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PROMISING THE RIGHT PRIZE

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ABSTRACT

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JEL Classification: D82, H57, O31 and O38
Keywords: innovation race, market commitment mechanism, mechanism design, prizes and sorting

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Promising the right prize

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Abstract

Prizes are often awarded to encourage research on products deemed of vital importance. We present a mechanism which can, in situations where the innovators are better informed about the difficulty of the research, tailor perfectly the expected reward to the expected research costs. The idea is to let the first successful inventor trade off the risk of having a competitor share the reward in exchange for a higher prize. If the goal of the designer is to minimize the prize awarded whilst encouraging innovators to conduct research, such a mechanism achieves the first best.

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1. INTRODUCTION

Prizes have often been used to encourage research on products considered of high social importance and for which private incentives are insufficient. Examples date back to the seventeenth century when a prize was offered in France for developing a workable water turbine.[1] There has recently been a renewed interest in prizes as a means of encouraging innovation, in part due to controversies surrounding patent protection (see Boldrin and Levine (2008)). For instance, it has been advocated that prizes should be used to encourage research on vaccines for the developing world with small market potential (Kremer (2001a, 2001b)).[2] This idea is currently being tested on a large scale: in February 2007, 5 countries[3]

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[1]see Gallini and Scotchmer (2002) and Maurer and Scotchmer (2004) for other examples and Brunt et al (2010) for systematic historical evidence.

[2]A similar idea was advocated for agricultural techniques for tropical countries (Kremer and Zwane (2002))

[3]Canada, Italy, Russia, Norway and UK.

and the Bill and Melinda Gates Foundation, committed \$1.5 billion for a pilot action to encourage the development and production of vaccines for pneumococcal diseases.[4]

An important concern of the donors is to minimize the prize awarded, whilst still encouraging research (potentially by several firms if the final good is deemed of sufficient importance). Indeed, the donors are either using taxpayer money or are concerned that the funds could be used for other projects. In the case of vaccines for developing countries, the donation for research on pneumococcal diseases is a trial of the system and more widespread political support is needed if the idea is to be applied to other diseases, such as malaria.

Ideally, the donors would like the prize to be in the same order of magnitude as the research costs. However, prizes are usually defined at the start of the process, when the initiative is announced. At that point in time, donors, who have limited knowledge about the difficulty of the research, find it difficult to tailor the prize to the expected research costs.[5] It is therefore conceivable that, if the research is easy, the profits of the winner will be "excessive" compared to the incurred costs.

In this paper we propose a simple mechanism which can achieve the dual goals of encouraging several innovators to conduct research and of minimizing the prize awarded. The idea of the mechanism is the following. The first inventor of an acceptable product is offered a choice between different menus (T, P), where P is the reward and T (that we will call invention window) is such that if a second innovator invents within T of the first, they both share the reward. The first inventor could for instance be given the choice between the following menus: (0 months, 1 billion) and (18 months, 1.6 billion). With the first menu he takes no risk and receives 1 billion for sure. With the second, if the other innovator does not succeed within 18 months he receives 1.6 billion, but if the competitor is successful within the innovation window, he has to give him 0.8 billion at that date.[6] The first inventor therefore needs to tradeoff the risk of loosing part of the market to a competitor in exchange for a higher reward.

We study this mechanism in a model where two innovators can invest in research by spending both a sunk and a flow cost. Success is then determined, for both inventors, by a Poisson arrival process of speed λ.[7] We assume that there is asymmetric information between the mechanism designer and the innovators regarding the speed of the research. The innovators know λ whereas the designer only knows the distribution from which it is

[4]The final plan adopted is not exactly a prize. The commitment is to guarantee a co-payment for each unit sold on the market. We discuss this in more details in section 7.

[5]Typically the donors won't be as well informed as the innovators about the difficulty of the research.

[6]Note that the mechanism works with any number n of innovators. If n innovators invent within T of the first, they each get $\frac{P}{n}$. We show in section 4 that our main result also holds in that case.

[7]The speed λ characterizes the difficulty of the research. It is a characteristic of the particular product being researched and is therefore a type shared by both innovators.

drawn.

In the context of this model, we show that if the designer does not face too much uncertainty about the difficulty of the research and if the up front sunk cost is not too small, the designer can offer menus trading off prize against innovation window, such that the the innovators obtain no expected surplus at the start of the process, regardless of the speed λ of the research. We refer to this result as perfect compensation: the innovators optimally choose a prize that perfectly compensates them for their expected research costs at date zero. If the goal of the designer is to minimize the expected reward paid under the condition that both innovators invest in research, such a menu achieves the first best.

This result rests on a useful correlation between the cost of research and the probability of quasi simultaneous invention. For instance, if the inventor knows that obtaining a vaccine for malaria is relatively easy, he does not expect to spend excessive resources on research. However, if he is the first to invent, he also realizes there is a high risk that his competitor succeeds soon after him. He is thus ready to accept a lower prize in exchange for a smaller invention window. Under certain conditions, this lower prize can be tailored to exactly compensate for the lower research cost.

In section 4, we examine the robustness of this result with respect to several assumptions. First, we show that perfect compensation of expected date zero costs is also possible if there are n potential innovators with access to the same research process. This result also rests on the correlation between the difficulty of the research and the probability that one of the competitors invents within the window.

Second, we show that if the innovators are heterogenous in their capacity to conduct research, the result has to be qualified. In the case of two innovators where one is more efficient than the other, we show that the mechanism can perfectly compensate the least efficient innovator at date zero, but will leave expected profits to the more efficient one.

Finally, we show that our result does depend on the assumption that the initial fixed cost of research is relatively large. Indeed, this assumption ensures that, if the prize is sufficient to encourage research at date zero, it is also sufficient to guarantee that innovators continue research after observing the success of their competitor, since the fixed cost has already been sunk. The fact that the second innovator keeps investing in research exerts a crucial competitive pressure that makes sorting between the different types possible. In section 4.3 we consider the case of no fixed cost and show that the innovators obtain positive expected profits at date zero. The mechanism we propose nevertheless still improves the donors'welfare compared to a situation with a single prize.

Of course, the designer does not necessarily want to minimize the prize at all costs. A more standard definition of welfare is the expected discounted surplus, net of research costs incurred by innovators and net of the social cost of raising the prize. Maximizing such a welfare function is identical to the goal of minimizing the prize under the constraint that sufficient research is conducted, if the designer puts no weight on the innovator's welfare and if the surplus from the invention is sufficiently large. We argued previously that this simplified objective appears reasonable in the case of sufficiently important inventions. Nevertheless, in section 5, we consider the general welfare function and derive conditions under which an optimally designed menu will include a strictly positive innovation window.

It is important to note that there are other solutions to encourage innovators to reveal their information about the difficulty of the research, such as initially running an auction. We discuss in detail in section 6 these alternative solutions and papers related to them. They generally rely on initially selecting an innovator and letting the chosen innovator conduct the research. Consider the case of the auction. Such a solution creates two problems, that we do not face in the mechanism we introduce. First, there is a moral hazard issue: the winner of the auction needs to be provided with incentives to effectively conduct the research. Second, the looser in the auction does not invest, which leads to a later discovery date.

We believe the second problem is relevant when prizes are awarded for important inventions. In such situations, it is often socially valuable to encourage several firms to work in parallel. Indeed, there typically exists a marginally decreasing relation between effort and success and furthermore, different teams can adopt very different research strategies aimed at achieving the same goal. Consider for instance a system aimed at increasing research on vaccines for malaria. A solution that places all the hopes of the donors in a single firm, does not seem appropriate.

The goal of our paper, is also to provide a practical mechanism, based on the introduction of a new tool, the invention window, that allows the designer to tailor the prize to the expected research costs. We show in section 7 that this idea can be easily implemented in practice and discuss more specifically the case of vaccines for developing countries. We present the model in section 2 and derive our main result in section 3. In section 4 we consider the three robustness checks previously mentioned. In section 5, we consider a general welfare function for the designer. Finally, in section 6 we discuss the related literature. All the proofs can be found in the appendix.

2. Model

A mechanism designer wants to encourage research on a particular product. Two risk neutral innovators can conduct that research.[8] They do so if a sufficient prize is promised to

[8]The argument is extended to n innovators in section 4.1.

them.[9] We first describe the research technology and then provide details on the mechanism used by the designer to reward innovators.

2.1 Research

The research process is the following. Time is continuous and we consider a discount rate r, identical for the innovators and the designer. Innovators need to pay a sunk research cost F at the start of the research and a constant flow cost c thereon. Such an investment produces a discovery according to a Poisson arrival process of speed λ.[10] We assume that if the innovator stops paying the flow cost c, the probability of discovery drops to zero.

There is asymmetric information between the innovators and the designer. The speed of the research is known to the innovators but unknown to the designer: λ thus represents the type of the innovators. It is common knowledge that λ is drawn from a distribution f with support $[\lambda_L, \lambda_H]$.

It is important to note that both innovators share the same type. The speed λ is a characteristic of the particular invention. It measures for instance the difficulty of obtaining a vaccine for malaria. We therefore implicitly suppose that innovators are equally efficient in conducting the research. We consider the case of heterogenous innovators in section 4.2.

We impose the following assumption on the research process:

Assumption 1: $F > \frac{c}{2\lambda_L + r}$

Assumption 1 states that the initial sunk cost is relatively high compared to the flow cost. We show in section 3 that this assumption guarantees that if an innovator finds it profitable to initially invest in research, he will also find it profitable to continue searching even after his competitor has succeeded. We show in section 4.3 how the results are modified when this assumption is relaxed.

2.2 The designer

The designer wants to maximize total welfare defined as follows:

$$\underbrace{E[d]\ S}_{discounted\ social\ value\ of\ invention} \quad - \quad \underbrace{\alpha\ [2E[C] + 2F]}_{expected\ cost\ for\ innovators} \quad - \quad \underbrace{\nu\ E[d]\ E[P]}_{expected\ cost\ of\ raising\ prize}$$

where

[9] We abstract from other means of obtaining profits from the discovery. It could be the case that there is insufficient market potential for the product, as in the example of vaccines for developing countries.

[10] see Loury (1979) and Lee and Wilde (1980) for the introduction of these types of models

- S is the social surplus from the invention and $E[d]$ is the discounted time until expected discovery date[11]

- $E[C]$ is the expected research flow cost incurred by each innovator (derived in lemma 2) and α is a weight in the welfare function (the designer might put more weight on the social value of the invention than on the costs incurred by firms)

- $E[P]$ is the expected prize paid. The payment of the reward requires raising distortionnary taxes. ν measures the unit cost of each dollar of reward.

The designer can use the following mechanism to achieve her goal: she offers the first inventor a menu of prizes that we denote (T, P). P is the amount of the prize and T is the innovation window. Suppose for instance that the first inventor succeeds at time t and chooses a menu (T_0, P_0). If the competitor invents before $t + T_0$ (before the end of the innovation window), the first inventor needs to share his prize. Specifically he gives $P_0/2$ to the second inventor as soon as that innovator succeeds. If the competitor does not invent during that period, the first inventor obtains the full prize P_0. We suppose that the success of the innovators is publicly observable: it could for instance be announced by the designer.[12]

One of the main results of this paper is to show that these menus can be designed such that the prize chosen by innovators in equilibrium, compensates them exactly for their expected research costs at the start of the process. In the case where the objective of the designer is to minimize the prize awarded under the constraint that both innovators invest in research, these menus clearly achieve the first best. In sections 3 and 4 we consider this to be the designer's objective. The following conditions are sufficient for the general objective to reduce to minimizing the prize, provided both innovators invest:

1. the designer does not take into account the research cost of the innovators ($\alpha = 0$)

2. the social value of the invention is sufficiently large that both innovators should be encouraged to conduct research regardless of the speed of the research

As we pointed out in the introduction, there are several reasons why this objective of minimizing the prize awarded under the condition that sufficient research is conducted, is natural in the case of research prizes. It could correspond to a political feasibility requirement. For instance, if it proves to be very easy to obtain a vaccine for pneumococcal diseases and a very large prize is paid, it will be difficult to justify such a mechanism to spur research

[11]For instance, we show in section 3, that if both innovators invest in research, $E[d] = \frac{2\lambda}{2\lambda+r}$

[12]Our results are robust if this assumption is relaxed as long as the expiration of the innovation window is publicly announced.

on other vaccines, such as a vaccine for malaria. In this context a mechanism that minimizes the amount of the prize while encouraging research is needed. We however consider, in section 5, results for the general objective.

3. Perfect compensation of innovators

3.1 The mechanism

In this section, we focus on the case where the goal of the designer is to propose a menu that minimizes the expected payments to innovators under the condition that it is

1. incentive compatible for the first inventor at invention date
2. provides incentives for both innovators to invest in research at date zero.

We define $R_W[P(\widehat{\lambda}), T(\widehat{\lambda}), \lambda]$ the expected reward of an innovator of type λ who has won the race and reports type $\widehat{\lambda}$. The expected date zero profit of a type λ is denoted $E\Pi_0(\lambda, P(\lambda), T(\lambda))$. From the revelation principle, we can concentrate on the following mechanism:

$$\begin{cases} \text{Min} \quad E_\lambda\ [P(\lambda)] \quad \text{subject to:} \\ \\ \qquad R_W[P(\lambda), T(\lambda), \lambda] \geq R_W[P(\widehat{\lambda}), T(\widehat{\lambda}), \lambda] \;\; \forall(\lambda, \widehat{\lambda}) \quad (IC) \\ \\ \qquad E\Pi_0(\lambda, P(\lambda), T(\lambda)) \geq 0 \;\; \forall\lambda \qquad (IR) \end{cases}$$

We insist on the fact that this program differs from a standard mechanism design problem due to the timing of choices. In the classical definition of the problem, the incentive compatibility and the individual rationality constraints need to be satisfied at the same date. They are therefore defined on the same profit function. In the case we are considering, incentive compatibility needs to be satisfied at the invention date. However the individual rationality constraint is defined at date zero: the menu is intended to provide research incentives to the innovators at the start of the game. In section 6, we compare this to the approach taken by other papers in the literature.[13]

[13] We discuss in section 6 papers that describe the patent system as a mechanism design problem (Scotchmer (1999), Cornelli and Schankerman (1999) and Hopenhayn and Mitchell (2001). All these papers adopt the standard timing. Note that due to the timing, our problem can be reexpressed as a model with type dependant reservation utility. Jullien (2000) conducts a theoretical analysis of the properties of the optimal contracts when the reservation utility is type dependent. Our paper examines a more specific model with two agents sharing the same type and our goal is to show that perfect compensation of all types is possible.

3.2 Perfect compensation

In this section we determine some properties of the optimal mechanism and derive our main results. We first calculate the profits of innovators (both at date zero and at invention date) that determine the incentive compatibility and individual rationality constraints.

An innovator who has succeeded knows that the competitor faces the same research process of speed λ. If the competitor keeps investing until the expiration of the invention window, the inventor's expected reward is:

$$P - \frac{P}{2}\int_0^T \lambda e^{-\lambda t} e^{-rt}\, dt$$

He collects P initially and has to share the prize if the competitor succeeds within the innovation window T. We thus obtain the following result:

Lemma 1: An innovator of type λ who is first to invent expects, at invention date, revenues $R_W = P - \frac{\lambda}{\lambda+r}\frac{P}{2}(1 - e^{-(\lambda+r)T})$ from menu (T,P) if the competitor keeps investing in research until the expiration of the innovation window.

We notice that when the research process is fast, the benefits from choosing a small innovation window (small value of T) are large. Such a choice reduces the risk that the competitor shares the prize.

The next step is to determine the expected profits at the start of the race of an inventor who knows that both he and his competitor will choose menu (T,P) once they succeed. If after observing the success of the competitor, the looser of the race keeps investing, then the expected date zero profits are given by:

$$E\Pi_0(\lambda) = \frac{\lambda}{2\lambda + r}(R_W + R_L) - F - E[C]$$

where:

- $R_W = P - \frac{\lambda}{\lambda+r}\frac{P}{2}(1 - e^{-(\lambda+r)T})$ is the expected reward of the winner of the contest
- $R_L = \frac{\lambda}{\lambda+r}\frac{P}{2}(1 - e^{-(\lambda+r)T})$ is the expected reward of the loser (ex ante both innovators are equally likely to be winner or looser)
- $E[C]$ is the expected flow cost of research (value derived in the appendix).

We show in lemma 2 that, under Assumption 1, the looser of the race indeed pursues his research effort after observing the success of his competitor.

Lemma 2: Under Assumption 1:

(a) if innovators of type λ choose menu (T,P) once they succeed, their expected profits at date zero are: $E\Pi_0(\lambda, P, T) = \frac{\lambda}{2\lambda+r}P - F - \frac{c}{\lambda+r}[1 - \frac{\lambda}{2\lambda+r}e^{-(\lambda+r)T}]$

(b) if an innovator of type λ finds it profitable to start research at date zero ($E\Pi_0(\lambda, P, T) \geq 0$) then he finds it profitable to continue research after his competitor has succeeded and before the innovation window expires.

On the one hand, after success of a competitor, investing the flow cost c in research becomes less attractive as the best outcome is to obtain half the prize. On the other hand the sunk cost F has already been paid at date zero. Assumption 1 guarantees that the initial sunk cost is large enough so that the second effect dominates and leads to result (b): if the prize P is large enough to encourage investment in research at date zero it will also be large enough to encourage an innovator to continue research even after observing the success of his competitor.

From result (a) we note that the expected total revenue ($\frac{\lambda}{2\lambda+r}[R_W + R_L] = \frac{\lambda}{2\lambda+r}P$) does not depend on T. Indeed, at the start of the race both innovators are equally likely to be first or second. The amount that is shared by the first inventor with the second therefore has ex ante a zero expected value. The marginal effect of a larger window is thus very different on the ex ante profits $E\Pi_0$ (that define the IR constraint) than on the ex post reward R_W once an inventor knows he was first to invent (that define the IC constraints). This has important consequences for the mechanism we consider.

Given the results of lemma 1 and 2, we can reexpress the mechanism design problem as:

$$\begin{cases} \text{Min} \quad E_\lambda \left[P(\lambda)\right] \quad \text{subject to:} \\ \\ \quad P(\lambda)(\lambda + 2r + \lambda e^{-(\lambda+r)T(\lambda)}) \geq P(\lambda')(\lambda + 2r + \lambda e^{-(\lambda+r)T(\lambda')}) \quad (IC) \\ \quad \frac{\lambda}{2\lambda+r}P(\lambda) - F - \frac{c}{\lambda+r}[1 - \frac{\lambda}{2\lambda+r}e^{-(\lambda+r)T(\lambda)}] \geq 0 \;\; \forall\lambda \qquad (IR) \end{cases}$$

We show in the following proposition that the solution to this problem has an important characteristic: if the designer does not face too much uncertainty regarding the speed of the research process, all innovators, given the prize they will choose at invention date, expect zero profits at date zero, regardless of their type (what we refer to as perfect compensation of all types).[14]

[14]We note that the compensation is exact only in expectation. It can very well be that the inventor succeeds very quickly and therefore obtains positive rents.

Proposition 1: If the interval $[\lambda_L, \lambda_H]$ is not too large, the optimal mechanism allows for perfect compensation, i.e is such that all types expect zero profits at the start of the research process: ($\forall\lambda \in [\lambda_L, \lambda_H]$, $E\Pi_0(\lambda) = 0$).

This result rests on a useful correlation between the cost of research and the probability of quasi simultaneous invention. For instance, if the inventor is a high type (high value of λ), he knows that the research will be quick and cost little. However, he also realizes that if he is the first to invent, there is a high risk that the competitor succeeds soon after. He is thus ready to accept a lower prize in exchange for a smaller invention window. For a certain range of types this lower prize can be tailored precisely to balance the lower research cost.

Perfect compensation is however not possible for large ranges of potential types. Consider in particular the case of very long research processes (small value of λ). To compensate such an innovator perfectly, the reward awarded needs to be large.[15] However incentive compatibility does not allow rewards to increase very fast. Indeed profits at invention date are proportional to $P(\lambda)(\lambda + 2r + \lambda e^{-(\lambda+r)T(\lambda)})$. If rewards increase fast, higher types are certainly willing to take more risk. Therefore, when the range of possible types is too large, perfect compensation is not possible.

4. Robustness

We examine the robustness of our main result with respect to some of the assumptions. In section 4.1 we study the case of n potential innovators. In section 4.2 we allow for heterogeneity in the capacity to conduct research. Finally in section 4.3 we examine the role of Assumption 1 that assumes that the fixed cost of research is high. To simplify the exposition, the results in this section are presented for the case of no discounting, $r = 0$.

4.1 Large number of potential innovators

The results in section 3 were derived under the assumption that only two innovators had the ability to develop the product. In this section we generalize these results to the case of n innovators. We still suppose they all face the same Poisson research process of speed λ. The mechanism is quasi identical. When a first innovator succeeds, he chooses a prize P and an innovation window T. If k other innovators succeed within this window, they all obtain a reward $\frac{P}{k+1}$.[16]

[15] Profits at date zero are given by $E\Pi_0(\lambda) = \frac{\lambda}{2\lambda+r}P - F - \frac{c}{\lambda+r}[1 - \frac{\lambda}{2\lambda+r}e^{-(\lambda+r)T}]$.

[16] The assumption of no discounting simplifies the calculation as we do not need to keep track of all invention dates.

The expected reward for the winner of the race is given by:

$$R_W = \sum_{j=0}^{n-1} C_{n-1}^j \; q^j (1-q)^{(n-1)-j} \; \frac{P}{j+1}$$

where $q = 1 - e^{-\lambda T}$ is the probability that each individual innovator succeeds within the window. We show that the results in lemma 1 generalize as follows:

Lemma 3: An innovator of type λ who is first to invent expects revenues $R_W = \frac{P}{qn}(1 + e^{-\lambda n T})$ from menu (T,P) if the competitors keep investing in research until the expiration of the innovation window.

We then generalize the result of lemma 2. The expected reward for the innovators that did not win the race is given by:

$$R_L = q \sum_{j=0}^{n-2} C_{n-2}^j \; q^j (1-q)^{(n-2)-j} \; \frac{P}{j+2}$$

A looser succeeds with probability q and his reward is then determined by how many other innovators succeeded within the window. Overall at date zero, the expected revenue is $\frac{1}{n} R_W + \frac{n-1}{n} R_L = \frac{P}{n}$. As in the case of two innovators, the expected reward does not depend on the type λ. Indeed the prize P is paid for sure to the innovators, and given the fact all n innovators are symmetric, they all expect a fraction $1/n$ of the total revenue.

To guarantee that innovators keep investing in research after observing the success of one of them, we impose the following assumption:[17]

Assumption 2: $F > \frac{c}{\lambda_L}$

and show that under this assumption, the result of lemma 2 generalizes in the following way

Lemma 4: Under Assumption 2:

(a) if innovators of type λ choose menu (T,P) once they succeed, their expected profits at date zero are: $E\Pi_0(\lambda, P, T) = \frac{P}{n} - \frac{c}{\lambda}[1 - \frac{n-1}{n} e^{-\lambda T}] - F$

(b) if an innovator of type λ finds it profitable to start research at date zero ($E\Pi_0(\lambda, P, T) \geq 0$), he also finds it profitable to continue research after his competitor has succeeded and

[17]Note that this is a sufficient assumption to obtain the results. There might be less restrictive constraints that would imply the same property.

before the innovation window expires.

In the case of n innovators, the mechanism design problem is therefore:

$$\begin{cases} \text{Min} \;\; E_\lambda \left[P(\lambda)\right] \quad \text{subject to:} \\ \\ \quad P(\lambda)(1+e^{-\lambda n T(\lambda)}) \geq P(\lambda')(1+e^{-\lambda n T(\lambda')}) \;\; \forall(\lambda,\lambda') \quad (IC) \\ \quad \frac{P(\lambda)}{n}\frac{c}{\lambda}[1-\frac{n-1}{n}e^{-\lambda T(\lambda)}] - F \geq 0 \;\; \forall\lambda \qquad (IR) \end{cases}$$

and Proposition 1 generalizes as follows:

Proposition 2: If the interval $[\lambda_L, \lambda_H]$ is not too large, the optimal mechanism is such that all types expect zero profits at the start of the research process.

We find that the general result obtained in the case of two innovators, still holds for a larger number of participants in the race. The intuition is identical: for easy research processes, the expected cost will be low, but at invention date, the probability that at least one of the competitors invents within a short period of time is high. A short window in association with a low prize then becomes attractive.

4.2 Heterogenous innovators

In previous sections we considered cases where innovators all faced exactly the same research process. In this section we reconsider our results in the case where some innovators are more efficient at conducting research than others. Note that we return to the case of two innovators.

Specifically we study the following variant of our initial model. There are two potential innovators and we assume that innovator LE is less efficient than innovator E. Both innovators conduct research on the same product and the difficulty of the research process is characterized by a parameter λ. We describe λ as the type of the product. The research process for the less efficient innovator LE, when the product is of type λ, follows a Poisson process of parameter λ while for the more efficient innovator E faces a Poisson process of parameter $\mu\lambda$ where $\mu > 1$. We therefore maintain the idea that even though the innovators might be more or less efficient, the difficulty of the research is still partly determined by the characteristics of the product.[18]

[18]If there is absolutely no correlation between the research processes of the two innovators, our mechanism

In the framework of this model we find that the result of Proposition 1, still holds, for the less efficient innovators, regardless of the type of the product λ. However, the more efficient innovators expect strictly positive profits at date zero.

Proposition 3: If the interval $[\lambda_L, \lambda_H]$ is not too large, the optimal mechanism is such that less efficient innovators expect zero profits at the start of the process, when the type of the product is in $[\lambda_L, \lambda_H]$. The more efficient innovators obtain strictly positive expected profits at date zero.

The intuition for this result is as follows. The reward for a less efficient innovator LE if he wins the race is given by $R_{W,LE} = \frac{P}{2}(1 + e^{-\lambda\mu T})$ while the reward for the more efficient innovator is $R_{W,E} = \frac{P}{2}(1 + e^{-\lambda T})$. We note that, at invention date, the reward is determined by the probability that the competitor invents within the window and thus only depends on the type of the competitor. In particular, an innovator LE when the product is of type λ and an innovator E when the product is of type $\lambda\mu$ choose the same contract at invention date. However, the expected profit at date zero is much higher for the more efficient innovator. It is therefore clear that perfect compensation of both type of innovators at date zero is not feasible.

Proposition 3 nevertheless indicates that perfect compensation of the less efficient innovators is possible if the uncertainty on the type of the product is not too large. The choice of contract at invention date is determined by the competitor's research process while the expected reward at the start of the research is partly determined by the own research process. The fact that both research processes are linked to the underlying type of the product, maintains the correlation between a fast research and a high risk that the competitor invents soon after. The invention window can therefore still be used to sort between the different types.

4.3 Research with no fixed cost

Assumption 1 plays an important role in the results of section 3. In this section we examine in more detail this exact role. We study the particular case where there is no fixed cost (F=0) for which Assumption 1 is clearly violated.

Lemma 5 shows that under Assumption 1, if the prize is high enough to encourage research at the start of the game, it is also high enough to encourage an innovator to continue research after observing the success of his competitor. On the contrary, when $F = 0$, we

has no effect. However, it appears sensible that innovators conducting research on the same topic will face similar difficulties.

show that the reverse result is true.

Lemma 5: If $F = 0$ then:

(a) the expected profits of the second innovator after observing the success of his competitor is $[\frac{P}{2} - \frac{c}{\lambda}][1 - e^{-\lambda T}]$ if the first chooses menu (T, P)

(b) if an innovator of type λ finds it profitable to continue research after his competitor has succeeded, he also finds it profitable to start research at date zero ($E\Pi_0(\lambda) \geq 0$).

(c) the expected profits of innovators of type λ at date zero are $\frac{P}{2} - \frac{c}{2\lambda}$ if both innovators are expected to stop research after observing the success of their competitor.

The designer therefore has two options. She can offer a prize sufficient to encourage the second innovator to stay in the race after success of the first innovator. This allows to sort among the different types. The prize in that case needs to be such that $\frac{P}{2} \geq \frac{c}{\lambda}$ (lemma 3 (a) and (b)). The second option is not to sort between types and award a prize that is just sufficient to encourage research at date zero. We assumed that the social value of the product is sufficiently large that the designer always wants to encourage research. Therefore in this second case, as no sorting is possible, the designer needs to ensure that the prize is high enough to compensate for the research costs under the worst case scenario where the type is λ_L: $\frac{P}{2} \geq \frac{c}{2\lambda_L}$. This logic leads to the results of Proposition 2.

Proposition 4: If $F = 0$ the optimal mechanism is such that:

(a) Types $\lambda \in [\lambda_L, 2\lambda_L]$ pool on one contract that pays a prize $P = \frac{c}{\lambda_L}$ and they stop their research effort after observing the success of their competitor.

(b) Types $\lambda \geq 2\lambda_L$ obtain prize $P(\lambda) = \frac{2c}{\lambda}$ that leads the second innovator to pursue research even after observing the success of his competitor.

Proposition 4 shows that perfect compensation of expected research costs (as in Proposition 1) is not feasible anymore when the fixed cost is small. Indeed, the ability to sort between types rests on the threat that the competitor may succeed within the innovation window. However, given the result of lemma 5, encouraging the competitor to continue research requires paying a higher prize. The designer will therefore choose either not to sort between types (case a) or pay the higher prize that ensures that the second innovator keeps searching after success of the first (case b). In both cases innovators enjoy strictly positive expected profits at the start of the process.

5. General welfare function

In the previous sections, to focus on the result that perfect compensation is possible, we considered a simple objective function for the designer: minimize the prize awarded under the constraint that both innovators invest in research. We argued it appears to be a reasonable objective for important inventions. In this section we consider the general objective, introduced in section 2:

$$max \quad \underbrace{E[d]\ S}_{discounted\ social\ value\ of\ invention} \quad - \quad \underbrace{\alpha\ [2E[C]+2F]}_{expected\ cost\ for\ innovators} \quad - \quad \underbrace{\nu\ E[d]\ E[P]}_{expected\ cost\ of\ raising\ prize}$$

We focus on the case where the social surplus S from the invention is sufficiently large so that both innovators should conduct the research. We can present a simplified expression for the designer's objective:

Lemma 6: The objective of the designer, for a given speed of research λ, is to maximize:

$$\tfrac{2\lambda}{2\lambda+r}[S-\nu P]-\alpha 2\tfrac{c}{\lambda+r}[1-\tfrac{\lambda}{2\lambda+r}e^{-(\lambda+r)T}]-2\alpha F$$

From the result of Lemma 6, we see that the designer will choose menus of prizes (P,T) to minimize:

$$E_\lambda\left[\tfrac{2\lambda}{2\lambda+r}\nu P-\alpha 2\tfrac{c}{\lambda+r}e^{-(\lambda+r)T}\right]$$

since the other parts of the objective function do not depend on P or T. The objective of the designer is therefore to minimize a weighted function of the prize and of the costs of innovators.

Proposition 5: If $\frac{\alpha}{\nu}$ is small enough, the designer offers menus such that certain types choose strictly positive innovation window $T>0$ in equilibrium.

On the one hand, as was shown in the previous sections, including a positive innovation window has value since it can allow for revelation of information that enables the designer to decrease the prize awarded. On the other hand, it has a cost since the second innovator continues his research efforts until the expiration of the window. The benefits are increasing in ν, the social cost of funds, and the costs increasing in α, the weight applied to the innovators' welfare.

The case $\alpha=0$ was considered in the previous sections and we showed that the innovation window was used in the optimal menu. In the case $\nu=0$, the only concern of the designer is to minimize the cost for innovators and the designer will therefore not include an innovation window. Therefore, there exists an intermediate value of the ratio $\frac{\alpha}{\nu}$ such that for smaller values of the ratio, the designer offers menus that allow for at least partial sorting of types.

We note that even in cases where the innovation window is used, the tradeoff between minimizing the prize and controlling the cost for innovators means that the designer does not necessarily try to achieve perfect compensation. It might be in that case beneficial to pay higher prizes to limit the costs incurred by innovators.

6. Related literature

In this paper we present a mechanism, based on competition by second inventors, that forces innovators to reveal their private information about the difficulty of the research. In this section we describe related literature that (1) examines other means of encouraging revelation (2) adopts a mechanism design approach to the formulation of policies to encourage innovation (3) studies the treatment of independent inventors in the case of patents.

As we pointed out in the introduction, there are alternative methods to encourage innovators to reveal their information before they engage in research. Scotchmer (1999) shows, based on Cremer McLean (1988), that if the information of the different firms is sufficiently correlated, penalties for reporting a signal distant from the signal of the other firms, can force participants to report truthfully. Another alternative is to initially run an auction where firms bid for the minimum prize for which they are ready to conduct the research (argument related to Kremer (1998)). Both these mechanisms would elicit truthful revelation.

Our paper can be viewed as providing an alternative approach that appears practical. Note that the mechanisms previously mentioned can face practical hurdles. In particular, they require having a well identified start date where the auction would be run or the penalties imposed. All interested innovators would need to participate at this stage. On the contrary, the mechanism we propose in this paper appears to be more flexible: innovators can join later in the process and their participation guarantees revelation at invention date. Furthermore, in the case of the auction, after the initial revelation, only one innovator conducts the research. As pointed out in the introduction, this can be problematic for two reasons: (1) moral hazard issues (2) the fact that simultaneous research by several innovators might be socially desirable.

There is a large literature taking a mechanism design approach in settings where the designer wants to encourage research. An essential feature of our mechanism, compared to most papers, is that it uses ex post information at invention date. The mechanism considered in Gandal and Scotchmer (1993) shares this feature. The authors consider the problem of revelation of information about abilities inside a joint venture. The optimal mechanism defines payments from the loser to the winner at success date and thus relies on ex post information. The focus of their paper is however very different as they determine how to implement a mechanism that leads to investments by the least cost firm and sharing of information about the value of the innovation. Furthermore, they allow for payments between participants but with no transfers to the designer.

Several papers have formally defined the organization of the patent system as a mechanism design problem. Scotchmer (1999) and Cornelli and Schankerman (1999) show that renewal fees can be used to sort between ideas of different value. Indeed, the higher value ideas are renewed the longest. Hopenhayn and Mitchell (2001) show that menus trading off length against breadth can often be more efficient in sorting between different types than using renewal fees.

In all the papers previously mentioned, the incentive compatibility and the individual rationality constraints are defined at the same date and therefore depend on the same profit functions.[19] One of the important differences in the mechanism we examine is that the incentive compatibility is defined at the invention date whereas the individual rationality constraint is defined at the start of the research process. This distinction is essential since we examine an innovation race with multiple prizes. At the invention date, the first inventor knows he succeeded before his competitor whereas ex-ante he is unsure whether he will finish first or second. This distinction was not relevant in these other papers where the problem was the attribution of a patent to a single firm of uncertain type.

Our mechanism relies on the competition from a second independent inventor. There has recently been a renewed interest in the treatment of independent inventions in the context of intellectual property (Maurer and Scotchmer (2002), Shapiro (2006), Denicolo and Franzoni (2007), Kultti and Takalo (2008), Henry (2010)). In Henry (2010) we show that letting second inventors share the patent if they succeed sufficiently quickly after the first inventor, can increase welfare. We suggest at the end of the paper that menus trading off length or breadth against invention window could be useful. In the current paper, we show, for the case of of research prizes, the much stronger result that trading off innovation window against value of the prize, can tailor the prize perfectly to the expected research cost at the start of the process.

7. Discussion

We conclude by discussing more specifically some implementation issues in the case of vaccines for diseases affecting mostly developing countries. The initial idea was that participating firms should be promised that a fixed number of doses would be purchased at a

[19]For instance in Scotchmer (1999) the private information is the cost c and the value v of the innovation, yielding instantaneous profits $\pi(c, v)$ (research does not involve a lag, it succeeds immediately). Both constraints are defined on that same profit function.

fixed price in case of success. In exchange, the participating firms would commit to sell extra doses at a low price. This is referred to as the Advanced Market Commitment (AMC).

As described in the introduction, a trial of the idea is being conducted for pneumococcal diseases.[20] Whereas the initial suggestion, was effectively a prize (fixed quantity at a fixed price), the current implementation does not commit to a quantity, but promises a co-payment for all the units sold on the market up to a certain quantity. There is therefore no minimum guarantee.

Nevertheless, the problem studied in this paper is still very relevant. The co-payment cannot be too high compared to the research investment if donors are to be convinced to provide funding for other diseases. A version of our mechanism could be implemented: the first inventor would trade-off the level of the co-payment against the risk of a large window and having to share his allocated quantity with a second inventor.

We acknowledge that the design of the pneumococcal AMC, based on co-payments, was intended to let the market choose the most appropriate vaccine among those proposed by the different inventors and that our mechanism gives an advantage to the first inventor. It could be interesting, in further work, to examine in more detail the balance between these two objectives: minimize the reward versus let the market select the most appropriate products.

Berndt and al. (2009), in the case of vaccines, argue that firms need strictly positive profits, comparable to what they can obtain in the developed world for other drugs, in order to invest in research. We note that this is not incompatible with our mechanism. Our result can be reexpressed as follows: regardless of the difficulty of the research, all innovators can be granted the same expected amount at date zero. This fixed amount does not however need to be zero and can be tailored following Berndt's recommendation. The concern expressed in Berndt and al. (2009) does not however remove the need to avoid granting excessive rewards in the case of an easy research process.

One of the practical concerns in implementing this mechanism is to avoid the risk of copying during the length of the innovation window T. We note however that the successful product would not be immediately available. The designer would need to first check that it indeed meets the requirements and do some clinical trials to test for potential side effects. The risk of reverse engineering from a commercialized product would thus not exist for a relatively short innovation window. The designer would thus mostly need ways to protect against illegal copying. Note that such illegal copying could be very damaging for the public image of a firm, such as a pharmaceutical firm found stealing a competitor's discovery.

Another potential concern is the possibility of collusion. This would be particularly

[20]Note that the research on these vaccines is at a late stage, as opposed to the case of vaccines for malaria. The goal of the commitment is to encourage investments in development and production.

relevant in the case of repeated use of the mechanism for different diseases.[21] Examining more in depth this issue could make for an interesting extension. We can also mention that, as in the case of copying, the risk for the public image of firms found colluding, might be too great to justify the reward.

Finally, we point out that Kremer (2001b) also mentions the question of adapting the reward to the research cost. He suggests that the prize should be increased through time. One of the concerns is however that the innovators might withhold their discovery after succeeding in the hope of getting a higher prize. The growth rate would therefore need to be smaller than the interest rate. The comparison between this solution and the mechanism we introduce could be the object of interesting future work. Given our previous remarks, this comparison would presumably depend on functional assumptions on research costs and would therefore become an empirical issue.

It appears that the main practical implementation hurdles can be overcome and that the mechanism, based on the innovation window, could be fairly easily introduced. We show that it allows the designer to minimize the prize paid while still encouraging research by several participants. Given the importance of obtaining vaccines for these neglected disease, such a variation that can increase the political attractiveness of advance market mechanisms, should be considered seriously.

Appendix

Lemma 1: The reward for the winner of the race is:

$$R_W = P - \frac{P}{2}\int_0^T \lambda e^{-\lambda t} e^{-rt}\, dt \quad = P - \frac{\lambda}{\lambda + r}\frac{P}{2}(1 - e^{-(\lambda+r)T})$$

Lemma 2: (a) we calculate the expected profits of an innovator at date zero who knows that both he and his competitor will choose menu (T, P) at invention date and that they will both continue investing after observing the success of their competitor (we show in result (b) that this is indeed the chosen behavior under Assumption 1).

The expression for expected profits is given in the main text:

$$E\Pi_0(\lambda) = \frac{\lambda}{2\lambda + r}(R_W + R_L) - F - E[C]$$

We have $R_W = P - \frac{\lambda}{\lambda+r}\frac{P}{2}(1 - e^{-(\lambda+r)T})$and $R_L = \frac{\lambda}{\lambda+r}\frac{P}{2}(1 - e^{-(\lambda+r)T})$.

[21] In the case of a single use of the mechanism it is hard to see how collusion can be implemented without side payments, that can make detection easier.

When research is conducted from date zero to date t, the flow cost incurred is:

$$\int_0^t ce^{-rx}dx = \frac{c}{r}[1 - e^{-rt}]$$

Therefore the expected flow cost at the start of the process is:

$$\begin{aligned} E[C] = & \int_0^{+\infty} \int_t^{+\infty} \frac{c}{r}[1 - e^{-rt}]\ \lambda e^{-\lambda t} \lambda e^{-\lambda t_2}\ dt\ dt_2 \\ & + \int_0^{+\infty} \int_t^{t+T} \frac{c}{r}[1 - e^{-rt_2}]\ \lambda e^{-\lambda t} \lambda e^{-\lambda t_2}\ dt\ dt_2 \\ & + \int_0^{+\infty} \int_{t+T}^{+\infty} \frac{c}{r}[1 - e^{-r(t+T)}]\ \lambda e^{-\lambda t} \lambda e^{-\lambda t_2}\ dt\ dt_2 \end{aligned}$$

The first term corresponds to the case where the innovator under consideration is the first to invent. The second case is one where the competitor invents first but the innovator under consideration invents soon after. Finally the last case is one where the inventor under consideration is too late (in this case he stops research at $t + T$).

Calculations of this integral yields:

$$E[C] = \frac{c}{\lambda + r}[1 - \frac{\lambda}{2\lambda + r} e^{-(\lambda+r)T}]$$

and

$$E\Pi_0(\lambda) = \frac{\lambda}{2\lambda + r}P - F - \frac{c}{\lambda + r}[1 - \frac{\lambda}{2\lambda + r} e^{-(\lambda+r)T}]$$

(b) The expected profits of an innovator at date t when his competitor has just succeeded and chosen a menu (T, P):

$$\begin{aligned} \Pi_2 = R_L - \int_0^T \frac{c}{r}[1 - e^{-rt}]\lambda e^{-\lambda t} dt - \int_T^{+\infty} \frac{c}{r}[1 - e^{-rT}]\lambda e^{-\lambda t} dt \\ = R_L - \frac{c}{\lambda + r}[1 - e^{-(\lambda+r)T}] \\ = [\frac{P}{2} - \frac{c}{\lambda}]\frac{\lambda}{\lambda + r}[1 - e^{-(\lambda+r)T}] \end{aligned}$$

So if $\frac{P}{2} \geq \frac{c}{\lambda}$ the second innovator keeps investing in research until the expiration of the invention window. We find that for all values of λ,

$$E\Pi_0(\lambda) \geq 0 \Leftrightarrow \frac{\lambda}{2\lambda + r} P \geq F + \frac{c}{\lambda + r}[1 - \frac{\lambda}{2\lambda + r} e^{-(\lambda+r)T}]$$
$$\Rightarrow \frac{\lambda}{2\lambda + r} P \geq F + \frac{c}{\lambda + r}[1 - \frac{\lambda}{2\lambda + r}]$$

According to assumption 1: $F > \frac{c}{2\lambda + r}$, so:

$$E\Pi_0(\lambda) \geq 0 \Rightarrow \frac{\lambda}{2\lambda + r} P \geq \frac{c}{2\lambda + r} + \frac{c}{\lambda + r}[1 - \frac{\lambda}{2\lambda + r}]$$
$$\Rightarrow \frac{P}{2} \geq \frac{c}{\lambda}$$

We have therefore shown result (b): if the prize is high enough to encourage initial investments in research ($E\Pi_0(\lambda) > 0$) then it is high enough to encourage an innovator to keep searching even after observing the success of his competitor ($\Pi_2 > 0$).

Proposition 1: The goal of this proof is to show that a menu of contracts that satisfy both (IR) and (IC) constraints can also achieve perfect compensation.

First we simplify the incentive compatibility constraint. According to lemma 1, an innovator of type λ who has succeeded and chooses contract $(T(\lambda'), P(\lambda'))$ can expect a reward: $R_W(\lambda, \lambda') = \frac{1}{2(\lambda+r)}\ P(\lambda')\ [\lambda + 2r + \lambda e^{-(\lambda+r)T(\lambda')}]$

An incentive compatible contract will satisfy: (a) $\frac{\partial R_W}{\partial \lambda'}(\lambda, \lambda) = 0$ and (b) $\frac{\partial^2 R_W}{\partial (\lambda')^2}(\lambda, \lambda) \leq 0$

Condition (a) (the first order condition) can be reexpressed as:

$$\frac{\partial P}{\partial \lambda}\ (\lambda + 2r + \lambda e^{-(\lambda+r)T})\ -\ \frac{\partial T}{\partial \lambda}\ \lambda(\lambda + r) P e^{-(\lambda+r)T}\ =\ 0 \tag{1}$$

Note that in an optimal mechanism, if an inventor obtains a higher prize, he has to accept in exchange a longer innovation window (i.e sign($\frac{\partial P}{\partial \lambda}$) = sign ($\frac{\partial T}{\partial \lambda}$)).

We now study condition (b), the second order condition

$$\frac{\partial^2 \pi}{\partial \lambda^2}(\lambda, \lambda) = \frac{\partial^2 P}{\partial \lambda^2}[\lambda + 2r + \lambda e^{-(\lambda+r)T}]\ -\ 2\lambda(\lambda + r)\frac{\partial P}{\partial \lambda}\frac{\partial T}{\partial \lambda} e^{-(\lambda+r)T}$$
$$-\lambda(\lambda + r)\frac{\partial^2 T}{\partial \lambda^2} P e^{-(\lambda+r)T}\ +\ (\lambda + r)^2 \lambda (\frac{\partial T}{\partial \lambda})^2 P e^{-(\lambda+r)T}$$

We follow the classic procedure to simplify this condition: first we take derivatives of condition (1) with respect to λ and replace in the previous expression. Condition (b) becomes:

$$-\frac{\partial P}{\partial \lambda}\left[1+e^{-(\lambda+r)T}-T\lambda e^{-(\lambda+r)T}\right] \;+\; \frac{\partial T}{\partial \lambda}P\left[(2\lambda+r)e^{-(\lambda+r)T}-T\lambda(\lambda+r)e^{-(\lambda+r)T}\right]\leq 0$$

From condition (1) we use the fact that:

$$\frac{\partial P}{\partial \lambda} \;=\; \frac{\partial T}{\partial \lambda}\;P\;\frac{\lambda(\lambda+r)e^{-(\lambda+r)T}}{\lambda+2r+\lambda e^{-(\lambda+r)T}}$$

and replace in the previous expression to obtain the following condition:

$$\frac{\partial T}{\partial \lambda}\;\left[(2\lambda+r)(\lambda+2r)\;-\;\lambda(\lambda+r)\;+\;\lambda^2 e^{-(\lambda+r)T}\;-\;\lambda T(\lambda+r)(\lambda+2r)\right]\;\leq\;0 \qquad (2)$$

We want to show that there exists an incentive compatible contract such that innovators have, regardless of their type, zero expected profits at the start of process i.e, $\forall\lambda$, $\frac{\lambda}{2\lambda+r}P-F-\frac{c}{\lambda+r}[1-\frac{\lambda}{2\lambda+r}e^{-(\lambda+r)T}]=0$.

We assume that this condition is satisfied and take the derivative:

$$\frac{\partial P}{\partial \lambda} \;=\; -\frac{r}{\lambda^2}F \;-\; c\frac{2\lambda^2+2\lambda r+r^2}{\lambda^2(\lambda+r)^2} \;+\; \frac{c}{(\lambda+r)^2}e^{-(\lambda+r)T} \;+\; \left[T+(\lambda+r)\frac{\partial T}{\partial \lambda}\right]\frac{c}{\lambda+r}e^{-(\lambda+r)T} \quad (3)$$

For an incentive compatible mechanism, we have shown that condition (1) needs to be satisfied. Condition (1) implies that:

$$\frac{\partial P}{\partial \lambda} \;=\; \frac{\partial T}{\partial \lambda}\;P\;\frac{\lambda(\lambda+r)e^{-(\lambda+r)T}}{\lambda+2r+\lambda e^{-(\lambda+r)T}}$$

We use the fact that $\frac{\lambda}{2\lambda+r}P-F-\frac{c}{\lambda+r}[1-\frac{\lambda}{2\lambda+r}e^{-(\lambda+r)T}]=0$ to obtain

$$\frac{\partial P}{\partial \lambda} \;=\; \frac{\partial T}{\partial \lambda}\;\left[\frac{\lambda(\lambda+r)e^{-(\lambda+r)T}}{\lambda+2r+\lambda e^{-(\lambda+r)T}}\right]\;\left[\frac{2\lambda+r}{\lambda}F+\frac{c}{\lambda+r}(\frac{2\lambda+r}{\lambda}-e^{-(\lambda+r)T})\right]$$

Using this expression we can simplify condition (3)

$$\begin{aligned}
&\frac{\partial T}{\partial \lambda}\;\left[\frac{1}{\lambda+2r+\lambda e^{-(\lambda+r)T}}\right]\;\left[(\lambda+r)(2\lambda+r)e^{-(\lambda+r)T}F+e^{-(\lambda+r)T}c(\lambda-r)-2\lambda c e^{-2(\lambda+r)T}\right]\\
&\qquad\qquad = \;-\frac{r}{\lambda^2}F \;-\; c\frac{2\lambda^2+2r\lambda+r^2}{\lambda^2(\lambda+r)^2} \;+\; \frac{c}{(\lambda+r)^2}e^{-(\lambda+r)T} \;+\; \frac{Tc}{\lambda+r}e^{-(\lambda+r)T}
\end{aligned}$$

We first examine the sign of the left hand side of this expression. The left hand side is a positive constant multiplied by:

$$(\lambda+r)(2\lambda+r)F \ + \ c(\lambda-r) \ - \ 2\lambda c \ = \ (\lambda+r)(2\lambda+r)F \ - \ c(\lambda+r)$$

Because of assumption 1 ($F > \frac{c}{2\lambda+r}$), this expression is positive. We now sign the right hand side of the equation. The right hand side is of the same sign as:

$$L = -(\lambda+r)^2 F \ - \ c(2\lambda^2+2r\lambda+r^2) \ + \ c\lambda^2 e^{-(\lambda+r)T} \ + \ Tc(\lambda+r)\lambda^2 e^{-(\lambda+r)T}$$

We have

$$\begin{aligned} L \ \leq \ & -c(2\lambda^2+2r\lambda+r^2) \ + \ c\lambda^2 e^{-(\lambda+r)T} \ + \ Tc(\lambda+r)\lambda^2 e^{-(\lambda+r)T} \\ & \leq \ c\lambda^2 \ \left[-2+e^{-(\lambda+r)T}+T(\lambda+r)e^{-(\lambda+r)T}\right] \end{aligned}$$

We have, for $x \geq 0$, $e^{-x}+xe^{-x} \leq 1$ and therefore the right hand side is negative.

We therefore find that $\frac{\partial T}{\partial \lambda} \leq 0$.

We have shown that an incentive compatible mechanism satisfies condition (2). Since $\frac{\partial T}{\partial \lambda} \leq 0$, this condition implies:

$$(2\lambda+r)(\lambda+2r) \ - \ \lambda(\lambda+r) \ + \ \lambda^2 e^{-(\lambda+r)T} \ - \ \lambda T(\lambda+r)(\lambda+2r) \ \geq \ 0 \tag{4}$$

If the interval $[\lambda_L, \lambda_H]$ is not too large, constraint (4) can always be satisfied. We have therefore shown that for small intervals of possible values for λ, there exists a set of incentive compatible contracts such that all types have zero expected profits at date zero. This proves the main result of Proposition 1.

Lemma 3: the reward for the winner of the race is given by

$$R_W = \sum_{j=0}^{n-1} C_{n-1}^{j} \ q^j (1-q)^{(n-1)-j} \ \frac{P}{j+1}$$

with $q = 1 - e^{-\lambda T}$. We have $\frac{1}{j+1} C_{n-1}^{j} = \frac{1}{n} C_n^{j+1}$ and thus:

$$
\begin{aligned}
R_W &= \frac{P}{n} \sum_{j=0}^{n-1} C_n^{j+1} \ q^j (1-q)^{(n-1)-j} \\
&= \frac{P}{qn} \left[\sum_{k=0}^{n} C_n^k \ q^k (1-q)^{n-k} \ - (1-q)^n \right] \\
&= \frac{P}{qn} \left[1 \ - (1-q)^n \right] = \frac{P}{qn} \left[1 - e^{-\lambda n T} \right]
\end{aligned}
$$

Lemma 4: (a) we calculate the expected profits of an innovator at date zero who knows that both he and his competitors will choose menu (T, P) at invention date and that they will all continue investing after observing the success of one of their competitors (we show in result (b) that this is indeed the chosen behavior under Assumption 2).

The expected profits at date zero are:

$$
E\Pi_0(\lambda) = \frac{1}{n} R_W + \frac{n-1}{n} R_L - F - E[C]
$$

We have $R_W = \frac{P}{n}(1 + e^{-\lambda n T})$. We first derive the expression for R_L:

$$
R_L = q \sum_{j=0}^{n-2} \ C_{n-2}^j \ q^j (1-q)^{(n-2)-j} \ \frac{P}{j+2}
$$

We have $C_{n-2}^j \ \frac{1}{j+2} = \frac{1}{n(n-1)} \ C_n^{j+2} \ (j+1)$. Thus

$$
R_L = \frac{Pq}{n(n-1)} \left[\sum_{j=0}^{n-2} (j+2) \ C_n^{j+2} \ q^j (1-q)^{(n-2)-j} \ - \sum_{j=0}^{n-2} \ C_n^{j+2} \ q^j (1-q)^{(n-2)-j} \right]
$$

We simplify this expression

$$
\sum_{j=0}^{n-2} (j+2) C_n^{j+2} \ q^j (1-q)^{(n-2)-j} = \frac{1}{q^2} \sum_{k=2}^{n} k C_n^k \ q^k (1-q)^{n-k} = \frac{1}{q^2} \left[nq - nq(1-q)^{n-1} \right]
$$

Similarly

$$
\sum_{j=0}^{n-2} C_n^{j+2} \ q^j (1-q)^{(n-2)-j} = \frac{1}{q^2} \sum_{k=2}^{n} C_n^k \ q^k (1-q)^{n-k} = \frac{1}{q^2} \left[1 - nq(1-q)^{n-1} - (1-q)^n \right]
$$

Overall we find

$$R_L = P\frac{1}{qn(n-1)}[nq - 1 + e^{-\lambda nT}]$$

and

$$\frac{1}{n}R_W + \frac{n-1}{n}R_L = \frac{P}{n}$$

We now calculate the expected cost. We use the fact that the minimum of $n-1$ exponential variables of parameter λ is an exponential variable of parameter $(n-1)\lambda$. Given that property, the calculation is very similar to the calculation of the expected cost in the case of two imitators:

$$\begin{aligned} E[C] = & \int_0^{+\infty}\int_t^{+\infty} ct\ \lambda e^{-\lambda t}(n-1)\lambda e^{-(n-1)\lambda t_2}\ dt\ dt_2 \\ & + \int_0^{+\infty}\int_t^{t+T} ct_2\ (n-1)\lambda e^{-(n-1)\lambda t}\lambda e^{-\lambda t_2}\ dt\ dt_2 \\ & + \int_0^{+\infty}\int_{t+T}^{+\infty} c(t+T)\ (n-1)\lambda e^{-(n-1)\lambda t}\lambda e^{-\lambda t_2}\ dt\ dt_2 \end{aligned}$$

We use repeatedly the following property:

$$\int_0^{+\infty} ct\ \lambda e^{-n\lambda t}\ dt\ = \frac{c}{\lambda n^2}$$

to obtain our final result:

$$E[C] = \frac{c}{\lambda}[1 - \frac{n-1}{n}e^{-\lambda T}]$$

and

$$E\Pi_0(\lambda) = \frac{P}{n} - \frac{c}{\lambda}[1 - \frac{n-1}{n}e^{-\lambda T}] - F$$

(b) Under Assumption 2, $F > \frac{c}{\lambda}$. Therefore $E\Pi_0(\lambda) \geq 0$ implies that $\frac{P}{n} \geq \frac{c}{\lambda}$. We show that this will imply that innovators keep investing after the success of one of them. The expected profits of such an innovator are:

$$\Pi_2 = R_L - \int_0^T ct\lambda e^{-\lambda t} - \int_T^{+\infty} cT\lambda e^{-\lambda t} = P\frac{1}{qn(n-1)}[nq - 1 + e^{-\lambda nT}] - \frac{c}{\lambda}[1 - e^{-\lambda T}]$$

We consider the function $\Pi_2(x) = P\frac{1}{qn(n-1)}[n(1-e^{-x}) - 1 + e^{-nx}] - \frac{c}{\lambda}[1-e^{-x}]$. The derivative is given by $P\frac{1}{q(n-1)}[e^{-x} - e^{-nx}] - \frac{c}{\lambda}e^{-x}$. Given that $e^{-y} > ne^{-ny}$, if $P\frac{1}{q(n-1)} - \frac{c}{\lambda} > \frac{1}{n}P\frac{1}{q(n-1)}$ then $\Pi_2(x)$ is an increasing function of x. It can be shown that the fact that $\frac{P}{n} \geq \frac{c}{\lambda}$ implies that property. Therefore $\Pi_2(x)$ is an increasing function of x. Furthermore $\Pi_2(0) = 0$.

We have therefore shown that under Assumption 2, $E\Pi_0(\lambda) \geq 0$ implies $\Pi_2 \geq 0$

Proposition 2: We follow the lines of the proof of Proposition 1 and generalize the equations to the case of n innovators.

The first order condition of the optimization problem (condition (1) for Prop 1) becomes

$$\frac{\partial P}{\partial \lambda}(1 + e^{-n\lambda T}) - n\lambda\frac{\partial T}{\partial \lambda}Pe^{-n\lambda T} = 0 \tag{5}$$

The second order condition (condition (2) for Prop 1) becomes

$$\frac{\partial T}{\partial \lambda}[1 + e^{-n\lambda T} - n\lambda T)] \leq 0 \tag{6}$$

We want to show that there exists an incentive compatible contract such that $\forall\lambda$, $\frac{P}{n} = \frac{c}{\lambda}[1 - \frac{n-1}{n}e^{-\lambda T}] + F$.

We follow the same procedure as in Proposition 1. We assume that the previous condition is satisfied, take the derivative and simplify the expression using the first order condition (5). As in the proof of Proposition 1 we find that this implies that $\frac{\partial T}{\partial \lambda} \leq 0$.

For an incentive compatible mechanism, condition (6) also needs to be satisfied and is equivalent to

$$1 + e^{-\lambda nT} - n\lambda T \geq 0$$

This condition is satisfied if and only if

$$\lambda T \leq \frac{x^*}{n} \tag{7}$$

where x^* is such that $1 + e^{-x^*} - x^* = 0$

Note that as in Proposition 1, the interval $[\lambda_L, \lambda_H]$ cannot be too large to achieve perfect compensation for all types.

Proposition 3: We follow the lines of the proofs of Proposition 1 and Proposition 2. First we need to calculate expected profits.

Consider a product of type λ. The expected reward of a less efficient innovator at date zero is given by:

$$E[R_0]_{LE} = \frac{1}{\mu+1}R_{W,LE} + \frac{\mu}{\mu+1}R_{L,LE}$$
$$= \frac{1}{\mu+1}\frac{P}{2}(1+e^{-\lambda\mu T}) + \frac{\mu}{\mu+1}\frac{P}{2}(1-e^{-\lambda T})$$
$$= \frac{P}{2}\left[1+\frac{1}{\mu+1}e^{-\lambda\mu T} - \frac{\mu}{\mu+1}e^{-\lambda T}\right]$$

We now calculate the expected cost at date zero of a less efficient innovator

$$E[C]_{LE} = \int_0^{+\infty}\int_t^{+\infty} ct\ \lambda e^{-\lambda t}(\lambda\mu)e^{-\lambda\mu t_2}\ dt\ dt_2$$
$$+ \int_0^{+\infty}\int_t^{t+T} ct_2\ (\lambda\mu)e^{-\lambda\mu t}\lambda e^{-\lambda t_2}\ dt\ dt_2$$
$$+ \int_0^{+\infty}\int_{t+T}^{+\infty} c(t+T)\ (\lambda\mu)e^{-\lambda\mu t}\lambda e^{-\lambda t_2}\ dt\ dt_2$$

Similar calculations as those performed in section 3 lead to:

$$E[C]_{LE} = \frac{c}{\lambda}\left[1-\frac{\mu}{\mu+1}e^{-\lambda T}\right]$$

As indicated in the main text, the incentive compatibility for an efficient type when the product is of type $\lambda\mu$ is the same as the incentive compatibility constraint for a less efficient innovator when the product is of type λ. Furthermore the individual rationality constraints for the more efficient innovators are implied by the constraints for the less efficient ones. The problem is therefore as if the designer was facing only less efficient innovators. We can write the mechanism design problem as:

$$\begin{cases} \text{Min} \quad E_\lambda\ [P(\lambda)] \quad \text{subject to:} \\ \\ \quad P(\lambda)(1+e^{-\lambda\mu T(\lambda)}) \geq P(\lambda')(1+e^{-\lambda\mu T(\lambda')}) \ \ \forall(\lambda,\lambda') \quad (IC) \\ \quad \frac{P(\lambda)}{2}\left[1+\frac{1}{\mu+1}e^{-\lambda\mu T(\lambda)} - \frac{\mu}{\mu+1}e^{-\lambda T(\lambda)}\right] - \frac{c}{\lambda}\left[1-\frac{\mu}{\mu+1}e^{-\lambda T(\lambda)}\right] - F \geq 0 \ \ \forall\lambda \qquad (IR) \end{cases}$$

The first order condition of the optimization problem (condition (1) for Proposition 1) becomes

$$\frac{\partial P}{\partial \lambda}(1+e^{-\mu\lambda T})-\mu\lambda\frac{\partial T}{\partial \lambda}Pe^{-\mu\lambda T}=0$$

The second order condition (condition (2) for Prop 1) becomes

$$\frac{\partial T}{\partial \lambda}[1+e^{-\mu\lambda T}-\mu\lambda T)]\leq 0$$

Finally, the condition for perfect compensation, implies as in the previous propositions that $\frac{\partial T}{\partial \lambda}\leq 0$.

The second order condition can therefore be reexpressed:

$$1+e^{-\mu\lambda T}-\mu\lambda T\geq 0$$

This condition is satisfied if and only if

$$\lambda T\leq \frac{x^*}{\mu}$$

where x^* is such that $1+e^{-x^*}-x^*=0$

Note that as in Proposition 1, the interval $[\lambda_L,\lambda_H]$ cannot be too large to achieve perfect compensation of less efficient innovators for all types of products λ.

Lemma 5: (a) This result was already derived in the proof of lemma 2 (b).

(b) An innovator of type λ finds it profitable to continue research after his competitor has succeeded and before the innovation window expires if $P\geq\frac{2c}{\lambda}$.

In lemma 2 we showed that $E\Pi_0(\lambda)=\frac{P}{2}-\frac{c}{\lambda}+e^{-\lambda T}\frac{c}{2\lambda}$ (for $F=0$). So $P\geq\frac{2c}{\lambda}\Rightarrow E\Pi_0(\lambda)>0$.

(c) The expected profits of innovators of type λ at date zero if both innovators are expected to stop research after observing the success of their competitor are

$$\int_0^{+\infty}\int_t^{+\infty}[P-ct]\lambda e^{-\lambda t}\lambda e^{-\lambda t_2}\ dt\ dt_2\ -\int_0^{+\infty}\int_{t_2}^{+\infty}ct\lambda e^{-\lambda t}\lambda e^{-\lambda t_2}\ dt\ dt_2$$
$$=\frac{P}{2}-\frac{c}{2\lambda}$$

Proposition 4: As explained in the main text, the designer can either:

- not sort between types and guarantee that even the slowest type conducts research by paying a prize $P_0 = \frac{c}{\lambda_L}$

- pay a prize such that innovators continue searching after observing the success of their competitor. In that case $P = \frac{2c}{\lambda}$

For types such that $\frac{2c}{\lambda} > \frac{c}{\lambda_L}$ the designer prefers not to sort. He offers the menu $(T = +\infty, P_0 = \frac{c}{\lambda_L})$. A type $\lambda \in [\lambda_L, 2\lambda_L]$ knows that his competitor (who has the same type) will stop searching after observing his success and he therefore chooses this menu as indicated in result (a).

For types such that $\lambda \geq 2\lambda_L$ the designer sorts between types. We now examine the conditions under which sorting is possible. The program is the same as the one examined in Proposition 1 except that the (IR) constraint becomes $P \geq \frac{2c}{\lambda}$.

Lemma 6: Using calculations for expected research costs from the proof of lemma 2, we can show that the objective of the designer is to maximize:

$$\tfrac{2\lambda}{2\lambda+r}[S-\nu P]-\alpha 2\tfrac{c}{\lambda+r}[1-\tfrac{\lambda}{2\lambda+r}e^{-(\lambda+r)T}]-2\alpha F$$

Proposition 5: We can simplify the objective given in lemma 6, by eliminating the terms that depend neither on P nor on T. We define this simplified objective $W(P,T)$

$$W(P,T)\ =\ E_\lambda\left[\tfrac{2\lambda}{2\lambda+r}\nu P-\alpha 2\tfrac{c}{\lambda+r}e^{-(\lambda+r)T}\right]$$

If the innovation window is not used, no sorting between types is possible and since we assumed that for any speed of research, the designer wants to encourage research by both innovators, she has to offer a prize sufficient for innovators to invest even if the research is the most difficult. We denote this prize $P_{max} = \frac{2\lambda_{min}+r}{\lambda_{min}}F\ + \frac{c}{\lambda_{min}}$. The simplified expression of welfare $W(P_{max},0)$ of the designer with no sorting is therefore:

$$\nu E_\lambda[\tfrac{2\lambda}{2\lambda+r}]P_{max}-\alpha 2E_\lambda[\tfrac{c}{\lambda+r}]$$

Suppose that for a value of the ratio $\frac{\alpha}{\nu} = d_0$, a strictly positive window is used for a range of types A. Then we must have:

$$\nu\ E_{\{\lambda\in A\}}\left[\frac{2\lambda}{2\lambda+r}\nu P_\lambda\right]\ -\ \alpha 2\ E_{\{\lambda\in A\}}\left[\frac{c}{\lambda+r}e^{-(\lambda+r)T(\lambda)}\right]\ >$$
$$\nu\ E_{\{\lambda\in A\}}\left[\frac{2\lambda}{2\lambda+r}\right]P_{max}\ -\ \alpha 2\ E_{\{\lambda\in A\}}\left[\frac{c}{\lambda+r}\right]$$

If such a menu is used for a $\frac{\alpha}{\nu} = d_0$, the condition is easier to satisfy for a smaller value of the ratio $\frac{\alpha}{\nu}$ and the designer therefore uses the innovation window in this case. Given that for $\alpha = 0$, we know that T is used, we have shown the result of Proposition 5.

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