wage and hours presented in the appendix do not clarify the qualitative features of the relationship. The other measure of productivity is positively correlated with lagged GNP and with hours in half of the cases, while the other measure of real wage is positively correlated with lagged GNP in 9 of the 12 cases, and with hours in all but one case.

done if there is the prospect of a reversal of the cycle in the near future. If the reversal of the cycle is expected to happen, say, 5-6 years in the future, the cost of keeping idle workers may well exceed the benefit of not having to rehire and retrain new workers in order to expand production when demand picks up. Hence, if one hopes to find evidence of labor hoarding via the simple correlation measure employed here, one should look for detrending methods which emphasize short cyclical fluctuations (say 2-3 years), where this phenomenon may be prevalent.

Among the available methods, there are two procedures which essentially produce the required outcome: HP4 and FOD. These procedures give, independently of the productivity series used, a negative although small lagged correlation with GNP. For filters which extract slightly longer cycles (UC, HP1600, FREQ1 and FREQ2) the correlation is still negative but it is closer to zero, while in the case of SEGM it is positive but only marginally so. Finally, filters which extract long cycles (LT and the three multivariate methods) induce positive correlation between productivity and lagged GNP, regardless of the productivity measure used.

Two conclusions can be drawn from the above discussion. First, because alternative detrending methods extract different types of information from the data, and because the labor hoarding hypothesis does imply restrictions on the length of the cycles to be examined, one should disregard the results obtained with those methods which pick up relationships which are well outside the acceptable band of fluctuations. Second, once the class of detrending methods is appropriately selected, one finds that, within standard business cycle frequencies, the sign of the correlation changes as we move from short to medium cycles. Although the change is not large, it is suggestive of the instabilities which may occur within business cycle frequencies and which are completely neglected by analyses which consider one detrending method only. For the case of labor hoarding, this instability conforms with economic intuition. For other cases, switches of this type warrant careful theoretical examination.

4.5 Is the Cycle Driven by Supply or by Demand?

I conclude this section by examining the implications of the patterns of impulse responses discussed in section 3.2.4 for questions concerning the generation of cycles in the economy. Although the exercise is incomplete because I do not attempt to fully identify the behavioral disturbances of the system, it is useful to stress once again that, lacking a precise definition of what business cycles are, one can by shrewdly search across detrending methods and produce results that confirm different prior expectations, without altering the set of variables or the time span used.

The two patterns of responses discussed in section 3.2.4 seem to agree with two different theoretical characterizations of business cycle fluctuations. The first pattern fits a RBC tale: a temporary shock to output increases labor demand, so that hours and the real wage go up within a year's time. As the real wage increases, consumption increases and investment follows. Since the average productivity increases more than the real wage, profits increase and payments to holders of capital rise as well (average product of capital = GNP/capital is clearly positive in the first stages of the cycle). Therefore the real return per unit of capital invested increases. This increase is correlated with the increase in hours. Hence hours move together with this measure of the real rate of return, a result which is consistent with the RBC emphasis on intertemporal substitution of labor. In addition, the responses of productivity are approximately coincident with the responses of GNP, a result which goes against the labor-hoarding explanation of business cycle fluctuations.

The second pattern of responses, on the other hand, fits the neoKeynesian perspective better. A one standard error shock in GNP instantaneously increases consumption by about 1.2 times that amount and, because of wealth effects, decreases the amount of hours worked. To achieve this consumption increase, the economy depletes the capital stock and invests negative amounts. At least in the first phase of the cycle, the response of the average productivity of labor is negatively related to (and lags) output responses, a pattern which fits the labor-hoarding story discussed in section 4.4. The demand driven expansion caused by the increase in consumption induces a further increase in output in the short run, possibly through the use of idle capacity or overtime and this drives hours and real wages up. When the consumption boom is exhausted, previous decisions are reverted: agents enjoy increasing amounts of leisure pushing hours below their long run path in the medium run, investments decrease and the deterioration of the capital stock is reversed. The reconstruction of the capital stock is completed in about 8 quarters and convergence to its steady state path occurs after about 15 quarters. Finally, because the capital stock is countercyclical, the real interest rate is large and positive in the first few quarters of the cycle. Despite large interest rates and real wage movements, hours move, relatively speaking,by only a small amount, a result which agrees with recent neoKeynesian descriptions of the business cycle (see e.g. Mankiw (1989)).

While one need not agree with the exact details of the stories provided here, it is clear that there are two characterizations of the transmission of GNP shocks which are consistent with contrasting theories of business cycles fluctuations.

5 Conclusions

This paper examines how different detrending methods affect summary statistics for the cyclical components of some US real variables. I compare the properties of the cyclical components of seven variables (GNP, Consumption, Investment, Hours, Real Wage, Productivity and Capital) obtained using seven univariate (Hodrick-Prescott (HP), Beveridge-Nelson (BN), Linear (LT), Segmented (SEGM), First Order Differencing (FOD), Unobservable Components (UC), Frequency Domain Masking (FD)) and three multivariate (Common deterministic trend (MLT), One dimensional index (MFD) and Cointegration (COIN)) detrending techniques for seasonally adjusted data over the sample 1955-1986. For each method I report moments of the data, the short term cross correlations and the impulse response function when GNP is shocked and examine the robustness of some stylized facts of the business cycle.

The paper documents a wide range of outcomes with little agreement in the qualitative and the quantitative properties of the second moments, even among those methods which extract cycles of comparable length from the data. I find that higher moments are less sensitive to the issue of detrending methods but these statistics are seldom considered by business cycle researchers. Finally, the paper shows that the qualitative response to a shock in GNP can result in two broad patterns: one consistent with RBC stories and one with neoKeynesian tales. Quantitatively speaking, the length of a typical cycle, the size of the peak responses and the persistence of the variables strongly depend on the detrending procedure employed.

A few conclusions can be drawn from the exercise. First, the practice of solely employing the HP1600 filter in compiling business cycle statistics is dangerous. The HP1600 filter produces results which are similar those obtained with conventional band-pass filters (e.g. frequency domain masking the low frequency components of the data) and concentrates the attention of the researcher on cycles of average length of 4-6 years. However, there are some instances, viz. the productivity series, where a high proportion of the variability appears to be connected with cycles of slightly longer length and this high portion of variability is simply neglected in analyses which exclusively use the HP1600 filter. The choice of the researcher to concentrate on cycles of 4-6 years length may also induce either extreme second order properties in the detrended data and misdirect theoretical research trying to cope with them (see e.g. Hansen's (1985) effort to remedy Kydland and Prescott's (1982) failure to replicate the variability of hours or Christiano's (1988) attempt to replicate with the magnitude of investment volatility) or inappropriately characterize certain phenomena (e.g. labor hoarding). A more interactive relationship between theory and practice could be very useful here. Theory may indicate which cycles it wants to explain and therefore implicitly select a class of procedures which extract this type of cycles in the data and empirical practice should indicate whether this theoretical choice leaves out important features of the data or produces distortions of various kind.

Second, since there are no quantitative stylized facts and very few qualitative features of the data which are robust across detrending methods and frequencies, the practice of building theoretical models whose numerical versions quantitatively match business cycle facts warrants a reconsideration. Because the major differences occur around business cycle frequencies, numerical exercises should at least be enlarged to provide results obtained with filters which emphasize different business cycle periodicities and theoretical work should try to explain why certain variables display a differential behavior within business cycle frequencies, e..g. why productivity is procyclical when cycles of 5 years are considered but is almost acyclical for cycles of 3 year period.

Third, the results obtained with multivariate detrending methods which have their base in dynamic economic theory are different from those obtained with statistically based univariate procedures. Because, at least with the data set used here, there is very weak evidence of common (deterministic or stochastic) trends among the variables considered, caution should be exercised in imposing theoretical restrictions which are far from being satisfied in the data.

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Table 1
Standard Deviations
Sample, 55,3-86,3

Method	GNP	Consumption as % of GNP	Investment as % of GNP	Hours as % of GNP	Real Wage as % of GNP	Productivity as % of GNP	Capital as % of GNP
HP1600	1.76	0.49	2.82	1.06	0.70	0.49	0.61
HP4	0.55(*)	0.48(*)	2.70(*)	0.89(*)	0.65(*)	0.69(*)	0.14(*)
FOD	1.03(*)	0.51(*)	2.82(*)	0.91(*)	0.98(*)	0.67(*)	0.63(*)
BN	0.43(*)	0.75(*)	3.80(*)	1.64(*)	2.18(*)	1.14(*)	2.64(*)
UC	0.38(*)	0.34(*)	6.72(*)	4.14(*)	2.24	4.09(*)	1.22(*)
LT	4.03(*)	0.69(*)	2.16(*)	0.69(*)	1.71(*)	1.00(*)	1.56(*)
SEGM	2.65(*)	0.52(*)	3.09(*)	1.01(*)	1.10(*)	0.54(*)	0.97(*)
FREQ1	1.78	0.46	3.10	1.20	1.07(*)	0.66(*)	1.41(*)
FREQ2	1.14(*)	0.44(*)	3.00(*)	1.16(*)	1.11	0.69	1.26(*)
MLT	6.01(*)	0.67(*)	2.36(*)	0.46(*)	1.21(*)	1.00(*)	1.05(*)
MFD	3.47(*)	0.98(*)	2.65(*)	1.14(*)	1.27(*)	0.72(*)	1.85(*)
COIN	4.15(*)	0.71(*)	3.96(*)	0.75(*)	1.68(*)	1.09(*)	1.30(*)

Note: A "*" indicates rejection at the 5% confidence level of the null hypothesis that the variance of the cyclical component of the series is identical to the variance of the cyclical component obtained with the HP1600 filter.

Table 2
Cross-Correlations
Sample 55,3-86,3

	Lag	Methods											
		HP1600	HP4	FOD	BN	UC	LT	SEGM	FREQ1	FREQ2	MLT	MFD	COIN
C-GNP	-1	0.75	0.16(*)	0.35(*)	0.23(*)	0.79	0.90(*)	0.76	0.75	0.68	0.93(*)	0.79	0.82(*)
	0	0.75	0.31(*)	0.46(*)	0.42(*)	0.74	0.91(*)	0.81(*)	0.73	0.69	0.96(*)	0.84(*)	0.83(*)
	1	0.62	0.01(*)	0.21(*)	0.38(*)	0.61	0.89(*)	0.76(*)	0.57	0.52(*)	0.93(*)	0.85(*)	0.82(*)
I-GNP	-1	0.76	0.07(*)	0.25(*)	-0.08(*)	0.82	0.73	0.78	0.73	0.58(*)	-0.26(*)	0.81	0.28(*)
	0	0.91	0.65(*)	0.71(*)	0.45(*)	0.45(*)	0.77(*)	0.86	0.86	0.85	-0.26(*)	0.84	0.30(*)
	1	0.84	0.28(*)	0.39(*)	0.38(*)	0.73(*)	0.76(*)	0.79	0.80	0.85	-0.26(*)	0.80	0.31(*)
H-GNP	-1	0.67	0.11(*)	0.28(*)	0.27(*)	0.01(*)	0.25(*)	0.71	0.63	0.48(*)	0.17(*)	0.71	0.16(*)
	0	0.88	0.73(*)	0.75(*)	0.72(*)	0.17(*)	0.34(*)	0.85	0.83	0.80	0.22(*)	0.77(*)	0.24(*)
	1	0.90	0.44(*)	0.54(*)	0.30(*)	0.28(*)	0.37(*)	0.86	0.85	0.87	0.23(*)	0.78(*)	0.27(*)
W/P-GNP	-1	0.81	0.12(*)	0.34(*)	0.05(*)	0.80	0.89(*)	0.79	0.73	0.59(*)	0.79	0.64(*)	0.89(*)
	0	0.81	0.49(*)	0.69(*)	0.52(*)	0.85	0.92(*)	0.88	0.84	0.81	0.82	0.67(*)	0.91(*)
	1	0.63	-0.30(*)	0.42(*)	0.45(*)	0.79(*)	0.90(*)	0.84(*)	0.75(*)	0.79(*)	0.81(*)	0.65	0.90(*)
APL-GNP	-1	0.26	0.12(*)	0.04(*)	-0.06(*)	0.20	0.78(*)	0.33	0.11(*)	0.36	0.80(*)	0.17	0.76(*)
	0	0.10	0.49(*)	0.45(*)	-0.16(*)	0.06	0.76(*)	0.25(*)	-0.02	0.08	0.89(*)	0.14(*)	0.74(*)
	1	-0.25	-0.30	-0.30	-0.09(*)	-0.07(*)	0.70(*)	0.05(*)	-0.27(*)	-0.28	0.85(*)	0.06(*)	0.68(*)
H-W/P	-1	0.43	-0.08(*)	0.39	0.48	-0.16(*)	0.06(*)	0.79(*)	0.53(*)	0.65(*)	0.06(*)	0.19(*)	0.12(*)
	0	0.67	0.39(*)	0.68	0.64	-0.05(*)	0.10(*)	0.85(*)	0.67	0.80(*)	0.10(*)	0.23(*)	0.17(*)
	1	0.80	0.34(*)	0.53(*)	0.21(*)	0.05(*)	0.13(*)	0.86	0.69(*)	0.73	0.11(*)	0.24(*)	0.20(*)
H-APL	-1	-0.55	-0.36(*)	-0.37(*)	-0.13(*)	-0.95(*)	-0.38(*)	-0.41(*)	-0.67(*)	-0.73(*)	-0.25(*)	-0.53(*)	-0.49
	0	-0.24	-0.22	-0.24	-0.79(*)	-0.97(*)	-0.34	-0.29	-0.55(*)	-0.52(*)	-0.23	-0.50(*)	-0.46(*)
	1	-0.07	0.12(*)	0.00	-0.11	-0.89(*)	-0.26(*)	-0.13	-0.34(*)	-0.18	-0.19(*)	-0.43(*)	-0.39(*)

Note: A "*" indicates a rejection at the 5% confidence level of the null hypothesis that the correlation coefficient in the cell is identical to correlation coefficient obtained using the HP1600 filter.

Table 3
Skewness
Sample, 55,3-86,3

Method	GNP	Consumption	Investment	Hours	Real Wage	Productivity	Capital
HP1600	-0.024	-0.034	-0.367	-0.400(*)	-0.310	-0.235	-0.247
HP4	0.174	0.196	0.058	0.303	0.065	-0.186	0.082
FOD	-0.045	-0.322	-0.367	-0.328	0.048	0.006	-0.351
BN	-0.243	-0.141	-0.402(*)	0.326	-0.165	-0.415(*)	-0.259
UC	-0.028	-0.207	-0.342	-0.179	0.155	0.384	-0.220
LT	-0.114	-0.253	-0.460(*)	-0.389	-0.059	0.143	-0.320
SEGM	0.086	-0.322	-0.459(*)	-0.350	0.050	0.085	4.490(*)
FREQ1	-0.048	0.090	-0.316	-0.310	-0.209	-0.147	-0.187
FREQ2	0.156	0.056	-0.104	-0.252	0.026	0.139	0.584(*)
MLT	-0.210	-0.283	-0.478(*)	-0.385	-0.032	0.038	-0.320
MFD	0.125	-0.269	-0.309	-0.275	0.022	0.193	0.383
COIN	-0.146	-0.239	-0.423(*)	-0.376	0.188	0.025	-0.226

Note: A "*" indicates a rejection at the 5% level of the null hypothesis that the skewness coefficient in each cell is identical to the skewness coefficient obtained under normality.

Table 4
Excess Kurtosis
Sample 55,3-86,3

Method	GNP	Consumption	Investment	Hours	Real Wage	Productivity	Capital
HP1600	0.066	-0.077	1.382(*)	0.953	0.613	-0.068	0.949
HP4	-0.131	-0.616	0.512	0.455	0.115	0.134	0.395
FOD	-0.222	-0.220	0.788	0.111	0.063	-0.050	0.660
BN	-0.026	-0.576	0.906	0.087	0.126	0.560	0.625
UC	-0.153	-0.568	0.781	-0.050	0.630	0.062	0.758
LT	0.206	0.162	1.089(*)	0.938	0.575	-0.269	1.051(*)
SEGM	0.438	0.428	1.002(*)	0.744	0.769	0.605	38.08(*)
FREQ1	-0.068	0.490	1.336(*)	0.649	0.671	-0.191	0.517
FREQ2	0.464	-0.265	0.234	-0.052	-0.126	-0.518	1.570(*)
MLT	0.120	0.041	0.935	0.938	0.497	-0.259	1.051(*)
MFD	-0.048	-0.189	0.829	0.641	0.553	-0.477	0.599
COIN	-0.065	0.064	0.906	0.561	0.653	-0.264	0.854

Note: A "*" indicates a rejection at the 5% level of the null hypothesis that the value of the excess kurtosis in each cell is identical to the value appearing under normality.

Table 5
Summary Statistics for the Impulse Response Function
Sample 55,3-86,3

Method	Cycle Length	Consumption	Investment	Hours	Real Wage	Productivity	Capital
		Size and Location of The Peak Response					
HP1600	20	2 0.28	3 1.93	3 0.76	1 0.46	1 2.00	6 0.30
HP4	8	1 0.17	1 1.50	1 0.58	1 0.37	1 1.70	2 0.05
FOD	6	1 0.25	1 1.50	1 0.57	1 0.53	1 1.82	4 0.11
BN	8	1 0.30	1 2.10	1 1.24	1 1.42	1 0.84	5 2.18
UC	21	1 0.23	1 6.02	5 2.38	4 1.18	1 0.98	6 0.54
LT	48	3 0.26	3 1.80	3 0.79	3 0.56	1 2.03	44 0.54
SEGM	19	1 0.24	4 1.86	3 0.77	1 0.52	1 2.02	6 0.36
FREQ1	17	4 0.30	3 1.98	3 0.82	3 0.60	1 2.10	4 0.31
FREQ2	12	4 1.12	4 10.20	4 3.25	4 2.08	4 2.06	25 1.53
MLT	48	2 0.26	2 1.84	3 0.79	3 0.52	1 1.83	7 0.28
MFD	39	2 0.28	4 1.91	2 0.81	3 0.55	1 1.67	44 0.36
COIN	24	3 1.32	4 6.23	4 3.18	6 1.78	1 0.74	10 0.42

Note: Cycle length measures the span of time, in quarters, needed to complete a cycle in GNP. If multiple peaks occur, size and location refer to the first peak.

Appendix

Table A.1 reports the second order properties of alternative measures of consumption, hours and of productivity . We consider two consumption series (total consumption and consumption of durables), labor input as measured by establishment survey data and the productivity series LBOUT obtained from the Citibase Tape which measures private productivity. The statistics obtained with multivariate filters are computed by substituting one variable at a time in the original system, i.e. for each detrending method I detrended four different systems: one with total consumption in place of consumption of nondurables and services and the other five original variables, one with consumption of durables, investment in plants and equipments and the other four original variables, one with establishment hours and the other five original variables and one with productivity in place of the real wage series. Since with all detrending methods no skewness or excess kurtosis appears for these series, these higher moments are not presented.

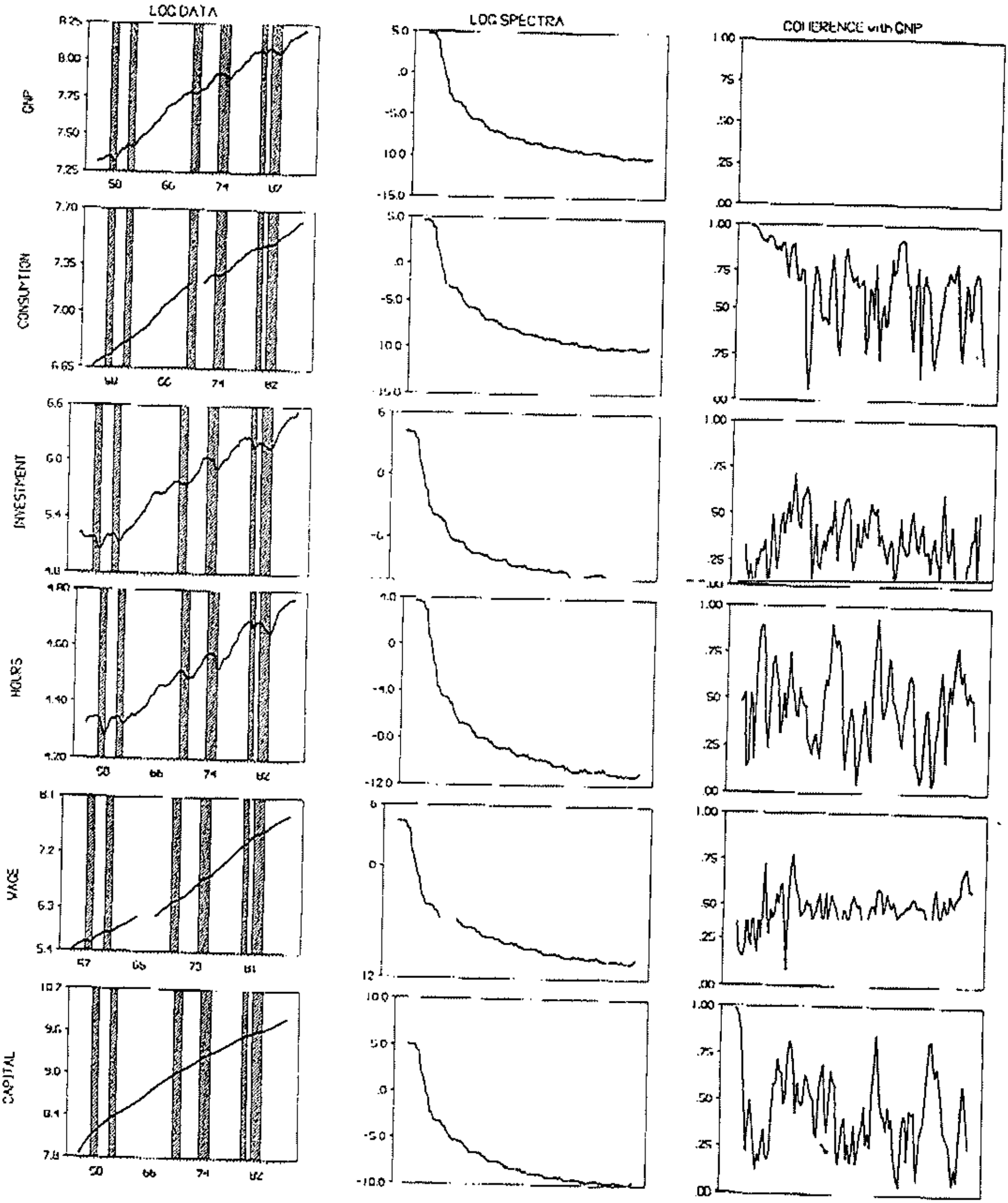
Figures A.1 through A.4 plot the cyclical component for nine series (GNP, Consumption, Investment, Hours, 2 measures of Real Wage, 2 measures of Productivity and Capital) obtained with the 12 different detrending methods. Shaded areas in the GNP boxes represent NBER recessions.

Table A.1
Standard Errors, as a percentage of GNP Standard Errors
Seasonally adjusted data

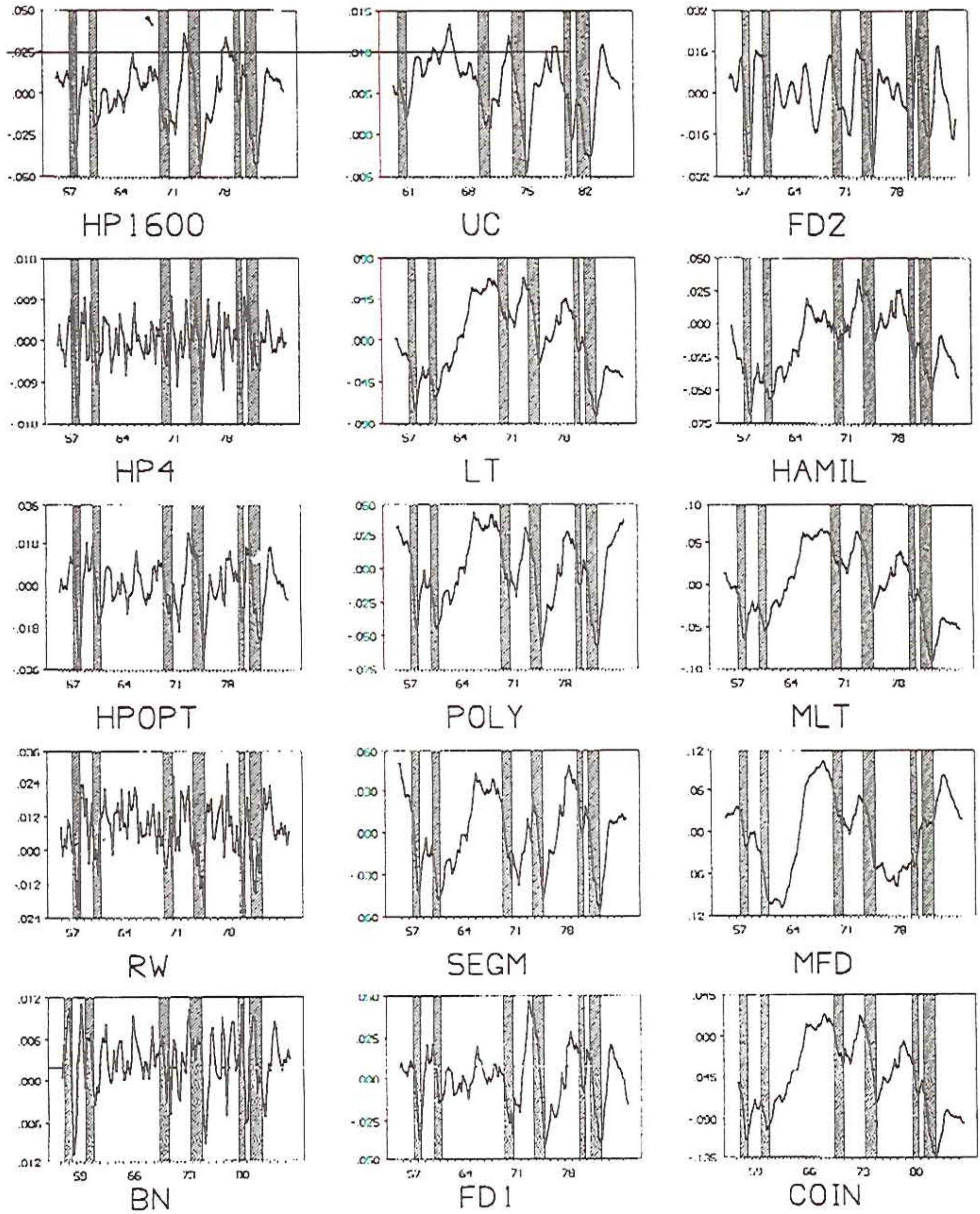
Method	Total Consumption	Consumption of Durables	Establishment Hours	Productivity	Real Wage
HP1600	0.69	4.89	0.82	4.84	0.78
HP4	0.71	5.02	0.84	5.51	0.78
RW	0.69	3.92	0.79	0.78	0.87
BN	0.76	3.98	0.58	0.72	2.92
UC	0.40	3.01	4.90	1.38	2.48
LT	0.77	1.97	0.62	1.15	1.80
SEGM	0.73	3.63	0.83	0.67	1.64
FREQ1	0.69	4.71	0.68	0.69	1.16
FREQ2	0.68	3.01	0.73	0.71	1.31
MLT	1.33	4.76	0.75	0.77	1.77
MFD	0.82	5.03	0.86	0.76	3.36
COIN	0.85	6.05	0.90	1.04	1.78

Table A.2
Cross-Correlations
Sample 55,3-86,3

		Methods											
	Lag	HP1600	HP4	FOD	BN	UC	LT	SEGM	FREQ1	FREQ2	MLT	MFD	COIN
CTOT-GNP	-1	0.81	0.01(*)	0.35(*)	0.90(*)	0.81	0.91(*)	0.77	0.82	0.78	0.92(*)	0.85	0.89
	0	0.81	0.50(*)	0.64(*)	0.90(*)	0.76	0.92(*)	0.81	0.80	0.82	0.94(*)	0.87	0.90(*)
	1	0.72	-0.15(*)	0.22(*)	0.87(*)	0.56(*)	0.88(*)	0.72	0.60(*)	0.64	0.93(*)	0.84(*)	0.87(*)
CDUR-GNP	-1	0.73	0.67	0.33(*)	0.30(*)	0.24(*)	0.32(*)	0.41(*)	0.70	0.66(*)	0.32(*)	0.82(*)	0.83(*)
	0	0.77	0.72	0.42(*)	0.31(*)	0.63(*)	0.39(*)	0.51(*)	0.75	0.77	0.81	0.90(*)	0.89(*)
	1	0.60	0.62	0.31(*)	0.22(*)	0.47(*)	0.45(*)	0.36(*)	0.66	0.72(*)	0.12(*)	0.78(*)	0.88(*)
H-GNP	-1	0.70	0.00(*)	0.27(*)	0.18(*)	-0.38(*)	0.05(*)	0.72	0.61(*)	0.43(*)	0.70	0.81	-0.01(*)
	0	0.86	0.41(*)	0.57(*)	0.25(*)	-0.35(*)	0.12(*)	0.82	0.76(*)	0.69(*)	0.74(*)	0.84	0.02(*)
	1	0.87	0.11(*)	0.42(*)	0.29(*)	-0.28(*)	0.15(*)	0.81	0.76(*)	0.64(*)	0.75(*)	0.82	0.04(*)
W/P-GNP	-1	0.35	0.04(*)	0.08(*)	-0.20(*)	0.05(*)	-0.53(*)	0.18(*)	0.30	0.39	-0.36(*)	0.50(*)	-0.58(*)
	0	0.64	0.71	0.56	0.28(*)	0.20(*)	-0.46(*)	0.29(*)	0.50(*)	0.66	-0.31	0.54(*)	-0.52(*)
	1	0.73	0.41(*)	0.36(*)	0.20(*)	0.27(*)	-0.41(*)	0.35(*)	0.55(*)	0.73(*)	-0.30(*)	0.57(*)	-0.47(*)
APL-GNP	-1	0.77	0.18(*)	0.21(*)	0.05(*)	0.72	0.85	0.58(*)	0.50(*)	0.66(*)	0.56(*)	0.30(*)	0.46(*)
	0	0.90	0.56(*)	0.59(*)	0.49(*)	0.62(*)	0.82	0.53(*)	0.42(*)	0.47(*)	0.55(*)	0.28(*)	0.74(*)
	1	0.75	-0.06(*)	-0.05(*)	0.37(*)	0.39(*)	0.76	0.34(*)	0.13(*)	0.09(*)	0.53(*)	0.20(*)	0.58(*)
H-W/P	-1	0.79	0.39(*)	0.50(*)	0.41(*)	0.82	0.55(*)	0.62(*)	0.70(*)	0.75	0.55(*)	0.82	0.59(*)
	0	0.82	0.84	0.76(*)	0.54(*)	0.88	0.55(*)	0.63(*)	0.76	0.89	0.55(*)	0.81	0.59(*)
	1	0.62	0.33(*)	0.42(*)	-0.06(*)	0.67	0.49(*)	0.56	0.64	0.78(*)	0.49(*)	0.78(*)	0.53(*)
H-APL	-1	0.58	0.26(*)	-0.17(*)	0.27(*)	-0.52(*)	0.27(*)	-0.08(*)	-0.27(*)	-0.33(*)	-0.27(*)	-0.38(*)	-0.40
	0	0.82	0.76	0.11(*)	0.40(*)	-0.43(*)	-0.21	0.07(*)	-0.07(*)	-0.07(*)	-0.21	-0.34(*)	-0.46(*)
	1	0.84	0.36(*)	0.22(*)	0.29(*)	-0.31(*)	-0.13(*)	0.20(*)	0.13(*)	0.24(*)	-0.13(*)	-0.27(*)	-0.43(*)



LOG DATA
LOG SPECTRA
COHERENCE with GNP
GNP
CONSUMTION
INVESTMENT
HOURS
WAGE
CAPITAL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SEGM
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BN
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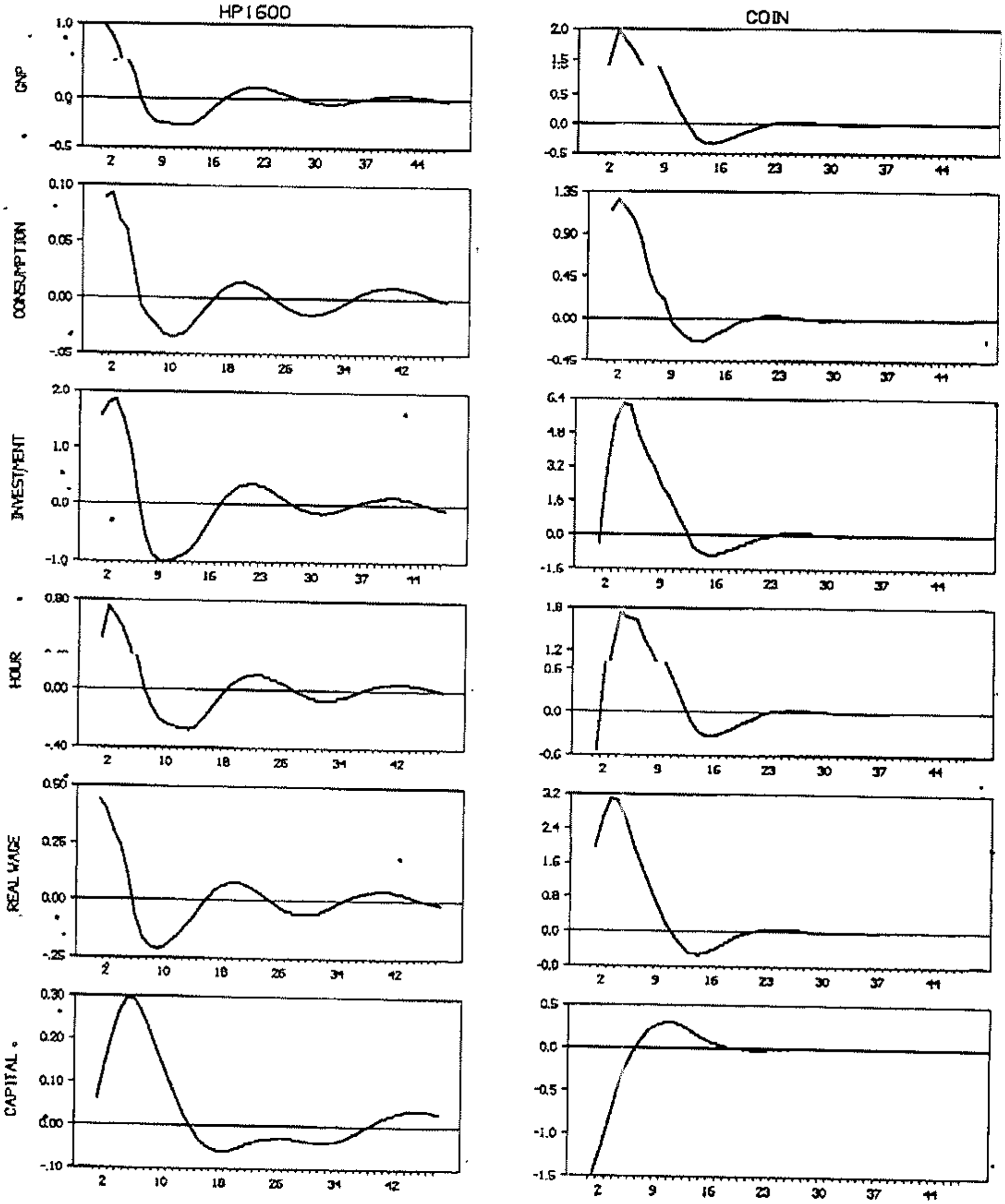

HP1600
COIN
GNP
CONSUMPTION
INVESTMENT
HOUR
REAL WAGE
CAPITAL