DISCUSSION PAPER SERIES

No. 7764

HIERARCHICAL TRADE

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INDUSTRIAL ORGANIZATION and INTERNATIONAL TRADE AND REGIONAL ECONOMICS

Centre for Economic Policy Research

www.cepr.org

Available online at: **www.cepr.org/pubs/dps/DP7764.asp**

ISSN 0265-8003

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Discussion Paper No. 7764
March 2010

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CEPR Discussion Paper No. 7764

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ABSTRACT

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We study the allocation of commodities through a two-stage hierarchy of competitive markets. Groups or countries trade at global prices while individuals within a group trade at local prices. We identify the free trade and the autarky equilibrium as polar cases. We show that no other two-stage market equilibria exist if the commodity space is two-dimensional. An example demonstrates that other, so-called intermediate equilibria exist for three dimensional commodity spaces. We give two existence proofs for intermediate equilibria in higher dimensions. Each proof provides an explicit construction of special classes of intermediate equilibria. Finally, we consider the consequences for welfare and trade flows when some countries control the agency that organizes trade at the global level.

JEL Classification: D43, D50 and F10
Keywords: control of trading platforms, hierarchical trade and intermediate equilibria

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Submitted 22 March 2010

We thank Clive Bell, Switgard Feuerstein, David Orden, Till Requate, seminar participants in Heidelberg and at Virginia Tech, and two referees for insightful comments. Hans Haller gratefully acknowledges the hospitality and support of ETH Zurich, Chair of Macroeconomics: Innovation and Policy (MIP).

1 Introduction

Motivation

Resource allocation mechanisms have often a hierarchical structure. The simplest case is a two-stage mechanism where goods are first allocated among groups of agents and then allocated among the individuals in each group. For instance, multi-member households may participate as entities in markets and decide on the distribution of the consumption bundles within the household afterwards. The most prominent example is provided by international trade where the groups are nations and the individuals are consumers and producers. Individuals in a nation face a domestic price system. At the same time, overall trade balances among nations are equalized at global price systems which may differ from the local prices, if significant barriers to international trade exist in the form of tariffs and quantitative restrictions that drive a wedge between local and global prices. Trade within and between trading blocks such as the European Union and NAFTA constitutes another example of a two-stage allocation scheme — with countries playing the role of individuals (agents). Finally, many governmental (political, public) decision processes follow a hierarchical pattern, distinguishing between "internal" and "external" decisions, between local (regional) and global (central) levels as well.

The aim of this paper is to study in a general equilibrium framework the simultaneous allocation of commodities through a two-stage hierarchy of competitive markets. Members of a group of people (a household, nation, etc.) trade within the group at an internal price system. However, internal markets need not be cleared and the group can act as a trading block vis-à-vis the rest of the world. The group's trade with the rest of the world occurs through an external trade agency at an external price system. Balancing of the group's external trade budget and market clearing across groups have to be achieved at the external price system. Internal and external prices can differ since we assume that goods arbitrage across groups is limited.

Our main assumption is motivated by a significant body of empirical literature which has shown that large differences in the prices of tradeable goods, such as cars, are a persistent phenomenon across countries and even within Europe [e.g. Verboven (1996)]. Lutz (2004) identifies arbitrage barriers in the car market as a main reason why prices differ across countries. Arbitrage barriers can arise from transaction costs and search costs.

Model and Results

We begin by devising a framework to study hierarchical market equilibria where groups trade at the global price system while individuals within a group trade at local price systems. We are interested in possible equilibrium price systems. First, we identify the free trade and the autarky equilibrium as polar cases. As a rule, free trade yields efficient allocations of commodities whereas autarky leads to inefficient allocations. Therefore, the question arises if there is the possibility of an **intermediate outcome** that Pareto dominates an autarky equilibrium allocation, but which itself is Pareto dominated by some other feasible allocation, though not necessarily by an equilibrium allocation.

In the main part of the paper, we aim for existence results for intermediate equilibria. A first finding indicates that equilibria that are intermediate to free trade and autarky generally do not exist, even under standard assumptions that guarantee competitive equilibria within each group of the economy. We show that essentially no other market equilibria than free trade and autarky exist if the commodity space is two-dimensional: The world is divided into a free trade zone and an autarky zone. An example demonstrates that intermediate equilibria exist for three-dimensional commodity spaces.

We go on to provide two existence proofs for intermediate equilibria in higher dimensions. They also show how one can construct special classes of intermediate equilibria. First, if the number of economic groups is at least three and smaller than the number of goods traded, an intermediate equilibrium can be constructed by dividing the economy into two distinct exchange economies. The existence proof can easily be extended to type economies. Second, if the dimension of the commodity space is at least three and the market demand of every group can be generated as the demand function of its representative consumer, intermediate equilibria can be shown to exist and can be constructed by restricting trade in a particular good within each group.

Finally, we outline plausible refinements of the notion of intermediate equilibrium that rationalize particular types of equilibria by specifying decision rules for the external agency. We outline several conceivable ways to formulate such rules. It turns out that non-trivial hierarchical equilibria continue to exist, but the indeterminacy is reduced. The refined equilibrium notion opens up a variety of interesting applications, relating the power to control the external agency at the global level to trade flows and welfare of countries.

Relation to the Literature

In this paper, we develop a hierarchical model of international trade. The trade literature [see Dixit and Norman 1980, or Bell 2006] assumes that all nations and their constituents trade in a single market place, though each nation may face a different price system due to tariffs or subsidies. We view trade within and trade among groups as activities that take place in two different market places. Consumers in a group trade at local prices whereas the group's trade with the rest of the world occurs in a global market place and at a global price system. Each individual faces a budget constraint when trading in the local market place. In addition, the group faces a budget constraint when trading in the global market place. We shall demonstrate that different market places for global and local trade and, consequently, double budget constraints play an important role for equilibrium existence, the occurrence of wedges between local and global prices and welfare considerations.

As an example consider the standard setting of conventional trade theory with a two-dimensional commodity space. Here we observe a stark contrast between single and double budget constraints. In his seminal paper, Yun (1995) obtains a continuous path from a distortionary equilibrium to the optimum. For two-stage market equilibria, we obtain a dichotomy, either autarky or free tree, and consequently the absence of a continuous path between the two.

The existence of dual market places and the occurrence of price distortions is reminiscent of several non-Walrasian market theories. We now discuss the parallels and differences between our model and strategic market games, fixed price equilibria and incomplete markets.

Strategic market games. Koutsougeras (2003) develops a strategic market game where a commodity can be traded in multiple "trading posts". He obtains instances of violation of "the law of one price", even though all traders can trade in all places. Attempts to take advantage of the apparent arbitrage opportunities are to no avail, because traders' actions impact on prices in such a way that the attempts make the opportunities disappear. Since the traders are aware of this effect, the price discrepancies are sustainable in equilibrium. In our model, consumers are price-takers and confined to their own market place, a very different non-Walrasian environment. The common feature is the existence of multiple market places and wedges between prices in different locations.

Fixed price equilibria. Drèze (1975) shows that under standard assumptions, a pure exchange economy with price rigidities has an equilibrium with uniform rationing of net trades. In particular, for every fixed strictly positive price vector, there exists an equilibrium with uniform rationing of net trades. Herings (1998) provides conditions so that for every fixed strictly positive price vector, there exists a continuum of equilibria with quantity rationing. Hence there exists multiplicity of equilibria with respect to equilibrium prices and multiplicity given equilibrium prices. This indeterminacy prevails even if all consumers face the same price system and the same rationing scheme. Citanna et al. (2001) extend the analysis to economies with production and find that a continuum of supply constrained equilibria can exist at Walrasian prices. In contrast, multiplicity of equilibria in our model is driven by price wedges between local markets on the one hand and local markets and the global market on the other hand.

Incomplete markets. It is well known from the study of allocation under uncertainty and a comparison of complete versus incomplete markets that the difference between single and multiple budget constraints is a significant one. Like Debreu's (1970) result in the case of complete markets, Geanakoplos and Polemarchakis (1986) show generic existence and finiteness of competitive equilibria for finite pure exchange economies with finitely many states of nature and real numéraire assets. In stark contrast, Geanakoplos and Mas-Colell (1989) find that with finitely many states of nature and financial assets, there tends to exist a continuum of equilibria exhibiting real indeterminacy. Two equilibria may only differ in the "price levels" in various states, yet yield different allocations. In our model, the analogue of the latter cannot happen: If all local equilibrium price systems are collinear, then the equilibrium allocation also arises as a free trade equilibrium allocation. As a rule, there are only finitely many such allocations — and they are Pareto optimal. In the incomplete market context, the transferability of wealth across states proves crucial. In our framework, markets are complete from an individual's perspective in that the individual can trade all commodities (assets) and is not concerned about wealth transfers across markets. Markets are also complete, yet distorted vis-à-vis the global market, in inter-group trade.

Following the foregoing agenda, the paper is organized as follows. In the next section, we relate the paper to the literature. In sections 2 and 3, we introduce the formal framework. In particular, we define two-stage market equilibria. Free trade and autarky are identified as the polar cases. In section 4 we show that no intermediate equilibrium exists if the dimension of the commodity space is two. In section 5, we analyze an example with three consumers and three commodities with an explicitly calculated two-stage market equilibrium, which is intermediate in the sense that the equilibrium allocation Pareto dominates the initial endowment allocation (autarky in this case). In section 6, we establish existence of intermediate equilibria by using two different methods of constructing intermediate equilibria. We also discuss the kinds of indeterminacy of intermediate equilibria. In section 7, we outline plausible refinements of the notion of intermediate equilibrium that rationalize particular types of equilibria by specifying decision rules for the external agency. In section 8, we discuss the relationship between trade in single and in dual market places. Section 9 concludes.

2 Two-Stage Market Allocations

We consider a model of a finite pure exchange economy where commodities, consumer characteristics and allocations are standard. The distinguishing feature of the model is the allocation mechanism, a two-stage hierarchy of markets.

2.1 Commodities, Consumers, and Allocations

There exists a finite number $\ell \geq 1$ of commodities. Thus the commodity space is $\mathbb{R}^\ell$. Each commodity is a private good.

There is a finite population of **consumers** or **individuals**, represented by a set I. A generic consumer is denoted by i or j. Each consumer $i \in I$ has consumption set $X_i = \mathbb{R}^\ell_+$. The **endowment** of i is a commodity bundle $w_i \in \mathbb{R}^\ell_{++}$. For a given price system $p \in \mathbb{R}^\ell$, $B_i(p) = \{x_i \in X_i | px_i \leq pw_i\}$ denotes i's budget set. Individual i has continuous, convex and monotonic **preferences** on X_i represented by a utility function $U_i : X_i \longrightarrow \mathbb{R}$.

An **allocation** of commodities assumes the form $\mathbf{x} = (x_i)_{i \in I}$ and belongs to the allocation space $\mathcal{X} \equiv \prod_{j \in I} X_j$. In $\mathbf{x} \in \mathcal{X}$, the consumption bundle $x_i \in X_i$ is assigned to individual $i \in I$.

2.2 Groups and Two-Stage Markets

The population I is partitioned into **groups** or **nations**, i.e., there exists a partition P of I into non-empty subsets. P has generic elements h and consists of H groups frequently labelled $h = 1, \ldots, H$. For each group $h \in P$, set $\mathcal{X}_h = \prod_{i \in h} X_i$, the consumption set for group h. $\mathcal{X}_h$ has generic elements $\mathbf{x_h} = (x_i)_{i \in h}$. If $\mathbf{x} \in \mathcal{X}$ is a commodity allocation, then consumption for group h is $\mathbf{x_h} = (x_i)_{i \in h}$, the restriction of $\mathbf{x} = (x_i)_{i \in I}$ to h. Thus group h attains the group consumption $\mathbf{x_h} \in \mathcal{X}_h$. We set $w_h \equiv \sum_{i \in h} w_i$, the social endowment of group h.

Next we define two-stage market equilibria. The definition is based on the given partition P of the consumer population I into groups. For $i \in I$, $P(i)$ denotes the group to which individual i belongs.

Def.: A **two-stage market equilibrium** is a tuple $(p; (q_h)_{h \in P}; \mathbf{x})$ such that

(1) $p \in \mathbb{R}^\ell$ is an external (global, world) price system;

(2) $q_h \in \mathbb{R}^\ell$ is an internal (local, domestic) price system for each $h \in P$;

(3) $\mathbf{x}$ is an allocation of commodities to consumers;

(4) $x_i \in \arg\max\{U_i(y_i) \,|\, y_i \in B_i(q_h)\}$
for each individual $i \in I$ and household (group, country) $h = P(i)$;

(5) $p \cdot (\sum_{i \in h} x_i - \sum_{i \in h} w_i) \leq 0$ for each $h \in P$;

(6) $\sum_{i \in I} x_i = \sum_{i \in I} w_i$.

2.3 Discussion of the Equilibrium Concept

The central idea is that individuals can only trade freely within their group (household, country) h, taking the internal price system q_h as given. This condition is formalized as (4). Under our assumptions on preferences, individual budget constraints are binding and Walras' Law holds group by group:

(7) $q_h z_h = 0$

where $z_h \equiv \sum_{i \in h} x_i - \sum_{i \in h} w_i$ is the group's aggregate excess demand. The fact that individuals can only trade within their respective group does not necessarily mean that the group's internal market has to be cleared. Rather the group h as a trading block can have a non-zero net trade z_h with the rest of the world. In external trade, the group takes the external price system p as given and is subject to an external budget constraint. This condition is reflected in (5). Finally, (6) is the formal expression of the global market clearing condition. Conditions (5) and (6) imply for each group h balancing of its external trade account:

(8) $pz_h = 0.$

Note that the equilibrium allocation of a group has to satisfy *two budget constraints:* the budget constraint with respect to the local prices that enter the individual budgets, giving rise to (7), and a second budget constraint with respect to global prices, giving rise to (8). This reflects our view of trade in two different market places.[1]

Our general assumptions guarantee the existence of Walrasian equilibria for an economy. Assuming strictly positive endowments and monotonic, convex, and continuous preferences for each individual consumer suffices. Ensuring equilibrium existence facilitates the discussion of efficiency of two-stage market equilibria. Moreover, the assumptions on preferences imply that all individuals and groups exhaust their budgets and hence the budget balancing conditions (7) and (8) hold.

[1]In a longer version of this paper, two extensions are introduced. We allow for trade deficits, ($pz_h > 0$), or a trade surpluses ($pz_h < 0$). We also allow that government tariff revenues are distributed it in a lump-sum fashion among the constituents of the country. More precisely, the government net tariff revenue is channeled to consumers as lump-sum transfers or will be raised by lump-sum taxes in the case of net subsidies. This requires to modify the individual budget constraints accordingly and thus $q_h z_h \neq 0$. Details are available upon request.

3 Free Trade and Autarky

To begin with, we can state the following proposition for non-trivial P, i.e., $1 < |P| < |I|$:

Proposition 1 *For generic consumer characteristics, there are at least two two-stage market equilibria.*

Proof:
For a proof, we construct a "free trade equilibrium" which is Pareto optimal and an "autarky equilibrium" which is not Pareto optimal.

Free Trade Equilibrium: Under our standard assumptions, there exists a Walrasian equilibrium $(\hat{p}; \hat{\mathbf{x}})$ for the entire economy. Set $\hat{q}_h = \hat{p}$ for $h \in P$, and $\hat{\mathbf{x}} = (\hat{x}_i)_{i \in I}$. Then $(\hat{p}; (\hat{q}_h)_{h \in P}; \hat{\mathbf{x}})$ is a two-stage market equilibrium and $\hat{\mathbf{x}}$ is a Pareto-optimal allocation corresponding to a "free trade equilibrium".

Autarky Equilibrium: Also under standard assumptions, there exists a Walrasian equilibrium $(q_h^*; x_h^*)$ for the sub-economy formed by the members of any group $h \in P$. Now fix a family $(q_h^*; x_h^*), h \in P$, of such "local" equilibria, choose an arbitrary $p^* \in \mathbb{R}^\ell$ and set $\mathbf{x}^* = (x_i^*)_{i \in I}$. Then $(p^*; (q_h^*)_{h \in P}; \mathbf{x}^*)$ constitutes a two-stage market equilibrium. It is an "autarky equilibrium" where each group has zero external trade since $\mathbf{x}_\mathbf{h}^* = w_h$. As a rule, the internal equilibrium price systems q_h^*, $h \in P$, are not collinear and the equilibrium allocation x^* is not Pareto-optimal.

The existence of these two particular equilibria establishes the claim of the proposition. (q.e.d.)

4 The Two-Dimensional Case

After having established two distinguished types of equilibria — a free trade equilibrium that is Pareto-optimal and an autarky equilibrium — an obvious question is whether there is room for intermediate degrees of equilibrium inefficiencies and how their properties differ from trade with price distortions in a single market place. As will become clear, the existence and nature of intermediate equilibria depends crucially on the dimension of the commodity space. We first examine exchange economies with two goods that are reminiscent of most of the classical international trade models.

Proposition 2 *Suppose $\ell = 2$. Then, at a two-stage market equilibrium with $p \gg 0$, the world is divided into an autarkic trade zone and a free trade zone. One of the zones may be empty.*

Proof:
Because of our assumptions on preferences, a two-stage market equilibrium requires

$$q_h x_i = q_h w_i;$$
$$p \sum_{i \in h} x_i = p \sum_{i \in h} w_i;$$
$$\sum_{i \in I} x_i = \sum_{i \in I} w_i.$$

Hence, we also have

$$q_h \sum_{i \in h} (x_i - w_i) = p \sum_{i \in h} (x_i - w_i) = 0.$$

If $\sum_{i \in h} x_i \neq \sum_{i \in h} w_i$, then both q_h and p are orthogonal to $\sum_{i \in h}(x_i - w_i) \neq 0$; hence they are collinear. But since $p \gg 0$ and $q_h > 0$, this implies that the two price systems are identical up to normalization. Therefore with prices restricted to the unit simplex, $\sum_{i \in h} x_i = \sum_{i \in h} w_i$ (autarky) or $q_h = p$ (free trade) prevails for each country h. (q.e.d.)

Thus, with a two-dimensional commodity space, no intermediate equilibria exist, except possibly equilibria that divide the world into an autarkic and a free trade zone. The basic geometric intuition is as follows: A group's aggregate consumption has to lie on two budget lines through the group's social endowment bundle, the line given by local prices and the line given by global prices. This can only happen if the two lines coincide — that is, the two price systems are collinear — or if the group's aggregate consumption is located at the intersection of the two lines — which means it is the group's social endowment bundle. This observation also suggests that non-trivial intermediate equilibria might exist with at least three commodities, since then the intersection of two budget hyper-planes through the same endowment bundle has at least dimension one. This will be taken up in the following section.

The above conclusion has been reached independently by Bell (2006, p. 50f) who argues in terms of the government budget: "In the two-good case considered here, which wholly conforms to the textbook case, imposing a tariff while denying the government the use of lump-sum transfers [to achieve zero net revenue] can only be done if there are no transactions to tax." This is in sharp contrast to classical international trade theory, where the existence of competitive equilibria is guaranteed under general conditions [see e.g. Dixit and Norman (1980)] even when significant, but not totally prohibitive, barriers to international trade exist in the form of tariffs and non-tariff restrictions.

The contrast between single and double budget constraints is further accentuated by the existence and non-existence, respectively, of a continuous path from autarky to free trade. Ideally, one would like to trace a continuous path from an inefficient, distorted equilibrium (e.g. autarky) to the optimal free trade equilibrium, perhaps even continuously improving welfare along the path. With single budget constraints, the important work of Yun (1995) shows that, indeed, a unique path can be constructed, beginning at a distortion equilibrium, going through proportional changes of price distortions, and ending at the targeted optimum. With double budget constraints, the non-existence of intermediate equilibria rules out any path between the two polar outcomes.

5 An Example

Before we establish the existence and nature of intermediate equilibria for three- or higher-dimensional commodity spaces, we illustrate the concepts with an example. Suppose $\ell = 3$, $|I| = H = 3$. Thus, each group contains exactly one individual. For every individual i the utility function is given by

$$(10) \qquad U_i = U_h = \frac{1}{3}\ln(x_h^1) + \frac{1}{3}\ln(x_h^2) + \frac{1}{3}\ln(x_h^3), \quad i = h = 1, 2, 3$$

x_h^k denotes the consumption of the k-th good by individual h. The endowments are given by

$$\begin{aligned} w_1 &= (1, 0, 0); \\ w_2 &= (0, 1, 0); \\ w_3 &= (0, 0, 1). \end{aligned}$$

The autarky solution leaves every individual with his endowments. Due to symmetry, it is obvious that free trade is characterized by $p^1 = p^2 = p^3 = 1$ and

$$\begin{aligned} x_1 &= (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}), \\ x_2 &= (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}), \\ x_3 &= (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}). \end{aligned}$$

To calculate an intermediate equilibrium, we normalize prices so that $p^1 = 1, q_1^1 = 1, q_2^2 = 1, q_3^3 = 1$. q_h^k denotes the local price for good k in group h. The corresponding excess demand vector amounts to

$$\begin{aligned} z_1 &= (-\frac{2}{3}, \frac{1}{3q_1^2}, \frac{1}{3q_1^3}); \\ z_2 &= (\frac{1}{3q_2^1}, -\frac{2}{3}, \frac{1}{3q_2^3}); \\ z_3 &= (\frac{1}{3q_3^1}, \frac{1}{3q_3^2}, -\frac{2}{3}). \end{aligned}$$

Market clearing and the global budget constraint yield:

$$q_2^1 + q_3^1 = 2q_2^1 q_3^1 \tag{11}$$

$$q_1^2 + q_3^2 = 2q_1^2 q_3^2 \tag{12}$$

$$q_1^3 + q_2^3 = 2q_1^3 q_2^3 \tag{13}$$

$$p_2 q_1^3 + p_3 q_1^2 = 2q_1^2 q_1^3 \tag{14}$$

$$q_2^3 + p_3 q_2^1 = 2p_2 q_2^3 q_2^1 \tag{15}$$

$$q_3^2 + p_2 q_3^1 = 2p_3 q_3^1 q_3^2 \tag{16}$$

Let us choose $q_1^2 = 2, q_1^3 = 3$. Solving the system of equations immediately yields $q_3^2 = \frac{2}{3}, q_2^3 = \frac{3}{5}$ and the remaining equations as:

$$q_2^1 + q_3^1 = 2q_2^1 q_3^1 \tag{17}$$

$$3p_2 + 2p_3 = 12 \tag{18}$$

$$3 + 5p_3 q_2^1 = 6p_2 q_2^1 \tag{19}$$

$$2 + 3p_2 q_3^1 = 4p_3 q_3^1 \tag{20}$$

$p_3 = \frac{5}{2}, p_2 = \frac{7}{3}, q_3^1 = \frac{2}{3}, q_2^1 = 2$ is a solution of this reduced system. The resulting intermediate equilibrium allocation is:

$$x_1 = (\frac{1}{3}, \frac{1}{6}, \frac{1}{9}) \tag{21}$$

$$x_2 = (\frac{1}{6}, \frac{1}{3}, \frac{5}{9}) \tag{22}$$

$$x_3 = (\frac{1}{2}, \frac{1}{2}, \frac{1}{3}) \tag{23}$$

This allocation differs from autarky and free trade. Thus, the allocation together with the global and local prices constitute an intermediate equilibrium. Note that the third individual is better off in the intermediate equilibrium than under free trade. However, the allocation in the intermediate equilibrium is not Pareto-efficient. Individuals would like to trade again in one market place, starting from the allocations they received in the intermediate equilibrium.

The example can also be used to demonstrate the differences and similarities between traditional trade theory in single market places and the perspective of hierarchical trade in local and global market places. Let us denote the price distortion of the local price for commodity k in group h by τ_h^k:

$$\tau_h^k = \frac{q_h^k}{p^k} - 1$$

τ_h^k can be positive or negative. In traditional trade theory, the price distortion is often related to the existence of tariffs, subsidies, quantitative restrictions, transportation costs, search costs, imperfect competition or non-convertibility of currencies. Let us denote the distortion vector of country h by $\tau_h = (\tau_h^1, \tau_h^2, \tau_h^3)$. $\tau^k = (\tau_1^k, \tau_2^k, \tau_3^k)$ is the distortion vector of good k. In our example, the commodity distortion vectors amount to:

$$\tau^1 = (0, 1, -\frac{1}{3}) \tag{24}$$

$$\tau^2 = (-\frac{1}{7}, -\frac{4}{7}, -\frac{5}{7}) \tag{25}$$

$$\tau^3 = (\frac{1}{5}, -\frac{19}{25}, -\frac{3}{5}) \tag{26}$$

The corresponding country distortion vectors amount to:

$$\tau_1 = (0, -\frac{1}{7}, \frac{1}{5}) \tag{27}$$

$$\tau_2 = (1, -\frac{4}{7}, -\frac{19}{25}) \tag{28}$$

$$\tau_3 = (-\frac{1}{3}, -\frac{5}{7}, -\frac{3}{5}) \tag{29}$$

Obviously, the intermediate equilibrium also defines an equilibrium in which nations trade in one market place, while domestic consumers face the domestic prices $q_h^k = p^k(1 + \tau_h^k)$.

We assume that the distortions represent tariffs and subsidies and the government net tariff revenue will be channeled to consumers as lump-sum transfers or will be raised by lump-sum taxes in the case of net subsidies. We claim that the intermediate equilibrium has a companion single market equilibrium with the same distortions τ_h^k and the same allocations. This follows from the following observations. At distortions τ_h^k and global prices p^k of the intermediate equilibrium domestic consumers choose an excess demand equal to that in the intermediate equilibrium if lump-sum transfers were zero. And thus markets would clear. But since $\sum_{k=1}^{3} p^k\,(x_h^k - w_h^k) = 0$ and $\sum_{k=1}^{3} q_h^k\,(x_h^k - w_h^k) = 0$ for all countries, in the intermediate equilibrium we have

$$\sum_{k=1}^{3} \tau_h^k\, p^k\,(x_h^k - w_h^k) = 0.$$

Therefore, at equilibrium distortions and global prices of the intermediate equilibrium, the government budget is balanced in the single market equilibrium and thus lump-sum transfers are indeed zero which validates our assertion. Obviously, the companion single-market equilibrium can be derived directly by fixing the price distortions and solving for the equilibrium prices and allocations.

Intermediate equilibria require a budget constraint at global prices that is absent from trade in one market place. While an intermediate equilibrium has an associated single-market place equilibrium, the situation changes and becomes more complex if we start with existing wedges between local and global prices. In section 8 we explore the circumstances under which single-market equilibria have corresponding two-stage market equilibria. Moreover, in our working paper Gersbach and Haller (2007) we discuss when given trade frictions in a subset of countries create endogenously price distortions in other countries in two-stage market equilibria. This property is absent in single-market equilibria.

6 Existence and Indeterminacy of Intermediate Equilibria

In this section we establish existence of intermediate equilibrium allocations for three- or higher dimensional commodity spaces. The propositions also demonstrate how one can construct special classes of intermediate equilibria. At the end of this section, we illustrate what types of indeterminacy of intermediate equilibria can arise.

6.1 Existence

In order to avoid pathological cases we assume throughout this subsection that autarky and free trade differ and that under free trade, each group has a non-zero net trade and each individual attains a strictly positive consumption bundle.

Proposition 3 *Suppose $\ell \geq 4$ and $2 < H < \ell$. Then an intermediate two-stage equilibrium exists for generic consumer characteristics.*

Proof:
Let $x_h(q_h) = \sum_{i \in h} x_i(q_h)$ for $h \in P$, $q_h \in \mathbb{R}^\ell_+$ denote the aggregate demand

vector of group h at prices q_h. Finally, denote the excess demand of group h at prices q_h by $z_h(q_h) = x_h(q_h) - w_h$. Let us divide the set of groups P into two non-empty subsets P^1 and P^2 ($P = P^1 \cup P^2$). Consider two pure exchange economies E^1 and E^2. In E^1 (E^2) consumers belong to groups in P^1 (P^2), respectively. Take two corresponding equilibria with price vectors $\mathfrak{p}^1$ and $\mathfrak{p}^2$ and allocations denoted by $\mathbf{x_{P^1}}$ and $\mathbf{x_{P^2}}$. Generically, $\mathfrak{p}^1$ or $\mathfrak{p}^2$ differs from both autarky and full trade prices. Consider for each group h the orthogonal complement of the equilibrium excess demand vector,

$$c_h^{\perp} = \{y_h \in \mathbb{R}^{\ell} | y_h \cdot z_h(\mathfrak{p}^1) = 0 \text{ if } h \in P^1, y_h \cdot z_h(\mathfrak{p}^2) = 0 \text{ if } h \in P^2, \text{ resp.}\big\}.$$

In general, $c_h^{\perp}$ has dimension $\ell - 1$ or dimension ℓ. Consider the intersection $\bigcap_h c_h^{\perp}$ taken over all groups h in P. This intersection has at least dimension 1 since we have at most $\ell - 1$ orthogonal complements (as $H < \ell$) and one intersection operation reduces the dimension at most by one. Now take any vector $p \neq 0$ in $\bigcap_h c_h^{\perp}$. We claim that $(p; (q_h = \mathfrak{p}^1)_{h \in P^1}, (q_h = \mathfrak{p}^2)_{h \in P^2}; \mathbf{x_{P^1}}, \mathbf{x_{P^2}})$ is a two-stage market equilibrium. $\mathfrak{p}^1$ and $\mathfrak{p}^2$ generate local price systems for the corresponding groups. Moreover, $(\mathbf{x_{P^1}}, \mathbf{x_{P^2}})$ is an allocation of commodities. By construction, we have $p \cdot \left(\sum_{i \in h} x_i - \sum_{i \in h} w_i\right) = 0$. Since market clearing prevails for both sub-economies P^1 and P^2, overall market clearing follows. (q.e.d.)

Proposition 3 can easily be extended to type economies. A type economy in our context is defined as follows. Two groups are of the same type if their excess demand function is identical. The most natural case occurs when both groups contain the same number of individuals and each individual in one group has an identical counterpart with respect to endowments and preferences in the other group. We obtain

Proposition 4 *Suppose $\ell \geq 3$, and M types of groups with $2 \leq M < \ell$. Then, generically, an intermediate two-stage equilibrium exists.*

Proof:
We can apply the same construction as in Proposition 3 with one additional consideration. When dividing the economy into two non-empty subsets, all groups of the same type have to be put into one exchange economy. Then, we have at most $\ell - 1$ different orthogonal complements of the equilibrium consumption vector, since each group of the same type has the same equilibrium vector. Again, by considering the intersection of all complements, we can find the global prices. (q.e.d)

Given that one may distinguish between hundreds if not thousands of commodities, the conditions $H < \ell$ and $M < \ell$ are quite plausible. Still, it is not always necessary to impose $H < \ell$ or $M < \ell$ in order to establish generic existence of intermediate equilibria:

Proposition 5 *Suppose $\ell > 2$ and that all groups are singletons. Suppose further that for each $i \in I$, interior consumption bundles are preferred to boundary ones, the utility function is concave and differentiable in the interior of X_i, and w_i belongs to the interior of X_i. Then, generically, intermediate equilibria exist.*

Proof: Here we identify individual i with group $\{i\}$ and, accordingly, label both individuals and groups by $h = 1, \ldots, H$.

Consider the following exchange economy, denoted by E. Individual h is allowed to trade except in one arbitrarily chosen commodity k_h. Consider a corresponding equilibrium of E, denoted $(p, \mathbf{x}^*)$. For each individual h, let

$$q_h = \operatorname{grad} U_h(\mathbf{x_h^*}).$$

Then, q_h is a supporting price system for group (individual) h at $\mathbf{x_h^*} \gg 0$. We claim that $(p; (q_h)_{h \in P}; \mathbf{x}^*)$ is an intermediate two-stage equilibrium. We first observe that p and q_h typically differ. Since generically, each group has a non-zero net trade under free trade, we can choose the commodity in which group h is not allowed to trade in E, so that the excess demand $\mathbf{x}_\mathbf{h}^* - \omega_h$ differs from that under free trade. Hence, the allocation under free trade is different from $\mathbf{x}^*$. Clearly, $\mathbf{x}^*$ is a feasible allocation of commodities. Since $\ell > 2$ and every group is only restricted in trading of one commodity, it follows that, as a rule, groups have non-zero net trades with the rest of the world in the exchange economy E. Thus the allocation $\mathbf{x}^*$ also differs from autarky. The incorporation of the non-tradeable commodity into the budget constraint does not matter in E and, therefore, we have $p \cdot z_h^* = 0$ for the group's excess demand $z_h^* = \sum_{i \in h}(x_i^* - \omega_i)$.

In the next step we show that $q_h z_h^* = 0$. Suppose that group h is not allowed to trade in commodity $k_h \in \{1, \ldots, \ell\}$ in the exchange economy E. Because of $\mathbf{x}_\mathbf{h}^* \gg 0$ and the hypothesized properties of the utility functions, there exists a scalar $\lambda_h > 0$ such that

$$\frac{\partial U_h(\mathbf{x}_\mathbf{h}^*)}{\partial x_h^k} = \lambda_h p^k \quad \text{for } k \neq k_h. \tag{30}$$

Equation (30) characterizes the first-order conditions for an interior competitive equilibrium in E. Furthermore,

$$\frac{\partial U_h(\mathbf{x}_\mathbf{h}^*)}{\partial x_h^k} = q_h^k \quad \text{for all k.} \tag{31}$$

Hence, we obtain:

$$q_h z_h^* = q_h^{k_h} z_h^{*k_h} + \lambda_h \sum_{k \neq k_h} p^k z_h^{*k} = 0.$$

The first term is zero because group h did not trade in commodity k_h. The second term is zero since this represents the budget constraint of group h in the exchange economy E. Finally, market clearing is guaranteed since all groups participate in the exchange economy E. Thus $(p, (q_h)_{h \in P}; \mathbf{x}^*)$ is an intermediate equilibrium, that is a two-stage market equilibrium with the desired properties: no group enjoys (quasi-)free trade or autarky. (q.e.d)

Notice that in the intermediate two-stage market equilibrium constructed in the proof, all commodities are tradeable, despite the fact that the construction is based on an artificial economy E where every group cannot trade a specific commodity k_h.

In traditional international trade theory, the central results such as the law of comparative advantage and Heckscher-Ohlin theorems are sensitive to dimensionality and survive only as correlations or in an average sense in higher dimensions. Existence of equilibria including distortions is, however, not sensitive to dimensionality [see e.g. Ethier (1984)]. For trade in two market places, dimensionality is decisive. We obtain non-trivial intermediate equilibria only for three- or higher dimensional commodity spaces. Since extensive empirical work suggests that low dimensionality may be inadequate (Leamer and Levinsohn 1995), we expect such intermediate equilibria to exist as a rule.

We have phrased the central existence theorem in terms of one-person groups. However, the existence result can be extended to groups containing an arbitrary number of consumers, as long as there exists a representative consumer for each group, that is, the aggregate demand function of each group is generated by the demand function of its representative consumer. Then we can apply the same arguments as above to establish the existence of intermediate equilibria for groups with an arbitrary number of individuals.

6.2 Indeterminacy

As is well known, a finite pure exchange economy can have multiple Walrasian equilibrium allocations. But as a rule, the economy is regular and has a finite number of equilibrium allocations. See for instance Propositions 17.D.5 and 17.D.2 in Mas-Colell, Whinston and Green (1995). In our model, the possibility of two kinds of indeterminacy of intermediate equilibria are given. First, there is the possibility of significant nominal or price indeterminacy. The world market price systems supporting a particular equilibrium allocation may span a multi-dimensional subspace of the commodity space, an indeterminacy that is not eliminated by price normalization. Second, there can be a continuum of equilibrium allocations and utilities. We briefly illustrate both kinds of indeterminacy.

6.2.1. Nominal indeterminacy. Suppose $2 < H < \ell - 1$ in Proposition 3. Then in the proof of Proposition 3, one can choose $p \neq 0$ in $\bigcap_h c_h^\perp$ and the latter has at least dimension 2.

6.2.2. Real indeterminacy. Rewrite equations (11) – (16) as

$$F(q_1^2, q_1^3; q_2^1, q_3^1, q_2^3, q_3^2, p_2, p_3) = 0$$

where $F : \mathbb{R}^8 \longrightarrow \mathbb{R}^6$. Then at the intermediate equilibrium values $(q_1^2, q_1^3; q_2^1, q_3^1, q_2^3, q_3^2, p_2, p_3) = (2, 3; 2, 2/3, 2/3, 3/5, 5/2, 7/3)$,

$$D_{(q_2^1, q_3^1, q_2^3, q_3^2, p_2, p_3)} F = \begin{pmatrix} -1/3 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & -5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 2 \\ -1 & 0 & -9 & 0 & -8/3 & 2 \\ 0 & -3/10 & 0 & -19/9 & 2/3 & -4/5 \end{pmatrix}.$$

Since this sub-Jacobian matrix has full rank and F is continuously differentiable, the implicit function theorem applies at $y_1^* = (q_1^2, q_1^3) = (2, 3)$

and $y_2^* = (q_2^1, q_3^1, q_2^3, q_3^2, p_2, p_3) = (2, 2/3, 2/3, 3/5, 5/2, 7/3)$: There exist open neighborhoods $\mathcal{U}_1$ of y_1^* and $\mathcal{U}_2$ of y_2^* and a continuously differentiable mapping $\varphi : \mathcal{U}_1 \longrightarrow \mathcal{U}_2$ such that for every $y_1 \in \mathcal{U}_1$, $y_2 = \varphi(y_1)$ is the unique element of $\mathcal{U}_2$ satisfying $F(y_1; y_2) = 0$. Therefore, every price system $q_1 = (1, y_1)$ with $y_1 \in \mathcal{U}_1$ is part of a two-stage market equilibrium price system in our main example. Different y_1 yield a different local price system q_1 and different equilibrium consumption for consumer 1 (group 1, country 1). For y_1 sufficiently close to y_1^*, the corresponding two-stage market equilibrium is an intermediate equilibrium. Finally, choosing $y_1 = \lambda \cdot y_1^*$, with λ varying in a sufficiently small neighborhood of 1, generates a continuum of equilibrium utility levels for consumer 1 (group 1, country 1).

Also note that as a by-product of the analysis in 7.3.2, we obtain a family of intermediate equilibria parametrized by $\varepsilon \in (0, 1)$ such that the equilibrium welfare of group 3 increases and that of groups 1 and 2 decreases as ε increases.

7 Equilibrium Refinements

Until now we have not specified any objective of the external agency other than market clearing. This can leave several degrees of freedom for market clearing price systems and can generate significant indeterminacy of intermediate equilibria when commodity spaces are higher dimensional. In subsection 6.2, we found the possibility of price indeterminacy that cannot be eliminated by price normalization and the possibility of real indeterminacy that can produce a continuum of potential equilibrium utility levels.

In this section we outline plausible refinements of the notion of intermediate equilibrium that rationalize particular types of equilibria and reduce or eliminate indeterminacy of intermediate equilibria. Our main emphasis lies on the effects of control of the external agency by a single country. We will also outline some further extensions.

7.1 Control of External Agency by a Single Country

In practice, the operations of external agencies are often located in particular countries. For instance, Switzerland is famous for hosting platforms for global commodity trading while platforms for agricultural products are operating in the US. To capture such aspects we denote by h^c the country that controls the external agency. A plausible equilibrium refinement is as follows:

Def.: A **two-stage market equilibrium** with control of the external agency by country h^c is a tuple $(p; (q_h)_{h \in P}; \mathbf{x})$ such that:

(32) $(p; (q_h)_{h \in P}; \mathbf{x})$ is a two-stage market equilibrium.

(33) There exists no other two-stage market equilibrium $(\hat{p}; (\hat{q}_h)_{h \in P}; \hat{\mathbf{x}})$ with
$U_i(\hat{x_i}) \geq U_i(x_i)\ \forall i \in h^c$,
$U_j(\hat{x_j}) > U_j(x_j)$ for some $j \in h^c$.

Essentially, we require that the external agency controlled by country h^c will select global prices such that the ensuing allocation is optimal for group h^c given the hierarchical nature of trade. Optimality for group h^c means that there is no other two-stage equilibrium that is Pareto superior for the individuals in group h^c.

Equilibrium refinements can be too stringent at times. In our context, there exists a free trade equilibrium under standard assumptions and, consequently, there exists at least one two-stage market equilibrium. Let us normalize prices so that the price systems q_{h^c} belong to the unit price simplex in $\mathbb{R}^\ell$. Suppose that the set of price systems q_{h^c} which are part of a two-stage market equilibrium is a compact subset Q_{h^c} of the unit price simplex in $\mathbb{R}^\ell$. Further assume that each consumer $i \in h^c$ has a continuous indirect utility function $V_i : Q_{h^c} \longrightarrow \mathbb{R}$. Finally, assume that group h^c aims at maximizing a utilitarian social welfare function of the form $W_{h^c} = \sum_{i \in h^c} a_i U_i$. Then there exists a $q^*_{h^c} \in Q_{h^c}$ that maximizes $\sum_{i \in h^c} a_i V_i(q_{h^c})$ on Q_{h^c}. By definition, $q^*_{h^c}$ is associated with a two-stage market equilibrium $(p^*; (q_h)_{h \in P}; \mathbf{x})$

where $q_{h^c} = q^*_{h^c}$ and country h^c is in control of the external agency. Two qualifications are warranted: First, the equilibrium $(p^*; (q_h)_{h\in P}; \mathbf{x})$ may be a free trade equilibrium if there is no two-stage market equilibrium at which group h^c fares better than under free trade. Second, setting the global price system equal to p^* may bring about the desired equilibrium $(p^*; (q_h)_{h\in P}; \mathbf{x})$, but there may exist other two-stage market equilibria that are consistent with the global price system p^* and are less desirable for group h^c.

While equilibrium refinements tend to reduce indeterminacy, they do not a priori yield uniqueness. We next illustrate, however, how two-stage market equilibria with control by a single group can indeed yield uniqueness of equilibria. Moreover, the ensuing equilibrium differs from free trade.

7.2 An Example

We reconsider our main example from section 5 and assume that one country, say country 3, controls the external agency. We observe:

Fact 1 *Suppose that a two-stage equilibrium with control by country 3 exists. Then it differs from autarky and free trade.*

The fact follows from the observation made in section 5. By selecting $p_2 = \frac{7}{3}$ and $p_3 = \frac{5}{2}$ country 3 induces a two-stage market equilibrium with $q_1^2 = 2,\ q_1^3 = 3,\ q_2^1 = 2,\ q_2^3 = \frac{3}{5},\ q_3^1 = \frac{2}{3},\ q_3^2 = \frac{2}{3}$ and equilibrium allocation

$$\begin{aligned} x_1 &= \left(\frac{1}{3}, \frac{1}{6}, \frac{1}{9}\right); \\ x_2 &= \left(\frac{1}{6}, \frac{1}{3}, \frac{5}{9}\right); \\ x_3 &= \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{3}\right). \end{aligned}$$

Hence, in this two-stage market equilibrium group 3 is strictly better off than in autarky and free trade. As the intermediate equilibrium with control by the third country has to generate at least the same utility for the third group as in this arbitrarily chosen example, it must differ from free trade and autarky which proves fact 1.

We next show how the equilibrium refinement yields uniqueness, up to price normalization, in our main example. Using the equilibrium conditions from section 5, the problem of the external agency controlled by country 3 can be formulated as follows:

$$\max_{p_2,p_3} \{U_3\} = \max_{p_2,p_3} \left\{ \frac{1}{3} \ln \left(\frac{1}{3q_3^1} \right) + \frac{1}{3} \ln \left(\frac{1}{3q_3^2} \right) + \frac{1}{3} \ln \left(\frac{1}{3} \right) \right\}$$
$$s.t. \quad (11), (12), (13), (14), (15), (16).$$

We obtain:

Fact 2 *There exists a unique two-stage equilibrium with control by country 3.*

Proof:
To prove fact 2 we construct an intermediate equilibrium with intuitive properties which will turn out to be the unique two-stage equilibrium with control by country 3.
Step 1: Suppose that country 3 consumes the same amount of good 1 and good 2 in the two-stage equilibrium in which it controls the external agency.[2] Using $q_3^1 = q_3^2$ in (16) yields

$$q_3^1 = q_3^2 = \frac{1 + p_2}{2p_3}$$

Hence the remaining problem is

[2] As q_3^1 and q_3^2 are treated symmetrically in the objective function and in the constraints, and the objective function is logarithmic, the property $q_3^1 = q_3^2$ also follows formally from the maximization problem.

$$\max_{p_2,p_3}\{U_3\} = \max_{p_2,p_3}\left\{ \frac{2}{3}\ln\left(\frac{2p_3}{3(1+p_2)}\right) + \frac{1}{3}\ln\left(\frac{1}{3}\right)\right\}$$
$$s.t. \quad (11),(12),(13),(14),(15),(16)$$

which is equivalent to

$$\max_{p_2,p_3}\left\{\ln\left(\frac{p_3}{1+p_2}\right)\right\} \quad s.t. \ (11)-(16).$$

Step 2: Suppose that group 1 and group 2 are treated symmetrically regarding consumption of the third good, i.e. $q_1^3 = q_2^3$. From (13) we obtain $2q_1^3 = 2(q_1^3)^2$ and $q_1^3 = q_2^3 = 1$. From (14) with $q_1^3 = 1$ follows

$$\begin{aligned} p_2 + q_1^2 p_3 &= 2q_1^2 \text{ and} \\ q_1^2 &= \frac{p_2}{2-p_3}. \end{aligned}$$

This yields the constraint: $p_3 \leq 2$.
Step 3: From (15) with $q_2^3 = 1$ follows

$$\begin{aligned} 1 + q_2^1 p_3 &= 2q_2^1 p_2 \text{ and} \\ q_2^1 &= \frac{1}{2p_2 - p_3}. \end{aligned}$$

This yields the constraint: $p_3 \leq 2p_2$.
Step 4: The remaining problem is

$$\max_{p_2,p_3}\left\{\ln\left(\frac{p_3}{1+p_2}\right)\right\} \quad s.t. \ p_3 \leq 2, \ p_3 \leq 2p_2.$$

The second constraint will hold as an equality as the objective function is monotonically increasing (decreasing) in p_3 (p_2) respectively. Hence, we are left with

$$\max_{p_3} \left\{ \ln \left(\frac{2p_3}{2+p_3} \right) \right\} \quad s.t. \;\; p_3 \leq 2.$$

As the objective function is monotonically increasing in p_3 we obtain $p_3 = 2$. As a consequence we obtain equilibrium prices

$$\begin{aligned} p =&(1,1,2), \\ q_1 =&(1,\infty,1), \\ q_2 =& (\infty, 1, 1) \,, \\ q_3 =& \left(\frac{1}{2}, \frac{1}{2}, 1\right) , \end{aligned}$$

and allocation

$$\begin{aligned} x_1 =& \left(\frac{1}{3}, 0, \frac{1}{3}\right) ; \\ x_2 =& \left(0, \frac{1}{3}, \frac{1}{3}\right) ; \\ x_3 =& \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right) . \end{aligned}$$

Step 5: We note that, indeed, the solution above is the optimal intermediate equilibrium for group 3. At any system of internal prices individuals in group 1, group 2 and group 3 will supply $\frac{2}{3}$ of their endowments to local markets. In the intermediate equilibrium we just constructed, group 3 receives all commodities offered by the first and second group, respectively. Hence given the hierarchical nature of trade the utility of group 3 cannot be improved further as no more commodities are traded in the markets.

We observe that two of the local equilibrium prices are infinite. Hence, the derived two-stage market equilibrium can at most be understood as the limit of a sequence of equilibria constructed as follows: Assume a finite upper bound on local and global prices and allow the upper bound to become larger and larger. In subsection 7.3.3 we derive two-stage market equilibria with control of group 3 when there are limits on the price differential between local and global goods. For a given limit on price differentials we will obtain a unique two-stage market equilibrium with finite prices.

7.3 Extensions

The equilibrium concept allows for further extensions. We outline several directions. The specific numerical illustrations refer to the main example of section 5 again.

7.3.1 Control by many Countries

First, we could allow that an arbitrary set of countries controls the external agency. We denote by H^c the set of groups controlling the external agency and by I^c the union of those groups. The second property of the equilibrium definition in 7.1 has to be replaced by:

(33′) There exists no other two-stage market equilibrium $(\hat{p}; (\hat{q}_h)_{h \in P}; \hat{\mathbf{x}})$ s.t.

$$U_i(\hat{x}_i) \geq U_i(x_i) \ \forall\, i \in I^c;$$
$$U_j(\hat{x}_j) > U_j(x_j) \text{ for some } j \in I^c.$$

The interpretation is that no other two-stage market equilibrium exists which would be a Pareto improvement for the set of individuals in groups that control the external agency. The following fact is obvious:

Fact 3 *The unique two-stage market equilibrium with control of the external agency by group 3 is also a two-stage market equilibrium with control of the external agency by H^c if group 3 belongs to H^c.*

Several remarks are in order. First, if for instance groups 1 and 2 control the external agency, the two-stage equilibrium with control by country 3 is not an equilibrium anymore as already free trade would be better for groups 1 and 2. Second, even if all countries control the external agency free trade may not necessarily emerge as the equilibrium with control by group 3 continues to be an equilibrium. However, the first welfare theorem implies that free trade constitutes a two-stage equilibrium when all countries jointly control the external agency.

7.3.2 Bargaining by Countries

We could further specify a detailed decision process regarding the selection of global prices when several countries control the external agency. Again we can illustrate this for the example outlined in section 5 by assuming that e.g. groups 1 and 2 control the external agency and bargain about global prices via the Nash bargaining solution. Let us also assume that the bargaining power is the same among groups. We obtain for this case:

Fact 4 *Suppose there exists a lower bound $\varepsilon > 0$ for the relative global market price of good 3 in any two-stage market equilibrium. Then there exists a unique two-stage market equilibrium with control of the external agency and symmetric Nash bargaining by groups 1 and 2. This equilibrium differs from autarky and free-trade.*

Proof:
Step 1: To construct the two-stage equilibrium with control by groups 1 and 2, we denote by $\mathfrak{P}$ the set of global prices such that for every $p \in \mathfrak{P}$, there exists a two-stage market equilibrium $(p; (q_h)_{h \in P}; \mathbf{x})$. Assuming that free trade is the default option, the problem facing the stakeholders of the external agency is maximization of the Nash product

$$\max_{p\in\mathfrak{P}} \left\{U_1(p) - \ln\left(\frac{1}{3}\right)\right\}^{\frac{1}{2}} \left\{U_2(p) - \ln\left(\frac{1}{3}\right)\right\}^{\frac{1}{2}}.$$

As we shall see this procedure selects a two-stage equilibrium in which countries 1 and 2 obtain the same utility.

Step 2: We assume that group 1 and group 2 are treated symmetrically as they have the same bargaining power: symmetry is expressed by

$$p_2 = p_1 = 1,\ q_1^3 = q_2^3.$$

The property $q_1^3 = q_2^3$ and (13) yield $q_1^3 = 2(q_2^3)^2$ which leads to $q_1^3 = q_2^3 = 1$. From (14) we obtain $1 + p_3 q_1^2 = 2q_1^2$ and hence

$$q_1^2 = \frac{1}{2 - p_3}$$

which yields the constraint $p_3 \le 2$. From (15) we obtain $1 + p_3 q_2^1 = 2q_2^1$ which yields

$$q_2^1 = \frac{1}{2 - p_3} = q_1^2.$$

Step 3: The remaining problem is given by

$$\max_{p\in\mathfrak{P}} \left\{\frac{2}{3}\ln\left(\frac{1}{3}\right) + \frac{1}{3}\ln\left(\frac{2 - p_3}{3}\right) - \ln\left(\frac{1}{3}\right)\right\},$$

as the objective functions of group 1 and group 2 are identical as functions of p_3. We assume that the smallest possible price p_3 is ε where $\varepsilon > 0$ can be arbitrarily small. Then, group 1 and 2 will choose $p_3 = \varepsilon$ to maximize their common objective function. Given $p_3 = \varepsilon$ we obtain

$$q_2^1 = q_1^2 = \frac{1}{2 - \varepsilon}.$$

From (11) we get

$$q_3^1 = \frac{q_2^1}{2q_2^1 - 1} = \frac{1}{\varepsilon}.$$

From (12) we get

$$q_3^2 = \frac{q_1^2}{2q_1^2 - 1} = \frac{1}{\varepsilon}.$$

Finally, equation (16) reads

$$2\frac{1}{\varepsilon} = 2\left(\frac{1}{\varepsilon}\right)^2 \varepsilon$$

and thus holds.

Step 4: To sum up, the two-stage equilibrium with control by groups 1 and 2 is characterized by

$$\begin{aligned} p &= (1, 1, \varepsilon), \\ q_1 &= \left(1, \frac{1}{2-\varepsilon}, 1\right), \\ q_2 &= \left(\frac{1}{2-\varepsilon}, 1, 1\right), \\ q_3 &= \left(\frac{1}{\varepsilon}, \frac{1}{\varepsilon}, 1\right), \end{aligned}$$

and the allocation

$$\begin{aligned} x_1 &= \left(\frac{1}{3}, \frac{2-\varepsilon}{3}, \frac{1}{3}\right), \\ x_2 &= \left(\frac{2-\varepsilon}{3}, \frac{1}{3}, \frac{1}{3}\right), \\ x_3 &= \left(\frac{\varepsilon}{3}, \frac{\varepsilon}{3}, \frac{1}{3}\right). \end{aligned}$$

7.3.3 Limits on Price Differentials

Third, the set of equilibria could also be constrained by limits on the price differentials between local and global prices. For instance we could assume that all local and global prices have to satisfy

$$|q_h^k - p^k| < \Delta \ \forall k, \ \forall h$$

for some $\Delta > 0$. This would reflect the fact that goods arbitrage sets in when local and global prices diverge too much. Goods arbitrage constraints can be combined with the refinements of the equilibrium notion when some groups control the external agency.

As an illustration, we consider again the example in which group 3 controls the external agency. Let us assume $\Delta = 1$. Then the two constraints

$$|q_2^1 - p_1| \leq 1, \ |q_1^2 - p_2| \leq 1$$

and

$$q_1^2 = \frac{p_2}{2 - p_3}, \ q_2^1 = \frac{1}{2p_2 - p_3}$$

imply $p_3 \leq 2p_2 - \frac{1}{2}$ and $p_3(1 + p_2) \leq 2 + p_2$.

We calculate the maximal value of p_3 that fulfills both constraints. Using $p_2 = \frac{p_3}{2} + \frac{1}{4}$ from the first constraint yields $2p_3^2 + 3p_3 - 9 \leq 0$. The maximal value of p_3 satisfying this constraint is $p_3 = \frac{3}{2}$. This implies $p_2 = 1, q_1^2 = 2, q_2^1 = 2$. The entire equilibrium is described by the price system

$$\begin{aligned} p &= \left(1, 1, \frac{3}{2}\right), \\ q_1 &= (1, 2, 1), \\ q_2 &= (2, 1, 1), \\ q_3 &= \left(\frac{2}{3}, \frac{2}{3}, 1\right), \end{aligned}$$

and the allocation

$$x_1 = \left(\frac{1}{3}, \frac{1}{6}, \frac{1}{3}\right),$$
$$x_2 = \left(\frac{1}{6}, \frac{1}{3}, \frac{1}{3}\right),$$
$$x_3 = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{3}\right).$$

We observe that constraints from goods arbitrage limit the utility group 3 can achieve by controlling the external agency.

As a second illustration, let again $\Delta = 1$. Imposing the conditions $|q_2^1 - p_1| \leq 1$, $|q_3^1 - p_1| \leq 1$ in 7.3.2 yields $\varepsilon \geq 1/2$. Hence constraints from goods arbitrage limit the Nash-bargained utility level groups 1 and 2 can achieve when they control the external agency.

7.4 Profit oriented external agencies

A totally different approach is to consider the external agency as an institution that demands a fixed fee f $(0 < f < 1)$ from each trade and maximizes profits arising from external trade. The equilibrium notion can be adapted to this case by substituting condition (5) by

$$(5') \quad p\left(\sum_{i\in h} \max\{z_i, 0\}(1+f) + \sum_{i\in h} \frac{\min\{z_i,0\}}{1+f}\right) \leq 0 \text{ for each } h \in P,$$

where $z_i = x_i - w_i$ is the excess demand of consumer i.

To close the model we would have to require further that a two-stage market equilibrium maximizes trade volume and thereby the profit of the external agency.

8 Hierarchical Trade versus Trade in One Market Place

Hierarchical trade differs from traditional trade theory in terms of the organization of markets. However, as already suggested in the example, an equivalence between the set of two-stage market equilibria and the set of single-market equilibria exists in terms of distortions. Let us denote the price distortion for commodity k in group h, for a given intermediate equilibrium $(p; (q_h)_{h \in P}; \mathbf{x})$ by τ_h^k. Thus:

$$\tau_h^k = \frac{q_h^k}{p^k} - 1$$

Moreover, let τ_h be the price distortion vector of group h. Let $(p^n; (\tau_h)_{h \in P}; \mathbf{y})$ denote a competitive equilibrium in which each nation trades in a single market place, where $\mathbf{y}$ is the equilibrium allocation, p^n is the world market price vector and where group h faces the distortion vector τ_h and thus the price vector $p^n(1 + \tau_h)$. Then the following proposition holds:

Proposition 6 *Suppose $(p; (q_h)_{h \in P}; \mathbf{x})$ is a two-stage market equilibrium. Then there is a single market equilibrium with distortions τ_h, $p^n = p$, $\mathbf{y} = \mathbf{x}$, and zero lump-sum transfers. Conversely, a single market equilibrium with zero lump-sum transfers has a corresponding two-stage market equilibrium with identical allocation and distortions.*

The proof is obvious. As demonstrated in our working paper (Gersbach and Haller 2007), the situation is in general different if we start with a single market equilibrium. Given a set of price distortions for a proper subset of groups, there are three possible cases. First, no corresponding two-stage-market equilibrium may exist. Second, wedges between local and global prices for the other groups may arise and thus the allocation and distortions in the two-stage market equilibrium will differ in general from those in the one-stage market equilibrium. Third, when lump-sum transfers are zero in the single-market equilibrium, there exists a two-stage market equilibrium with the same allocation and distortions as the single market equilibrium.

9 Conclusion

We have developed a simple model of trade in different market places. There remain several interesting open questions. First, how many intermediate equilibria are there in general? Specifically, if the number of commodities exceeds two, are there sufficient conditions for the existence of a continuous path from autarky to free trade? Second, how is the analysis affected by the incorporation of producers? Whereas the introduction of local producers seems to cause mainly notational complications, the modeling of import-export enterprises and multinational corporations constitutes a much more formidable challenge.

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