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BARGAINING CUM VOICE

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ABSTRACT

Bargaining cum Voice

We propose a formal concept of the power of voice in the context of a simple model where individuals form groups and trade in competitive markets. Individuals use outside options in two different ways. Actual outside options reflect the possibility to exit or to join other existing groups. Hypothetical outside options refer to hypothetical groups that are ultimately not formed. Articulation of hypothetical outside options in the bargaining process determines the relative bargaining power of the members of a group, which constitutes an instance of the power of voice. The adopted equilibrium concept, *competitive equilibrium with free group formation and bargaining cum voice*, endogenizes the outside options as well as the power of voice. We establish existence of such equilibria and we explore their properties.

JEL Classification: D13, D5 and D71
Keywords: articulation of outside options, bargaining, competitive equilibria, group formation and power of voice

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1 Introduction

Power has always been a prominent theme in the social sciences. The meaning of power can include everything from the ability to keep oneself alive to the ability of government to arrest people. A central conception of power is an individual's capacity to influence decisions taken by a group he or she belongs to.

The influential work of Hirschman (1970) has provided a convenient and compelling way of thinking about the power pertaining to collective decisions of social organizations. He distinguishes between power derived from the exit option and power based on voice. Although these concepts have been widely applied and alluded to, the role of voice has proven extremely difficult to conceptualize.

In this paper we propose a concept of voice power in a model of asymmetric Nash bargaining within groups.[1] The essential idea is as follows. In a society, members of a group use outside options in two different ways. First, the possibility to exit or to join other existing groups determines the **actual outside options** and a fortiori the reservation utilities of group members. Second, **hypothetical outside options** affect relative bargaining power in existing groups. Namely, individuals also reason with reference to hypothetical groups, whose formation would require that several individuals break away from their existing groups and form a new group. The hypothetical possibility of forming such new groups is articulated in the bargaining process — even though those groups are ultimately not formed. The best possible hypothetical scenarios for each person determine the relative bargaining power in existing groups. This impact of articulating one's conceivable opportunities in hypothetical groups is called the "power of voice".

Our model of voice power presumes a model of endogenous group formation. A comprehensive treatment of the formation, composition, stability, and decision making of socio-economic groups warrants a formal framework that incorporates the allocation of commodities to individuals and of individuals to groups. In Gersbach and Haller (2003), we analyze a general equilibrium model with multi-member households where such a dual allocation is brought about by three interacting mechanisms, each operating at a particular level of aggregation:

(A) Individual decisions are made to join or leave households.

(B) Collective decisions within households determine the consumption plans of household members.

[1] Hirschman considers voice as a mechanism of recuperation and a means of influence. Here we focus on the second function.

(C) Competitive exchange across households achieves a feasible allocation of resources.

Whereas in Gersbach and Haller (2003), group or household decisions satisfy collective rationality à la Chiappori (1988, 1992), we consider here a special case of the Gersbach and Haller (2003) model where group decisions are asymmetric Nash bargaining outcomes and the relative bargaining power within a group reflects the power of voice. Thus the main innovation concerns (B). But group decision making interacts with (A) group formation and (C) competitive exchange: The bargaining outcome of the group depends on the feasible and affordable outcomes for the group, hence on market prices. It also depends on reservation utilities (which reflect actual outside options that are functions of other existing groups and market prices) and on relative bargaining power (which depends on hypothetical outside options). In turn, group decisions determine market excess demand and, consequently, market clearing prices. Finally, group stability requires a favorable comparison of group decisions with actual outside options.

Actual Outside Options. When dealing with group formation and group stability, one of the most critical modeling assumptions is how much choice between groups an individual has. Here we consider a finite pure exchange economy with variable group (coalition, household) structure and focus on two types of actual outside options available to group members:

(EO) Under the "exit option", an individual belonging to a non-singleton group can decide to leave that group and become a single.

(JO) With the "joining option", an individual can leave its current group — or cease to be single if applicable — and join another already existing group.

Whether an individual is capable of and interested in exercising one of its actual outside options depends on the prevailing group structure and market conditions. It benefits from exercising the exit option if it fares better as an individual consumer at the going prices. The individual has an incentive and opportunity to exercise the joining option if there exist another group and an affordable consumption plan for the augmented group (consisting of the members of the other group plus the joining individual) so that nobody in the augmented group is made worse off and the individual is better off.

Equilibrium Conditions. In our model, an allocation consists of a group structure — that is a partition of the population into groups — and an allocation of commodities to individual consumers. We shall adopt from Gersbach and Haller (2003) the concept of **competitive equilibrium with free group formation (CEFG)** which requires 1. collective rationality of group decisions; 2. market clearing; 3. that no individual can benefit from exit, i.e., benefit from exercising the exit option; 4. that no individual can benefit from joining another group, i.e., benefit from exercising the joining option. Condition 4 is akin to individual stability in the sense of Drèze and Greenberg (1980). The conjunction of conditions 3 and 4 constitutes the weak version of individual stability in the sense of Bogomolnaia and Jackson (2002). If group decisions are the outcomes of asymmetric Nash bargaining and a group member's reservation utility represents its best actual outside option, then equilibrium conditions 1, 3, and 4 will be automatically satisfied.

Hypothetical Outside Options. There are hypothetical groups of which a particular individual would be a member that differ from the groups resulting from the individual exercising the exit or joining option. For each of these groups, Kalai and Smorodinsky's (1975) bargaining solution is constructed by means of an ideal point that assigns an individual its maximal attainable utility as a group member. For a member of an actually formed group, we take as hypothetical outside option value the maximum of the individual's maximal attainable utilities in each of the hypothetical groups which the individual might consider. In contrast to the actual outside options, the hypothetical outside options do not serve as reservation utilities or threat points in our bargaining model. Instead, they reflect articulated rather than actual opportunities and determine relative bargaining power in existing groups. We implicitly assume that decision makers can use hypothetical outside options as arguments when they bargain over consumption bundles. Thus, "power of voice" manifests itself in the transformation of hypothetical outside options into relative bargaining power and, consequently, bargaining outcomes. The specific way in which such outside options impact the bargaining outcome is axiomatized through the "voice impact function" introduced in subsection 3.5 and examined in section 5.

At a more abstract level, our paper suggests a way to formalize how discussion among individuals can bring about a consensus. The role of communication in reaching a consensus in democratic societies has been stressed a great deal in political science (see e.g. Elster 1998) and philosophy (see e.g. Habermas 1987). In our context, discussion enables each side to convince the other of the feasibility or plausibility of potential best alternatives. Each individual assesses the feasibility of hypothetical outside options of other group members. We assume that this deliberation and discussion transforms the best hypothetical outside alternative of one individual into concessions by the other individual(s) and thus into relative bargaining power. Still, the hypothetical outside options form the basis for complaints rather than threats and it proves difficult to explain why voice — which may be merely cheap talk — would have any impact on material collective decisions. Therefore, we refrain from explaining why voice does have an impact and settle for the more modest objective of demonstrating how voice might have an impact. In addition, the restriction to asymmetric Nash bargaining with endogenous bargaining power can serve as a useful refinement or selection criterion for CEFG. For CEFG tend to be numerous when they exist.[2] The equilibria considered in the current paper are "CEFG with bargaining cum voice" and, as a rule, constitute a proper subset of the set of all CEFG if the latter is non-empty.

In the next section, we discuss the related literature. In section 3, we present the general equilibrium model and the new concept of a competitive equilibrium with free group formation (CEFG) and bargaining cum voice. Section 4 deals with existence and properties of bargaining cum voice solutions for a given group. In section 5, we examine the properties of voice impact functions which transform group members' voiced complaints into relative bargaining power. In section 6, we analyze a simple illustrative example. Section 7 offers final remarks.

2 Related Literature

We are going to assume a finite population of individuals (consumers, economic agents) and finitely many private consumption goods. An equilibrium outcome of our model consists of a price system and a feasible allocation. The latter has two components, a group structure and a feasible allocation of commodities. A group structure is a partition of the population into groups (coalitions, clubs, households). Group decision making and group stability are obviously related. First, we will comment on the sizeable literature on group formation in cooperative game theory and economics. Second, we will report on the related large literature on axiomatic bargaining theories. Third, we will offer some remarks on values and power indices.

2.1 Values

Shapley (1953) assumes transferable utility and proposes a "value" that measures for each player the expected utility of participating in the game. For super-additive games, the Shapley value satisfies individual rationality or, in our context, the equilibrium condition 3 that no individual should benefit from exit. However, the Shapley value as

[2] In general, CEFG need not exist. See Gersbach and Haller (2003).

originally conceived has several shortcomings as regards our purposes:

- The Shapley value does not yield a non-trivial group structure. It is assumed that ultimately the grand coalition is formed.
- Large groups (households, clubs) may be undesirable. In that case, super-additivity of the characteristic function and individual rationality may be violated.
- As a rule, non-transferable utility obtains in our context. There are several ways to extend the Shapley value to NTU games; see in particular Hart and Mas-Colell (1996).

There exist basically two approaches to define the "value of a coalition structure" of a TU (tranferable utility) game. The Aumann-Drèze value [Aumann and Drèze (1974)] determines for each element of the coalition structure the Shapley value of the corresponding subgame and, hence, ignores the opportunities arising from coalitions not contained in the coalition at hand. The Owen value [Owen (1977, 1995)] assumes that ultimately the grand coalition is formed. For further details, see the lucid exposition in Kurz (1988).

The Shapley value has also been suggested as an allocation or arbitration mechanism. There the premise is that the parties involved agree to a system of axioms characterizing the value and accept the utility allocation given by the value in the game at hand. Still, the above shortcomings may render the premise unlikely or the value non-applicable in our context.

2.2 Models of coalition formation

Hedonic coalitions, matching, assignment games. Our concept of CEFG with bargaining cum voice can be applied to a variety of models of coalition formation, including models with hedonic coalitions [e.g. Greenberg (1978), Bogomolnaia and Jackson (2002), Banerjee *et al.* (2001)], matching [e.g. Gale and Shapley (1962), Alkan (1988), Roth and Sotomayor (1990)], assignment games [e.g. Shapley and Shubik (1972), Roth and Sotomayor (1990)]. In the cited literature, markets are inactive and relative prices are irrelevant, simply because there exits at most one tradable commodity. A noteworthy exception are Drèze and Greenberg (1980) who combine the concepts of individual stability and price equilibrium, but confine the analysis of their most comprehensive model to an instructive example.

The research on group or coalition formation has highlighted that it is ultimately unclear how deviations from a proposed group structure should be modelled. Standard solutions such as Nash stability, individual or coalitional stability ignore any possible further deviations and thus may be myopic and may lack credibility.[3] Deviations can be followed by further deviations and thus it is plausible to allow a deviating coalition to reason about the ultimate consequences of its deviation. Such a reasoning of credibility and foresight has been initiated by von Neumann and Morgenstern's (1944) stable set and Harsanyi's (1974) indirect dominance. More recent formalizations of farsightedness and solution concepts include Greenberg (1990), Chwe (1994), Xue (1998), Diamantoudi and Xue (2003), and Barberà and Gerber (2003). They show that the answers depend on the behavioral characteristics of the individuals and that there are various plausible ways to formulate how deviations might induce further deviations. Barberà and Gerber (2007) show that no solution to coalition formation games can satisfy a set of plausible axioms.

Given these potential difficulties, we assume in the present paper that decision makers use hypothetical outside options as arguments when they bargain over consumption bundles. The way in which such uncertain outside options impact the bargaining outcome is axiomatized through the "voice impact function" introduced in subsection 3.4. While "bargaining cum voice" formulated in that way constitutes a novel concept, our general economic model of pure exchange among endogenously formed groups and its concept of a CEFG share several features with previous models of coalition formation, most notably individual preferences and stability concepts. The similarity of equilibrium conditions 4 and 3 & 4, respectively, and individual stability in the sense of Drèze and Greenberg (1980) on the one hand and of Bogomolnaia and Jackson (2002) on the other hand has been pointed out earlier. Next we turn to similarities in the restrictions on preferences.

Notice that in a general equilibrium model of a pure exchange economy, coalition formation is only important if there exist externalities among members of a group or across groups. Like the literature, we rule out inter-group externalities and focus on intra-group externalities. In principle, there can be three kinds of intra-group externalities (externalities within a group): (i) consumption externalities, that is, a group member is affected by the consumption of fellow group members; (ii) group externalities, that is, a group member cares about the composition or size of the group; (iii) endowment externalities, that is, the group's endowment with resources differs from

[3]Moreover, in hedonic coalition formation games, the core may be empty. Banerjeee et al. (2001), Bogomolnaia and Jackson (2002), Alcalde and Revilla (2004), Pápai (2004), Dimitrov and Sung (2007), and Iehlé (2007) provide conditions for the non-emptiness of the core.

the sum of the endowments its members would have as singleton groups.

In Gersbach and Haller (2003), a consumption plan for group h assumes the form $\mathbf{x_h} = (x_i)_{i \in h}$ where x_i is the consumption bundle of group member i. Preferences are represented by $\mathcal{U}_i(\mathbf{x_h}; h)$. Consumption externalities mean that $\mathcal{U}_i(\mathbf{x_h}; h)$ depends on the private consumption x_j for some $j \in h \setminus \{i\}$. Here we follow Drèze and Greenberg (1980) and rule out consumption externalities by assuming for each individual a utility function with two arguments: the individual's private consumption bundle and the group to which the individual belongs. Formally, individual i attains utility $U_i(x_i, h)$ if i consumes the bundle x_i and belongs to group h. Drèze and Greenberg (1980) refer to the dependence on h as the "hedonic aspect" of i's preferences. Bogomolnaia and Jackson (2002) among others call i's preferences "purely hedonic" if U_i depends only on h. In Gersbach and Haller (2003) and in the sequel, we use the terminology "group externality" in lieu of "hedonic aspect" and "pure group externalities" in lieu of "purely hedonic preferences".

Clubs. Club models also deal with an endogenous partition of the population into groups. Some models allow for the competitive market allocation of multiple private goods as well.[4] There are a variety of descriptive features distinguishing between club models and our model. First and foremost, in traditional club theory, the benefit of a club to a member is determined by its membership profile and/or the provision of local public goods (club goods) or abstract club projects. In serving this purpose, the club incurs a resource cost which it tries to recoup through the collection of admission fees. Procurement of private goods remains an individual decision. In contrast, the members of a group or household in our model — and if applicable, in models of hedonic coalitions, matching and assignment games — face a common budget constraint and reach a collective decision regarding the consumption of private goods. In Gersbach and Haller (2009), we clarify in a more systematic way the relationship between our general equilibrium models with multi-member groups or households and club models with multiple private goods.

2.3 Multilateral bargaining and consistency

Multilateral bargaining. Our paper is also related to the theory of multilateral bargaining problems, when there are potential gains from forming coalitions but there is conflict over which coalition to form and how to distribute gains. The idea of antagonistic outside options appears already in Rochford (1984) who focuses on selections

[4]See in particular Cole and Prescott (1997), Ellickson (1979), Ellickson, Grodal, Scotchmer, and Zame (1999, 2001), Gilles and Scotchmer (1997, 1998), Wooders (1988, 1989, 1997).

from the core. Bennett (1988, 1997) has pursued the idea further and has developed an intriguing approach to multilateral bargaining problems.[5] She considers an agreement within a coalition as a solution for the intra-coalitional bargaining process, if the agreement is consistent with the bargaining processes in all other coalitions. The outside option of an individual is the utility the individual would obtain from the agreement in his best alternative coalition.

Our model shares the one important feature with Bennett (1997) that certain (hypothetical) outside options may not be disagreement outcomes because they are not jointly compatible. Our approach differs in other important aspects from the theory of Bennett (1988, 1997). In contrast to her, we consider outside options in a dual role for the bargaining process in a particular coalition. Coalitions belonging to the outcome and, thus, coalitions that will actually form determine outside options in the narrow sense. Hypothetical coalitions, that is those that ultimately are not formed, play a different role. They are used in the speeches of members in a particular coalition in order to articulate potential alternatives. Then the best hypothetical outside alternatives (or maximal complaints) determine the relative bargaining power inside the coalition.

Consistency. Suppose the solution of a cooperative game induces a "reduced game" on any subset of players and the solution concept is also defined for the reduced games. Then the solution satisfies the consistency principle or reduced game property if the solution for each reduced game is the restriction of the solution of the initial game to the respective subset of players. There exist several definitions of reduced games. For details, see Maschler (1990), Thomson (1990), and Tadenuma (1992). In a similar vein, the axiomatic theory of bargaining with a variable number of agents surveyed by Thomson and Lensberg (1989) imposes consistency requirements on bargaining solutions: Their population monotonicity axiom and their multilateral stability axiom relate the solution of a bargaining problem with a large player set to the solution of sub-problems. Now consider two different groups or coalitions g and h with a common member i. Consistency relates the outcomes for g and h to the outcome(s) for $g \cup h$. Hence the outcomes for g and h are indirectly related via the outcome(s) for $g \cup h$. In contrast, in a CEFG with bargaining cum voice, the outcome(s) in g are directly affected by the opportunities or outcomes in h by way of outside options — and vice versa, mutatis mutandis. Moreover, a CEFG comprises an endogenous group structure whereas so far applications of consistency have been confined to scenarios that result in the formation of the grand coalition.

[5]Other analyses of multilateral bargaining problems have been proposed by Kalai and Samet (1985), Chatterjee et al. (1993) and Bennett and van Damme (1991). Bell (1991) provides a subtle discussion of the role of power and outside options in rural societies.

2.4 Power indices

A priori voting power as commonly measured by the Banzhaf index [Banzhaf (1965)] or the Shapley-Shubik index [Shapley and Shubik (1954)] concerns binary decisions of a voting body.[6] It does not predict how the body would decide in a concrete case. Nor does it apply to collective choice problems with a continuum of alternatives, which is typical for the consumption choices groups have to make in our model. The voting situations which are the domain of power indices can be represented as simple cooperative or coalitional games and the Shapley-Shubik index is the restriction of the Shapley value to these games. The suitability of the Shapley value for our purposes has been discussed in subsection 2.1.

3 General Equilibrium Model

We consider a finite pure exchange economy with an endogenous group structure that partitions the population of individual consumers into groups (households, clubs, etc.). Members of a multi-person group have their own individual preferences. Group decisions are the outcome of asymmetric Nash bargaining within each group, where relative bargaining weights are determined by hypothetical outside options, a manifestation of the "power of voice", our main innovation.

3.1 Consumer Characteristics and Allocations

In this subsection, we describe the basic structure of the model: consumers, group structures, commodities, endowments, allocations, and preferences.

Consumers and Group Structures. We consider a finite population of **individuals or consumers**, represented by a set $I = \{1, \ldots, n\}$. A generic consumer is denoted i, j or k. The population I is partitioned into groups, i.e., there exists a partition P of I into non-empty subsets referred to as groups. For a consumer $i \in I$, $P(i)$ denotes the unique element of P (unique group in P) to which i belongs.

A potential group of consumers is any non-empty subset of the population I. A generic group is denoted g or h. $\mathcal{G} = \{g \subseteq I | g \neq \emptyset\}$ denotes the set of all potential groups. $\mathcal{G}(2) = \{g \subseteq I | \, |g| > 1\}$ denotes the set of all potential non-singleton groups. For $i \in I$, $\mathcal{G}_i = \{g \subseteq I | i \in g\}$ denotes the set of all potential groups which have i as a

[6] See Felsenthal and Machover (1998) for an elaborate discussion of the pros and cons of various power indices.

member.

We call any partition P of I a **group structure in** I. We treat the group structure as an object of endogenous choice. Groups are endogenously formed so that some group structure P is ultimately realized. Consequently, our **consumer allocation space** is $\mathcal{P}$, the set of all group structures in I.

Commodities, endowments, and allocations. The commodity space is $\mathbb{R}^\ell$ with $\ell \geq 1$. The commodity allocation space is $\mathcal{X} \equiv \prod_{j \in I} X_j$ where $X_j = \mathbb{R}^\ell_+$ is the consumption set of individual i. The consumption bundle of a generic individual i is denoted by x_i. Let $\mathbf{x} = (x_i), \mathbf{y} = (y_i)$ denote generic elements of $\mathcal{X}$. For $h \in \mathcal{G}$, define $\mathcal{X}_h = \prod_{i \in h} X_i$ with generic elements $\mathbf{x_h} = (x_i)_{i \in h}$. If $\mathbf{x} \in \mathcal{X}$ is an allocation, then for $h \in \mathcal{G}$, group consumption is given by $\mathbf{x_h} = (x_i)_{i \in h} \in \mathcal{X}_h$.

Group h is endowed with a commodity bundle $\omega_h \in \mathbb{R}^\ell$, $\omega_h > 0$, which may differ from $\sum_{i \in h} \omega_{\{i\}}$. Such differences may reflect costs of forming groups or represent a rudimentary form of group production. If the group structure is P, then the aggregate or social endowment is $\omega_P = \sum_{h \in P} \omega_h$.

An allocation is a pair $(\mathbf{x}; P) \in \mathcal{X} \times \mathcal{P}$, consisting of a commodity allocation $\mathbf{x}$ and a group structure P. Thus an allocation specifies the consumption bundle and group membership of each individual. An allocation $(\mathbf{x}; P)$ is feasible if $\sum_i x_i = \omega_P$.

Preferences. A generic individual $i \in I$ has:

- consumption set $X_i = \mathbb{R}^\ell_+$;
- preferences $\succsim_i$ on $X_i \times \mathcal{G}_i$ represented by a utility function $U_i : X_i \times \mathcal{G}_i \to \mathbb{R}$.

In general, individual i cares about her private consumption $x_i \in X_i$ and the composition of the group $g \in \mathcal{G}_i$ she belongs to. We assume that $U_i(x_i; g)$ is non-decreasing, concave and continuous in x_i.

The dependence on g reflects group externalities or "hedonic aspects". In Gersbach and Haller (2003), we also consider consumption externalities within groups, that is dependence of an individual's welfare on the consumption of fellow group members. As noted above, the current model further allows for endowment externalities, that is in general, $\omega_h \neq \sum_{i \in h} \omega_{\{i\}}$.

3.2 Competitive equilibrium with free group formation (CEFG)

The notion of voice power will act as a selection device for competitive exchange among groups. Hence, we first need to define an equilibrium notion in which the power of voice can be embedded. Among the several conceivable ways to formulate an equilibrium state of a model with variable group structure, we follow Gersbach and Haller (2003) and adopt the concept of a competitive equilibrium with free group formation.

Budget Constraints. Consider a group $h \in \mathcal{G}$ and a price system $p \in \mathbb{R}^\ell$. For $\mathbf{x_h} = (x_i)_{i \in h} \in \mathcal{X}_h$, denote total group expenditure $p * \mathbf{x_h} := p \cdot \sum_{i \in h} x_i$. As p and $\mathbf{x_h}$ are of different dimension for multi-member groups, we use the $*$-product in lieu of the familiar inner product. Then h's **budget set** is defined as $B_h(p) = \{\mathbf{x_h} \in \mathcal{X}_h \ : p * \mathbf{x_h} \leq p \cdot \omega_h\}$. We now define the **efficient budget set** $EB_h(p)$ by:

■ $\mathbf{x_h} \in EB_h(p)$ if and only if $\mathbf{x_h} \in B_h(p)$ and there is no $\mathbf{y_h} \in B_h(p)$ such that

(i) $U_i(y_i; h) \geq U_i(x_i; h)$ for all $i \in h$ and

(ii) $U_i(y_i; h) > U_i(x_i; h)$ for some $i \in h$.

CEFG. We briefly review the definition of a competitive equilibrium with free group formation in the current context. First define a **state** of the economy as a triple $(p, \mathbf{x}; P)$ such that $p \in \mathbb{R}^\ell$ is a price system and $(\mathbf{x}; P) \in \mathcal{X} \times P$ is an allocation, i.e. $\mathbf{x} = (x_i)_{i \in I}$ is an allocation of commodities and P is an allocation of consumers (a group structure, a partition of the population into groups). A state $(p, \mathbf{x}; P)$ is a **competitive equilibrium with free group formation (CEFG)** if it satisfies the following conditions:

1. $\mathbf{x_h} \in EB_h(p)$ for all $h \in P$.
2. $\sum_i x_i = \omega_P$.
3. There are no $h \in P$, $i \in h$ and $y_i \in B_{\{i\}}(p)$ such that $U_i(y_i; \{i\}) > U_i(x_i; h)$.
4. There are no $h \in P$, $g \in P$, $i \in h \setminus g$, and $\mathbf{y_{g \cup \{i\}}} \in B_{g \cup \{i\}}(p)$ such that $U_j(y_j; g \cup \{i\}) \geq U_j(x_j; g)$ for all $j \in g$ and $U_i(y_i; g \cup \{i\}) > U_i(x_i; h)$.

Condition 1 reflects collective rationality. Efficient choice by the group refers to the individual consumption and welfare of its members, not merely to the aggregate consumption bundle of the group. Condition 2 requires market clearing. Condition 3 stipulates that no individual wants to leave a group and participate as a one-member group in the market at the going equilibrium prices, i.e., nobody wants to exercise the exit option. Condition 4 requires that no individual can leave a group and can propose a feasible consumption allocation to the members of a new group, created by the individual and another already existing group, which makes nobody in the new group worse off and the particular individual better off at the going equilibrium prices, i.e., no individual benefits from exercising the joining option.

3.3 Outside options

With more than one commodity, a group's attainable commodity allocations and utility allocations may depend on the prevailing price system. Since asymmetric Nash bargaining and in particular bargaining cum voice require compact choice sets, we consider strictly positive price systems which yield compact budget sets.[7] Then let $p \in \mathbb{R}^\ell, p \gg 0$ be a price system, $g \in \mathcal{G}$ a potential group and

$$\mathcal{V}_g(p) = \{V = (V_i)_{i \in g} \in \mathbb{R}^g |\, \exists\, \mathbf{x_g} = (x_i)_{i \in g} \in B_g(p) :\, V_i = U_i(x_i, g) \text{ for all } i \in g\}$$

the set of utility allocations attainable by group g at the price system p. Outside options are contingent on a state of the economy $(p, \mathbf{x}; P)$: Let $i \in I$ be an individual and $g = P(i)$ be the group i belongs to. Let $P^c(i)$ denote the complement of $P(i)$, the set of individuals who are not members of $P(i)$.

Let us stress the dual role of outside options. Agents operate from a given partition (group structure) P. First, they consider whether to exit or to join another existing group. If $|P(i)| > 1$, denote $P_J(i) \equiv P \setminus \{P(i)\} \cup \{\emptyset\}$, the set of groups i might join, including the empty set in case of exit. Second, agents reason with reference to hypothetical groups which are ultimately not formed. If $|P(i)| > 1$, denote $P_H(i) \equiv \{h \subseteq P^c(i) : h \neq \emptyset, h \notin P\}$, the set of potential groups h such that $h \cup \{i\}$ constitutes a hypothetical group for individual i.

Actual outside option(s): Suppose $|P(i)| > 1$. For $h \in P_J(i)$, let

$$a_i(p, h) = \sup\{V_i |\, \exists\, V = (V_j)_{j \in h \cup \{i\}} \in \mathcal{V}_{h \cup \{i\}}(p) :\, V_j \geq U_j(x_j, P(j)) \text{ for all } j \in h\}, \quad (1)$$

with $\sup \emptyset = -\infty$. Then $a_i(p, \emptyset) \in \mathbb{R}$ and

$$A_i(p, P) = \max_{h \in P_J(i)} a_i(p, h) \in \mathbb{R}$$

[7]Note that standard sufficient conditions for the existence of CEFG imply strictly positive equilibrium prices.

is i's **actual outside option value**. $A_i(p, P)$ is the maximal utility i can expect from exit or joining an existing group h when the members of h are guaranteed their status quo utilities. We sometimes use $V_i^0 \equiv A_i(p, P)$ to simplify exposition.

Formula (1) can also be applied to any consumer k with $|P(k)| \geq 1$ and $h = \emptyset$, in which case $a_k(p, \emptyset)$ constitutes k's stand-alone utility that will be used to define hypothetical outside options.

Hypothetical outside option(s): Suppose $|P(i)| > 1$. For $h \in P_H(i)$ let

$$\hat{v}_i(p, h) = \sup\{V_i | \exists V = (V_j)_{j \in h \cup \{i\}} \in \mathcal{V}_{h \cup \{i\}}(p) : V_k \geq a_k(p, \emptyset) \text{ for all } k \in h\}. \qquad (2)$$

$\hat{v}_i(p, h)$ is the maximum utility for i in the hypothetical group $h \cup \{i\}$ if the members of h are guaranteed their stand-alone utilities. Next let us define

$$\widehat{V}_i(p, P) = \max_{h \in P_H(i)} \hat{v}_i(p, h).$$

$\widehat{V}_i(p, P)$ is i's **hypothetical outside option value**.

Notice that $\hat{v}_i(p, h)$ might serve as i's aspiration level à la Kalai and Smorodinsky (1975) for bargaining in group $h \cup \{i\}$. Yet in our context, it impacts upon i's bargaining weight in group g where groups g and $h \cup \{i\}$ have only i in common.

3.4 Nash bargaining

We are now ready to specify the bargaining solution. An efficient consumption decision for group g in state $(p, \mathbf{x}; P)$ with $g \in P$ and $p \gg 0$ is obtained as the solution of a Nash bargaining problem

$$\max \prod_{i \in g} (V_i - V_i^0)^{\beta_i(g)} \text{ s.t. } (V_i)_{i \in g} \in \mathcal{V}_g(p) \text{ and } V_i \geq V_i^0 \text{ for all } i \in g. \qquad (3)$$

The bargaining weight $\beta_i(g) \geq 0$ measures the **relative bargaining power** of individual i within group g. The weights satisfy $\sum_{i \in g} \beta_i(g) = 1$. If the constraint set is empty, then g would be unstable — and $(p, \mathbf{x}; P)$ cannot be a CEFG state. If there exists $(V_i)_{i \in g} \in \mathcal{V}_g(p)$ with $V_i > V_i^0$ for all $i \in g$ and if $\beta_i(g) > 0$ for all $i \in g$, then problem (3) has a unique solution $(V_i^*)_{i \in g}$ and there exists $\mathbf{x}_\mathbf{g}^* \in B_g(p)$ such that $V_i^* = U_i(x_i^*; g)$ for $i \in g$. Now suppose that, indeed, $\mathbf{x}_\mathbf{g} = \mathbf{x}_\mathbf{g}^*$ and $\beta_i(g) > 0$ for $i \in g$. Then in state $(p, \mathbf{x}; P)$, group g satisfies equilibrium condition 1 and its members satisfy equilibrium conditions 3 and 4.

If the constraint set is non-empty and $\beta_i(g) > 0$ for all $i \in g$, but $(V_i)_{i\in g} \in \mathcal{V}_g(p)$ implies $V_i = V_i^0$ for some $i \in g$, then not every solution $(V_i^*)_{i\in g}$ of (3) may be associated with an element of $EB_g(p)$. But at least one is. In case $\mathcal{V}_g(p) = \{(V_i^0)_{i\in g}\}$, the problem has the unique solution $(V_i^0)_{i\in g}$ which will do. Otherwise, there exists a maximal number m such that $1 \leq m < |g|$ and $(V_i)_{i\in g} \in \mathcal{V}_g(p)$ implies $V_i = V_i^0$ for at least m consumers $i \in g$. We can choose $(V_i')_{i\in g} \in \mathcal{V}_g(p)$ such that $V_i' = V_i^0$ for exactly m consumers $i \in g$. Let h be the set of members of g with $V_i' > V_i^0$. Maximization of the Nash product $\prod_{i\in h}(V_i - V_i^0)^{\beta_i(g)}$ subject to $(V_i)_{i\in g} \in \mathcal{V}_g(p)$ and $V_i \geq V_i^0$ for all $i \in g$ yields a solution that will do.

Endogenous bargaining power. Next we express formally the power of voice. The gist of the novel concept is that group members voice complaints, if their hypothetical outside option value exceeds their current utility, and members with stronger complaints enjoy greater relative bargaining power. Suppose the current state is $(p, \mathbf{x}; P)$, group g belongs to the prevailing group structure P and $|g| > 1$. Then the complaint of individual $i \in g$ is $C_i = \max\{0, \widehat{V}_i(p, P) - U_i(x_i; g)\}$. Let us assume momentarily that $C_i > 0$ for all $i \in g$, that is, every group member has a **valid complaint**. Then the complaints are transformed into relative bargaining weights by means of a **voice impact function** $\varphi : [0, 1] \to [0, 1]$ and group g solves problem (3) with weights

$$\beta_i(g) = \varphi\left(\frac{C_i}{\sum_{j\in g} C_j}\right). \tag{4}$$

φ is assumed to satisfy $\varphi(0) = 0$, $\varphi(1) = 1$, $\varphi' > 0$. Moreover, $\sum_{j\in g} \beta_j(g) = 1$ has to hold. The voice impact function thus reflects the impact of voice: The larger the complaint by an individual relative to the complaints by the other group members, the higher is the individual's bargaining power.

If all group members have valid complaints, then all $\beta_i(g)$ are positive. If in addition, there exists $(V_i)_{i\in g} \in \mathcal{V}_g(p)$ with $V_i > V_i^0$ for all $i \in g$, then the solution to (3) is associated with an element in $EB_g(p)$. If $C_i \geq 0$ for all group members and at least one member has a valid complaint, then (4) can still be applied, but only some of the solutions to (3) may be associated with an element in $EB_g(p)$. If no group member has a valid complaint, then we set $\beta_i(g) = 1/|g|$ for all $i \in g$. The mixed case where some group members have valid complaints while others have not, is quite plausible after further reflection. Namely, suppose for example that one member of the group holds all the bargaining power and the other members get zero consumption. The other individuals may still prefer to stay in the group because of strong positive group externalities — so that not only their complaints lack validity, but also their actual outside options are dominated by the status quo.

CEFG with bargaining cum voice. A state $(p, \mathbf{x}; P)$ is a **competitive equilibrium with free group formation (CEFG) and bargaining cum voice** if it satisfies conditions 1 to 4 defining a CEFG and there exist bargaining weights $\beta_i(g)$ for $i \in g \in P \cap \mathcal{G}(2)$ such that the following two conditions hold:

5. $V_i^* = U_i(x_i; g), i \in g$, defines a solution $(V_i^*)_{i\in g}$ of (3) for each $g \in P \cap \mathcal{G}(2)$.
6. Equation (4) determines the bargaining weights $\beta_i(g), i \in g$, for each $g \in P \cap \mathcal{G}(2)$ with some valid complaints. Further $\beta_i(g) = 1/|g|$ if no member of g has a valid complaint.

4 Basic Feedback Mechanism

Here we outline and analyze the basic feedback mechanism that incorporates the impact of voice. Notice that in a CEFG with bargaining cum voice, the bargaining weights in (3) are given by (4) unless all complaints are invalid. Conversely, in equilibrium, the complaints $C_i = \max\{0, \widehat{V}_i(p, P) - U_i(x_i; g)\}$ in (4) can be rewritten $C_i = \max\{0, \widehat{V}_i(p, P) - V_i^*)\}$ where $(V_i^*)_{i\in g}$ is a solution of (3). We call a **bargaining cum voice solution** for group g a pair $((V_i^*)_{i\in g}, (\beta_i(g))_{i\in g})$ that is simultaneously determined by (3) and (4).

For the remainder of this section, we are going to take a state $(p, \mathbf{x}; P)$ as given and focus on some group $g \in P \cap \mathcal{G}(2)$.

4.1 Existence of bargaining cum voice solutions

Here we explore existence of a bargaining cum voice solution for group $g \in P \cap \mathcal{G}(2)$. We shall show that under certain conditions, there exist $(V_i^*)_{i\in g} \in \mathcal{V}_g(p)$ and $\beta_i(g) > 0$ for $i \in g$ such that $(V_i^*)_{i\in g}$ is the unique solution of (3) given the $\beta_i(g)$ and the $\beta_i(g)$ are determined by (4) given $C_i = \max\{0, \widehat{V}_i(p, P) - V_i^*)\}$. To this end, set

$$\widetilde{\mathcal{V}}_g(p) = \{V = (V_i)_{i\in g} \in \mathcal{V}_g(p) \,|\, V_i \geq V_i^0 = A_i(p, P) \text{ for all } i \in g\}.$$

Then $\widetilde{\mathcal{V}}_g(p)$ is a convex and compact subset of $\mathbb{R}^g$. Let $\gg$ denote component-wise strict inequality.

We obtain:

Proposition 1 *Suppose that:*

(i) $\exists\,(V_i)_{i\in g} \in \mathcal{V}_g(p) : (V_i)_{i\in g} \gg (V_i^0)_{i\in g}$.

(ii) $(\widehat{V}_i(p,P))_{i\in g} \gg (V_i)_{i\in g}$ *for all* $(V_i)_{i\in g} \in \widetilde{\mathcal{V}}_g(p)$.

Then there exist $(V_i^*)_{i\in g} \in \widetilde{\mathcal{V}}_g(p)$ *and* $(\beta_i^*(g))_{i\in g} \gg 0$ *that solve the bargaining cum voice problem and, thus, endogenize relative bargaining power for group* g.

Proof. Since $\widetilde{\mathcal{V}}_g(p)$ is compact, it follows from (ii) that there exist $\overline{C} > 0$ and $\underline{C} > 0$ such that $(\overline{C},\ldots,\overline{C}) \gg (\widehat{V}_j(p,P) - V_j)_{j\in g} \gg (\underline{C},\ldots,\underline{C})$ and

$$\varphi\left(\frac{\widehat{V}_i(p,P) - V_i}{\sum_{j\in g}[\widehat{V}_j(p,P) - V_j]}\right) > \varepsilon \equiv \varphi\left(\frac{\underline{C}}{|g|\cdot\overline{C}}\right) > 0$$

for all $(V_j)_{j\in g} \in \widetilde{\mathcal{V}}_g(p)$ and $i \in g$. For such $\overline{C}$, $\underline{C}$, and ε let us set

$$\Delta_g^\varepsilon = \{(\beta_i(g))_{i\in g} \in \mathbb{R} \mid \sum_{i\in g}\beta_i(g) = 1 \text{ and } \beta_i(g) > \varepsilon \text{ for all } i \in g\}.$$

Then Δ_g^ε is a convex and compact but possibly empty simplex. Let us choose $\overline{C}$ large enough and $\underline{C}$ small enough so that $1/|g| > \varepsilon > 0$ — which is possible because φ is continuous and strictly increasing and $\varphi(0) = 0$. Then Δ_g^ε has non-empty relative interior. Next consider the continuous function $\Psi : \widetilde{\mathcal{V}}_g(p) \times \Delta_g^\varepsilon \to \mathbb{R}$ defined by

$$\Psi(V,\beta(g)) = \prod_{i\in g}(V_i - V_i^0)^{\beta_i(g)}$$

for $V = (V_j)_{j\in g} \in \widetilde{\mathcal{V}}_g(p)$ and $\beta(g) = (\beta_i(g))_{i\in g} \in \Delta_g^\varepsilon$. For a given $\beta(g)$, we obtain the objective function in (3). Further consider the continuous correspondence Γ from Δ_g^ε to $\widetilde{\mathcal{V}}_g(p) \times \Delta_g^\varepsilon$ given by $\beta(g) \longmapsto \widetilde{\mathcal{V}}_g(p) \times \{\beta(g)\}$ for $\beta(g) \in \Delta_g^\varepsilon$. For each $\beta(g) \in \Delta_g^\varepsilon$, the problem

$$\max_{V\in\widetilde{\mathcal{V}}_g(p)} \Psi(V,\beta(g))$$

has a unique solution $\psi_1(\beta(g))$. Application of Berge's maximum theorem applied to Ψ and the constraint correspondence Γ shows that $\psi_1(\beta(g))$ is a continuous function of $\beta(g)$. Conversely, $(V_j)_{j\in g} \longmapsto \left(\varphi([\widehat{V}_j(p,P) - V_j]/[\sum_{i\in g}(\widehat{V}_i(p,P) - V_i)])\right)_{j\in g}$ defines a continuous mapping ψ_2 from $\widetilde{\mathcal{V}}_g(p)$ to Δ_g^ε. By Brouwer's fixed point theorem, the composition $\psi_2\circ\psi_1$ has a fixed point $\beta^*(g) = (\beta_i^*(g))_{i\in g}$. $\beta^*(g)$ and $(V_i^*)_{i\in g} = \psi_1(\beta^*(g))$ have the asserted properties. □

Remarks.

(a) If g is a two-person group, say $g = \{1, 2\}$ and any two efficient boundary points $(V_1, V_2), (V_1', V_2')$ of $\widetilde{\mathcal{V}}_g(p)$ satisfy $[V_1 > V_1' \iff V_2 < V_2']$, then $(V_i^*)_{i \in g}$ and $(\beta_i^*(g))_{i \in g}$ in Proposition 1 are unique. For suppose that there are two different solutions $((V_i^*)_{i \in g}, (\beta_i^*(g))_{i \in g})$ and $((V_i^{**})_{i \in g}, (\beta_i^{**}(g))_{i \in g})$. Then
$\beta_i^*(g) > \beta_i^{**}(g) \Rightarrow [V_i^* > V_i^{**}, V_{-i}^* < V_{-i}^{**}] \Rightarrow [C_i^* < C_i^{**}, C_{-i}^* > C_{-i}^{**}] \Rightarrow \beta_i^*(g) < \beta_i^{**}(g)$, a contradiction. The condition on boundary points is met if the utility functions U_1 and U_2 are strictly increasing in x_1 and x_2, respectively.

(b) If $\widetilde{\mathcal{V}}_g(p)$ is non-empty, but (i) does not hold, then as indicated earlier, we can proceed with a suitably chosen subgroup h of g and instead of (3) solve the problem $\max \prod_{i \in h} (V_i - V_i^0)^{\beta_i(g)}$ subject to $(V_i)_{i \in g} \in \mathcal{V}_g(p)$ and $V_i \geq V_i^0$ for all $i \in g$.

(c) Hypothesis (ii) and the distinction between valid and invalid (zero) complaints become obsolete if each individual is granted a minimal complaint $\theta > 0$ so that $C_i = \max\{\theta, \widehat{V}_i(p, P) - U_i(x_i; g)\}$.

4.2 Properties of bargaining cum voice solutions

The Nash bargaining solution is the only solution that satisfies a series of desirable properties: individual rationality, Pareto optimality, independence of affine transformations, and independence of irrelevant alternatives. We next discuss whether these properties are preserved by bargaining cum voice. For this purpose we introduce the following property.

Independence of Common Affine Transformations:

> Let $(p, \mathbf{x}; P)$ be a state and $g \in P \cap \mathcal{G}(2)$ be a group so that hypotheses (i) and (ii) of Proposition 1 are satisfied. A bargaining solution for g with feasible set $\widetilde{\mathcal{V}}_g(p)$ is independent of common affine transformations if for any $c \in \mathbb{R}$ and $d > 0$ the solution is transformed the same way when all utilities are affinely transformed:
>
> $$U_i^{\bullet} = c + dU_i \text{ for all } i \in g. \tag{5}$$

Then we obtain:

Proposition 2 *The bargaining cum voice solution satisfies individual rationality, independence of common affine transformations, Pareto optimality, and independence of irrelevant alternatives.*

Proof. Individual rationality and Pareto optimality are fulfilled as the solution maximizes a Nash product. Independence of common affine transformations holds since a solution $(V_i^*)_{i\in g} \in \widetilde{\mathcal{V}}_g(p)$ and $(\beta_i^*(g))_{i\in g} \gg 0$ in Proposition 1 gets transformed into $V^{\bullet *} = (V_i^{\bullet *})_{i\in g} = (c+dV_i^*)_{i\in g}$ and $(\beta_i^*(g))_{i\in g} \gg 0$ after a common affine transformation (5). Namely,

(a) a solution $V^* = (V_i^*)_{i\in g}$ of (3) is replaced by $V^{\bullet *} = (V_i^{\bullet *})_{i\in g} = (c + dV_i^*)_{i\in g}$ after a common affine transformation (5) (because asymmetric Nash bargaining solutions satisfy independence of affine transformations) and

(b) $\widehat{V}_i^{\bullet}(p, P) - V_i^{\bullet *} = d \cdot [\widehat{V}_i(p, P) - V_i]$ for $i \in g$ so that (4) yields the same bargaining weights after the transformation (5).

Independence of irrelevant alternatives follows from the simultaneous solution of (3) and (4). □

Two important remarks are in order:

Dependence on arbitrary affine transformations: Bargaining cum voice does not satisfy independence of arbitrary transformations, because the voice impact function is affected when utility units of agents are changed by different scale parameters, that is, when $U_i^{\bullet} = c_i + d_i U_i$ with $d_i \neq d_j$ for some $i, j \in g$. In the voice impact function we compare the maximal complaints of both agents which necessarily requires cardinal information on preferences and interpersonal comparison of utilities.

Dependence on hypothetical outside option values: The notion independence of irrelevant alternatives refers solely to actual choices of the group g. By construction, the bargaining outcome does depend on hypothetical outside option values as well. When we showed independence of common affine transformations, we used the fact that hypothetical outside option values are subject to the same common affine transformation.

5 Properties of Voice Impact Functions

Regarding the question which functions φ qualify as voice impact functions, there is a striking difference between two-person groups and groups with more than two members.

5.1 Two-person groups

Without loss of generality, consider $g = \{1, 2\}$. We can set $\beta = \beta_1(g)$ and $1 - \beta = \beta_2(g)$. Instead of $\varphi : [0, 1] \to [0, 1]$, it proves productive and convenient to postulate a voice impact function $f : \mathbb{R}_+ \to \mathbb{R}_+$ so that

$$\beta = f(C_1/C_2) \tag{6}$$

if both group members have valid complaints, that is, $C_1 > 0$ and $C_2 > 0$.[8] We note that with $x = \frac{C_1}{C_2}$, we have $f(x) = \varphi(\frac{x}{1+x})$. In turn, with $y = \frac{C_1}{C_1+C_2}$, we get $\varphi(y) = f(\frac{y}{1-y})$. f should have the following properties:

(A1) $f(0) = 0$.

(A2) $f(x) + f(1/x) = 1$.

(A3) $\lim_{x\to\infty} f\big(x\big) = 1$.

(A4) $f' > 0$.

Axioms A1, A3 are boundary conditions. The condition A2 supposes that both group members are equally able in transforming hypothetical but possible utility gains from forming other groups into bargaining power through articulation of their aspirations or complaints. Axiom A4 captures the power of voice.

Let us examine the family of voice impact functions:

Fact 1

(a) *Any voice impact function f is fully determined by its restriction to $x \in [0, 1]$, since $f(x) = 1 - f(1/x)$ for $x > 1$. Moreover, $f(1) = 1/2$. Conversely, any differentiable function $f : [0, 1] \to \mathbb{R}_+$ with $f(0) = 0, f(1) = 1/2$ and $f' > 0$ can be extended to a voice impact function by setting $f(x) = 1 - f(1/x)$ for $x > 1$.*

(b) *(6) is the equivalent of (4). $\varphi : [0, 1] \to [0, 1]$ can be obtained from f via $\varphi(y) = f\left(\frac{y}{1-y}\right)$. It satisfies $\varphi' > 0$ and*

$$\varphi(y) + \varphi(1 - y) = f\left(\frac{y}{1-y}\right) + f\left(\frac{1-y}{1-(1-y)}\right) = f(x) + f\left(\frac{1}{x}\right) = 1.$$

By (a), there is a large family of φ that qualify.

[8] The definition can be extended to the cases $C_i = 0$ by setting $\beta = 1$ if $C_1 > 0, C_2 = 0$, $\beta = 0$ if $C_1 = 0, C_2 > 0$, and $\beta = 1/2$ if $C_1 = 0, C_2 = 0$.

Examples

- $f(x) = \dfrac{x}{1+x}$ corresponds to $\varphi(y) = y$.

- $f(x) = 2 \cdot \left(\dfrac{x}{1+x}\right)^2$ for $x \leq 1$ yields
  $f(x) = 1 - f(1/x) = 1 - 2 \cdot \left(\dfrac{1/x}{1+1/x}\right)^2 = 1 - 2 \cdot \left(\dfrac{1}{1+x}\right)^2$ for $x \geq 1$.
  This corresponds to $\varphi(y) = 2 \cdot \left(\dfrac{y/(1-y)}{1+y/(1-y)}\right)^2 = 2y^2$ for $y \leq 1/2$;
  $\varphi(y) = 1 - 2 \cdot \left(\dfrac{1}{1+y/(1-y)}\right)^2 = 1 - 2 \cdot (1-y)^2$ for $1/2 < y \leq 1$.

5.2 Groups with more than two members

By Fact 1, there exist many voice impact functions for two-person groups. In sharp contrast, there exists only one voice impact function for groups with more than two members. Consider a group g with $|g| \geq 3$ and relative bargaining weights $\beta_i(g), i \in g$, given by (4). The function φ is assumed to satisfy $\varphi(0) = 0$, $\varphi(1) = 1$, $\varphi' > 0$. Moreover, $\sum_{j \in g} \beta_j(g) = 1$ has to hold. We obtain

Proposition 3 *Suppose $|g| \geq 3$ and φ is applicable to group g in various economic environments so that any vector of complaints $(C_i)_{i \in g} \in \mathbb{R}^g_+$ can occur. Then φ is the identity function.*

Proof. Let $|g| = m \geq 3$. Without restriction, we label individuals so that $g = \{1, \ldots, m\}$. Consider vectors of complaints $(C_i)_{i \in g} \in \mathbb{R}^g_+$ where at least one group member j has a valid complaint, i.e., $C_j > 0$. Set $z_i = C_i/(\sum_j C_j)$, the relative complaint of member i and $z = (z_1, \ldots, z_m)$. Next let $y_1 > 0, y_2 > 0, y_3 > 0$ satisfy $y_1 + y_2 + y_3 = 1$. Then $z = (y_1, y_2, y_3, 0, \ldots, 0)$ and $z' = (0, y_1 + y_2, y_3, 0, \ldots, 0)$ can occur. Note that all values z_j and z'_j for $j = 4, \ldots, m$ have been set to zero if $m > 3$. Since $\varphi(0) = 0$, the requirement $\sum_{j=1}^m \beta_j(g) = 1$ implies

$$\varphi(y_1) + \varphi(y_2) = 1 - \varphi(y_3) = \varphi(y_1 + y_2). \tag{7}$$

Let $\mathbb{N}$ denote the set of positive integers (natural numbers). Repeatedly applying (7), we obtain $\varphi(kx) = k\varphi(x)$ for every $x \in (0,1)$ and $k \in \mathbb{N}$ as long as $kx < 1$. Similarly

$\varphi(\frac{x}{K}) = \frac{1}{K}\varphi(x)$ because of $K\varphi(\frac{x}{K}) = \varphi(x)$ for any $K \in \mathbb{N}$. Taking both properties together yields

$$\varphi\left(\frac{k}{K}\right) = \frac{k}{K}\varphi(1) = \frac{k}{K} \text{ for all } k, K \in \mathbb{N} \text{ with } k < K.$$

Since the rational numbers in $[0, 1]$ are dense in $[0, 1]$ and φ is continuous, the assertion follows. □

6 An Illustrative Example

We analyze an example of an exchange economy where individuals can form groups in which they benefit from group externalities. Furthermore, a group's economic environment is going to depend on the price system which in turn is determined by market clearing conditions.

6.1 Primitive Data

In this subsection, we describe the primitive data of our example. We consider a population of four individual consumers which is the minimum number needed to illustrate the power of voice. Thus, the population is represented by the set $I = \{1, \ldots, 4\}$. A generic consumer is again denoted by i or j. We assume that there exist two commodities for private consumption. Hence $\ell = 2$ and each individual $i \in I$ has consumption set $X_i = \mathbb{R}^2_+$. The consumption bundle of individual i is denoted by $x_i = (x_i^1, x_i^2)$ or $y_i = (y_i^1, y_i^2)$.

Preferences are represented by functions of the specific form

$$U_i(x_i; h) = \begin{cases} \gamma \, \ln x_i^1 + (1-\gamma) \, \ln x_i^2, & \text{in case } \ h = \{i\}; \\ \gamma \, \ln x_i^1 + (1-\gamma) \, \ln x_i^2 + s_{ij}, & \text{in case } \ h = \{i, j\} \ \text{ with } \ i \neq j; \\ \gamma \, \ln x_i^1 + (1-\gamma) \, \ln x_i^2 - t, & \text{in case } \ |h| = 3 \ \text{ or } \ |h| = 4; \end{cases}$$

where $\gamma \in (0, 1)$, $t > 0$, $s_{ij} \geq 0$ for $i \neq j$.

Decisions within actually formed groups are reached by way of bargaining cum voice, based on (3), (6), and the voice impact function $f(x) = x/(x+1)$.

We further assume individual endowments $w_i = (w_i^1, w_i^2)$ and that group formation is costless and does not affect endowments. Therefore, a potential group h is endowed with the commodity bundle $w_h \in \mathbb{R}^2$ given by the sum of the endowments of all participating individuals: $w_h = \sum_{i \in h} w_i$. Consequently, the social or aggregate endowment is independent of the group structure and equals $w_S = w_I = \sum_{i \in I} w_i$.

Note that forming a three-person or four-person group exerts negative group externalities of $-t$ on everybody in that group. Hence, given the assumption on endowments, such large groups will never be formed in a competitive equilibrium with free group formation (CEFG): Given such a large group and any feasible commodity allocation, at least one group member rather goes single.

6.2 Equilibria with Free Group Formation

Bargaining cum voice will act as a selection device for competitive exchange among groups. In order to demonstrate this, we first characterize equilibria with free group formation (CEFG). Recall that we can neglect group structures where any group size is larger than 2. Accordingly, only group structures with two two-person groups prevail in CEFG.

Commodity prices $p = (p_1, p_2)$ are normalized so that $p_1 = 1$. We can represent the efficient decisions of a two-person group $h = \{i, j\}, i < j$, by assuming that the group maximizes a utilitarian social welfare function

$$W_h = \alpha_h U_i(x_i; h) + (1 - \alpha_h)\, U_j(x_j; h)$$

subject to the budget constraint. The number α_h $(0 \leq \alpha_h \leq 1)$ is the utilitarian weight of individual i in group h. In the remainder of this subsection we treat α_h as parametrically given. In the next subsection, the weight α_h will be endogenized — and will give rise to endogenous Nash bargaining weights. Given any p_2, identical homothetic preferences with respect to consumption imply that group demand as well as individual consumption bundles will be linear in income. Hence, we obtain

Fact 2 *CEFG exist and have the following properties:*

(i) *Two two-person groups are formed.*

(ii) *The equilibrium price* p_2^* *is given by* $p_2^* = (1 - \gamma) \cdot w_S^1 / [\gamma \cdot w_S^2]$, *with associated nominal social wealth* $y_S^* = w_S^1 + p_2^* w_S^2$ *and nominal income* $y_g^* = w_g^1 + p_2^* w_g^2$ *and* $y_h^* = w_h^1 + p_2^* w_h^2$ *for groups* g *and* h*, respectively.*

(iii) *The equilibrium allocation for a group structure P, say $P = \{h, g\}$ with $h = \{1, 2\}$ and $g = \{3, 4\}$, is characterized by two numbers α_h and α_g ($0 < \alpha_h < 1$, $0 < \alpha_g < 1$) and given by*

$$\begin{array}{rclrcl} x_1^* & = & \alpha_h (y_h^*/y_S^*) w_S, & x_2^* & = & (1-\alpha_h)(y_h^*/y_S^*) w_S; \\ x_3^* & = & \alpha_g (y_g^*/y_S^*) w_S, & x_4^* & = & (1-\alpha_g)(y_g^*/y_S^*) w_S. \end{array}$$

To establish the boundaries for the numbers α_h and α_g, we observe that we can neglect the joining option. Forming three-person groups does not create positive group externalities for the entrant and destroys existing benefits of group formation. Hence, exit dominates joining in all conceivable deviations from the CEFG candidate.

The exit option for individual i yields utility

$$U_i(x_i^0(p_2^*)) = \gamma \ \ln\left(\gamma(w_i^1 + p_2^* w_i^2)\right) + (1-\gamma) \ \ln\left((1-\gamma)\left(\frac{w_i^1 + p_2^* w_i^2}{p_2^*}\right)\right)$$

where $x_i^0(p_2^*)$ is i's demand as a single at the price system $(1, p_2^*)$, which establishes

Fact 3
For a typical group structure that can qualify for a CEFG, say $P = \{h, g\}$ with $h = \{1, 2\}$ and $g = \{3, 4\}$, there exist $\underline{\alpha}_h < \overline{\alpha}_h$ and $\underline{\alpha}_g < \overline{\alpha}_g$ such that a CEFG with the properties described in Fact 2 exists if and only if

$$\underline{\alpha}_h \leq \alpha_h \leq \overline{\alpha}_h \text{ and } \underline{\alpha}_g \leq \alpha_g \leq \overline{\alpha}_g.$$

6.3 Bargaining cum voice ("voice power")

The remaining question is how α_h and α_g are determined — whose answer leads to the selection of a particular CEFG via bargaining cum voice. Like in our general model, we presume that every group member expresses the utility that he could achieve in a hypothetical group, i.e., in a group that does not currently exist and cannot be formed by exit or by joining another group. The potential gains relative to current utility that group members can identify in their speeches will then determine relative bargaining power through the power of voice.

We commence with the bargaining problem in a particular group, say $h = \{1, 2\}$. Suppose that a CEFG as characterized in Fact 2 prevails and h's demand is obtained by solving a bargaining problem of the form (3):

$$\max \left\{ N_h = \prod_{i \in g} (V_i - V_i^0)^{\beta_i(g)} \right\} \text{ s.t. } (V_i)_{i \in g} \in \mathcal{V}_g(p) \text{ and } V_i \geq V_i^0 = A_i(p, P) \text{ for all } i \in g.$$

To determine unique values of α_h and $\beta_h = \beta_1(h)$, we proceed in two steps. In the FIRST STEP, we determine the utilitarian weight α_h so that the maximizer of W_h also maximizes the Nash product N_h for a given value of β_h. In the SECOND STEP, we determine the value of β_h for a given α_h through the power of voice — through equation (4), to be precise. We will find a unique pair (α_h^*, β_h^*) that yields the solution of the group bargaining problem and is consistent with bargaining cum voice.

FIRST STEP. Because of Fact 2, we can adopt the following simplifying notation. With α_h as the weight of the first member in the actual group h in the utilitarian welfare function, we can express the various utilities as follows:

$$\begin{aligned}
V_1 = V_1(\alpha_h) &:= U_1(x_1; h) = \gamma \ln\{\alpha_h \gamma y_h^*\} + (1-\gamma)\ln\{\alpha_h(1-\gamma)y_h^*/p_2^*\} + s_{12} \\
V_2 = V_2(\alpha_h) &:= U_2(x_2; h) = \gamma \ln\{(1-\alpha_h)\gamma y_h^*\} + (1-\gamma)\ln\{(1-\alpha_h)(1-\gamma)y_h^*/p_2^*\} + s_{21} \\
V_1^0 &:= U_1(x_1^0(p_2^*)) = \gamma \ln\{\gamma y_1^*\} + (1-\gamma)\ln\{(1-\gamma)y_1^*/p_2^*\} \\
V_2^0 &:= U_2(x_2^0(p_2^*)) = \gamma \ln\{\gamma y_2^*\} + (1-\gamma)\ln\{(1-\gamma)y_2^*/p_2^*\} \\
V_3^0 &:= U_3(x_3^0(p_2^*)) = \gamma \ln\{\gamma y_3^*\} + (1-\gamma)\ln\{(1-\gamma)y_3^*/p_2^*\} \\
V_4^0 &:= U_4(x_4^0(p_2^*)) = \gamma \ln\{\gamma y_4^*\} + (1-\gamma)\ln\{(1-\gamma)y_4^*/p_2^*\}
\end{aligned}$$

For any given $\beta_h \in (0,1)$, the bargaining problem is well defined and can be solved for the utilitarian weights. Taking $\ln N_h$ and maximizing with respect to α_h yields the first-order condition:

$$\begin{aligned}
&\beta_h \frac{1}{V_1(\alpha_h) - V_1^0} \left\{\gamma \frac{1}{\alpha_h} + (1-\gamma)\frac{1}{\alpha_h}\right\} \\
-&(1-\beta_h)\frac{1}{V_2(\alpha_h) - V_2^0} \left\{\gamma \frac{1}{1-\alpha_h} + (1-\gamma)\frac{1}{1-\alpha_h}\right\} = 0
\end{aligned}$$

or

$$\beta_h \frac{1}{(V_1(\alpha_h) - V_1^0)\alpha_h} - (1-\beta_h)\frac{1}{(V_2(\alpha_h) - V_2^0)(1-\alpha_h)} = 0. \tag{8}$$

SECOND STEP. We determine β_h as a function of the utilitarian weight α_h through the power of voice. An individual $i = 1, 2$ can imagine being in a two-person group $\{i, 3\}$ or $\{i, 4\}$. We concentrate on the group $\{i, 3\}$. Forming a group with individual 3 constitutes the more attractive hypothetical group if $v_{i3} > v_{i4} > 0$, $i = 1, 2$, and $v_{3i} > v_{4i} > 0$, $i = 1, 2$ — which we are going to assume for the remainder of this subsection.[9]

[9] One might further impose the requirement that hypothetical groups must be credible in the sense of Chwe (1994). The further assumptions $v_{3i} > v_{34}$ and $v_{4i} > v_{43}$ for each $i = 1, 2$ would satisfy the Chwe criterion for hypothetical groups.

Now suppose individual i imagines a group allocation such that individual 3 just obtains his utility as a single. Then $\widehat{V}_i = \widehat{V}_i(p^*, P)$, the maximal hypothetical utility for individual i, is given by

$$\begin{aligned} \widehat{V}_i = \widehat{V}_i(p^*, P) &= \gamma \ln \left\{ \gamma \overline{\alpha}_{\{i,3\}} \left(w_i^1 + w_3^1 + p_2^*(w_i^2 + w_3^2) \right) \right\} \\ &+ (1-\gamma) \ln \left\{ (1-\gamma)\overline{\alpha}_{\{i,3\}} \left(\frac{w_i^1 + w_3^1 + p_2^*(w_i^2 + w_3^2)}{p_2^*} \right) \right\} + s_{i3}, \end{aligned}$$

where $\overline{\alpha}_{\{i,3\}}$ is determined by

$$\begin{aligned} V_3^0 &= \gamma \ln \left\{ \gamma (1 - \overline{\alpha}_{\{i,3\}}) \left(w_i^1 + w_3^1 + p_2^*(w_i^2 + w_3^2) \right) \right\} \\ &+ (1-\gamma) \ln \left\{ (1-\gamma)(1 - \overline{\alpha}_{\{i,3\}}) \left(\frac{w_i^1 + w_3^1 + p_2^*(w_i^2 + w_3^2)}{p_2^*} \right) \right\} + s_{3i}. \end{aligned}$$

Note that $\overline{\alpha}_{\{i,3\}}$ is the highest possible weight individual i can have in household $\{i, 3\}$ without forcing the exit of individual 3. It is obvious that $\overline{\alpha}_{\{i,3\}}$ and $\widehat{V}_i(p^*, P)$ are uniquely determined. Running through the same exercise for group $g = \{3, 4\}$, when individuals imagine forming groups with the first individual, yields hypothetical outside option $\widehat{V}_i(p^*, P)$, $i = 3, 4$.

We note that the hypothetical outside option values cannot be used as threat points or reservation utilities in intra-group bargaining, but nevertheless can have an impact: The utilities $\widehat{V}_i(p^*, P)$ are used in the speeches of existing groups to express their members' aspirations and we further assume that these aspirations translate into relative bargaining power in existing groups. Hence, the relative bargaining power must be consistent with the hypothetical utility gains that individuals can articulate — which themselves depend on relative bargaining power. In the current example, those gains are the complaints $C_i = \max\{0, \widehat{V}_i(p^*, P) - U_i(x_i; h)\} = \max\{0, \widehat{V}_i(p^*, P) - V_i(\alpha_h)\}$ for $i \in h$ and $C_i = \max\{0, \widehat{V}_i(p^*, P) - V_i(\alpha_g)\}$ for $i \in g$. By (6), the complaints C_i translate into relative bargaining power $\beta_h = f(C_1/C_2)$ and $\beta_g = f(C_3/C_4)$. With $f(x) = x/(x+1)$, we obtain for group h (if 1 or 2 has a valid complaint):

$$\left.\begin{aligned} \beta_h &= \frac{\widehat{V}_1(p^*, P) - V_1(\alpha_h)}{\widehat{V}_1(p^*, P) - V_1(\alpha_h) + \widehat{V}_2(p^*, P) - V_2(\alpha_h)}; \\ 1 - \beta_h &= \frac{\widehat{V}_2(p^*, P) - V_2(\alpha_h)}{\widehat{V}_1(p^*, P) - V_1(\alpha_h) + \widehat{V}_2(p^*, P) - V_2(\alpha_h)}. \end{aligned}\right\} \tag{9}$$

Using (9) to substitute β_h and $1 - \beta_h$ in (8) yields:

$$\frac{\widehat{V}_1(p^*, P) - V_1(\alpha_h)}{(V_1(\alpha_h) - V_1^0)\alpha_h} = \frac{\widehat{V}_2(p^*, P) - V_2(\alpha_h)}{(V_2(\alpha_h) - V_2^0)(1 - \alpha_h)} \tag{10}$$

We obtain:

Proposition 4
Suppose there exists $\alpha_h \in [\underline{\alpha}_h, \overline{\alpha}_h]$ *such that* $\widehat{V}_1(p^*, P) > V_1(\alpha_h)$ *and* $\widehat{V}_2(p^*, P) > V_2(\alpha_h)$. *Then there exist unique values* $\alpha_h^* \in [\underline{\alpha}_h, \overline{\alpha}_h]$ *and* $\beta_h^* \in (0, 1)$ *that solve the group optimization problem and are consistent with bargaining cum voice. The value* α_h^* *is determined by* (10) *— and* β_h^* *is given by* (9).

The proof of proposition 4 follows immediately from the observation that the left-hand side of (10) is strictly decreasing in α_h whereas the right-hand side of (10) is strictly increasing in α_h. Moreover, for $\alpha_h \to \underline{\alpha}_h$ $(1 - \alpha_h \to 1 - \overline{\alpha}_h)$ the left-hand side (right-hand side) becomes infinite. Finally, by construction, the solution of the group optimization problem in terms of α_h lies in the interval $[\underline{\alpha}_h, \overline{\alpha}_h]$, which guarantees that no individual would want to exit.

Proposition 4 shows how exit and voice power interact in determining the group allocation. *Ceteris paribus* considerations yield:

Corollary 1

$$\frac{\partial \alpha_h^*}{\partial \widehat{V}_1} > 0, \quad \frac{\partial \alpha_h^*}{\partial \widehat{V}_2} < 0; \; \frac{\partial \alpha_h^*}{\partial V_1^0} > 0, \quad \frac{\partial \alpha_h^*}{\partial V_2^0} < 0.$$

We note that exit and voice power uniquely determine the group allocation. Given that equilibrium prices are independent of α_h^* and β_h^* we obtain:

Proposition 5
Suppose there exist utilitarian weights $\alpha_h \in [\underline{\alpha}_h, \overline{\alpha}_h]$ *and* $\alpha_g \in [\underline{\alpha}_g, \overline{\alpha}_g]$ *such that* $\widehat{V}_1 > V_1(\alpha_h)$, $\widehat{V}_2 > V_2(\alpha_h)$, $\widehat{V}_3 > V_3(\alpha_g)$, *and* $\widehat{V}_4 > V_4(\alpha_g)$. *Then for* $P = \{\{1, 2\}, \{3, 4\}\}$, *there exists a unique CEFG of the form* (p^*, x^*, P) *that is consistent with bargaining cum voice.*

Remark. Suppose all consumers have identical homothetic preferences for consumption, represented by a utility function which is differentiable, strictly concave and strictly increasing on $\mathbb{R}^{\ell}_{++}$. Then an analogue of Fact 2 holds. Next consider household $h = \{1, 2\}$, say, with voice impact function f. Then given β_h and the equilibrium price system determined in Fact 2, maximization of the Nash product N_h yields utilitarian welfare weight α_h for consumer 1 in h as a continuous function ϕ_1 of β_h. On the other hand, (6) determines β_h as a continuous function ϕ_2 of α_h. By Brouwer's fixed point theorem, the composition mapping $\phi_1 \circ \phi_2$ has a fixed point α_h^*. Hence there exist $\alpha_h^* \in [0,1]$ and $\beta_h^* \in [0,1]$ that solve the group optimization problem and are consistent with bargaining cum voice. The value of β_h^* is obtained via (6). However, application of the fixed point theorem does not yield uniqueness.

6.4 Specific Numerical Values

By choosing specific numerical values for some or all of the exogenous model parameters, we can (a) illustrate the workings of Proposition 5, in particular demonstrate the possibility of comparative statics; (b) demonstrate that the hypotheses of Propositions 4 and 5 can be met.

To this end, we consider the following two assumptions:

(A) $\gamma = 1 - \gamma = 1/2\,, \quad w_1 = w_2 = w_3 = w_4 = (1,1)$ so that the endowments of the particular two-person households are $w_g = (2,2)$ and $w_h = (2,2)$.

(B) There exists $s_0 > 0$ such that $s_{ij} = s_0$ for all $i \neq j$.

Suppose (A). Then $p_2^* = 1$ and moreover:

$$
\begin{aligned}
V_1(\alpha_h) &= \frac{1}{2}\{\ln(2\alpha_h) + \ln(2\alpha_h)\} + s_{12} = \ln(2\alpha_h) + s_{12},\\
V_2(\alpha_h) &= \frac{1}{2}\{\ln[2(1-\alpha_h)] + \ln[2(1-\alpha_h)]\} + s_{21} = \ln[2(1-\alpha_h)] + s_{21},\\
V_1^0 &= \frac{1}{2}\{\ln 1 + \ln 1\} = 0,\\
V_2^0 &= 0,\\
\widehat{V}_1 &= \frac{1}{2}\left\{\ln[2\overline{\alpha}_{\{1,3\}}] + \ln[2\overline{\alpha}_{\{1,3\}}]\right\} + s_{13} = \ln[2\overline{\alpha}_{\{1,3\}}] + s_{13},\\
V_3^0 &= 0 = \frac{1}{2}\left\{\ln(2(1-\overline{\alpha}_{\{1,3\}})) + \ln(2(1-\overline{\alpha}_{\{1,3\}}))\right\} + s_{31},\\
\widehat{V}_2 &= \ln[2\overline{\alpha}_{\{2,3\}}] + s_{23},\\
V_3^0 &= 0 = \frac{1}{2}\left\{\ln(2(1-\overline{\alpha}_{\{2,3\}}) + \ln(2(1-\overline{\alpha}_{\{2,3\}})\right\} + s_{32}.
\end{aligned}
$$

This implies:

$$
\begin{aligned}
2(1-\overline{\alpha}_k) &= \mathrm{e}^{-s_{31}}\,, \quad \widehat{V}_1 = \ln(2 - \mathrm{e}^{-s_{31}}) + s_{13},\\
2(1-\overline{\alpha}_{k'}) &= \mathrm{e}^{-s_{32}}\,, \quad \widehat{V}_2 = \ln(2 - \mathrm{e}^{-s_{32}}) + s_{23}.
\end{aligned}
$$

Using (10), we find that the group allocation satisfies:

$$\frac{\ln\left(2-\mathrm{e}^{-s_{31}}\right)+s_{13}-\ln(2\alpha_h)-s_{12}}{\alpha_h(\ln(2\alpha_h)+s_{12})}=\frac{\ln\left(2-\mathrm{e}^{-s_{32}}\right)+s_{23}-\ln[2(1-\alpha_h)]-s_{21}}{(1-\alpha_h)\left(\ln[2(1-\alpha_h)]+s_{21}\right)}$$

This equation determines α_h^* uniquely. We obtain

Corollary 2 *Suppose* $s_{12}=s_{21}$ *and the existence of* α_h *such that* $\widehat{V}_1>V_1(\alpha_h)>0$ *and* $\widehat{V}_2>V_2(\alpha_h)>0$*. Then there exists a unique value of* α_h^**. Moreover,* α_h^* *is increasing in* $\ln\left(2-e^{-s_{31}}\right)+s_{13}$ *and decreasing in* $\ln\left(2-e^{-s_{32}}\right)+s_{23}$*, with* $\alpha_h^*=1/2$ *if and only if* $\ln\left(2-e^{-s_{31}}\right)+s_{13}=\ln\left(2-e^{-s_{32}}\right)+s_{23}$.

Intuitively, the higher s_{13} is relative to s_{23} and the higher s_{31} is relative to s_{32}, the larger becomes the relative bargaining power of the first individual since her power of voice is comparatively larger.

Suppose (A) and (B) so that there prevails total symmetry among individuals. Then each CEFG with bargaining cum voice and normalized price system is of the form $(p^*,\mathbf{x}^*;P^*)$ with $P^*=\{g,h\}$, $|g|=|h|=2$, $p^*=(1,1)$, $x_i^*=w_i$ for all $i\in I$, utilitarian weights $\alpha_g^*=\alpha_h^*=1/2$, and bargaining weights $\beta_g^*=\beta_h^*=1/2$. Moreover, in equilibrium, $\widehat{V}_i>V_i(\alpha_{P(i)}^*)>V_i^0$ for all i. The inequalities persist when the model parameters deviate slightly from the symmetry conditions (A) and (B).

7 Final Remarks

We have proposed and examined a concept of voice power. Numerous issues deserve further attention. Model variations might be considered where even with invalid complaints, individuals always preserve some bargaining power. Apart from incorporating voice power in more general models, a more detailed behavioral foundation of our concept should be taken up in future research.

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