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ABSTRACT

Incentives to Innovate and the Decision to Go Public or Private

We model the impact of public and private ownership structures on firms' incentives to choose innovative projects. Innovation requires the exploration of new ideas with potential advantages but unknown probability of success. We show that it is optimal to go public when firms wish to exploit the current technology and to go private when firms wish to explore new ideas. This result follows from the fact that privately-held firms are less transparent to outside investors than publicly-held firms. In private firms, insiders can time the market by choosing an early exit strategy when they learn bad news. This option makes insiders more tolerant of failures and thus more inclined to choose innovative projects. In public firms, an early exit strategy is less valuable because there is less information asymmetry about cash flows. In such firms, prices of publicly-traded securities react quickly to good news, providing insiders with incentives to choose conventional but safer projects in order to cash in early when good news arrive. Extensions to the model allow us to incorporate other drivers of the decision to go public or private, such as liquidity and cost of capital. Our model rationalizes recent evidence linking private equity to innovation and creative destruction and also generates new predictions concerning the determinants of going public and going private decisions.

JEL Classification: G2, G3 and O3

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1. Introduction

There is evidence that private firms are more innovative than publicly-traded firms. Using patent citation data, Lerner, Sorensen and Strömberg (2008) find that firms invest in more influential innovations after being acquired by private equity funds. Popov and Roosenboom (2009) find that private equity investment increases the number of patents in a panel sample of firms from 21 European countries. Private equity is also associated with corporate restructuring, changes in strategic direction, and creative destruction. For example, Davis et al. (2009) find that firms acquired by private equity funds fire workers and shut down existing establishments, but also engage more in mergers and acquisitions and create jobs in new establishments. They conclude that private equity is a catalyst for creative destruction.¹

We introduce a model in which the choice of ownership structure of the firm—either public or private—affects managers’ incentives to innovate. Our key contribution is to show that private ownership creates incentives for innovation, while public ownership creates incentives against innovation. Because we allow for an endogenous choice of ownership structure, the model also provides a novel explanation for the decision to go public or private. We find that the decision to go public or private is affected by the profitability of innovative versus conventional projects.

In our model, a risk-neutral insider chooses between a conventional project and an innovative project (*exploitation of existing ideas or exploration of new ideas*; March, 1991). Both projects generate cash flow streams in two consecutive periods. The insider has an option to liquidate his stake early (i.e. before final cash flows are realized) by trading on the basis of his private information. We first show that, under private ownership, if the insider can time the market by choosing an early exit strategy after bad news, the insider becomes more tolerant of early failures and thus more inclined to choose the innovative project. In the model, this *tolerance-for-failure effect* is the key driver of innovation in private companies.²

¹See Strömberg (2009) for a review of the literature on private equity and innovation.

²Evidence consistent with the tolerance-for-failure effect is provided by Acharya and Subramanian (2009), who show empirically that innovation is more prevalent in countries with debtor-friendly bankruptcy codes,

Exiting early constitutes a real option to the insider. This option is valuable only when the insider's actions do not fully reveal his private information. An early exit does not fully reveal the insider's private information for two reasons. First, cash flows of private firms may not be fully observable, thus outside investors may be poorly informed about interim cash flows generated by projects. Second, the insider may suffer a liquidity shock, thus outside investors cannot know whether trading is motivated by information or liquidity.

Under public ownership, cash flows are observable and thus an early exit after bad news is not profitable. Therefore, there is no tolerance for failures in public companies. Furthermore, market prices of public securities react quickly to good news. This is known to create incentives for short-termist behavior (e.g. Stein, 1989). A rational pressure towards quick results arises in our model because good news are quickly incorporated into market prices. Thus, the insider may prefer the conventional project because it has a higher expected probability of an early success. We show that the unique equilibrium under public ownership involves choosing the conventional project with some positive probability, even when innovation is ex ante efficient.

Our model shows that incentives in public firms are biased towards conventional projects, while incentives in private firms are biased towards innovative projects. Consequently, the optimal structure of ownership—public or private—changes with the firm's life cycle depending on whether exploitation of existing ideas or exploration of new ideas is optimal.

We interpret our model as a theory of the evolution of ownership structures. It is usually believed that exploration is very important early in a firm's life. Our model thus predicts that it is optimal to start private to maximize incentives to explore. Our model would also view going private decisions as complements to risky restructurings. Every time a firm needs to reinvent itself, it makes sense to do it out of the public eye. Major company restructurings involving radical changes in strategy are departures from the conventional, and thus more

and by Acharya, Baghai and Subramanian (2009), who show that more stringent labor laws lead to more innovation inside firms.

properly motivated under private equity.³

Our model sheds light on a number of controversial issues raised in the empirical literature on the real effects of venture capital and buyout investments. Kaplan and Strömberg (2008) review this evidence and conclude that private equity investment creates value not only because of tax benefits and the exploitation of mispricings in debt and equity markets, but also by affecting corporate behavior, such as operations and investments. But the evidence on the role of private equity on innovation remains controversial. For example, while there is some agreement that venture capital investment positively affects patenting activity,⁴ others argue that this does not imply increases in productivity (Ueda and Hirukawa, 2008). Furthermore, there is the question of whether “innovation follows VC investment” or “VC investment follows innovation” (Mollica and Zingales, 2007; Hirukawa and Ueda, 2008).

Our model’s predictions can reconcile many of such disparate results. In particular, our model suggests that, when studying the relation between private equity and innovation, it is important to distinguish between venture capital and public-to-private buyouts. Our model’s main empirical implications are as follow.

- Firms become more innovative after public-to-private transitions, as the evidence suggests (Lerner, Strömberg, Sorensen, 2008; Davis et al., 2009). But, because the ownership mode is endogenous in the model, it is also true that more innovative industries attract more private equity investment.
- There can be too much innovation in private companies: innovation does not necessarily increase productivity or profitability in privately-held companies (Ueda and Hirukawa, 2008).
- VC investment does not necessarily lead to an increase in innovation activities, because

³For an alternative incentive-based theory of the life cycle of speculative industries, see Biais, Rochet, and Woolley (2009).

⁴Lerner and Kortum (2000) provide evidence that venture capital backing is positively related to patent count. In their data, VC backing has a much larger impact on patents than corporate R&D. Ueda and Hirukawa (2008) confirm such findings in an updated sample.

venture capitalists usually invest in companies that are already private.

- On the other hand, in our model, public-to-private transitions are necessary for innovation to occur; in this case, innovation follows PE (buyout) investment. Furthermore, if PE firms have unique skills in identifying promising companies, PE-backed private companies are more successful in their innovations than non-PE-backed ones.

Our theory also has implications for future empirical work on private equity and innovation. First, it highlights that controlling for the type of the transition (public-to-private versus private-to-private) is at least as important as controlling for the type of investment (buyout versus venture capital).⁵ Second, although the discussion in the empirical literature usually focuses on sorting out the direction of causality, our theory suggests a unifying explanation for both hypotheses (“innovation follows private equity” and “private equity follows innovation”). It might be more fruitful to pursue empirical strategies that allow for the testing of these two possibilities simultaneously.

There is an emerging theoretical and empirical literature on the role of ownership structures and financing choices on corporate innovation. An early example is Aghion and Tirole (1994); more recent works include Aghion, Van Reenen, and Zingales (2009), Atanassov, Nanda, and Seru (2007), Belenzon, Berkovitz, and Bolton (2009), Bhattacharya and Guriev (2006, 2009), and Fulghieri and Sevilir (2009). These papers focus on related but different questions, such as the impact of capital structure, governance, organization, and ownership concentration on corporate innovation.

Our model is also closely related to two different theoretical literatures: (1) models of interactions between stock prices and incentives in firms and (2) models of the decision to

⁵Strömberg (2007) shows that most of the PE transactions are private-to-private. Boucly, Sraer and Thesmar (2009) provide empirical evidence on the importance of distinguishing between different types of transitions. They show that private-to-private LBOs are followed by periods of growth, while no such effect is observed in public-to-private LBOs (but profitability increases in this case). Because in our model firms are not financially constrained, we derive no explicit implications for firm growth. However, if we assume that growth is associated with periods of exploitation of existing technologies, our model is compatible with private-to-public transitions being followed by periods of growth while public-to-private transitions being followed by periods of restructuring but no immediate growth.

go public or private.

There is an extensive literature examining the role of stock prices in guiding corporate investment decisions and affecting insiders' incentives more generally. An incomplete list includes Holmström and Tirole (1993), Khanna, Slezak, and Bradley (1994), Dow and Gorton (1997), Faure-Grimaud and Gromb (2004), Almazan, Banerji, and Motta (2008), and Edmans (2009).

Our model is also related to the work of Stein (1989), who develops a model of rational managerial short-termism driven by the stock market. In his model, firms take actions to boost current earnings at the cost of lower future earnings in an attempt to mislead the market. In equilibrium managers are stuck with an inefficient strategy. The same logic is present in our model. If the firm is public, a manager may choose the conventional project even when the innovative project has a higher net present value, because the former has a higher expected probability of generating high earnings in the short run. But our model also shows the other side of the story. If the firm is private and thus free from pressure to boost current earnings, it will put too much emphasis on future cash flows. Without the stock market punishing short-run falls in earnings, managers rationally become biased towards innovative projects, which are risky but very profitable if successful. This bias may lead them to *inefficient long-termism*: innovation may be chosen even when it is inferior to conventional methods. Thus, our model provides a more balanced view of market incentives: while managers of public firms may focus excessively on current earnings, managers of private firms may focus excessively on future earnings. The best structure thus depends on the nature of the projects available to the firm at a particular time.

Our paper is also related to a large literature on the choice between public and private structures, including Boot, Gopalan, and Thakor (2006), Shah and Thakor (1988), Chemmanur and Fulghieri (1999), Zingales (1995), and Pagano and Roel (1998). None of these papers consider incentives for innovation as a determinant of ownership structures.

More closely related to our model is the work of Maksimovic and Pichler (2001). In their

model, firms may choose between a new or an existing technology and then decide whether to finance future rounds of investment with either public or private offerings. Public offerings are assumed to be cheaper, but they reveal information about the industry prospects to potential competitors.⁶ Thus, firms may strategically delay finance or resort to private offerings to prevent entry. Their model is concerned with the effect of technological uncertainty at the industry level on the mode and timing of financing. Our model is concerned with the effect of the financing mode (private or public) on firms' internal incentives to choose between different technologies. Thus, our model allows us to address a different question: Should the decision to go public or private depend on the profitability of new versus old technologies?

The paper is structured as follows. We present the basic model in Section 2, discuss the going public or private decision in Section 3, develop extensions in Section 4, and conclude with a discussion of empirical applications in Section 5. All proofs are in the Appendix.

2. The model

2.1. Setup

A risk-neutral insider initially holds all shares of a firm. The insider has no initial wealth, is protected by limited liability, and has outside utility normalized to zero. We view the insider as a manager-entrepreneur who founded the firm and owns it fully. Because the identity of the manager making the decisions is not important in our model, we assume that the founder stays as manager regardless of how many shares he sells to other investors. The results are identical if the founder is replaced by a newly-hired professional manager.

⁶Spiegel and Tookes (2009) develop and estimate a dynamic oligopoly model that incorporates some of the trade-offs in Maksimovic and Pichler's (2001) model.

2.1.1. Technology

The insider has to decide between two projects, projects 1 and 2, at two consecutive dates, dates 0 and 1. Each project has two possible outcomes: success or failure. Success yields earnings S and failure yields earnings F , $S > F$. We call project 1 *exploitation of existing ideas* and project 2 *exploration of new ideas*. This setup is similar to Manso (2009).

If the insider chooses project 1, the conventional project, there is a probability p of success. The probability p is known to everyone. If the insider chooses project 2, the innovative project, the probability of success is q , which is unknown. It is only possible to learn about q if the insider chooses project 2. We assume that

$$E[q|F] < E[q] < E[q|S]. \quad (1)$$

The expectation of success q increases if project 2 is successful in period 1 and decreases if project 2 fails in period 1.

The insider will only consider choosing the innovative project if it has a chance of improving upon the old method. Thus, we assume also that $E[q|S] > p$ to eliminate the trivial case in which project 1 always strictly dominates project 2. On the other hand, the insider always chooses the innovative project if $E[q]$, the unconditional probability of success before ever trying the project, is higher than p . The interesting case is when $E[q] < p$. To economize on algebra and notation, define δ and θ such that $\delta p = E[q]$ and $\theta p = E[q|S]$. These assumptions imply that $0 < \delta < 1$ and $1 < \theta < 1/p$. To summarize,

$$\delta p = E[q] < p < E[q|S] = \theta p. \quad (2)$$

Equations (1) and (2) encapsulate all characteristics of project 2. From (1), project 2 is exploratory because it is only possible to know more about the new method by trying it out. From (2), project 2 is promising because its probability of success is higher than

the probability of success of project 1 if project 2 is successful in period 1. We can think of radical methods that look unlikely to work but that would greatly improve the current method if they do work. The interpretation of δ and θ is that a method is more radical the smaller δ is and the higher θ is.

Total profits (gross of any initial investment costs) are given by the undiscounted sum of earnings of the two dates, $\pi = x_1 + x_2$, where x_t is equal to F or S . We assume that earnings are only liquid at date 2. That is, earnings x_1 are realized at date 1 but dividends based on x_1 are paid at date 2 (as when sales are on trade credit so that earnings x_1 are simply accounts receivables). More generally, we wish to capture a situation in which it is possible to learn a signal x_1 at date 1 about future profits of the firm, although such cash flows have not yet materialized. We call x_1 earnings in date 1 for simplicity of exposition, but it can also be understood as “a signal in date 1 about earnings in date 2.”

The insider makes an initial investment I , paid in cash, to produce positive earnings by investing in either project. Without this initial investment, all earnings are equal to zero regardless of the project chosen.

The insider may switch from one project to the other after observing x_1 . If the insider initially chooses to exploit the old method, the option to switch has zero value. If the initial choice is to explore the new method, however, to maximize firm value the insider switches to project 1 after observing $x_1 = F$. The option to switch is valuable under exploration. If the new method is tried out but fails, the insider returns to the old method. Figure 1 provides a visual summary of the technology taking into account the option to switch.

To simplify notation, we make $F = 0$ and $S = 1$, without loss of generality. Under exploitation (project 1), the expected market value of the firm (gross of initial investment costs) is

$$v_1 = 2p. \tag{3}$$

We always write the value of the firm gross of initial investment costs, unless we say otherwise. The value of the firm takes into account the two periods of operation.

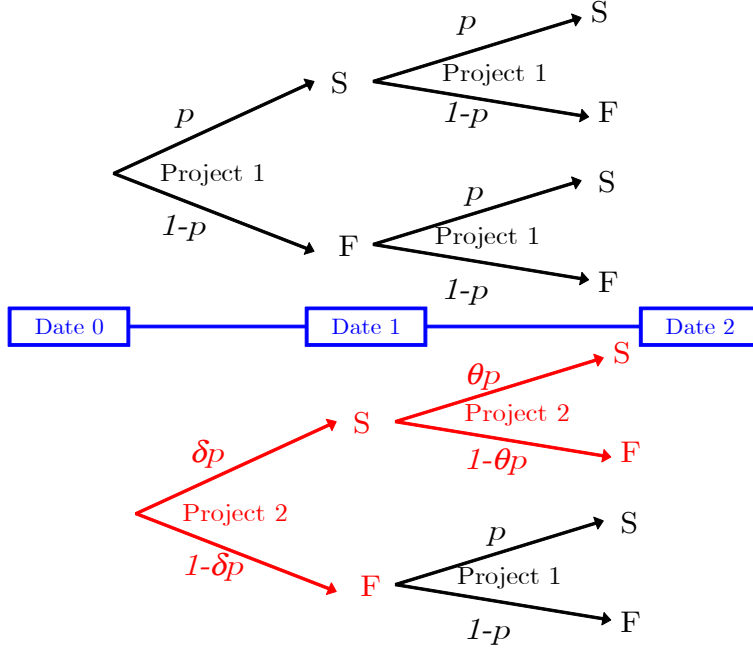


Figure 1. Technology: Earnings and probabilities associated with each initial project choice.

If the insider chooses exploration (project 2), in case of success in date 1, the firm continues to use the new method. In case of failure, the firm returns to the old method (project 1). The expected market value of the firm under exploration is

$$v_2 = p \{1 + \delta [1 + p(\theta - 1)]\}. \quad (4)$$

The innovative project (project 2) is ex ante preferable to the conventional project (project 1) if and only if $v_2 - v_1 \geq 0$. We have

$$v_2 - v_1 > 0 \text{ if and only if } \delta [1 + p(\theta - 1)] > 1. \quad (5)$$

2.1.2. Liquidity and financial market frictions

The key financial market friction in our model is the existence of a demand for liquid assets caused by (unmodeled) borrowing constraints. We model the insider's preference for liquid

assets by assuming that he has a utility function as in Diamond and Dybvig (1983),

$$U(c_1, c_2) = \begin{cases} c_1 & \text{with probability } \mu, \\ c_2 & \text{with probability } 1 - \mu, \end{cases} \quad (6)$$

where c_t is consumption at date t . This reduced-form approach is common in microeconomic models of liquidity shocks (see e.g. Freixas and Rochet, 1997). With probability μ , a liquidity shock forces the insider to consume at date 1. With probability $1 - \mu$, there is no liquidity shock and dividends and consumption are synchronized at date 2. We can think of liquidity shocks as representing different types of consumers. Insiders that do not suffer a liquidity shock are called *late consumers*. Insiders that suffer a liquidity shock are *early consumers*.⁷

For liquidity shocks to have a real impact on decisions, we need to assume that the insider faces borrowing constraints. The assumption of limited liability eliminates uncollateralized borrowing. The assumption of zero initial wealth implies that the insider has no initial collateral. We need to assume further that the insider cannot borrow using the securities issued against the firm's cash flows as collateral.⁸

Liquid securities such as cash can be stored from one period to the following at no cost. There is no discounting nor systematic risk in the economy.

2.1.3. Project financing

The insider must sell securities backed by future cash flows to finance the initial investment I , as the insider has no initial wealth. The insider may sell securities to private or public investors. The initial investment I is observable to all and contractible. Thus, the investment I must occur for sure if the insider sells securities to raise funds for investments.

We initially assume that the only securities available are share contracts. This is for

⁷We interpret the liquidity shock as any reason for the insider to sell other than private information, including portfolio rebalancing, tax considerations and behavioral biases. For evidence of such motives to trade, see Kallunki, Nilsson, and Hellström (2009).

⁸Although we state this as an assumption, it is possible to endogenize borrowing constraints fully by introducing additional moral hazard considerations to the problem.

simplicity of exposition. Capital structure choices are relevant in our model (that is, the model is not in a Modigliani-Miller world), but they do not change the qualitative results about the choice between private and public ownership structures. In Section 4 we discuss the robustness of the results to different contracting assumptions and to the introduction of other securities, such as debt.

2.1.4. Differences between private and public ownership structures

The key results of our model depend only on one difference between private and public ownership: the possibility to observe interim earnings x_1 . Under public ownership, we assume that interim earnings x_1 are observable by everyone. Under private ownership, in contrast, only the insider and the incumbent private investors observe x_1 . Future private investors do not observe x_1 either. These assumptions capture the fact that public companies are more transparent. Public companies are subject, for example, to tighter regulatory disclosure requirements such as quarterly earnings reporting and comprehensive annual reports, to analyst coverage and, perhaps most importantly, to the aggregation of dispersed information into the stock price.

For the sake of realism and to permit the analysis of different trade-offs, we also allow for other differences between the two structures, such as the cost of capital and liquidity costs. These enrich the model but are not necessary for any of the qualitative results linking innovation incentives and going public or private decisions.

We assume that there are transaction costs associated with raising funds for investment through an IPO.⁹ We capture the costs of issuing public equity by a parameter $c_{pub} \in (0, 1)$, such that each dollar sold in public offerings yields only c_{pub} to the firm. A high c_{pub} means a low discount.

⁹Lee et al. (1996) estimate that administrative and underwriting costs are usually about 11% of the IPO proceeds. More importantly, IPO underpricing can create much higher costs, with total costs reaching the 20-30% range (Ritter, 1987). Seasoned Equity Offerings (SEOs) are less costly, but discounts are also common, with a typical negative stock price reaction after announcements of equity offerings of 3% (Asquith and Mullins, 1986), to which direct costs of roughly 7% should be added (Lee et al., 1996).

Raising capital through private equity also involves transaction costs. We denote by $c_{priv} \in (0, 1)$ the discount factor associated with private securities. This parameter is likely to change with changes in the institutional environment and the state of the economy. For example, when interest rates are relatively low, private equity funds can borrow cheaply and thus going private becomes less costly for the firm, as required returns fall. Private equity booms are thus associated with high levels of c_{priv} .

We make no assumptions with respect to the relative cost of public equity capital $c_{priv} - c_{pub}$. Thus, our model allows for situations in which funds for investment are cheaper if financed by public securities ($c_{pub} > c_{priv}$) as well as cases in which being private reduces the cost of capital ($c_{pub} < c_{priv}$).¹⁰

A traditional justification for going public is to create liquidity for insiders' shareholdings (see for example Chemmanur and Fulghieri, 1999, and Ritter and Welch, 2002). For example, a founder may value the option of selling his stake quickly on the market should the need arise. If the firm is privately held, the founder may have to negotiate with a few private investors. Especially in cases in which the founder suffers a liquidity shock and needs to sell quickly, the bargaining power of the founder may be compromised if the firm is private. In contrast, in public markets the founder may be able to sell more easily his own shares through organized markets (provided compliance with insider trading regulations). To capture a potential liquidity advantage of public equity, we assume that each dollar in shares sold by the insider in date 1 (the liquidity shock period) yields only $k \leq 1$ if the company is private. No such discount happens if the firm is public. In most of the analysis that follows, for simplicity we assume no liquidity discount when the insider sells his own shares ($k = 1$). In the robustness section (Section 4), we fully consider the case in which $k < 1$.

¹⁰In Chemmanur and Fulghieri's (1999) model of the decision to go public, the cost of capital in public firms reflects the trade-off between the liquidity of public securities and the information production costs associated with the duplication of monitoring efforts by public investors. Our model can incorporate such effects in reduced form by changing c_{pub} and c_{priv} according to which effect dominates.

2.1.5. The structure of information and timing of events

At date 0, the insider decides to sell a fraction $1 - \alpha$ of the shares to either private investors or public markets. The insider needs to raise at least I in cash to pay for the initial investment cost. After paying I , the insider chooses project 1 or 2. Outsiders cannot observe which project was chosen.

At date 1, the insider observes the first realization of earnings $x_1 \in \{0, 1\}$ and then chooses project 1 or project 2, which again is unobservable by outsiders. The insider then learns his type. If the insider is an early consumer, he sells all shares he owns α regardless of the market valuation of the shares. If the insider is a late consumer, he may sell some of the shares or keep them until date 2. After observing whether the insider places orders to sell or keeps the shares, the market forms a price for the shares.

At date 2, the second-period earnings $x_2 \in \{0, 1\}$ are realized, the shareholders receive dividends $x_1 + x_2$, and the firm is liquidated. The liquidation value is normalized to zero. Figure 2 illustrates the time line.

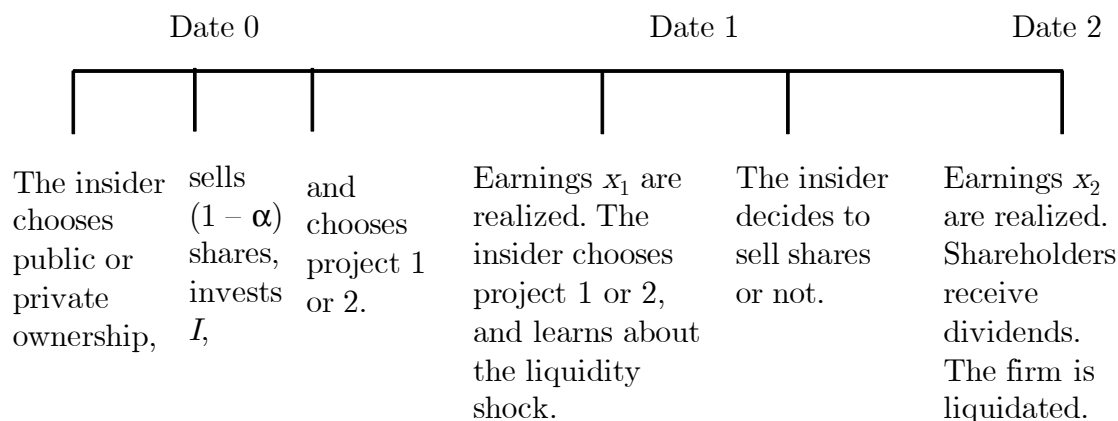


Figure 2. Time line.

2.2. Private ownership

We start the analysis by first considering the case of private ownership; that is, at date 0 the insider has chosen to sell $1 - \alpha$ shares to private investors. We take α as exogenous for now and then work backwards to find the optimal α .

After $1 - \alpha$ shares are sold, at the end of date 0, the insider chooses either project 1 or 2. Recall that the project choice is private information to the insider. The intuition is that, although investments may be observable, the insider has unique information or expertise that allows him to assess the characteristics of the available projects. This is a natural assumption, which is consistent with the view that managers' unique expertise may be essential for investment decisions.

Let $\sigma \in [0, 1]$ be the probability that the insider chooses project 2 (innovation). We allow from the outset for the possibility of equilibria involving mixed strategies. Intuitively, an equilibrium with strictly mixed strategies could also be interpreted as the choice of an "intermediate project," which is more innovative than project 1 but not as radical as project 2. Our goal in this section is to compute the equilibrium project choice σ^* under private ownership.

2.2.1. Selling behavior at date 1

Recall that at the end of date 1, after observing x_1 , the insider chooses whether to keep or sell his shares. We assume that the current private investors cannot buy out the insider.¹¹ Thus, if the insider sells, the buyers are either new private investors or public investors in an IPO. Because the identity of the new investors is irrelevant in our model, we simply say

¹¹Our results do not change qualitatively under the much weaker assumption that there is some positive probability that current private investors cannot offer liquidity insurance to insiders. There are many reasons for that being the case. One possibility is that all capital committed to a private equity fund has already been used. Even if there is still capital available, fund covenants may prevent the investment of more than a certain fraction of fund capital in a single firm (Kaplan and Strömberg, 2009). Fund covenants and restrictions to raising additional capital can be rationalized as potential solutions to agency conflicts between general partners (fund managers) and limited partners (Axelson, Strömberg, and Weisbach, 2009). Finally, it could also be that funds need to exit early in order to produce evidence of good performance and raise more capital (Gompers, 1996).

that the insider sells shares to the market.

We first consider how the market updates its beliefs after observing the insider selling shares at date 1. Let m be the posterior probability that the insider had a liquidity shock conditional on the insider selling shares at date 1. A low m means that the market believes that the insider is selling shares because the firm is overvalued, while a high m means that the market believes that a liquidity shock is probably the main reason why the insider sells.¹²

A liquidity shock forces the insider to sell his shares. Without a liquidity shock, the insider chooses whether to sell or not to sell. We thus need to characterize when an insider without a liquidity shock chooses to sell. The following lemma describes the insider's behavior when earnings are $x_1 = 1$.

Lemma 1 *In the private ownership case, a late-consumer insider never sells shares at date 1 after observing a success ($x_1 = 1$).*

Intuitively, a late-consumer insider would only sell shares in date 1 if he believes that these shares are overvalued. Market rationality rules out excessive overvaluation (i.e. shares sold at prices that are not compatible with the selling behavior of the insider), thus prices at date 1 are never high enough to make an insider with good news sell rather than keep his shares.

Now let $b \in [0, 1]$ be the probability that a late-consumer insider sells shares after observing $x_1 = 0$.¹³ By Bayes's rule, rational market beliefs imply

$$m = \Pr(\textit{Shock} \mid \textit{Sale}) = \frac{\Pr(\textit{Sale} \mid \textit{Shock}) \Pr(\textit{Shock})}{\Pr(\textit{Sale})}. \quad (7)$$

The inputs for this formula are as follows. In an equilibrium in which the probability of

¹²If we treat m as an exogenous parameter, we can perform comparative statics with respect to market beliefs; a low m is equivalent to a "cold market" while a high m is equivalent to a "hot market." On the other hand, by treating m as endogenous, as we do in this paper, hot and cold markets still exist, but they are driven by fundamentals rather than sentiment.

¹³Because b can only be non-zero if $x_1 = 0$, whether project 1 or 2 were chosen is immaterial for the decision to sell, thus b does not need to be conditional on project choice. For brevity, we omit the proof of this claim; this proof is available upon request.

choosing project 2 is σ , the unconditional probability of selling shares at date 1 is

$$\Pr(Sale) = \mu + (1 - \mu) [\sigma (1 - \delta p) b + (1 - \sigma) (1 - p) b]. \quad (8)$$

Conditional on having a liquidity shock, the insider sells with probability $\Pr(Sale | Shock) = 1$. Since the probability of a liquidity shock is μ , we have

$$m(\sigma, b) = \frac{\mu}{\mu + (1 - \mu) [\sigma (1 - \delta p) b + (1 - \sigma) (1 - p) b]}. \quad (9)$$

The equilibrium value of shares if the market holds rational beliefs is

$$V(\sigma, b) = m(\sigma, b) [\sigma v_2 + (1 - \sigma) v_1] + [1 - m(\sigma, b)] p. \quad (10)$$

After a failure, the best option is to switch to project 1. Conditional on $x_1 = 0$, the expected value of the firm is p . A necessary condition for selling shares with positive probability is

$$V(\sigma, b) \geq p. \quad (11)$$

The next lemma characterizes the equilibrium behavior of the insider if $x_1 = 0$.

Lemma 2 *In the private ownership case, a late-consumer insider sells shares with probability $b = 1$ at date 1 after observing a failure ($x_1 = 0$).*

Intuitively, the insider always sells after $x_1 = 0$ because the market assigns a positive probability to $x_1 = 1$. This belief is rational because the insider could have received a liquidity shock and be forced to sell.

From Lemma 2, the equilibrium probability of selling after a failure is equal to 1 (as we will see later, results change if we allow for liquidity costs, i.e. $k < 1$). As $b^* = 1$ in equilibrium, we write the equilibrium value of shares sold at date 1 for each σ as $V(\sigma) \equiv V(\sigma, 1)$.

A key aspect of the private ownership case is the possibility of selling shares in date

1 after observing a failure. A late-consumer insider only sells shares in date 1 if they are overvalued. This may happen in equilibrium because the market does not observe x_1 and thus cannot distinguish between a liquidity-motivated sale and an opportunistic sale. This information asymmetry creates a valuable option for a late-consumer insider.

Let $T(\sigma) \equiv V(\sigma) - p$ denote the intrinsic value of the *option to exit early* for a late-consumer insider conditional on $x_1 = 0$. Selling shares is a real option to the insider. The value of the underlying asset is $V(\sigma)$ —the market value of shares in equilibrium—while the exercise price of the option (the opportunity cost of selling in date 1) is p . Lemma 2 implies that $T(\sigma) > 0$. Note that $T(\sigma)$ is a function of both the fundamental parameters and the equilibrium strategy and beliefs.

2.2.2. Project choice at date 0

Now that we know how the insider behaves and how the market sets the price of shares in date 1, we can go back to date 0 to analyze the choice between the projects 1 and 2 (the conventional or the innovative project). Suppose that the market expects project 2 to be chosen with probability σ . In this case, the expected value as of date 0 of each share held by the insider if he chooses project 2 is given by

$$u_2(\sigma) \equiv \mu V(\sigma) + (1 - \mu) [(1 - \delta p)(p + T(\sigma)) + \delta p(1 + \theta p)]. \quad (12)$$

To understand this expression, recall that at date 0 the insider does not yet know his type. He knows that with probability μ he will be an early consumer and be forced to sell. With probability $1 - \mu$ he is a late consumer and has the option to sell after a failure.

Similarly, the expected value of one share when the insider chooses project 1 at date 0 while the market expects that project 2 is chosen with probability σ is:

$$u_1(\sigma) \equiv \mu V(\sigma) + (1 - \mu) [(1 - p)(p + T(\sigma)) + p(1 + p)]. \quad (13)$$

For an equilibrium with a positive probability of exploration (project 2) to occur ($\sigma > 0$), choosing project 2 at date 0 must be incentive compatible for the insider. That is, we need $u_2(\sigma) \geq u_1(\sigma)$ to guarantee incentive compatibility. Similarly, if $\sigma < 1$ in equilibrium, we need $u_2(\sigma) \leq u_1(\sigma)$. The next proposition summarizes the incentive compatibility constraints.

Proposition 1 *Incentive Compatibility under Private Ownership.*

1. *An equilibrium in which the insider chooses project 2 (exploration of new ideas) with strictly positive probability ($\sigma > 0$) exists only if the following incentive compatibility condition holds:*

$$v_2 - v_1 + p(1 - \delta)T(\sigma) \geq 0. \quad (14)$$

2. *An equilibrium in which the insider chooses project 1 (exploitation of old ideas) with strictly positive probability ($\sigma < 1$) exists only if the following incentive compatibility condition holds:*

$$v_2 - v_1 + p(1 - \delta)T(\sigma) \leq 0. \quad (15)$$

The main intuition for the incentive effects of private ownership on innovation can be grasped from the incentive compatibility (IC) condition (14). The first part of this condition, $v_2 - v_1$, shows the efficiency incentives for choosing the innovative project. We call this the *efficiency effect*. This effect is fully determined by the technology and it can be either positive or negative. In a first best world, the efficiency effect would determine which project is chosen.

Proposition 1 shows that there is a second force pushing towards innovation, which is given by

$$p(1 - \delta)T(\sigma) = (1 - \delta p)T(\sigma) - (1 - p)T(\sigma) > 0. \quad (16)$$

Because the innovative project has a higher probability of failure than the conventional project, the expected value of the option to exit early is higher when innovation is chosen

$$((1 - \delta p) T(\sigma) > (1 - p) T(\sigma)).$$

The value of the option to exit early $T(\sigma)$ reflects the fact that the private ownership structure displays a higher degree of *tolerance for failure* than the first-best benchmark. Tolerance for failure has been shown to be a key feature of optimal incentive schemes for innovation (Manso, 2009). The key insight of our model is that tolerance for failure is more valuable for innovation because the option to exit early is exercised more often when exploration is chosen. Thus, the *tolerance-for-failure effect* is always positive.

The option to exit early nudges the insider towards choosing the more innovative project. When innovation is efficient from a technological perspective ($v_2 - v_1 \geq 0$), this extra incentive for innovation is not necessary; the IC is not binding.

More interesting is the case of $v_2 - v_1 < 0$. This is a situation in which innovation is inefficient. Proposition 2 below shows that, for a set of parameters, innovation may be chosen despite being inefficient. This happens because the option value of an early exit is strictly positive. If the tolerance-for-failure effect dominates the (negative) efficiency effect, the private ownership structure creates incentives to innovate even when it would be optimal to choose the conventional project.

The next proposition fully characterizes all equilibria under private ownership for any set of parameters.

Proposition 2 *A unique equilibrium always exists and is given by:*

1. *If $v_2 \geq v_1$, then $\sigma^* = 1$ (exploration for sure if innovation is efficient).*
2. *If $v_2 < v_1$, then σ^* is uniquely given by*

$$\sigma^* = \begin{cases} 1 & \text{if } \frac{v_1 - v_2}{p(1-\delta)} \leq T(1), \\ T^{-1}\left(\frac{v_1 - v_2}{p(1-\delta)}\right) & \text{if } \frac{v_1 - v_2}{p(1-\delta)} \in (T(1), T(0)), \\ 0 & \text{if } \frac{v_1 - v_2}{p(1-\delta)} \geq T(0). \end{cases} \quad (17)$$

Figure 3 illustrates the three possible cases when $v_1 - v_2 > 0$. The flat dashed lines represent different values for $v_1 - v_2$. The R_1 line represents a case in which $v_1 - v_2$ is sufficiently high. In such a case, the (absolute value of the) efficiency effect is large and dominates the tolerance-for-failure effect, implying that the first-best action $\sigma^* = 0$ is chosen in equilibrium. The R_2 line represents an intermediate value for $v_1 - v_2$, for which a given probability of innovation $\sigma^* \in (0, 1)$ makes the insider indifferent between projects 1 and 2. That means that the efficiency effect is fully offset by the tolerance-for-failure effect, and the unique equilibrium must involve some inefficient amount of innovation. Finally, the R_3 line is a case where $v_1 - v_2$ is positive but small, so that the option to exit early is so valuable to the insider compared to $v_1 - v_2$ that the insider always makes the inefficient project choice in equilibrium.

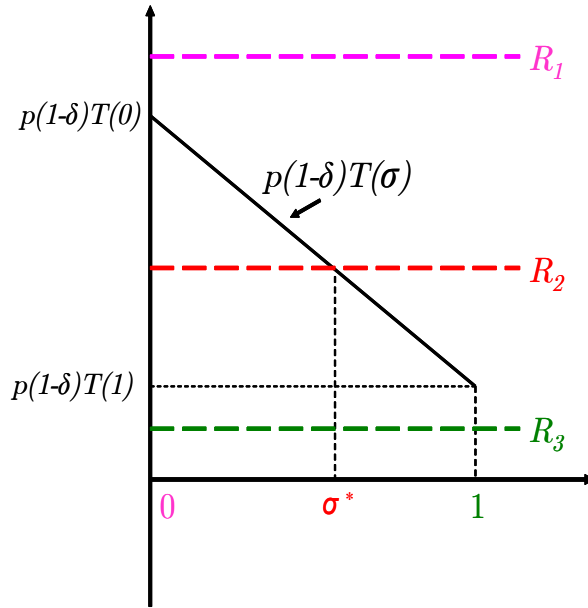


Figure 3. Equilibrium σ^* when $v_1 > v_2$.

In sum, our model shows that the private ownership structure is biased towards innovation. This bias is welcome when $v_1 < v_2$ but may lead to inefficiencies if $v_1 > v_2$. Earnings opacity, typical in privately-owned firms, gives an exit option to the insider. It is profitable to sell before a bad signal about the value of the firm becomes public. The exit option is

available regardless of the project chosen, innovative or conventional. But the exit option is more valuable under innovation because the probability of failure is higher.

2.2.3. Comparative statics

When $v_2 - v_1 \geq 0$, $\sigma^* = 1$. As a result, changes in the fundamental parameters do not affect the equilibrium, as long as $v_2 - v_1$ remains non-negative. The interesting case for comparative statics is when $v_1 - v_2 > 0$.

Suppose that we have $\sigma^* \in (0, 1)$, that is, the R_2 case in Figure 3. In this case, σ^* is defined implicitly by

$$v_2 - v_1 + p(1 - \delta)T(\sigma^*) = 0.$$

To perform comparative statics with respect to the fundamental parameters of the model, we first define

$$G(\sigma^*, \mu, \theta, \delta) = v_2 - v_1 + p(1 - \delta)T(\sigma^*).$$

Since $T(\sigma^*) = V(\sigma^*) - p$,

$$G(\sigma^*, \mu, \theta, \delta) = v_2 - v_1 + p(1 - \delta) \{m(\sigma^*, 1) [p + \sigma^* (v_2 - v_1)]\}. \quad (18)$$

We have

$$\frac{\partial G}{\partial \sigma^*}(\sigma^*, \mu, \theta, \delta) = p(1 - \delta) \left\{ \frac{\partial m(\sigma^*, 1)}{\partial \sigma^*} [p + \sigma^* (v_2 - v_1)] + (v_2 - v_1) m(\sigma^*, 1) \right\} < 0,$$

because $\frac{\partial m(\sigma^*, 1)}{\partial \sigma^*} < 0$ and $v_2 - v_1 < 0$. Define $\Delta \equiv \frac{\partial G}{\partial \sigma^*}(\sigma^*, \mu, \theta, \delta)$. Using the implicit function theorem, we have

$$\frac{\partial \sigma^*}{\partial \mu} = - \frac{p(1 - \delta) \frac{\partial m}{\partial \mu} [p + \sigma^* (v_2 - v_1)]}{\Delta} > 0, \quad (19)$$

because $\frac{\partial m}{\partial \mu} > 0$. Intuitively, by making it easier for a late-consumer insider to disguise his trade as a liquidity shock, innovation becomes more attractive and in equilibrium there is

more of it.

Increases in δ and θ increase the NPV of innovation. Thus, we would expect that the equilibrium amount of innovation should also increase. This is indeed true:

$$\frac{\partial G}{\partial \theta}(\sigma^*, \mu, \theta, \delta) = p(1 - \delta) [p^2\delta + p(1 - \delta)m(\sigma^*, 1)p^2\delta\sigma^*] > 0$$

which implies

$$\frac{\partial \sigma^*}{\partial \theta} = -\frac{p^2(1 - \delta)\delta [1 + p(1 - \delta)\sigma m(\sigma^*, 1)]}{\Delta} > 0, \quad (20)$$

and (after some algebra it can be shown that)

$$\begin{aligned} \frac{\partial G}{\partial \delta}(\sigma^*, \mu, \theta, \delta) &= p[1 + p(\theta - 1)] - pm(\sigma^*, 1)[p + \sigma^*(v_2 - v_1)] + \\ & p(1 - \delta) \left\{ \frac{\partial m(\sigma^*, 1)}{\partial \delta} [p + \sigma^*(v_2 - v_1)] + \sigma^*p[1 + p(\theta - 1)] \right\} > 0, \end{aligned}$$

which implies

$$\frac{\partial \sigma^*}{\partial \delta} = -\frac{\frac{\partial G}{\partial \delta}(\sigma^*, \mu, \theta, \delta)}{\Delta} > 0 \quad (21)$$

2.2.4. The value of being private

Now we compute the expected value of the firm to the insider taking into account the value of shares initially sold to private investors. Because the insider needs to raise I to finance the investment, if the market expects σ^* to occur in equilibrium, then the revenue from selling shares must satisfy

$$(1 - \alpha) [\sigma^*v_2 + (1 - \sigma^*)v_1] \geq \frac{I}{c_{priv}}. \quad (22)$$

Due to the trading costs implied by $c_{priv} < 1$, the insider will only sell the minimum number of shares that allows him to invest. To avoid uninteresting cases in which the investment can never be financed, we assume that $I \in (0, c_{priv} \min\{v_1, v_2\})$. That is, the firm's cost of capital is never so high so that funds for investment cannot be raised. Under this assumption,

the equilibrium insider's stake is uniquely given by

$$\alpha^* = 1 - \frac{I}{c_{priv}[\sigma^*v_2 + (1 - \sigma^*)v_1]}. \quad (23)$$

Finally, the ex ante value of the firm to the insider under private ownership is given by

$$\begin{aligned} W_{private} &= \alpha^* [\sigma^*u_2(\sigma^*) + (1 - \sigma^*)u_1(\sigma^*)] \\ &= \sigma^*v_2 + (1 - \sigma^*)v_1 - \frac{I}{c_{priv}}. \end{aligned} \quad (24)$$

The first two terms of this expression represent the expected outcome from the project decision and the third term is the initial investment cost. Notice that $W_{private}$ may differ from the first best in a full information, frictionless economy both because the equilibrium level of innovation σ^* is excessive compared to the first best (our results imply that under private ownership there is never too little innovation) and because raising funds for investing is costly ($c_{priv} < 1$), which generates deadweight costs.

2.3. Public ownership

In this section we consider the case of public ownership, i.e. there are $1 - \alpha$ of shares floating in the market. As in the case of private ownership, the insider sells his shares at date 1 if there is a liquidity shock. The difference between the public case and the private case is due to the transparency of earnings. Earnings x_1 can be observed by all investors.

2.3.1. Selling behavior at date 1

The analysis of the equilibrium is similar to the private ownership case. As before, the insider chooses project 2 (exploration) with probability $\sigma \in [0, 1]$. Regardless of the project chosen, the expected market value of the firm when $x_1 = 0$ is p because there is no information asymmetry between the insider and the market. Earnings transparency means that the

market always knows when $x_1 = 0$. The market also knows that project 1 is always chosen after $x_1 = 0$. Although the market does not know which project was chosen at date 0, that knowledge is not value relevant when $x_1 = 0$. Thus, shares are always fairly valued when $x_1 = 0$ and the insider gains nothing by selling shares. We can assume that the insider sells or keeps his shares when $x_1 = 0$; the equilibrium payoffs are not affected by this choice.

The insider may however choose to sell shares after $x_1 = 1$. Although the market knows that $x_1 = 1$ has occurred, the market does not know which project was chosen at date 0. If project 1 was chosen, the expected value of the firm is $1 + p$. If project 2 was chosen, the expected value of the firm is $1 + \theta p$. Thus, the insider is always weakly better off when the market believes that project 2 was initially chosen. That creates a value-relevant information asymmetry, which may distort the incentives of the insider when making project choice decisions.

The next two lemmas characterize the behavior of a late-consumer insider after $x_1 = 1$.

Lemma 3 *In the public ownership case, a late-consumer insider never sells shares at date 1 after observing a success ($x_1 = 1$) if project 2 was chosen.*

Therefore, the insider never sells voluntarily at date 1 after exploration. The intuition is that, if project 2 was chosen, after $x_1 = 1$ the firm is always sold with a discount because the market can never be certain that project 2 was chosen.

Lemma 4 *In the public ownership case, in equilibrium a late-consumer insider weakly prefers to sell shares after $x_1 = 1$ if project 1 was chosen.*

If the insider chooses project 1 (exploitation) and obtains a success, the insider trades with probability 1 (to simplify the exposition, we assume that the insider sells in case of indifference). Selling after $x_1 = 1$ if the insider chooses project 1 is always profitable as long as the market assigns some probability to project 2. Given Lemmas 3 and 4, without the possibility of a liquidity shock, trading after $x_1 = 1$ would always reveal the choice of

project. Liquidity shocks allow insiders who choose project 1 to trade after $x_1 = 1$ without revealing the choice of the project. In equilibrium, late consumer insiders who have chosen project 1 pool with early consumer insiders.

2.3.2. Project choice in date 0

In equilibrium, the market must have correct beliefs and thus must assign probability σ to project 2 being chosen. When the market observes a success and the insider sells shares, the market assigns probability s that project 2 has been chosen. The difference between σ and s is that σ is the unconditional probability of choosing project 2 while s is the probability of project 2 being chosen given that the insider sells shares and the market observes $x_1 = 1$:

$$s \equiv \Pr(\text{Project 2} \mid \text{Sale}, x_1 = 1) = \frac{\Pr(\text{Sale}, x_1 = 1 \mid \text{Project 2}) \Pr(\text{Project 2})}{\Pr(\text{Sale}, x_1 = 1)}. \quad (25)$$

The inputs for this formula are as follows. From Lemma 3, the insider only sells after choosing project 2 and $x_1 = 1$ if he suffers a liquidity shock:

$$\Pr(\text{Sale}, x_1 = 1 \mid \text{Project 2}) = \mu\delta p. \quad (26)$$

From Lemmas 3 and 4, the probability of selling and $x_1 = 1$ is

$$\Pr(\text{Sale}, x_1 = 1) = (1 - \sigma)p + \sigma\delta p\mu. \quad (27)$$

Finally, the unconditional probability of project 2 is σ . Therefore, equilibrium beliefs must be

$$s(\sigma) = \frac{\sigma\mu\delta}{(1 - \sigma) + \sigma\mu\delta}. \quad (28)$$

Given such beliefs, the market value of shares sold after a success is

$$V(\sigma) = 1 + s(\sigma)\theta p + (1 - s(\sigma))p = 1 + p + s(\sigma)p(\theta - 1). \quad (29)$$

We now calculate the expected gains for the insider from choosing either project 1 or project 2. As before, the expected value of one share if the insider chooses project 1 is given by

$$u_1(\sigma) = pV(\sigma) + (1-p)p. \quad (30)$$

If the insider chooses project 1, the probability of success is p . In the case of a success, the insider sells and obtains $V(\sigma)$. If there is a failure, the market value of the firm becomes p as the best project to choose in date 1 is project 1, again with probability p of success.

The expected gain per share for the insider from choosing project 2 (exploration) is

$$u_2(\sigma) = \delta p [\mu V(\sigma) + (1-\mu)(1+\theta p)] + (1-\delta p)p. \quad (31)$$

At date 1, the probability of success is δp . In case of success, the insider only sells if there is a liquidity shock, which happens with probability μ . If the insider is not forced to sell (no liquidity shock), he keeps his shares until date 2 and selects project 2, now with expected probability θp of success.

The next proposition fully characterizes the unique equilibrium.

Proposition 3 *In the public ownership case, there is a unique equilibrium probability of exploration $\sigma^* \in [0, 1)$ given by*

$$\sigma^* = \frac{s^*}{\mu\delta + s^*(1-\mu\delta)}, \quad (32)$$

where

$$s^* = \max \left\{ \frac{(v_2 - v_1) - \delta\mu p^2(\theta - 1)}{p^2(\theta - 1)(1 - \delta\mu)}, 0 \right\}. \quad (33)$$

Proposition 3 implies that an equilibrium with full innovation, $\sigma = 1$, is never possible. If the market expects exploration with probability $\sigma = 1$, then choosing exploitation becomes a dominant strategy. The insider increases the probability of success by exploiting and then selling the shares at date 1. The strategy $\sigma = 1$ could only be an equilibrium if there was no

liquidity shock, $\mu = 0$. In this case, the market knows that there is a sale in case of success only if the insider exploited. The insider cannot disguise exploitation. Therefore, the insider chooses to explore if $\mu = 0$.

The following corollary facilitates the comparison with the private case.

Corollary 1 *The unique equilibrium is such that*

1. *If $v_1 - v_2 \geq -\delta\mu p^2(\theta - 1)$, then $\sigma^* = 0$ (always exploits).*
2. *If $v_1 - v_2 < -\delta\mu p^2(\theta - 1)$, then $\sigma^* \in (0, 1)$.*

When the conventional project is efficient, $v_1 > v_2$, then the insider chooses the conventional project with probability 1. On the other hand, if $v_2 > v_1$, however greater v_2 is, the insider never chooses to explore with probability one. In fact, the insider may choose project 1 with probability 1 even though $v_2 > v_1$.

These results show that public ownership creates a bias against innovation. But it always induces the efficient project choice when $v_1 > v_2$.

2.3.3. Comparative statics

For the comparative statics with respect to the fundamental parameters, first note that σ^* is strictly increasing in s^* when the solution is interior. Assuming an interior solution for simplicity, we have

$$\frac{\partial \sigma^*}{\partial \theta} = \frac{1 - \delta}{p(1 - \delta\mu)} \frac{1}{(\theta - 1)^2} \frac{\partial \sigma^*}{\partial s^*} > 0. \quad (34)$$

Intuitively, an increase in θ makes innovation more valuable and increases the amount of innovation.

For δ , we have a similar effect:

$$\frac{\partial \sigma^*}{\partial \delta} = \frac{p(\theta - 1)(1 - \mu) + (1 - \mu)}{p(\theta - 1)(1 - \delta\mu)^2} \frac{\partial \sigma^*}{\partial s^*} > 0. \quad (35)$$

An increase in δ makes exploration less risky because it increases the expected probability of success. As in the case of θ , an increase in δ increases the amount of innovation because it makes innovation more valuable.

Finally, we have

$$\frac{\partial \sigma^*}{\partial \mu} = -\delta(1-\delta) \frac{1 + (\theta - 1)p}{p(\theta - 1)(1 - \delta\mu)^2} \frac{\partial \sigma^*}{\partial s^*} < 0. \quad (36)$$

If liquidity shocks occur very often, it is easier for the insider to hide the choice of project 1. Frequent liquidity shocks make the market believe that the insider is selling because of a liquidity shock, and not because of success under exploitation. As it is easier to hide the choice of exploitation, the incentives to choose innovation decrease.

In sum, exploiting the old method is better when θ or δ are low, or when μ is high.

2.3.4. The value of being public

We now compute the expected value of the firm to the insider, taking into account the value of shares initially sold to public investors. As before, we have

$$\alpha^* = 1 - \frac{I}{c_{pub}[\sigma^*v_2 + (1 - \sigma^*)v_1]}. \quad (37)$$

Thus, the ex ante value of the firm to the insider is given by

$$\begin{aligned} W_{public} &= \alpha^* [\sigma^*u_2(\sigma^*) + (1 - \sigma^*)u_1(\sigma^*)] \\ &= \sigma^*v_2 + (1 - \sigma^*)v_1 - \frac{I}{c_{pub}}. \end{aligned} \quad (38)$$

In the public ownership case, inefficiencies may arise because there is too little innovation in equilibrium (i.e. $\sigma^* < 1$ when $v_2 > v_1$) and because raising funds through equity offerings is costly ($c_{pub} < 1$).

3. The decision to go public or private

We now analyze the decision to go public or private. The insider chooses to go private or public depending on whether or not $W_{private}$ is greater than W_{public} . To differentiate between the two cases, let $\sigma_{private}$ denote the private-ownership equilibrium and σ_{public} denote the public-ownership equilibrium. To simplify notation, define the relative cost advantage of public offerings compared to private offerings as

$$a = \frac{1}{c_{priv}} - \frac{1}{c_{pub}} = \frac{c_{pub} - c_{priv}}{c_{priv}c_{pub}}. \quad (39)$$

If public offerings are cheaper than private offerings ($c_{pub} > c_{priv}$), then $a > 0$.

Proposition 4 follows immediately from the comparison of $W_{private}$ with W_{public} .

Proposition 4 *The private ownership structure is preferable to the public ownership structure if and only if*

$$(\sigma_{private} - \sigma_{public})(v_2 - v_1) \geq aI. \quad (40)$$

The choice between public and private is driven by two considerations. The first one is the main novelty of our model: the choice between public versus private depends on the relative efficiency of innovative projects, $v_2 - v_1$. The second one, summarized in a , is the relative cost of capital advantage of public offerings compared to private offerings.

Notice that $\sigma_{private} - \sigma_{public} \geq 0$ for any set of parameters. Thus, going private is more attractive than going public when innovation is efficient ($v_2 - v_1 > 0$). In fact, if we shut down the other effect by setting $a = 0$, whether innovation is efficient or not is the only consideration for the choice of ownership structure, as shown in the next corollary.

Corollary 2 *Let $a = 0$.*

1. *If innovation is efficient ($v_2 > v_1$), the insider goes private.*
2. *If the conventional project is efficient ($v_2 < v_1$), the insider strictly prefers to go public if $\frac{v_1 - v_2}{p(1-\delta)} < T(0)$ and is indifferent between going public or private if $\frac{v_1 - v_2}{p(1-\delta)} \geq T(0)$.*

3. *If both projects are equivalent ($v_2 = v_1$), the insider is indifferent between going public or private.*

4. Robustness and extensions

While presenting our main results, we have made many simplifying assumptions to facilitate the exposition. In this section, we discuss the robustness of our model to relaxing some of these assumptions. We also show that some simple extensions lead to additional implications that also have empirical content.

4.1. Contracting

We have assumed that the firm can only issue one type of securities: straight share contracts. This assumption is not essential for the qualitative results of the paper. Although expanding the contracting space increases the number of instruments the insider can use to maximize firm value, they do not fully eliminate inefficiencies that may arise when the insider cannot choose between public and private ownership forms. We illustrate this fact with an example that allows the company to have debt in its capital structure.

4.1.1. Debt

Our goal here is to understand whether the availability of debt securities makes the choice of ownership irrelevant. For the sake of brevity, we focus only on the public case when $v_2 - v_1 > 0$; that is, there is a public market for the firm's shares but the firm can also borrow to finance some or all of its investments (debt can be either public or private). In this case, debt is likely to have an impact on incentives to innovate. The asset substitution effect (Jensen and Meckling, 1976) makes risky projects more attractive when there is debt. Therefore, this effect could offset the public ownership bias against innovation. We investigate here whether this conjecture is true.

Suppose that the firm finances its investment fully with debt with face value D , a zero-coupon long-term bond, to be paid in the end of period 2.¹⁴ If debt is not paid in full, bondholders seize the company's cash flows. It is trivial to show that nothing changes from the previous analysis if $D \leq S = 1$, thus here we focus on the interesting case in which $D \in (1, 2)$. In this case, default occurs unless the firm observes two successes in a row.

Suppose that the insider chooses project 2 with probability σ' . Given that in equilibrium the market's belief that the insider has chosen 2 must be σ' , the insider will not trade voluntarily after choosing project 2 and $x_1 = 1$. However, the insider will trade with probability 1 if he used project 1 and $x_1 = 1$. When the market observes $x_1 = 1$ and there are shares being sold, market prices in equilibrium are

$$V(\sigma') = s(\sigma')\theta p(2 - D) + (1 - s(\sigma'))p(2 - D). \quad (41)$$

Simplifying,

$$V(\sigma') = [1 + s(\sigma')(\theta - 1)]p(2 - D). \quad (42)$$

Thus, the per share expected utility from choosing project 1 is

$$u_1(\sigma') = pV(\sigma'), \quad (43)$$

while the per share expected utility from project 2 is

$$u_2(\sigma') = \mu\delta pV(\sigma') + (1 - \mu)\delta\theta p^2(2 - D). \quad (44)$$

For the insider to be willing to randomize between 1 and 2, we need $s(\sigma') = s'$ where

$$s' \equiv \frac{(1 - \mu)\delta\theta - (1 - \mu\delta)}{(\theta - 1)(1 - \mu\delta)} > 0.$$

¹⁴As it will become clear, financing the initial investment fully with debt is the optimal financing choice unless the cost of debt capital is higher than the cost of equity capital.

Crucially, we have $s' < 1$, implying that the first best ($s = 1$) cannot be implemented in this case.

To find out the effect of debt on the probability of innovation, we need to compare s' with

$$s^* = \max \left\{ \frac{\delta [1 + p(\theta - 1)(1 - \mu)] - 1}{p(\theta - 1)(1 - \delta\mu)}, 0 \right\}. \quad (45)$$

Algebra shows that

$$s' - s^* = \max \left\{ \frac{(1 - \delta)(1 - p)}{p(\theta - 1)(1 - \delta\mu)}, 0 \right\} > 0, \quad (46)$$

which implies that debt increases the amount of innovation in the public case when $v_2 > v_1$. Thus, if firms want to innovate more but remain public, it is optimal to lever up. However, we also find that $s' < 1$, so the public ownership structure with debt is still inferior to the private ownership case when $v_2 > v_1$. In sum, the capital structure is not a perfect substitute for the ownership structure in providing incentives to innovation.

4.1.2. Other contractual arrangements

In our model, as long as the insider can choose between public and private structures ex ante, the first-best outcomes are always achieved. Thus, any other contractual solution can at best replicate what the choice of ownership mode does. Although a full analysis of the optimal contractual solution is beyond the scope of this paper,¹⁵ the case in which debt contracts are allowed illustrates the limitations of contractual solutions that do not involve an optimal choice of ownership mode.

¹⁵Such an analysis can be done as in Manso (2009), with three important modifications: (i) Diamond-Dybvig preferences, (ii) free trading of securities at date 1 (i.e. the possibility of exiting the contract at date 1), and (iii) different levels of transparency of date 1 cash flows (public versus private).

4.2. Illiquid private securities

As discussed in Subsection 2.1.4, private securities are probably more difficult to unload than public securities because private securities are not traded in centralized markets. To capture the relative illiquidity of private securities, we now assume that, for each dollar sold in shares at date 1 if the firm is private, the insider only pockets $k < 1$.

Most of the analysis of the private case remains unchanged. In particular, Lemma 1 is not affected: also for $k < 1$, an insider never sells if there is a success.

The probability of selling after a failure, however, changes. With $k < 1$, the necessary condition for selling shares after a failure changes to

$$kV(\sigma, b) \geq p. \quad (47)$$

Because $V(\sigma, b) > p$, we have $kV(\sigma, b) > p$ for k close enough to 1. Thus, the insider sells for sure after a failure if the market for private securities is liquid enough: as k approaches 1, eventually we get $b = 1$. If, on the other hand, the market in date 1 is very illiquid (k close to zero), then a late-consumer insider never sells: $b = 0$. For intermediate values of k , the equilibrium is in strictly mixed strategies, with $b \in (0, 1)$ and b increasing in k . The next Lemma formalizes these results.

Lemma 5 *In the private ownership case with $k \in (0, 1)$, a late-consumer insider sells shares with equilibrium probability $b(\sigma)$ at date 1 after observing $x_1 = 0$, where*

$$b(\sigma) = \begin{cases} 1 & \text{if } k \geq k_1(\sigma) \\ \mu \frac{k[v_1 + \sigma(v_2 - v_1)] - p}{(1-k)(1-\mu)[1-p + \sigma p(1-\delta)]p} & \text{if } k_2(\sigma) < k < k_1(\sigma) \\ 0 & \text{if } k \leq k_2(\sigma) \end{cases}$$

where

$$k_1(\sigma) \equiv \frac{\mu p + (1 - \mu) [1 - p + \sigma p (1 - \delta)] p}{\mu [v_1 + \sigma (v_2 - v_1)] + (1 - \mu) [1 - p + \sigma p (1 - \delta)] p},$$

$$k_2(\sigma) \equiv \frac{p}{v_1 + \sigma (v_2 - v_1)}.$$

The threshold values k_1 and k_2 define three regions for the behavior of the insider, as shown in Figure 4. In Region 3, the insider never sell shares. In Region 2, the insider plays a strictly mixed strategy on selling shares. The probability b of selling after failure increases with market liquidity. If the market for private securities is liquid enough, $k \geq k_1$, as shown in Region 1, then the insider sells after a failure with probability 1.

Figure 4 also illustrates the effect of the liquidity shock on the equilibrium strategy. If μ increases, k_1 decreases: a late-consumer insider sells shares with probability 1 for a larger set of values for k . That is, the insider sells even with a less liquid market. Intuitively, if μ increases, it becomes easier for the insider to disguise a failure behind a liquidity shock. The insider has more incentives to sell.

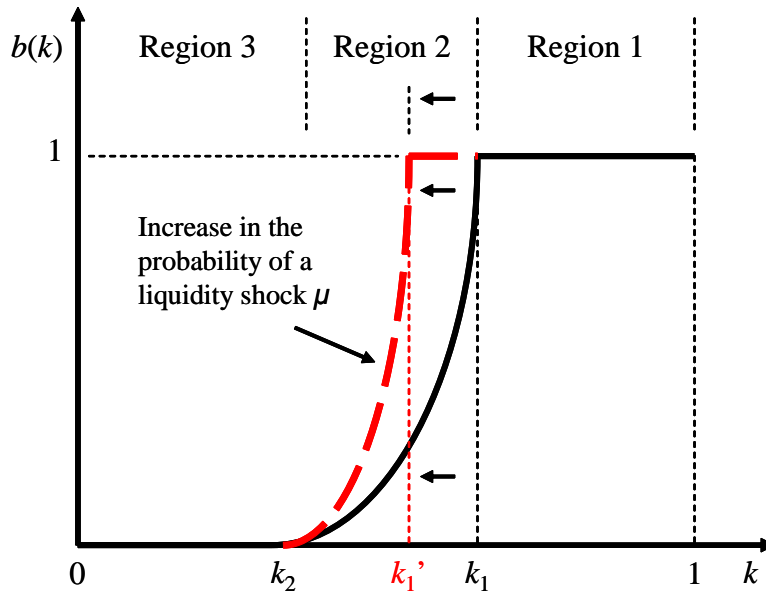


Figure 4. $b(k)$: probability of a late-consumer insider selling shares after $x_1 = 0$.

We redefine T (the intrinsic value of the option to exit early for a late-consumer insider) as

$$T(\sigma, k) = \max \{kV(\sigma) - p, 0\}. \quad (48)$$

Notice now that this option has zero value if the underlying $kV(\sigma)$ is low, which may happen either because the market for private securities is very illiquid (low k) or because the market is “cold,” i.e. the market believes that when an insider sells shares, $x_1 = 0$ is very likely (μ is low). In terms of the regions in Figure 4, we find that $T(\sigma)$ is strictly positive in Region 1, while zero in Regions 2 and 3.

The next proposition generalizes our previous results to the case where $k \leq 1$.

Proposition 5 *In the private ownership case with $k \leq 1$, an equilibrium always exists and is given by:*

1. *If $v_1 - v_2 < 0$, then $\sigma^*(k) = 1$.*
2. *If $v_1 - v_2 > 0$, then σ^* is uniquely given by*

$$\sigma^*(k) = \begin{cases} 1 & \text{if } \frac{v_1 - v_2}{p(1-\delta)} \leq T(1, k) \\ T^{-1}\left(\frac{v_1 - v_2}{p(1-\delta)}\right) & \text{if } \frac{v_1 - v_2}{p(1-\delta)} \in (T(1), T(0, k)) \\ 0 & \text{if } \frac{v_1 - v_2}{p(1-\delta)} \geq T(0, k) \end{cases}$$

3. *If $v_1 - v_2 = 0$, then $\sigma^* \in \arg \min_{\sigma \in [0,1]} T(\sigma, k)$.*

The bias of the private ownership structure towards innovation also appears here: if k is close to 1, the insider may choose the innovative project with certainty even though $v_1 > v_2$ and $k < 1$. In terms of the structure of the equilibrium, the difference between our benchmark case of $k = 1$ and the case in which $k < 1$ is the possibility of an equilibrium as in case 3 of the proposition above. In such a case, an equilibrium σ^* is not unique. This case only happens when $v_1 = v_2$.

4.2.1. The decision to go private or public with $k < 1$

Because there are no liquidity costs associated with selling shares at date 1 in the case of public ownership, the ex ante value of the firm to the insider with the public structure does not change. On the other hand, the ex ante value of the firm to the insider under private ownership changes to

$$W_{private}(k) = \sigma_{private}v_2 + (1 - \sigma_{private})v_1 - \frac{I}{c_{priv}} - L(k) \quad (49)$$

where

$$L(k) \equiv \alpha_{private} \{ \mu + (1 - \mu) [\sigma_{private}(1 - \delta p) + (1 - \sigma_{private})(1 - p)] \} V(\sigma_{private})(1 - k). \quad (50)$$

The new term $L(k)$ represents the expected costs of illiquidity associated with the sale of shares at date 1. This represents another source of deadweight losses associated with private ownership: the trading of shares because of liquidity shocks or privileged information is costly because private securities are illiquid.

The choice between public and private is then modified to include this cost. Now the private ownership structure is preferable to the public ownership structure if and only if

$$(\sigma_{private} - \sigma_{public})(v_2 - v_1) \geq aI + L(k). \quad (51)$$

where a is as defined in (39). In particular, when $c_{priv} = c_{pub}$ ($a = 0$), we have the following proposition.

Proposition 6 *If $c_{priv} = c_{pub}$, for any level of $k \in [0, 1)$, we have*

1. *If $v_2 \leq v_1$, the insider always chooses the public structure.*
2. *If $v_2 > v_1$, there is a unique $k^* \in (0, 1)$ such that the insider chooses the public structure if $k < k^*$ and chooses the private structure if $k \geq k^*$.*

The novel result of this section is quite intuitive. If private securities are less liquid than public ones ($k < 1$), the insider faces a trade-off when $v_2 > v_1$: the private structure provides appropriate incentives to innovate but imposes liquidity costs. Thus, if these liquidity costs are sufficiently large ($k < k^*$), the insider prefers to choose the public structure even though it leads to too little innovation. If we think of k as representing the (inverse of the) costs of selling private securities (such as IPO costs), our model suggests that innovation is fostered by the development of IPO markets (i.e. increases in k).

5. Conclusions

Our results suggest that public and private firms invest in fundamentally different ways. Private firms take more risks, invest more in new products and technologies, and pursue more radical innovations. Private firms are more likely to choose projects that are complex, difficult to describe, and untested. Organizational change is also more likely under private ownership. Mergers and acquisitions, divestitures, and changes in organizational structure and management practices are more easily motivated under private ownership.

On the other hand, public firms choose more conventional projects. Their managers appear short-sighted; they care too much about current earnings. They find it difficult to pursue complex projects that the market does not appear to understand well. Public firms go private after bad shocks, when it is clear that their business models are no longer working and a restructuring is needed.

Anecdotal and systematic evidence corroborates the link between private ownership and innovative change. Firms that go private pursue more influential innovations (Lerner, Sorensen, and Strömberg, 2008) and engage more in organizational change (Davis et al., 2009). There is also some evidence that private equity owned firms introduce innovations in management practices (Bloom, Sadun, and Van Renssen, 2008).

Moon (2006) describes the acquisition by Morgan Stanley Capital Partners of an oil and

gas subsidiary of a utility that was undergoing a restructuring. The company had good long term-prospects according to independent analysts, but faced several years of negative cash flows due to the restructuring efforts. Although finding strategic buyers for the company seemed the most logical solution, none of the public firms in the industry appeared to be willing to deal with the complexity of the business and with its negative cash flows. Private equity investors, on the other hand, were keen to deal with this uncertainty and with the prospect of negative cash flows in the short run.

There are still some untested implications of our model. Our model predicts that cash-flow volatility should be higher in private than public firms. Private firms should be more profitable during technological revolutions, while public firms should be more valuable in mature but growing industries.

Our model also has implications for the decision to go public or private. Firms are likely to go public after a technological breakthrough, that is, when it makes sense to exploit a newly discovered technology. Firms are likely to go private after suffering permanent negative productivity shocks, that is, when their existing technologies or business models become permanently unprofitable. Chemmanur, He, and Nandy (2007) find that firms go public at the peak of their productivity and then performance declines after going public. This is consistent with firms going public only after perfecting a new technology; they become public in the “harvesting” period. The model also explains why companies go private when performance is particularly poor.

Finally, we note that there are many directions to which the model can be extended. Our model emphasizes two important effects – short termism and (lack of) tolerance for failures – that make public firms ill suited to pursue innovations. But one could also argue, along the lines of Burkart, Gromb, and Panunzi (1997), that the “hands-off” approach of public shareholders is necessary to foster managerial initiative, and may counteract the effects we emphasize here. This is a promising avenue for future theoretical and empirical explorations.

6. Appendix—Proofs

Lemma 1.

Proof. Conditional on the market observing the insider selling his shares, define $h \equiv \Pr(x_1 = 0 \mid \text{sale})$ and $s \equiv \Pr(\text{project 2} \mid x_1 = 1)$. A rational market should value each share sold by the insider at date 1 at

$$V = hp + (1 - h)[s(1 + \theta p) + (1 - s)(1 + p)]. \quad (52)$$

Furthermore, as a minimum rationality requirement, the market must believe that the insider is (weakly) more likely to sell after a failure than after a success. This is because, for any given price of shares sold at date 1, if the insider chooses to sell when $x_1 = 1$, then he should also sell if $x_1 = 0$. Thus, if the market observes shares being sold, the lowest possible weakly rational value for h is $1 - p$ (that is, the market believes that both types sell shares with the same probability and project one was chosen).

We now need to consider two cases.

(a) Project 1 was chosen at date 0. A late-consumer insider would only sell after observing $x_1 = 1$ if $V \geq 1 + p$. This condition is easier to satisfy when the market believes that project 2 was chosen when $x_1 = 1$, i.e. $s = 1$. Setting $s = 1$ implies

$$V = 1 + \theta p - h[1 + p(\theta - 1)] \geq 1 + p. \quad (53)$$

This condition is easier to satisfy when h is low. The lowest possible rational h occurs when $h = 1 - p$ in which case the condition becomes

$$\theta p - (1 - p)[1 + p(\theta - 1)] \geq p. \quad (54)$$

This expression is easier to satisfy when θ is high. For any given p , the maximum θ is $1/p$.

Thus, a necessary condition for it to hold is

$$p^2 - 2p + 1 \leq 0, \quad (55)$$

which implies $p = 1$, which is ruled out by assumption. Thus, we have that $V < 1 + p$.

(b) Project 2 was chosen at date 0. After a success, the best option is to stick with project 2. The condition for selling is thus $V \geq 1 + \theta p$. But we prove in (a) that $V < 1 + p$. As $\theta > 1$, this implies $V < 1 + \theta p$. ■

Lemma 2.

Proof. For any given pair of market beliefs (s, b) , the insider sells with probability 1 if $V(s, b) > p$. That is, if $m(s, b)[sv_2 + (1 - s)v_1] + [1 - m(s, b)]p > p$. As $\mu > 0$ (and so is $m > 0$), this expression holds for any (s, b) because $v_1 > p$ and $v_2 > p$. ■

Proposition 1.

Proof. The insider chooses the project after selling $1 - \alpha$ shares of the company. His goal is thus to maximize the value of his equity stake $\alpha u(\sigma)$, where $u(\sigma)$ is the value of the firm given that the market expects σ . The incentive compatibility constraint for the insider to choose project 2 when the market expects project 2 to be chosen with probability σ is $\alpha u_2(\sigma) \geq \alpha u_1(\sigma)$, which implies

$$(1 - \delta p)(p + T(\sigma)) + \delta p(1 + \theta p) \geq (1 - p)(p + T(\sigma)) + p(1 + p). \quad (56)$$

This constraint simplifies to condition (14). Reversing the inequalities proves part 2. ■

Proposition 2.

Proof. To prove each case it is sufficient to find out the value of σ that satisfy the incentive compatibility constraints.

Case 1. Because $T(\sigma) > 0$, then the IC for project 1 cannot be satisfied. The IC for project

2, on the other hand, is trivially satisfied. Therefore, $\sigma^* = 1$ is the only equilibrium.

Case 2. Suppose that $v_1 - v_2 > 0$. Notice that

$$V'(\sigma) = \frac{\partial m(\sigma, 1)}{\partial \sigma} [v_1 + \sigma(v_2 - v_1) - p] + (v_2 - v_1)m(\sigma, 1), \quad (57)$$

which is strictly negative when $v_1 - v_2 > 0$ because

$$\frac{\partial m(\sigma, 1)}{\partial \sigma} < 0 \text{ and } v_1 + \sigma(v_2 - v_1) - p > 0. \quad (58)$$

Thus, the highest possible value for the option to exit is

$$T(0) = \frac{\mu p}{\mu + (1 - \mu)(1 - p)}. \quad (59)$$

As a result, if $v_1 - v_2 \geq p(1 - \delta)T(0)$ then the unique equilibrium occurs when $\sigma = 0$.

Now, $T(\sigma)$ is minimized at $\sigma = 1$, so

$$T(1) = \frac{\mu v_2 + (1 - \mu)(1 - \delta)p}{\mu + (1 - \mu)(1 - \delta)} - p = \frac{\mu p \delta [1 + p(\theta - 1)]}{\mu + (1 - \mu)(1 - \delta)}. \quad (60)$$

As $T(\sigma)$ is decreasing in $\sigma \in [0, 1]$, its inverse T^{-1} is well defined in that domain. If

$$v_1 - v_2 < p(1 - \delta)T(0) \text{ and } v_1 - v_2 > p(1 - \delta)T(1) \quad (61)$$

then there exists a unique $\sigma^* \in [0, 1]$ such that

$$\sigma^* = T^{-1}\left(\frac{v_1 - v_2}{p(1 - \delta)}\right). \quad (62)$$

■

Lemma 3.

Proof. Because rational market beliefs imply that shares sold after $x_1 = 1$ can be valued

at most at $1 + \theta p$, the late-consumer insider strictly prefers to keep his shares unless the market believes that $\sigma = 1$. However, $\sigma = 1$ cannot be an equilibrium. If the market believed that $\sigma = 1$, then the insider would choose to exploit instead, sell in case of success, and obtain an expected payoff $p(1 + \theta p) + (1 - p)p > \delta p(1 + \theta p) + (1 - \delta p)p$. (Recall that the market observes $x_1 = 1$ but cannot observe the project.) Therefore, $\sigma = 1$ cannot be an equilibrium. Thus, if an equilibrium exists, it must be that $\sigma < 1$ and the insider in the public structure never sells in case of success. ■

Lemma 4.

Proof. Because rational market beliefs imply that shares sold after $x_1 = 1$ can be valued at least at $1 + p$, the late-consumer insider strictly prefers to sell his shares unless the market believes that $\sigma = 0$, in which case he is indifferent between selling or not selling. ■

Proposition 3.

Proof. For the insider to be willing to randomize between projects 1 and 2, we must have equal expected gains from both projects, that is

$$pV(\sigma) + (1 - p)p = \delta p[\mu V(\sigma) + (1 - \mu)(1 + \theta p)] + (1 - \delta p)p. \quad (63)$$

Solving for s ,

$$s^* = \frac{\delta[1 + p(\theta - 1)] - \delta\mu p(\theta - 1) - 1}{p(\theta - 1)(1 - \delta\mu)}, \quad (64)$$

as long as the numerator is positive. If negative, the equilibrium s^* is zero, because project 1 always gives higher payoffs than project 2. Notice that s is always strictly lower than 1 (the difference between the numerator and the denominator is $-(1 - \delta)[1 + p(\theta - 1)]$).

Using (28), $\sigma^* = s/[\mu\delta + s(1 - \mu\delta)]$ when $s > 0$, and $\sigma = 0$ when $s = 0$: there is a one-to-one mapping between σ and s . ■

Proposition 4.

Proof. Immediately from the comparison of $W_{private}$ with W_{public} . ■

Lemma 5.

Proof. $V(b, \sigma)$ can be rewritten as

$$V(b, \sigma) = \frac{\mu[v_1 + \sigma(v_2 - v_1)] + (1 - \mu)[1 - p + \sigma p(1 - \delta)]bp}{\mu + (1 - \mu)[1 - p + \sigma p(1 - \delta)]b}.$$

The proof has three parts.

(1) For $b = 1$ to be an equilibrium strategy for the insider we need that $kV(\sigma, 1) \geq p$.

$$k \frac{\mu[v_1 + \sigma(v_2 - v_1)] + (1 - \mu)[1 - p + \sigma p(1 - \delta)]p}{\mu + (1 - \mu)[1 - p + \sigma p(1 - \delta)]} \geq p. \quad (65)$$

If $b = 1$, then the condition for selling is

$$k \geq \frac{\mu p + (1 - \mu)[1 - p + \sigma p(1 - \delta)]p}{\mu[v_1 + \sigma(v_2 - v_1)] + (1 - \mu)[1 - p + \sigma p(1 - \delta)]p} \equiv k_1(\sigma). \quad (66)$$

Because $v_1 + \sigma(v_2 - v_1) > p$, then $k_1(\sigma)$ is strictly lower than 1. Thus, if $k > k_1(\sigma)$, $b = 1$ is an equilibrium strategy.

(2) For $b = 0$ to be an equilibrium strategy for the insider we need that $kV(\sigma, 0) \leq p$.

Similar algebra reveals that this condition is equivalent to

$$k \leq \frac{p}{v_1 + \sigma(v_2 - v_1)} \equiv k_2(\sigma).$$

Algebra shows that $0 < k_2(\sigma) < k_1(\sigma)$.

(3) If $k \in (k_2(\sigma), k_1(\sigma))$, an equilibrium must be in strictly mixed strategies. Imposing the condition $kV(\sigma, b(\sigma)) = p$ leads to

$$b(\sigma) = \mu \frac{k[v_1 + \sigma(v_2 - v_1)] - p}{(1 - k)(1 - \mu)[1 - p + \sigma p(1 - \delta)]p}. \quad (67)$$

Simple substitution shows that $b(\sigma) = 0$ if $k = k_2(\sigma)$ and $b(\sigma) = 1$ if $k = k_1(\sigma)$.

Furthermore, $b(\sigma)$ is strictly increasing in k

$$\frac{\partial b}{\partial k} = \mu \frac{v_1 + \sigma(v_2 - v_2) - p}{p(1 - \mu)(1 - k)^2(1 - p + p\sigma(1 - \delta))} > 0. \quad (68)$$

Thus, $b(k) \in (0, 1)$ for $k \in (k_2, k_1)$. ■

Proposition 5.

Proof. To prove each case it is sufficient to find out the values of σ that satisfy the incentive compatibility constraints.

Case 1. Because $T(\sigma) \geq 0$, then the IC for project 1 cannot be satisfied. The IC for project 2 is trivially satisfied, thus $\sigma^* = 1$ is the only equilibrium.

Case 2. Suppose that $v_1 - v_2 > 0$. Suppose that there is an equilibrium with $\sigma > 0$. From the incentive compatibility constraints, it must be that $T(\sigma) > 0$. That implies $b(\sigma) = 1$. Thus, for $\sigma > 0$ we have

$$V(\sigma) = m(\sigma, 1)[v_1 + \sigma(v_2 - v_1)] + [1 - m(\sigma, 1)]p. \quad (69)$$

Notice that

$$V'(\sigma) = \frac{\partial m(\sigma, 1)}{\partial \sigma} [v_1 + \sigma(v_2 - v_1) - p] + (v_2 - v_1)m(\sigma, 1). \quad (70)$$

which is strictly negative when $v_1 - v_2 \geq 0$ because

$$\frac{\partial m(\sigma, 1)}{\partial \sigma} < 0, v_1 + \sigma(v_2 - v_1) - p > 0. \quad (71)$$

Thus, the highest possible value for the option to exit is $T(0)$. Thus, if

$$v_1 - v_2 \geq p(1 - \delta)T(0) \quad (72)$$

then the unique equilibrium occurs when $\sigma = 0$.

To analyze the other cases, first we define σ_L as

$$\sigma_L = \min \left\{ V^{-1} \left(\frac{p}{k} \right), 1 \right\}. \quad (73)$$

That is, σ_L is the lowest value of $\sigma \in [0, 1]$ that minimizes $T(\sigma, k)$. Note that, as $T(\sigma)$ is strictly decreasing for $\sigma \in [0, \sigma_L]$, its inverse T^{-1} for a given k is well defined in that domain.

If

$$v_1 - v_2 < p(1 - \delta)T(0, k) \text{ and } v_1 - v_2 > p(1 - \delta)T(1, k) \quad (74)$$

then there exists a unique $\sigma^* \in [0, \sigma_L]$ such that

$$\sigma^* = T^{-1} \left(\frac{v_1 - v_2}{p(1 - \delta)} \right). \quad (75)$$

Finally, if

$$v_1 - v_2 \leq p(1 - \delta)T(1, k). \quad (76)$$

then only equilibrium σ is given by $\sigma^* = 1$.

Case 3: $v_1 - v_2 = 0$. If $\sigma_L = 1$, then for any $\sigma < 1$ we have $T(\sigma, k) > 1$, thus the only equilibrium occurs with $\sigma^* = 1$. If $\sigma_L < 1$, then $T(\sigma, k) = 0$ for any $\sigma \in [\sigma_L, 1]$, proving the result. ■

Proposition 6.

Proof. Define

$$u_{priv}(k) \equiv \sigma_{private}u_2(k) + (1 - \sigma_{private})u_1(k) \quad (77)$$

$$u_{pub} \equiv \sigma_{public}v_2 + (1 - \sigma_{public})v_1. \quad (78)$$

Note that u_{pub} does not depend on k , as k affects the sale of shares only in the private case.

Let $w(k) \equiv W_{private} - W_{public}$. The insider chooses the private structure if $w(k) > 0$. With

$c_{priv} = c_{pub}$, the expression of $w(k)$ simplifies to $w(k) = u_{priv}(k) - u_{pub}$.

Part 1. Trivial

Part 2. If $v_2 > v_1$ then, by proposition 3 (after some algebra):

$$\sigma_{public} = \begin{cases} \frac{(v_2 - v_1) - \delta \mu p^2 (\theta - 1)}{(v_2 - v_1)(1 - \delta \mu)} & \text{if } \mu < \mu_L \equiv \frac{v_2 - v_1}{\delta p^2 (\theta - 1)} \\ 0 & \text{if } \mu \geq \mu_L. \end{cases} \quad (79)$$

Because $\sigma_{private} = 1$, we have that $u_{priv} = v_2 - L(k)$. To show that there exists a $k^* \in (0, 1)$ such that the insider chooses the private structure iff $k \geq k^*$, it suffices to show that the function $w(k) = W_{private}(k) - W_{public}$ has the following properties: $w(0) \leq 0$, $w(k)$ is nondecreasing and continuous, and $w(1) \geq 0$.

(i) $w(0) \leq 0$. Consider first the case of $\mu \geq \mu_L$. In such a case,

$$w(0) = W_{private}(0) - W_{public} = (1 - \mu) v_2 - v_1. \quad (80)$$

Because this function is decreasing in μ , it achieves a maximum at $\mu = \mu_L$, in which case it becomes

$$\left(1 - \frac{v_2 - v_1}{\delta p^2 (\theta - 1)}\right) v_2 - v_1 = -(v_2 - v_1) \frac{(1 + \delta)}{\delta p (\theta - 1)} < 0. \quad (81)$$

Thus $w(0)$ is also negative for any $\mu \geq \mu_L$.

What about $\mu < \mu_L$? In this case, we have

$$w(0) = (1 - \mu) v_2 - v_1 + \frac{(v_2 - v_1) - \delta \mu p^2 (\theta - 1)}{(1 - \delta \mu)} \quad (82)$$

Differentiating with respect to μ yields

$$\frac{\partial w(0)}{\partial \mu} = -v_2 + \frac{\delta p^2 (\theta - 1) (1 - \delta \mu) - \delta [(v_2 - v_1) - \delta p^2 (\theta - 1) \mu]}{(1 - \delta \mu)^2} = \quad (83)$$

$$= p \frac{\delta (p + 1) (\theta - 1) [(1 - \delta) - (1 - \delta \mu)^2] - (1 - \delta \mu)^2}{(1 - \delta \mu)^2} < 0. \quad (84)$$

Thus, the highest value of $w(0)$ occurs when $\mu \rightarrow 0$:

$$\lim_{\mu \rightarrow 0} w(0) = v_2 - v_2 = 0, \quad (85)$$

which implies that $w(0) < 0$ for all $\mu > 0$.

(ii) $w(k)$ is nondecreasing and continuous. We have to consider the different regions in which $b = 0$, $0 < b < 1$, and $b = 1$. In Region 3 ($k < k_2(1)$), we have $u_2(k, \mu) = \mu k v_2 + (1 - \mu) v_2$, which is increasing in k . In Region 2 ($k_2(1) \leq k \leq k_1(1)$), we have $u_2(k) = \mu p + (1 - \mu) v_2$, which is constant in k . In Region 1 ($k > k_1(1)$), we have $u_2(k) = k v_2 + (1 - \mu) \delta p (1 + \theta p) (1 - k)$, which is increasing in k . Thus, $u_2(k)$ is increasing in regions 1 and 3, and constant in region 2. Therefore, $w(k)$ is nondecreasing (continuity is easily verified).

(iii) $w(1) \geq 0$. This is trivially verified: $w(1) = v_2 - \sigma_{pub} v_2 - (1 - \sigma_{pub}) v_1 \geq 0$.

As a result there exists a k^* such that $w(k^*) = 0$.

To prove uniqueness, we have to rule out $w(k) = 0$ for $k \in [k_2, k_1]$. As $u_{priv}(k)$ is constant in this region, we only need to show that $w(k_2) < 0$. If $\mu \geq \mu_L$, then $u_{pub} = v_1$. So, $w(k_2, \mu) = \mu p + (1 - \mu) v_2 - v_1$, which is decreasing in μ . Substituting the expression of μ_L , we have that $w(k_2, \mu_L) = -\frac{v_2 - v_1}{p(\theta - 1)} < 0$ and so $w(k_2, \mu) < 0$. If $\mu < \mu_L$, then $w(k_2, \mu) = \mu p + (1 - \mu) v_2 - v_1 - \frac{(v_2 - v_1) - \delta p^2 (\theta - 1) \mu}{(1 - \delta \mu)}$. We have $w(k_2, \mu) = 0$ trivially if $\mu = 0$, which is ruled out by assumption. For $\mu > 0$, we have $w(k_2, \mu) < 0 \Leftrightarrow \mu < 1$, which is always true. Therefore, $w(k_2, \mu) < 0$ for all μ , which implies that $k^* \notin [k_2, k_1]$. As $w(k, \mu)$ is nondecreasing for $k \leq k_1(1)$ and is increasing for $k > k_1(1)$, we have a unique k^* and $k^* > k_1(1)$. ■

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