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ABSTRACT

Reconciling VAR-based and Narrative Measures of the Tax-Multiplier

The currently available empirical evidence shows remarkable differences between various estimates of the effects on U.S. output of an exogenous shift in Federal tax liabilities. Shocks identified via the narrative method imply a multiplier of about three over an horizon of three years. Tax shocks identified in fiscal VAR models deliver a much smaller multiplier of about one. Is this heterogeneity real, or is it simply the result of different approaches to the identification of exogenous shifts in taxes? Or of different specifications of the empirical model used to estimate the tax multiplier? In this paper we reconcile this apparently contradictory evidence by showing that the large multiplier obtained via the narrative identification methods is generated by the choice of a limited information approach in their estimation and not by the different nature of the shocks. Using the shocks identified by narrative methods in a multivariate dynamic model delivers estimates of the tax multiplier very much in line with those obtained in the traditional fiscal VAR approach.

JEL Classification: E62 and H60
Keywords: fiscal policy, government budget constraint, public debt and VAR models

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1 Introduction

The size of the tax multiplier is without any doubt a crucial economic policy issue. The currently available empirical evidence shows remarkable differences between various estimates of the effects on U.S output of an exogenous shift in Federal tax liabilities. Is this heterogeneity real, or is it simply the result of different approaches to the identification of exogenous shifts in taxes? Or of different specifications of the empirical model used to estimate the tax multiplier?

Romer and Romer (forthcoming, R&R in what follows) identify tax shocks from the "narrative" of Presidential speeches and Congressional records. This analysis allows them to separate legislated changes in taxes between those they consider endogenous (induced by short-run counter-cyclical concerns or adopted as a response to changes in government spending) and those they judge exogenous (associated with a political shift, or adopted in response to the state of government debt, or in the attempt to raise long-run economic growth). For the post World War II period they find a multiplier significantly greater than one: a tax increase of 1% of U.S. GDP reduces output over the next three years by nearly 3%.

The fiscal VAR approach identifies tax shocks in linear multivariate dynamic model either by exploiting the fact that it typically takes longer than one quarter for discretionary fiscal policy to respond to news in macroeconomic variables, and using institutional information on the elasticities of tax revenues and government spending to macro variables (Blanchard and Perotti 2002, Perotti, 2008, from here onwards we refer to both articles as B&P), or by imposing restrictions on the sign of impulse responses that reflect institutional information (Mountford and Uhlig, 2002) or even by relying on a Choleski ordering (Fatas and Mihov,2001). None of these identification schemes, however, is capable of delivering tax multipliers of the size estimated by R&R. The typical estimate of the tax multiplier delivered by fiscal VARs over the same time period analyzed by R&R is of about one.

There are also important difference in the structural stability of these estimates in the post World War II period. The B&P results for the entire sample (1960 to 2001) average very different responses before and after 1980, (as shown in Perotti 2008). In the first part of the sample tax cuts have a positive and significant effect on output, with a multiplier only slightly smaller compared with R&R (around 2.6 at a three year horizon). After 1980, however, the effect turns negative and significant with a multiplier that is similar in absolute value. On the contrary the R&R evidence on the size of tax multipliers is stable over the two subsamples.

The contrasting evidence is summarized in Figure 1, where we report the effect on output of an exogenous shift in taxes equivalent to 1% of U.S. GDP as computed by R&R and as derived form the application of the B&P identification scheme to a closed-economy fiscal VAR which includes government expenditure, government receipts, output growth, inflation, and the average cost of servicing the debt.

R&R claim that these differences are the result of the failure of structural VAR's to identify truly exogenous shifts in taxes. Figure 2 shows that the exogenous shocks identified by the two alternative methods are indeed quite different. Their correlation over the entire sample is 0.21 and the two identification strategies lead to a substantial disagreement as to when the largest policy shifts occurred. It thus looks as if the difference in the estimates of the tax multiplier depends on the different approaches used in identifying tax shocks which indeed produce different time series for such shocks.

Can this be the reason? Not necessarily. "Shocks" measure exogenous shifts in fiscal policy, but there is no reason why such measure should be unique. Different identification approaches could produce different time series of tax shocks, each exogenous and thus each legitimate. However, alternative valid instruments, although different, could very well deliver the same estimate of the tax multiplier, as they should.

The debate on alternative ways of identifying fiscal shocks is reminiscent of a similar debate on the identification of monetary policy shocks. Rudebusch (1998) criticizes the VAR-based analysis of the effects of monetary policy shocks pointing out that shocks identified from structural VARs—typically from a regression of the Fed funds rate on an assortment of macro variables and therefore via a recursive identification scheme between macroeconomic variables and monetary policy—bear little correlation with shocks to the Fed funds rate derived from forward-looking financial markets (the Fed funds future). He thus concludes that monetary policy shocks identified from a VAR make no sense. Sims (1998) replies observing that in a multivariate framework measures of the same variables that bear little correlation with one another can produce identical transmission mechanisms. He suggests, as an example, the measurement of the effects of supply shocks in a simple demand-supply model. Two variables can shift the supply function, weather and insect density. Consider two possible alternative instrumenting models: each excludes one instrument. As both supply shifters are valid instruments, the two models will produce valid and equivalent estimates of the structural parameters, despite their different specification.

Thus, the relevant test for the establishing the difference between shocks estimated in a VAR and outside the VAR is the inclusion of tax shocks identified outside the VAR into the VAR model to evaluate difference between the impulse responses computed using different shocks in the same empirical model. In the case of monetary policy shocks VAR and non-VAR shocks (as shown by Bagliano and Favero 1999) deliver the same description of the monetary transmission mechanism.

The evidence, reported in Figure 1, of a difference between the multipliers associated with the R&R and the B&P measures of exogenous shifts in taxes is not decisive. The different impulse responses reported in Figure 1 are not only based on different shocks but also on different models. R&R estimate fiscal multipliers using a limited information framework: a univariate regression of GDP growth on the identified exogenous tax shocks. The multiplier estimated by B&P is derived form the application of their identification scheme to a closed-economy fiscal VAR which includes the five variables indicated above. How much of the these differences in the tax multiplier depend on the different empirical models used to estimate it?

We construct an answer proceeding in two steps:

1. First we show that the equation R&R estimate to compute the tax multiplier can be interpreted as a truncated MA representation of the equation for output growth in a VAR. The truncations happens along two dimensions: the number of lags is finite, and no shocks other than tax shocks are included. This suggests three potential reasons why the R&R approach yields an estimate of the multiplier that differs from that obtained from a VAR:

   (a) the adoption of a limited information rather than a full information approach;

   (b) both models overlook the government intertemporal budget constraint. The consequences of such an omission could be different in the two empirical models;

   (c) fiscal foresight—that neither R&R nor B&P explicitly consider—could have different implications in the two estimation strategies.

2. Next we compare the multiplier obtained using the two different time series of tax shocks in a common VAR framework.

When we move from different-shocks, different-models to different-shocks, common-model we find tax multipliers that are almost identical and whose size is about 1.0.

2 VAR-based and Narrative Measures of the effect of tax shocks

In this section we describe exactly how we have derived the evidence reported in Figure 1 and Figure 2.

2.1 VAR Approach

We consider the type of structural VAR estimated in B&P. Tax multipliers are obtained estimating a vector autoregression of the form [1]:

$$\begin{aligned} \mathbf{Z}_t &= \mathbf{C}_1 \mathbf{Z}_{t-1} + \mathbf{u}_t \\ \mathbf{Z}_t' &= \left[\begin{array}{ccccc} g_t & t_t & y_t & \Delta p_t & i_t \end{array} \right] \end{aligned} \tag{1}$$

where i_t is the average nominal cost of financing the debt , Δy_t is real GDP growth, Δp_t is inflation, t_t and g_t are, respectively, (the logs of) government revenues and government expenditure net of interest. We have chosen this particular set of variables to be able to fully describe the debt dynamics[2]. This seems to be a natural choice for a minimal set of variables to be included in the analysis of the effects of fiscal policy.

VAR's identify fiscal shocks imposing restrictions that allow to recover uniquely the structural shocks of interest from the reduced form residuals, $\mathbf{u}$. The innovations in the reduced form equations for taxes and government spending, u_t^g and u_t^t, contain three terms: *(i)* the response of taxes and government spending to fluctuations in macroeconomic variables, such as output and inflation, that is implied by the presence of automatic stabilizers; *(ii)* the discretionary response of fiscal policy to news in macro variables, and *(iii)* truly exogenous shifts in taxes and spending, the shocks we wish to identify. B&P exploit the fact that it typically takes longer than a quarter for discretionary fiscal policy to respond to news in macroeconomic variables: at quarterly frequency the contemporaneous discretionary response of fiscal policy to macroeconomic data can thus be assumed to be zero. To identify the component of

[1] For simplicity we consider a first order VAR. VARs of any order can be reparameterised as a first order VAR, through a stacked representaton.

[2] In fact, we choose variable in order to be able to reconstructl exactly the debt dynamics, given an inital condition for the debt to GDP ratio. So our choice of variables is slightly different from that of B&P. See the Data Appendix for a full description of the construction of our data-set.

u_t^g and u_t^t which corresponds to automatic stabilizers they use institutional information on the elasticities of tax revenues and government spending to macroeconomic variables. They thus identify the structural shocks to g and t by imposing on the matrices $\mathbf{A}$ and $\mathbf{B}$ that determine the mapping from the VAR innovations $\mathbf{u}$ to the structural shocks $\mathbf{e}$ ($\mathbf{Au}_t = \mathbf{Be}_t$) the following restrictions:

$$\begin{bmatrix} 1 & 0 & a_{gy} & a_{g\Delta p} & a_{gi} \\ 0 & 1 & a_{ty} & a_{t\Delta p} & a_{ti} \\ a_{31} & a_{32} & 1 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & 1 & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & 1 \end{bmatrix} \begin{bmatrix} u_t^g \\ u_t^t \\ u_t^y \\ u_t^{\Delta p_t} \\ u_t^i \end{bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 & 0 \\ 0 & 0 & b_{33} & 0 & 0 \\ 0 & 0 & 0 & b_{44} & 0 \\ 0 & 0 & 0 & 0 & b_{55} \end{bmatrix} \begin{bmatrix} e_t^g \\ e_t^t \\ e_t^1 \\ e_t^2 \\ e_t^3 \end{bmatrix}$$

where e_t^i $(i = 1, 2, 3)$ are non-fiscal shocks and have no structural interpretation. Since a_{gy}, $a_{g\Delta p}$, a_{gi}, a_{ty}, $a_{t\Delta p}$ and a_{ti} are identified using external information [3], there are only 15 parameters to be estimated. As there are also 15 different elements in the variance-covariance matrix of the 5-equation VAR innovations, the model is just identified. The e_t^i $(i = 1, 2, 3)$ are derived by imposing a recursive scheme on the bottom three rows of $\mathbf{A}$ and $\mathbf{B}$; however, the identification of the two fiscal shocks– the only ones that we shall use to compute impulse responses–is independent of this assumption. Finally, the identification assumption imposes $b_{12} = 0$. [4] What we report in Figure 1 under the label "Blanchard-Perotti impulse responses" are the responses of the level of output to a one period shock in e_t^t of the size of one per cent of GDP. In Figure 2 we report the time-series of e_t^t.

[3] The elasticities of taxes and government spending with respect to output, inflation and interest rates used in the identification have been updated in Perotti (2007) and are

Elasticities of government revenues and expenditures						
	a_{gy}	$a_{g\Delta p}$	a_{gi}	a_{ty}	$a_{t\Delta p}$	a_{ti}
Entire sample	0	-0.5	0	1.85	1.25	0
1960:1-1979:4	0	-0.5	0	1.75	1.09	0
1980:1-2006:2	0	-0.5	0	1.97	1.40	0

[4] B&P provide robustness checks for this assumption by setting $b_{21} = 0$ and estimating b_{12}. We have also experimented with this alternative option. In practice, as the top left corner of the $\mathbf{B}$ matrix is not statistically different from a diagonal matrix, the assumption $b_{12} = 0$ is irrelevant to determine the shape of impulse response functions.

2.2 The Narrative Approach

R&R construct a time-series of tax shocks without the need to estimate a model. They refer to the narrative record, such as presidential speeches and Congressional reports, to identify the size, timing, and principal motivation for all major postwar tax policy actions. They then classify legislated changes into endogenous (those induced by short-run countercyclical concerns and those taken because of change in government spending) and exogenous (those that are responses to the state of government debt or to concerns about long-run economic growth). Having constructed a time series of exogenous shifts in taxes, e_{t-i}^{RR}—where each e_{t-i}^{RR} measures the impact of a tax change at the time it was implemented $(t-i)$ on tax liabilities at time t—R&R measure their effect on output, y_t, estimating, using quarterly data and ordinary least squares, a single equation of the form

$$\Delta y_t = a + \sum_{i=0}^{M} b_i e_{t-i}^{RR} + \epsilon_t \tag{2}$$

Exogenous tax shocks are measured as a percentage of GDP. So the response of the level of output at time $t+i$ to a one-period shock of the size of 1% of GDP is measured by the sum of the b_i coefficients. This is what we report in Figure 1 under the label R&R. Figure 2 reports the time series of e_{t-i}^{RR}. As in R&R we have chosen $M = 12$.

3 Different Shocks, Different Models, Different Impulse Responses

To understand the narrative approach in terms of the SVAR start from the structural representation of the VAR:

$$\mathbf{A}\mathbf{Z}_t = \mathbf{C}\mathbf{Z}_{t-1} + \mathbf{B}\mathbf{e}_t. \tag{3}$$

The MA representation of (3) is

$$\mathbf{Z}_t = \Gamma(L)\mathbf{e}_t \tag{4}$$

where $\Gamma(L) \equiv \frac{A^{-1}B}{1-A^{-1}CL}$.

The MA representation is not directly estimated in the VAR approach, but it is derived by inversion after having estimated (3) .

We can rewrite the infinite MA representation of the VAR as follows

$$\begin{aligned}
\mathbf{Z}_t &= \sum_{i=0}^{M} \Gamma_0 \Gamma_1^i \mathbf{e}_{t-i} + \Gamma_1^{M+1} \mathbf{Z}_{t-M+1} \\
\Gamma_0 &\equiv A^{-1}B, \ \Gamma_1 \equiv A^{-1}C.
\end{aligned}$$

By extracting from the above system the equation for output growth we have

$$\begin{aligned}
\Delta y_t &= \sum_{i=0}^{M} e^t \Gamma_0 \Gamma_1^i \mathbf{e}_{t-i} + \sum_{i=0}^{M} \left(\iota - e^t\right) \Gamma_0 \Gamma_1^i \mathbf{e}_{t-i} + \Gamma_1^{M+1} \mathbf{Z}_{t-M+1} \qquad (5) \\
e^t &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \\
\iota &= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}.
\end{aligned}$$

Compare now (5) with the equation estimated by R&R, it is clear from the above discussion that R&R use a limited information approach, while the VAR uses a system approach. Under what conditions will the two approaches deliver the same estimate of the tax multiplier?

3.1 Different instruments, same shocks

We start by considering the case the VAR based and the narrative approach are two different ways of finding valid instruments for tax shocks. When the VAR is a valid approximation for the unknown Data Generating Process, two necessary conditions must be satisfied for (2) and (3) to deliver the same estimates of the impulse responses of output to an exogenous tax shock :

- the "tax shocks" e_{t-i}^{RR} are orthogonal to any other shock in $\boldsymbol{\varepsilon}_t$ that might influence output growth,
- the "tax shocks" e_{t-i}^{RR} are orthogonal to $\mathbf{Z}_{t-M+1}$.

Orthogonality of the tax shocks to any other shock in $\boldsymbol{\varepsilon}_t$ is the identifying assumption in R&R: from an analysis of the extensive discussion in the narrative record of why each e_{t-i}^{RR} action was taken, R&R conclude that *"most actions had a single predominant motivation, and that some of those motivations are unrelated to other factors likely to have important effects on output growth (and to any other tax responses policymakers may have been making to those factors at around the same time)"*.

The second condition, however is unlikely to be satisfied, for the following reason. R&R classify as exogenous those legislated tax changes that "*are responses to the* *state of government debt or to concerns about long-run economic growth*". Given that debt dynamics is fully determined by the variables normally included in $\mathbf{Z}$ in SVAR model-based analysis of fiscal policy, the orthogonality of e_{t-i}^{RR} to $\mathbf{Z}_{t-M+1}$ does not seem to be warranted by the R&R identification strategy.

Consider the government intertemporal budget constraint:

$$
\begin{aligned}
d_t &= \frac{1+i_t}{(1+x_t)} d_{t-1} + \frac{\exp(g_t) - \exp(t_t)}{\exp(y_t)} \\
& x_t \equiv \Delta p_t + \Delta y_t + \pi_t \Delta y_t
\end{aligned}
\tag{6}
$$

where i_t is the average nominal cost of financing the debt , Δy_t is real GDP growth, Δp_t is inflation, t_t and g_t are, respectively, (the logs of) government revenues and government expenditure net of interest. From (6) it is immediately obvious that the dynamics of the debt is fully determined at any point in time by the dynamics of the variables normally included in the vector $\mathbf{Z}$, moreover the relationship between the debt and the variables in $\mathbf{Z}$ is non-linear. Therefore, the orthogonality the "tax shocks" e_{t-i}^{RR} to $\mathbf{Z}_{t-M+1}$ is violated and the moving average representation of any variable in the system in terms of an (infinite) sum of structural shocks is not valid anymore.

As a matter of fact, debt and the Intertemporal Government Budget Constraint (IGBC) has been systematically excluded from empirical investigations of the effects of fiscal policy—not only by R&R, but essentially by the entire empirical literature (Edelberg et al, 1999, Blanchard and Perotti, 2002, Mountford Uhlig, 2002, Fatàs and Mihov, 2001 among other). Only very recently Chung and Leeper(2007) have adopted an empirical model that explicitly considers the government intertemporal budget constraint via cross -equation restrictions derived from a log-linearized version of (6)

This omission is surprising for theoretical and empirical reasons.

- From the empirical point of view, Bohn (1998), using a century of U.S data, finds a positive correlation between the government surplus and the Federal debt—a result which suggests that U.S. fiscal policy reacts to the level of the debt ratio.[5] If fiscal variables respond to the level of the debt, then the analysis of the impact of fiscal shocks should be conducted by explicitly recognizing a role for debt and the stock-flow identity linking debt and deficits. The response

[5] Corsetti et al.(2009) show that the impact of a shift in public spending depends on expectations about offsetting fiscal measures in the future.

of the economy to a fiscal shock clearly depends on the dynamic impact on the debt of such shocks. Such a response will be very different depending on whether the shock produces a path of debt that is stable or tends to become explosive.

- From the theoretical point of view, the omission of a debt feedback from estimated fiscal VARs is surprising also because the equilibrium structural models used to analyze the effects of fiscal policy are typically solved by imposing the government intertemporal budget constraint and are simulated under the assumption that the real value of the debt in the hands of the public must equal the expected present value of government surpluses.

One justification for omitting the debt level is that the effects of this variable are captured by all other variables included in a fiscal VAR. The difference is that the debt dynamics equation is non-linear, while the VAR is linear. Whether or not including the debt ratio directly in the VAR makes a difference thus depends on how good an approximation the linear version of (6) is. This requires in particular that the debt to GDP ratio is stationary. In this case impulse responses to fiscal shocks are to be interpreted as the response of the economy computed at the mean of the stationary government debt to GDP ratio.

To sum-up the omission of debt has two implications for the mapping between the fiscal VAR and its truncated MA representation adopted by R&R.

- The importance of the debt in the fiscal reaction function strongly contradicts the orthogonality of e_{t-i}^{RR} to $\mathbf{Z}_{t-M+1}$ and therefore the possibility of obtaining empirically the same multipliers in the limited information R&R approach and in the full information VAR approach.
- If the linear approximation of the IGBC is invalid, then it will be still possible to estimate a VAR with the IGBC but this VAR will not have a MA representation.

3.2 Different instruments, different shocks

In the previous Section we have considered the shocks identified from the VAR and from the narrative approach as two alternative procedures for instrumenting the same exogenous shocks. The difference in the instruments arises from the fact that the narrative shocks are derived independently form any statistical model. Instead the VAR-based evidence is obviously model dependent and its validity relies on the assumption that the agents' and the econometrician's information sets are aligned. Leeper, Walker and Sun Chun Yang (2008) point out that fiscal foresight could cause a misalignment of the two information sets, thus making it impossible to extract meaningful shocks to taxes from statistical innovation in the VAR.

Fiscal foresight happens when agents, at some point in time, receive signals on the taxes they will face in the future. This is very likely given legislative and implementation lags in tax policy. To understand the implication of fiscal foresight consider, as an example, the simplest RBC model, adapted from Leeper et al. (2008). The model is log linearized, with log preferences, inelastic labour supply and complete depreciation of capital. A proportional tax is levied against income and used for lump-sun transfers on a period by period basis, There is no government spending. The economy is subject to two shocks: an exogenous technological shocks e_t^A and a tax shock, e_{t-q}^t. News about future tax rates arrives q periods before the new rates are implemented.

The equilibrium conditions are the following:

$$\begin{aligned} \frac{1}{C_t} &= \alpha\beta E_t \left(1-\tau_{t+1}\right) \frac{1}{C_{t+1}} \frac{Y_{t+1}}{K_t} \\ C_t + K_t &= Y_t = A_t K_{t-1}^{\alpha} \\ \tau_t &= \tau \exp\left(e_{t-q}^t\right) \\ A_t &= \exp\left(e_t^A\right) \end{aligned}$$

This reduces (after log-linearization) to a bivariate model for capital and technology

$$\theta E_t k_{t+1} - \left(1+\alpha\theta\right) k_t + \alpha k_{t-1} = \rho E_t \tau_{t+1} - e_{A,t}$$

$$\theta = \alpha\beta\left(1-\tau\right), \ \rho = \tau\frac{1-\theta}{1-\tau}$$

After solving the model we obtain the following representation:

$$\begin{aligned} k_t &= \alpha k_{t-1} + e_t^A - \ \rho \sum_{i=0}^{\infty} \theta^i E_t e_{t-q+1+i}^t \\ a_t &= e_t^A \end{aligned}$$

Consider now estimating a bivariate VAR in a_t, k_t and retrieving the two shocks from the VAR innovations. As the equilibrium looks different for different degrees of fiscal foresight, the outcome of this procedure would clearly be affected by it.

- In the case of no fiscal foresight the ($q = 0$) the equilibrium is

$$\begin{aligned} k_t &= \alpha k_{t-1} + e_t^A \\ a_t &= e_t^A \end{aligned}$$

  and a VAR in a_t, k_t would feature stochastic singularity, as only one shock will drive the two variables.

- In the case of one-oeriod fiscal foresight, $q = 1$, the equilibrium is

$$\begin{aligned} a_t &= e_t^A \\ k_t &= \alpha k_{t-1} + e_t^A - \rho e_t^t \end{aligned}$$

  and a Choleski identification for the innovations in the VAR in a_t, k_t would allow to correctly identify the structural shocks of interest.

- In the case of two-periods fiscal foresight, $q = 2$, the equilibrium is

$$\begin{aligned} a_t &= e_t^A \\ k_t &= \alpha k_{t-1} + e_t^A - \rho \left(e_{t-1}^t + \theta e_t^t \right) \end{aligned}$$

  and it would not be possible to identify the structural shocks of interest from the VAR innovations. In fact, for any $q \geqq 2$ we have non-invertibility of the moving average component of the time series of k_t (see Hansen and Sargent, 1991, Lippi and Reichlin, 1994).

Note that in the presence of fiscal foresight, the VAR identification is hopeless, while the narrative approach is in principle still able to identify tax shocks. Mertens and Ravn (2008) use the R&R measure of shifts in taxes and distinguish between those that were anticipated and those that were not. Their findings confirm the large multiplier reported by R&R and show that anticipated and unanticipated shifts in taxes have similar effects—though anticipated tax cuts, before they are implemented, tend to have a contractionary effect on output.

4 Different shocks, same models, same impulse responses.

To understand the relative effect of all the potential sources of discrepancies between the R&R tax multipliers and the traditional VAR based tax multiplier we propose to re-estimate the impact of tax shocks within a general unifying framework.

We adopt the following empirical specification

$$\begin{aligned} \mathbf{Z}_t &= \sum_{i=1}^{k} \mathbf{C}_i \mathbf{Z}_{t-i} + \delta_i e_t^{RR} + \boldsymbol{\gamma}_i \left(d_{t-1} - d^*\right) + \mathbf{u}_t \qquad (7) \\ d_t &= \frac{1+i_t}{\left(1+\Delta p_t\right)\left(1+\Delta y_t\right)} d_{t-1} + \frac{\exp\left(g_t\right) - \exp\left(t_t\right)}{\exp\left(y_t\right)} \end{aligned}$$

where $\mathbf{Z}_t$ includes the five variable in the fiscal VAR: the nominal rate of interest (the average cost of the Federal debt), output growth (the first difference of the log of real GDP), inflation (the first difference of the log of the price level), (the logs of) government receipts and government expenditure net of interest. $\mathbf{u}_t$ are reduced form innovations.

The specification of (7) allows us to address all points discussed in the previous section.

- The R&R shocks are directly included in the model as observable structural shocks. So impulse responses to them are obtained in a full-information framework whose underlying MA representation, when existent (in the case $\boldsymbol{\gamma}_i = 0$), is infinite and not truncated as in R&R.

- Second, in (7) macroeconomic variables are assumed to respond not to the distance of the debt-to-GDP ratio from a target level d^*. Such debt-feedback mirrors that estimated in Bohn (1998). As in Bohn we take 0.35, as the target value for d^*. As shown in Figure A1, this is also the average debt level in our sample.

- As we introduce the debt level into the VAR, we need to make it endogenous, otherwise impulse response functions would be computed assuming a constant debt ratio, thus ruling out the very reason why debt is included in the first place—namely to allow macro variables to respond to the effect of the fiscal shock on the level of the debt. The way to make the debt ratio endogenous is to add to the model the equation that describes how it evolves over time as a function of the path of all other variables, *i.e.* the government's intertemporal budget constraint. [6]

The introduction of the IGBC makes (7) non- linear, so constructing an MA representation of $\mathbf{Z}_t$ is no longer possible. However, the computation of impulse responses is still possible by going through the following steps:

[6] Note that the budget constraint is an identity: it does not add new parameters to be estimated, nor new shocks to be identified.

- generate a baseline simulation for all variables by solving (7) dynamically forward (this requires setting to zero all shocks for a number of periods equal to the horizon up to which impulse responses are needed),
- generate an alternative simulation for all variables by setting to one–just for the first period of the simulation–the structural shock of interest, and then solve dynamically forward the model up to the same horizon used in the baseline simulation,
- compute impulse responses to the structural shocks as the difference between the simulated values in the two steps above. (Note that these steps, if applied to a standard VAR, would produce standard impulse responses. In our case they produce impulse responses that allow for both the feedback from d_{t-i} to $\mathbf{Z}_t$ and for the endogeneity of d_t modelled via (6),
- compute confidence intervals via bootstrap methods.[7]

4.1 The empirical evidence

The tax multipliers obtained from the estimation of (7) are illustrated in Figures 3-4.

Figure 3 compares the effect on output of an e_t^{RR} tax shock equivalent to one per cent of U.S. GDP estimated using, alternatively, (2), model (7) without the IGBC ($\gamma_i = 0$), and model (7) with the IGBC. Estimating the effect of R&R tax shocks using the VAR rather than a single equation framework delivers a response of output that is much smaller than that reported by R&R and very similar to that delivered by traditional fiscal VAR and reported in Figure 1. The impact of a tax shock on output growth estimated in a VAR never exceeds one per cent. The VAR also highlights the instability of the effects of tax shocks between the periods preceding and following 1980: the impact of tax shocks in the first sub-sample is larger and significantly different from the impact in the second sub-sample, where it is not significantly different from zero.

The results in Figure 3 show that the differences between the impulse responses obtained from the estimation of a single equation and those obtained within a system

[7]Bootstrapping requires saving the residuals from the estimated VAR and then iterating the following steps: a) re-sample from the saved residuals and generate a set of observation for $\mathbf{Y}_t$ and d_t, b) estimate the VAR and identify strucutral shocks, c) compute impulse responses going thorough the steps described in the text, d) go back to step 1. By going thorugh 1,000 iterations we produce bootstrapped distributions for impulse responses and compute confidence intervals.

framework only appear after a few quarters, and not on impact. This is a clear symptom that the single-equation framework fails to capture some significant simultaneity.

Note that the model augmented with debt and the non-linear debt dynamics equation produces results which are very similar to those obtained by including the R&R shocks in a traditional fiscal VAR. Figure 3 confirms that when the R&R measure of tax shocks is considered within a multiple equation model, rather than in a single equation framework, the estimated multipliers are much smaller. However, while simultaneity is important, we find no major empirical difference between a non-linear model with an explicit debt dynamics equation and a linearized model where the effect of debt is captured by its components.

Interestingly, the impulse responses based on the linearized model and on the non-linear model with debt differ in the second subsample where the effect of an exogenous increase in taxes affects negatively and significantly output growth (with a peak effect of about 0.5 per cent), while the same effect is never significantly different from zero in the model without debt.

Figure 4 completes our evidence by reporting the results obtained when re-running the R&R regression augmented it with $\mathbf{Z}_{t-M+1}$

$$\Delta Y_t = a + \sum_{i=0}^{M} b_i e_{t-i}^{RR} + \mathbf{C}_i^{M+1}\mathbf{Z}_{t-M+1} + e_t \tag{8}$$

This is a robustness check R&R do not perform, since the one they report only uses information dated up to time M. Figure 4 reports the effect of tax shocks as computed originally by R&R alongwith those based on the augmented regression (8) over the full sample 1950:1-2006:2. The Figure shows that the truncation has an effect on the size of the multiplier after the 8th quarter. The multiplier estimated using the augmented equation gets very close [8] to the one delivered by the inclusion of the R&R shocks in a fiscal VAR. Interestingly, the R^2 increases from 0.09 in the original R&R specification to 0.17 in the augmented specification.

5 Conclusions

We have estimated the multiplier associated with the shifts in taxes constructed by R&R without imposing that tax shocks are uncorrelated with past macroeconomic

[8] The small remaining difference between the impulse responses can be rationalized on the ground that, following B&P (2002), we specify our fiscal VAR in the (log) levels of the macroeconomic variables.

outcomes. We find a much smaller multiplier: 1, rather than 3 at a three-year horizon. We also find that the multiplier changes significantly before and after 1980, when the impact of tax shocks becomes not significantly different from zero.

We have also estimated the multiplier keeping track of the effect of tax shocks on the level of the debt-GDP ratio. We have done this allowing for the non-linearity which arises from the government budget constraint. We find that, while in general not very important, this non-linearity makes a difference after 1980, when the response of fiscal variables to the level of debt becomes stronger.

The methodology we have developed to analyze the impact of tax shocks by keeping track of the non-linear budget constraint, could be used in other settings. For instance, the discussions on the importance of including capital as a slow-moving variable to capture the relation between productivity shocks and hours worked (see e.g. Christiano et al, 2005 and Chari et al. 2005) could benefit from an estimation technique that tracks the dynamics of the capital stock generated by the relevant shocks. The same applies to open economy models that study, for instance, the effects of a productivity shock on the current account and that typically omit a feedback from the stock of external debt to macroeconomic variables.

This approach could also be used in the analysis of the effects of tax shocks on debt sustainability, an issue which cannot be addressed in the context of a VAR that fails to keep track of the debt dynamics.

6 References

Bagliano, Fabio and C. Favero [1999]: "Information from financial markets and VAR measures of monetary policy", *European Economic Review*, 43, 825-837.

Blanchard, Olivier and R. Perotti [2002]: "An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output", *Quarterly Journal of Economics*

Bohn, Henning [1998]: "The Behaviour of U.S. public debt and deficits", *Quarterly Journal of Economics,* 113, 949-963.

Burnside, Craig, M. Eichenbaum and J.D.M. Fisher [2003]: "Fiscal Shocks and Their Consequences", NBER Working Paper No 9772

Cashell, Brian W. [2006]: "The Federal Government Debt: its size and economic significance" CRS Report for Congress.

Chari V.V., P.J. Kehoe and E.R. McGrattan [2005]: "A Critique of Structural VARs Using Business Cycle Theory", Federal Reserve Bank of Minneapolis Research Department Staff Report 364

Christiano, Lawrence J., M.Eichenbaum and R.Vigfusson [2005]: "Assessing Structural VARs", mimeo

Chung Hess and D.Leeper [2007] "What has financed government debt?" NBER WP 13425

Corsetti G., A. Meier, and G. Muller [2009] "Fiscal Stimulus with Spending Reversals", mimeo

Edelberg, Wendy, M. Eichenbaum and J. D.M. Fisher [1999]: "Understanding the Effects of a Shock to Government Purchases", *Review of Economics Dynamics*, pp.166-206, 41

Fatás, Antonio and I. Mihov [2001]: "The Effects of Fiscal Policy on Consumption and Employment: Theory and Evidence", mimeo, INSEAD

Hansen, L. P., and T. J. Sargent (1991): "Two Difficulties in Interpreting Vector Autoregressions," in Rational Expectations Econometrics, ed. by L. P. Hansen, and T. J. Sargent, pp. 77–119. Westview Press, Boulder, CO.

Leeper D., T. Walker and Sun Chun Yang [2008]: "Fiscal Foresight:analytics and Econometrics", mimeo

Lippi, M., and L. Reichlin (1994): "VAR Analysis, Nonfundamental Representations, Blaschke Matrices," Journal of Econometrics, 63(1), 307–325.

Mertens, Karl and Morten O. Ravn [2008]: "The Aggregate Effects of Anticipated and Unanticipated U.S. Tax Policy Shocks: Theory and Empirical Evidence", mimeo.

Mountford, Andrew and H. Uhlig [2002]: "What Are the Effects of Fiscal Policy Shocks?" CEPR Discussion Paper 3338

Perotti, Roberto [2008]: "In Search of the Transmission Mechanism of Fiscal policy"; *NBER Macroeconomic Annual*, forthcoming

Romer, Christina and David H. Romer [forthcoming]: The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks", *American Economic Review.*

Rudebusch, Glenn D, 1998. "Do Measures of Monetary Policy in a VAR Make Sense?," International Economic Review, Department of Economics, University of Pennsylvania and Osaka University Institute of Social and Economic Research Association, vol. 39(4), pages 907-31, November.

Sims C.(1996) "Comment on Glenn Rudebusch's "Do Measures of Monetary Policy in a VAR Make Sense?" available from http://www.princeton.edu/~sims/

7 Data Appendix

y_t is (the log of) real GDP per capita, Δp_t is the log difference of the GDP deflator. Data for the stock of U.S. public debt and for population are from the FRED database (available on the Federal Reserve of St.Louis website,also downloaded on December 7th 2006). Our measure for g_t is (the log of) real per capita primary government expenditure: nominal expenditure is obtained subtracting from total Federal Government Current Expenditure (line 39, NIPA Table 3.2) net interest payments at annual rates (obtained as the difference between line 28 and line 13 on the same table). Real per capita expenditure is then obtained by dividing the nominal variable by population times the GDP chain deflator. Our measure for t_t is (the log of) real per capita government receipts at annual rates (the nominal variable is reported on line 36 of the same NIPA Table).

The R&R tax shocks start in 1947, while our data only start in 1950:1 because data for total governemnt spending are available on a consistent basis only from 1950:1. We thus exclude the exogenous shocks that occurred between January 1947 and December 1949.

Our approach requires that the debt-dynamics equation in (7) tracks the path of d_t accurately: we thus need to define the variables in this equation with some care. The source for the different components of the budget deficit and for all macroeconomic variables are the NIPA accounts (available on the Bureau of Economic Analysis website, downloaded on December 7th 2006). The average cost servicing the debt, i_t, is obtained by dividing net interest payments by the federal government debt held by the public (FYGFDPUN in the Fred database) at time $t-1$. The federal government debt held by the public is smaller than the gross federal debt, which is the broadest definition of the U.S. public debt. However, not all gross debt represents past borrowing in the credit markets since a portion of the gross federal debt is held by trust funds–primarily the Social Security Trust Fund, but also other funds: the Trust Fund for Unemployment Insurance, the Highway Trust Fund, the pension fund of federal employees, etc.. The assets held by these funds consist of non-marketable debt.[9] We thus exclude it from our definition of federal public debt. We are unable to build the debt series back to 1947:1, the start of the Romer and Romer sample, because, as mentioned above, data for total governemnt spending, needed to buld the debt series, are available on a consistent basis only from 1950:1

[9] Cashell (2006) notes that "this debt exists only as a book-keeping entry, and does not reflect past borrowing in credit markets."

Figure A-1 reports, starting in 1970:1 (the first quarter for which the debt data are available in FRED), this measure of the debt held by the public as a fraction of GDP (this is the dotted line). We have checked the accuracy of the debt dynamics equation in (7) simulating it forward from 1970:1 (this is the continuous line in Figure A-1). The simulated series is virtually super-imposed to the actual one: the small differences are due to approximation errors in computing inflation and growth rates as logarithmic differences, and to the fact that the simulated series are obtained by using seasonally adjusted measures of expenditures and revenues. Based on this evidence we have used the debt dynamics equation to extend d_t back to 1950:1.

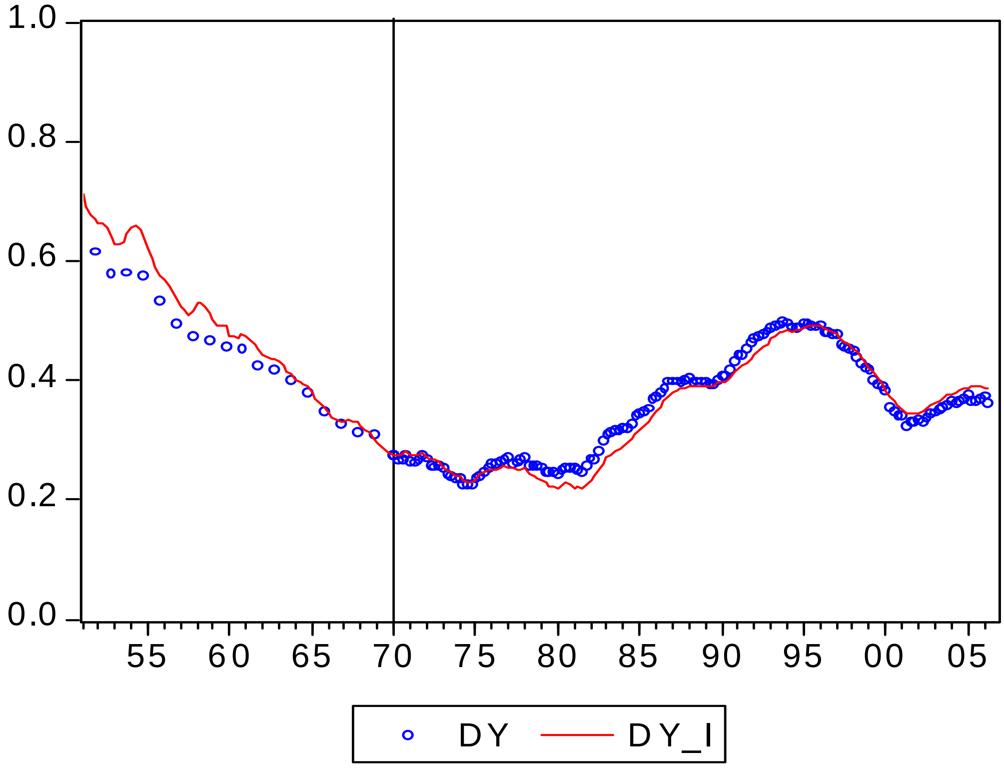

Figure A1: Actual (DY) and simulated (DY_I) (dynamically backward and forward starting in 1970:1) debt-GDP ratio. Actual data are observed at quarterly frequency from 1970 onwards and at annual frequency from 1970 backward. The simulated data are constructed using the government intertemporal budget constraint (2) with observed data and initial conditions given by the debt-to-GDP ratio in 1970:1.

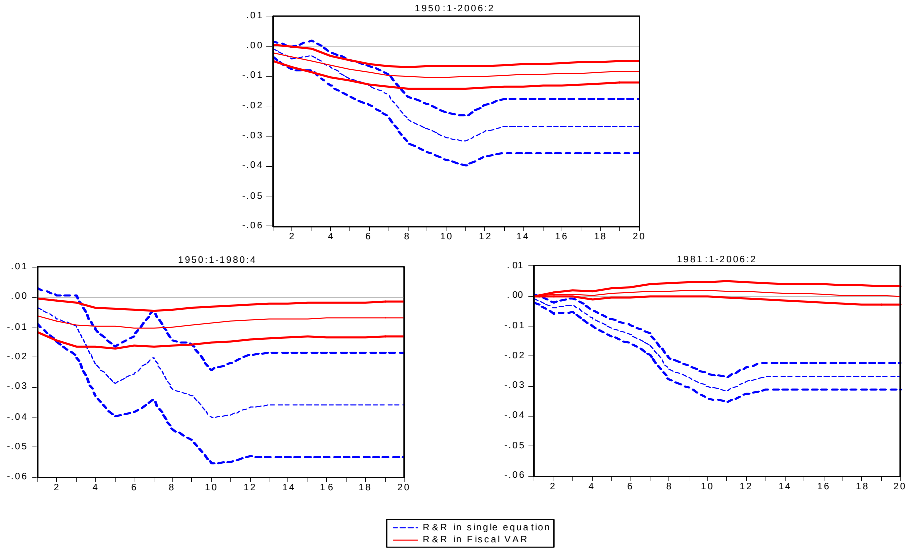

Figure 1: Different estimates of structural tax shocks in the narrative and the SVAR approaches

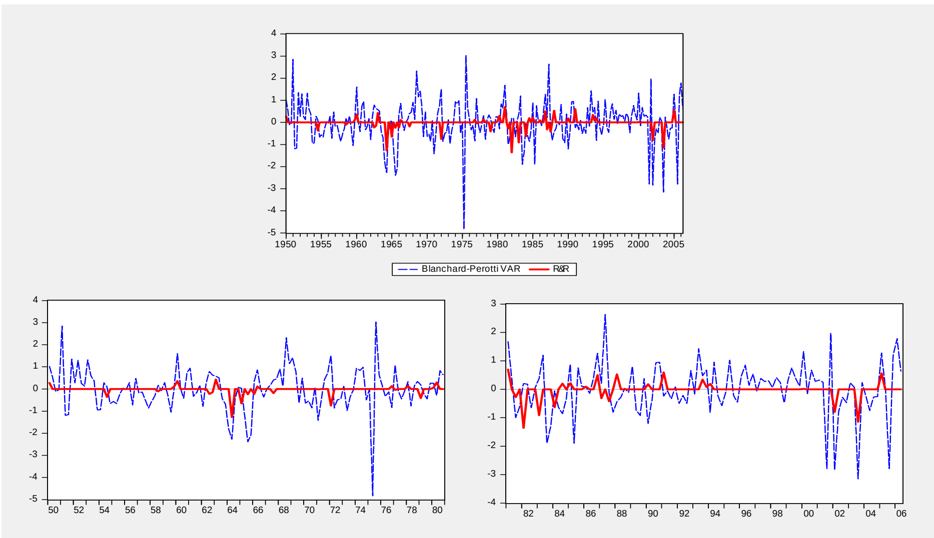

Figure 2: Different estimates of structural tax shocks in the narrative and the VAR approaches

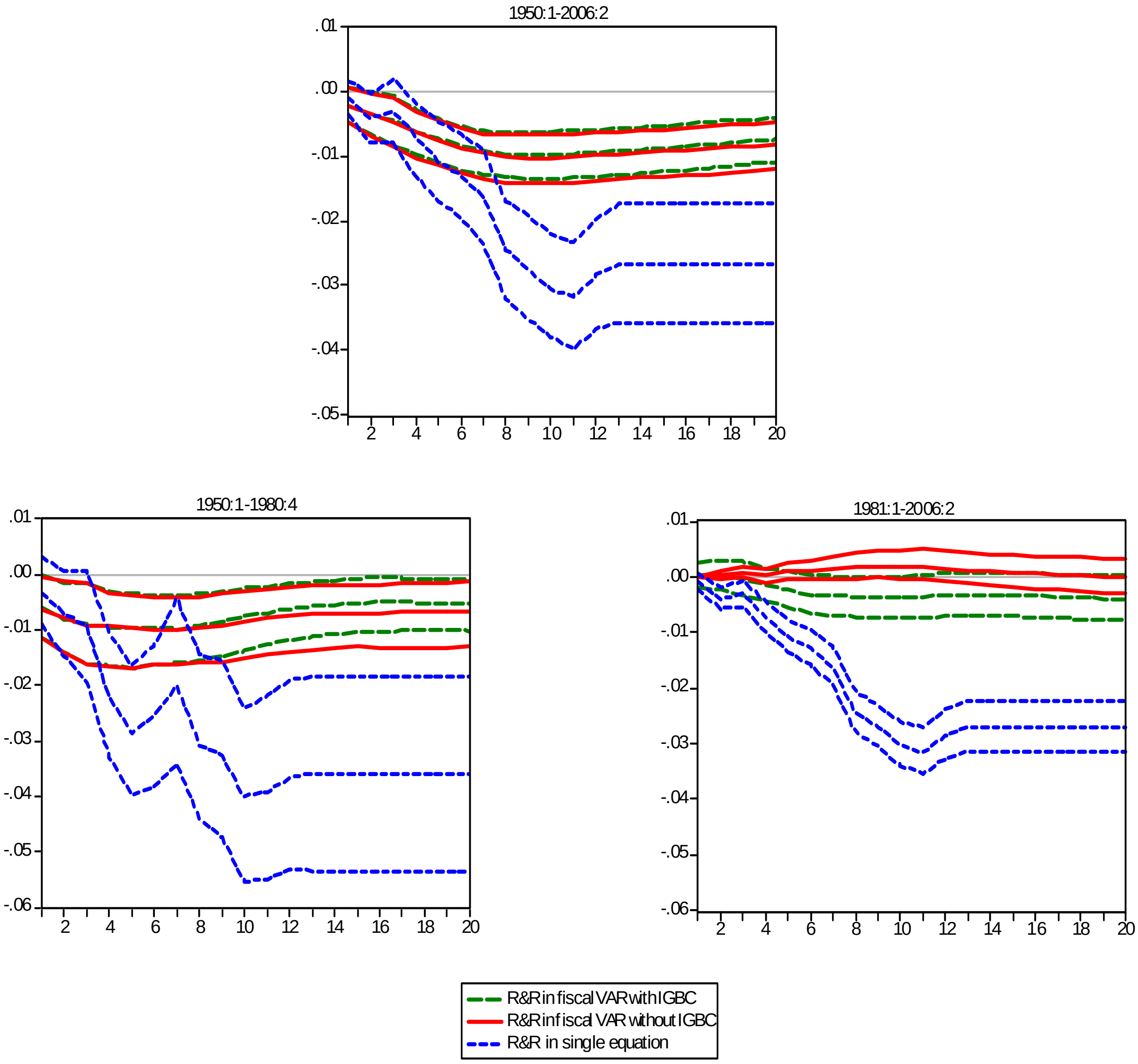

Figure 3: Estimated Impact of an Exogenous Tax Increase of 1% of GDP on GDP with the R&R framework and with the Fiscal VAR framework, with and without the IGBC

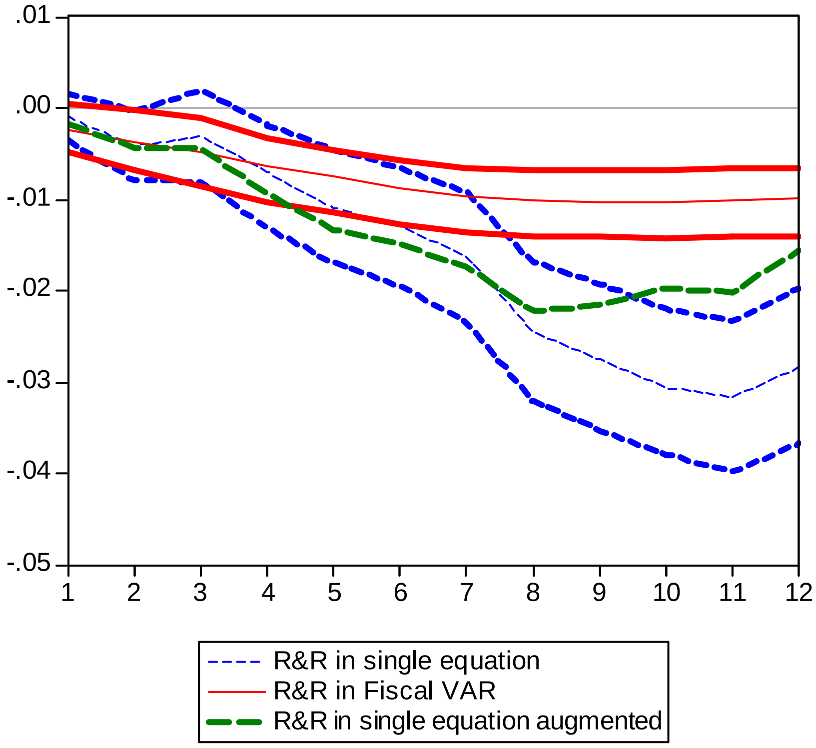

Figure 4: Different Shocks, Same Models, Same Impulse Responses