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ABSTRACT

Is Economic Recovery a Myth? Robust Estimation of Impulse Responses*

There is a lively debate on the persistence of the current banking crisis' impact on GDP. Impulse Response Functions (IRF) estimated by Cerra and Saxena (2008) suggest that the effects of earlier crises were long-lasting. We show that standard estimates of IRFs are highly sensitive to misspecification of the underlying data generation process. Direct estimation of IRFs by a methodology similar to Jorda's (2005) local projection method is robust to misspecifications of the data generation process but yields biased estimates when country fixed effects are added. We propose a simple method to deal with this bias, which we apply to panel data from 99 countries for the period 1974-2001. Our estimates suggest that an average banking crisis leads to an output loss of around 10 percent with little sign of recovery. GDP losses from banking crises are more severe for African countries and economies in transition.

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1 Introduction

The demise of Lehman Brothers left the world economy in a state of disarray. Output in most OECD countries has declined by an order of magnitude of 5 percent, a number unseen since World War II and the Great Depression. Will output recover from this shock in the next five to ten years, or will (part of) the output loss be permanent? The Council of Economic Advisors (CEA) has a clear view on this issue: *"A key fact is that recessions are followed by rebound. Indeed, if periods of lower-than-normal growth were not followed by periods of higher-than-normal growth, the unemployment rate would never return to normal."* (Cited in Greg Mankiw's blog of March 3, 2009).

This view is not shared by all economists. For example, in his joint paper with John Campbell, Mankiw shows that there is little mean reversion in output (Campbell and Mankiw, 1987) . By itself, this does not automatically imply that the effect of a banking crisis will be permanent. GDP can be mixture of random processes, some of which have a unit root, while others do not. Banking crises may affect the non-unit root components of GDP, an idea paraphrased in the following passage by Paul Krugman: "*I always thought the unit root thing involved a bit of deliberate obtuseness - it involved pretending that you didn't know the difference between, say, low GDP growth due to a productivity slowdown like the on that happened from the 1973 to 1995, on one side, an low GDP growth due to a severe recession."* (Krugman's blog of March 3, 2009.) For Krugman, the conclusion that recessions have permanent effects is implied by the fact that long-run productivity growth follows a random walk. Contrary to productivity growth, short-run fluctuations in the business cycle have largely temporary effects.

A recent paper by Valerie Cerra and Sweta Saxena (2008) that goes under ominous tittle "The Myth of Economic Recovery" seems to settle the debate in favour of Mankiw's view. The authors estimate a dynamic model of GDP growth as a function of lagged growth rates and a dummy for the occurrence of a banking crisis. They then iterate it several lags ahead, using their regression estimates, to obtain the impulse-response function (IRF) of GDP to a banking crisis event. As the tittle of their paper suggests, the authors find strong persistence of the initial negative effect of banking crises.

However, their claim has not remained undisputed. Adam Elbourne and Bert Smid (2009) show that slight changes in the specification of Cerra and Saxena have a substantial effect on the estimated impulse response of a banking crisis. Also, Xiaoming Cai and Wouter J. den Haan (2009) question the conclusion that financial crises have permanent effects, arguing that it may be driven by the use of one-type-shock models. Since GDP itself is (close to) a random walk, there is at least one type of shocks that has permanent effects, and hence, a one-type-shock model will predict that any shock has permanent effects. They therefore propose that one should estimate the effects of banking crises on GDP components rather than the aggregate.

The current paper addresses both problems – the sensitivity of iterative IRFs to misspecifications in the underlying data generation process (Elbourne and Smid, 2009) as well as the restriction that different types of shocks affect GDP in the same way (Cai and Den Haan, 2009), implicit in the one-type shock models. Our proposed solution to these problems is to estimate the IRFs directly from the data, by regressing future values of GDP on the current GDP level and its lags, and the banking crisis dummy. We apply a methodology similar to Oscar Jorda's (2005) adapted for the special case of rare events such as banking crises. We show that our approach yields consistent estimates of the IRFs for any ARMA process of GDP, even when its structure is misspecified. Moreover, by directly estimating the IRFs from the data, our approach allows us to relax the assumption that various types of shocks have the same IRF, thus addressing the concerns of Cai and Den Haan (2009).

A potential weakness of our approach, to which iterative IRF calculation is also not immune, is that when country fixed effects are added in the regression model to account for unobserved heterogeneity in GDP levels, the persistency of the banking crisis effect on output is underestimated. (It is well known (Arellano and Bond, 1991) that adding fixed effects leads to a downward bias in the estimate of the persistence of shocks.) We develop a method to correct for this bias, applicable for analysis of the impact of rare events that can be captured by a dummy variable. The empirical application shows that it works well. On average, a banking crisis reduces GDP by up to 10 percent after five years on average, with little sign of recovery afterwards, though the impact in Africa (and some transition economies) is much larger than in Non-Africa – 14 percent for Africa compared to 6 percent for Non-Africa.

The setup of the paper is as follows. Section 2 discusses the theory of the estimation of IRFs, and Section 3 presents the empirical evidence. Section 4 concludes.

2 Robust estimation of IRF

2.1 The simplest model: AR(1)

Our approach is to start with the simplest possible model and then to add complications one by one. Consider the following AR(1) model:

$$y_{i,t+1} = \alpha_0 + \alpha_{0!}t + \alpha_1 y_{i,t} + \alpha_2 d_{i,t} + u_{i,t+1}, \tag{1}$$

where $y_{i,t}$ is log GDP per capita for country i in year t, $d_{i,t}$ is a dummy variable that takes the value of 1 at the start of a banking crisis and zero otherwise, and the error term $u_{i,t+1}$ is an i.i.d. random variable, $u_{i,t+1} \sim N(0, \sigma^2)$; in particular, banking crises are independent of all forward and backward realizations of the error term: $\mathrm{E}[u_{i,s}d_{i,t}] = 0, \forall s, t$. The parameter $\alpha_{0!}$ captures a deterministic time trend. We assume that banking crises arrive randomly at a rate λ, so that $\mathrm{E}[d_{i,t}] = \lambda$, see Cerra and Saxena (2008) for some evidence on this issue.[1] Consider a k-period ahead forecast $y_{i,t(k)}$ conditional on the information available on time t:

$$\begin{aligned} y_{i,t(k)} &= \beta_{0k} + \beta_{0!k}t + \beta_{1k}y_{i,t} + \beta_{2k}d_{i,t}, \qquad (2) \\ y_{i,t(k)} &\equiv \mathrm{E}\left[y_{i,t+k}|y_{i,s}, d_{i,s}, s \leq t\right], \\ \beta_{0k} &= \alpha_0\alpha_k^* + \alpha_2\lambda\left(\alpha_k^* - \alpha_1^{k-1}\right), \beta_{0!k} = \alpha_{0!}\alpha_k^*, \beta_{1k} = \alpha_1^k, \beta_{2k} = \alpha_2\alpha_1^{k-1}, \\ \alpha_k^* &\equiv \sum_{j=1}^{k}\alpha_1^{j-1}. \end{aligned}$$

For $k = 1$, we obtain model (1). Define the IRF of a banking crisis as a function of forecast period $k, k \geq 1$ as

$$\mathrm{IRF}\left(k\right) = \mathrm{E}\left[y_{i,t+k}|y_{i,t}, d_{i,t} = 1\right] - \mathrm{E}\left[y_{i,t+k}|y_{i,t}, d_{i,t} = 0\right]$$

Hence, $\mathrm{IRF}(k) = \beta_{2k}$. The standard approach to calculating β_{2k} is to estimate the parameters of model (1) and then use forecast rule (2). An alternative approach would be to directly estimate a forecast rule for each $k > 0$:

$$\begin{aligned} y_{i,t+k} &= \beta_{0k} + \beta_{0!k}t + \beta_{1k}y_{i,t} + \beta_{2k}d_{i,t} + v_{i,t(k)}, \qquad (3) \\ v_{i,t(k)} &= \sum_{j=0}^{k-1}\alpha_1^j u_{i,t+k-j-1} + \sum_{j=1}^{k-1}\alpha_1^{j-1}\left(d_{i,t+k-j} - \lambda\right). \end{aligned}$$

Hence $v_{it(k)} \equiv y_{i,t+k} - y_{it(k)}$, and $v_{it(1)} = u_{it}$. The latter approach can be interpreted as estimating a reduced form model for forecasting. The coefficients β_{2k} give direct estimates of the IRF of a banking crisis. These estimates are consistent and unbiased since the error term $v_{i,t(k)}$ is uncorrelated to the explanatory variables. All 'endogenous' variables – realizations of $y_{i,t+j}$ and $d_{i,t+j}$ between the date of forecasting t and the date $t + k$ for which log GDP per capita is forecasted – have been eliminated by recursive substitution of model (1) and by replacing $d_{i,t+j}$ with its expectation.

[1] Strictly speaking, this assumption cannot be true, since a new banking crisis cannot arrive as long as an old banking crisis is still going on. However, the arrival rate of banking crisis is too low for this to be a serious problem.

This approach has two advantages above the standard approach of calculating the IRF analytically using forecast rule (2). The first advantage is the ease of computing standard errors of the IRF. The calculation of the standard errors of the coefficients in forecast rule (2) is cumbersome, since the coefficients β_{mk} are non-linear functions of the underlying coefficients α_m, increasingly so the further ahead the forecast. By directly estimating model (3), on the other hand, one obtains the standard errors of β_{mk} straight away[2].

Direct estimation, however, involves some loss in efficiency. In the appendix to this paper we prove for the case of $\alpha_0 = \alpha_{0!} = \alpha_2 = 0$ that the variance of the IRF satisfies:

$$\text{plim}\left((T-1)\text{Var}\left[\widehat{\alpha}_1^k\right]\right) = k^2\left(1-\alpha_1^2\right)\alpha_1^{2(k-1)}$$

when the IRF is calculated analytically, and

$$\text{plim}\left((T-k)\text{Var}\left[\widehat{\beta}_{1k}\right]\right) = 1-\alpha_1^{2k} + 2\left(k-1\right) - 2\frac{\alpha_1^2-\alpha_1^{2k}}{1-\alpha_1^2}$$

when the IRF is estimated directly from the data. Figure 1 plots the variances of $\widehat{\alpha}_1^k$ (the boxes) and $\widehat{\beta}_{1k}$ (the crosses), and their ratio (the circles) for $\alpha_1 = 0.95$. The efficiency loss is not large, especially for small k, and when compared to the large effects of specification errors (see figure 2 below). Moreover, the analytical method of calculating the variance of the IRFs can be extremely sensitive to tail risk in the estimation. When α_1 is close to unity, and when by some freak of nature, $\widehat{\alpha}_1$ is estimated to be greater than unity, the estimation error in the IRF explodes. A direct estimation of the IRF does not suffer from this problem.

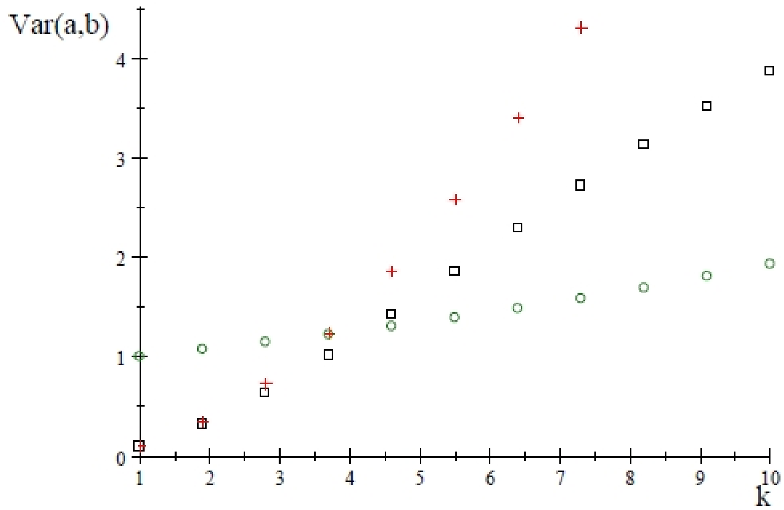
The second advantage of estimating the IRF directly from model (3) is its robustness to misspecifications of model (1). As a simple example, consider the case where $\alpha_0 = \alpha_{0!} = \alpha_2 = 0$, but instead of AR(1) the true data generating process is AR(3)

$$y_{i,t+1} = \alpha_1 y_{i,t} + \alpha_3 y_{i,t-2} + u_{i,t+1}. \tag{4}$$

For the sake of the argument, we consider the case when the coefficient for $y_{i,t-1}$ is zero and the coefficient for $y_{i,t-2}$ is much smaller in absolute value than that for the $y_{i,t}$: $|\alpha_3| \ll |\alpha_1|$; in fact, α_3 is so much smaller that it is insignificant and the econometrician therefore decides to use

[2] Even though $u_{i,t}$ is i.i.d. by assumption, $v_{i,t(k)}$ for $k > 1$ are correlated over time, since the effect of shocks in either $u_{i,t+m}$ or $d_{i,t+m}, 1 < m < k$ are picked up in $v_{i,t(k)}$. Hence, one has to apply White's robust standard errors.

Figure 1: The variances of the estimates of $\widehat{\alpha}_1^k$ (box) and $\widehat{\beta}_{1k}$ (cross), and their ratio (circles)



the AR(1) specification. The probability limit of the estimated value of α_1 is equal to[3]

$$\text{plim}\left(\widehat{\alpha}_1\right) = \frac{\alpha_1}{1 - \alpha_3\left(1 - \alpha_1\right)}.$$

Figure 2 plots the IRF for the true AR(3) model (the line with boxes) and the estimated AR(1) model (the line with crosses) for the case that $\alpha_1 = 0.95$ and $\alpha_3 = -0.10$. The estimated AR(1) model yields a much higher persistence. If one uses the forecast rule (2) one will greatly

[3] We leave out the subscript i for convenience. Consider the general AR(3) model:

$$y_{t+1} = \alpha_1 y_t + \alpha_2 y_{t-1} + \alpha_3 y_{t-2} + u_{t+1},$$

Consider the variance of the left and right hand side of three versions of the equation above, first the equation as it stands, second the equation with $\alpha_1 y_t$ brought to the left hand side, and third the version with $\alpha_1 y_t + \alpha_2 y_{t-1}$ brought to the left hand side. This yields a system of three equations:

$$: \text{Var}\left(y_t\right) = \left(\alpha_1^2 + \alpha_2^2 + \alpha_3^2\right)\text{Var}\left(y_t\right) + 2\alpha_2\left(\alpha_1 + \alpha_3\right)\text{Cov}\left(y_{t+1}, y_t\right) + 2\alpha_1\alpha_3\text{Cov}\left(y_{t+1}, y_{t-1}\right) + \sigma^2,$$
$$: \left(1 + \alpha_1^2\right)\text{Var}\left(y_t\right) - 2\alpha_1\text{Cov}\left(y_{t+1}, y_t\right) = \left(\alpha_2^2 + \alpha_3^2\right)\text{Var}\left(y_t\right) + 2\alpha_2\alpha_3\text{Cov}\left(y_{t+1}, y_t\right) + \sigma^2,$$
$$: \left(1 + \alpha_1^2 + \alpha_2^2\right)\text{Var}\left(y_t\right) - 2\alpha_1\left(1 + \alpha_2\right)\text{Cov}\left(y_{t+1}, y_t\right) + 2\alpha_2\text{Cov}\left(y_{t+1}, y_{t-1}\right) = \alpha_3^2\text{Var}\left(y_t\right) + \sigma^2.$$

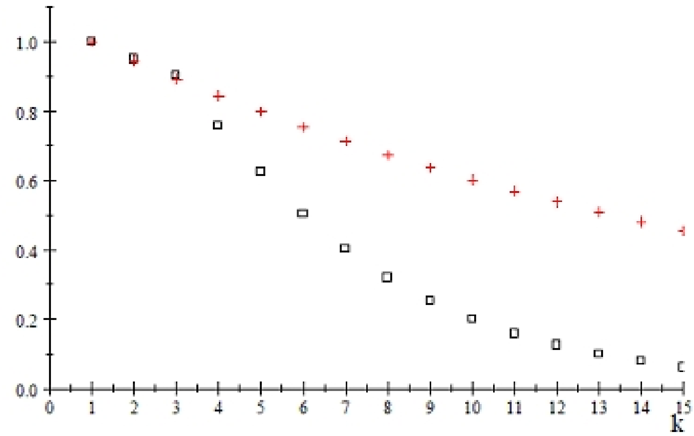
The system can be solved for Var(y_t) ,Cov(y_{t+1}, y_t) , and Cov(y_{t+1}, y_{t-1}). We have:

$$\widehat{\alpha}_1 = \frac{\text{Cov}\left(y_{t+1}, y_t\right)}{\text{Var}\left(y_t\right)}.$$

Taking $\lim \alpha_2 \rightarrow 0$ yields the expression in the text.

overstate the persistence of the shock, whereas estimating equation (3) directly yields a consistent estimate.

Figure 2: The IRFs from the AR(3) (box) and AR(1) (cross) models



A form of misspecification that is particularly relevant for model (1) is the implicit assumption that the persistence of shocks in d_{it} and u_{it} is the same, since both affect future values of $y_{i,t+k}$ via the term $\alpha_1 y_{it}$. Consider the following extreme example where the effect of a banking crisis dies out in a year:

$$\begin{aligned} y_{i,t+1} &= x_{i,t+1} + z_{i,t+1}, \\ z_{i,t+1} &= \alpha_1 z_{i,t} + u_{i,t+1}, \\ x_{i,t+1} &= \alpha_2 \left(d_{i,t} - \lambda\right). \end{aligned}$$

If instead, the econometrician estimated model (1) (under the restriction $\alpha_0 = \alpha_{0!} = 0$) and used forecast rule (2) for prediction, then he would find[4]

$$\operatorname{plim}\left(\widehat{\alpha}_1\right) = \frac{\alpha_1}{1 + \lambda\left(1-\lambda\right)\alpha_2^2\left(1-\alpha_1^2\right)\sigma^{-2}}.$$

[4]Since

$$\begin{aligned} \operatorname{Var}\left[x_t\right] &= \lambda\left(1-\lambda\right)\alpha_2^2, \\ \operatorname{Cov}\left[x_t, x_{t-1}\right] &= 0, \\ \operatorname{Var}\left[z_t\right] &= \left(1-\alpha_1^2\right)^{-1}\sigma^2, \\ \operatorname{Cov}\left[z_t, z_{t-1}\right] &= \alpha_1\left(1-\alpha_1^2\right)^{-1}\sigma^2, \\ \operatorname{plim}\left(\widehat{\alpha}_1\right) &= \frac{\operatorname{Cov}\left[x_t, x_{t-1}\right] + \operatorname{Cov}\left[z_t, z_{t-1}\right]}{\operatorname{Var}\left[x_t\right] + \operatorname{Var}\left[z_t\right]}. \end{aligned}$$

For example, if $\alpha_1 = 0.95$ and $\lambda(1-\lambda)\alpha_2^2(1-\alpha_1^2)\sigma^{-2} = 0.50$, the econometrician would conclude that the rate of decay of the effect of a banking crisis would be $1 - 0.95/1.5 = 0.36$, while in fact the rate of decay is unity, which is the answer he would have got if he had used the direct approach of estimating equation (3).

2.2 A generalization of the argument

The argument that direct estimation of the IRF is robust to misspecification can be generalized. Consider an ARMA(R, M) equivalent of model (1), where we also allow for lags of d_{it}:

$$y_{i,t+1} = \alpha_0 + \alpha_{0!}t + \sum_{r=1}^{R} \alpha_{1r} y_{i,t+1-r} + \sum_{l=1}^{L} \alpha_{2l} d_{i,t+1-l} + \sum_{m=0}^{M} \alpha_{3m} u_{i,t+1-m}, \quad (5)$$

Set $\alpha_{30} = 1$ as a convenient normalisation of $\mathrm{Var}[u_{i,t}]$. For $R = 1, L = 1, M = 0$, model (5) is equal to model (1) with $\alpha_1 = \alpha_{11}$ and $\alpha_2 = \alpha_{21}$. We assume that αs are such that the process is stationary. Given this assumption, the lagged values of $y_{i,t+1}$ can be eliminated from the model by infinite recursive substitution of lagged versions of equation (5):

$$y_{i,t+1} = \gamma_0 + \gamma_{0!}t + \sum_{l=1}^{\infty} \gamma_{2l}\left(d_{i,t+1-l} - \lambda\right) + \sum_{m=0}^{\infty} \gamma_{3m} u_{i,t+1-m}, \quad (6)$$

where the γ-parameters can be expressed as functions of the α-parameters; $\gamma_{30} = \alpha_{30} = 1$. $y_{i,t+1}$ can therefore be written as a linear function of all past shocks $d_{i,t} - \lambda$ and $u_{i,t}$ from now till minus infinity. Hence, since $\mathrm{E}[d_{i,t} - \lambda] = \mathrm{E}[u_{i,t}] = 0$,

$$y_{i,t(k)} = \gamma_0 + \gamma_{0!}(t + k - 1) + \sum_{l=k}^{\infty} \gamma_{2k}\left(d_{i,t+k-l} - \lambda\right) + \sum_{m=k-1}^{\infty} \gamma_{3m} u_{i,t+k-m}. \quad (7)$$

The shocks hitting GDP between time t and time $t + k$ will not affect the forecasted level of output, $y_{it(k)}$, since they are uncorrelated to past shocks. By applying the definition of the IRF of a banking crisis, it is easy to see that $\mathrm{IRF}(k) = \gamma_{2k}$. For $R = 1, L = 1, M = 0$, we have $\gamma_{2k} = \beta_{2k}$. We can now reverse the recursive substitution procedure, by solving equation (5) for u_{it} and using that equation to eliminate u_{it} and its lags from equation (7), resulting in the following model:

$$y_{i,t+k} = \delta_{0k} + \delta_{0!k}t + \sum_{r=1}^{\infty} \delta_{1kr} y_{i,t+1-r} + \sum_{l=1}^{\infty} \delta_{2kl} d_{i,t+1-l} + v_{i,t(k)}, \quad (8)$$

where the δ-parameters can be expressed as functions of the γ-parameters, and $v_{i,t(k)}$ is the error term uncorrelated with the other right-hand-side variables. This error term can be written as a function of the shocks $u_{i,s}$ and $d_{i,s} - \lambda$ for $t \leq s < t+k$ as follows:

$$v_{i,t(k)} = \sum_{l=1}^{k-1} \gamma_{2l}^{l-1} \left(d_{i,t+k-l} - \lambda\right) + \sum_{m=0}^{k-1} \gamma_{3m}^{m} u_{i,t+k-m} \tag{9}$$

Again, $v_{i,t(1)} = u_{i,t+1}$. For an AR(R) model, this equation can be simplified to

$$y_{i,t+k} = \delta_{0k} + \delta_{0!k} t + \sum_{r=0}^{R} \delta_{1kr} y_{i,t-r} + \sum_{l=0}^{L} \delta_{2kl} d_{i,t-l} + v_{i,t(k)}, \tag{10}$$

Using $\alpha_{30} = 1$, it is easy to show that

$$\delta_{2k1} = \gamma_{2k}.$$

Hence, OLS on equation (10) yields a consistent estimate of IRF$(k) = \delta_{2k1}$. Suppose that the econometrician uses model (3) instead of model (10). Then, he would obtain consistent estimates of γ_{2k}, since lagged values of $y_{i,t}$ are linear functions of lagged values of $d_{i,t}$ and $u_{i,t}$ only and since these lagged shocks are uncorrelated to $d_{i,t}$. Direct estimation of the IRF is therefore robust to misspecification of either R, or L, or M.

2.3 Fixed effects

Model (5) assumes the GDP per capita in all countries to be at the same level. To relax this assumption, country fixed effects may be added to the model. It is well known (Arellano and Bond, 1991) that estimating model (5) with fixed effects by OLS yields biased estimates, as the fixed effect soaks up part of the persistence of shocks, hence underestimating $|\alpha_1|$. Part of this bias spills over to the estimate of the banking crisis effect, α_2. Consider a simplified version of model (5) with fixed effects:

$$y_{i,t+1} = \alpha_{0i} + \alpha_1 y_{i,t} + \alpha_2 d_{i,t} + u_{i,t+1}$$

The fixed effects estimation involves running OLS on the original model specified in deviations from country means:

$$\widetilde{y}_{it1} = \alpha_1 \widetilde{y}_{it0} + \alpha_2 \widetilde{d}_{it0} + \widetilde{u}_{it1}, \tag{11}$$

where $\widetilde{z}_{its} = z_{i,t+s} - \overline{z}_{is}$, and $\overline{z}_{is}$ is the mean of variable z over the period $t = 1+s, T$. Let us concentrate on the bias to α_2, for which purpose we partition equation (11) using Frisch-Waugh-Lovell theorem and estimate

$$M\widetilde{y}_{it1} = \alpha_2 M\widetilde{d}_{it0} + M\widetilde{u}_{it1}, \tag{12}$$

where $M = I - \widetilde{y}_{it0}(\widetilde{y}'_{it0}\widetilde{y}_{it0})^{-1}\widetilde{y}'_{it0}$ is the orthogonal projection matrix (so that $M\widetilde{y}_{it0} = 0$). After some algebra, the expected value of the OLS estimate of α_2 will be

$$\mathrm{E}(\hat{\alpha}_2) = \alpha_2 - \widetilde{d}'_{it0}\widetilde{y}_{it0}(\widetilde{y}'_{it0}\widetilde{y}_{it0})^{-1}\overline{y}'_{i0}\overline{u}_{i1} > \alpha_2 \tag{13}$$

Assuming a negative effect of banking crisis on output, that is $\alpha_2 < 0$ and $\widetilde{d}'_{it0}\widetilde{y}_{it0} < 0$, and noting that $\overline{y}'_{it}\overline{u}_{i,t+1} > 0$ by definition, equation (13) implies that adding fixed effects to dynamic model (5) leads to the estimate of the effect of banking crisis being attenuated. However, a relatively long T and further lags of dependent variable in our regressions will mitigate this bias (Phillips and Sul, 2007; Judson and Owen, 1999).

Yet, there is another bias which is specific to our case as we will now demonstrate. For the ease of illustration, we consider the simple AR(1) model (equation (1)), although the same argument will apply to the more general model (5). Rewrite model (1) and its forecast rule (3) allowing for country fixed effects, α_{0i}, and for the arrival rate of banking crises, λ_i, to differ by country:

$$y_{i,t+k} = \beta_{0ik} + \beta_{0!k}t + \beta_{1k}y_{i,t} + \beta_{2k}d_{i,t} + v_{i,t(k)}. \tag{14}$$

Similarly, one can adapt equation (3), and the AR(1) equivalents of equations (6), and (10) by replacing β_{0k} by β_{0ik}, γ_0 by γ_{0i}, and δ_0 by δ_{0i} respectively and by replacing λ by λ_i. We shall refer to these equations below as if we have allowed for a country-specific fixed effect.

Consider a forecast k periods ahead made at time t. Taking the expectation of $y_{i,t+k}$ over all shocks $u_{is}, t < s < t+k$ over the forecast period, $t+1$ to $t+k$, but retaining the realisations of banking crisis, $d_{i,s}$ for $s \leq t+k-1$ yields:

$$y^*_{i,t(k)} = \beta_{0i} + \beta_{0!1}t + \beta_{1k}y_{i,t} + \beta_{2k}d_{i,t} + \sum_{l=1}^{k-1}\beta_{2l}d_{i,t+k-l}.$$

That is, a forecast k periods ahead made at time t will not only be affected by the banking crises happened at or before t, but also by banking crises happening at any future date within the forecast interval. In equation (10), $\sum_{l=1}^{k-1}\beta_{2l}d_{i,t+k-l}$, which ends up in the error term $v_{i,t(k)}$, is offset by term λ, so that $\mathrm{E}[v_{i,t(k)}] = 0$. However, in equation (14) transformed into deviations from country means the error term $\widetilde{v}_{i,t(k)}$ no longer has the expected value of zero, since

$$\mathrm{E}\left[\overline{v}_{i,t+k}\right] = \frac{1}{T-k}\sum_{l=1}^{k-1}\beta_{2l}d_{i,t+k-l} < 0. \tag{15}$$

(The last inequality is based on the assumption the the effect of banking crisis on output is negative.) The quantity in (15) will be soaked up by the estimated fixed effects, resulting in a reduction (in absolute value) of the estimate for β_{2k} to compensate[5]. As can be seen, the bias to the fixed effects estimates increases with k.

We can eliminate the bias in (15) by retaining the forward lags of d_{it} in the forecast equation (10):

$$y_{i,t+k} = \varepsilon_{0ik} + \varepsilon_{0!k}t + \sum_{r=0}^{R} \delta_{1kr} y_{i,t-r} + \sum_{l=1}^{L} \delta_{2kl} d_{i,t-l} + \sum_{l=1}^{k} \gamma_{2l} d_{i,t+l-1} + v^*_{i,t(k)}. \tag{17}$$

where the ε-parameters can be expressed as functions of the underlying α-parameters, and $v^*_{i,t(k)}$ is the error term $v_{i,t(k)}$ conditional on the information available at time t and the information on the occurrence of banking crises between t and $t+k$.

Interestingly, equation (17) yields estimates, not only of γ_{2k}, but also of γ_{2l} for $l = 1, k-1$. This suggests a further step, where all γ_{2k}'s are estimated in one simultaneous regression. An alternative estimator of IRF(k) is obtained by eliminating all lags of $y_{i,t}$ from the right hand side of equation (17) (cf. equation (7)):

$$y_{i,t+k} = \varepsilon_{0(k)i} + \varepsilon_{0!}t + \sum_{k=1}^{\infty} \gamma_{2k} d_{i,t-k} + v^*_{i,t(k)}. \tag{18}$$

The advantage of estimating equation (18) rather than equation (17) is that (18) yields estimates of γ_{2k} free from the bias due to the lagged dependent variable. The disadvantage is that controlling for y_{it} and its lags would improve the precision of the estimate of γ_{2k}. Equations (17) and (18) yield independent estimates for γ_{2l} that should be equal conditional on the model. The equality of these estimates could therefore be used for a specification test. How many lags of the banking crisis dummy to include in the feasible specification of model (18) is an empirical question. In our regression analysis, we observe that the estimated γ_{2k}'s stabilise when the number of lags of $d_{i,t}$ exceeds 18.

[5] An alternative to transforming equation (14) into deviations from the country means is to difference it and run OLS on

$$y_{i,t+k} - y_{i,t} = \beta_{0!k} + \beta_{1k} \triangle y_{i,t} + \beta_{2k} \triangle d_{i,t} + v_{i,t(k)} \tag{16}$$

This transformation will not solve the problem either, since the error term, $v_{i,t(k)}$, will include all the banking crises happening between t and k leading to the same problem as in the fixed effects case considered earlier.

3 Empirical estimates of the IRFs

3.1 Data

Our dataset is compiled from several sources. The data on banking crises come from Gerard Caprio and Daniela Klingebiel (2003), observations available from 1974 to 2001. Penn World Tables (Heston, Summers and Aten, 2006) provide GDP data from 1960 onwards. The complete data are available for a panel of 99 countries. The average length of a time series depends on the forecast length and ranges from 24 (one year after a banking crisis) to 15 observations (ten years after). There have been 89 banking crises within our sample. The majority of countries (56) had one banking crisis, twenty-eight countries had no crisis, thirteen countries had two, one country had three, and one four banking crises. There is a significant negative correlation ($r = -0.35$) between the frequency of banking crises and a country's GDP level, suggesting that a country-specific banking crisis arrival rate λ_i is a useful generalization. Our assumption that the likelihood of a banking crisis is time-invariant seems to be tolerated by the data as there is no significant time trend.

3.2 The impulse response functions estimates

In table 1 we report estimates of the IRFs to a banking crisis from the various specifications we considered above. We start with the specification in levels without and with country fixed effects (equations 10 and 14), proceeding then to the specification in growth rates (equation 16), with each specification estimated with and without the leads of the banking crisis dummy. Every specification includes four lags of log GDP or its growth rate, four lags of the banking crisis dummy ($R = L = 4$), and a linear time trend for the specifications in levels. We test the choice of these lag lengths and find them adequate (see table 3 below).

All specifications paint a similar picture of the GDP loss within the first few years after the start of a banking crisis, but diverge thereafter. The differences in the IRF estimates for the effect of banking crisis observed in the medium to long run can be explained through our models developed in the previous sections. The specifications with leads of the banking crisis dummy produce larger estimates than those without, because of the effects of the crises happening outside the forecast base period between R and $T-k-1$. These effects are captured by banking crisis leads, when these are included, or absorbed by the constant term or fixed effects when not, biasing them downward and leading to underestimated IRFs to compensate or the downward bias in the constant term. The difference in estimates between the specifications with and without banking crisis leads increases towards the end of the forecast

Table 1: Impulse response estimates for the effect of a banking crisis on output

Specification	GDP in levels eq. (10)	GDP in levels	GDP in levels eq. (14)	GDP in levels eq. (17)	GDP growth rate eq. (16)	GDP growth rate	GDP in levels eq. (18)
k							
1	-0.035***	-0.035***	-0.032***	-0.032***	-0.033***	-0.034***	-0.050*
2	-0.066***	-0.069***	-0.058***	-0.063***	-0.062***	-0.065***	-0.063*
3	-0.071***	-0.079***	-0.060***	-0.072***	-0.067***	-0.074***	-0.091**
4	-0.077***	-0.090***	-0.064***	-0.084***	-0.074***	-0.088***	-0.106**
5	-0.069***	-0.088***	-0.056***	-0.088***	-0.066***	-0.087***	-0.102*
6	-0.069***	-0.096***	-0.057***	-0.100***	-0.069***	-0.099***	-0.109*
7	-0.084***	-0.117***	-0.069***	-0.122***	-0.084***	-0.120***	-0.138**
8	-0.058*	-0.097***	-0.041**	-0.107***	-0.051	-0.093***	-0.132*
9	-0.051	-0.089**	-0.030	-0.103***	-0.047	-0.091**	-0.122
10	-0.023	-0.063	-0.007	-0.085**	-0.021	-0.068	-0.093
fixed effects	no	no	yes	yes	no	no	yes
banking crisis leads	no	yes	no	yes	no	yes	yes

Note: from this table onwards, estimates significant at 1%, 5% and 10% levels are marked ***, ** and *, respectively.

period, again as predicted, because the bias due to banking crises over the forecast base period increases with k.

The other source of the bias to the IRF estimates is due to the inclusion of lags of the dependent variable. This bias appears when we include country fixed effects, as can be seen by comparing the specifications in GDP levels with and without fixed effects. The difference between the two specifications is not entirely due to the lagged dependent variable bias, since when country fixed effects are present, as is most certainly the case here, the specification which ignores them unduly attributes their influence to the observed variable – the banking crisis dummy – thus overestimating its effect. However, that the specification in growth rates, which fares only slightly better than that in levels with fixed effects in terms of the bias, brings the IRF estimates closer to the levels without fixed effects specification suggests that the lagged dependent variable bias is the major reason for the results in columns 1 and 3 to differ. The specification where lags of GDP are replaced with a long series of banking crisis dummies is free from this bias, and as a result is estimates are larger. This specification is also free from the bias due to non-inclusion of the banking crises outside the forecast base period that we addressed in the previous paragraph, since it includes information on practically all the banking crises that happened. Therefore, the estimates from this specification are the most unbiased of all, albeit least precise.

The IRF estimates in table 1 imply that banking crises lead to large and prolonged output losses, and that recovery, if any, is a distant and uncertain prospect. The following figure 3 illustrates the 95% confidence interval bounds for our preferred specification (18). We see that, even though a complete recovery is within the range of possibilities, sustained losses are more likely. The point estimate for the cumulative GDP loss ten years after the crisis of around 9 percent. To put this estimate into perspective, consider that the median growth rate observed in our sample is roughly 3% per year, so that the median economy would have grown by 34% over ten years. Thus, a single episode of a banking crisis can cost, on average, about a quarter of country's long-term growth potential.

To appreciate how statistically important banking crises are in explaining the variation in output, we calculate in table 2 the share of residual variance in log GDP explained by the banking crisis dummies in the specification with fixed effects and banking crises leads. (Doing the same exercise with our preferred specification is statistically incorrect, since all explainable variance in output would be explained through banking crises, provided we look long enough back.) We find that, start-

Figure 3: Predicted GDP loss from a banking crisis, and its 95% confidence interval.

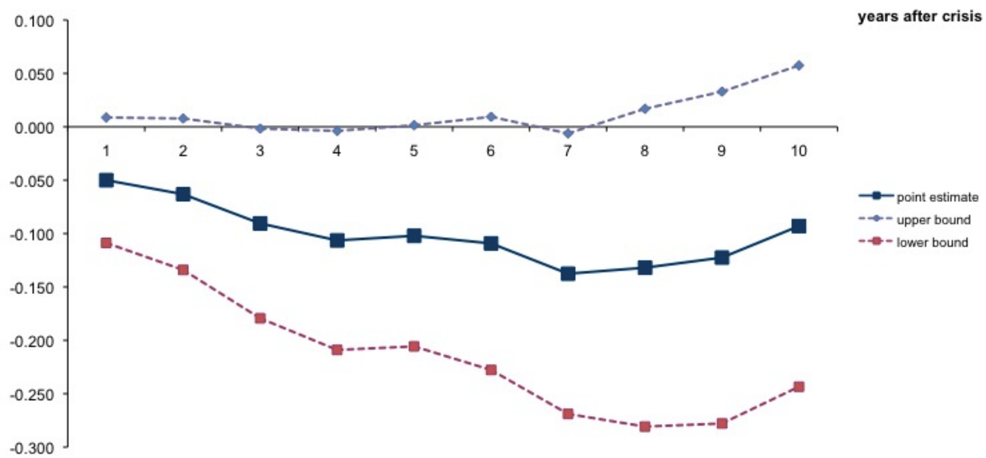


Table 2: Share of residual variance in log GDP explained by banking crisis

k		k	
1	0.010	6	0.063
2	0.019	7	0.072
3	0.027	8	0.081
4	0.038	9	0.087
5	0.050	10	0.087

ing with a humble 1% in the next year, banking crises explain just under 9% of log GDP's residual variance in the long run. For a relatively rare event such as a banking crisis, this share is quite high.

3.3 Robustness checks

As our model predicts, our regression results are fairly robust to the choice of lag length. In table 3 we test whether further lags of GDP improve the explanatory power of the model. The restrictions to a lag length of four hold at reasonable levels of significance for all specifications except for the case of levels with fixed effects. The robustness of our estimates to the number of lags included in the model contradicts with the result reported by Elbourne and Smid (2009), that IRFs are sensitive to the number of lags. However, their result is obtained using standard recursive method for calculating IRF's, and the robustness of our results to lag length may be thought of as yet another illustration of the robustness of our method. Moreover, for our preferred specification with a potentially infinite number of lags of the banking crisis dummy the choice of lag length is not even an issue.

Our results are also robust to the assumption that part of the observed negative effect of a banking crisis on output is in fact a correction of excessive growth in the period of an expectations bubble. We can test whether there is some currency in this view by including future banking crises in model (18). The results in table 4 do not suggest a major problem with the interpretation of our regression results as estimated GDP losses from banking crisis, rather than a cyclical correction, since the estimates for the GDP effects from future banking crises are not nearly significant.

Finally, because banking crises sometimes coincide with war, social unrest, major political reform or other disturbing events which consequences for output are hard to isolate, our estimates may be vulnerable to a lack of appropriate controls. We can, however, rerun our regressions on the sample excluding Africa and some transition countries known to be hard hit by these problems. We therefore divide our sample into the subsamples of countries of the African continent (41 countries), countries in transition (4) and the rest, and run our preferred specification on the African and the rest subsamples separately (table 5 and figure 4). Looking at Figure 4 we observe that African countries lose more output than the rest of the sample but experience stronger recovery at the end of the ten-year period. The IRFs for the African subsample are also much less precisely estimated than the rest of the sample; hence when we exclude them the estimates become more precise as compared with table 1.

4 Concluding remarks

The impact of banking crises on future GDP have been heavily debated by economists. The evidence in Cerra and Saxena (2008) shows strong persistence in the effect of banking crisis, but their results have been disputed by other economist, see, among others, Krugman's blog, Elbourne and Smid (2009) and Cai and Den Haan (2009). The problem is that their methodology is sensitive to misspecifications of their dynamic model, which in practice are always there. In particular, for longer forecast periods, these specification errors are likely to have substantial effects. We apply an alternative method which is robust to these specification errors. We find indeed that our regression results are robust to the number of lags of GDP and the banking crisis dummy added in the model.

Table 3: GDP lag restrictions test statistics ($p-$values) by specification and forecast period

Specification	GDP in levels eq. (10)	GDP in levels	GDP in levels eq. (14)	GDP in levels eq. (17)	GDP growth rate eq. (16)	GDP growth rate
k						
1	0.751	0.739	0.004	0.287	0.577	0.577
2	0.750	0.723	0.000	0.378	0.907	0.893
3	0.918	0.908	0.000	0.874	0.960	0.949
4	0.763	0.792	0.000	0.424	0.968	0.949
5	0.972	0.993	0.050	0.697	0.994	0.990
6	0.659	0.768	0.004	0.513	0.883	0.872
7	0.671	0.871	0.000	0.347	0.869	0.827
8	0.672	0.886	0.000	0.276	0.369	0.326
9	0.147	0.266	0.000	0.283	0.043	0.040
10	0.059	0.152	0.000	0.386	0.023	0.017
fixed effects	no	no	yes	yes	no	no
banking crisis leads	no	yes	no	yes	no	yes

Table 4: Output loss before the start of a banking crisis

Dependent variable	log GDP	
Years *before* crisis	Point estimate	Std. deviation
1	0.026	0.033
2	0.018	0.042
3	0.004	0.042

Note: results based on specification (18)

Although we have applied a different method, our results are consistent with the findings of Cerra and Saxena (2008) that the loss from a banking crisis is likely to last for a long time, especially for richer countries. Our findings suggest that an average banking crisis may cause an output loss of around 9 percent over a period of ten years from its start. Even though estimates for more distant future are increasingly imprecise, output is unlikely to return to its pre-crisis path. The consequences of banking crises vary by country. Thus, comparing the IRF estimation results on the two sub-samples of our data – Africa versus the rest – we find that in African countries banking crises are quite devastating, whereas in non-African countries (which are mainly rich world because of long time series required for our method) output loss is not as strong.

Table 5: Impulse response estimates by subsample of countries

	African countries (41)		The rest, excluding transition countries (54)	
Specification	eq. (17)	eq. (18)	eq. (17)	eq. (18)
Years after crisis				
1	-0.033**	-0.040	-0.028***	-0.024
2	-0.068**	-0.075	-0.053***	-0.021
3	-0.084***	-0.121	-0.056***	-0.041*
4	-0.108***	-0.124	-0.062***	-0.059**
5	-0.104**	-0.103	-0.069***	-0.094***
6	-0.118**	-0.138	-0.081***	-0.088***
7	-0.153***	-0.198	-0.087***	-0.088***
8	-0.132**	-0.195	-0.076***	-0.082***
9	-0.137**	-0.188	-0.066***	-0.061**
10	-0.108	-0.124	-0.063***	-0.071***

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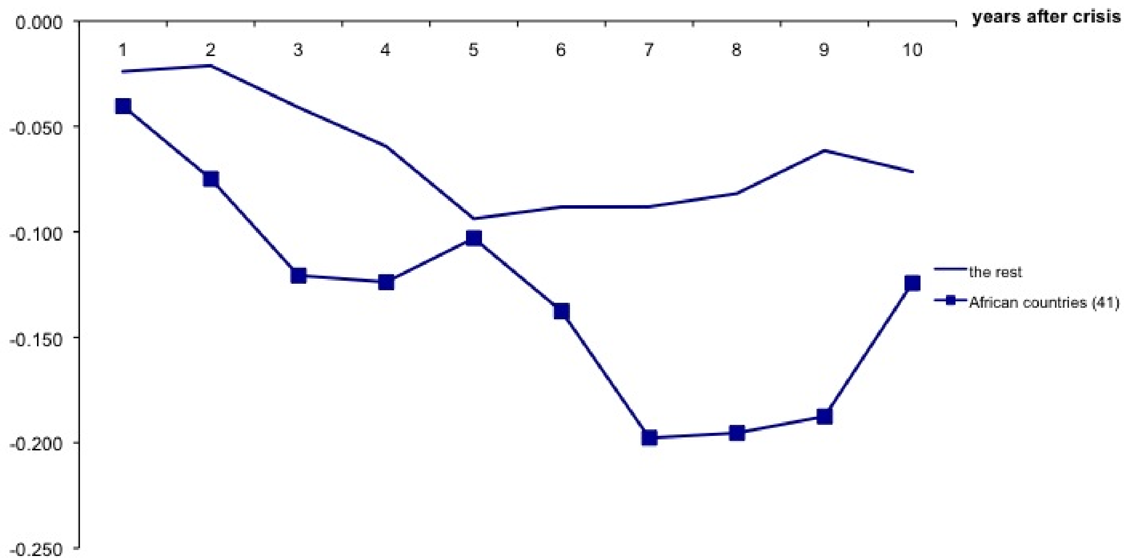
Figure 4: Estimated IFRs by subsample of countries



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5 Appendix

We drop the subscript i throughout this appendix to simplify notation. We have:

$$\begin{aligned} y_{t+1} &= \alpha_1 y_t + u_t, \\ \operatorname{Var}(y_t) &= \sigma^2 \left(1-\alpha_1^2\right)^{-1}, \\ \operatorname{Cov}(y_t, y_{t-1}) &= \sigma^2 \alpha_1^2 \left(1-\alpha_1^2\right)^{-1}, \\ y_t &= \sum_{j=-\infty}^{-1} \alpha_1^{-j-1} u_{t+j}, \\ v_{t(k)} &= \sum_{j=0}^{k-1} \alpha_1^{k-j-1} u_{t+j}. \end{aligned}$$

The variance estimator for α_1 reads:

$$\operatorname{plim}\left[(T-1)\operatorname{Var}(\widehat{\alpha}_1)\right] = \sigma^2 \operatorname{Var}(y_t)^{-1} = \left(1-\alpha_1^2\right), \tag{19}$$

The variance of $\widehat{\alpha}_1^k$ satisfies:

$$\begin{aligned} \operatorname{Var}\left(\widehat{\alpha}_1^k\right) &= \operatorname{E}\left(\widehat{\alpha}_1^{2k}\right) - \operatorname{E}^2\left(\widehat{\alpha}_1^k\right); \\ \operatorname{E}\left(\widehat{\alpha}_1^k\right) &= \lim_{t\to 0} \frac{d^k}{dt^k} \exp\left[\alpha_1 t + \frac{1}{2}\operatorname{Var}(\widehat{\alpha}_1)\, t^2\right]. \end{aligned}$$

where we use the moment generating function of the normal distribution in the second line, see Mood, Graybill, and Boes (1974, p.540). We have:

k	1	2	3
$\operatorname{E}\left(\widehat{\alpha}_1^k\right)$	α_1	$\operatorname{Var}(\widehat{\alpha}_1) + \alpha_1^2$	$3\operatorname{Var}(\widehat{\alpha}_1)\,\alpha_1 + \alpha_1^3$
$\operatorname{Var}\left(\widehat{\alpha}_1^k\right)$	$\operatorname{Var}(\widehat{\alpha}_1)$	$2\operatorname{Var}(\widehat{\alpha}_1)^2 + 4\operatorname{Var}(\widehat{\alpha}_1)\,\alpha_1^2$	$15\operatorname{Var}(\widehat{\alpha}_1)^3 + 36\operatorname{Var}(\widehat{\alpha}_1)^2\,\alpha_1^2 + 9\operatorname{Var}(\widehat{\alpha}_1)\,\alpha_1^4$

k	4	6
$\operatorname{E}\left(\widehat{\alpha}_1^k\right)$	$3\operatorname{Var}(\widehat{\alpha}_1)^2 + 6\operatorname{Var}(\widehat{\alpha}_1)\,\alpha_1^2 + \alpha_1^4$	$15\operatorname{Var}(\widehat{\alpha}_1)^3 + 45\operatorname{Var}(\widehat{\alpha}_1)^2\,\alpha_1^2 + 15\operatorname{Var}(\widehat{\alpha}_1)\,\alpha_1^4 + \alpha_1^6$

Substitution of the expression for $\operatorname{Var}(\widehat{\alpha}_1)$ and taking limits yields:

$$\operatorname{plim}\left[(T-1)\operatorname{Var}\left(\widehat{\alpha}_1^k\right)\right] = k^2\left(1-\alpha_1^2\right)\alpha_1^{2(k-1)}.$$

The variance of $\widehat{\beta}_{1k}$ satisfies:

$$\begin{aligned} &: \operatorname{Var}\left(\widehat{\beta}_{1k}\right) = \left(X'X\right)^{-1} X'VX \left(X'X\right)^{-1} \\ &= (T-k)^{-2}\operatorname{Var}(y_t)^{-2}\operatorname{E}\left[(T-k)\left(y_t^2 v_{t(k)}^2\right) + 2\sum_{j=1}^{k-1}(T-k-j)\left(y_t v_{t(k)} v_{t+j(k)} y_{t+j}\right)\right] \end{aligned}$$

$$
\begin{aligned}
&= \sigma_u^{-4} \frac{\left(1-\alpha_1^2\right)^2}{T-k} \times \\
&\quad \mathrm{E}\left[\begin{array}{c} \left(\sum_{r=-\infty}^{-1} \alpha_1^{-r-1} u_{t+r}\right)^2 \left(\sum_{l=0}^{k-1} \alpha_1^{k-l-1} u_{t+l}\right)^2 + \\ 2\sum_{j=1}^{k-1} \frac{T-k-j}{T-k} \sum_{r=-\infty}^{-1} \sum_{s=-\infty}^{j-1} \sum_{l=0}^{k-1} \sum_{m=j}^{k-1+j} \alpha_1^{2k-r-s-l-m-4} u_{t+r} u_{t+s} u_{t+l} u_{t+m} \end{array}\right] \\
&= \sigma_u^{-4} \frac{\left(1-\alpha_1^2\right)^2}{T-k} \alpha_1^{2k} \times \\
&\quad \mathrm{E}\left[\sum_{r=-\infty}^{-1} \sum_{l=0}^{k-1} \alpha_1^{-2(r+l+2)} u_{t+r}^2 u_{t+l}^2 + 2\sum_{j=1}^{k-1} \frac{T-k-j}{T-k} \sum_{r=-\infty}^{-1} \sum_{l=j}^{k-1} \alpha_1^{-2(r+l+2)} u_{t+r}^2 u_{t+l}^2\right] \\
&= \frac{\left(1-\alpha_1^2\right)^2}{T-k} \alpha_1^{2k} \sum_{r=-\infty}^{-1} \alpha_1^{-2(r+1)} \left[\sum_{l=0}^{k-1} \alpha_1^{-2(l+1)} + 2\sum_{j=1}^{k-1} \frac{T-k-j}{T-k} \sum_{l=j}^{k-1} \alpha_1^{-2(l+1)}\right] \\
&= \frac{1}{T-k} \left[1-\alpha_1^{2k} + 2\sum_{j=1}^{k-1} \frac{T-k-j}{T-k} \left(1-\alpha_1^{2(k-j)}\right)\right].
\end{aligned}
$$

The second line uses $X'X = (T-k)\mathrm{Var}(y_t)$ and writes the covariance matrix $X'VX$ as the sum of the terms on the main diagonal and two times the $k-1$ diagonal along both sides of the main diagonal that account for the MA part of the error term. The third line use the expression for $\mathrm{Var}(y_t)$ and expresses $v_{t(k)}$ and y_t in terms of (lags of) u_t. The fourth line uses the fact that $\mathrm{E}(u_t u_{t+k}) = 0$ for $k \neq 0$, so that we can drop these terms. The fifth line takes the expectation. The final lines is straightforward algebra. Hence, we can write:

$$
\begin{aligned}
\operatorname{plim}\left[(T-k)\mathrm{Var}\left(\widehat{\beta}_{1k}\right)\right] &= 1-\alpha_1^{2k} + 2\sum_{j=1}^{k-1}\left(1-\alpha_1^{2(k-j)}\right) \\
&= 1-\alpha_1^{2k} + 2\left(k-1\right) - 2\frac{\alpha_1^2-\alpha_1^{2k}}{1-\alpha_1^2}.
\end{aligned}
$$