DISCUSSION PAPER SERIES

No. 7690

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INTERNATIONAL MACROECONOMICS

Centre for Economic Policy Research

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CEPR Discussion Paper No. 7690

February 2010

ABSTRACT

Financial Development and the Patterns of International Capital Flows

We develop a tractable, two-country, overlapping-generations model and show that cross-country differences in financial development can explain three recent empirical patterns of international capital flows: Financial capital flows from relatively poor to relatively rich countries while foreign direct investment flows in the opposite direction; net capital flows go from poor to rich countries; despite its negative net international investment position, the US receives a positive net international investment income. We also explore the welfare and distributional effects of international capital flows and show that the direction of capital flows may change along the convergence process of a developing country.

Matsuyama (*Econometrica* 2004) argues that, in the presence of credit market imperfections, financial market globalization may lead to a steady-state equilibrium in which fundamentally identical countries end up with different levels of per-capita output. We show that this *symmetry-breaking* property depends crucially on the assumption that investment operates on the extensive rather than the intensive margin.

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1 Introduction

Standard international macroeconomics predicts that capital flows from capital-rich countries, where the marginal return on investment is low, to capital-poor countries, where the marginal return is high. Furthermore, there should be no difference between gross and net capital flows, as capital movements are unidirectional.

The patterns of international capital flows observed in the past 20 years, however, stand in stark contrast to these predictions (Lane and Milesi-Ferretti, 2001, 2006, 2007). First, since 1998, the average per-capita income of countries running current account surpluses has been below that of the deficit countries, i.e., net capital flows have been "uphill" from poor to rich countries (Prasad, Rajan, and Subramanian, 2006, 2007). Second, many developing economies, including China, Malaysia, and South Africa, are net importers of foreign direct investment (hereafter, FDI) and net exporters of financial capital at the same time, while developed countries such as France, the United Kingdom, and the United States exhibit the opposite pattern (Ju and Wei, 2007). Third, despite its negative net international investment position since 1986, the U.S. has been receiving a positive net investment income until 2005 (Gourinchas and Rey, 2007; Hausmann and Sturzenegger, 2007; Higgins, Klitgaard, and Tille, 2007).

Recent research offers two main explanations to these empirical facts. Devereux and Sutherland (2009) and Tille and van Wincoop (2010) focus on the cross-country risk-sharing investors can achieve by diversifying their portfolios globally. International portfolio investment is determined by the cross-correlation patterns of aggregate shocks at the country level. These models do not distinguish between FDI and portfolio equity investment and, therefore, offer no explanation for the second pattern.

The other strand of literature focuses on domestic financial market imperfections (Aoki, Benigno, and Kiyotaki, 2009; Caballero, Farhi, and Gourinchas, 2008; Smith and Valderrama, 2008). Matsuyama (2004) shows that, in the presence of credit market imperfections, financial market globalization may lead to a steady-state equilibrium in which fundamentally identical countries end up with different levels of per capita output, a result he calls "symmetry breaking". Furthermore, financial capital flows from poor to rich countries in the steady state. However, Matsuyama (2004) does not address FDI flows. Mendoza, Quadrini, and Ríos-Rull (2009) analyze the joint determination of financial capital flows and FDI in a heterogeneous-agent model with uninsurable idiosyncratic endowment and investment risks. The precautionary savings motive plays the crucial role. Ju and Wei (2007) show in a static model that, when both FDI and financial capital flows are allowed, all financial capital leaves the country where credit market imperfections are more severe, while FDI flows into this country. Thus, capital mobility allows investors to fully *bypass* the underdeveloped financial system.

Our paper extends the second strand of literature and provides a tractable, two-country, overlapping-generations model to explain the three recent empirical facts. Our model builds on the notion that individuals in an economy differ in the productivity (Kiyotaki and Moore, 1997). In the frictionless case, all capital would be operated by the most productive individuals and, the rates of return on loan and equity capital would be equal to the marginal return on investment. Due to financial frictions, however, individuals are subject to borrowing constraints. The constraint on the aggregate credit demand has a general equilibrium effect, keeping the rate of return on loans (hereafter, the loan rate) lower and the rate of return on equity capital (hereafter, the equity rate) higher than the marginal return on investment.[1] Thus, financial frictions distort the two interest rates and generate an equity premium in this deterministic model.

Following Matsuyama (2004), we take the tightness of the borrowing constraints as a measure of a country's level of financial development. In a more financially developed country, credit contracts can be enforced and borrowers can be monitored more effectively. Thus, the individuals can borrow more from financial institutions. The two countries in our model differ fundamentally only in the level of financial development. Under international financial autarky (hereafter, IFA), interest rates depend on two factors. First, a lower aggregate capital-labor ratio implies a higher marginal return on investment and a higher equity and loan rate. We call this the *neoclassical* effect, because it arises from the concavity of the neoclassical production function with respect to the capital-labor ratio. Second, for a given capital-labor ratio, a lower level of financial development implies a lower aggregate credit demand, which leads to a lower loan rate and a higher equity rate. We call this the *credit-demand* effect of financial development. Financial frictions distort only the interest rates but not production efficiency under IFA in our model.[2] In the steady state, financial development affects the interest rates only via the credit-demand channel but not the neoclassical channel. That is, the loan rate is higher while the equity rate is lower in the more financially developed country.

Under full international capital mobility, the more financially developed country receives net capital inflows and becomes richer than the less financially developed country in the steady state. Net capital flows are "uphill" from the poor to the rich country.[3] In the

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[1] The overlapping-generations framework together with certain assumptions ensures that the aggregate credit supply is perfectly inelastic to the loan rate. Thus, we can isolate the effect of financial frictions on the aggregate credit demand and on the interest rates. Caballero, Farhi, and Gourinchas (2008) assume that agents have a constant probability of death, which has the similar effect.

[2] This is due to the assumption that the less productive users of capital do not have an alternative production technology except inelastically lending to the productive individuals. In von Hagen and Zhang (2009), we relax this assumption and, as a result, financial frictions distort production efficiency as well as interest rates. Our qualitative results still hold but the model then becomes less tractable.

[3] "Uphill" capital flows occur between two countries with the same level of financial development in

steady state, financial capital to flow from the poor to the rich country, while FDI flows in the opposite direction. Since the rich country receives a higher return on its FDI than it pays on its foreign debts, it gets a positive net investment income despite its negative net position of international investment. Essentially, the more financially developed country "exports" its financial services in the form of two-way capital flows and receives a positive net reward. As our first contribution, we show that cross-country differences in financial development can explain the three empirical facts.

Ju and Wei (2007) assume cross-country differences in factor endowments, financial developments, corporate governance, and property rights protection to generate two-way capital flows. In contrast, difference in financial development alone can do that in our model. The static model of Ju and Wei (2007) is useful for analyzing the immediate impacts of capital account liberalization, but not the transitional and long-run effects, while our overlapping-generations model facilitates the short-run and the long-run analyses. In the absence of uncertainty, international capital flows vanish in Devereux and Sutherland (2009); Mendoza, Quadrini, and Ríos-Rull (2009); Tille and van Wincoop (2010). In contrast, international capital flows still exist in our deterministic model. This way, we show that differences in financial development rather than uncertainty are the fundamental factor driving the recent patterns of international capital flows.[4] In this sense, our model shares the similar feature as Caballero, Farhi, and Gourinchas (2008). However, Caballero, Farhi, and Gourinchas (2008) assume that foreign direct investors from the more financially developed country have the advantage in capitalizing the investment revenue in the host country, while we do not need this extra assumption.

We also analyze a scenario where one country is more financially developed and in its steady state, while the other country is less financially developed and below its steady state before capital account liberalization. In so doing, we study the interactions of capital flows and the economic convergence of the second country. We show that, if the initial capital-labor ratio in the second country is very low, the neoclassical effect dominates the credit-demand effect so that the loan rate is higher in the second country than in the first country under IFA and so is the equity rate. Upon capital account liberalization, both financial capital and FDI flow into the poor country. Thus, capital flows are one-way and "downhill". As the capital-labor ratio in the poor country grows over time, the credit-demand effect begins to dominate the neoclassical effect and financial capital flows "uphill", but its size may still be dominated by that of "downhill" FDI. Thus, gross capital flows are two-way and net capital flows are "downhill". In both cases, capital account liberalization facilitates net capital flows into the poor country, which enhances

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Matsuyama (2004), while two countries differ in the level of financial development in our model.

[4]Aggregate or idiosyncratic uncertainty is important for the quantitative purpose in Devereux and Sutherland (2009); Mendoza, Quadrini, and Ríos-Rull (2009); Tille and van Wincoop (2010).

capital accumulation and speeds up its convergence.

As the capital-labor ratio in the poor country grows further, "uphill" financial capital flows begin to dominate "downhill" FDI. Thus, gross capital flows are two-way and net capital flows are "uphill". Since net capital outflows hamper capital accumulation, the poor country eventually converges to a steady state with a lower capital-labor ratio than under IFA. As our second contribution, we show that for a developing country which is below its steady state, the patterns of capital flows may change and even reverse along its convergence process. More importantly, capital account liberalization may offer this country a faster capital accumulation in the short run but at the cost of a lower output level in the long run. In order to reduce or avoid the long-run cost, this country should liberalize capital account together with promoting financial development.

Financial capital flows affect the owners of credit capital and equity capital in the opposite way and so do FDI flows. Due to transitional effects, capital flows also affect the intergenerational income distribution. By pointing out its distributional effect, our model offers an explanation for why capital account liberalization often encounters both support and opposition in a given country.

We assume that the mass of individuals who can produce is fixed in each country, while the size of each production project is endogenously determined. Thus, investment occurs on the intensive margin instead of the extensive margin as in Matsuyama (2004). We show that, under various forms of capital mobility, countries with identical fundamentals have the same, unique, and stable steady state. As our third contribution, we show that Matsuyama's *symmetry-breaking* property depends critically on the assumption of a fixed size of individual projects and thus, investment occurs on the extensive margin. In our model, countries with identical fundamentals except the level of financial development have the same steady-state output level under IFA, but they have different steady-state levels of output under capital mobility. Thus, capital mobility also breaks the symmetry in our model, but it does so for the reason different from that in Matsuyama (2004).

The rest of the paper is structured as follows. Section 2 sets up the model under IFA. Section 3 proves the properties of the steady state and the patterns of international capital flows under capital mobility. Section 4 concludes with the main findings. Appendix collects the technical proofs and relevant discussions.

2   The Model under International Financial Autarky

We use an overlapping-generations model closely related to Matsuyama (2004). The world economy consists of two countries, Home (H) and Foreign (F). There are two types of goods, a final good, which is internationally tradable and serves as the numeraire, and a capital good, which is not traded internationally. The price of the capital good in

country $i \in \{H, F\}$ and period t is denoted by v_t^i. The final good can be either consumed or transformed into capital goods. At the beginning of each period, final goods Y_t^i are produced with capital goods K_t^i and labor L_t^i in a Cobb-Douglas fashion. Capital goods fully depreciate after production. Capital goods and labor are priced at their respective marginal products in terms of final goods. To summarize,

$$Y_t^i = \left(\frac{K_t^i}{\alpha}\right)^{\alpha} \left(\frac{L_t^i}{1-\alpha}\right)^{1-\alpha}, \qquad \text{where} \qquad \alpha \in (0, 1), \qquad (1)$$

$$v_t^i K_t^i = \alpha Y_t^i \qquad \text{and} \qquad w_t^i L_t^i = (1 - \alpha) Y_t^i. \qquad (2)$$

There is no uncertainty in the economy. In this section, we assume that capital flows are not allowed between the two countries.

In both countries, the population consists of two generations, the old and the young, which live for two periods each. There is no population growth and the population size of each generation in each country is normalized to one. Agents consume only when old. Young agents are endowed with a unit of labor which they supply inelastically to the production of final goods $L_t^i = 1$ at the wage rate w_t^i in period t. Each generation consists of two types of agents of mass η and $1 - \eta$, respectively, which we call *entrepreneurs* and *workers*. Only young entrepreneurs are endowed with the productive projects and it takes one period to produce capital goods using final goods.

Consider any particular worker born in period t. With no other investment opportunity available to him[5], the worker lends his entire labor income inelastically to the credit market at a gross interest rate of r_t^i in period t to finance his consumption in period $t + 1$,

$$c_{t+1}^{i,w} = w_t^i r_t^i. \qquad (3)$$

Consider any particular entrepreneur born in period t. The entrepreneur invests i_t^i units of final goods into his project in period t and produces $R i_t^i$ units of capital goods in period $t + 1$. Given the gross loan rate of r_t^i, he finances the investment i_t^i with the debt $z_t^i = i_t^i - w_t^i$ and the equity capital, w_t^i. Due to limited commitment problems, however, he can borrow only against a fraction of the project revenues,

$$r_t^i z_t^i = r_t^i(i_t^i - w_t^i) \leq \theta^i R i_t^i v_{t+1}^i. \qquad (4)$$

As in Matsuyama (2004, 2007, 2008), the level of financial development in country i is measured by $\theta^i \in (0, 1]$. θ^i is higher in countries with more sophisticated financial and legal systems, better creditor protection, and more liquid asset market. Thus, θ^i captures

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[5]Excluding workers from other savings alternatives facilitates the closed-form solution, but it may seem implausible. von Hagen and Zhang (2009) show that allowing workers to have other investment opportunity does not change our results qualitatively but the model becomes less tractable.

a wide range of institutional factors.[6] We assume that the two countries differ only in the level of financial development, i.e., $0 < \theta^H < \theta^F \leq 1$.

Let $\lambda_t^i \equiv \frac{i_t^i}{w_t^i}$ denote the investment-equity ratio of the entrepreneurial project and I_t^i denotes the aggregate project investment in country i and period t. Under IFA, the credit market equilibrium condition,

$$\eta(i_t^i - w_t^i) = (1 - \eta)w_t^i, \quad \Rightarrow \quad I_t^i = \eta i_t^i = w_t^i, \tag{5}$$

implies that aggregate labor income in period t is invested by young entrepreneurs. Thus, the investment-equity ratio is constant at $\lambda_t^i = \frac{1}{\eta}$ and the level of financial development θ^i does not affect aggregate investment. Intuitively, the aggregate credit demand is lower in the country with a lower level of financial development. Given the perfectly inelastic aggregate credit supply, the credit market clears at a lower loan rate.

After repaying his debt in period $t + 1$, the entrepreneur gets $Ri_t^i v_{t+1}^i - r_t^i z_t^i$ as net return. The equity rate is defined as the rate of return on the equity capital (w_t^i),

$$\Gamma_t^i \equiv \frac{Ri_t^i v_{t+1}^i - r_t^i z_t^i}{w_t^i} = Rv_{t+1}^i + (Rv_{t+1}^i - r_t^i)\frac{(1 - \eta)}{\eta} \geq r_t^i. \tag{6}$$

Intuitively, for each unit of equity capital invested in the project, the entrepreneur gets Rv_{t+1}^i as the marginal return. Additionally, he can borrow $(\lambda_t^i - 1) = \frac{(1-\eta)}{\eta}$ units of debt which gives him an extra rate of return, $Rv_{t+1}^i - r_t^i$. The term $(Rv_{t+1}^i - r_t^i)\frac{(1-\eta)}{\eta}$ captures the *leverage* effect. In equilibrium, the equity rate should be no less than the loan rate; otherwise, he would rather lend than borrow. The inequality in (6) is equivalent to $r_t^i \leq Rv_{t+1}^i$ and can be considered as his participation constraint.

If $r_t^i < Rv_{t+1}^i$, the entrepreneur borrows to the limit, i.e., he finances the investment i_t^i using $z_t^i = \frac{\theta^i Ri_t^i v_{t+1}^i}{r_t^i}$ units of debt and w_t^i units of equity capital in period t. After repaying the debt in period $t + 1$, he gets $(1 - \theta^i)Ri_t^i v_{t+1}$ as the project return. Given the investment-equity ratio at $\lambda_t^i \equiv \frac{i_t^i}{w_t^i} = \frac{1}{\eta}$, the equity rate has a closed-form solution,

$$\Gamma_t^i = \frac{(1 - \theta^i)Ri_t^i v_{t+1}^i}{w_t^i} = \frac{(1 - \theta^i)Rv_{t+1}^i}{\eta}. \tag{7}$$

Combining equations (6) and (7), we get a closed-form solution for the loan rate,

$$r_t^i = \frac{\theta^i Rv_{t+1}^i}{1 - \eta}. \tag{8}$$

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[6]The pledgeability, θ, can be argued in various forms of agency costs, e.g., costly state verification by Townsend (1979), inalienable human capital by Hart and Moore (1994), or unobservable project (effort) choices by Holmstrom and Tirole (1997). In order to compare our results with Matsuyama (2004), we minimize the deviation of our model setting from his by choosing this simplest form of borrowing constraints. The pledgeability of individual projects may depend on idiosyncratic features. As we focus on the aggregate implications of financial development, we assume that entrepreneurs investing in country i are subject to the same θ^i for simplicity.

If $r_t^i = Rv_{t+1}^i$, the entrepreneur does not borrow to the limit. According to equation (6), the equity rate is equal to the loan rate, $\Gamma_t^i = r_t^i = Rv_{t+1}^i$. Lemma 1 summarizes the interest rate patterns with respect to the level of financial development.

Lemma 1. *Let $\bar{\theta} \equiv 1 - \eta$. For $\theta^i \in (\bar{\theta}, 1]$, the borrowing constraints are not binding and $\Gamma_t^i = r_t^i = Rv_{t+1}^i$; for $\theta^i \in (0, \bar{\theta})$, the borrowing constraints are binding and $\Gamma_t^i = \frac{(1-\theta^i)Rv_{t+1}^i}{\eta} > Rv_{t+1}^i > \frac{\theta^i Rv_{t+1}^i}{1-\eta} = r_t^i$.*

Given the labor income w_t^i, the entrepreneur chooses the project investment i_t^i in period t to maximize his consumption in period $t + 1$,

$$c_{t+1}^{i,e} = v_{t+1}^i R i_t^i - r_t^i z_t^i = w_t^i \Gamma_t^i, \tag{9}$$

subject to the borrowing constraint (4) and the participation constraint (6). Note that only one of the two constraints can be strictly binding in equilibrium.

Since aggregate labor income is invested in the entrepreneurial projects in period t, aggregate output of capital goods available for production in period $t + 1$ is

$$K_{t+1} = R I_t^i = R w_t^i. \tag{10}$$

The market-clearing condition for final goods in period t is

$$C_t^i + I_t^i = Y_t^i, \tag{11}$$

where $C_t^i = \eta c_t^{i,e} + (1-\eta) c_t^{i,w}$ is the aggregate consumption of the old generation in period t. We measure the social welfare of the generation born in period t and country i using its aggregate consumption when old, C_{t+1}^i.

Definition 1. *Given the level of financial development θ^i, a market equilibrium in country $i \in \{H, F\}$ under IFA is a set of allocations of workers, $\{c_t^{i,w}\}$, entrepreneurs, $\{i_t^i, z_t^i, c_t^{i,e}\}$, and aggregate variables, $\{Y_t^i, K_t^i, I_t^i, C_t^i, w_t^i, v_t^i, r_t^i, \Gamma_t^i\}$, satisfying equations (1)-(5) and (9)-(11) as well as Lemma 1.*

Since the size of the working population is normalized at one, the capital-labor ratio coincides with the aggregate capital stock. Thus, K_t^i also denotes the capital-labor ratio.

According to equations (1), (2), and (10), the model dynamics can be characterized by a nonlinear first-order difference equation on wages,

$$w_{t+1}^i = (1-\alpha)Y_{t+1}^i = \left(\frac{K_{t+1}^i}{\rho}\right)^\alpha = \left(\frac{Rw_t^i}{\rho}\right)^\alpha, \quad \text{where} \quad \rho \equiv \frac{\alpha}{1-\alpha}. \tag{12}$$

Given $\alpha \in (0, 1)$, the phase diagram of wages is concave and starts from the origin. Its slope, $\frac{dw_{t+1}^i}{dw_t^i} = \alpha \left(\frac{R}{\rho}\right)^\alpha (w_t^i)^{\alpha-1}$, converges to $+\infty$ for $w_t^i \to 0$ and to 0 for $w_t^i \to +\infty$. Thus, there exists a unique and stable non-zero steady state with the wage at,

$$w_{IFA} = \left(\frac{R}{\rho}\right)^\rho, \tag{13}$$

where a variable with subscript IFA denotes its steady-state value under IFA. According to equations (12) and (13), the wage dynamics are independent of the level of financial development θ^i and, thus, the wage converges to the same steady state in the two countries. So do aggregate output and capital.

According to Lemma 1, for $\theta^i \in [1-\eta, 1]$, the two interest rates are equal to the marginal return on investment, $r_t^i = \Gamma_t^i = Rv_{t+1}^i = R^\alpha \rho^{1-\alpha^2}(K_t^i)^{\alpha(\alpha-1)}$, depending negatively on the capital-labor ratio, K_t^i. Thus, the two interest rates are higher in the country with a lower capital-labor ratio. We call this the *neoclassical* effect, because it arises from the concavity of the neoclassical production function with respect to the capital-labor ratio. The neoclassical effect is independent of the level of financial development.

For $\theta^i \in (0, 1-\eta)$, besides the neoclassical effect, the loan rate is affected positively by the level of financial development, $r_t^i = Rv_{t+1}^i \frac{\theta^i}{(1-\eta)}$. Given the capital-labor ratio, the loan rate is higher in the country with a higher θ, reflecting the general equilibrium effect of the larger aggregate credit demand. We call this the *credit-demand* effect of financial development, captured by $\frac{\theta^i}{(1-\eta)} \in (0,1)$. According to equation (6), besides the neoclassical effect, the equity rate is also affected by the leverage effect. Given the capital-labor ratio, the spread $(Rv_{t+1}^i - r_t^i)$ is lower in the country with a higher θ so that, with the debt-equity ratio constant at $\frac{(1-\eta)}{\eta}$, the leverage effect is smaller. The equity rate is thus affected negatively by the level of financial development, $\Gamma_t^i = Rv_{t+1}^i \frac{(1-\theta^i)}{\eta}$.

Under IFA, the financial frictions in our model do not distort production efficiency, but they distort the two interest rates [7] In so doing, financial frictions have a distributional effect on the welfare of borrowers (entrepreneurs) and lenders (workers).

Aggregate labor income is invested in the entrepreneurial projects in period t, $I_t^i = w_t^i = (1-\alpha)Y_t^i$, and aggregate output of capital goods has the value of $v_{t+1}^i K_{t+1}^i = \alpha Y_{t+1}^i$ in period $t+1$. In the steady state, $Y_{t+1}^i = Y_t^i = Y^i$, and the marginal return on investment is $v^i R = \frac{v^i K^i}{w^i} = \frac{\alpha Y^i}{(1-\alpha)Y^i} = \rho$. Plugging it into Lemma 1, we get the steady-state pattern of interest rates, which is summarized in Proposition 1.

Proposition 1. *For $\theta^i \in (\bar{\theta}, 1]$, the two interest rates are independent of the level of financial development, $r^i = \Gamma^i = \rho$; for $\theta^i \in (0, \bar{\theta})$, the loan rate rises and the equity rate declines in the level of financial development, $r^i = \frac{\theta^i \rho}{1-\eta} < \rho < \Gamma^i = \frac{(1-\theta^i)\rho}{\eta}$.*

Figure 1 shows the steady-state pattern of output, wages, and interest rates, with the horizontal axis denoting $\theta \in (0, 1]$, where $\theta^U \equiv \bar{\theta} = 1 - \eta$. Since financial frictions in our

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[7]The equity premium, $\Gamma_t - r_t > 0$, in the case of $\theta^i \in (0, \bar{\theta})$ arises from two factors, i.e., the difference in productivity and the binding borrowing constraints. For $\theta \in (0, \bar{\theta})$, the constraint on aggregate credit demand keeps the loan rate lower than the marginal return on investment. The equity premium is the reward to entrepreneurs' advantage in productivity. For $\theta \in (\bar{\theta}, 1]$, the unconstrained aggregate credit demand raises the loan rate to the marginal return on investment and the equity premium vanishes.
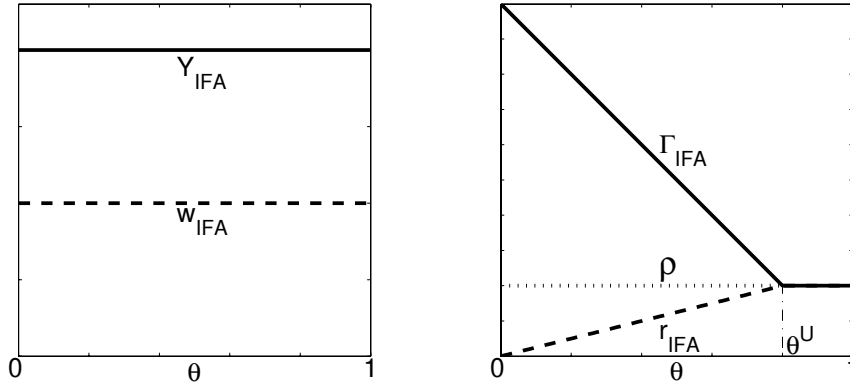
Figure 1: Steady-State Pattern under IFA

model does not affect production efficiency, aggregate output is independent of θ^i and the marginal return on investment is constant at ρ. Thus, the cross-country loan rate (equity rate) differentials only depend on the credit-demand (leverage) effect.

According to Proposition 1, for $\theta^H \in [0, \bar{\theta})$ and $\theta^F \in (\theta^H, 1]$, the loan rate is lower while the equity rate higher in country H than in county F; for $\bar{\theta} \leq \theta^H < \theta^F \leq 1$, the borrowing constraints are not binding in the two countries so that the credit-demand effect and the leverage effect are muted. Then, the two interest rates coincide with the marginal return on investment, which is same in the two countries.

3   International Capital Mobility

We consider three scenarios of capital mobility, *free mobility of financial capital* under which individuals are allowed to lend abroad but entrepreneurs are not allowed to make direct investments abroad, *free mobility of FDI* under which entrepreneurs are allowed to make direct investments abroad but individuals are not allowed to lend abroad, and *full capital mobility* under which individuals are allowed to lend abroad and entrepreneurs are allowed to make direct investments abroad.

Without loss of generality, we study the case of $0 < \theta^H < \theta^F \leq \bar{\theta}$, where the borrowing constraints are binding in the two countries in the steady state under three scenarios of capital mobility. Appendix B provides a general analysis for $\theta^i \in (0, 1)$, where $i \in \{H, F\}$.

In subsections 3.1, 3.2, and 3.3, we assume that the two countries are in the steady state under IFA before capital mobility is allowed from period $t = 0$ on. We analyze capital flows in the short run and in the long run between two countries with the same initial output level. The results on the uniqueness and stability of the market equilibrium before and after capital mobility do not depend on this assumption.

In subsection 3.4, we assume that country F is in the steady state and country H is below the steady state under IFA before capital mobility is allowed from period $t = 0$ on.

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We study the interactions between capital flows and economic convergence of country H.

Let Υ_t^i and Ω_t^i denote the aggregate outflows of financial capital and FDI from country i in period t, respectively, with negative values indicating capital inflows. Financial capital outflows reduce the domestic credit supply, $(1 - \eta)w_t^i - \Upsilon_t^i$, while FDI outflows reduce the aggregate equity capital available for the domestic investment, $\eta w_t^i - \Omega_t^i$. Therefore, FDI flows raise the aggregate credit demand in the host country and reduce that in the parent country.[8] With these changes, the analysis in section 2 carries through for the cases of capital mobility, due to the linearity of preferences, projects, and borrowing constraints.

3.1 Free Mobility of Financial Capital

The Cobb-Douglas production function implies that

$$v_{t+1}^i = (w_{t+1}^i)^{-\frac{1}{\rho}} \quad \text{and} \quad I_t^i = \frac{K_{t+1}^i}{R} = \frac{\rho}{R}(w_{t+1}^i)^{\frac{1}{\alpha}}. \tag{14}$$

Free mobility of financial capital equalizes the loan rates across the border, $r_t^H = r_t^F = r_t^*$. Given the domestic equity capital ηw_t^i, the aggregate domestic investment is,

$$I_t^i = \lambda_t^i \eta w_t^i = \frac{\eta w_t^i}{1 - \frac{\theta^i R v_{t+1}^i}{r_t^*}}. \tag{15}$$

Using equations (14) and (15) to substitute away I_t^i and v_{t+1}^i, we get

$$\eta w_t^i = \frac{\rho}{R}(w_{t+1}^i)^{\frac{1}{\alpha}} - \frac{\theta^i \rho}{r_t^*} w_{t+1}^i. \tag{16}$$

3.1.1 Existence, Uniqueness, and Stability of the Steady State

Let a variable with subscript FCF denote its steady-state value under free flows of financial capital.

Proposition 2. *Given the world loan rate r_{FCF}^*, there exists a unique and stable non-zero steady state with the wage rate at $w_{FCF}^i = w_{IFA} \left[\eta + (1 - \eta)\frac{r_{IFA}^i}{r_{FCF}^*} \right]^\rho$.*

The solid line and the dash-dotted line in the left panel of figure 2 show the phase diagrams of wages under IFA and under free mobility of financial capital, respectively, given a fixed world loan rate at $r_t^* = r_{IFA}^i$. In both cases, wages converge monotonically and globally to a unique and stable steady state (point A).

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[8]In the case of debt default, the project liquidation value depends on the efficiency of the legal institution, the law enforcement, and the asset market in the host country. Thus, we assume that entrepreneurs making FDI borrow only from the host country and are subject to the borrowing constraints there. Alternatively, we can assume that entrepreneurs may borrow only in their parent country no matter where they invest, since the financial institutions in their parent country have better information on the credit record, social network, and business activities of the entrepreneurs. The realistic case should be a hybrid of these two. Our results hold under the two alternative assumptions.
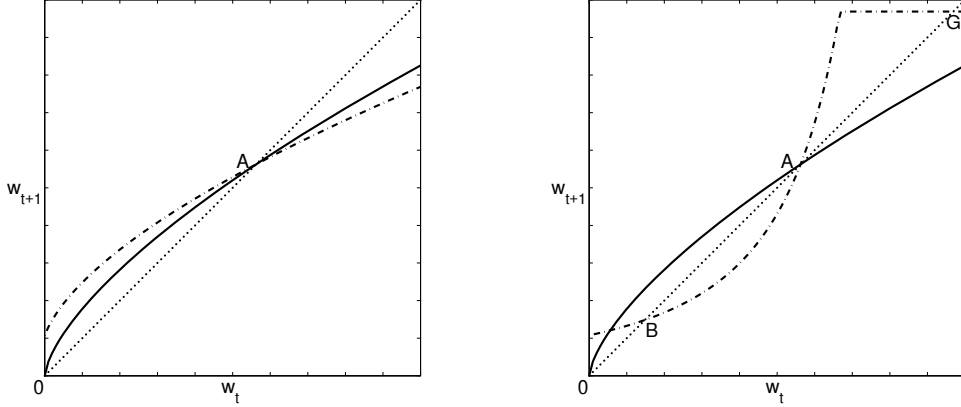
Figure 2: The Phase Diagrams of Wage

Matsuyama (2004) assumes that the size of every production project is fixed at $i_t^i = 1$, while the mass of individuals in a country who become entrepreneurs is endogenously determined. He shows that, at a given world loan rate, free mobility of financial capital may lead to an equilibrium with multiple steady states. In contrast, we assume that the mass of entrepreneurs in a country is fixed at η, while the investment size of any project i_t^i is endogenously determined. Since his model and ours differ only in this one aspect, it is straightforward to illustrate Matsuyama's result in the current framework.

The borrowing constraints, if binding, take the same form in both models,

$$ r_t^*(1 - \frac{w_t^i}{i_t^i}) = \theta^i R v_{t+1}^i = \theta^i R (w_{t+1}^i)^{-\frac{1}{\rho}}. \tag{17} $$

Lemma 2. *Given the world loan rate r_t^*, for $w_t^i \in [0, 1 - \theta^i]$, the phase diagram of wages in Matsuyama (2004) described by $r_t^*(1 - w_t^i) = \theta^i R (w_{t+1}^i)^{-\frac{1}{\rho}}$ is strictly convex, and w_{t+1}^i monotonically increases in w_t^i with an intercept on the vertical axis at $w_{t+1}^i = \left[\frac{\theta^i R}{r_t^*} \right]^\rho$; for $w_t^i > 1 - \theta^i$, the phase diagram of wages is flat with $w_{t+1}^i = \left(\frac{R}{r_t^*} \right)^\rho$.*

The solid line in the right panel of figure 2 shows the phase diagram of wages under IFA in Matsuyama (2004), which is the same as in our model and gives rise to the unique and stable steady state at point A. The dash-dotted line shows the phase diagram under free mobility of financial capital in his model, given a fixed world loan rate $r_t^* = r_{IFA}^i$. The phase diagram is convex for wages below a threshold value. Thus, the steady state at point A becomes unstable under free mobility of financial capital, because the slope of the phase diagram at point A is larger than one. There are two stable steady states at points B and G. This implies that countries with the identical fundamentals (including θ) and, thus, the same steady state under IFA may end up with different levels of income under free mobility of financial capital. Thus, Matsuyama (2004) claims that, in the presence of credit market imperfections, financial capital flows may result in the symmetry breaking.[9]

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[9]The symmetry-breaking property depends on the specific value of the world loan rate and the steady-

According to equation (17), given a world loan rate and a fixed size of project investment as in Matsuyama (2004), a marginal increase in the current wage reduces the credit demand of each borrower, $(1 - w_t^i)$, and the debt-investment ratio, $\frac{z_t^i}{i_t^i} = (1 - \frac{w_t^i}{i_t^i}) = (1 - w_t^i)$. More domestic individuals can borrow at the prevailing world loan rate and produce. If the current wage w_t^i exceeds the level corresponding to point A, the debt-investment ratio will decrease and more individuals will become entrepreneurs. The higher the current wage w_t^i, the larger the expansion of aggregate investment and, consequently, the larger the increase in aggregate output and the wage in the next period. This explains the convexity of the phase diagram of wages in Matsuyama's model.

In contrast, given a constant world loan rate and a fixed mass of entrepreneurs in our model, a marginal increase in the current wage enables entrepreneurs to borrow and invest more. According to equation (17), the increase in the current investment i_t^i partially offsets the negative effect of a marginal increase in the current wage w_t^i on the debt-investment ratio, $\frac{z_t^i}{i_t^i} = (1 - \frac{w_t^i}{i_t^i})$, and then on the wage in the next period, w_{t+1}^i. The higher the current wage, the smaller the investment expansion and, consequently, the smaller the increase in aggregate output and the wage in the next period. This explains the concavity of the phase diagram of wages and the uniqueness of the steady state in our model.

3.1.2 Interest Rates and Capital Flows

Proposition 3. *There exists a unique world loan rate r_t^* that clears the world credit market every period. In the steady state, $r_{FCF}^* \in (\underline{r}^*, r_{IFA}^F)$, where $\underline{r}^* \equiv \frac{r_{IFA}^H + r_{IFA}^F}{2}$.*

Intuitively, given the steady-state loan rates in the two countries under IFA, $r_{IFA}^H < r_{IFA}^F$, the steady-state loan rate under free mobility of financial capital lies between them.

Proposition 4. *Under free mobility of financial capital, if the borrowing constraints are binding in country i, $\Gamma_t^i = \frac{(1-\theta^i)\rho}{\eta} \frac{w_{t+1}^i}{w_t^i}$. In the steady state, $\Gamma_{FCF}^i = \frac{(1-\theta^i)\rho}{\eta} = \Gamma_{IFA}^i$.*

Given the binding borrowing constraints, entrepreneurs use $\frac{z_t^i}{i_t^i} = \frac{\theta^i R v_{t+1}^i}{r_t^*}$ units of loans and $\frac{w_t^i}{i_t^i} = \frac{(1-\theta^i)R v_{t+1}^i}{\Gamma_t^i}$ units of equity capital to finance a unit of investment in period t.

$$1 = \frac{z_t^i}{i_t^i} + \frac{w_t^i}{i_t^i} = \frac{\theta^i R v_{t+1}^i}{r_t^*} + \frac{(1 - \theta^i) R v_{t+1}^i}{\Gamma_t^i} \quad \Rightarrow \quad \frac{1 - \theta^i}{\Gamma_t^i} = \frac{1}{R v_{t+1}^i} - \frac{\theta^i}{r_t^*}. \tag{18}$$

Given θ^i, financial capital flows affect the equity rate in two ways. Consider country H. First, financial capital outflows raise the loan rate and the decline in the spread tends to reduce the equity rate. Second, financial capital outflows have a general equilibrium effect, i.e., the decline in the current aggregate investment reduces aggregate output in period $t + 1$ and the price of capital goods rises, which tends to raise the equity rate.

state equilibrium may be unique under other values of world loan rate. See Matsuyama (2004) for details.

Financial capital outflows in period $t = 0$ reduce the labor income in period $t = 1$, $w_1^i < w_0^i = w_{IFA}$, and the equity rate is lower in period $t = 0$, $\Gamma_0^i = \frac{(1-\theta^i)\rho}{\eta} \frac{w_1^i}{w_0^i} < \frac{(1-\theta^i)\rho}{\eta} = \Gamma_{IFA}^i$, according to Proposition 4. Thus, in period $t = 0$, the first effect dominates the second and the equity rate is lower than the steady-state level under IFA. As the economy converges to the new steady state, the price of capital goods rises further and the equity rate converges back to the initial level, because the initial effect on the spread is fully offset by the neoclassical effect over time.

Proposition 5. *In the steady state, financial capital flows from country H to country F,* $\Upsilon_{FCF}^H > 0 > \Upsilon_{FCF}^F$, *where* $\Upsilon_{FCF}^i = (r_{FCF}^* - r_{IFA}^i) \frac{(1-\eta)w_{FCF}^i}{r_{FCF}^*}$ *and* $i \in \{H, F\}$.

In the steady state, financial capital outflows from country i are proportional to the steady-state loan-rate differentials under free mobility of financial capital and under IFA. Since $r_{IFA}^H < r_{FCF}^* < r_{IFA}^F$, country H (F) witnesses financial capital outflows (inflows).

3.1.3 Production and Welfare

From period $t = 0$ on, financial capital flows reduce (raise) aggregate investment in country H (F). Thus, from period $t = 1$ on, aggregate output in country H (F) is lower (higher) than before period $t = 0$, $Y_t^H < Y_{IFA} < Y_t^H$.

Proposition 6. *From period $t = 1$ on,* $Y_t^H + Y_t^F < 2Y_{IFA}$.

Before period $t = 0$, aggregate production in the two countries is efficient and identical. From period $t = 0$ on, the cross-country resource reallocation due to financial capital flows lead the world economy away from the efficient allocation. Due to the concave aggregate production with respect to the capital-labor ratio at the country level, the world output is lower than before period $t = 0$, according to the Jensen's inequality. This also explains the world output losses in Matsuyama (2004). More generally, this is a typical result of the theory of second best. Given domestic financial frictions, capital account liberalization causes financial capital to flow to the country with the higher loan rate rather than to the country with the higher marginal product of capital.

Since financial frictions do not affect production efficiency, aggregate output in the steady state is same in the two countries under IFA, even if the two countries have the different levels of financial development. International capital flows break the initial symmetry in the two countries in our model similar as in Matsuyama (2004) but for different reasons. In particular, the levels of financial development in the two countries are same in Matsuyama (2004) but they are different in our model.

The welfare of individuals born in period t and country i is measured by their consumption in period $t+1$. According to Proposition 4, the welfare of entrepreneurs is given as $c_{t+1}^{i,e} = w_t^i \Gamma_t^i = w_{t+1}^i \frac{(1-\theta^i)\rho}{\eta}$, reflecting the joint effects of financial capital flows on labor

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income and on the equity rate. From period $t = 0$ on, due to financial capital flows, the labor income falls in country H and rises in country F in period $t+1$, $w_{t+1}^H < w_{IFA} < w_{t+1}^F$. Thus, entrepreneurs born in country H (F) are strictly worse (better) off than before period $t = 0$, implying that entrepreneurs in the less (more) financially developed country have a strong incentive to oppose (support) policies favoring financial capital mobility.

The welfare of workers born in period t and country i is given by $c_{t+1}^{i,w} = w_t^i r_t^i$. Given the predetermined labor income w_0^i, workers born in country H (F) and period $t = 0$ are better (worse) off, $c_1^{i,w} = w_0^i r_0^*$, due to the rise (decline) in the loan rate, $r_{IFA}^H < r_0^* < r_{IFA}^F$. From period $t = 1$ on, financial capital flows affect workers born in country H (F) and period t, $c_{t+1}^{i,w} = w_t^H r_t^*$, negatively (positively) through labor income, $w_t^H < w_{IFA} < w_t^F$ and positively (negatively) through the loan rate, $r_{IFA}^H < r_t^* < r_{IFA}^F$.

Proposition 7. *In comparison with the steady state under IFA, free mobility of financial capital makes entrepreneurs in country H (F) worse (better) off, while the welfare effects on workers and at the country level depend on the parameters, especially the levels of financial development in the two countries.*

Table 1 summarizes the long-run welfare impacts on workers under various parameter constellations[10], where $\kappa \equiv \frac{(\rho-1)(1-\eta)}{\eta}$.

Table 1: The Long-Run Welfare Impacts on Workers

	$\kappa \in (-\infty, \frac{\theta^H+\theta^F}{2\theta^F}]$	$\kappa \in (\frac{\theta^H+\theta^F}{2\theta^F}, 1]$	$\kappa \in (1, \frac{\theta^F}{\theta^H}]$	$\kappa \in (\frac{\theta^F}{\theta^H}, \infty)$
$c_{FCF}^{H,w} - c_{IFA}^{H,w}$	+	+	?	-
$c_{FCF}^{F,w} - c_{IFA}^{F,w}$	-	?	+	+

If $\kappa \in (-\infty, \frac{\theta^H+\theta^F}{2\theta^F}]$, entrepreneurs (workers) in country H lose (benefit) from financial capital flows in the long run as well as in the short run. Similar results exist for country F. Thus, free mobility of financial capital may affect different individuals in the same country in the opposite ways.

Since free mobility of financial capital reduces the world output in the steady state, its welfare effect at the world level is negative and no public transfer policy can achieve a world-level Pareto improvement in comparison with the case of IFA.

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[10]The conventional values of parameters should be used with cautions in the OLG model. As we focus here on the qualitative results, we do not take specific positions on the parameter values but offer the full range of possibilities. Readers who are interested in this issue may decide on which interval κ may belong to. For example, if the capital share is set at $\alpha = \frac{1}{3}$, we get $\kappa < 0$. Thus, workers in country F are worse off in the long run. Boyd and Smith (1997) set $\alpha = 0.65$ in some example. If so, together with a reasonable population share of entrepreneurs $\eta = 20\%$, we may get $\kappa > 1$. In this case, workers in country F are better off in the long run.

Proposition 8. *Workers of different generations born in the same country may be affected by financial capital flows in opposite ways during the transitional process from IFA to free mobility of financial capital.*

Financial capital outflows make workers born in country H and period $t = 0$ better off. According to Table 1, if $\kappa \geq \frac{\theta^F}{\theta^H}$, the decline in labor income dominates the rise in the loan rate in the long run so that workers are worse off in the long run. Workers of early and later generations born in country F are also affected in the opposite way. Thus, free mobility of financial capital may have opposite welfare effects across generations.

3.2 Free Mobility of FDI

The analysis for free mobility of FDI yields a mirror image of that for free mobility of financial capital. We summarize the results here and appendix A contains the full analysis.

Under free mobility of FDI, there exists a unique and stable steady state with the wage at $w^i_{FDI} = w_{IFA} \left[1 - \eta + \eta \frac{\Gamma^i_{IFA}}{\Gamma^*_{FDI}} \right]^\rho$, where a variable with subscript FDI denotes its steady-state value under free flows of FDI. In the steady state, the world equity rate is $\Gamma^*_{FDI} \in (\underline{\Gamma}^*, \Gamma^H_{IFA})$, where $\underline{\Gamma}^* \equiv \frac{\Gamma^H_{IFA} + \Gamma^F_{IFA}}{2}$; FDI flows from country F to country H, $\Omega^H_{FDI} < 0 < \Omega^F_{FDI}$, where $\Omega^i_{FDI} = (\Gamma^*_{FDI} - \Gamma^i_{IFA}) \frac{\eta w^i_{FDI}}{\Gamma^*_{FDI}}$ and $i \in \{H, F\}$. The loan rate has a closed-form solution, $r^i_t = \frac{\theta^i \rho}{(1-\eta)} \frac{w^i_{t+1}}{w^i_t}$, with the steady-state value $r^i_{FDI} = \frac{\theta^i \rho}{1-\eta} = r^i_{IFA}$.

Initially, aggregate production in the two countries is efficient and identical. From period $t = 0$ on, FDI flows raise (reduce) aggregate investment in country H (F) and aggregate output in country H (F) is higher (lower). According to the Jensen's inequality, the cross-country output gap implies a lower world output under free mobility of FDI.

From period $t = 0$ on, FDI flows make the labor income in country H (F) higher (lower) than its initial value in period $t + 1$, $w^H_{t+1} > w_{IFA} > w^F_{t+1}$. Workers born in country H (F) are better (worse) off than those born before period $t = 0$, $c^{i,w}_{t+1} = w^i_t r^i_t = w^i_{t+1} \frac{\theta^i \rho}{(1-\eta)}$. Thus, workers in the less (more) financially developed country have a strong incentive to support (oppose) policies favoring international mobility of FDI.

Given the predetermined labor income w^i_0, entrepreneurs born in country H (F) and period $t = 0$ are worse (better) off, $c^{i,e}_1 = w^i_0 \Gamma^*_0$, due to the decline (rise) in the equity rate, $\Gamma^H_{IFA} > \Gamma^*_0 > \Gamma^F_{IFA}$. From period $t = 1$ on, FDI flows affect entrepreneurs born in country H (F), $c^{i,e}_{t+1} = w^i_t \Gamma^*_t$, positively (negatively) through labor income, $w^H_t > w_{IFA} > w^F_t$ and negatively (positively) through the equity rate, $\Gamma^H_{IFA} > \Gamma^*_t > \Gamma^F_{IFA}$. Table 2 summarizes the long-run welfare impacts on entrepreneurs, where $\mu \equiv \frac{(\rho-1)\eta}{(1-\eta)}$.

In comparison with the steady state under IFA, the country-level welfare effect is ambiguous. Since it reduces the world output in the steady state, its world-level welfare effect is negative and no public transfer policy can achieve a world-level Pareto improvement.

Table 2: The Long-Run Welfare Impacts on Entrepreneurs

	$\mu \in (-\infty, \frac{1-\theta^H+1-\theta^F}{2(1-\theta^H)}]$	$\mu \in (\frac{1-\theta^H+1-\theta^F}{2(1-\theta^H)}, 1]$	$\mu \in (1, \frac{1-\theta^H}{1-\theta^F}]$	$\mu \in (\frac{1-\theta^H}{1-\theta^F}, \infty)$
$c_{FDI}^{H,e} - c_{IFA}^{H,e}$	-	?	+	+
$c_{FDI}^{F,e} - c_{IFA}^{F,e}$	+	+	?	-

3.3 Full Capital Mobility

Full capital mobility equalizes the loan rates and the equity rates across the border, $r_t^H = r_t^F = r_t^*$ and $\Gamma_t^H = \Gamma_t^F = \Gamma_t^*$. Using equation (14) to substitute away v_{t+1}^i from equation (18), we get

$$(w_{t+1}^i)^{\frac{1}{\rho}} = \frac{R}{\rho}\left[\frac{(1-\theta^i)\rho}{\Gamma_t^*} + \frac{\theta^i\rho}{r_t^*}\right], \quad \text{with} \quad \frac{\partial w_{t+1}^i}{\partial \Gamma_t^*} < 0, \quad \frac{\partial w_{t+1}^i}{\partial r_t^*} < 0. \tag{19}$$

3.3.1 Existence, Uniqueness, and Stability of the Steady State

Let a variable with subscript FCM denote its steady-state value under full capital mobility.

Proposition 9. *Given the world interest rates r_{FCM}^* and Γ_{FCM}^*, there is a unique and stable non-zero steady state with the wage at $w_{FCM}^i = w_{IFA}\left[\frac{(1-\theta^i)\rho}{\Gamma_{FCM}^*} + \frac{\theta^i\rho}{r_{FCM}^*}\right]^\rho$.*

3.3.2 Interest Rates and Capital Flows

Before period $t = 0$, the loan rate (the equity rate) is lower (higher) in country H than in country F. From period $t = 0$ on, the initial interest rate differentials drive the flows of financial capital and FDI.

Proposition 10. *There exists a unique world loan rate and a unique world equity rate that clear the world credit market and the world equity market, respectively, in every period. In the steady state, $\Gamma_{FCM}^* \in (\Gamma_{IFA}^F, \underline{\Gamma}^*)$ and $r_{FCM}^* \in (\underline{r}^*, r_{IFA}^F)$.*

The steady-state loan rate and equity rate under full capital mobility should lie between their respective values under IFA.

Proposition 11. *In the steady state, financial capital flows from country H to country F, $\Upsilon_{FCM}^H > 0 > \Upsilon_{FCM}^F$, FDI flows in the opposite direction, $\Omega_{FCM}^H < 0 < \Omega_{FCM}^F$, and net capital flows are from country H to country F, $\Upsilon_{FCM}^H + \Omega_{FCM}^H > 0 > \Upsilon_{FCM}^F + \Omega_{FCM}^F$, with $\Upsilon_{FCM}^i = (r_{FCM}^* - r_{IFA}^i)\frac{(1-\eta)w_{FCM}^i}{r_{FCM}^*}$, $\Omega_{FCM}^i = (\Gamma_{FCM}^* - \Gamma_{IFA}^i)\frac{\eta w_{FCM}^i}{\Gamma_{FCM}^*}$, and $i \in \{H, F\}$.*

In the steady state, financial capital (FDI) outflows from country i have the same functional form as under free mobility of financial capital (FDI). Since the credit market in country F has a larger capacity, net capital flows are from country H to country F.

3.3.3 Production and Welfare

Proposition 12. *In the steady state, net capital flows keep aggregate output in country H (F) lower (higher) than its steady-state value under IFA, $Y_{FCM}^H < Y_{IFA} < Y_{FCM}^F$. The world output is lower than under IFA, $Y_{FCM}^H + Y_{FCM}^F < 2Y_{IFA}$.*

The world output losses depend on net capital flows rather than gross capital flows.

Proposition 13. *In the steady state, country F has a negative net international investment position, $\Upsilon_{FCM}^F + \Omega_{FCM}^F < 0$, but it receives a positive net investment income, $(r_{FCM}^* - 1)\Upsilon_{FCM}^F + (\Gamma_{FCM}^* - 1)\Omega_{FCM}^F > 0$.*

Given the positive equity premium, $\Gamma_t^* > r_t^*$, country F earns a higher return on its direct investments abroad than its pays out on foreign debts. Although closed-form solutions of the interest rates do not exist, we can still prove that $r_{FCM}^* \Upsilon_{FCM}^i + \Gamma_{FCM}^* \Omega_{FCM}^i = 0$. Thus, the net investment income of country F,

$$(r_{FCM}^* - 1)\Upsilon_{FCM}^F + (\Gamma_{FCM}^* - 1)\Omega_{FCM}^F = -(\Upsilon_{FCM}^F + \Omega_{FCM}^F) = \Upsilon_{FCM}^H + \Omega_{FCM}^H,$$

is fully financed by net capital outflows from country H. Intuitively, country F has a competitive advantage in financial intermediation. By exporting financial services via two-way capital flows, it receives a positive net investment income.

Proposition 14. *In the steady state, due to the decline (rise) in labor income and the equity rate in country H (F), entrepreneurs in country H (F) are worse (better) off than in the steady state under IFA. In addition, country H (F) as a whole is worse (better) off.*

Full capital mobility is never an option for country H to make a Pareto improvement upon the steady-state allocation under IFA. In contrast, full capital mobility is a good option for country F to make a Pareto improvement, if implemented with some properly designed public transfer policies. Net capital flows widen the cross-country output gap, which generates the world output losses. In this case, full capital mobility can never achieve a Pareto improvement at the world level.

3.4 Capital Mobility and Economic Convergence

We analyze here how full capital mobility affects the economic convergence of country H, if it is initially below its steady state before period $t = 0$. The properties of the market equilibrium under full capital mobility proved in subsection 3.3 do not depend on whether any country is initially in the steady state. Thus, the convergence path of country H is always stable and unique under full capital mobility as well as under IFA.

Figure 3 plots the capital-labor ratio in country H against the degree of financial development and provides an overview of the directions of gross and net capital flows

during the convergence process to the steady state. The upper bound of figure 3 and the solid line represent the steady-state values of the capital-labor ratio under IFA and under full capital mobility, K_{IFA} and K_{FCM}^H, respectively.
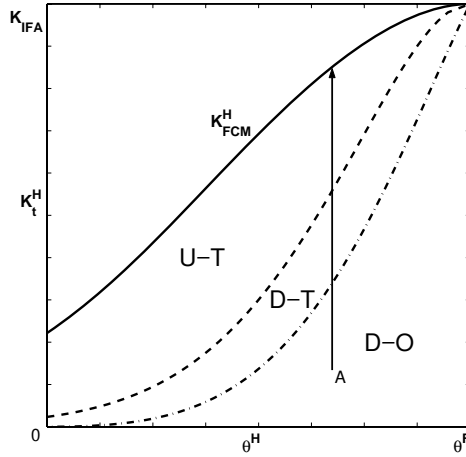

Figure 3: Full Capital Mobility between Initially Poor and Rich Countries

Suppose that the level of financial development in country H is constant over time $0 < \theta^H < \theta^F$ and that country H's initial capital-labor ratio is at point A. Lemma 1 implies that, under IFA, the equity rate is strictly higher in country H than in country F. With full capital mobility, FDI flows unambiguously "downhill" from country F to country H and the equity rates equalize. The direction of financial capital flows, however, depends on the initial capital-labor ratio, K_0^H.

Given K_0^H at the initial level at point A, the neoclassical effect dominates the credit demand effect. Under IFA, the loan rate is higher in country H than in country F. With full capital mobility, both financial capital and FDI flow "**D**ownhill" and **O**ne-way.

Over time, the capital-labor ratio in country H rises along the arrow and eventually crosses a threshold value given by the dash-dotted line. The neoclassical effect is then dominated by the credit demand effect. Under IFA, the loan rate would be lower in country H than in country F. With full capital mobility, financial capital flows "uphill" and FDI flows "downhill". However, net capital flows are still "**D**ownhill", while gross capital flows are **T**wo-way.

As the capital-labor ratio in country H grows further, it crosses a second threshold value given by the dotted line. Financial capital still flows "uphill" and FDI flows "downhill". However, net capital flows become "**U**phill" while gross capital flows are **T**wo-way. Eventually, the capital-labor ratio reaches its steady state given by the solid line with two-way gross capital flows and "uphill" net capital flows. In this sense, the phenomenon of two-way capital flows is a feature of middle-income rather than low-income economies.

Now we may address the costs and benefits of capital account liberalization for a

developing country converging to its steady state. Suppose that its initial capital-labor ratio is at point A. Without international capital flows, it would gradually converge to the level K_{IFA}. With international capital flows, both financial capital and FDI flow into this country, which speeds up its capital accumulation in the short run. As the capital-labor ratio rises over time and moves into region **D-T**, financial capital flows change the direction from "downhill" to "uphill". However, the country still receives net capital inflows and accumulates capital at a faster speed than under IFA. However, as the capital-labor ratio enters into region **U-T**, financial capital outflows exceed FDI inflows and capital accumulation is slower than under IFA. Finally, it converges to a steady state with a capital-labor ratio smaller than under IFA, $K_{FCM}^H < K_{IFA}$. Starting from a sufficiently low level of output, capital account liberalization offers a developing country the short-run benefit of faster capital accumulation but at the long-run cost of a lower level of output. Since K_{FCM}^H increases in θ^H, the developing country, when liberalizing capital account, should promote financial development so as to avoid the long-run cost.

4   Conclusion

We develop a tractable, two-country, overlapping-generations model and show that cross-country differences in financial development can explain three recent empirical characteristics of international capital flows which have been puzzling. We also show that capital account liberalization policies may offer a developing country the short-run benefit of faster capital accumulation but at the long-run cost of a lower level of output. In order to reduce the cost and exploit the benefit, the developing country should promote its level of financial development when liberalizing capital account.

We take the level of financial development as given and analyze how differences in financial development affect capital flows. An obvious and important question is how economic growth and various forms of capital flows shape the institutional infrastructures, e.g., the level of financial development. We leave this issue for future research.

References

AOKI, K., G. BENIGNO, AND N. KIYOTAKI (2009): "Adjusting to Capital Account Liberalization," Working Paper.

BOYD, J. H., AND B. D. SMITH (1997): "Capital Market Imperfections, International Credit Markets, and Nonconvergence," *Journal of Economic Theory*, 73(2), 335–364.

CABALLERO, R., E. FARHI, AND P.-O. GOURINCHAS (2008): "An Equilibrium Model of "Global Imbalances" and Low Interest Rates," *American Economic Review*, 98(1), 358–93.

DEVEREUX, M., AND A. SUTHERLAND (2009): "A Portfolio Model of Capital Flows to Emerging Markets," *Journal of Development Economics*, 89(2), 181–193.

GOURINCHAS, P.-O., AND H. REY (2007): "From World Banker to World Venture Capitalist: U.S. External Adjustment and The Exorbitant Privilege," in *G7 Current Account Imbalances: Sustainability and Adjustment*, ed. by R. Clarida. NBER Conference Volumn, The University of Chicago Press.

HART, O., AND J. MOORE (1994): "A Theory of Debt Based on the Inalienability of Human Capital," *The Quarterly Journal of Economics*, 109(4), 841–79.

HAUSMANN, R., AND F. STURZENEGGER (2007): "The missing dark matter in the wealth of nations and its implications for global imbalances," *Economic Policy*, 22(51), 469 – 518.

HIGGINS, M., T. KLITGAARD, AND C. TILLE (2007): "Borrowing Without Debt? Understanding the U.S. International Investment Position," *Business Economics*, 42(1), 17.

HOLMSTROM, B., AND J. TIROLE (1997): "Financial Intermediation, Loanable Funds, and the Real Sector," *Quarterly Journal of Economics*, 112(3), 663–691.

JU, J., AND S.-J. WEI (2007): "Domestic Institutions and the Bypass Effect of Financial Globalization," NBER Working Papers No 13148.

KIYOTAKI, N., AND J. MOORE (1997): "Credit Cycles," *Journal of Political Economy*, 105(2), 211–48.

LANE, P., AND G. M. MILESI-FERRETTI (2001): "The external wealth of nations: measures of foreign assets and liabilities for industrial and developing countries," *Journal of International Economics*, 55(2), 263–294.

――― (2006): "The External Wealth of Nations Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities, 1970-2004," IIS Discussion Paper No. 79, Trinity College Dublin.

――― (2007): "A Global Perspective on External Positions," in *G7 Current Account Imbalances: Sustainability and Adjustment*, ed. by R. Clarida. The University of Chicago Press.

MATSUYAMA, K. (2004): "Financial Market Globalization, Symmetry-Breaking, and Endogenous Inequality of Nations," *Econometrica*, 72(3), 853–884.

――― (2007): "Credit Traps and Credit Cycles," *American Economic Review*, 97(1), 503–516.

――― (2008): "Aggregate Implications of Credit Market Imperfections," in *NBER Macroeconomics Annual 2007*, ed. by D. Acemoglu, K. Rogoff, and M. Woodford, vol. 22. MIT Press.

MENDOZA, E. G., V. QUADRINI, AND J.-V. RÍOS-RULL (2009): "Financial Integration, Financial Deepness, and Global Imbalances," *Journal of Political Economy*, 117(3), 371–416.

PRASAD, E. S., R. RAJAN, AND A. SUBRAMANIAN (2006): "Patterns of International Capital Flows and their Implications for Economic Development," in *Proceedings of the 2006 Jackson Hole Symposium*, pp. 119–158. Federal Reserve Bank of Kansas City.

――― (2007): "Foreign Capital and Economic Growth," *Brookings Papers on Economic Activity*, 38(1), 153–230.

SMITH, K. A., AND D. VALDERRAMA (2008): "Why Do Emerging Economies Import Direct Investment and Export Savings? A Story of Financial Underdevelopment," working paper.

TILLE, C., AND E. VAN WINCOOP (2010): "International Capital Flows," *Journal of International Economics*, forthcoming.

TOWNSEND, R. M. (1979): "Optimal Contracts and Competitive Markets with Costly State Verification,," *Journal of Economic Theory*, 21(2), 265–293.

VON HAGEN, J., AND H. ZHANG (2009): "Financial Development and Patterns of International Capital Flows," working paper.

Appendix: not for publication

A Free Mobility of FDI

The equity rates are equalized across the border, $\Gamma_t^H = \Gamma_t^F = \Gamma_t^*$. According to the credit market equilibrium, domestic equity capital and investment in country i are

$$\eta w_t^i - \Omega_t^i = \frac{(1-\eta)w_t^i}{\lambda_t^i - 1} \ \text{ and } \ I_t^i = \lambda_t^i(\eta w_t^i - \Omega_t^i) = \frac{\lambda_t^i(1-\eta)w_t^i}{\lambda_t^i - 1} \ \Rightarrow \ \frac{\theta^i R v_{t+1}^i}{r_t^i} = \frac{(1-\eta)w_t^i}{I_t^i}.$$

Thus, the project-financing equation can be transformed into

$$1 = \frac{\theta^i R v_{t+1}^i}{r_t^i} + \frac{(1-\theta^i)R v_{t+1}^i}{\Gamma_t^*}, \quad \Rightarrow \quad 1 = \frac{(1-\eta)w_t^i}{I_t^i} + \frac{(1-\theta^i)R v_{t+1}^i}{\Gamma_t^*}. \tag{20}$$

Using equation (14) to substitute away v_{t+1}^i and I_t^i in equation (20), we get

$$(1-\eta)w_t^i = \frac{\rho}{R}(w_{t+1}^i)^{\frac{1}{\alpha}} - \frac{(1-\theta^i)\rho w_{t+1}^i}{\Gamma_t^*}. \tag{21}$$

A.1 Existence, Uniqueness, and Stability of the Steady State

Let a variable with subscript FDI denote its steady-state value under free flows of FDI

Proposition 15. *There exists a unique and stable non-zero steady state with the wage rate at* $w_{FDI}^i = w_{IFA}\left[1 - \eta + \eta\frac{\Gamma_{IFA}^i}{\Gamma_{FDI}^*}\right]^\rho$.

The solid line and the dash-dotted line in figure 4 show the phase diagrams of wages under IFA and under free mobility of FDI, respectively, given a fixed world equity rate at $\Gamma_t^* = \Gamma_{IFA}^i$. In both cases, the wage converges monotonically and globally to the unique and stable steady state (point A).
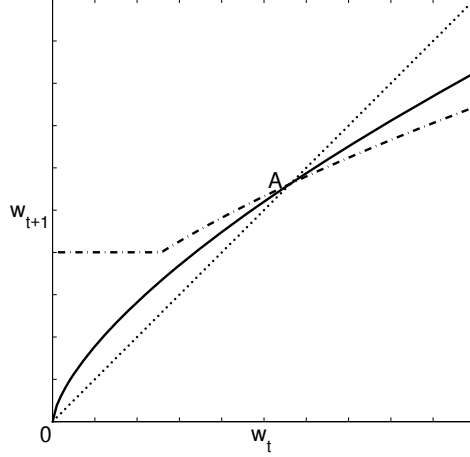
Figure 4: The Phase Diagram of Wages

A.2 Interest Rates and Capital Flows

Proposition 16. *There exists a unique world equity rate that clears the world equity market every period. In the steady state, $\Gamma_{FDI}^* \in (\underline{\Gamma}^*, \Gamma_{IFA}^H)$, where $\underline{\Gamma}^* \equiv \frac{\Gamma_{IFA}^H + \Gamma_{IFA}^F}{2}$.*

Given the steady-state equity rate in the two countries under IFA, $\Gamma_{IFA}^H > \Gamma_{IFA}^F$, the steady-state equity rate under free mobility of FDI should lie between them.

Proposition 17. *Under free mobility of FDI, if the borrowing constraints are binding in country i, $r_t^i = \frac{\theta^i \rho}{(1-\eta)} \frac{w_{t+1}^i}{w_t^i}$. In the steady state, $r_{FDI}^i = \frac{\theta^i \rho}{1-\eta} = r_{IFA}^i$.*

The proof resembles that of Proposition 4. In the steady-state, the equity-rate effect and the price-of-capital effect cancel out so that the loan rate is same as under IFA.

Proposition 18. *In the steady state, FDI flows from country F to country H, $\Omega_{FCF}^H < 0 < \Omega_{FCF}^F$, where $\Omega_{FDI}^i = (\Gamma_{FDI}^* - \Gamma_{IFA}^i) \frac{\eta w_{FDI}^i}{\Gamma_{FDI}^*}$ and $i \in \{H, F\}$.*

In the steady state, FDI outflows from country i are proportional to the steady-state equity-rate differentials under free mobility of FDI and under IFA. Since $\Gamma_{IFA}^H > \Gamma_{FCF}^* > \Gamma_{IFA}^F$, country H (F) has FDI inflows (outflows).

A.3 Production and Welfare

From period $t = 0$ on, FDI flows raise (reduce) aggregate investment in country H (F). Thus, from period $t = 1$ on, aggregate output in country H (F) is higher (lower) than before period $t = 0$, $Y_t^H > Y_{IFA} > Y_t^H$.

Proposition 19. *From period $t = 1$ on, $Y_t^H + Y_t^F < 2Y_{IFA}$.*

The proof follows that of Proposition 6. As FDI flows widen the cross-country output gap, the world output is lower than under IFA, due to the Jensen's Inequality. The welfare impacts are discussed briefly in subsection 3.2 and summarized in Proposition 20.

Proposition 20. *In comparison with the steady state under IFA, free mobility of FDI makes workers in country H (F) better (worse) off, while the welfare impacts on entrepreneurs and at the country level depend on the parameters.*

Proposition 21. *Entrepreneurs of different generations born in the same country may be affected by FDI flows in opposite ways during the transitional process from IFA to free mobility of FDI.*

Entrepreneurs born in country H and period $t = 0$ are worse off, due to the decline in the equity rate. According to Table 2, for $\frac{(\rho-1)\eta}{(1-\eta)} \geq 1$, and the rise in labor income dominates the decline in the equity rate in the long run and entrepreneurs born in country H are better off in the long run. Entrepreneurs of early and later generations born in country F may also be affected differently. Thus, free mobility of FDI may have opposite welfare effects across generations.

B   Threshold Values under Capital Mobility

B.1   Free Mobility of Financial Capital

Proposition 22. *Given $\theta^H \in (0, \bar{\theta})$, there exists $\bar{\theta}^F_{FCF} \in (\bar{\theta}, 1 - \frac{\theta^H \eta}{1-\eta})$ as the function of θ^H such that for $\theta^F \in (\theta^H, \bar{\theta}^F_{FCF})$, the borrowing constraints are binding in both countries in the steady state; for $\theta^F \in (\bar{\theta}^F_{FCF}, 1]$, the borrowing constraints are binding in country H but not in country F and the economic allocation is same as that in the case of $\theta^F = \bar{\theta}^F_{FCF}$.*

Figure 5 illustrates these results. The horizontal and vertical axes denote $\theta^H \in (0, 1]$ and $\theta^F \in (0, 1]$, respectively.

For $\theta^H = \theta^F$, i.e., the parameters on the 45 degree line, the loan rates are same in the two countries under IFA. For $\theta^H \in [\bar{\theta}, 1]$ and $\theta^F \in [\bar{\theta}, 1]$, i.e., the parameters in region A, the loan rates are equal to the marginal return on investment, which is same in the two countries, according to Proposition 1. In these two cases, there are no financial capital flows even if allowed. The curve splitting regions B and D represents the threshold value $\bar{\theta}^F_{FCF}$ as the function of θ^H described by equation (43). For the parameters on the curve, the equity rate in country F is equal to the world loan rate, $\Gamma^F_{FCF} = \frac{(1-\bar{\theta}^F_{FCF})\rho}{\eta} = r^*_{FCF}$. Similarly, the curve splitting region B' and D' represents the threshold value $\bar{\theta}^H_{FCF}$ as the function of θ^F. For the parameters on the curve, the equity rate in country H is equal to the world loan rate, $\Gamma^H_{FCF} = \frac{(1-\bar{\theta}^H_{FCF})\rho}{\eta} = r^*_{FCF}$.
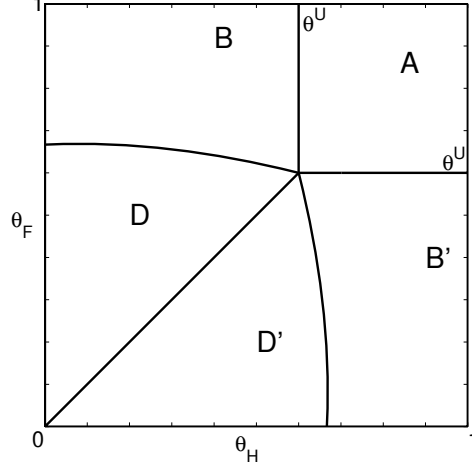
Figure 5: Free Mobility of Financial Capital: Threshold Values

Table 3: Financial Capital Flows and Equity Premium in the Steady State

Region	A	B	B'	D	D'
Υ^H	0	$\Upsilon^H(\theta^H) > 0$	$\Upsilon^H(\theta^F) < 0$	$(0, \Upsilon^H(\theta^H))$	$(\Upsilon^H(\theta^F), 0)$
$\Gamma^H - r^*$	0	$+$	0	$+$	$+$
$\Gamma^F - r^*$	0	0	$+$	$+$	$+$

Table 3 summarizes the steady-state pattern of financial capital flows and the equity premium in the five regions of figure 5. Note that $\Upsilon^F = -\Upsilon^H$. $\Upsilon^H(\theta^i)$ implies that given the parameters in region B and B', financial capital flows depend only on θ^i not on θ^m, where $i, m \in \{H, F\}$ and $i \neq m$. The borrowing constraints are strictly binding only if the equity premium is positive.

B.2 Free Mobility of FDI

Proposition 23. *If $\eta \in \left[\frac{2^\rho}{1+2^{(\rho+1)}}, 1\right)$, given $\theta^H \in (0, \bar{\theta})$, there exists $\bar{\theta}_{FDI}^F \in (\bar{\theta}, 1)$ as the function of θ^H such that for $\theta^F \in (\theta^H, \bar{\theta}_{FDI}^F)$, the borrowing constraints are binding in country F in the steady state; for $\theta^F \in (\bar{\theta}_{FDI}^F, 1]$, the borrowing constraints are not binding in country F and the economic allocation is same as that in the case of $\theta^F = \bar{\theta}_{FDI}^F$.*

If $\eta \in (0, \frac{2^\rho}{1+2^{\rho+1}})$, there exists $\underline{\theta}^H$ such that given $\theta^H \in [\underline{\theta}^H, \bar{\theta})$, there exists $\bar{\theta}_{FDI}^F \in (\bar{\theta}, 1)$ as the function of θ^H such that for $\theta^F \in (\theta^H, \bar{\theta}_{FDI}^F)$, the borrowing constraints are binding in country F in the steady state; for $\theta^F \in (\bar{\theta}_{FDI}^F, 1]$, the borrowing constraints are not binding in country F and the economic allocation is same as that in the case of $\theta^F = \bar{\theta}_{FDI}^F$. Given $\theta^H \in (0, \underline{\theta}^H)$, the borrowing constraints are always binding in country F for $\theta^F \in (\theta^H, 1]$.

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Figure 6 illustrates these results in the cases of $\eta < \frac{2^\rho}{1+2^{\rho+1}}$ and $\eta > \frac{2^\rho}{1+2^{\rho+1}}$ respectively. The horizontal and vertical axes denote $\theta^H \in (0,1]$ and $\theta^F \in (0,1]$, respectively.
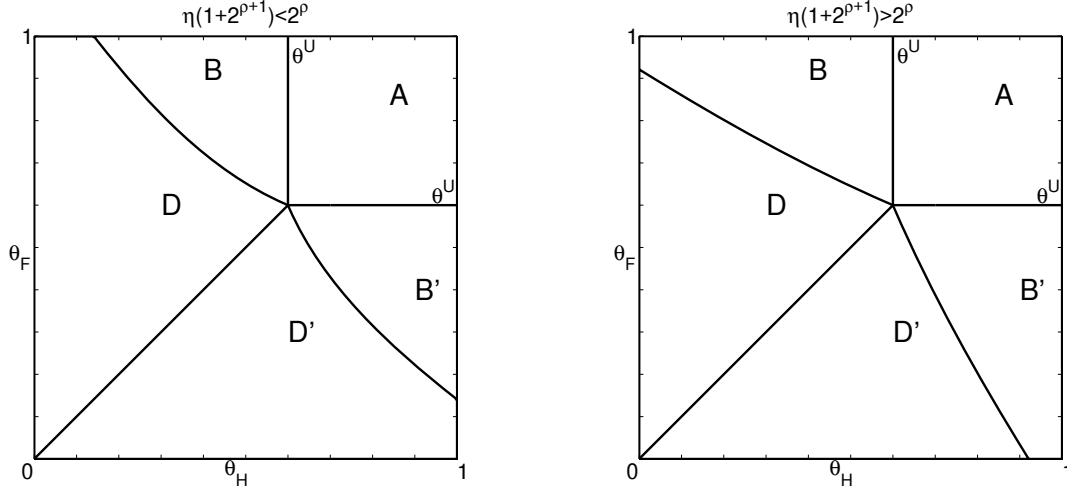

Figure 6: Free Mobility of FDI: Threshold Values

For $\theta^H = \theta^F$, i.e., the parameters on the 45 degree line, the equity rate is same in the two countries under IFA. For $\theta^H \in [\bar{\theta}, 1]$ and $\theta^F \in [\bar{\theta}, 1]$, i.e., the parameters in region A, according to Proposition 1, the equity rates are equal to the marginal return on investment, which is same in the two countries. In these two cases, there are no FDI flows even if allowed. The curve splitting regions B and D represents the threshold value of $\bar{\theta}_{FDI}^F$ as the function of θ^H described by equation (44). For the parameters on the curve, the loan rate in country F is equal to the world equity rate $r_{FDI}^F = \frac{\bar{\theta}_{FDI}^F \rho}{1-\eta} = \Gamma_{FDI}^*$. Similarly, the curve splitting regions B' and D' represents the threshold value of $\bar{\theta}_{FDI}^H$ as the function of θ^F. For the parameters on the curve, the loan rate in country H is equal to the world equity rate, $r_{FDI}^H = \frac{\bar{\theta}_{FDI}^H \rho}{1-\eta} = \Gamma_{FDI}^*$.

Table 4: FDI Flows and Equity Premium in the Steady State

Region	A	B	B'	D	D'
Ω^H	0	$\Omega^H(\theta^H) < 0$	$\Omega^H(\theta^F) > 0$	$(\Omega^H(\theta^H), 0)$	$(0, \Omega^H(\theta^F))$
$\Gamma^H - r^*$	0	$+$	0	$+$	$+$
$\Gamma^F - r^*$	0	0	$+$	$+$	$+$

Table 4 summarizes the steady-state values of FDI flows and the equity premium in the five regions. Note that $\Omega^F = -\Omega^H$. $\Omega^H(\theta^i)$ implies that given the parameters in region B and B', FDI flows depend only on θ^i not on θ^m, where $i, m \in \{H, F\}$ and $i \neq m$. The borrowing constraints are strictly binding only if the equity premium is positive.

B.3 Full Capital Mobility

Proposition 24. *Given $\theta^H \in (\max\{1 - 2\eta, 0\}, 1 - \eta)$, there exists a threshold value $\bar{\theta}^F_{FCM} = 2(1 - \eta) - \theta^H$ such that for $\theta^F \in (\theta^H, \bar{\theta}^F_{FCM})$, the borrowing constraints are binding in both countries in the steady state; for $\theta^F \in (\bar{\theta}^F_{FCM}, 1]$, the world loan rate and equity rate are same as the marginal return to investment, $\Gamma^* = r^* = \rho$, in the steady state, the borrowing constraints are not binding in both countries, and the economic allocation is same as that in the case of $\theta^F = \bar{\theta}^F_{FCM}$.*

Figure 7 illustrates the results. The horizontal and vertical axes denote $\theta^H \in (0, 1]$ and $\theta^F \in (0, 1]$, respectively.
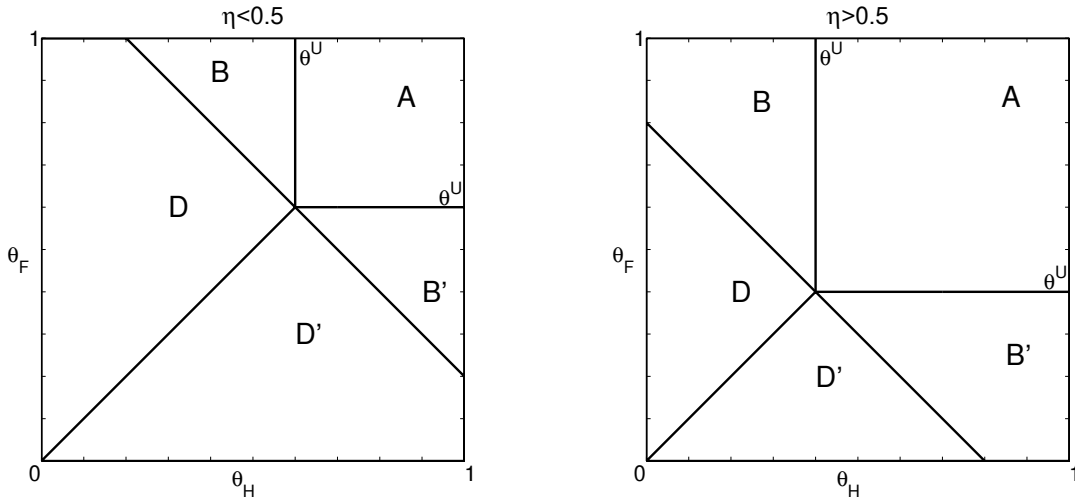

Figure 7: Full Capital Mobility: Threshold Values

For $\theta^H = \theta^F$, i.e., the parameters on the 45 degree line, the loan rates are same in the two countries under IFA and so are the equity rates. For $\theta^H \in [\bar{\theta}, 1]$ and $\theta^F \in [\bar{\theta}, 1]$, i.e., the parameters in region A, according to Proposition 1, the loan rate and the equity rate under IFA are equal to the marginal return on investment, which is same in the two countries, $r^i_{IFA} = \Gamma^i_{IFA} = \rho$. In these two cases, there are no financial capital flows or FDI even if allowed. The line splitting region B and D represents the threshold value of $\bar{\theta}^F_{FCM}$ as the function of θ^H, while the line splitting region B' and D' represents the threshold value of $\bar{\theta}^H_{FCM}$ as the function of θ^F. For the parameters on the two lines, the world loan rate is equal to the world equity rate, $r^* = \Gamma^* = \rho$.

Table 5 summarizes the steady-state pattern of capital flows and the equity premium in the five regions of figure 7. Note that $\Upsilon^F = -\Upsilon^H$ and $\Omega^F = -\Omega^H$. $\Upsilon^H(\theta^i)$ and $\Omega^H(\theta^i)$ implies that given the parameters in region B and B', financial capital flows and FDI depend only on θ^i not on θ^m, where $i, m \in \{H, F\}$ and $i \neq m$. The borrowing constraints are strictly binding only if the equity premium is positive.

Table 5: Capital Flows and Equity Premium in the Steady State

Region	A	B	B'	D	D'
Υ^H	0	$\Upsilon^H(\theta^H) > 0$	$\Upsilon^H(\theta^F) < 0$	$(0, \Upsilon^H(\theta^H))$	$(\Upsilon^H(\theta^F), 0)$
Ω^H	0	$\Omega^H(\theta^H) < 0$	$\Omega^H(\theta^F) > 0$	$(\Omega^H(\theta^H), 0)$	$(0, \Omega^H(\theta^F))$
$\Omega^H + \Upsilon^H$	0	0	0	$+$	$-$
$\Gamma^* - r^*$	0	0	0	$+$	$+$

C Proofs of Propositions

Proof of Propositions 2

Proof. Take the world loan rate r_t^* as given. According to equation (16), w_{t+1}^i can be considered as a function of w_t^i. For $w_t^i \in [0, \frac{(1-\theta^i)\rho}{R\eta}\left(\frac{R}{r_t^*}\right)^{\frac{1}{1-\alpha}}]$, take the first derivative of equation (16) with respect to w_t^i,

$$\eta = \left[\frac{\rho}{R\alpha}(w_{t+1}^i)^{\frac{1}{\rho}} - \frac{\theta^i \rho}{r_t^*}\right]\frac{dw_{t+1}^i}{dw_t^i}. \tag{22}$$

According to equation (16), for $w_t^i = 0$, there is a non-zero solution of $w_{t+1}^i = \left[\frac{\theta^i R}{r_t^*}\right]^\rho$. The slope of the phase diagram at the point $(0, \left[\frac{\theta^i R}{r_t^*}\right]^\rho)$ is $\frac{r_t^* \eta}{\theta^i} > 0$. In other words, $w_{t+1}^i \geq \left(\frac{\theta^i R}{r_t^*}\right)^\rho$. Thus, according to equation (22), the phase diagram of wages has the positive slope, $\frac{dw_{t+1}^i}{dw_t^i} > 0$. Take the second derivative of equation (16) with respect to w_t^i,

$$0 = \left[\frac{\rho}{R\alpha}(w_{t+1}^i)^{\frac{1}{\rho}} - \frac{\theta^i \rho}{r_t^*}\right]\frac{dw_{t+1}^i}{d^2 w_t^i} + \left(\frac{dw_{t+1}^i}{dw_t^i}\right)^2 \frac{1}{R\alpha}(w_{t+1}^i)^{\frac{1-2\alpha}{\alpha}}, \quad \Rightarrow \quad \frac{dw_{t+1}^i}{d^2 w_t^i} < 0.$$

Thus, the phase diagram of wages is concave for $w_t^i \in [0, \frac{(1-\theta^i)\rho}{R\eta}\left(\frac{R}{r_t^*}\right)^{\frac{1}{1-\alpha}}]$, and w_{t+1}^i monotonically increases in w_t^i with an intercept on the vertical axis at $w_{t+1}^i = \left[\frac{\theta^i R}{r_t^*}\right]^\rho$.

For $w_t^i > \frac{(1-\theta^i)\rho}{R\eta}\left(\frac{R}{r_t^*}\right)^{\frac{1}{1-\alpha}}$, the marginal return on investment is equal to the world loan rate, $Rv_{t+1}^i = r_t^*$, and, thus, entrepreneurs do not borrow to the limit. The phase diagram of wages $w_{t+1}^i = (v_{t+1}^i)^{-\frac{1}{\rho}} = \left(\frac{R}{r_t^*}\right)^\rho$ is flat and independent of w_t^i.

The phase diagram of wages is continuous and concave. It crosses the 45 degree line only once and from the left. There exists a stable and unique non-zero steady state. $\square$

Proof of Lemma 2

Proof. Take the world loan rate r_t^* as given. For $w_t^i \in (0, 1 - \theta^i]$ and $i_t^i = 1$, take the first and second derivatives of equation (17) with respect to w_t^i,

$$\frac{dw_{t+1}^i}{dw_t^i} = \frac{\rho r^*}{\theta^i R}(w_{t+1}^i)^{\frac{1}{\alpha}} > 0, \quad \text{and,} \quad \frac{dw_{t+1}^i}{d^2 w_t^i} = \frac{\rho r^*}{\theta^i R}\frac{1}{\alpha}(w_{t+1}^i)^{\frac{1}{\rho}}\frac{dw_{t+1}^i}{dw_t^i} > 0. \tag{23}$$

The phase diagram of wages is convex for $w_t^i \in (0, 1 - \theta^i]$. By setting $w_t^i = 0$ in equation (17), we get the vertical intercept of the phase diagram of wages at $w_{t+1}^i = \left[\frac{\theta^i R}{r_t^*}\right]^{\rho}$. For $w_t^i > 1 - \theta^i$, the marginal return on investment is equal to the world loan rate, $R v_{t+1}^i = r_t^*$, and, thus, entrepreneurs do not borrow to the limit. The phase diagram of wages $w_{t+1}^i = (v_{t+1}^i)^{-\frac{1}{\rho}} = \left(\frac{R}{r_t^*}\right)^{\rho}$ is flat and independent of w_t^i. $\qquad \square$

Proof of Proposition 3

Proof. The world loan rate is determined by the identity of financial capital flows, $\Upsilon_t^H + \Upsilon_t^F = 0$. We first prove the existence of a unique world loan rate clearing the world credit market every period and then derive the world loan rate in the steady state.

Suppose that the borrowing constraints are binding in country i. Given the predetermined w_t^i, equation (16) shows that w_{t+1}^i is a function of r_t^*. Take the first derivative of equation (16) with respect to r_t^*,

$$0 = \left[\frac{\rho}{R\alpha}(w_{t+1}^i)^{\frac{1}{\rho}} - \frac{\theta^i \rho}{r_t^*}\right]\frac{dw_{t+1}^i}{dr_t^*} + \frac{\theta^i \rho}{(r_t^*)^2}.$$

As shown in the proof of Proposition 2, $w_{t+1}^i \geq \left(\frac{\theta^i R}{r_t^*}\right)^{\rho}$ so that the term in the square bracket is positive. An increase in the world loan rate enhances financial capital outflows and reduces domestic investment. Thus, the wage in the next period declines, $\frac{dw_{t+1}^i}{dr_t^*} < 0$.

Capital outflows represent the gap between domestic savings and investment,

$$\Upsilon_t^i = w_t^i - I_t^i = w_t^i - \frac{\rho}{R}(w_{t+1}^i)^{\frac{1}{\alpha}} = (1 - \eta)w_t^i - \frac{\rho}{r_t^*}\theta^i w_{t+1}^i. \qquad (24)$$

The world credit market equilibrium implies

$$\Upsilon_t^H + \Upsilon_t^F = 0, \quad \Rightarrow \quad (1 - \eta)(w_t^H + w_t^F) = \frac{\rho}{r_t^*}\left(\theta^H w_{t+1}^H + \theta^F w_{t+1}^F\right). \qquad (25)$$

The loan rate in country H is lower than in country F before period $t = 0$, $r_{IFA}^H < r_{IFA}^F$. The world loan rate in period $t \geq 0$ must be $r_t^* \in (r_{IFA}^H, r_{IFA}^F)$. The proof is by contradiction. If $r_t^* > r_{IFA}^F > r_{IFA}^H$, w_{t+1}^i would be lower than under IFA in both countries, as $\frac{dw_{t+1}^i}{dr_t^*} < 0$. Thus, equation (25) would not hold. The same argument applies to the case of $r_t^* < r_{IFA}^H < r_{IFA}^F$. Since w_{t+1}^i monotonically decreases with r_t^*, there is a unique solution $r_t^* \in (r_{IFA}^H, r_{IFA}^F)$ that clears the world credit market and financial capital flows from country H to F, given the predetermined w_t^H and w_t^F.

In the next step, we assume that there exists a unique world loan rate in the steady state and then prove its uniqueness.

Given r^*, Proposition 2 shows that the wage in the steady state is $w^i = \left(\frac{R}{\rho}\right)^{\rho}\left(\eta + \frac{\theta^i \rho}{r^*}\right)^{\rho}$. According to equation (24), financial capital flows in the steady state are

$$\Upsilon^i = w^i\left[(1 - \eta) - \frac{\theta^i \rho}{r^*}\right] \quad \Rightarrow \quad \Upsilon^i = (r^* - r_{IFA}^i)\frac{(1 - \eta)w^i}{r^*} \qquad (26)$$

According to equation (25), the world loan rate in the steady state is determined by

$$\Upsilon^H + \Upsilon^F = 0, \quad \text{or,} \quad \frac{\frac{\theta^F \rho}{1-\eta} - r^*}{r^* - \frac{\theta^H \rho}{1-\eta}} = \frac{w^H}{w^F}, \quad \text{or,} \quad \frac{r^F_{IFA} - r^*}{r^* - r^H_{IFA}} = \left(\frac{\eta r^* + \theta^H \rho}{\eta r^* + \theta^F \rho} \right)^\rho. \tag{27}$$

For $0 < \theta^H < \theta^F \le \bar{\theta}$, the right-hand side of equation (27) is less than one,

$$\frac{r^F_{IFA} - r^*}{r^* - r^H_{IFA}} < 1, \quad \Rightarrow \quad r^* \in (\underline{r}^*, r^F_{IFA}).$$

Let $\aleph(r^*) \equiv \frac{r^F_{IFA} - r^H_{IFA}}{r^* - r^H_{IFA}} - 1$ and $\Re(r^*) \equiv \left[1 - \frac{(\theta^F - \theta^H)\rho}{\eta r^* + \theta^F \rho} \right]^\rho$ denote the left-hand and the right-hand sides of equation (27) as the functions of r^*. For $r^* \in (\underline{r}^*, r^F_{IFA})$,

$$\aleph'(r^*) < 0 < \Re'(r^*),$$
$$\aleph(r^* = \underline{r}^*) = 1 > \Re(r^* = \underline{r}^*),$$
$$\aleph(r^* = r^F_{IFA}) = 0 < \Re(r^* = r^F_{IFA}).$$

Thus, $\aleph(r^*)$ decreases while $\Re(r^*)$ increases monotonically in r^*; the two functions cross once and only once at $r^* \in (\underline{r}^*, r^F_{IFA})$. Therefore, there exists a unique steady state. $\square$

Proof of Proposition 4

Proof. Under free mobility of financial capital, if the borrowing constraints are binding in country i, the equity rate is $\Gamma^i_t = \frac{(1-\theta)R v^i_{t+1}}{1 - \frac{\theta R v^i_{t+1}}{r^*_t}}$. Using equations (14) and (15), we can rewrite the equity rate as $\Gamma^i_t = \frac{(1-\theta^i)R v^i_{t+1}}{\left[\frac{\eta w^i_t}{\frac{\rho}{R}(w^i_{t+1})^{\frac{1}{\alpha}}} \right]} = \frac{(1-\theta^i)\rho}{\eta} \frac{w^i_{t+1}}{w^i_t}$. $\square$

Proof of Proposition 5

Proof. See the proof of Proposition 3 and equation (26). $\square$

Proof of Proposition 6

Proof. Let $a_t \equiv \frac{w^H_t + w^F_t}{2w_{IFA}}$ and $b_t \equiv \frac{w^F_t - w^H_t}{2w_{IFA}} + \frac{\Upsilon^H_t}{w_{IFA}}$, where $t = 0, 1, 2, 3, ...$ According to Proposition 5 and the aggregate resource constraint in country H, $0 < \Upsilon^H_t < w^H_{FCF}$, we get $b_t \in (0, a_t)$. In period $t \ge 0$, the aggregate project investment in country H and in country F are $I^H_t = w^H_t - \Upsilon^H_t = (a_t - b_t)w^i_{IFA}$ and $I^F_t = w^F_t + \Upsilon^H_t = (a_t + b_t)w^i_{IFA}$, respectively. Given the share of capital goods in the aggregate production, $\alpha \in (0, 1)$, and $b_t \in (0, a_t)$, the world-average wage in period $t+1$ can be reformulated into a condensed form with the following property,

$$\frac{w^H_{t+1} + w^H_{t+1}}{2} = \left(\frac{R}{\rho} \right)^\alpha \left[\frac{(I^H_t)^\alpha + (I^F_t)^\alpha}{2} \right] \Leftrightarrow a_{t+1} = \frac{(a_t - b_t)^\alpha + (a_t + b_t)^\alpha}{2} < (a_t)^\alpha, \tag{28}$$

where the last inequality sign results from the Jensen's Inequality. The wage in period $t = 0$ is same in the two countries, $w_0^H = w_0^F = w_{IFA}$, and, thus, $a_0 = 1$. From period 0 on, financial capital flows are allowed. According to the inequality in equation (28), we get $a_1 < 1$. For $t = 1, 2, 3, ...$, given $b_t \in (0, a_t)$, we have $a_{t+1} < (a_t)^\alpha$ and, thus, the time series of a_t is below 1, or equivalently, $\frac{w_t^H + w_t^F}{2} < w_{IFA}$. Thus, the world output is smaller than before period $t = 0$, $Y_t^H + Y_t^F = \frac{w_t^H + w_t^F}{1-\alpha} < \frac{2w_{IFA}}{1-\alpha} = Y_{IFA}^H + Y_{IFA}^F$. $\qquad\square$

Proof of Proposition 7

Proof. If the borrowing constraints are binding, the steady-state workers' consumption is

$$c^{i,w} = w^i r^* = \left(\frac{R}{\rho}\right)^\rho \left(r^*\eta + \theta^i\rho\right)^\rho (r^*)^{1-\rho},$$

$$\frac{d\ln c^{i,w}}{dr^*} = \frac{r^*\eta + \theta^i\rho - \theta^i\rho^2}{(r^*\eta + \theta^i\rho)r^*}.$$

As an analytical solution of the world loan rate is not obtainable, we provide sufficient conditions for the welfare changes as follows.

Let $\kappa \equiv \frac{(\rho-1)(1-\eta)}{\eta}$. Evaluate $\frac{d\ln c^{H,w}}{dr^*}$ at $r^* = r_{IFA}^H$ and $r^* = r_{IFA}^F$. For $\kappa \leq 1$, $\frac{d\ln c^{H,w}}{dr^*}|_{r^*=r_{IFA}^F} > \frac{d\ln c^{H,w}}{dr^*}|_{r^*=r_{IFA}^H} \geq 0$ implies that workers born in country H is better off in the long run than before period $t = 0$ since the positive loan rate effect dominates the negative wage effect; for $\kappa \geq \frac{\theta^F}{\theta^H}$, $\frac{d\ln c^{H,w}}{dr^*}|_{r^*=r_{IFA}^H} < \frac{d\ln c^{H,w}}{dr^*}|_{r^*=r_{IFA}^F} \leq 0$ implies that workers born in country H is worse off in the long run since the negative wage effect dominates; for $\kappa \in (1, \frac{\theta^F}{\theta^H})$, the numerical solutions are needed for the welfare evaluation.

Evaluate $\frac{d\ln c^{F,w}}{dr^*}$ at $r^* = \underline{r}^*$ and $r^* = r_{IFA}^F$. For $\kappa \leq \frac{\theta^H + \theta^F}{2\theta^F}$, $\frac{d\ln c^{F,w}}{dr^*}|_{r^*=r_{IFA}^F} > \frac{d\ln c^{F,w}}{dr^*}|_{r^*=\underline{r}^*} \geq 0$ implies that workers born in country F is worse off in the long run than before period $t = 0$ since the negative loan rate effect dominates the positive wage effect; for $\kappa \geq 1$, $\frac{d\ln c^{F,w}}{dr^*}|_{r^*=r_{IFA}^F} < \frac{d\ln c^{F,w}}{dr^*}|_{r^*=\underline{r}^*} \leq 0$ implies that workers born in country F is better off in the long run since the positive wage effect dominates; for $\kappa \in (\frac{\theta^H + \theta^F}{2\theta^F}, 1)$, the numerical solutions are required for the welfare evaluation.

Social welfare in country i in the steady state is

$$C^i \equiv \eta c^{i,e} + (1 - \eta)c^{i,w} = w^i[\eta\Gamma^i + (1 - \eta)r^*]$$

$$= \left(\frac{R}{\rho}\right)^\rho \left(r^*\eta + \theta^i\rho\right)^\rho (r^*)^{-\rho}[(1 - \theta^i)\rho + (1 - \eta)r^*],$$

$$\frac{d\ln C^i}{dr^*} = \frac{\eta\rho}{r^*\eta + \theta^i\rho} - \frac{\rho}{r^*} + \frac{1 - \eta}{(1 - \theta^i)\rho + (1 - \eta)r^*}.$$

Evaluate $\frac{d\ln C^H}{dr^*}$ at $r^* = r_{IFA}^H$ and $r^* = r_{IFA}^F$. For $\rho \in (0, \frac{\theta^H}{1-\eta}]$, $\frac{d\ln C^H}{dr^*}|_{r^*=r_{IFA}^F} > \frac{d\ln C^H}{dr^*}|_{r^*=r_{IFA}^H} \geq 0$ implies that the workers' welfare gains dominate the welfare losses of entrepreneurs and hence, country H as a whole benefits from free mobility of financial capital; for $\rho \in [\frac{\theta^F[\theta^H + \eta(\theta^F - \theta^H)]}{(1-\eta)\theta^H[1+(\theta^F - \theta^H)]}, \infty)$, $\frac{d\ln C^H}{dr^*}|_{r^*=r_{IFA}^H} < \frac{d\ln C^H}{dr^*}|_{r^*=r_{IFA}^F} \leq 0$ implies that both workers

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and entrepreneurs are worse off or the workers' welfare gains are dominated by the welfare losses of entrepreneurs and hence, country H as a whole loses from free mobility of financial capital; for $\rho \in (\frac{\theta^H}{1-\eta}, \frac{\theta^F[\theta^H+\eta(\theta^F-\theta^H)]}{(1-\eta)\theta^H[1+(\theta^F-\theta^H)]})$, the numerical solutions are required for the welfare evaluation.

Evaluate $\frac{d\ln C^F}{dr^*}$ at $r^* = \underline{r}^*$ and $r^* = r_{IFA}^F$. For $\rho \in (0, \frac{\frac{\theta^H+\theta^F}{2(1-\eta)}\left[\theta^F - \frac{\eta(\theta^F-\theta^H)}{2}\right]}{\theta^F\left[1-\frac{(\theta^F-\theta^H)}{2}\right]}]$, $\frac{d\ln C^F}{dr^*}|_{r^*=r_{IFA}^F}$ $\geq \frac{d\ln C^F}{dr^*}|_{r^*=\underline{r}^*} \geq 0$ implies that both workers and entrepreneurs are worse off or the workers' welfare gains are dominated by the welfare losses of entrepreneurs and hence, country F as a whole loses from free mobility of financial capital; for $\rho \in [\frac{\theta^F}{1-\eta}, \infty)$, $\frac{d\ln C^F}{dr^*}|_{r^*=\underline{r}^*} < \frac{d\ln C^F}{dr^*}|_{r^*=r_{IFA}^F} \leq 0$ implies that the workers' welfare gains dominate the welfare losses of entrepreneurs and hence, country H as a whole benefits from free mobility of financial capital; for $\rho \in (\frac{\frac{\theta^H+\theta^F}{2(1-\eta)}\left[\theta^F - \frac{\eta(\theta^F-\theta^H)}{2}\right]}{\theta^F\left[1-\frac{(\theta^F-\theta^H)}{2}\right]}, \frac{\theta^F}{1-\eta})$, the numerical solutions are required for the welfare evaluation. $\square$

Proof of Proposition 9

Proof. According to equation (19), w_{t+1}^i is determined only by Γ_t^* and r_t^*. The phase diagram of wages is flat and crosses the 45 degree line only once and from the left. $\square$

Proof of Proposition 10

Proof. The world equity rate Γ_t^* is determined by the identity of FDI flows, $\Omega_t^H + \Omega_t^F = 0$ and the world loan rate r_t^* by $\Upsilon_t^H + \Upsilon_t^F = 0$. We first prove the existence of a unique world equity (loan) rate and clearing the world equity (credit) market every period. Then, we derive the world interest rates in the steady state.

According to the domestic credit market equilibrium and the Cobb-Douglas production function, FDI and financial capital flows are solved as

$$\lambda_t^i\left(\eta w_t^i - \Omega_t^i\right) = \frac{\lambda_t^i}{\lambda_t^i - 1}\left[(1-\eta)w_t^i - \Upsilon_t^i\right] = I_t^i = \frac{\rho}{R}(w_{t+1}^i)^{\frac{1}{\alpha}}, \tag{29}$$

$$\Omega_t^i = \eta w_t^i - \frac{(1-\theta^i)\rho}{\Gamma_t^*}w_{t+1}^i, \tag{30}$$

$$\Upsilon_t^i = (1-\eta)w_t^i - \frac{\theta^i\rho}{r_t^*}w_{t+1}^i, \tag{31}$$

$$\Omega_t^i + \Upsilon_t^i = w_t^i - I_t^i = w_t^i - \frac{\rho}{R}(w_{t+1}^i)^{\frac{1}{\alpha}}. \tag{32}$$

Given the world interest rates at Γ_t^* and r_t^* and the predetermined labor income w_t^i, w_{t+1}^i is uniquely determined under full capital mobility and so are Υ_t^i and Ω_t^i. Take first derivative

of equations (30) and (31) with respect to the two interest rates, respectively,

$$\frac{d\Omega_t^i}{d\Gamma_t^*} = \frac{(1-\theta^i)\rho}{(\Gamma_t^*)^2}w_{t+1}^i - \frac{(1-\theta^i)\rho}{\Gamma_t^*}\frac{dw_{t+1}^i}{d\Gamma_t^*} > 0, \qquad \frac{d\Omega_t^i}{dr_t^*} = -\frac{(1-\theta^i)\rho}{\Gamma_t^*}\frac{dw_{t+1}^i}{dr_t^*} > 0,$$

$$\frac{d\Upsilon_t^i}{dr_t^*} = \frac{\theta^i\rho}{(r_t^*)^2}w_{t+1}^i - \frac{\theta^i\rho}{r_t^*}\frac{dw_{t+1}^i}{dr_t^*} > 0, \qquad \frac{d\Upsilon_t^i}{d\Gamma_t^*} = \frac{\theta^i\rho}{r_t^*}\frac{dw_{t+1}^i}{d\Gamma_t^*} > 0.$$

Υ_t^i and Ω_t^i increase monotonically in Γ_t^* and r_t^*. By contradiction, we can prove the existence and the uniqueness of the world interest rates as $\Gamma_t^* \in (\Gamma_{IFA}^F, \Gamma_{IFA}^H)$ and $r_t^* \in (r_{IFA}^H, r_{IFA}^F)$. Given w_t^i, the world interest rates Γ_t^* and r_t^* are uniquely determined by the two equilibrium conditions, i.e., $\Omega_t^H + \Omega_t^F = 0$ and $\Upsilon_t^H + \Upsilon_t^F = 0$,

$$\eta(w_t^H + w_t^F) = \frac{\rho}{\Gamma_t^*}\left[(1-\theta^H)R^\rho\left(\frac{1-\theta^H}{\Gamma_t^*} + \frac{\theta^H}{r_t^*}\right)^\rho + (1-\theta^F)R^\rho\left(\frac{1-\theta^F}{\Gamma_t^*} + \frac{\theta^F}{r_t^*}\right)^\rho\right],$$

$$(1-\eta)(w_t^H + w_t^F) = \frac{\rho}{r_t^*}\left[\theta^H R^\rho\left(\frac{1-\theta^H}{\Gamma_t^*} + \frac{\theta^H}{r_t^*}\right)^\rho + \theta^F R^\rho\left(\frac{1-\theta^F}{\Gamma_t^*} + \frac{\theta^F}{r_t^*}\right)^\rho\right].$$

In the next step, we assume that there exists a unique world loan rate and a unique world equity rate in the steady state and then prove that they are indeed unique.

In the steady state, $w_{t+1}^i = w_t^i$. According to equations (30) and (31), the equilibrium conditions of FDI and financial capital flows, $\Omega^H + \Omega^F = \Upsilon^H + \Upsilon^F = 0$, are rewritten as,

$$\frac{\eta - \frac{(1-\theta^F)\rho}{\Gamma^*}}{\frac{(1-\theta^H)\rho}{\Gamma^*} - \eta} = \frac{w^H}{w^F} = \frac{\frac{\theta^F\rho}{r^*} - (1-\eta)}{(1-\eta) - \frac{\theta^H\rho}{r^*}}, \tag{33}$$

$$\frac{(\theta^F - \theta^H)\rho}{(\rho - \eta\Gamma^*) - \theta^H\rho} - 1 = \frac{(\theta^F - \theta^H)\rho}{(1-\eta)r^* - \theta^H\rho} - 1, \tag{34}$$

$$(\rho - \eta\Gamma^*) - \theta^H\rho = (1-\eta)r^* - \theta^H\rho, \quad \Rightarrow \quad \Gamma^* = \frac{\rho}{\eta} - \frac{1-\eta}{\eta}r^*. \tag{35}$$

In the case of the binding borrowing constraints, $\frac{\partial \ln w^i}{\partial \theta^i} = \frac{\rho(\Gamma^* - r^*)}{r^* + \theta^i(\Gamma^* - r^*)} > 0$ implies $w^H < w^F$. According to equation (33),

$$\frac{\eta - \frac{(1-\theta^F)\rho}{\Gamma^*}}{\frac{(1-\theta^H)\rho}{\Gamma^*} - \eta} = \frac{\frac{\theta^F\rho}{r^*} - (1-\eta)}{(1-\eta) - \frac{\theta^H\rho}{r^*}} = \frac{w^H}{w^F} < 1, \quad \Rightarrow \quad \Gamma^* < \underline{\Gamma}^* \quad \text{and} \quad r^* > \underline{r}^*.$$

Thus, the steady-state values of the world equity rate and the world loan rate are $\Gamma^* \in (\Gamma_{IFA}^F, \underline{\Gamma}^*)$ and $r^* \in (\underline{r}^*, r_{IFA}^F)$, respectively.

Substitute equation (35) and $w^i = R^\rho\left[\frac{1-\theta^i}{\Gamma_t^*} + \frac{\theta^i}{r_t^*}\right]^\rho$ into equation (33), r^* solves,

$$\frac{(\theta^F - \theta^H)}{(1-\eta)\frac{r^*}{\rho} - \theta^H} - 1 = \left[1 - \frac{(\theta^F - \theta^H)}{\frac{\eta}{\frac{\rho}{r^*} - 1} + \theta^F}\right]^\rho. \tag{36}$$

Let $\aleph(r^*) \equiv \frac{(\theta^F - \theta^H)}{(1-\eta)\frac{r^*}{\rho} - \theta^H} - 1$ and $\Re(r^*) \equiv \left[1 - \frac{(\theta^F - \theta^H)}{\frac{\eta}{\frac{\rho}{r^*} - 1} + \theta^F}\right]^\rho$ denote the functions of r^* defined by the left-hand and the right-hand sides of equation (36). Given $\theta^F > \theta^H$ and

$r^* \in (\underline{r}^*, r^F_{IFA})$, we get

$$\aleph'(r^*) < 0 < \Re'(r^*),$$
$$\aleph(r^* = \underline{r}^*) = 1 > \Re(r^* = \underline{r}^*),$$
$$\aleph(r^* = r^F_{IFA}) = 0 < \Re(r^* = r^F_{IFA}).$$

In the steady state, there exists a unique world loan rate $r^* \in (\underline{r}^*, r^F_{IFA})$ that solves equation (36). So does the world equity rate, according to equation (35). □

Proof of Proposition 11

Proof. See the proofs of Propositions 10 and 12. □

Proof of Proposition 12

Proof. According to equations (30) and (31), the steady-state values of FDI and financial capital flows are $\Omega^i = (\Gamma^* - \Gamma^i_{IFA}) \frac{\eta w^i}{\Gamma^*}$ and $\Upsilon^i = (r^* - r^i_{IFA}) \frac{(1-\eta)w^i}{r^*}$, respectively. Since $r^* \in (\underline{r}^*, r^F_{IFA})$, financial capital flows from country H to country F, $\Upsilon^H > 0 > \Upsilon^F$; since $\Gamma^* \in (\Gamma^F_{IFA}, \underline{\Gamma}^*)$, FDI flows in the opposite direction, $\Omega^H < 0 < \Omega^F$. The direction of capital flows is same as under free mobility of FDI and financial capital, respectively.

According to equations (32), net capital flows are $\Omega^i + \Upsilon^i = w^i \left[1 - \frac{\rho}{R}(w^i)^{\frac{1}{\rho}} \right]$ in the steady state. The identity of net capital flows $\Omega^H + \Upsilon^H + \Omega^F + \Upsilon^F = 0$ implies

$$\sum_{i \in \{H,F\}} w^i \left[1 - \frac{\rho}{R}(w^i)^{\frac{1}{\rho}} \right] = 0 \quad \Rightarrow \quad \left[1 - \frac{\rho}{R}(w^H)^{\frac{1}{\rho}} \right] \left[1 - \frac{\rho}{R}(w^F)^{\frac{1}{\rho}} \right] \leq 0. \quad (37)$$

If $\Gamma^* > r^*$, the borrowing constraints are binding and the steady-state wage is lower in country H than in country F, $w^H \leq w^F$. Thus, $1 - \frac{\rho}{R}(w^H)^{\frac{1}{\rho}} > 1 - \frac{\rho}{R}(w^F)^{\frac{1}{\rho}}$. According to equation (37), $1 - \frac{\rho}{R}(w^H)^{\frac{1}{\rho}} > 0 > 1 - \frac{\rho}{R}(w^F)^{\frac{1}{\rho}}$ and net capital flows are from country H to country F in the steady state, $\Omega^H + \Upsilon^H > 0 > \Omega^F + \Upsilon^F$.

If $\Gamma^* = r^*$, the borrowing constraints are not binding and the steady-state wage is same in the two countries with zero net capital flows, $w^i = \left(\frac{R}{\rho} \right)^\rho$ and $\Omega^i + \Upsilon^i = 0$. Economic allocation is almost same as under IFA except that the interest rates in country H are different, $\Gamma^* = \rho < \Gamma^H_{IFA}$ and $r^* = \rho > r^H_{IFA}$. □

Proof of Proposition 13

Proof. According to equations (30) and (31), in the steady state, $r^* \Upsilon^i + \Gamma^* \Omega^i = w^i[(1-\eta)r^* + \eta\Gamma^*] - \rho w^i$. According to equation (35), we get $r^* \Upsilon^i + \Gamma^* \Omega^i = 0$. This way, as a net debtor, $\Upsilon^F + \Omega^F < 0$, country F receives a positive net investment income, $NII^F \equiv (r^*-1)\Upsilon^F + (\Gamma^*-1)\Omega^F = 0 - (\Upsilon^F + \Omega^F) > 0$. Intuitively, the net interest income received by entrepreneurs from investing abroad, $|(\Gamma^*-1)\Omega^F|$ dominates the net interest income paid to foreign workers, $|(r^*-1)\Upsilon^F|$, due to the positive equity premium. □

Proof of Proposition 14

Proof. According to Proposition 12, $w_{FCM}^H \leq w_{IFA}^i \leq w_{FCM}^F$. Given the world equity rate $\Gamma_{FCM}^* \in (\Gamma_{IFA}^F, \underline{\Gamma}^*)$, entrepreneurs born in country H (F) are worse (better) off in the long run than before period $t = 0$, due to the declines (increases) in the wage and in the equity rate, $c^{i,e} = w^i \Gamma^i$.

In the steady state, social welfare in country i is $C^i = \eta c^{i,e} + (1 - \eta)c^{i,w} = w^i[\eta \Gamma^* + (1 - \eta)r^*]$. According to equation (35), social welfare is proportional to aggregate labor income, $C^i = w^i \rho$. Due to net capital flows, aggregate investment in country H (F) is lower and so are the aggregate labor income and social welfare. $\qquad \square$

Proof of Proposition 15

Proof. Take the world equity rate Γ_t^* as given. According to equation (21), w_{t+1}^i is considered as a function of w_t^i. For $w_t^i \in [0, \frac{\theta^i \rho}{R(1-\eta)} \left(\frac{R}{\Gamma_t^*}\right)^{\frac{1}{1-\alpha}})$, the marginal return on investment is equal to the world equity rate, $Rv_{t+1}^i = \Gamma_t^*$, and, thus, entrepreneurs do not borrow to the limit. The phase diagram of wages is flat at $w_{t+1}^i = \left(\frac{R}{\Gamma_t^*}\right)^\rho$, independent of w_t^i.

For $w_t^i \geq \frac{\theta^i \rho}{R(1-\eta)} \left(\frac{R}{\Gamma_t^*}\right)^{\frac{1}{1-\alpha}}$, take the first derivative of equation (21) with respect to w_t^i,

$$1 - \eta = \left[\frac{\rho}{R\alpha}(w_{t+1}^i)^{\frac{1}{\rho}} - \frac{(1-\theta^i)\rho}{\Gamma_t^*} \right] \frac{dw_{t+1}^i}{dw_t^i}. \tag{38}$$

For $w_t^i = \frac{\theta^i \rho}{R(1-\eta)} \left(\frac{R}{\Gamma_t^*}\right)^{\frac{1}{1-\alpha}}$, there is a non-zero solution $w_{t+1}^i = \left(\frac{R}{\Gamma_t^*}\right)^\rho$. The slope of the phase diagram at the point $(\frac{\theta^i \rho}{R(1-\eta)} \left(\frac{R}{\Gamma_t^*}\right)^{\frac{1}{1-\alpha}}, \left(\frac{R}{\Gamma_t^*}\right)^\rho)$ is $\frac{\Gamma_t^* \eta}{\rho[\frac{1}{\alpha} - (1-\theta^i)]} > 0$. In other words, $w_{t+1}^i \geq \left(\frac{\theta^i R}{r^*}\right)^\rho$. Thus, according to equation (38), the phase diagram has the positive slope, $\frac{dw_{t+1}^i}{dw_t^i} > 0$. Take the second derivative of equation (21) with respect to w_t^i,

$$0 = \left[\frac{\rho}{R\alpha}(w_{t+1}^i)^{\frac{1}{\rho}} - \frac{(1-\theta^i)\rho}{\Gamma_t^*} \right] \frac{dw_{t+1}^i}{d^2 w_t^i} + \left(\frac{dw_{t+1}^i}{dw_t^i} \right)^2 \frac{1}{R\alpha}(w_{t+1}^i)^{\frac{1-2\alpha}{\alpha}}, \quad \Rightarrow \quad \frac{dw_{t+1}^i}{d^2 w_t^i} < 0.$$

The phase diagram of wages is concave for $w_t^i > \frac{\theta^i \rho}{R(1-\eta)} \left(\frac{R}{\Gamma_t^*}\right)^{\frac{1}{1-\alpha}}$ and w_{t+1}^i monotonically increases in w_t^i.

The phase diagram of wages is continuous and concave. It crosses the 45 degree line only once and from the left. There exists a stable and unique non-zero steady state. $\quad \square$

Proof of Proposition 16

Proof. The world equity rate is determined by the identity of FDI flows, $\Omega_t^H + \Omega_t^F = 0$. We first prove the existence of a unique world equity rate clearing the world equity market every period and then derive the world equity rate in the steady state.

Suppose that the borrowing constraints are binding in country i. Given the predetermined w_t^i, equation (21) shows that w_{t+1}^i is a function of Γ_t^*. Take the first derivative of equation (21) with respect to Γ_t^*,

$$0 = \left[\frac{\rho}{R\alpha} (w_{t+1}^i)^{\frac{1}{\rho}} - \frac{(1-\theta^i)\rho}{\Gamma_t^*} \right] \frac{dw_{t+1}^i}{dr_t^*} + \frac{(1-\theta^i)\rho}{(\Gamma_t^*)^2}.$$

As shown in the proof of Proposition 15, $w_{t+1}^i \geq \left(\frac{R}{\Gamma_t^*} \right)^\rho$ so that the term in square brackets is positive. An increase in the world equity rate enhances FDI outflows and reduces the domestic investment. Thus, the wage in the next period declines, $\frac{dw_{t+1}^i}{d\Gamma_t^*} < 0$.

Capital outflows represent the gap between domestic savings and investment,

$$\Omega_t^i = w_t^i - I_t^i = w_t^i - \frac{\rho}{R}(w_{t+1}^i)^{\frac{1}{\alpha}} = \eta w_t^i - \frac{\rho}{\Gamma_t^*}(1-\theta^i)w_{t+1}^i. \tag{39}$$

The world equity market equilibrium implies

$$\Omega_t^H + \Omega_t^F = 0, \quad \Rightarrow \quad \eta(w_t^H + w_t^F) = \frac{\rho}{\Gamma_t^*} \left[(1-\theta^H)w_{t+1}^H + (1-\theta^F)w_{t+1}^F \right]. \tag{40}$$

The equity rate in country H is higher than in country F before period $t = 0$, $\Gamma_{IFA}^H > \Gamma_{IFA}^F$. Given the predetermined wage w_t^H and w_t^F, the world equity rate in period t must be $\Gamma_t^* \in (\Gamma_{IFA}^F, \Gamma_{IFA}^H)$ and FDI flows from country F to country H. The proof is by contradiction similar as in the proof of Proposition 3.

In the next step, we assume that there exists a unique world equity rate in the steady state and then prove its uniqueness.

Given Γ^*, Proposition 15 shows the steady-state wage $w^i = \left(\frac{R}{\rho} \right)^\rho \left[(1-\eta) + \frac{(1-\theta^i)\rho}{\Gamma^*} \right]^\rho$. According to equation (39), FDI flows in the steady state are

$$\Omega^i = w^i \left[\eta - \frac{(1-\theta^i)\rho}{\Gamma^*} \right] \quad \Rightarrow \quad \Omega^i = (\Gamma^* - \Gamma_{IFA}^i)\frac{\eta w^i}{\Gamma^*}. \tag{41}$$

According to equation (40), the world equity rate in the steady state is determined by

$$\Omega^H + \Omega^F = 0, \quad \Rightarrow \quad \frac{\Gamma^* - \frac{(1-\theta^F)\rho}{\eta}}{\frac{(1-\theta^H)\rho}{\eta} - \Gamma^*} = \frac{w^H}{w^F}, \quad \Rightarrow \quad \frac{\Gamma^* - \Gamma_{IFA}^F}{\Gamma_{IFA}^H - \Gamma^*} = \left[\frac{\Gamma^* + \frac{1-\theta^H}{1-\eta}\rho}{\Gamma^* + \frac{1-\theta^F}{1-\eta}\rho} \right]^\rho. \tag{42}$$

For $\theta^H \in (0, \bar{\theta})$ and $\theta^F > \theta^H$, the right-hand side of equation (42) is larger than one,

$$\frac{\Gamma^* - \Gamma_{IFA}^F}{\Gamma_{IFA}^H - \Gamma^*} > 1, \quad \text{or} \quad \Gamma^* \in (\underline{\Gamma}^*, \Gamma_{IFA}^H).$$

Let $\aleph(\Gamma^*) \equiv \frac{\Gamma_{IFA}^H - \Gamma_{IFA}^F}{\Gamma_{IFA}^H - \Gamma^*} - 1$ and $\Re(\Gamma^*) \equiv \left[1 + \frac{(\theta^F - \theta^H)\rho}{(1-\eta)\Gamma^* + (1-\theta^F)\rho} \right]^\rho$ denote the left-hand and the right-hand sides of equation (42) as the functions of Γ^*. For $\Gamma^* \in (\underline{\Gamma}^*, \Gamma_{IFA}^H)$,

$$\aleph'(\Gamma^*) > 0 > \Re'(\Gamma^*),$$

$$\aleph(\Gamma^* = \underline{\Gamma}^*) = 0 < \Re(\Gamma^* = \underline{\Gamma}^*),$$

$$\aleph(\Gamma^* = \Gamma_{IFA}^H) \to \infty > \Re(\Gamma^* = \Gamma_{IFA}^H).$$

$\aleph(\Gamma^*)$ decreases while $\Re(\Gamma^*)$ increases monotonically in Γ^*; the two functions cross once and only once at $\Gamma^* \in (\underline{\Gamma}^*, \Gamma^H_{IFA})$. Thus, there exists a unique non-zero steady state. $\qquad\square$

Proof of Proposition 18

Proof. See the proof of Proposition 16 and equation (41). $\qquad\square$

Proof of Proposition 20

Proof. If the borrowing constraints are binding, the steady-state consumption of entrepreneurs is

$$c^{i,e} = w^i \Gamma^* = \left(\frac{R}{\rho}\right)^\rho \left[\Gamma^*(1-\eta) + (1-\theta^i)\rho\right]^\rho (\Gamma^*)^{1-\rho},$$

$$\frac{d\ln c^{i,e}}{d\Gamma^*} = \frac{\Gamma^*(1-\eta) + (1-\theta^i)\rho(1-\rho)}{[\Gamma^*(1-\eta) + (1-\theta^i)\rho]\Gamma^*}.$$

As the analytical solution of the world equity rate is not obtainable, we provide the sufficient conditions of welfare changes as follows.

Let $\mu \equiv \frac{(\rho-1)\eta}{(1-\eta)}$. Evaluate $\frac{d\ln c^{H,e}}{d\Gamma^*}$ at $\Gamma^* = \Gamma^H_{IFA}$ and $\Gamma^* = \underline{\Gamma}^*$. For $\mu \leq \frac{(1-\theta^H)+(1-\theta^F)}{2(1-\theta^H)}$, $\frac{d\ln c^{H,e}}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} > \frac{d\ln c^{H,e}}{d\Gamma^*}|_{\Gamma^*=\underline{\Gamma}^*} \geq 0$ implies that entrepreneurs born in country H is worse off in the long run than under before period $t = 0$, since the negative equity rate effect dominates the positive wage effect; for $\mu > 1$, $\frac{d\ln c^{H,e}}{d\Gamma^*}|_{\Gamma^*=\underline{\Gamma}^*} < \frac{d\ln c^{H,e}}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} \leq 0$ implies that entrepreneurs born in country H is better off in the long run since the positive wage effect dominates; for $\mu \in (\frac{(1-\theta^H)+(1-\theta^F)}{2(1-\theta^H)}, 1)$, the numerical solutions are required for the welfare evaluation.

Evaluate $\frac{d\ln c^{F,e}}{d\Gamma^*}$ at $\Gamma^* = \Gamma^H_{IFA}$ and $\Gamma^* = \Gamma^F_{IFA}$. For $\mu \leq 1$, we get $\frac{d\ln c^{F,e}}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} > \frac{d\ln c^{F,e}}{d\Gamma^*}|_{\Gamma^*=\Gamma^F_{IFA}} \geq 0$, implying that entrepreneurs born in country F is better off in the long run since the positive equity rate effect dominates the negative wage effect; for $\mu \geq \frac{1-\theta^H}{1-\theta^F}$, we get $\frac{d\ln c^{F,e}}{d\Gamma^*}|_{\Gamma^*=\Gamma^F_{IFA}} < \frac{d\ln c^{F,e}}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} \leq 0$, implying that entrepreneurs born in country F is worse off in the long run since the the negative wage effect dominates; for $\mu \in (1, \frac{1-\theta^H}{1-\theta^F}, 1)$, the numerical solutions are required for the welfare evaluation.

Social welfare of country i in the steady state is

$$C^i \equiv \eta c^{i,e} + (1-\eta)c^{i,w} = w^i[\eta\Gamma^* + (1-\eta)r^i]$$

$$= \left(\frac{R}{\rho}\right)^\rho \left[\Gamma^*(1-\eta) + (1-\theta^i)\rho\right]^\rho (\Gamma^*)^{-\rho}(\eta\Gamma^* + \theta^i\rho),$$

$$\frac{d\ln C^i}{d\Gamma^*} = \frac{(1-\eta)\rho}{\Gamma^*(1-\eta) + (1-\theta^i)\rho} - \frac{\rho}{\Gamma^*} + \frac{\eta}{\eta\Gamma^* + \theta^i\rho}.$$

Evaluate $\frac{d\ln C^H}{d\Gamma^*}$ at $\Gamma^* = \Gamma^H_{IFA}$ and $\Gamma^* = \underline{\Gamma}^*$. For $\rho \in (0, \frac{(2-\theta^H-\theta^F)[2-\theta^H-\theta^F+\eta(\theta^F-\theta^H)]}{2(1-\theta^H)[(2-(\theta^F-\theta^H)]}]$, $\frac{d\ln C^H}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} > \frac{d\ln C^H}{d\Gamma^*}|_{\Gamma^*=\underline{\Gamma}^*} \geq 0$ implies that the welfare loss of entrepreneurs dominates the welfare gains of workers and hence, country H as a whole loses in the long run from

free mobility of FDI; for $\rho \in [\frac{1-\theta^H}{\eta}, \infty)$, $\frac{d\ln C^H}{d\Gamma^*}|_{\Gamma^*=\underline{\Gamma}^*} < \frac{d\ln C^H}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} \leq 0$ implies that both workers and entrepreneurs are better off or the workers' welfare gains dominate the welfare losses of entrepreneurs and hence, country H as a whole benefits in the long run from free mobility of FDI; for $\rho \in (\frac{(2-\theta^H-\theta^F)[2-\theta^H-\theta^F+\eta(\theta^F-\theta^H)]}{2(1-\theta^H)[(2-(\theta^F-\theta^H)]}, \frac{1-\theta^H}{\eta})$, the numerical solutions are required for the welfare evaluation.

Evaluate $\frac{d\ln C^F}{d\Gamma^*}$ at $\Gamma^* = \Gamma^H_{IFA}$ and $\Gamma^* = \Gamma^F_{IFA}$. For $\rho \in (0, \frac{1-\theta^H}{\eta}]$, $\frac{d\ln C^F}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} > \frac{d\ln C^F}{d\Gamma^*}|_{\Gamma^*=\Gamma^F_{IFA}} \geq 0$ implies that the welfare gains of entrepreneurs dominates the welfare losses of workers and hence, country F as a whole benefits in the long run from free mobility of FDI; for $\rho \in [\frac{(1-\theta^H)[1-\theta^H-\eta(\theta^F-\theta^H)]}{\eta(1-\theta^F)(1+\theta^F-\theta^H)}, \infty)$, $\frac{d\ln C^F}{d\Gamma^*}|_{\Gamma^*=\Gamma^F_{IFA}} < \frac{d\ln C^F}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} \leq 0$ implies that both workers and entrepreneurs are worse off or the welfare gains of entrepreneurs is dominated by the welfare losses of workers and hence, country F as a whole loses in the long run from free mobility of FDI; for $\rho \in (\frac{1-\theta^F}{\eta}, \frac{(1-\theta^H)[1-\theta^H-\eta(\theta^F-\theta^H)]}{\eta(1-\theta^F)(1+\theta^F-\theta^H)})$, the numerical solutions are required for the welfare evaluation. $\square$

Proof of Proposition 22

Proof. According to Proposition 4, if the borrowing constraints are binding in the two countries under free mobility of financial capital, the steady-state equity rate $\Gamma^i = \frac{(1-\theta^i)\rho}{\eta}$ has the same form as under IFA. Given $\theta^H \in (0, \bar{\theta})$ and $\theta^F = \bar{\theta}^F_{FCF}$, the equity rate in country F is equal to the world loan rate and the borrowing constraints are weakly binding in the steady state, $\Gamma^F = \frac{\rho(1-\bar{\theta}^F_{FCF})}{\eta} = r^*$. Thus, $\bar{\theta}^F_{FCF}$ is the solution to the following equation,

$$\frac{\bar{\theta}^F_{FCF} - \frac{1-\eta}{\eta}(1-\bar{\theta}^F_{FCF})}{\frac{1-\eta}{\eta}(1-\bar{\theta}^F_{FCF}) - \theta^H} = (1 - \bar{\theta}^F_{FCF} + \theta^H)^\rho. \tag{43}$$

Let $\aleph(\bar{\theta}^F_{FCF}) \equiv \frac{\bar{\theta}^F_{FCF} - \theta^H}{\frac{1-\eta}{\eta}(1-\bar{\theta}^F_{FCF}) - \theta^H} - 1$ and $\Re(\bar{\theta}^F_{FCF}) \equiv (1 - \bar{\theta}^F_{FCF} + \theta^H)^\rho$ denote the left-hand and the right-hand sides of equation (43) as the functions of $\bar{\theta}^F_{FCF}$. For $\bar{\theta}^F_{FCF} \in (\bar{\theta}, 1 - \frac{\theta^H \eta}{1-\eta})$,

$$\aleph'(\bar{\theta}^F_{FCF}) > 0 > \Re'(\bar{\theta}^F_{FCF}),$$
$$\aleph(\bar{\theta}^F_{FCF} = \bar{\theta}) = 0 < (\eta + \theta^H)^\rho = \Re(\bar{\theta}^F_{FCF} = \bar{\theta}),$$
$$\aleph(\bar{\theta}^F_{FCF} = 1 - \frac{\theta^H \eta}{1-\eta}) \to +\infty > \left(\frac{\theta^H}{1-\eta}\right)^\rho = \Re(\bar{\theta}^F_{FCF} = 1 - \frac{\theta^H \eta}{1-\eta}).$$

Thus, $\aleph(\bar{\theta}^F_{FCF})$ monotonically increases while $\Re(\bar{\theta}^F_{FCF})$ monotonically decreases in $\bar{\theta}^F_{FCF}$; the two functions cross once and only once for $\bar{\theta}^F_{FCF} \in (\bar{\theta}, 1 - \frac{\theta^H \eta}{1-\eta})$. Therefore, the threshold value of $\bar{\theta}^F_{FCF} \in (\bar{\theta}, 1 - \frac{\theta^H \eta}{1-\eta})$ exists and is unique.

For $\theta^F \in (\bar{\theta}^F_{FCF}, 1]$, $\Gamma^F = r^*$ in the steady state and the borrowing constraints are not binding in country F. The economic allocation is same as in the case of $\theta^F = \bar{\theta}^F_{FCF}$. $\square$

Proof of Proposition 23

Proof. If the borrowing constraints are binding in the two countries under free mobility of FDI, the steady-state loan rate $r^i = \frac{\theta^i \rho}{(1-\eta)}$ has the same form as under IFA. Suppose that given $\theta^H \in (0, \bar{\theta})$ and $\theta^F = \bar{\theta}^F_{FDI}$, the borrowing constraints are binding and the loan rate in country F is equal to the world equity rate, $r^F = \frac{\theta^F \rho}{1-\eta} = \Gamma^*$. Substitute it into equation (42),

$$\frac{\eta - (1 - \bar{\theta}^F_{FDI})}{(1-\eta)(1-\theta^H) - \theta^F \eta} = (1 + \bar{\theta}^F_{FDI} - \theta^H)^\rho. \tag{44}$$

It can be shown for $\eta \in [\frac{2^\rho}{1+2^{\rho+1}}, 1)$, given $\theta^H \in (0, \bar{\theta})$, there exist a $\bar{\theta}^F_{FDI} \in (\bar{\theta}, 1)$ that solve equation (44). For $\eta \in (0, \frac{2^\rho}{1+2^{\rho+1}})$, there exists a $\underline{\theta}^H$ that solves equation (45),

$$\frac{\eta}{(1-\eta)(1-\theta^H) - \eta} = (2 - \theta^H)^\rho. \tag{45}$$

For $\theta^H \in [\underline{\theta}^H, \bar{\theta})$, there exists $\bar{\theta}^F_{FDI}$ that solves equation (44); for $\theta^H \in (0, \underline{\theta}^H)$, the borrowing constraints are always binding in country F for $\theta^F \in (\theta^H, 1]$. $\square$

Proof of Proposition 24

Proof. Suppose that for $\theta^H \in (\max\{1 - 2\eta, 0\}, 1 - \eta)$ and $\theta^F = \bar{\theta}^F_{FCM}$, the borrowing constraints are binding and the loan rate is equal to the equity rate in both countries. According to equation (35), $\Gamma^* = r^* = \rho$. The wage is same in the two countries, $w^i = \left(\frac{R}{\eta}\right)^\rho$. According to equation (33),

$$\frac{\frac{\theta^F \rho}{r^*} - (1 - \eta)}{(1-\eta) - \frac{\theta^H \rho}{r^*}} = \frac{w^H}{w^F} = 1, \quad \Rightarrow \quad \bar{\theta}^F_{FCM} = 2(1-\eta) - \theta^H. \tag{46}$$

For $\theta^F \in (\bar{\theta}^F_{FCM}, 1)$, the borrowing constraints are not binding and the loan rate is equal to the equity rate at $\Gamma^* = r^* = \rho$. $\square$