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James R. Markusen

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James R. Markusen, University of Colorado, Boulder and CEPR

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Centre for Economic Policy Research
53–56 Gt Sutton St, London EC1V 0DG, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Expansion of Trade at the Extensive Margin: A General Gains-from-Trade Result and Illustrative Examples*

The basic gains-from-trade theorem makes a stark comparison between completely free trade and complete autarky. This paper is motivated by recent evidence that trade has greatly expanded on the extensive margin (aka fragmentation, offshoring) by adding newly traded goods and services and that much of this new trade is in intermediates. I provide an extension of existing gains-from-trade results by allowing trade in an added set of final and/or intermediate goods. As seems generally understood, a sufficient condition for *all* countries to gain from fragmentation is that the relative *world* prices of initially-trade goods don't change. However, trade costs break the strict link between domestic and world prices in my approach and this results in interesting subtleties as initially-traded goods change their trade status following fragmentation. I illustrate these results by applying them to two recent and quite specific formulations of expansion at the extensive margin: Grossman and Rossi-Hansberg (2008) and Markusen and Venables (2007). Symmetry in two senses results in gains for all countries: countries are relatively symmetric in size and the newly-traded goods are relatively symmetric in their factor intensities with respect to the world endowment ratio.

JEL Classification: F10
Keywords: extensive margin, fragmentation, offshoring, trade in tasks and vertical specialization

James R. Markusen
Department of Economics
University of Colorado at Boulder
Campus Box 256
Boulder
Colorado 803-4-0256
USA
Email:
james.markusen@colorado.edu

* This paper is a revision of a long-stalled earlier version circulated and presented in 2006 and 2007 under the title “Expansion of trade at the extensive margin: welfare and trade-volume consequences”. The paper was presented at the Athens ETSG and at CESifo Munich in September 2007 and in a couple of other places I can’t remember. I appreciate comments received then and added comments, suggestions and references are most welcome now.

Submitted 14 April 2010

1. Introduction

There has been a lot of recent interest in the expansion of trade at the extensive margin, in which innovations in communications, transportation, and institutions have permitted a wider range of goods and services to be traded. This added trade at the extensive margin goes by a variety of names including "fragmentation", "vertical specialization", "trade in tasks", and "offshoring". These added goods and services have generally been modeled as intermediates, though there is no compelling reason to make such an assumption.

The basic gains-from-trade theorem which we all teach makes the stark comparison between completely free trade and complete autarky. While this is an important benchmark, no one claims that it is a very relevant comparison. Some generalizations are relatively straightforward, such as comparing restricted trade versus more liberal trade. In this latter case, requirements for gains are more demanding than in the simple free-trade versus autarky case: losses from liberalization are possible due to adverse world price changes.

The analysis and analytical tools needed to address fragmentation are rather different from those of traditional trade theory and computational analysis, in which a liberalization generates more trade in an existing set of goods and services. Applied general-equilibrium modeling has long suffered from failing to deal with trade at the extensive margin. Now we must focus on discrete changes in which liberalization switches some goods and services from non-traded to traded status and indeed some previously-traded goods could become non-traded or no longer produced in some countries. In analytical theory, many analyses use "cones" for example, which are fine in aiding our intuition about changes in specialization but relevant unit-value isoquants are going to move around with the general-equilibrium price changes that must accompany fragmentation. In short, analytical theory has been confounded by an inability to solve for world general equilibrium price changes and computational analysis by difficulties with "corner solutions" that are fundamental to understanding the consequences of fragmentation.

The purpose of this paper is to try to make progress in a more general approach to the gains from trade due to expansion of trade at the extensive margin. I begin with a general gains-from-trade result in which there are two sets of goods, traded and non-traded, any of which can be used as an intermediate and/or final good. An initially non-traded good could, for example, be produced by a single primary factor of production such as labor, making that good identical to a Grossman - Rossi-Hansberg (2008) "task". There are (or can be) trade costs on initially traded goods such that the relationship between domestic and world prices is not strictly pinned down by the latter: the domestic price differs according to whether the good is exported, imported, or non-traded. Liberalization takes the form of allowing trade in the non-traded goods. A given country may or may not start trading some of these goods, and any initially traded good may change its trade status.

The gains-from-trade result shows that a sufficient condition for all countries to gain from fragmentation is that the *world* relative prices of the initially-traded goods do not change. While this seems generally understood in models with costless trade, this severe condition is the weakest condition for Pareto improvements in standard models with no trade costs: with any change in world prices, the sufficient condition must fail for at least one country. With trade costs, this need not be the case since domestic prices of initially-trade goods may move differently from world prices as goods change their trade status. Pareto improving gains are possible in spite of some movement in world prices.[1]

The sufficient conditions still seem severe and so the paper then turns to two specific cases that have been analyzed in the last couple of years. The point is to get some idea about what sort of general circumstances might lead us to expect that gains from trade should hold for all countries and, equally importantly, what might be the "markers" for suspecting that a country might lose. Both examples use a simple Heckscher-Ohlin world with two initially-traded final goods and two primary factors, and no trade in intermediates. One formulation is a modification of Grossman and Rossi-Hansberg (2008), who model intermediates as "tasks", each task using one unit of a primary factor and each final good using all tasks. The two final goods differ in the amount of each labor task they use relative to the amount of each capital (skilled-labor) task they use. Then some subset of tasks, for example some of the labor-intensive tasks, become tradable. The other formulation is a modification of from Markusen and Venables (2007), with three intermediate goods producing the two final goods: a capital-intensive intermediate, a labor-intensive intermediate, and a middle intermediate. The capital-intensive final good uses the first two and the labor-intensive final good uses the last two. With only final goods initially traded, one or more of the intermediates then become tradable.

Both examples suggest that the sufficient conditions for gains and actual (numerical) gains are likely to occur to all countries when the countries and the fragmentation itself is relatively "symmetric". The countries are symmetric (by definition) when they are approximately the same size though differing in relative factor endowments. The fragmentation is symmetric when, for example, both a labor-intensive intermediate or task and a capital-intensive intermediate or task are both introduced into trade together. These two symmetries minimize or in the case of perfect symmetry eliminate terms-of-trade changes that violate the sufficient condition for gains. Numerical solutions in which one country loses always involve a deterioration in the terms of trade for the losing country as the general result requires, and always involve one of the two symmetries being violated. But the numerical solutions also emphasize the "sufficiency" part of the general result in that there are clear examples of where a country gains in spite of a significant terms-of-trade loss.

The final section is an adaptation of Markusen and Venables (2007) in which we propose a new geometric tool to analyze problems such as fragmentation in the presence of country-specific trade costs. Our "box" is a matrix of countries, with a country's factor endowments on one axis and a country's trade costs on the other. Every cell in the matrix is a distinct country, and all countries trade together simultaneously. Thus not only can a

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[1]The idea that gains moving from restricted (but positive) trade to more-liberal trade requires ruling out "on average" terms-of-trade losses is not new, though I don't know who to credit (a folk theorem?). I have had a proof of this in my PhD course notes since the late 1970s, but am not claiming credit for it. An excellent general discussion is found in Deardorff (2008) and the result is noted for trade in "tasks" in Baldwin and Robert-Nicoud (2010). Added references are welcome. A simple example of losses from liberalization: a large country that unilaterally drops its optimal tariff is going to be worse off due to the resulting terms-of-trade deterioration.

capital abundant country be compared to a labor-abundant country, a high-trade-cost capital-abundant country can be compared to a low-trade-cost capital-abundant country. The gains-from-trade result, or its violation, is nicely illustrated with this technique. Finally, all of these examples show that, due to trade costs, there are lots of cases where an initially traded final good becomes non-tradable and this is to the benefit of that country, illustrating another important feature of the gains-from-trade result.

2. A general gains-from-trade result

There are two sets of goods, X and Y goods. X are initially traded and Y are initially non-traded. p_i and q_i denote the domestic prices for X_i and Y_i. Any X or Y good may be used as an intermediate in any other good and may have no final use. A Y good could be produced with a single primary factor and used as an intermediate in all X goods, which is a Grossman and Rossi-Hansberg's (2008) "task".

We will use a simple trade-cost formulation from Markusen and Venables (2007). Assume the usual iceberg trade costs, and conceive of each country exporting and importing from a central entrepot. Costs are paid in both directions. Let superscript w denote world prices, and let t >1 denote the (gross) trade cost to our country in question. For one unit of a good exported, $1/t$ arrives, so if its world price at the entrepot is p^w, then price received for a unit of (unmelted) exports (E) must be p^w/t for the value to balance ($p^w(E/t)$ at the entrepot equals $(p^w/t)E$ exported). Similarly, an import good's domestic price is $p^w t$. With $t \geq 1$, the *domestic* price of an initially-tradable X good i must fall in the interval

$$p_i^w t \;\geq\; p_i \;\geq\; p_i^w / t \tag{1}$$

The domestic price p_i must lie at the left-hand boundary if it is an import good, at the right-hand boundary if it is an export good, and (weakly) in between if it is a non-traded good.

X_i and Y_i will denote the total (gross) output of these goods. The following notation is used for domestic intermediate use (of which some part might be imported X or Y).

XX_{ji}	intermediate use of X_j in X_i
YX_{ji}	intermediate use of Y_j in X_i
XY_{ji}	intermediate use of X_j in Y_i
YY_{ji}	intermediate use of Y_j in Y_i

$\sum_j p_j XX_{ji} \equiv pXX_i$	value of X intermediate use in X_i
$\sum_j q_j YX_{ji} \equiv qYX_i$	value of Y intermediate use in X_i
$\sum_j p_j XY_{ji} \equiv pXY_i$	value of X intermediate use in Y_i
$\sum_j q_j YY_{ji} \equiv qYY_i$	value of Y intermediate use in Y_i

There are m primary factors V in inelastic in supply, with domestic prices w_i.

V_{xim} factor m used in X_i V_{yim} factor m used in Y_i

$$\sum_m w_m V_{xim} \equiv w V_{xi} \qquad \sum_m w_m V_{yim} \equiv w V_{yi}$$

Let superscript f denote the quantity or *domestic* price of a good when trade in Y goods (fragmentation) is permitted and superscript n denote quantities and domestic prices when Y goods are not traded. Profit maximization in X industry i means that the profits for industry i at f-prices and f-quantities are (weakly) greatly than the value of any other feasible set of inputs and output at f-prices: in particular, the n-quantities which are feasible by definition

$$p_i^f X_i^f - p^f XX_i^f - q^f YX_i^f - w^f V_{xi}^f \geq p_i^f X_i^n - p^f XX_i^n - q^f YX_i^n - w^f V_{xi}^n \qquad (2)$$

Sum over all i industries

$$p^f X^f - p^f XX^f - q^f YX^f - w^f V_x^f \geq p^f X^n - p^f XX^n - q^f YX^n - w^f V_x^n \qquad (3)$$

Similarly for Y industries

$$q^f Y^f - p^f XY^f - q^f YY^f - w^f V_y^f \geq q^f Y^n - p^f XY^n - q^f YY^n - w^f V_y^n \qquad (4)$$

Add the (3) and (4) together, noting that total primary factor usage on both sides at prices w^f cancel out (on both sides of the equation usage sums to the (inelastic) total supply).

$$w^f V_x^f + w^f V_y^f = w^f V_x^n + w^f V_y^n = w^f V \qquad (5)$$

The sum of (3) and (4) then simplifies to:

$$\begin{aligned} &\left[p^f X^f - p^f XX^f - q^f YX^f + q^f Y^f - p^f XY^f - q^f YY^f\right] \geq \\ &\left[p^f X^n - p^f XX^n - q^f YX^n + q^f Y^n - p^f XY^n - q^f YY^n\right] \end{aligned} \qquad (6)$$

Rearrange the terms on the right-hand side of (6)

$$\begin{aligned} &\left[p^f X^f - p^f XX^f - q^f YX^f + q^f Y^f - p^f XY^f - q^f YY^f\right] \geq \\ &\left[p^f X^n - p^f XX^n - p^f XY^n\right] + \left[q^f Y^n - q^f YX^n - q^f YY^n\right] \end{aligned} \qquad (7)$$

The left-hand side of (7) is the value of net output in the f equilibrium and so, by the balance-of-trade constraint, is equal to the value of final consumption (subscript c) in the f equilibrium. Trade must balance at domestic prices ($p^w t$ for an import or p^w/t for an export). With no tariffs, the domestic price of any initially traded good will equal the world price corrected for trade costs. For any initially-non-traded good, production minus intermediate use equals consumption and so such a good cancels out of the balance-of-trade constraint at any price. The balance-of-trade constraint can thus be written in terms of *domestic* prices for all goods, traded and non-traded.[2] The left-hand side of (7) equals the value of consumption in the f equilibrium.

$$\left[p^f X^f - p^f XX^f - q^f YX^f + q^f Y^f - p^f XY^f - q^f YY^f\right] =$$

$$p^f X_c^f + q^f Y_c^f \equiv C^f \tag{8}$$

Y goods are non-trade in the n equilibrium, so the second term on the right-hand side of (7) (net output of Y good) is just the value of the consumption of Y goods in the n equilibrium evaluated at f prices.

$$q^f Y_c^n = \left[q^f Y^n - q^f YX^n - q^f YY^n\right] \tag{9}$$

Substitute (8) and (9) for the relevant terms in (7), then (7) becomes

$$\left[C^f\right] \geq \left[p^f X^n - p^f XX^n - p^f XY^n\right] + \left[q^f Y_c^n\right] \tag{10}$$

Trade balance in the n equilibrium is given by:

$$p^n X^n - p^n XX^n - p^n XY^n - p^n X_c^n = 0 \tag{11}$$

Add (11) to the right-hand side of (10), also add and subtract $p^f X_c^n$ from the right-hand side, and (10) becomes

$$\begin{aligned}\left[C^f\right] \geq \left[p^f X_c^n + q^f Y_c^n\right] + \left[p^f X^n - p^f XX^n - p^f XY^n - p^f X_c^n\right] \\ + \left[p^n X^n - p^n XX^n - p^n XY^n - p^n X_c^n\right]\end{aligned} \tag{12}$$

The first term on the right-hand side of (12) is the value of the n-equilibrium consumption at f-equilibrium prices which we can denote as:

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[2]The terminology here is not ideal, but I haven't come up with a better one. "Domestic" prices for trade goods here *are* world prices in the sense that they are the prices at which a country can trade. I am reserving the term "world" prices for p^w , the relative prices at the entrepot if you like. Defined this way, trade must balance at domestic prices as indicated. This would not be true with tariffs, in which case domestic prices do not equal the prices at which a country trades.

$$\left[C^{n}\right] \;\equiv\; \left[p^{f}X_{c}^{n} \;+\; q^{f}Y_{c}^{n}\right] \tag{13}$$

Finally, note that the second and third bracketed terms on the right-hand side of (12) are net exports of *X* goods in the n-equilibria: total output minus domestic intermediate use minus domestic consumption, the second term values at f-prices and the third term valued at n-prices. Let net exports be given by

$$\left[X^{k} \;-\; XX^{k} \;-\; XY^{k} \;-\; X_{c}^{k}\right] \;\equiv\; EX^{k} \qquad\qquad k = f, n \tag{14}$$

Then using (13) and (14), (12) becomes:

$$\left[C^{f}\right] \;\geq\; \left[C^{n}\right] \;+\; (p^{f} - p^{n})EX^{n} \tag{15}$$

Gains from allowing trade in *Y* goods makes the country better off if the left-hand side of (15) exceeds the first term on the right-hand side: the value of f-equilibrium consumption at f-equilibrium prices exceeds the value of n-equilibrium consumption at f-equilibrium prices; that is, f-equilibrium consumption is revealed preferred.

Suppose that fragmentation leaves relative *world* prices of *X* goods unchanged and refer back to (1). Consider an initially-exported good EX_{i}^{n} > 0, so that its initial *domestic* price is equal to 1/*t* times the world price: $p_{i}^{n} \;=\; p_{i}^{w}/t$, with the world price held constant by assumption after trade in Y is allowed.

$$\begin{array}{lll} (p_{i}^{f} \;-\; p_{i}^{n})(EX_{i}^{n}) \;=\; 0 & \qquad\text{good i continues to be exported} & \\ (p_{i}^{f} \;-\; p_{i}^{n})(EX_{i}^{n}) \;\geq\; 0 & \qquad\text{good i becomes non-traded} & \qquad (16) \\ (p_{i}^{f} \;-\; p_{i}^{n})(EX_{i}^{n}) \;\geq\; 0 & \qquad\text{good i switches to an import} & \end{array}$$

If export good i becomes non-traded or an import good, it’s price cannot be lower than the price at which it could be exported, $p_{i}^{n} \;=\; p_{i}^{w}/t$, and so $p_{i}^{f} \;\geq\; p_{i}^{n} \;=\; p_{i}^{w}/t$ for EX_{i}^{n} > 0 when i switches to an import or to non-traded. The argument and inequalities in (16) also hold for any initially imported good: $p_{i}^{f} \;\leq\; p_{i}^{n} \;=\; p_{i}^{w}t$ when EX_{i}^{n} < 0 initially and switches to an export or to non-traded..

Thus for any *X* good, that term in the right-hand summation in (15) must be non-negative. From (15), this is in turn a sufficient condition for

$$C^{f} \;\geq\; C^{n} \tag{17}$$

Trade in the f-equilibrium is revealed preferred to trade in the n-equilibrium for any/all countries.

General Result:

A *sufficient* condition for adding trade in *Y* goods to (weakly) improve the welfare of *all countries* is that the *world* relative prices of the *X* goods are unchanged.

A sufficient condition for adding trade in *Y* goods to improve the welfare of a given country is that the value of its initial net-export vector at post-*Y*-trade prices is positive: $p^f EX^n > p^n EX^n = 0$ (the initial net export vector would now generate a surplus; "on average", the terms-of-trade do not get worse).

Because $(p^f - p^n)EX^n > 0$ is defined at trade-cost-adjusted domestic prices, it is possible that this condition can be satisfied for all countries, contrary to the usual case where trade costs are zero.

Again, (a) this does *not* assume *domestic* prices are unchanged, since some goods may change their trade status. (b) the result covers Grossman and Rossi-Hansberg (2008) as a special case: *Y* goods are intermediate "tasks", with each *Y* produced with a single factor and each *Y* good used in all *X* goods.

In a formulation with two countries and *zero* trade costs, the inequality $(p^f - p^n)EX^n > 0$ cannot hold for both countries. Both price vectors must equal the world price vectors in each country, and one country's net export vector is equal and opposite in sign to the other country's vector. So in fact this term has an equal and opposite value in the two countries: the sufficient condition *cannot hold* for one of the countries. But it is worth noting that this need not be the case in the presence of trade costs and we will see the relevance later in the paper. Suppose that good i is initially exported from country e and imported in country m, and so the initial price relationships are

$$p_i^{en} t = p_i^w = p_i^{mn}/t \qquad \Rightarrow \qquad p_i^{en} t^2 = p_i^{mn} \tag{18}$$

A quick numerical example should make the point. Let $t^2 = 1.2$ (making t approximately 1.095) and suppose that the exporter's price is one and the importer's price is 1.2. If the good becomes non-traded, the domestic price might be 1.1 in each country, in which case the (initial) exporter's domestic price rises $(p_i^{ef} > p_i^{en})$ and the importer's price falls $(p_i^{mf} < p_i^{en})$: the sufficient condition in (16) holds as a *strict* inequality for *both* countries. With reference to the inequalities in (1), the importer's price moves in from the left-hand boundary and the exporter's price moves in from the right-hand boundary. This is not an arcane technical point: it will be highly relevant because fragmentation will permit some countries to stop importing expensive goods and just import the fragment of it that they are bad at producing themselves.

3. Example 1: based on Grossman and Rossi-Hansberg (2008)

It is nice to have some specifics to chew on; sufficient conditions in particular leave a lot of ambiguity and are puzzling in particular if they are unlikely to be satisfied by all country at the same time. The first specific case I will show is a much-simplified version of Grossman and Rossi-Hansberg (2008) (GRH). The simple version is shown in Figure 1. There are two final goods, X_1 and X_2 that produce utility, and two primary factors, L and K which themselves are non-traded. L is (endogneously) divided into two task, labeled TL_1 and TL_2 and similarly for capital. All goods require all tasks and a particular good requires TK_1 and TK_2 in identical amounts and there is no substitution between them and similarly for talks TL_1 and TL_2. Where the final goods X_1 and X_2 differ is in the amounts of labor versus capital tasks, and X_1 is assumed capital intensive. In the illustration of Figure 1, X_1 requires 60 each of both TK_1 and TK_2 and 40 each of both TL_1 and TL_2. X_2 is the symmetric mirror image, requiring 40 each of TK_1 and TK_2 and 60 each of TL_1 and TL_2.

The benchmark is that goods X_1 and X_2 can be traded while none of the tasks can be traded initially. I assume in the simulations that there are trade costs of five percent (t = 1.05) initially, partly to break factor-price equalization for similar countries. I assume that there is an elasticity of substitution of 1.0 between K and L nests for both goods and utility is Cobb-Douglas as well. Countries can differ in relative and/or absolute factor endowments and I will compute a series of outcomes over a world Edgeworth box as shown in Table 1. The focus is on country h whose origin is at the southwest corner of the box. Below the NW-SE diagonal country h is small and above the SW-NE diagonal it is capital abundant. The model is solved with GAMS, whose non-linear complementarity solver is extremely robust to corner solutions.

The first experiment in Table 1 is to allow costless trade in task TL_1 only, no trade in TL_2 or in K tasks, with results in panels A and B. Note that this is not the same as allowing labor mobility as strongly emphasized by GRH. Foreign labor can essentially enter to do task TL_1 but cannot do task TL_2, which must be done by domestic labor. Thus allowing trade in TL_1 may well not equalize labor returns across countries. The numbers in panel A are the proportional changes in the welfare of country h relative to the benchmark with X and Y traded at 5 percent costs (held constant in the experiment).

Most all welfare changes in Panel A are positive, except of course on the SW-NE diagonal (no trade). The changes are naturally bigger when country h is small. There are three points of loss which are shaded in Panel A. Panel B plots proportional changes in the relative price of country h's export good, so this is the change in p_1/p_2 above the NE-SW diagonal and p_2/p_1 below (so any plus is a terms-of-trade improvement and a minus is a loss for h). Comparing the points of loss in Panel A to the corresponding points in Panel B, we see that losses in the former are all points of terms-of-trade deterioration in Panel B as required by the general result above. However, we also see that this result is only a sufficient condition in that there are a number of points in Panel B where the terms-of-trade deteriorates yet welfare improves.

The intuition behind the points of loss in Panel A, due to the deterioration in the relative price of X_2, country h's export good at these points, is explained by a sort of monopoly-power in trade argument. Country h is somewhat small than f at these points, and has a favorable terms of trade (relative price of X_2) initially. But the movement of labor (in the factor content sense) to country f after liberalization allows f to expand production of X_2 more than it shrinks in country h, thus driving down the relative price of X_2. This terms-of-trade deterioration for h outweighs the trade-creation by exporting task TL_1.

Panels C and D of Figure 2 show the effects of a symmetric experiment in which both TL_1 and TK_1 become traded. All welfare changes in Panel C are positive and symmetric and, of course, they must also be the same for country f (if we look at cell (i,j) for country h, country f's change is given in cell (j,i)). However, recall that we mentioned above that differences in country size are also a form of asymmetry. Thus we see a number of cells in Panel D in which the relative price of country h's comparative-advantage good falls following fragmentation. The intuition lies in a monopoly or simple scarcity concept. The relatively small country has a terms-of-trade advantage when only X_1 and X_2 are traded. Allowing trade in tasks essentially erodes these scarcity rents and the relative price of the comparative-advantage good falls. Welfare rises nevertheless at all points in Panel C.

Another interesting feature of the results is that a large majority of the relative price changes in Panel B and Panel D of Figure 2 are positive.. How can this be? Surely in the symmetric case of Panel D there should be an equal number of negative and positive changes since the numbers are identically the changes in the (inverse) relative price in country f? The explanation lies in one of the final goods becoming non-traded as discussed in section 2, permitting a positive terms-of-trade (really the domestic price ratio) change in both countries.[3]

As an example, consider points in the two rows above the SW-NE diagonal in Panels B and D. In the first row above the diagonal, the trade cost is such that there is no trade in X_1 and X_2 initially, and so the opportunity to trade tasks must be welfare improving and the relative price of the comparative-advantage good must rise; that is rather trivial. Points two rows above the diagonal such as point (0.5, 0.3) involve trade in X_1 for X_2 initially. The opportunity to trade tasks leads to an elimination of trade in X_2 in Panel B: country h simply imports task TL_1 to complement its scarce labor supply, and produces X_2 more cheaply than it imported it under goods-trade only: $(p_1^f/p_2^f) > (p_1^n/p_2^n)$ which is an improvement in country h's relative price. Country f does better exporting TL_1 than in exporting X_2 and so this drives up the relative price of X_2 in country f: $(p_1^f/p_2^f) < (p_1^n/p_2^n)$ for country f which is an improvement in its relative price. With reference back to the sufficient conditions in (15) and (16), this is a clear case where the condition holds for both countries, as it does at a number of point in both the experiments of Panel B and Panel D: hence the "terms-of-trade" improvement for *both* countries (see footnotes 2 and 3).

[3]Perhaps it is misleading to use the phrase "terms of trade" when a good becomes non-traded; perhaps a better label would be "the relative price of the country's comparative advantage good", though this seems awkward. A feature of the general result is that it uses *domestic* prices (given no tariffs). Henceforth, "terms-of-trade" refers to the *domestic* price ratio. See also footnote 2.

4. Example 2: based on Markusen and Venables (2007)

A second example is a significant modification of Markusen and Venables (2007). Again, we start with a 2x2x2 Heckscher-Ohlin world, but intermediates are added. The model is shown in Figure 2, where there are three symmetric intermediate goods *A*, *B*, and *C*. As in GRH, these are pure intermediates and not used in final consumption. *A* is the most capital intensive and is used in X_1, *B* is in the middle and is used in both X_1 and X_2, and *C* is the most labor intensive and is used only in good X_2. At benchmark prices of one (countries identical), the capital/labor ratio in X_1 is 70/30 and that in X_2 is 30/70. Symmetry is built into the model quite deliberately.

The numerical model uses the same total factor endowments and preferences as GRH and Cobb-Douglas substitution is used the upper nest (between intermediate goods) in both this section and the previous one. Trade costs for final goods are 5 percent as in the previous section. So the two treatments are very similar except for the structure of intermediates. Experiments similar to those of Table 1 are shown in Table 2. Panels *A* and *B* consider a non-symmetric fragmentation in which free trade in *C* is added to the benchmark of trade in X_1 and X_2 only, and the results show some qualitative similarity to GRH. Panel A of Table 2 shows some points of welfare loss in approximately the same place as Panel A of Table 1 (GRH). Panels B in the two figures are also similar and show that the points of welfare losses in both cases are associated with a terms-of -trade deterioration as the general result requires. Again, the simulations indicate and emphasize that the general result is a sufficient condition, in that there are a couple points of terms-of-trade deterioration in which welfare nevertheless increases.

Panels C and D of Table 2 allow trade in both *A* and *C* intermediates, a symmetric fragmentation, so the results on welfare and price changes are symmetric. However, there are still point of welfare loss for country h (Panel C) and again we see that these are necessarily associated with adverse changes in the prices of the initially-traded goods. While the fragmentation itself is symmetric, the countries are of different size: as in the case of Panel C of Table 1 (GRH), country h is somewhat smaller is the points of welfare loss.

The welfare losses in Panels A and C of Table 2 have a subtle intuition; we concentrate on Panel A, for example cell (9,7) of Panel A where the welfare loss is 13.9 percent. Country h is initially specialized in X_2 and has an endowment ratio well suited to producing the integrated good by dividing its endowment between *B* and *C*. When *C* can be traded, country h exports *C* and country f has a significant expansion in its production of X_2. This pushes down the relative price of X_2 and, while country h gets a small increase in the price of *C* this is outweighed by a much bigger fall in the price of *B* (non-traded). The negative effect of the price change in X_2 (33 percent in Panel B) causes a fall in country h's welfare. This is somewhat easier to explain in a multi-country context, the subject of the next section, and we will pick up on the intuition at the end of that section.

5. An extension to a multi-country case

A well-understood limitation of the world Edgeworth or Dixit-Norman (1980) box technique is that it is limited to two countries. There is no sense in which there can be high trade-cost countries and low trade-cost countries. This section presents a multi-country generalization based again on Markusen and Venables (2007). The structure of production is the same as in the previous section. There are two final goods, X_1 and X_2, and three intermediate goods: *A*, *B*, and *C*. *A* and *B* are inputs into X_1 production and *B* and *C* are inputs into X_2 production. Factor intensities are the same as those shown in Figure 2 and all countries have identical and homothetic preferences, with shares 50/50.

There are many countries all of which trade together simultaneously, with each country identified by a double index, one referring to the country's endowment ratio and one referring to its country-specific trade cost (costs to and from the entrepot). Countries' endowments are evenly and symmetrically distribution along a line, with the most capital abundant country having endowments $K = 0.90$ $L = 0.10$ and the most labor abundant country having endowments $K = 0.10$ $L = 0.9$. There are an odd number of countries with the central country having an endowments $K = 0.5$ $L = 0.5$. A country's endowment is indexed by j.

Trade costs are country specific and apply to imports and exports from/to all countries. We could think of trade costs as being port costs only. The marginal cost of added distance is zero. Bilateral trade flows will thus not be determined, a limitation of the model. In addition to an endowment index j, a country has a trade-cost index i, which is common to all imports and exports. There are exactly i countries with endowment index j and j countries with trade-cost index i. Our countries form an ixj matrix, with exactly one country in each cell of the matrix. Unlike the world Edgeworth-box approach, all countries trade at once.[4]

We assume that the final goods X_1 and X_2 are always tradable at a country's country-specific trade cost (although autarky is computed as a benchmark). None, some, or all of the intermediates may be tradable, at each country's country-specific trade cost depending on the experiment. Primary factors are not tradable as noted above. "Fragmentation" is short-hand for the introduction of trade in some previously-non-traded intermediate. Referring back to the notion of asymmetries, in our example here there is essentially no asymmetry due to country size, since all countries will be a quite small share of the world endowment. But there can be asymmetries in the fragmentation itself in the sense that it is biased toward one final good or the other. In the present case, allowing trade in *B*, trade in *A* and *C*, or trade in *A*, *B*, and *C* are neutral or symmetric fragmentations. Allowing trade in *C* but not in *A* and *B* is an asymmetric fragmentation. As your intuition will likely suggest, the latter will increase the efficiency of X_2 production and will lower the price of X_2 relative to X_1 in equilibrium.

Figure 3 shows the experiment in which *A* and *C* become traded at each country's country-specific trade cost, shown in the Y axis (running front to back). Each cell of the

[4]The numerical model uses the GAMS MCP solver to solve 36,863 non-linear inequalities in the same number of complementary non-negative variables. Code written in Rutherford's higher-level MPS/GE language fits onto three pages using GAMS wonderful set features, available upon request. Fabulous. No, I have no financial connection with GAMS corporation.

Figure is a country, with most capitalabundant countries at the left and highest-trade-cost countries in the front; the back row of Figure 3 is a row of countries with zero trade costs (the view from this angle is better). The vertical axis plots the proportional welfare gains over trade in X_1 and X_2 only (not autarky). There are countries in the front middle (high-trade costs, near average endowments) that have zero gains because they do not trade before or after the liberalization or innovation that allows trade in *A* and *C*. The big gainers are the fringe countries in terms of endowments: either they stop producing *B* and specialize in *A* or *C* only (low-trade-cost fringe) or they leave autarky and stop producing intermediate good *A* or *C* (high-trade-cost fringe), importing their disadvantaged intermediate to combine with domestic good *B*. But interestingly, the countries with central relative endowments and relatively low trade costs gain.

The reasons for this are indicated in Table 3, where the price index of the zero-trade-cost country with the average world endowment is used as numeraire. When trade in *A* and *C* are allowed, fringe countries want to specialize more or completely in *A* or *C,* while the central countries have no incentive to specialize more in *B* at initial world prices. So it is basically a supply-demand issue: at initial relative prices p_1/p_2, there is excess supply of *A* and *C* and so their prices must fall relative to *C* to re-establish equilibrium. Table 3 notes that, while the relative prices of final goods don't change, the world price of *A* and *C* fall relative to *B*, which makes the central countries better off.

No one loses in this symmetric example with essentially identical country sizes as the general result suggests: relative world prices of X_1 and X_2 don't change. A few low-trade-cost countries have zero gains/losses, and these are countries which were well suited to integrated X_1 or X_2 production initially, and remain exporters of one final good and importers of the other after trade in *A* and *C* is allowed.

Figure 4 completes the analysis by showing the welfare gains from allowing trade in *C* (no trade in *A* or *B*) relative to trade in X_1 and X_2 only. *C* is the most labor-intensive intermediate and this is an opportunity for the labor-abundant countries. The supply to the world market of *C* by these countries pushes down the relative price of X_2 and *C* has a price below that of X_2, though its pre-fragmentation world price is not defined. Prices are shown in Table 3. Figure 4 shows that the most labor intensive countries are significant gainers. They become specialized in *C* and their *domestic* prices for *C* rise. Note that these countries are significant gainers in spite of the fact that the relative price of their initial export good X_2 falls, though this is not a violation of the sufficient condition for many of these countries: X_2 switches from being their export good to being non-traded or even imported and (16) is in fact satisfied despite the fall in the *world* price of the initially-exported good.[5]

The countries that experience losses are moderately labor-abundant countries, who produce and export good X_2 before and after the ability to trade C. The relative price of their initial export good falls, violating our sufficient condition, but they cannot escape this by switching to specializing in and exporting *C*. Thus they experience losses as indicated in the "basin of welfare losses" area of Figure 4.

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[5]The "plateau" in the back left region of Figure 4 is an area of welfare gains. These countries export X_1 before and after trade in *C* is allowed. They all get an equal terms-of-trade gain as p_1/p_2 rises (shown in Table 3).

6. Summary

The purpose of this paper is to trying to identify some general principles about the welfare effects of adding newly-traded goods and services to an existing set of traded goods. In my view, the existing theory literature is not very satisfactory on this, often because the tools applied do not allow the researcher to solve for world general equilibrium and world prices after fragmentation is allowed. Here I derive a general gains-from-trade result that gives benchmark sufficient conditions for added trade to be beneficial to one country or to all countries together. While this result has clear antecedents in the literature concerning going from partially liberal trade to more liberalized trade, an important innovation here is to add trade costs which allow domestic and world prices to differ and which allow some of the existing set of traded or tradable goods to move in and out of a country's trade vector following a liberalization.

Two specific examples are then examined which are simplifications and modifications of two recent papers, Grossman and Rossi-Hansberg (2008) and Markusen and Venables (2007). In both cases, results are consistent with the central result: a necessary condition for a country to lose is that it experiences a weighted adverse price change for its initially trade goods. At the same time, the sufficiency part of the general result is emphasized: there many cases in which a country gains substantially in spite of the sufficient condition failing. The role of trade costs, which imply that initially-traded goods often change trade status, is also illustrated and found to be important in these examples: for example, a country may stop importing an expensive good and only import the fragment of it that is costly to produce at home.

Results suggest that gains are likely to occur to all countries when the countries and the fragmentation itself is relatively "symmetric". The countries are symmetric (by definition) when they are approximately the same size. The fragmentation is symmetric when, for example, both a labor-intensive intermediate or task and a capital-intensive intermediate or task are both introduced into trade together. These two symmetries minimize or even eliminate terms-of-trade changes that violate the sufficient condition for gains.

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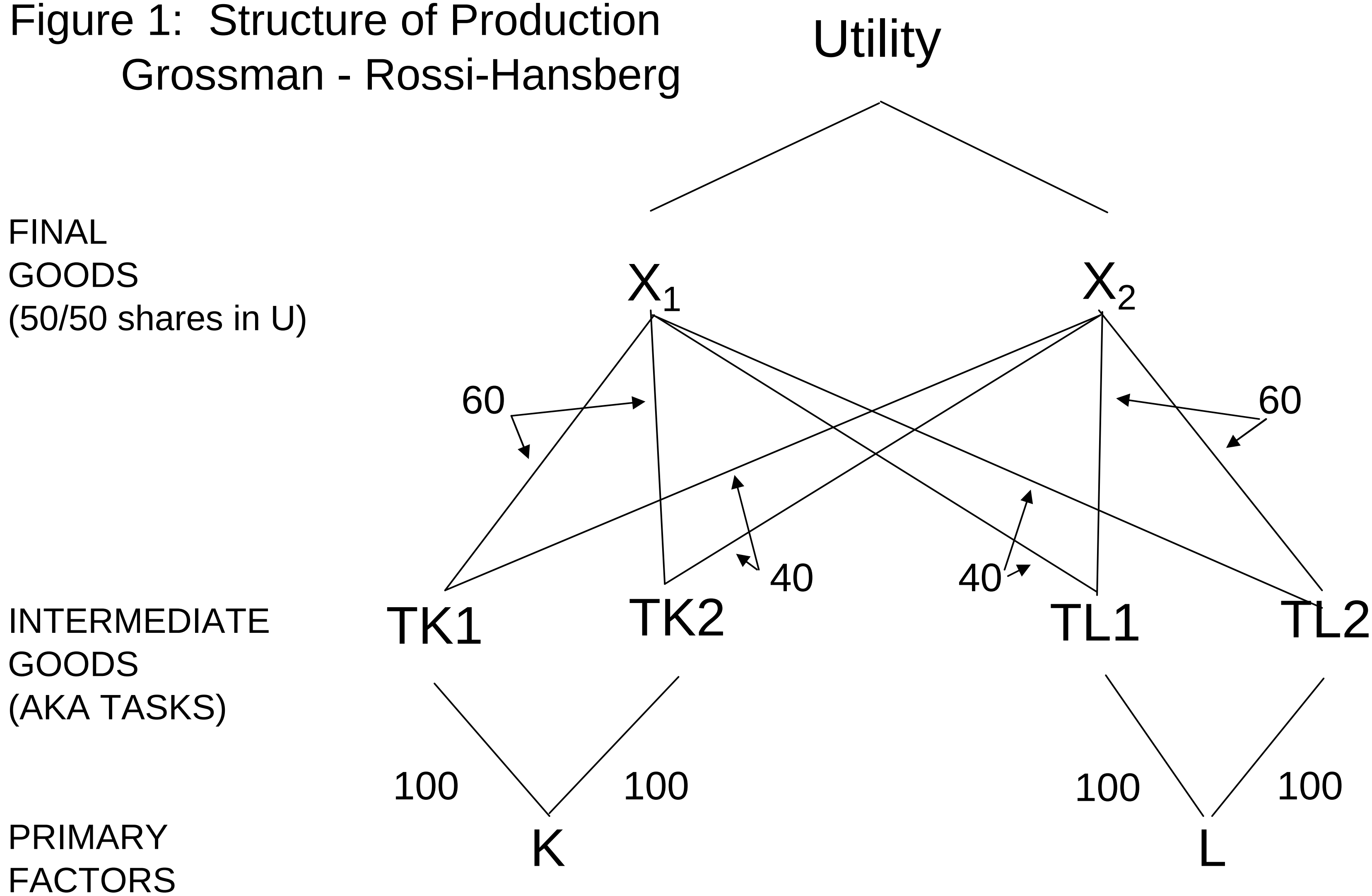
Figure 1: Structure of Production
Grossman - Rossi-Hansberg



FINAL GOODS	ALWAYS TRADABLE
INTERMEDIATE GOODS	NONE / SOME / ALL TRADABLE
PRIMARY FACTORS	NOT TRADABLE

Table 1: Fragmentation in the Grossman - Rossi-Hansberg Model

Panel A: proportional change in welfare of country h following trade in TL1

O_f

World endowment of capital	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.9	0.128	0.265	0.216	0.096	0.038	0.018	0.006	0.001	
0.8	0.113	0.127	0.143	0.048	0.020	0.007	0.002		0.001
0.7	0.181	0.065	0.055	0.022	0.010	0.002		0.001	0.001
0.6	0.156	0.070	0.030	0.014	0.003		0.002	0.006	0.007
0.5	0.125	0.056	0.022	0.004		0.003	0.008	0.010	0.010
0.4	0.091	0.036	0.007		0.004	0.013	0.019	0.016	0.014
0.3	0.077	0.015		0.007	0.020	0.027	0.024	0.049	0.019
0.2	0.039		0.014	0.034	0.040	0.045	-0.020	0.028	0.097
0.1		0.038	0.071	0.109	0.144	0.129	-0.020	-0.044	0.098

O_h

World endowment of labor

Panel B: proportional change in the price of h's export good with trade in TL1

O_f

World endowment of capital	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.9	-0.17	0.11	0.12	0.12	0.09	0.06	0.03	0.01	
0.8	-0.17	-0.04	0.10	0.09	0.05	0.03	0.02		0.01
0.7	-0.04	-0.07	0.04	0.04	0.03	0.03		0.02	0.01
0.6	-0.04	-0.04	0.05	0.04	0.03		0.02	0.02	0.03
0.5	-0.03	0.02	0.05	0.04		0.03	0.03	0.03	0.04
0.4	-0.03	0.06	0.05		0.04	0.04	0.04	0.04	0.04
0.3	0.00	0.06		0.05	0.05	0.04	0.02	0.08	0.04
0.2	0.07		0.06	0.06	0.03	0.00	-0.09	0.04	0.20
0.1		0.07	0.05	0.03	0.01	-0.02	-0.11	-0.10	0.20

O_h

World endowment of labor

Panel C: proportional change in welfare of country h following trade in TL1, TK1

O_f

World endowment of capital	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.9	0.229	0.244	0.174	0.075	0.038	0.017	0.007	0.001	
0.8	0.127	0.118	0.098	0.051	0.022	0.008	0.002		0.001
0.7	0.124	0.071	0.048	0.026	0.012	0.003		0.002	0.007
0.6	0.189	0.062	0.036	0.017	0.004		0.003	0.008	0.017
0.5	0.162	0.054	0.027	0.006		0.004	0.012	0.022	0.038
0.4	0.130	0.044	0.010		0.006	0.017	0.026	0.051	0.075
0.3	0.091	0.020		0.010	0.027	0.036	0.048	0.098	0.174
0.2	0.050		0.020	0.044	0.054	0.062	0.071	0.118	0.244
0.1		0.050	0.091	0.130	0.162	0.189	0.124	0.127	0.229

O_h

World endowment of labor

Panel D: proportional change in the price of h's export good with trade in TL1, TK1

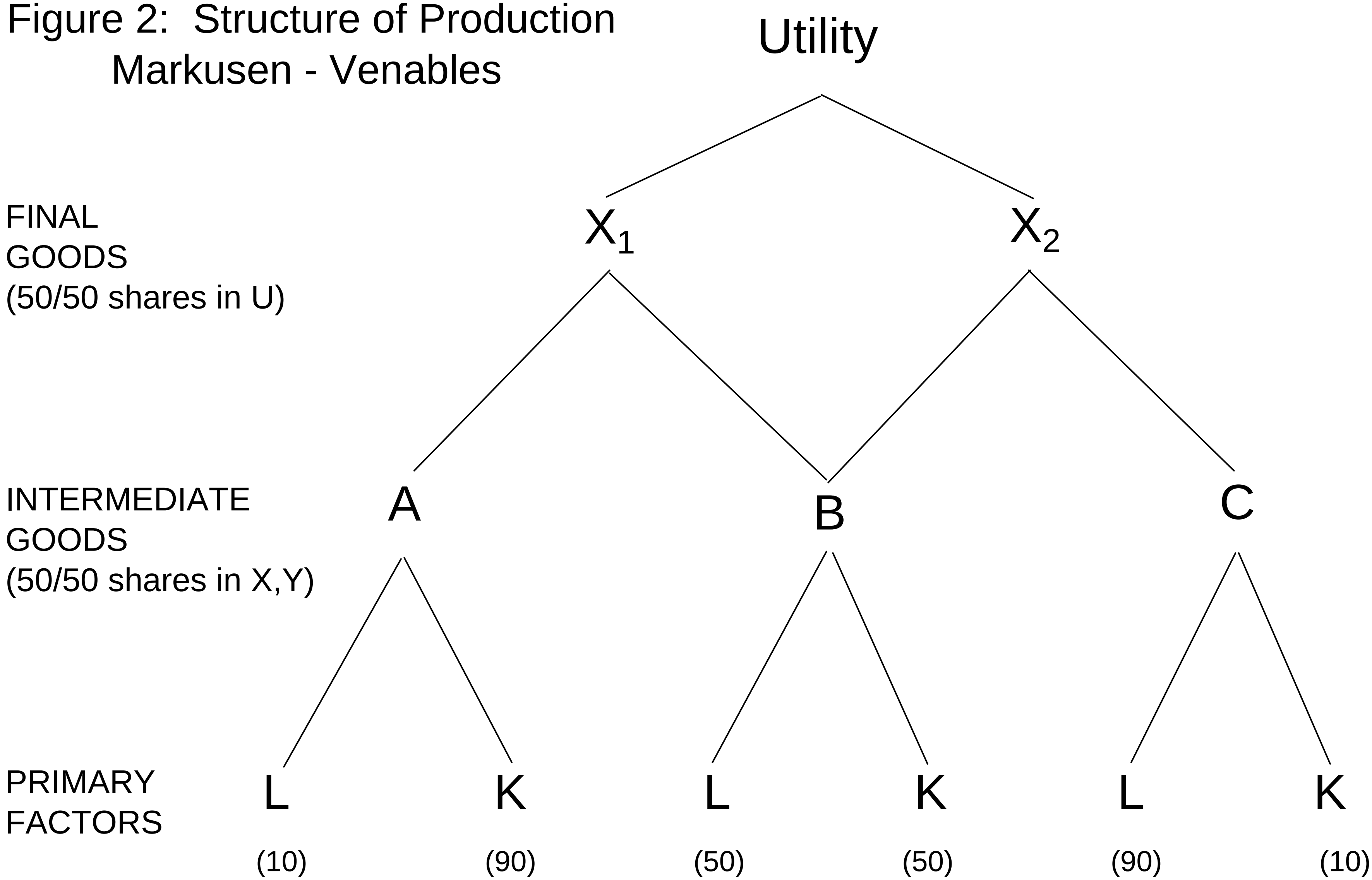
O_f

World endowment of capital	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.9	0.00	0.16	0.15	0.13	0.09	0.06	0.03	0.01	
0.8	-0.14	0.00	0.13	0.09	0.05	0.03	0.02		0.01
0.7	-0.13	-0.04	0.05	0.05	0.04	0.03		0.02	0.03
0.6	-0.04	0.01	0.05	0.05	0.04		0.03	0.03	0.06
0.5	0.00	0.05	0.06	0.05		0.04	0.04	0.05	0.09
0.4	0.03	0.07	0.06		0.05	0.05	0.05	0.09	0.13
0.3	0.07	0.08		0.06	0.06	0.05	0.05	0.13	0.15
0.2	0.09		0.08	0.07	0.05	0.01	-0.04	0.00	0.16
0.1		0.09	0.07	0.03	0.00	-0.04	-0.13	-0.14	0.00

O_h

World endowment of labor

Figure 2: Structure of Production
Markusen - Venables



FINAL GOODS	ALWAYS TRADABLE AT COUNTRY-SPECIFIC TRADE COST
INTERMEDIATE GOODS	NONE / SOME / ALL TRADABLE AT COUNTRY-SPECIFIC TRADE COST
PRIMARY FACTORS	NOT TRADABLE

Table 2: Fragmentation in the Markusen - Venables Model

Panel A: proportional change in welfare of country h following trade in C

O_f

World endowment of capital									
0.9	0.035	0.290	0.223	0.115	0.041	0.018	0.004	0.001	
0.8	0.000	0.036	0.102	0.044	0.007	0.004	0.001		0.000
0.7	0.000	0.000	0.043	0.011	0.005	0.002		0.001	-0.001
0.6	0.000	0.005	0.022	0.008	0.003		0.002	0.001	0.000
0.5	0.000	0.034	0.012	0.004		0.002	0.004	-0.011	0.000
0.4	0.000	0.021	0.006		0.003	0.006	0.002	-0.003	0.000
0.3	0.007	0.010		0.005	0.010	0.012	-0.026	0.000	0.000
0.2	0.022		0.009	0.016	0.019	-0.021	-0.086	-0.034	0.000
0.1		0.020	0.024	0.032	0.048	-0.024	-0.139	-0.204	-0.034
O_h	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

World endowment of labor

Panel B: proportional change in the price of h's export good with trade in C

O_f

World endowment of capital									
0.9	0.07	0.64	0.49	0.29	0.15	0.09	0.03	0.01	
0.8	0.00	0.07	0.21	0.10	0.03	0.02	0.02		0.01
0.7	0.00	0.00	0.07	0.03	0.03	0.02		0.01	-0.01
0.6	0.00	0.01	0.05	0.04	0.03		0.02	0.01	0.00
0.5	0.00	0.05	0.04	0.04		0.03	0.02	-0.05	0.00
0.4	0.00	0.05	0.04		0.03	0.03	0.01	-0.01	0.00
0.3	0.01	0.05		0.04	0.04	0.03	-0.06	0.00	0.00
0.2	0.05		0.05	0.04	0.04	-0.05	-0.17	-0.07	0.00
0.1		0.05	0.03	-0.02	-0.04	-0.14	-0.33	-0.39	-0.07
O_h	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

World endowment of labor

Panel C: proportional change in welfare of country h following trade in A and C

O_f

World endowment of capital									
0.9	0.024	0.261	0.197	0.115	0.041	0.016	0.003	0.001	
0.8	-0.156	0.024	0.075	0.026	0.006	0.005	0.002		0.001
0.7	-0.098	-0.023	0.024	0.014	0.007	0.003		0.002	0.003
0.6	-0.024	0.018	0.021	0.011	0.004		0.003	0.005	0.016
0.5	0.048	0.037	0.017	0.006		0.004	0.007	0.006	0.041
0.4	0.039	0.027	0.009		0.006	0.011	0.014	0.026	0.115
0.3	0.039	0.016		0.009	0.017	0.021	0.024	0.075	0.197
0.2	0.032		0.016	0.027	0.037	0.018	-0.023	0.024	0.261
0.1		0.032	0.039	0.039	0.048	-0.024	-0.098	-0.156	0.024
O_h	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

World endowment of labor

Panel D: proportional change in the price of h's export good with trade in A and C

O_f

World endowment of capital									
0.9	0.05	0.58	0.44	0.29	0.15	0.08	0.03	0.01	
0.8	-0.30	0.05	0.16	0.07	0.02	0.03	0.02		0.01
0.7	-0.23	-0.05	0.05	0.04	0.04	0.03		0.02	0.03
0.6	-0.14	0.03	0.05	0.05	0.04		0.03	0.03	0.08
0.5	-0.04	0.08	0.06	0.05		0.04	0.04	0.02	0.15
0.4	0.02	0.07	0.06		0.05	0.05	0.04	0.07	0.29
0.3	0.07	0.08		0.06	0.06	0.05	0.05	0.16	0.44
0.2	0.09		0.08	0.07	0.08	0.03	-0.05	0.05	0.58
0.1		0.09	0.07	0.02	-0.04	-0.14	-0.23	-0.30	0.05
O_h	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

World endowment of labor

Figure 3: Additional gains from allowing trade in A and C (no trade in B)

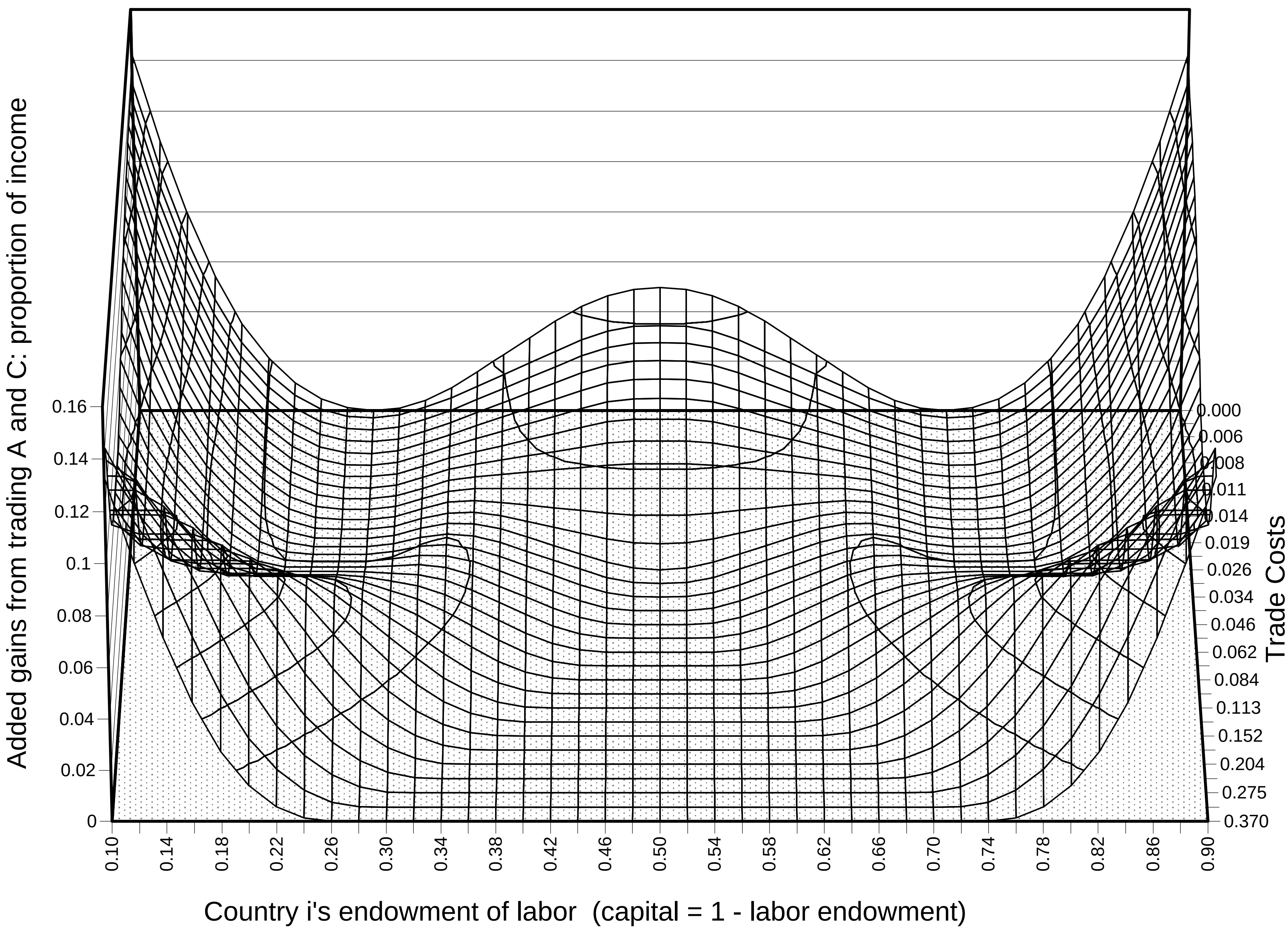


Figure 4: Additional gains from allowing trade in C (no trade in A or B)

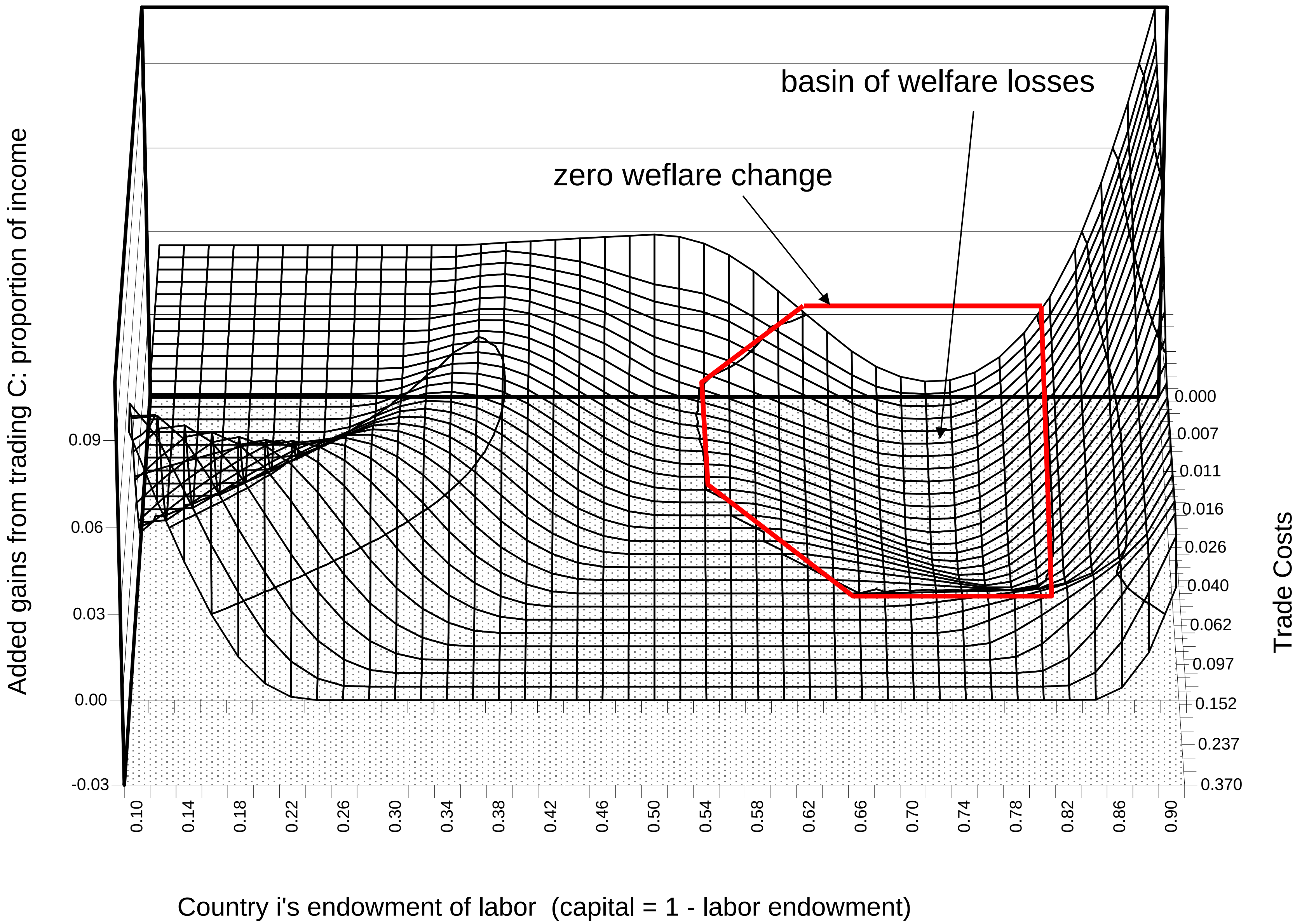


Table 3: World prices of p_1, p_2, p_a, and p_c in the multi-country fragmentation example

(for reference, p_b is the domestic price of B in the central (K/L = 1) zero-trade-cost country)

	p_1	p_2	p_a	p_c	p_b
Trade in X_1 and X_2 only	1.000	1.000			1.000
Add trade in A and C	1.000	1.000	0.952	0.952	1.000
Add trade in C only	1.025	0.976		0.925	1.000

The consumer price index $e(p_{1,}\ p_2)$ in the central (K/L = 1) zero-trade-cost country is numeraire

Thus all numbers shown are also the domestic prices of this central, free-trade country. All prices are World prices p^w except p_b, which is not traded in any of the scenarios

```
$TITLE: MULTI.GMS  for paper "Expansion of trade at the extensive margin..."
*  James Markusen author
*  model solves 36,863 non-linear inequalities in 36,863 unknows
*  each country designated by a two-dimensional set:
*  i = trade cost index  j = endowment index

SET     I   countries' trade-cost index    /1*31/,
        J   countries' endowment index     /1*41/,
        F   factors of production          /L, S/;

ALIAS  (F,FF);

PARAMETERS
 TC(I)                  trade cost X and Y (gross - one plus trade cost rate)
 TCA(I), TCB(I), TCC(I) trade costs for A B and C
 ENDOW(I,J,F)           endowment of country I-J of factor F

*  factor intentsities, all activities Cobb-Douglas

 AX          amount of A used in final good X
 BX          amount of B used in final good X
 BY          amount of B used in final good Y
 CY          amount of C used in final good Y
 FA(F)       primary factors used in intermediate A
 FB(F)       primary factors used in intermediate B
 FC(F)       primary factors used in intermediate C

*  indices used to show what each country produces in each "regime"

 RA(I,J)      if country i-j produces A then RA = 100  zero otherwise
 RX(I,J)      if country i-j produces A then RA = 10   zero otherwise
 RB(I,J)      if country i-j produces A then RA = 1    zero otherwise
 RY(I,J)      if country i-j produces A then RA = 0.1  zero otherwise
 RC(I,J)      if country i-j produces A then RA = 0.01 zero otherwise

*  regimes stored as numbers for each country i-j
*  AT - autarky, N - trade in X-Y only, AC - trade in X-Y-A-C, C trade X-Y-C

 REGAT(I,J)  equals RA(IJ)+RX(IJ)+RB(IJ)+RY(IJ)+RC(IJ) in the autarky eq
 REGN(I,J)   equals RA(IJ)+RX(IJ)+RB(IJ)+RY(IJ)+RC(IJ) with X-Y trade only
 REGAC(I,J)  equals RA(IJ)+RX(IJ)+RB(IJ)+RY(IJ)+RC(IJ) with X-Y and A and C trade
 REGC(I,J)   equals RA(IJ)+RX(IJ)+RB(IJ)+RY(IJ)+RC(IJ) with X-Y and C trade

 VOTN(I,J), VOTCAPN(I,J)  volume of trade and as a share of income X-Y trade
 VOTAC(I,J),VOTCAPAC(I,J) volume of trade and as a share of income X-Y-A-C trade
 VOTC(I,J), VOTCAPC(I,J)  volume of trade and as a share of income X-Y-C trade

 DVOTNAC(I,J) change in VOTCAP from adding A-C trade to X-Y trade
 DVOTNC(I,J)  change in VOTCAP from adding C trade to X-Y trade

 WELAT(I,J)   welfare of country i-j in autarky
 WELN(I,J)    welfare of country i-j with trade in X-Y only
 WELAC(I,J)   welfare of country i-j with trade in X-Y-A-C
 WELC(I,J)    welfare of country i-j with trade in X-Y-C

 DWELN(I,J)   change in welfare of country i-j from autarky to N (X-Y trade)
 DWELAC(I,J)  change in welfare of country i-j from N to AC
 DWELC(I,J)   change in welfare of country i-j from N to C

*  the star "*" is a GAMS wild card set designator "RES" is short for "RESULTS"
*  in the following, the wild card is a regime indicator (AT, N, AC, C)

 RESWEL(*,I,J) welfare of country i-j in regime *
 RESVOT(*,I,J) trade volume of country i-j in regime *
 RESREG(*,I,J) which goods country i-j produces in regime *

 WPRICE(*,*) first wild card: regime - second is world price of X-Y-A-B-C

*  these parameters will indicate if the model solves correctly

 MODELSTATAT, MODELSTATN, MODELSTATAC, MODELSTATC;

*  set factor intensities, all activities Cobb-Douglas

AX = .50;
BX = .50;
BY = .50;
CY = .50;
FA("L") = .20;
FA("S") = .80;
FB("L") = .50;
FB("S") = .50;
FC("L") = .80;
FC("S") = .20;
```

```
POSITIVE VARIABLES

* production acvitities

        X(I,J)     ! production level of X for country i-j
        Y(I,J)     ! production level of Y for country i-j
        A(I,J)     ! production level of A for country i-j
        B(I,J)     ! production level of B for country i-j
        C(I,J)     ! production level of C for country i-j
        EA(I,J)    ! exports of A by country i-j
        IA(I,J)    ! imports of A by country i-j
        EB(I,J)    ! exports of B by country i-j
        IB(I,J)    ! imports of B by country i-j
        EC(I,J)    ! exports of C by country i-j
        IC(I,J)    ! imports of C by country i-j
        EX(I,J)    ! exports of X by country i-j
        IX(I,J)    ! imports of X by country i-j
        EY(I,J)    ! exports of Y by country i-j
        IY(I,J)    ! imports of Y by country i-j
        XX(I,J)    ! production of X in i-j supplied to the domestic market
        YY(I,J)    ! production of Y in i-j supplied to the domestic market
        W(I,J)     ! welfare index for country i-j

* prices

        PX(I,J)       ! producer price (marginal cost) of X in country i-j
        PY(I,J)       ! producer price (marginal cost) of Y in country i-j
        PA(I,J)       ! producer price (marginal cost) of A in country i-j
        PB(I,J)       ! producer price (marginal cost) of B in country i-j
        PC(I,J)       ! producer price (marginal cost) of C in country i-j
        PCX(I,J)      ! consumer price of X in country i-j
        PCY(I,J)      ! consumer price of Y in country i-j
        PF(I,J,F)     ! price of factor F in country i-j
        PFA, PFB, PFC ! world price of A B C
        PFX, PFY      ! world price of X Y

* income and price of utility

        PW(I,J)       ! utility price index (real consumer price index) of i-j
        CONS(I,J)     ! income of representative consumer in country i-j;

EQUATIONS

* zero profit inequalities (marginal cost greater than or equal to price)

CX(I,J)     ! zero profits for X for country i-j
DY(I,J)     ! zero profits for Y for country i-j
CA(I,J)     ! zero profits for A for country i-j
CB(I,J)     ! zero profits for B for country i-j
CC(I,J)     ! zero profits for C for country i-j
CEA(I,J)    ! zero profits for exports of A by country i-j
CIA(I,J)    ! zero profits for imports of A by country i=j
CEB(I,J)    ! zero profits for exports of B by country i-j
CIB(I,J)    ! zero profits for imports of B by country i=j
CEC(I,J)    ! zero profits for exports of C by country i-j
CIC(I,J)    ! zero profits for imports of C by country i=j
CEX(I,J)    ! zero profits for exports of X by country i-j
CIX(I,J)    ! zero profits for imports of X by country i=j
CEY(I,J)    ! zero profits for exports of Y by country i-j
CIY(I,J)    ! zero profits for imports of Y by country i=j
CXX(I,J)    ! zero profits for production of X in i-j supplied to the domestic market
CYY(I,J)    ! zero profits for production of Y in i-j supplied to the domestic market
CW(I,J)     ! zero profits for utility in i-j

* market clearing inequalities

MPX(I,J)       ! supply-demand for X in country i-j
MPY(I,J)       ! supply-demand for Y in country i-j
MPA(I,J)       ! supply-demand for A in country i-j
MPB(I,J)       ! supply-demand for B in country i-j
MPC(I,J)       ! supply-demand for C in country i-j
MPCX(I,J)      ! consumer supply-demand X in country i-j
MPCY(I,J)      ! consumer supply-demand Y in country i-j
MPF(I,J,F)     ! supply-demand for factor F in country i-j

MPFA           ! world market for A
MPFB           ! world market for B
MPFC           ! world market for C
MPFX           ! world market for X
MPFY           ! world market for Y

* utility and income balance equations

MPW(I,J)       ! market for utility in country i-j
ICONS(I,J)     ! income of representative consumer in country i-j;
```

```
CX(I,J)..     100*PA(I,J)**AX * PB(I,J)**BX  =G=  100*PX(I,J);

DY(I,J)..     100*PB(I,J)**BY * PC(I,J)**CY  =G=  100*PY(I,J);

CA(I,J)..     100*PROD(F, PF(I,J,F)**FA(F)) =G= 100*PA(I,J);

CB(I,J)..     100*PROD(F, PF(I,J,F)**FB(F)) =G= 100*PB(I,J);

CC(I,J)..     100*PROD(F, PF(I,J,F)**FC(F)) =G= 100*PC(I,J);

CEA(I,J)..     100*PA(I,J)*TCA(I)  =G=  100*PFA;
CIA(I,J)..     100*PFA*TCA(I)      =G=  100*PA(I,J);
CEB(I,J)..     100*PB(I,J)*TCB(I)  =G=  100*PFB;
CIB(I,J)..     100*PFB*TCB(I)      =G=  100*PB(I,J);
CEC(I,J)..     100*PC(I,J)*TCC(I)  =G=  100*PFC;
CIC(I,J)..     100*PFC*TCC(I)      =G=  100*PC(I,J);
CEX(I,J)..     100*PX(I,J)*TC(I)   =G=  100*PFX;
CIX(I,J)..     100*PFX*TC(I)       =G=  100*PCX(I,J);
CEY(I,J)..     100*PY(I,J)*TC(I)   =G=  100*PFY;
CIY(I,J)..     100*PFY*TC(I)       =G=  100*PCY(I,J);
CXX(I,J)..     100*PX(I,J)         =G=  100*PCX(I,J);
CYY(I,J)..     100*PY(I,J)         =G=  100*PCY(I,J);

CW(I,J)..      200*PCX(I,J)**0.5 * PCY(I,J)**0.5  =G=  200*PW(I,J);


MPX(I,J)..     100*X(I,J) =G= 100*(XX(I,J) + EX(I,J)*TC(I));
MPY(I,J)..     100*Y(I,J) =G= 100*(YY(I,J) + EY(I,J)*TC(I));

MPA(I,J)..     100*(A(I,J) + IA(I,J) - EA(I,J)*TCA(I)) =G= X(I,J)*AX*100*PA(I,J)**AX * PB(I,J)**BX/PA(I,J);

MPB(I,J)..     100*(B(I,J) + IB(I,J) - EB(I,J)*TCB(I)) =G= X(I,J)*BX*100*PA(I,J)**AX * PB(I,J)**BX/PB(I,J)
                                                        + Y(I,J)*BY*100*PC(I,J)**CY * PB(I,J)**BY/PB(I,J);

MPC(I,J)..     100*(C(I,J) + IC(I,J) - EC(I,J)*TCC(I)) =G= Y(I,J)*CY*100*PC(I,J)**CY * PB(I,J)**BY/PC(I,J);

MPCX(I,J)..    100*(XX(I,J) + IX(I,J)) =G= W(I,J)*100*PCX(I,J)**0.5 * PCY(I,J)**0.5/PCX(I,J);
MPCY(I,J)..    100*(YY(I,J) + IY(I,J)) =G= W(I,J)*100*PCX(I,J)**0.5 * PCY(I,J)**0.5/PCY(I,J);

MPF(I,J,F)..   ENDOW(I,J,F) =G= A(I,J)*100*FA(F)*PROD(FF, PF(I,J,FF)**FA(FF))/PF(I,J,F)
                             +  B(I,J)*100*FB(F)*PROD(FF, PF(I,J,FF)**FB(FF))/PF(I,J,F)
                             +  C(I,J)*100*FC(F)*PROD(FF, PF(I,J,FF)**FC(FF))/PF(I,J,F);

MPFA..         SUM(I, SUM(J, 100*EA(I,J))) =G= SUM(I, SUM(J,100*IA(I,J)*TCA(I)));
MPFB..         SUM(I, SUM(J, 100*EB(I,J))) =G= SUM(I, SUM(J,100*IB(I,J)*TCB(I)));
MPFC..         SUM(I, SUM(J, 100*EC(I,J))) =G= SUM(I, SUM(J,100*IC(I,J)*TCC(I)));

MPFX..         SUM(I, SUM(J, 100*EX(I,J))) =G= SUM(I, SUM(J,100*IX(I,J)*TC(I)));
MPFY..         SUM(I, SUM(J, 100*EY(I,J))) =G= SUM(I, SUM(J,100*IY(I,J)*TC(I)));

MPW(I,J)..     200*W(I,J) =G=  CONS(I,J)/PW(I,J);
ICONS(I,J)..   CONS(I,J)  =E=  SUM(F, PF(I,J,F)*ENDOW(I,J,F));

MODEL MULTI / CX.X, DY.Y, CA.A, CB.B, CC.C,
 CEA.EA, CIA.IA, CEB.EB, CIB.IB, CEC.EC, CIC.IC,
 CEX.EX, CIX.IX, CEY.EY, CIY.IY, CXX.XX, CYY.YY, CW.W,
 MPX.PX, MPY.PY, MPA.PA, MPB.PB, MPC.PC, MPCX.PCX, MPCY.PCY, MPF.PF,
 MPFA.PFA, MPFB.PFB, MPFC.PFC, MPFX.PFX, MPFY.PFY, MPW.PW,
 ICONS.CONS/;


OPTION MCP=PATH;
OPTION LIMROW=0;
OPTION LIMCOL=0;
$OFFSYMLIST OFFSYMXREF OFFUELLIST OFFUELXREF
OPTION SOLPRINT = OFF;

* set starting values for variables

X.L(I,J) = 1; Y.L(I,J) = 1; A.L(I,J) = 0.5; B.L(I,J) = 1; C.L(I,J) = 0.5;

EA.L(I,J) = 0; IA.L(I,J) = 0; EB.L(I,J) = 0; IB.L(I,J) = 0; EC.L(I,J) = 0; IC.L(I,J) = 0;
EX.L(I,J) = 0; IX.L(I,J) = 0; EY.L(I,J) = 0; IY.L(I,J) = 0; XX.L(I,J) = 1; YY.L(I,J) = 1;

W.L(I,J) = 1;
PX.L(I,J) = 1; Y.L(I,J) = 1;  PA.L(I,J) = 1; PB.L(I,J) = 1; PC.L(I,J) = 1;
PCX.L(I,J) = 1; PCY.L(I,J) = 1; PF.L(I,J,F) = 1;
PFA.L = 1; PFB.L = 1; PFC.L = 1; PFX.L = 1; PFY.L = 1;
PW.L(I,J) = 1;  CONS.L(I,J) = 200;

*  choose the consumer price index of central country (average world endowment)
*  with zero trade cost to be numeraire

PW.FX("31","21") = 1;
```

```
* set endowments for country i-j (LOOP(j,  )
* everone and everything has prohibitive trade cost (LOOP(i,  )
* compute autarky equilibrium

LOOP(I,
LOOP(J,

ENDOW(I,J,"S") = (180+(160/40) - (160/40)*ORD(J));
ENDOW(I,J,"L") = (20-(160/40) + (160/40)*ORD(J));

TC(I) = 100;
TCA(I) = 100;
TCA(I) = 100;
TCB(I) = 100;
TCB(I) = 100;
TCC(I) = 100;
TCC(I) = 100;
);
);

MULTI.workspace = 25;
MULTI.ITERLIM = 20000;
SOLVE MULTI USING MCP;
MODELSTATAT = MULTI.MODELSTAT;

RA(I,J) = 100$(A.L(I,J) GT 0);
RX(I,J) = 10$(X.L(I,J) GT 0);
RB(I,J) = 1$(B.L(I,J) GT 0);
RY(I,J) = 0.1$(Y.L(I,J) GT 0);
RC(I,J) = 0.01$(C.L(I,J) GT 0);

REGAT(I,J) = RA(I,J) + RX(I,J) + RB(I,J) + RY(I,J) + RC(I,J) ;
WELAT(I,J) = W.L(I,J);
WPRICE("AT", "PX") = PFX.L;
WPRICE("AT", "PY") = PFY.L;
WPRICE("AT", "PA") = PFA.L;
WPRICE("AT", "PB") = PFB.L;
WPRICE("AT", "PC") = PFC.L;

*  allow trade in X-Y at each country's trade cost
*  compute regime N: trade in X-Y permitted

LOOP(I,

TC("31") = 1.0001;
TC(I)$(ORD(I) LT 31) = 1 + (1.16**(30 - ORD(I)))*0.005;

);

SOLVE MULTI USING MCP;
MODELSTATN = MULTI.MODELSTAT;

RA(I,J) = 100$(A.L(I,J) GT 0);
RX(I,J) = 10$(X.L(I,J) GT 0);
RB(I,J) = 1$(B.L(I,J) GT 0);
RY(I,J) = 0.1$(Y.L(I,J) GT 0);
RC(I,J) = 0.01$(C.L(I,J) GT 0);

REGN(I,J) = RA(I,J) + RX(I,J) + RB(I,J) + RY(I,J) + RC(I,J) ;
WELN(I,J) = W.L(I,J);
DWELN(I,J) = (WELN(I,J) -  WELAT(I,J))/WELAT(I,J);

VOTN(I,J) = (PX.L(I,J)*EX.L(I,J) + PCX.L(I,J)*IX.L(I,J) +
             PA.L(I,J)*EA.L(I,J) + PA.L(I,J)*IA.L(I,J) +
             PB.L(I,J)*EB.L(I,J) + PB.L(I,J)*IB.L(I,J) +
             PC.L(I,J)*EC.L(I,J) + PC.L(I,J)*IC.L(I,J) +
             PY.L(I,J)*EY.L(I,J) + PCY.L(I,J)*IY.L(I,J))/2;

VOTCAPN(I,J) = VOTN(I,J)/(PW.L(I,J)*W.L(I,J));
WPRICE("N", "PX") = PFX.L;
WPRICE("N", "PY") = PFY.L;
WPRICE("N", "PA") = PFA.L;
WPRICE("N", "PB") = PFB.L;
WPRICE("N", "PC") = PFC.L;

*  compute regime AC: trade in X-Y-A-C permitted

LOOP(I,
TCB(I) = 100;
TCA("31") = 1.00025;
TCA(I)$(ORD(I) LT 31) = 1.0001 + (1.16**(30 - ORD(I)))*0.005;
TCC("31") = 1.00025;
TCC(I)$(ORD(I) LT 31) = 1.0001 + (1.16**(30 - ORD(I)))*0.005;
);
```

```
SOLVE MULTI USING MCP;
MODELSTATAC = MULTI.MODELSTAT;

RA(I,J) = 100$(A.L(I,J) GT 0);
RX(I,J) = 10$(X.L(I,J) GT 0);
RB(I,J) = 1$(B.L(I,J) GT 0);
RY(I,J) = 0.1$(Y.L(I,J) GT 0);
RC(I,J) = 0.01$(C.L(I,J) GT 0);

REGAC(I,J) = RA(I,J) + RX(I,J) + RB(I,J) + RY(I,J) + RC(I,J) ;
WELAC(I,J) = W.L(I,J);
DWELAC(I,J) = (WELAC(I,J) -  WELN(I,J))/WELN(I,J);

VOTAC(I,J) = (PX.L(I,J)*EX.L(I,J) + PCX.L(I,J)*IX.L(I,J) +
            PA.L(I,J)*EA.L(I,J) + PA.L(I,J)*IA.L(I,J) +
            PB.L(I,J)*EB.L(I,J) + PB.L(I,J)*IB.L(I,J) +
            PC.L(I,J)*EC.L(I,J) + PC.L(I,J)*IC.L(I,J) +
            PY.L(I,J)*EY.L(I,J) + PCY.L(I,J)*IY.L(I,J))/2;

VOTCAPAC(I,J) = VOTAC(I,J)/(PW.L(I,J)*W.L(I,J));
DVOTNAC(I,J)= VOTCAPAC(I,J)- VOTCAPN(I,J);
WPRICE("AC", "PX") = PFX.L;
WPRICE("AC", "PY") = PFY.L;
WPRICE("AC", "PA") = PFA.L;
WPRICE("AC", "PB") = PFB.L;
WPRICE("AC", "PC") = PFC.L;

*  compute regime C: trade in X-Y-C permitted

LOOP(I,
TCA(I) = 100;
);

EA.L(I,J) = 0;
IA.L(I,J) = 0;
EB.L(I,J) = 0;
IB.L(I,J) = 0;
EC.L(I,J) = 1;
IC.L(I,J) = 1;

SOLVE MULTI USING MCP;
MODELSTATC = MULTI.MODELSTAT;

RA(I,J) = 100$(A.L(I,J) GT 0);
RX(I,J) = 10$(X.L(I,J) GT 0);
RB(I,J) = 1$(B.L(I,J) GT 0);
RY(I,J) = 0.1$(Y.L(I,J) GT 0);
RC(I,J) = 0.01$(C.L(I,J) GT 0);

REGC(I,J) = RA(I,J) + RX(I,J) + RB(I,J) + RY(I,J) + RC(I,J) ;
WELC(I,J) = W.L(I,J);
DWELC(I,J) = (WELC(I,J) -  WELN(I,J))/WELN(I,J);

VOTC(I,J) = (PX.L(I,J)*EX.L(I,J) + PCX.L(I,J)*IX.L(I,J) +
            PA.L(I,J)*EA.L(I,J) + PA.L(I,J)*IA.L(I,J) +
            PB.L(I,J)*EB.L(I,J) + PB.L(I,J)*IB.L(I,J) +
            PC.L(I,J)*EC.L(I,J) + PC.L(I,J)*IC.L(I,J) +
            PY.L(I,J)*EY.L(I,J) + PCY.L(I,J)*IY.L(I,J))/2;

VOTCAPC(I,J) = VOTC(I,J)/(PW.L(I,J)*W.L(I,J));
DVOTNC(I,J)= VOTCAPC(I,J)- VOTCAPN(I,J);
WPRICE("C", "PX") = PFX.L;
WPRICE("C", "PY") = PFY.L;
WPRICE("C", "PA") = PFA.L;
WPRICE("C", "PB") = PFB.L;
WPRICE("C", "PC") = PFC.L;

*  check if all four cases solve correctly

DISPLAY MODELSTATAT, MODELSTATN, MODELSTATAC, MODELSTATC;

*  collect results

RESWEL("DWELN", I,J)     = DWELN(I,J);
RESWEL("DWELAC", I,J)    = DWELAC(I,J);
RESWEL("DEWELC", I,J)    = DWELC(I,J);

RESVOT("VOTN", I,J)      = VOTCAPN(I,J);
RESVOT("DVOTAC", I,J)    = DVOTNAC(I,J);
RESVOT("DVOTC", I,J)     = DVOTNC(I,J);

RESREG("REGN", I,J)      = REGN(I,J);
RESREG("REGAC", I,J)     = REGAC(I,J);
RESREG("REGC", I,J)      = REGC(I,J);
```

```
*  dump results to EXCEL file called MARKUSEN


$LIBINCLUDE XLDUMP RESWEL  MARKUSEN SHEET1!A3
$LIBINCLUDE XLDUMP RESVOT  MARKUSEN SHEET2!A3
$LIBINCLUDE XLDUMP RESREG  MARKUSEN SHEET3!A3
$LIBINCLUDE XLDUMP WPRICE  MARKUSEN SHEET4!A3
```