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ABSTRACT

Keynes Meets Markowitz: The Trade-off Between Familiarity and Diversification

We develop a model of portfolio choice to nest the views of Keynes - who advocates concentration in a few familiar assets - and Markowitz - who advocates diversification across assets. We rely on the concepts of ambiguity and ambiguity aversion to formalize the idea of an investor's "familiarity" toward assets. The model shows that when an investor is equally ambiguous about all assets, then the optimal portfolio corresponds to Markowitz's fully diversified portfolio. In contrast, when an investor exhibits different degrees of familiarity across assets, the optimal portfolio depends on (i) the relative degree of ambiguity across assets, and (ii) the standard deviation of the estimate of expected return on each asset. If the standard deviation of the expected return estimate and the difference between the ambiguity about familiar and unfamiliar assets are low, then the optimal portfolio is composed of a mix of both familiar and unfamiliar assets; moreover, an increase in correlation between assets causes an investor to increase concentration in the assets with which they are familiar (flight to familiarity). Alternatively, if the standard deviation of the expected return estimate and the difference between the ambiguity of familiar and unfamiliar assets are high, then the optimal portfolio contains only the familiar asset(s) as Keynes would have advocated. In the extreme case in which the ambiguity about all assets and the standard deviation of the estimated mean are high, then no risky asset is held (non-participation). The model also has empirically testable implications for trading behavior: in response to a change in idiosyncratic volatility, the Keynesian portfolio always exhibits more trading than the Markowitz portfolio, while the opposite is true for a change in systematic volatility. In the equilibrium version of the model with heterogeneous investors who are familiar with different assets, we find that the risk premium of stocks depends on both systematic and idiosyncratic volatility, and that the equity risk premium is significantly higher than in the standard model without ambiguity.

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1 Introduction

John Maynard Keynes and Harry Markowitz personify two contrasting schools of thought concerning the process of portfolio selection. On the one hand, Keynes advocates allocating wealth in the few stocks about which one feels most favorably:

> The right method in investment is to put fairly large sums into enterprises which one thinks one knows something about [...]. It is a mistake to think that one limits one's risk by spreading too much between enterprises about which one knows little and has no reason for special confidence. [...] One's knowledge and experience are definitely limited and there are seldom more than two or three enterprises at any given time in which I personally feel myself entitled to put full confidence.[1]

Keynes is not alone in his view on portfolio selection. Loeb (1950), for example, advocates: "Once you obtain confidence, diversification is undesirable; diversification [is] an admission of not knowing what to do and an effort to strike an average." Moreover, the Keynesian view is far from old-fashioned: Warren Buffet, in his letter to the Berkshire Hathaway shareholders in 1991, espouses the same view and supports it by citing the above quote from Keynes.[2]

On the other hand, Markowitz (1952, p. 77), championing the concept of diversification, argues that it is inefficient to put a large holding in just a few stocks, and that an investor should instead diversify across a large number of stocks, as the following quote effectively summarizes:

> Diversification is both observed and sensible; a rule of behavior which does not imply the superiority of diversification must be rejected both as a hypothesis and as a maxim.

Even though Markowitz's idea of diversification has been accepted as one of the most fundamental tenets of modern financial economics, a vast body of empirical evidence that we review below suggests that investors do not hold diversified portfolios but rather invest heavily in only a few assets, as suggested by Keynes. However, because of the lack of an analytical characterization, the academic literature has so far paid relatively little attention to Keynes's view on portfolio selection, preferring instead the more extensively developed analytical framework supporting Markowitz's idea of diversification. Our objective in this paper is to fill this gap by developing a model that allows us to assess, both qualitatively and quantitatively, the different trade-offs advocated by Keynes and Markowitz.[3]

In particular, our goal is to understand the implications of Keynes's view for the portfolios selected by individual investors, and to answer the following questions. Under what circumstances should

[1] From Keynes's letter to F. C. Scott, 15 August 1934, Keynes (1983).

[2] Warren Buffet's letter is available at http://www.berkshirehathaway.com/letters/1991.html.

[3] The view of Keynes has been making a comeback also in the field of macroeconomics in the face of the recent financial crisis; see, for example, the article titled "The other-worldly philosophers" in the July 16, 2009 issue of *The Economist*.

investors hold only the assets they "feel confident about" (*familiar* assets), and when should they hold only the broadly diversified (and possibly *unfamiliar*) market portfolio? When they hold both, how should their portfolio be allocated between the familiar asset(s) and the market portfolio? How does this allocation depend on the volatility and correlation of asset returns? When the number of assets available for investment is large, are the benefits of investing in the familiar assets overwhelmed by the benefits from diversification? What are the implications of an imperfectly diversified portfolio, as Keynes recommends, for the asset risk premia required in equilibrium?

We model the lack of familiarity via the concepts of *ambiguity* (or uncertainty) in the sense of Knight (1921) and ambiguity *aversion*. In our model, the investor exhibits different degrees of ambiguity about the distributions of asset returns and considers "familiar" the asset(s) with the lowest level of ambiguity. Specifically, we build on the portfolio selection framework of an ambiguity averse investor developed in Garlappi, Uppal, and Wang (2007). The framework we develop has three attractive features: (i) it is simple and only a mild departure from the well understood Markowitz (1959) model; (ii) it has a solid axiomatic foundation; and (ii) it is capable of capturing in a parsimonious way several aspects of the observed evidence on household portfolios. Our framework can also be thought of as providing an analytic characterization of the concept of familiarity introduced by Huberman (2001).

A framework based on ambiguity aversion is appealing to model individual investment decisions. Experimental evidence shows that ambiguity aversion is particularly strong in cases where people feel that their competence in assessing the relevant probabilities is low and in comparative situations (Heath and Tversky (1991) and Fox and Tversky (1995)). The concepts of ambiguity and ambiguity aversion have lately received renewed attention in the aftermath of the financial crisis, triggered by the collapse of mortgage-related securities in August 2007. The common practice followed by financial institutions of "stress-testing" pricing models used for trading and hedging reveals preferences that cannot be reconciled with the Savage (1954) axioms of expected utility. Instead, by expressing concerns about "model uncertainty", financial institutions are behaving *as if* they are ambiguity averse (see Routledge and Zin (2009)).

To isolate the effect of ambiguity on portfolio selection, we consider an economy with N *identical* assets that differ only in the degree of uncertainty the investor exhibits toward the expected returns of each asset. We operationalize the concept of ambiguity as the size of the confidence interval each investor has for the statistical estimate of the mean of each asset return. This characterization of ambiguity, that dates back to Bewley (1988), allows us to "quantify" ambiguity as the size of the confidence interval for the estimate of an asset's expected return.

The structure of the optimal portfolio that emerges from our model depends on two quantities: (i) the standard deviation of the estimate of the expected return, and (ii) the relative degree of ambiguity across different assets. Specifically, we show that when an investor is equally ambiguous about all assets, then the optimal portfolio corresponds to Markowitz's fully diversified portfolio. In contrast, when the investor exhibits different degrees of ambiguity across assets, then the structure of the optimal portfolio depends on the *relative* degree of ambiguity and on the standard deviation of the estimate of expected return. If the standard deviation is low and the difference between the ambiguity levels of the familiar and unfamiliar assets is small, then the optimal portfolio is composed of a mix of both familiar and unfamiliar assets. Alternatively, if the standard deviation of the expected return estimate is high and the difference between the ambiguity levels of the familiar and unfamiliar assets is large, then the optimal portfolio contains *only* the familiar asset(s) as Keynes would have advocated. In the extreme case in which the ambiguity levels for all assets is high and the standard deviation of the estimated mean is also high, then the investor optimally decides not to hold any risky asset (*non-participation*).[4]

Several interesting implications emerge from the above structure of the optimal portfolio. First, in all cases, other than in the limiting case in which the optimal portfolio corresponds to Markowitz's fully diversified portfolio, the optimal portfolio will be exposed to idiosyncratic risk. Second, when the optimal portfolio contains both familiar and unfamiliar assets, the optimal holding of the familiar asset *increases* with the correlation across assets, a manifestation of the "flight to familiarity" phenomenon. The reason is that when correlations increase, the benefit from diversification from holding the unfamiliar asset together with the familiar asset decreases, and therefore the investor holds relatively more of the familiar asset. Thus, in our model we obtain a flight to familiarity uniquely from the ambiguity-aversion channel, which distinguishes our mechanism from liquidity-based theories such as Brunnermeier and Pedersen (2009) and Vayanos (2004). Third, a change in idiosyncratic risk induces more rebalancing in the ambiguity-averse portfolio than in the Markowitz portfolio, while a change in systematic risk has the opposite effect. Finally, in an equilibrium setting, where investors are heterogeneous in terms of the assets with which they are familiar, the equilibrium expected return depends on both systematic and idiosyncratic risk, as well as an ambiguity premium. This last result addresses a criticism that has frequently been leveled at behavioral economics that "it really hasn't shown in aggregate how it affects prices."[5]

Our quantitative assessment of the model reveals that reasonable differences in familiarity across assets can generate portfolios that are biased toward a few assets. For example, using data calibrated to the U.S. stock market, we find that an investor who uses a 1.0 standard-deviation bound on the

[4]Easley and O'Hara (2009) show how in such circumstances regulation can be effective.

[5]The quoted text is a statement by Myron Scholes as reported in the article "Efficiency and beyond" in the July 16, 2009 issue of *The Economist*.

estimate of the risk premium for the stock market and a 0.75 standard-deviation bound on the estimate of the expected excess return on the familiar stock will invest 20% in the familiar asset and 80% in the market. On the other hand, an investor who uses a 1.5 standard-deviation bound on the estimate of the equity risk premium will hold only the familiar asset and not invest at all in the market portfolio; thus, the model can explain why an investor might find it optimal to hold only two or three stocks. Furthermore, an investor who uses a 2.0 standard-deviation bound on the estimate of the risk premium for all assets will find it optimal not to participate in the stock market at all. Finally, in equilibrium we find that modest differences in familiarity across asset returns lead to portfolio holdings that are underdiversified, and, therefore, exposed to idiosyncratic risk. This idiosyncratic risk is reflected in the equity risk premium, which is substantially greater than in a model without ambiguity.

It is, of course, not surprising that familiarity toward some assets leads to a bias in portfolio holdings toward those assets. However, the contribution of this paper is: First, to provide a unified framework that nests the views of Keynes and Markowitz and to propose an empirically motivated measure of ambiguity that originates from classical statistics. Second, to explore the implications of the familiarity bias for (i) the riskiness of an investor's portfolio; (ii) the sensitivity of the portfolio to variations in asset-return correlations and the ensuing flight to familiarity effect; (iii) the opposite effects of systematic and idiosyncratic volatility on the trading intensity observed in the Keynes and Markowitz portfolios; and (iv) the equilibrium risk premium. Third, to show quantitatively that, for reasonable parameter values, the implications of the model are consistent with several empirical observations such as the low number of stocks held by investors; the "own-company stock," "home-country," and limited-stock-market-participation puzzles; the presence of idiosyncratic risk in equilibrium equity stock returns; and the magnitude of the equity risk premium.

We now describe the literature related to our work. Since the early work of Epstein and Wang (1994), many papers have relied on the concept of ambiguity (or Knightian uncertainty) to model an individual investor's attitude towards uncertain scenarios and explore its implications for asset prices (Dow and Werlang (1992), Cao, Wang, and Zhang (2005)); portfolio choice (Uppal and Wang (2003), Garlappi, Uppal, and Wang (2007)), regulation (Easley and O'Hara (2009)), liquidity (Easley and O'Hara (2008) and Routledge and Zin (2009)) and information processing (Epstein and Schneider (2004) and Illeditisch (2009)). Although some of the modeling features of our paper are shared with these papers, the main difference of our work is the focus on the trade-off between diversification and familiarity, and the implications of this tradeoff for trading behavior and asset prices.

Early evidence on the lack of diversification is provided by Blume and Friend (1975): using data on income-tax returns, they find that most investors hold only one or two stocks. Kelly (1995), relying on data from the Survey of Consumer Finances (SCF), finds that mean-variance efficiency does not describe very well the portfolio diversification of U.S. households: the median stockholder

owns a single publicly traded stock, often in the company where he works. Polkovnichenko (2005), also using data from SCF, finds that of the households that hold individual stocks directly, the median number of stocks held was two from 1983 until 2001, when it increased to three, and that poor diversification is often attributable to investments in employer stock, which is a significant part of equity portfolios. Barber and Odean (2000) and Goetzman and Kumar (2008) report similar findings of underdiversification based on data for individual investors at a U.S. brokerage. Calvet, Campbell, and Sodini (2007), based on detailed government records covering the entire Swedish population, find that some households are poorly diversified and bear significant idiosyncratic risk. A comprehensive summary of this aspect of household finance is provided in Campbell (2006, Section 3).

Typically, the few assets that investors do hold are ones with which they are "familiar." Huberman (2001) introduces the idea that people invest in familiar assets and provides evidence of this in a multitude of contexts. Massa and Simonov (2006) also find that investors tilt their portfolios away from the market portfolio and toward stocks that are geographically and professionally close to the investor, resulting in a portfolio biased toward familiar stocks. French and Poterba (1990), Cooper and Kaplanis (1994), and Tesar and Werner (1995) document that investors bias their portfolios toward "home equity" rather than diversifying internationally; Grinblatt and Keloharju (2001) and Massa and Simonov (2006) find this bias to be present among Finnish and Swedish investors, respectively; and Feng and Seasholes (2004) find that Chinese investors not only overweight local companies but also companies that are traded on a local exchange. Coval and Moskovitz (1999) show that the bias toward familiar assets is not just in the international portfolios of small investors, but also U.S. investment managers have a bias toward local equities that are geographically close to the managers.[6] A good summary of this literature is provided in Vissing-Jorgensen (2003, Section 4.2).

The literature has also proposed several other theories to rationalize the tendency of investors to hold only a small number of stocks in their portfolios. These include: the presence of fixed transaction costs (Brennan, 1975), awareness about only a subset of the available stocks (Merton, 1987), investors' preference for skewness (Kraus and Litzenberger, 1976; Conine and Tamarkin, 1981; Harvey and Siddique, 2000; Mitton and Vorkink, 2007; Barberis and Huang, 2008), employees' loyalty toward the company where they work (Cohen, 2009), rank-dependent preferences (Polkovnichenko, 2005), the desire of investors to construct portfolios in "layers" in order to obtain downside protection and upside potential (Shefrin and Statman, 2000), the desire for portfolio insurance in the presence of margin and short-sale constraints (Liu, 2009), and overconfidence (Odean, 1999).

Recently, information advantage has also been offered as a potential explanation for under-diversification in portfolio choice. Van Nieuwerburgh and Veldkamp (2008) develop a model of information *acqui-*

[6]Sarkissian and Schill (2004) find that the familiarity bias is not just in investment decisions but also in financing decisions: the decision to cross-list equity in a particular foreign market is driven by how familiar that market is rather than diversification considerations.

sition and portfolio choice which is capable of explaining portfolio concentration by exploiting the increasing return to scale from specializing in one asset. They use this model in Van Nieuwerburgh and Veldkamp (2006) to study the bias toward investing in own-company stock. and they extend this to a two-country general equilibrium setting in Van Nieuwerburgh and Veldkamp (2009) to study the "home-bias" puzzle. While appealing, an information-based explanation cannot explain entirely the observed bias toward familiar stocks, such as the "own-company" and "home-bias" puzzles. First, this explanation will hold true only if the investor believes that the expected return on the familiar asset (for example, own-company stock or domestic equity) will always be higher than that on other assets; but information about the familiar asset can also be negative, in which case the information-based explanation would recommend a reduced position in the familiar asset; and, in the case of sufficiently negative information about the familiar asset, the information-based explanation would recommend a short position. Second, our model can generate a portfolio with a small number of assets, while in the information-based explanation there is always a component of the Markowitz portfolio. Third, our model can explain non-participation. Finally, an information-based explanation would indicate that investors who hold portfolios that are biased toward a few assets outperform diversified portfolios, but the empirical evidence on this is mixed—for example, while Ivković and Weisbenner (2005) and Massa and Simonov (2006) find some support for this hypothesis, Calvet, Campbell, and Sodini (2007) and Goetzman and Kumar (2008) do not.

The remainder of this paper is organized as follows. In Section 2, we develop the theoretical framework that allows us to study the tradeoff among return, risk, and familiarity. In Section 3, we derive the optimal portfolio weights for an investor who exhibits familiarity toward some assets, and in Section 4 we study the implications of these weights for portfolio risk, trading intensity, and equilibrium returns. Our conclusions are presented in Section 5. Our main results are highlighted in propositions, and the proofs of all propositions are relegated to the Appendix.

2 A Model that Nests the Views of Keynes and Markowitz

In this section, we introduce a model of portfolio choice that incorporates an investor's ambiguity about the true distribution of each asset's return. We first describe our assumptions about the statistical properties of asset returns. We then formulate the portfolio problem of an investor in the classical setting of Markowitz (1959) and finally generalize it to the case of an investor who is averse to ambiguity.

We have made a conscious decision to consider a static, discrete-time model where all assets have the same expected return, volatility, and correlation with other assets, with the only difference across

assets being the degree of ambiguity an investor has about their returns.[7] Our choice of a static model set in discrete-time is dictated by the desire for simplicity; our choice of identical asset returns is driven by the desire to focus on the key characteristic of our framework: differences in familiarity (or ambiguity) across assets. It is however straightforward to extend the model to a setting where each asset has distinct first and second moments.

2.1 Asset returns

We consider a static economy with N identical risky assets and one risk-free asset. Let μ denote the common expected return in excess of the risk-free rate (risk premium), σ the common volatility, and $\rho \geq 0$ the common correlation across assets. The N-dimensional vector of expected excess returns is given by $\boldsymbol{\mu} = \mu \, \mathbf{1}_N$, where $\mathbf{1}_N$ is a $N \times 1$ vector of ones. The $N \times N$ variance-covariance matrix is

$$\Sigma = \begin{bmatrix} \sigma^2 & \text{cov} & \cdots & \text{cov} \\ \text{cov} & \sigma^2 & \cdots & \text{cov} \\ \vdots & \vdots & \ddots & \vdots \\ \text{cov} & \text{cov} & \cdots & \sigma^2 \end{bmatrix} = \begin{bmatrix} \sigma^2 & \rho\sigma^2 & \cdots & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 & \cdots & \rho\sigma^2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho\sigma^2 & \rho\sigma^2 & \cdots & \sigma^2 \end{bmatrix},$$

where $\text{cov} = \rho\sigma^2$ denotes the common covariance between the returns of any two assets. This simplified specification implies that if risky assets are to be held in equilibrium then the risk premium μ must be positive.

2.2 The Markowitz mean-variance portfolio problem

The investor's problem is to choose a vector of portfolio weights, $\pi = (\pi_1, \pi_2, \ldots, \pi_N)^\top$, for the available N risky assets. According to the classical mean-variance model (Markowitz (1952, 1959), and Sharpe (1970)), the optimal portfolio of a risk averse investor is given by the solution to the following optimization problem,

$$\max_{\pi} \; \pi^\top \boldsymbol{\mu} - \frac{\gamma}{2} \pi^\top \Sigma \, \pi, \tag{1}$$

where γ denotes the investor's risk-aversion parameter. If the investor knows the true parameters μ and Σ, the solution to this problem is

$$\pi = \frac{1}{\gamma} \Sigma^{-1} \boldsymbol{\mu} = \frac{1}{\gamma} \left(\frac{\mu}{\sigma^2 \left(1 + \rho(N-1)\right)} \right) \mathbf{1}_N, \tag{2}$$

where the last equality follows from imposing the properties of returns described in Section 2.1.

[7] The issue of portfolio choice in a dynamic setting when the agent is averse to ambiguity has already been addressed in the literature. The case of dynamic portfolio choice with only a single risky asset in a robust-control setting is addressed in Maenhout (2004); the case of multiple risky assets in a dynamic setting in which the investor is averse to ambiguity is considered in Chen and Epstein (2002), Epstein and Miao (2003), and Uppal and Wang (2003).

The *relative* weights in the portfolio of only-risky assets, that is, the fraction of the risky-asset portfolio invested in each asset, is

$$\omega_i = \frac{\pi_i}{\pi^\top \mathbf{1}_N} = \frac{1}{N}, \quad i = 1, \ldots, N. \tag{3}$$

The variance of this portfolio of only-risky assets is given by

$$\omega^\top \Sigma\, \omega = \frac{1}{N}\sigma^2 + \frac{N-1}{N}\text{cov}, \tag{4}$$

so that when the number of assets is large, the variance of the portfolio depends only on *systematic* risk, cov, and not on *idiosyncratic* risk, σ^2:

$$\lim_{N\to\infty} \omega^\top \Sigma\, \omega = \text{cov}. \tag{5}$$

2.3 The Keynesian portfolio problem

A fundamental assumption of the standard mean-variance portfolio selection problem formulated in (1) is that the investor knows the true expected returns and covariance matrix. In practice, however, the investor has to estimate these quantities. The estimation of the expected return is a notoriously difficult task, in comparison to the estimation of the second moments.[8] For simplicity we therefore assume that the investor has perfect knowledge of σ, the volatility of asset returns, and ρ, the correlation between asset returns, but is *ambiguous* about the true expected return.[9]

An intuitive way of modeling this ambiguity is to rely on results from classical statistics and consider the confidence interval for the estimator of the expected returns. Let $\hat{\mu}_n$ be the estimated value of the mean return of asset n obtained by using a return time series of length T, and

$$\sigma_{\hat{\mu}_n} \equiv \sqrt{\sigma_n^2/T}, \tag{6}$$

the standard deviation of the estimate $\hat{\mu}_n$.[10] Therefore, we can define the following *confidence interval* for the expected return μ_n:

$$\alpha_n\text{-confidence interval} = \left\{ \mu_n \,:\, \frac{(\mu_n - \hat{\mu}_n)^2}{\sigma^2_{\hat{\mu}_n}} \leq \alpha_n^2 \right\}, \tag{7}$$

[8] Merton (1980) describes the theoretical reason why estimating the mean is much more difficult than estimating second moments; empirical evidence for the resulting poor performance of the Markowitz portfolio is provided in Frost and Savarino (1986, 1988), Michaud (1989), Best and Grauer (1991), Litterman (2003), and DeMiguel, Garlappi, and Uppal (2009).

[9] In the literature, parameter uncertainty and estimation risk are also used to describe the problem the investor faces. Parameter uncertainty is often dealt with using a Bayesian approach in which the unknown parameters are treated as random variables that are "integrated out" while maximizing utility in order to find optimal portfolios. A Bayesian investor is *neutral* to ambiguity, since he is capable of aggregating the uncertainty about the parameters via a subjective prior. We are interested in the case of an investor who *cannot* form a unique prior on the uncertain parameters and is averse to this ambiguity. We adopt this model in contrast to the Bayesian approach because our model allows us to obtain zero holdings in some assets, a result that cannot be obtained in a Bayesian model.

[10] We consider the case where the length of time over which the mean return is estimated is the same for all assets. It is straightforward to extend the model to allow for different lengths of time over which the mean returns for different assets are estimated.

where α_n is the critical value determining the size, or level, of the confidence interval. The above expression suggests that the critical level α_n can be directly interpreted as a measure of the amount of ambiguity about the estimate of the expected return. A larger value of α_n will result in a larger confidence interval, and hence, a larger set of possible distributions to which the true returns may belong.[11] Moreover, if excess asset returns are Normally distributed and σ_n is known, the quantity $(\hat{\mu}_n - \mu_n)/\sigma_{\hat{\mu}_n}$ has a standard Normal distribution.[12]

If historical data is all the information an investor has, then the above confidence interval, which is based on historical data, is a good description of the ambiguity about the returns on that asset. If, in addition, the investor has further knowledge about a particular asset, say, through working for that company, then this familiarity would typically lead to a reduction in the ambiguity about this asset. Mathematically, this can be represented by a smaller α_n in (7). A smaller α_n is the result of familiarity *above and beyond* the information contained in historical data on returns. In the rest of our analysis, we will refer to *familiar* asset(s) as those for which the investor exhibits the lowest amount of ambiguity about the estimated expected return.

To capture both the presence of ambiguity and the aversion to it, we rely on the work of Gilboa and Schmeidler (1989), using the approach adopted in Garlappi, Uppal, and Wang (2007). We introduce two new components into the standard mean-variance portfolio selection problem in (1). First, we model the *presence* of ambiguity by imposing the confidence interval (7) as an *additional constraint* on the mean-variance optimization program. Second, to account for *aversion* to ambiguity, we impose that the investor chooses his portfolio by *minimizing* over the set of expected returns he considers plausible according to (7).[13] Thus, the extended mean-variance model takes the following form:

$$\max_{\pi} \min_{\boldsymbol{\mu}} \; \pi^\top \boldsymbol{\mu} - \frac{\gamma}{2} \pi^\top \Sigma \pi, \tag{8}$$

subject to

$$\frac{(\mu_n - \hat{\mu}_n)^2}{\sigma^2_{\hat{\mu}_n}} \leq \alpha_n^2, \quad n = 1, \ldots, N, \tag{9}$$

where α_n refers to the level of ambiguity associated with asset n. In the extreme case in which $\alpha_n = 0$ for all n, the optimal portfolio in (8) reduces to the mean-variance portfolio problem in (1) in which the estimated means, $\hat{\mu}_n$, are used as values for the expected returns, μ_n.

[11] Bewley (1988) originally formulated the argument that confidence intervals can be interpreted also as a measure of the level of ambiguity associated with the estimated parameters. For another recent paper that uses Bewley's characterization of Knightian uncertainty, see Easley and O'Hara (2008).

[12] In principal, instead of modeling ambiguity about each individual asset separately, one could model ambiguity about the joint distribution of stock returns; Garlappi, Uppal, and Wang (2007) show how this can be done. We do not use this more general formulation because our objective is to show that even a very simple specification can generate interesting implications for portfolio holdings and asset returns.

[13] Klibanoff, Marinacci, and Mukerji (2005) develop an alternative model of decision-making under uncertainty, which generates smooth demand functions, in contrast to the kinked demand functions generated by the maxmin specification that we use.

3 The Optimal Portfolio Weights in the Keynesian Model

In this section, we solve for the optimal portfolio weights of an investor who is familiar with one or more assets, and then explain how the optimal portfolio can be interpreted in terms of the views of both Markowitz and Keynes. The implications of these portfolio weights are examined in the next section.

To understand the solution to the problem in Equations (8) and (9), and to highlight the Markowitz effects of diversification and the Keynesian effect of familiarity, we start by considering in Section 3.1 the simple case where the investor may invest in only two risky assets, and the investor is more familiar with the first asset. Then, in Section 3.2, we consider the more general problem where there are $N > 2$ risky assets, and the investor is more familiar with one of these assets. In Section 3.3, we consider the case where the number of risky assets goes to infinity. In Section 3.4, we consider the case of N risky assets, which are divided into M classes, with the investor having a different level of familiarity for each of the M asset classes.

3.1 The case with only two risky assets

In the two-asset setting, for the limiting case with *no ambiguity* about asset returns, $\alpha_n \to 0$, and so

$$\mu_n = \hat{\mu}_n \equiv \mu, \quad n = 1, 2. \tag{10}$$

In this case, "Keynes meets Markowitz" and the optimal portfolio is given by the familiar Markowitz expression:

$$\begin{bmatrix} \pi_1 \\ \\ \pi_2 \end{bmatrix} = \frac{1}{\gamma}\Sigma^{-1}\hat{\boldsymbol{\mu}} = \frac{1}{\gamma}\frac{1}{\sigma^2(1+\rho)}\begin{bmatrix} \hat{\mu} \\ \\ \hat{\mu} \end{bmatrix}. \tag{11}$$

The portfolio weights above have the well-known properties that investment in the risky assets increases with expected return, $\hat{\mu}$, and decreases with risk aversion, γ, volatility of returns, σ, and the correlation between the returns on the two assets, ρ. And for the special case where the two assets are uncorrelated, $\rho = 0$, the expression for the weights reduces to:

$$\pi|_{\rho=0} = \frac{1}{\gamma}\frac{1}{\sigma^2}\begin{bmatrix} \hat{\mu} \\ \\ \hat{\mu} \end{bmatrix}. \tag{12}$$

On the other hand, an investor who is averse to ambiguity and has maxmin preferences acts conservatively (or pessimistically), so in his choice of μ_n he will use its lowest possible estimate within the confidence interval when deciding to go long an asset. Doing the inner minimization for the problem in (8)–(9), we see that the investor will use the following estimates for μ_n:

$$\mu_n = \hat{\mu}_n - \text{sign}(\pi_n)\sigma_{\hat{\mu}_n}\alpha_n, \quad n = \{1, 2\}. \tag{13}$$

This equation implies that if the weight of Asset n, π_n, were positive, then the estimated mean return, $\hat{\mu}_n$, should be reduced downward by the product of α_n, the ambiguity about Asset n, and $\sigma_{\hat{\mu}_n}$, the standard deviation of the estimate $\hat{\mu}_n$. This, of course, would reduce the magnitude of the position in Asset n. In our simplified setting, because the excess return on all the risky assets is positive, the investor will never choose to short any of the risky assets. In a more general setting, where all assets do not have the same return distribution, it is possible that the investor would like to be long some assets and short other assets. In that case, for assets that the investor would have shorted, the investor will use the *highest* possible estimate of μ_n within the confidence interval when deciding to go short that asset, thereby again *reducing* the magnitude of the position that is taken. Of course, there is also the possibility that if $\hat{\mu}_n > 0$, then, after subtracting $\sigma_{\hat{\mu}_n}\alpha_n$, the resulting portfolio weight becomes negative, or that if $\hat{\mu}_n < 0$, then after adding $\sigma_{\hat{\mu}_n}\alpha_n$ the resulting weight is positive. In this case, the optimal portfolio is $\pi_n = 0$.

Substituting (13) back into the problem in (8) gives

$$\max_{\pi} \left\{ \pi^\top \hat{\boldsymbol{\mu}} - \sum_{n}^{2} \text{sign}(\pi_n)\pi_n \sigma_{\hat{\mu}_n}\alpha_n - \frac{\gamma}{2}\pi^\top \Sigma \pi \right\}. \tag{14}$$

The optimal portfolio in (14) is given by the following proposition where we assume, without loss of generality, that the investor is relatively more familiar with the first asset, which we label as "Asset F." We use α_F to denote the level of ambiguity for the mean of the return distribution of Asset F, and α_U to denote the level of ambiguity for the other "unfamiliar" asset in the economy, with $\alpha_F \leq \alpha_U$.

Proposition 1 *In the case where only two risky assets are available, and $\alpha_F \leq \alpha_U$, the optimal portfolio is characterized by the following three cases:*

Case I. If $\frac{\hat{\mu}}{\sigma_{\hat{\mu}}} > \alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F)$:

$$\begin{bmatrix} \pi_F \\ \pi_U \end{bmatrix} = \frac{1}{\gamma}\frac{1}{\sigma^2(1+\rho)} \begin{bmatrix} \hat{\mu} - \sigma_{\hat{\mu}}\left(\alpha_F - \frac{\rho}{1-\rho}(\alpha_U - \alpha_F)\right) \\ \hat{\mu} - \sigma_{\hat{\mu}}\left(\alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F)\right) \end{bmatrix}. \tag{15}$$

Case II. If $\alpha_F < \frac{\hat{\mu}}{\sigma_{\hat{\mu}}} < \alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F)$:

$$\begin{bmatrix} \pi_F \\ \pi_U \end{bmatrix} = \frac{1}{\gamma\sigma^2} \begin{bmatrix} \hat{\mu} - \sigma_{\hat{\mu}}\alpha_F \\ 0 \end{bmatrix}. \tag{16}$$

Case III. If $\alpha_F > \frac{\hat{\mu}}{\sigma_{\hat{\mu}}}$:

$$\begin{bmatrix} \pi_F \\ \pi_U \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{17}$$

To understand the structure of the solution we start by studying the simpler situation in which there is zero correlation between the two assets, $\rho = 0$. In Case I above, for the case of zero correlation:

$$\pi|_{\rho=0} = \begin{bmatrix} \pi_F \\ \\ \pi_U \end{bmatrix} = \frac{1}{\gamma\sigma^2} \begin{bmatrix} \hat{\mu} - \sigma_{\hat{\mu}}\alpha_F \\ \\ \hat{\mu} - \sigma_{\hat{\mu}}\alpha_U \end{bmatrix}. \tag{18}$$

Comparing the expressions in (12) and (18), we see that in the presence of ambiguity about a particular risky asset n, the investment in that risky asset is reduced. Moreover, the investor will hold asset n only if $\hat{\mu}/\sigma_{\hat{\mu}} > \alpha_n$. Conversely, if the above inequality is not satisfied, that is, the ambiguity α_n about asset n is too large, then the investor will not hold asset n at all, even though holding this asset would allow the investor to diversify the portfolio. Clearly, as the level of ambiguity α_n increases, the ratio $\hat{\mu}/\sigma_{\hat{\mu}}$ that is required so that the investor is willing to hold the asset also increases; that is, either the mean return needs to increase or the standard deviation of the mean return needs to decrease.

When the two assets are correlated, from the expressions for the optimal portfolio weights in (15) we see that the investor will hold more of Asset F than Asset U if ambiguity about Asset F is less than that for Asset U: $\alpha_F < \alpha_U$; that is, the relative investment in these two asset classes depends on the relative ambiguity regarding their return distributions. This is because from Equation (15), if $\alpha_F < \alpha_U$, it follows that $(\alpha_F - \rho\alpha_U) < (\alpha_U - \rho\alpha_F)$, and hence, $\pi_F > \pi_U > 0$. Therefore, the portfolio is diversified across the two assets but has a *bias* toward the familiar asset. Only when the level of ambiguity in both assets is the same, $\alpha_F = \alpha_U$, is the proportion of wealth allocated to each asset the same, $\pi_F = \pi_U$.

Comparing the portfolio weights in (15) with those for the Markowitz portfolio in (11), we also see that the investor will hold Asset U (the unfamiliar asset) only if the ratio $\hat{\mu}/\sigma_{\hat{\mu}}$ for this asset is sufficiently large to offset the effect of its own ambiguity *and* the effect that comes from Asset F's correlation with Asset U:

$$\frac{\hat{\mu}}{\sigma_{\hat{\mu}}} > \alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F) = \alpha_U + \frac{\rho}{1-\rho}(\alpha_U - \alpha_F). \tag{19}$$

The quantity on the right-hand side of the above expression can be thought of as the "correlation-adjusted" ambiguity threshold to induce holding in Asset U. This thresholds account for the fact that, due to positive correlation, holding Asset F already entails partially holding Asset U. In the limit as $\rho \to 1$, the right-hand side of (19) goes to infinity, implying that the investor will not hold Asset U at all. The intuition for this result is that Asset F, the more familiar asset, has a higher ambiguity-adjusted return than the relatively unfamiliar Asset U (because $\alpha_F < \alpha_U$), and as the two assets become increasingly correlated, the diversification benefits from holding Asset U diminish. This is a prediction of the model that is consistent with empirical evidence. For instance, during a financial

crisis there is an increase in the correlations between returns on different assets, and at such times we often observe investors pulling out of foreign assets and going into domestic assets because they are more familiar with these assets.

3.2 The case with N risky assets and familiarity about only one asset

We now present the solution for the general case of $N > 2$ assets. Suppose the investor is relatively more familiar with only one of the N available assets, which we label as "Asset F." The investor has a level of ambiguity α_F for the mean of the return distribution of Asset F and a common level of ambiguity α_U for all the remaining $N-1$ "unfamiliar" assets in the economy, with $\alpha_F \leq \alpha_U$.[14] Because the remaining $N-1$ assets are identical in every respect, the optimal portfolio weights in these assets must be identical. We denote by π_F the portfolio holding in the familiar asset and by π_U the $(N-1)$-dimensional vector of holdings in the remaining unfamiliar assets. The following proposition characterizes the optimal portfolio choice of an ambiguity averse investor.

Proposition 2 *Let α_F be the level of ambiguity for the familiar asset and α_U the level of ambiguity common across the remaining $N-1$ assets, with $\alpha_F \leq \alpha_U$. Then the optimal portfolio weights, (π_F, π_U), of an investor who is averse to ambiguity are given by the following three cases:*
Case I. If $\frac{\hat{\mu}}{\sigma_{\hat{\mu}}} > \alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F)$:

$$\pi_F = \frac{1}{\gamma\sigma^2(1+\rho(N-1))}\left(\hat{\mu} - \sigma_{\hat{\mu}}\left(\alpha_F - (N-1)\frac{\rho}{1-\rho}(\alpha_U - \alpha_F)\right)\right) > 0, \tag{20}$$

$$\pi_U = \frac{1}{\gamma\sigma^2(1+\rho(N-1))}\left(\hat{\mu} - \sigma_{\hat{\mu}}\left(\alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F)\right)\right) \cdot \mathbf{1}_{N-1} > \mathbf{0}_{N-1}. \tag{21}$$

Case II. If $\alpha_F < \frac{\hat{\mu}}{\sigma_{\hat{\mu}}} < \alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F)$:

$$\pi_F = \frac{1}{\gamma\sigma^2}\left(\hat{\mu} - \sigma_{\hat{\mu}}\alpha_F\right) > 0, \tag{22}$$

$$\pi_U = \mathbf{0}_{N-1}. \tag{23}$$

Case III. If $0 < \frac{\hat{\mu}}{\sigma_{\hat{\mu}}} \leq \alpha_F$:

$$\pi_F = 0, \tag{24}$$

$$\pi_U = \mathbf{0}_{N-1}. \tag{25}$$

This proposition allows us to formalize the contrasting views of Keynes and Markowitz on portfolio choice. In Case I, the investor holds both the familiar asset (Asset F) and the unfamiliar assets (all

[14] As we show in Proposition 4 below, it is possible to generalize the model to more than two categories of assets with different degrees of ambiguity for each category.

assets other than Asset F). But, because $\alpha_F < \alpha_U$, the weight of Asset F in the portfolio exceeds that of each of the other assets. So, the investor holds familiar assets, as advocated by Keynes, but balances this investment by holding also a portfolio of all the other assets, as advocated by Markowitz.[15] This portfolio is biased toward familiar assets, and like in the two-asset case, the relative investment between the familiar asset and the unfamiliar assets depends on the relative ambiguity regarding the return distributions of these two asset classes, that is, α_F and α_U.

Case II of Proposition 2 corresponds to the setting where the investor is relatively familiar with a particular asset (that is, $\hat{\mu}/\sigma_{\hat{\mu}} > \alpha_F$) *and* sufficiently unfamiliar with all the other assets (that is, $\frac{\hat{\mu}}{\sigma_{\hat{\mu}}} \leq \alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F)$). In this case, the investor follows the advice of Keynes to the extreme: "The right method in investment is to put fairly large sums into enterprises which one thinks one knows something about. ... It is a mistake to think that one limits one's risk by spreading too much between enterprises about which one knows little and has no reason for special confidence." Thus, the household invests only in the familiar risky asset, $\pi_F > 0$, and not at all in the unfamiliar risky assets, $\pi_U = \mathbf{0}_{N-1}$.

In Case III of Proposition 2, the ambiguity about the expected return of the familiar is large (that is, $\hat{\mu}/\sigma_{\hat{\mu}} \leq \alpha_F$), and the ambiguity about the other assets is even larger; thus, $\pi_F = 0$ and also $\pi_U = \mathbf{0}_{N-1}$, which corresponds to the case of complete non-participation in the stock markets that has been documented empirically (see, for instance, Vissing-Jorgensen (2003) and Campbell (2006)). That is, if ambiguity is sufficiently high for all risky assets, the investor will choose not to invest in any of the risky assets, and instead will put his entire wealth in the risk-free asset.

Notice that because $\sigma_{\hat{\mu}} = \sigma/\sqrt{T}$, as $T \to \infty$, $\sigma_{\hat{\mu}} \to 0$, and so the optimal portfolio weights from Proposition 2 converge to the Markowitz portfolio weights, as can be verified from equations (20) and (21). This is intuitive: with an infinite amount of data, there is no ambiguity.

In the following corollary, we derive the expression for the *relative* weights in the familiar risky asset, $\omega_F = \frac{\pi_F}{\pi_F + \pi_U^\top \mathbf{1}_{N-1}}$, and in the other $(N-1)$ assets, $\omega_U = \frac{\pi_U}{\pi_F + \pi_U^\top \mathbf{1}_{N-1}}$.

Corollary 1 *The relative weights (ω_F, ω_U), of an investor who is averse to ambiguity are given by the following three cases:*

[15] Jeremy Siegel, in an article titled "Ben Bernanke's Favorite Stock," published on the web site Yahoo!Finance, reported that Bernanke held only one individual stock, Altria Group (formerly Phillip Morris); Chris Isidore in the article "Bernanke's Bucks," published on CNNMoney.com, reports that an overwhelming majority of Bernanke's holdings are in his TIAA-CREF account. Greenspan, on the other hand, has most of his wealth in Treasury bonds and bond funds, with only a small amount invested in individual stocks: besides the investment in General Electric stock held in his wife's 401(k) account, Greenspan's largest stock holdings are in Abbott Laboratories, Kimberly Clark, Anheuser Busch, and H.J. Heinz.

Case I. If $\frac{\hat{\mu}}{\sigma_{\hat{\mu}}} > \alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F)$,

$$\omega_F = \frac{1}{N} + \sigma_{\hat{\mu}} \frac{(\alpha_U - \alpha_F)\left(1 - \frac{1}{N} + \frac{\rho}{1-\rho}(N-1)\right)}{N\left(\hat{\mu} - \sigma_{\hat{\mu}}\alpha_F - \sigma_{\hat{\mu}}(\alpha_U - \alpha_F)\left(1 - \frac{1}{N}\right)\right)} > \frac{1}{N}, \tag{26}$$

$$\omega_U = \left(\frac{1}{N} - \sigma_{\hat{\mu}} \frac{(\alpha_U - \alpha_F)\left(\frac{1}{1-\rho} - \left(1 - \frac{1}{N}\right)\right)}{N\left(\hat{\mu} - \sigma_{\hat{\mu}}\alpha_F - \sigma_{\hat{\mu}}(\alpha_U - \alpha_F)\left(1 - \frac{1}{N}\right)\right)}\right) \cdot \mathbf{1}_{N-1} < \frac{1}{N} \cdot \mathbf{1}_{N-1}. \tag{27}$$

Case II. If $\alpha_F < \frac{\hat{\mu}}{\sigma_{\hat{\mu}}} \leq \alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F)$*:*

$$\omega_F = 1 \tag{28}$$

$$\omega_U = \mathbf{0}_{N-1}. \tag{29}$$

Case III. If $0 < \frac{\hat{\mu}}{\sigma_{\hat{\mu}}} \leq \alpha_F$*:*

$$\omega_F = 0, \tag{30}$$

$$\omega_U = \mathbf{0}_{N-1}. \tag{31}$$

As expected, relative to the benchmark Markowitz portfolio, the portfolio that combines the views of Keynes and Markowitz (Case I) overweighs the familiar asset $\omega_F > 1/N$ and underweighs the unfamiliar assets, that is, each element of ω_U is less than $1/N$.

To assess the magnitude of the result presented in the proposition above, we undertake the following quantitative exercise. We assume that the investor uses 100 years of annual data ($T = 100$) to estimate the excess return (risk premium) μ on each stock and that the number of available stocks is $N = 50$.[16] We take the estimated value $\hat{\mu}$ of the risk premium to be equal to 6% p.a. We specify the volatility of individual stock returns, σ, to be 30%.[17] This implies that the standard deviation of the estimate of mean returns is $\sigma_{\hat{\mu}} = \frac{\sigma}{\sqrt{T}} = \frac{0.30}{\sqrt{100}} = 0.03$. We assume that the correlation between stocks, ρ, is equal to 25%, which then implies that the volatility of the stock market is $\sqrt{\rho \times \sigma^2} = 15\%$ and that the market Sharpe ratio is 40%. We also assume that the investor's risk aversion is $\gamma = 2$.

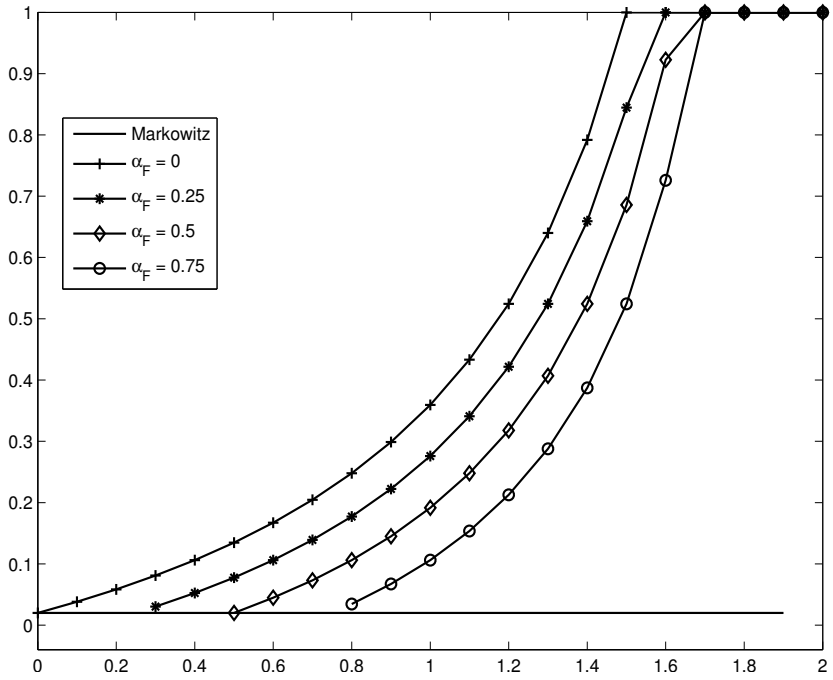
Our results are presented in Figure 1. The horizontal axis of the plot represents the level α_U of ambiguity about the unfamiliar assets. To better appreciate the meaning of a particular value of α_U, it may help to relate its magnitude to the size of the corresponding confidence interval. For example, under the assumption of normally distributed asset returns, a value for α_U of 0.25 corresponds to a 19%

[16] The assumption that we have 100 years of data is a conservative one because the effect of ambiguity decreases with T; in reality, one would be fortunate to find data for even 50 years that was generated by a stationary distribution.

[17] This value for individual firm volatility is in line with the estimates of Chan, Karceski, and Lakonishok (1999) who find that the average volatility of large firms is 28.3%, the average volatility of the average firm is 34.3%, and the average volatility of small firms is 46.6%.

Figure 1: Relative investment in the familiar asset

This figure plots the relative investment in the familiar asset, ω_F, as a function of the ambiguity about the unfamiliar asset, α_U. Each curve in the figure corresponds to a different level of ambiguity about the familiar asset, $\alpha_F = \{0.00, 0.25, 0.50, 0.75\}$. The horizontal solid line plots the relative investment in the familiar asset under the Markowitz model, where there is no ambiguity, and therefore the relative investment in the familiar asset is $1/50 = 0.02$.



confidence interval, $\alpha_U = 0.50$ to a 38% confidence interval, $\alpha_U = 1.00$ to a 68% confidence interval, $\alpha_U = 1.64$ to a 90% confidence interval, $\alpha_U = 1.96$ to a 95% confidence interval, and $\alpha_U = 2.56$ to a 99% confidence interval. The y-axis of the plot displays ω_F, the *relative* investment in the familiar asset as a proportion of the total investment in the $N = 50$ risky assets. The horizontal solid line labeled "Markowitz" shows the investment in the familiar asset when there is no ambiguity about any of the N assets. Each of the curved lines corresponds to a different level of ambiguity for the familiar asset, $\alpha_F = \{0.0, 0.25, 0.5, 0.75\}$.

The figure shows that there is substantial investment in the familiar stock even for low values of α_U. For instance, the leftmost curve for $\alpha_F = 0$ shows that even for $\alpha_U = 0.75$, the investment in the familiar stock exceeds 20%; for $\alpha_U = 1.0$, the investment in the familiar stock exceeds 30%; and, for $\alpha_U = 1.5$, the investment in the familiar stock is 100%. Looking at the rightmost curve for $\alpha_F = 0.75$, we see that if $\alpha_U = 1.5$, then the relative investment in the familiar stock exceeds 50%, and if $\alpha_U = 1.75$, then the relative investment in the familiar stock is 100%.

The trade-offs advocated by Keynes and Markowitz are particularly stark in the context of the decision to invest in own-company stock; that is, the empirical observation that if a firm's own stock is one of the assets available for investment in 401(k) defined-contribution pension plans, many employees invest a significant fraction of their discretionary contributions in their own company's stock. For instance, Benartzi (2001), Mitchell and Utkus (2002), and Meulbroek (2005) find that about 25%–30% of the discretionary contributions of employees are invested in stocks of their own company. From the experiment above, we observe that the model can generate substantial holdings of the familiar stock that is consistent with empirical observations.

3.3 The limiting case with an infinite number of risky assets

To gain further insight on the dichotomy between the views of Keynes and Markowitz on portfolio allocation, we analyze the optimal portfolio in an economy where the number of risky assets, N, goes to infinity. As the number of assets increases, the benefits of diversification increase. As the next proposition shows, these benefits of diversification are not sufficient to overcome the desire to hold a portfolio biased toward familiar assets. We limit our analysis to the case in which the investor optimally is holding both the familiar and unfamiliar asset (Case I of Proposition 2), because for Cases II and III the portfolio weights do not depend on N; and so, even when $N \to \infty$, the expressions for the portfolio weights are the same as those in (22) and (24), respectively, while the relative weights are the same as those in (28) and (30), respectively.

Proposition 3 *Assume the investor is less ambiguous about Asset F than about the rest of the assets, that is, $\alpha_F < \alpha_U$, and that $\frac{\hat{\mu}}{\sigma_{\hat{\mu}}} > \alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F)$ (Case I of Proposition 2). Then, as $N \to \infty$:*

1. *The optimal investment of an ambiguity averse investor in the familiar asset converges to:*

$$\lim_{N\to\infty} \pi_F = \frac{1}{\gamma\sigma^2(1-\rho)}\sigma_{\hat{\mu}}(\alpha_U - \alpha_F) > 0; \tag{32}$$

2. *The investment in each of the $N-1$ assets with which the investor is not familiar goes to zero:*

$$\lim_{N\to\infty} \pi_U = \mathbf{0}_{N-1}; \tag{33}$$

3. *The* total *investment in the $N-1$ assets with which the investor is not familiar converges to:*

$$\lim_{N\to\infty} \pi_U^\top \mathbf{1}_{N-1} = \frac{1}{\gamma\sigma^2\rho}\left(\hat{\mu} - \sigma_{\hat{\mu}}\left(\alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F)\right)\right); \tag{34}$$

4. *The relative portfolio weight ω_F converges to:*

$$\lim_{N\to\infty} \omega_F = \frac{\sigma_{\hat{\mu}}\frac{\rho}{1-\rho}(\alpha_U - \alpha_F)}{\hat{\mu} - \sigma_{\hat{\mu}}\alpha_U} > 0. \tag{35}$$

The above proposition shows that, as N approaches infinity, the weight for the more familiar asset approaches a positive constant that depends on the difference in ambiguity about this asset and all the other assets (Equation (32)). In contrast, Equation (33) shows that the *individual* weights for each of the other less-familiar assets approach zero as the number of assets increases, while the *total* weight in these $(N-1)$ unfamiliar assets approaches the quantity given in Equation (34).

In summary, familiarity about Asset F implies that the holding in this asset does *not* decrease to zero even as N tends to infinity, while the gains from diversification imply that an investor should hold only an infinitesimal amount in each of the remaining unfamiliar assets. As we will show below in Sections 4.3 and 4.4, this result will have implications for the effect of idiosyncratic risk on optimal portfolio rebalancing and also on the equilibrium risk premium.

3.4 The case with N risky assets and varying familiarity across M asset classes

In the above analysis, we considered the case where the N assets can be divided into only two classes, with Asset F in the first class and the remaining $N-1$ assets in the second class. The characterization of the optimal portfolio weights provided in Proposition 2 takes advantage of this particular feature. However, an investor may not always group assets into just two classes. A case of particular interest for its empirical relevance is that documented by Huberman and Jiang (2006) who, using a large dataset of individual portfolio choice in retirement accounts, find that investors tend to choose three to four funds out of the ones offered which, in their study, ranges from 4 to 59. This evidence, combined with the finding that the percentage of retirement account wealth invested in company stock is around thirty percent,[18] seems to suggests that investors do not view all assets other than their own-company stock as being the same in every respect.

Intuitively, the portfolio problem in which an investor categorizes assets into three (or more) separate classes with different degree of ambiguity is a simple generalization of the case of two separate asset classes analyzed above. In the next proposition, we characterize the optimal portfolio for an arbitrary number of asset classes about which the investor has varying degrees of familiarity.

Proposition 4 *Let us assume that the set of available N assets can be categorized into $M \leq N$ mutually exclusive asset classes, with each category containing N_m assets, $m = \{1, \ldots, M\}$ and $N_1 + \ldots + N_M = N$. Each asset class is characterized by degree of ambiguity $\alpha_1 < \alpha_2 < \ldots < \alpha_M$. Denote by π_m the portfolio weight in each of the assets belonging to the class $m = \{1, \ldots, M\}$, and the quantity*

$$s(m) = \alpha_m + \frac{\rho}{1-\rho} \sum_{j=1}^{m-1} N_j(\alpha_m - \alpha_j) > 0, \quad m = \{1, \ldots, M\}. \tag{36}$$

[18] See, for example, Benartzi (2001), Mitchell and Utkus (2002), and Meulbroek (2005).

Then, the optimal portfolio is characterized as follows.

Case I: If $\frac{\hat{\mu}}{\sigma_{\hat{\mu}}} > s(M)$*, then:*

$$\pi_m = \frac{1}{\gamma\sigma^2(1+\rho(N-1))}\left(\hat{\mu} - \sigma_{\hat{\mu}}\left(\alpha_m - \frac{\rho}{1-\rho}\sum_{j\neq m}^{M} N_j(\alpha_j - \alpha_m)\right)\right) > 0, \quad m = \{1,\ldots,M\}. \quad (37)$$

Case II: If $s(m) < \frac{\hat{\mu}}{\sigma_{\hat{\mu}}} \leq s(m+1)$*,* $m = \{1,\ldots,M-1\}$*, then:*

$$\begin{aligned}\pi_i &= \frac{1}{\gamma\sigma^2\left(1+\rho\left(\sum_{j=1}^{m} N_j - 1\right)\right)} \\ &\quad \times \left(\hat{\mu} - \sigma_{\hat{\mu}}\left(\alpha_i - \frac{\rho}{1-\rho}\sum_{j\neq i}^{m} N_j(\alpha_j - \alpha_i)\right)\right) > 0, \quad i = \{1,\ldots,m\}; \quad (38)\\ \pi_j &= 0, \quad j = \{m+1,\ldots,M\}. \quad (39)\end{aligned}$$

Case III: If $\frac{\hat{\mu}}{\sigma_{\hat{\mu}}} \leq s(1)$*, then:*

$$\pi_m = 0, \quad m = \{1,\ldots,M\}. \quad (40)$$

The above proposition is a natural generalization of Proposition 2 to the case of M asset classes with different levels of ambiguity. It can be verified that by setting $M = 2$ and $N = 1$ one recovers the optimal portfolio derived in Proposition 2. The general structure of the solution suggests that the portfolio weights are larger in the assets with smaller ambiguity, that is, $\pi_1 \geq \pi_2 \ldots \geq \pi_M$. In contrast, when the level of ambiguity is sufficiently large in a particular asset class, the optimal decision is not to hold assets in that class, as Cases II and III of the above proposition show.

We now examine whether our model can generate the pattern of asset allocation documented by Huberman and Jiang (2006) for reasonable parameter values. In order to analyze this, we divide the class of assets *other* than the familiar own-company stock into two separate sub-classes. We take the first sub-class to be composed of three identical assets, to capture the identical allocation across the chosen funds documented by Huberman and Jiang (2006). The remaining $N - 3 - 1$ assets are treated as distinct from own-company stock and from the funds, but otherwise identical and ex-ante indistinguishable from each other, and we assume that the total number of assets offered is $N = 50$. Using the same parameter values as before, we now determine under what values for α_1, α_2, and α_3 we obtain portfolio weights that are similar to the observed "30–70–0" asset allocation in own-company stocks, funds, and the rest of the assets, respectively.

In Table 1, we report the relative portfolio weights for the case with three asset classes: own-company stock, ω_1, three funds (with aggregate weight $\omega_2^\top \mathbf{1}_3$), and the remaining 46 $(= 50 - 3 - 1)$

Table 1: Relative portfolio weights when there are three asset classes

This table gives the relative investment in own-company stock, ω_1, the sum of the investment in three other assets, $\omega_2^\top \mathbf{1}_3$, and the investment in the remaining 46 $(= 50 - 3 - 1)$ assets, $\omega_3^\top \mathbf{1}_{46}$, as a function of the ambiguity about the assets in these three asset classes, α_1, α_2, and α_3. Each panel in the table corresponds to a different level of ambiguity about the familiar asset, $\alpha_1 = \{0.00, 0.25, 0.50, 0.75\}$. Each row corresponds to a different level of $\alpha_2 = \{0.75, 0.80, 0.85, 0.90\}$. The four sets of columns correspond to $\alpha_3 = \{0.90, 1.10, 1.25, 1.50\}$. All other parameter values are as described earlier in the text.

	$\alpha_3 = 0.90$			$\alpha_3 = 1.10$			$\alpha_3 = 1.25$			$\alpha_3 = 1.50$		
α_2	ω_1	$\omega_2^\top \mathbf{1}_3$	$\omega_3^\top \mathbf{1}_{46}$	ω_1	$\omega_2^\top \mathbf{1}_3$	$\omega_3^\top \mathbf{1}_{46}$	ω_1	$\omega_2^\top \mathbf{1}_3$	$\omega_3^\top \mathbf{1}_{46}$	ω_1	$\omega_2^\top \mathbf{1}_3$	$\omega_3^\top \mathbf{1}_{46}$
						Panel A: $\alpha_1 = 0.00$						
0.75	0.29	0.18	0.53	0.42	0.41	0.18	0.48	0.52	0.00	0.48	0.52	0.00
0.80	0.30	0.13	0.57	0.42	0.35	0.23	0.50	0.50	0.00	0.50	0.50	0.00
0.85	0.30	0.09	0.62	0.42	0.30	0.28	0.52	0.48	0.00	0.52	0.48	0.00
0.90	0.30	0.04	0.66	0.42	0.25	0.33	0.55	0.45	0.00	0.55	0.45	0.00
						Panel B: $\alpha_1 = 0.25$						
0.75	0.22	0.18	0.60	0.33	0.41	0.26	0.41	0.59	0.00	0.41	0.59	0.00
0.80	0.22	0.14	0.64	0.33	0.36	0.31	0.43	0.57	0.00	0.43	0.57	0.00
0.85	0.22	0.09	0.69	0.33	0.31	0.36	0.45	0.54	0.02	0.45	0.55	0.00
0.90	0.22	0.05	0.73	0.33	0.26	0.41	0.45	0.47	0.08	0.48	0.52	0.00
						Panel C: $\alpha_1 = 0.50$						
0.75	0.14	0.19	0.67	0.23	0.42	0.35	0.33	0.67	0.00	0.33	0.67	0.00
0.80	0.14	0.14	0.72	0.24	0.37	0.40	0.34	0.61	0.06	0.35	0.65	0.00
0.85	0.14	0.10	0.76	0.24	0.31	0.45	0.34	0.54	0.12	0.37	0.63	0.00
0.90	0.15	0.05	0.80	0.24	0.26	0.50	0.34	0.48	0.18	0.40	0.60	0.00
						Panel D: $\alpha_1 = 0.75$						
0.75	0.06	0.19	0.74	0.14	0.43	0.43	0.23	0.68	0.10	0.25	0.75	0.00
0.80	0.06	0.15	0.79	0.14	0.38	0.48	0.23	0.62	0.16	0.27	0.73	0.00
0.85	0.07	0.10	0.83	0.15	0.32	0.53	0.23	0.55	0.22	0.29	0.71	0.00
0.90	0.07	0.06	0.88	0.15	0.27	0.58	0.23	0.49	0.27	0.31	0.69	0.00

assets (with aggregate weight in these three assets relative to the total investment in the 50 risky assets, $\omega_3^\top \mathbf{1}_{46}$). For these three asset classes, we consider values of $\alpha_1 = \{0.00, 0.25, 0.50, 0.75\}$, $\alpha_2 = \{0.75, 0.80, 0.85, 0.90\}$, and $\alpha_3 = \{0.90, 1.10, 1.25, 1.50\}$. From Panel A of the table, in which $\alpha_1 = 0$, we see that when $\alpha_3 = 1.25$ or larger, there is no investment at all in the third asset class, and the investor chooses to invest in only the first asset (own-company stock) and the second asset (funds), holding about 50% in both the own-company stock and in the three funds. In Panels C and D, where α_1 is 0.50 and 0.75 respectively, we see that for several combinations of α_2 and α_3, we obtain an

investment of about 30% in own-company stock, 70% in the three funds, and zero in the remaining assets.

Overall, the findings suggest that in a standard asset allocation model, small changes in expected returns that fall well within confidence bounds used in traditional statistical inference can generate the documented asset-allocation holdings of pension-plan participants, with roughly one-third of their holdings invested in their own-company stock, the rest invested in a few mutual funds, and zero invested in other assets offered by the pension plan.

4 Implications for Portfolio Risk, Trading Intensity, and Risk Premium

In this section, we study the implications of the optimal portfolio derived above for portfolio risk, asset demand, and trading intensity. We also analyze the implication for the equilibrium equity risk premium in an economy populated by heterogeneous agents who exhibit different degrees of ambiguity toward the available assets.

4.1 Implications for portfolio risk

The fact that, in the presence of different degrees of ambiguity across assets, the optimal portfolio exhibits a tilt toward the more familiar asset classes has a natural consequence on the riskiness of such a portfolio, relative to the benchmark Markowitz portfolio. The next proposition formalizes this relation in an economy with an infinite number of assets. We limit our focus on the implications for the optimal portfolios in Cases I and II of Proposition 2, because in the non-participation situation of Case III, the portfolio variance is trivially equal to zero.

Proposition 5 *Assume the investor is less ambiguous about Asset F than about other assets, that is,* $\alpha_F < \alpha_U$. *As* $N \to \infty$, *the variance* $\omega^\top \Sigma \omega$ *of the risky part of the investor's portfolio is:*
Case I: If $\frac{\hat{\mu}}{\sigma_{\hat{\mu}}} > \alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F)$,

$$\lim_{N\to\infty} \omega^\top \Sigma \omega = \text{cov} + \frac{\rho^2\, \sigma^2\, \sigma_{\hat{\mu}}^2\, (\alpha_U - \alpha_F)^2}{(1-\rho)(\hat{\mu} - \sigma_{\hat{\mu}}\, \alpha_U)^2}. \tag{41}$$

Case II: If $\alpha_F < \frac{\hat{\mu}}{\sigma_{\hat{\mu}}} \leq \alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F)$,

$$\lim_{N\to\infty} \omega^\top \Sigma \omega = \text{cov} + (1-\rho)\sigma^2 = \sigma^2. \tag{42}$$

Recall from Equation (5) that the quantity $\text{cov} = \rho\sigma^2$ in the proposition above represents systematic risk and is equal to the variance of the Markowitz portfolio. Therefore, from Proposition 5 we

can conclude that in both Cases I and II, the portfolio variance is greater than what it would be in the case of the Markowitz portfolio. This is a direct consequence of the fact that in both these cases the investor's portfolio is less diversified than the Markowitz portfolio. The additional risk in Case I increases with the difference between the familiarity of the first asset relative to the other assets, and it also increases with the extent of idiosyncratic risk, σ^2. Case II is intuitive because the optimal portfolio ω contains only the familiar asset, and so the portfolio risk is equal to σ^2, the idiosyncratic risk of holding a single asset.

In Figure 2, we illustrate Proposition 5 using the same parameter values as before for the case where $N \to \infty$. From the proposition we know that the variance $\omega^\top \Sigma \omega$ of the risky part of the portfolio of an ambiguity-averse investor is larger than the variance of the benchmark Markowitz portfolio, cov. This is because the ambiguity-averse portfolio is more concentrated in the familiar asset than the Markowitz portfolio. Each curve in the figure corresponds to a different level of ambiguity α_F, while we vary α_U along the horizontal axis. From expression (41), we can see that portfolio variance is larger the greater the spread between α_F and α_U, that is, the more familiar is Asset F, relative to the remaining assets. This follows intuitively from the fact that the more familiar an asset is, the more biased the portfolio is toward that asset. In Figure 2, this can be seen by observing that the portfolios with lower values of α_F correspond to higher variance, for any given value of α_U. Furthermore, keeping α_F constant, the variance of a portfolio increases in the degree of ambiguity in the unfamiliar assets. This again follows from expression (41) and is a consequence of an increase in concentration as the spread between ambiguity about the two classes of assets increases.

The lower bound of all the lines in Figure 2 is cov, the variance of the Markowitz portfolio, while the dotted upper bound is σ^2, the variance of an individual asset. The portfolio that reflects the view of Keynes and Markowitz (Case I) corresponds to values of α_F and α_U such that $\hat{\mu}/\sigma_{\hat{\mu}} > \alpha_F + 1/(1-\rho)(\alpha_U - \alpha_F)$, or equivalently, for $\alpha_U \in [\alpha_F, \hat{\mu}/\sigma_{\hat{\mu}}(1-\rho) + \alpha_F]$. For values of $\alpha_U > \hat{\mu}/\sigma_{\hat{\mu}}(1-\rho) + \alpha_F$, the investor holds only the familiar Asset F (Case II) and hence the variance of his portfolio will be σ^2. The kink in Figure 2 corresponds to the boundary between Case I and Case II.

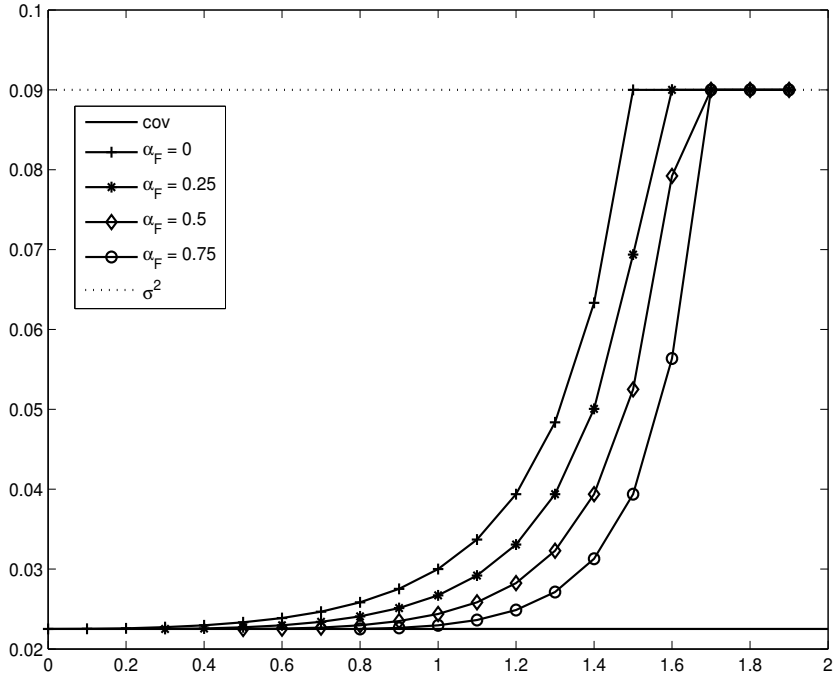
4.2 Effect of correlation on the demand for risky assets

The analytical expression for the optimal portfolio from Proposition 2 allows us to understand the effect of some of the key parameters on portfolio rebalancing. In this section we focus our attention on the correlation coefficient ρ, while in the next subsection we analyze the effects of idiosyncratic volatility and systematic volatility.

There is a substantial literature documenting that during a financial crisis, there is an increase in correlations between the returns of risky assets. We would like to understand the implications of this

Figure 2: Riskiness of the Keynesian portfolio

This figure plots the variance of the investor's portfolio, $\omega^\top \Sigma\, \omega$, as a function of the ambiguity about the unfamiliar asset, α_U. Each curve in the figure corresponds to a different level of ambiguity about the familiar asset, $\alpha_F = \{0.00, 0.25, 0.50, 0.75\}$. The horizontal solid line plots the variance of the portfolio under the Markowitz model, where there is no ambiguity, and, therefore, the portfolio risk is $cov = \rho\sigma^2 = 0.0225$. The upper bound is the risk of holding only a single asset, $\sigma^2 = 0.09$.



for the portfolio weights in our model. The next proposition shows that an increase in correlation leads to an increase in the holding of the familiar asset.

Proposition 6 *Let ω_F be the relative weight in the familiar Asset F for the optimal portfolio in Case I of Proposition 2, where the investor biases her portfolio toward Asset F but also invests in all the other risky assets. Then:*

$$\frac{\partial \omega_F}{\partial \rho} = \frac{(N-1)(\alpha_U - \alpha_F)\sigma_{\hat{\mu}}}{(1-\rho)^2\sigma^2\left(N\left(\hat{\mu} - \sigma_{\hat{\mu}}\alpha_U\right) + \sigma_{\hat{\mu}}(\alpha_U - \alpha_F)\right)} > 0, \tag{43}$$

and in the limit:

$$\lim_{N\to\infty} \frac{\partial \omega_F}{\partial \rho} = \frac{(\alpha_U - \alpha_F)\sigma_{\hat{\mu}}}{(1-\rho)^2\,\sigma^2\,\left(\hat{\mu} - \sigma_{\hat{\mu}}\,\alpha_U\right)} > 0. \tag{44}$$

To understand the implication of this result, note first that for the Markowitz portfolio in which the relative weights are constant and equal to $1/N$, the derivative of these weights with respect to ρ is zero. The above proposition shows that if there are differences in the ambiguity about expected returns of

assets, then when correlations increase, there is *flight to familiarity*; that is, the investor increases the holding of the familiar asset. From (44), we also see that the magnitude of this effect increases with the difference in ambiguity between the familiar asset and the other assets and with the correlation between the assets. The intuition for this result is simple. Given that all assets are identical, an increase in ρ means that the unfamiliar assets become progressively less useful for diversification; that is, the unfamiliar assets tend to behave more like the familiar asset; in addition, their expected returns are estimated less precisely. An ambiguity-averse investor will therefore invest relatively less in the unfamiliar assets. The implication of this result is interesting: the greater the difference in ambiguity across assets, the more concentration in the familiar asset one should expect in times of a financial crisis.

Figure 3: Flight to familiarity

This figure plots the derivative $\partial\omega_F/\partial\rho$, which gives the change in the relative weight of the familiar asset, ω_F, as one changes the correlation, ρ. This derivative is plotted as a function of the ambiguity about the unfamiliar asset, α_U. Each curve in the figure corresponds to a different level of ambiguity about the familiar asset, $\alpha_F = \{0.00, 0.25, 0.50, 0.75\}$. The horizontal solid line plots the derivative for the Markowitz portfolio, which is 0, because if there is no ambiguity then the relative portfolio weigh is $1/N$, which is insensitive to a chance in ρ.

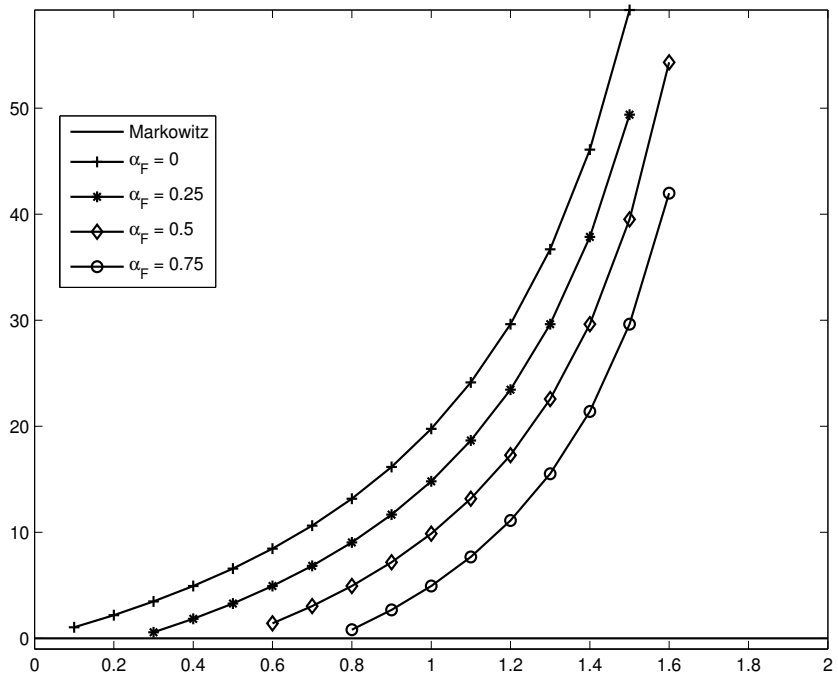


Figure 3 plots the derivative $\partial\omega_F/\partial\rho$ of the relative holdings of Asset F with respect to correlation, using the same parameter values as before. The figure displays the derivative $\partial\omega_F/\partial\rho$ in the region of α_U that gives rise to the portfolio combining the views of Keynes and Markowitz, that is, Case I of Proposition 2. From the figure we see that this derivative is (i) increasing in the degree of ambiguity

α_U about the unfamiliar asset, keeping α_F constant, and (ii) decreasing in the degree of ambiguity α_F about the familiar asset, keeping α_U constant. It is interesting to note the magnitude of this sensitivity. Under our benchmark parameter values we see that when the ambiguity level α_U is set to 1 standard deviation, the derivative $\partial\omega_F/\partial\rho$ is about 10 for $\alpha_F = 0.8$ and about 50 for $\alpha_F = 0$. This means that an increase of 1% in the correlation ρ induces an increase in the weight ω_F of $1\% \times 10 = 10\%$, when $\alpha_F = 0.8$, and $1\% \times 50 = 50\%$, when $\alpha_F = 0$. Naturally, the more familiar the investor is with Asset F, the stronger will be the flight to familiarity effect. Note finally that in the Markowitz case, relative portfolio weights are a constant, $1/N$, and hence, their derivative with respect to ρ is zero.

Table 2: Effect of correlation on portfolio weights

This table gives the relative investment in the familiar asset, ω_F, and in the remaining $N-1$ assets, ω_U, as a function of the ambiguity levels α_F and α_U. Each panel in the table corresponds to a different level of ambiguity about the familiar asset, $\alpha_F = \{0.00, 0.25, 0.50, 0.75\}$. Each row corresponds to a different level of $\alpha_U = \{0.75, 0.80, 0.85, 0.90\}$. The four sets of columns correspond to $\rho = \{0.25, 0.50, 0.70.0.90\}$. All other parameter values are as described earlier in the text.

	$\rho = 0.25$		$\rho = 0.50$		$\rho = 0.70$		$\rho = 0.90$	
α_U	ω_F	ω_U	ω_F	ω_U	ω_F	ω_U	ω_F	ω_U
				Panel A: $\alpha_F = 0.00$				
0.75	0.23	0.77	0.61	0.39	1.00	0.00	1.00	0.00
0.80	0.25	0.75	0.68	0.32	1.00	0.00	1.00	0.00
0.85	0.27	0.73	0.75	0.25	1.00	0.00	1.00	0.00
0.90	0.30	0.70	0.82	0.18	1.00	0.00	1.00	0.00
				Panel B: $\alpha_F = 0.25$				
0.75	0.16	0.84	0.42	0.58	0.94	0.06	1.00	0.00
0.80	0.18	0.82	0.47	0.53	1.00	0.00	1.00	0.00
0.85	0.20	0.80	0.54	0.46	1.00	0.00	1.00	0.00
0.90	0.22	0.78	0.60	0.40	1.00	0.00	1.00	0.00
				Panel C: $\alpha_F = 0.50$				
0.75	0.09	0.91	0.22	0.78	0.48	0.52	1.00	0.00
0.80	0.11	0.89	0.27	0.73	0.59	0.41	1.00	0.00
0.85	0.12	0.88	0.32	0.68	0.72	0.28	1.00	0.00
0.90	0.15	0.85	0.38	0.62	0.85	0.15	1.00	0.00
				Panel D: $\alpha_F = 0.75$				
0.75	0.02	0.98	0.02	0.98	0.02	0.98	0.02	0.98
0.80	0.03	0.97	0.06	0.94	0.12	0.88	0.39	0.61
0.85	0.05	0.95	0.11	0.89	0.22	0.78	0.79	0.21
0.90	0.07	0.93	0.16	0.84	0.33	0.67	1.00	0.00

The magnitude of the effect of correlation highlighted in Figure 3 is confirmed in the portfolio weights we report in Table 2. In each panel we report a different level of ambiguity in the familiar asset α_F; each row represents a different level of ambiguity α_U in the unfamiliar asset, and each pair of columns represents a different level of correlation, ρ. From the table, we see that an increase in correlation from $\rho = 50\%$ to $\rho = 75\%$ roughly doubles the holding of the familiar asset. Notice that for a high level of correlation, Case II in Proposition 2 (where the investor holds only the familiar asset) becomes more likely. This explains the prevalence of 100% holdings in the familiar asset when ρ is high. The larger the spread between α_F and α_U (Panels A and B), the stronger is the flight to familiarity effect.

4.3 Trading intensity in the Markowitz and Keynes models

When an investor is risk averse, an increase in the risk of an asset obviously leads to a decrease in the holding of that asset. An interesting extension of this insight is to distinguish between the effect of systematic and idiosyncratic risk. From traditional asset pricing theory we know that the relative weights in a well-diversified portfolio will not be affected by idiosyncratic risk because, by construction, a well-diversified portfolio contains *no* idiosyncratic risk. In our framework, however, optimal portfolios may exhibit concentration in familiar assets. Note that because systematic risk is $\text{cov} = \rho\sigma^2$, a change in idiosyncratic volatility σ does affect systematic volatility as well. Therefore, we treat the portfolio weights π as a function of σ and cov, that is, $\pi = \pi(\sigma, \text{cov})$, and then, to study the effect of idiosyncratic risk, we take derivatives with respect to σ, *keeping* cov *constant*.

In order to study differences in trading volume of the portfolio strategies advocated by Keynes and Markowitz, we analyze the effect on the change in the portfolio weights of a change in systematic volatility, cov, and idiosyncratic volatility, σ. We focus our analysis on the case in which the views of both Keynes and Markowitz are reflected in the portfolio weights, that is, Case I of Proposition 2, and for simplicity, we confine ourselves to the case of an economy with an infinite number of assets N.

The following proposition derives the sensitivity of portfolio weights to changes in systematic and idiosyncratic risk.

Proposition 7 *Let π_F and π_U be the portfolios in Case I of Proposition 2. Then, as $N \to \infty$:*

1. *The effect of* idiosyncratic *risk on the overall portfolio where the investor exhibits familiarity toward Asset F is given by:*

$$\lim_{N\to\infty}\left(\frac{\partial \pi_F}{\partial \sigma} + \frac{\partial}{\partial \sigma}\left(\pi_U^\top \mathbf{1}_{N-1}\right)\right) = -\frac{\alpha_U}{\gamma\sqrt{T}\text{cov}} = -\frac{\alpha_U}{\gamma\sqrt{T}\rho\sigma^2} < 0, \tag{45}$$

while for the Markowitz portfolio it is:

$$\lim_{N\to\infty} \frac{\partial}{\partial \sigma}\left(\pi^{\top}\mathbf{1}_N\right) = 0. \tag{46}$$

2. *The effect of* systematic *risk on the overall portfolio where the investor exhibits familiarity toward Asset F is given by:*

$$\lim_{N\to\infty}\left(\frac{\partial \pi_F}{\partial \text{cov}} + \frac{\partial}{\partial \text{cov}}\left(\pi_U^{\top}\mathbf{1}_{N-1}\right)\right) = -\frac{\hat{\mu}-\alpha_U\sigma_{\hat{\mu}}}{\gamma \text{cov}^2} = -\frac{\hat{\mu}-\alpha_U\sigma_{\hat{\mu}}}{\gamma\rho^2\sigma^4} < 0, \tag{47}$$

while for the Markowitz portfolio it is:

$$\lim_{N\to\infty}\frac{\partial}{\partial \text{cov}}\left(\pi^{\top}\mathbf{1}_N\right) = -\frac{\hat{\mu}}{\gamma \text{cov}^2} = -\frac{\hat{\mu}}{\gamma\rho^2\sigma^4} < 0. \tag{48}$$

The above proposition shows that the effects of changes in idiosyncratic risk and systematic risk are very different in the Keynesian and Markowitz models. A non-zero value for the derivative means that, as a consequence of a change in σ and cov, the investor will revise his portfolio which will result in trading. We take the absolute value of these derivatives as a simple measure of trading volume. Under this convention, see that the absolute value of the expression in Equation (45) is greater than that in (46), indicating that in response to a change in idiosyncratic risk the Keynesian portfolio always exhibits *more* trading than the Markowitz portfolio. In contrast, the absolute value of the expression in Equation (47) is smaller than that in (48), indicating that in response to a change in systematic risk the Keynesian portfolio exhibits *less* trading than the Markowitz portfolio.[19]

4.4 Implications of the optimal weights for asset returns in equilibrium

We conclude this section by studying the effect of ambiguity on the equilibrium risk premium. We consider an economy with N risky assets and one risk-free asset in zero-net supply. There are N agents who differ only in their degree of familiarity toward assets. We assume that agent j is familiar with asset j and treats the remaining assets as equally unfamiliar. We denote by α_j^j the degree of familiarity of agent j toward asset j, and by α_{-j}^j the common degree of familiarity of agent j toward the remaining $N-1$ assets. Other than the heterogeneity in their level of familiarity toward assets, the agents are identical in all other respects.

[19]There is a large literature on trading volume and its relation with asset prices and volatility; Karpoff (1987) is an excellent survey of the early work in this area. This literature, however, does not differentiate between idiosyncratic and systematic volatility. For example, Lo and Wang (2000) study the implication of two-fund separation, and more generally $(K+1)$-fund separation, for trading volume; Jones, Kaul, and Lipson (1994) and Chan and Fong (2000) examine the determinants of the positive volatility-volume relation; and Bessembinder and Seguin (1993) study at the individual stock level the determinants of the volatility-volume relation. To the best of our knowledge, this literature has not studied the differential response of investors who hold a diversified portfolio, as recommended by Markowitz, and of investors who hold a portfolio biased toward a few familiar assets, as per the view of Keynes, to changes in idiosyncratic and systematic volatilities.

Each agent solves a portfolio-choice problem of the form described in (8) subject to the constraint (9). We denote by π^j the optimal portfolio of agent j. Because the agents in the economy are symmetric in their attitude toward ambiguity, we will have that, for any two agents, i and j, the holding of the familiar and unfamiliar assets are the same, that is, $\pi_i^i = \pi_j^j \equiv \pi_F$ and $\pi_{-i}^i = \pi_{-j}^j \equiv \pi_U$. Hence, for all i, j, $\pi_i^i + (\pi_{-i}^i)^\top \mathbf{1}_{N-1} = \pi_j^j + (\pi_{-j}^j)^\top \mathbf{1}_{N-1} = \pi_F + \pi_U^\top \mathbf{1}_{N-1}$, which implies that each agent will have identical holding of the risk-free asset. Since the risk-free asset is in zero-net supply, the market clearing condition in equilibrium is $\pi_F + \pi_U^\top \mathbf{1}_{N-1} = 1$. In the following proposition we characterize the equilibrium expected return, denoted by $\hat{\mu}$, for the case in which assets are divided into only two classes based on their degree of ambiguity.

Proposition 8 *For each investor, let α_F be the level of ambiguity for the asset with which such an investor is familiar, and α_U the level of ambiguity common across the remaining $N-1$ assets, with $\alpha_F \leq \alpha_U$. Then the equilibrium excess return on each risky asset is as follows.*
Case I. If $\frac{\hat{\mu}}{\sigma_{\hat{\mu}}} > \alpha_U + \frac{\rho}{1-\rho}(\alpha_U - \alpha_F)$, in which case the investor holds both the familiar asset and the rest of the market, then:

$$\hat{\mu} = \frac{1}{N}\left(\gamma\rho\sigma^2(N-1) + \gamma\sigma^2 + \sigma_{\hat{\mu}}\alpha_U(N-1) + \sigma_{\hat{\mu}}\alpha_F\right), \tag{49}$$

and for the case where N is large:

$$\lim_{N\to\infty} \hat{\mu} = \gamma\rho\sigma^2 + \sigma_{\hat{\mu}}\alpha_U. \tag{50}$$

Case II. If $\alpha_F < \frac{\hat{\mu}}{\sigma_{\hat{\mu}}} \leq \alpha_U + \frac{\rho}{1-\rho}(\alpha_U - \alpha_F)$, in which case the investor holds only *the familiar asset and invests zero in all the other assets, then the equilibrium return is:*

$$\hat{\mu} = \gamma\sigma^2 + \sigma_{\hat{\mu}}\alpha_F. \tag{51}$$

Case III. If $0 < \frac{\hat{\mu}}{\sigma_{\hat{\mu}}} \leq \alpha_F$, in which case no risky assets are held, then this cannot be an equilibrium.[20]

From Equation (49) we see that the expected excess return has four components for Case I, in which the investor holds both the familiar asset and unfamiliar assets. The first term, $\gamma\rho\sigma^2(N-1)/N$, is the conventional risk premium. The second term is $\gamma\sigma^2/N$, which is the product of risk aversion and idiosyncratic risk, σ^2, and it is present because the investor holds a portfolio concentrated in the familiar asset. The third term, $\sigma_{\hat{\mu}}\ \alpha_U(N-1)/N$, is the ambiguity premium due to the ambiguity of the average investor about the unfamiliar assets. The fourth term, $\sigma_{\hat{\mu}}\alpha_F/N$, is the premium due to the ambiguity about the familiar asset. Note that the second and fourth terms are small when N

[20]Note that in our model all investors are symmetric; if investors were not symmetric, then one would have an equilibrium where some of them would choose not to participate in the stock market while others do.

is large, as can be seen from Equation (50), where only the first and third term are present. From Equation (50), we also see that the premium arising from ambiguity, given by the term $\sigma_{\hat{\mu}}\ \alpha_U$, can add substantially to the standard equity risk premium, $\gamma\sigma^2$. Under our choice of parameter values, the standard deviation of the estimate of mean returns is $\sigma_{\hat{\mu}} = \frac{\sigma}{\sqrt{T}} = \frac{0.30}{\sqrt{100}} = 0.03$. Therefore, even if α_U was assumed to be only 1, the premium arising from ambiguity that would be added to the standard equity premium is 0.03.

Case II gives the equilibrium equity risk premium for the extreme case where each investor in the economy holds only the familiar asset. From Equation (51) we see that the equity risk premium now depends on idiosyncratic volatility, σ^2, and on the ambiguity about the familiar asset, α_F.

In order for the above expression to represent an equilibrium, it is important to guarantee that the conditions defining Cases I and II be satisfied by $\hat{\mu}$. Substituting the expression for $\hat{\mu}$ from (50) into the condition defining Case I, we obtain that, for the limiting case where $N \to \infty$, Case I is satisfied in equilibrium if $\alpha_U < \alpha_F + \frac{\gamma\sigma^2}{\sigma_{\hat{\mu}}}$. Similarly, substituting the expression for $\hat{\mu}$ in (51) into the condition defining Case II, we see that Case II is satisfied if $\alpha_U > \alpha_F + \frac{\gamma\sigma^2}{\sigma_{\hat{\mu}}}$.

5 Conclusion

Even though Markowitz's portfolio theory dictates that an investor should invest in only a single fund of risky assets, which in equilibrium is the market portfolio, there is substantial evidence that rather than holding just the market portfolio or a well-diversified portfolio, investors hold a substantial amount in just a few assets, often those with which they are familiar. In this paper we reconcile this apparent contradiction between Markowitz's theory and the empirical evidence by introducing Keynes's view of investment into an otherwise standard Markowitz model of portfolio selection.

Our model incorporates the view of Keynes by introducing ambiguity about the true distribution of asset returns. and investors' aversion to this ambiguity, into the standard Markowitz portfolio-selection setting. The main feature of our model is that it allows investors to distinguish their ambiguity about one asset class relative to others. We show analytically that the model has the following implications, which are consistent with the stylized empirical observations. (i) In the presence of ambiguity about returns on the other assets, the investor holds a disproportionally large amount of the familiar asset (relative to the Markowitz model), but continues to invest in the market portfolio. (ii) The proportion of wealth allocated to the familiar asset increases with an increase in correlations between assets, implying a *flight to familiarity* effect. (iii) Investors who are familiar with a particular asset *and* sufficiently ambiguous about all other assets hold only the familiar asset, as Keynes would have advocated, and thus the median number of stocks held by the typical investor is quite small.

(iv) Investors who are sufficiently ambiguous about all risky assets do not participate at all in the equity market (*non-participation*). (vi) Finally, the risk premium of assets can depend on both systematic and idiosyncratic volatility.

The analysis in this paper suggests that perhaps we should not ignore Keynes's view of how to make investment decisions. We hope that our work will encourage others to explore other implications of the Keynesian view of portfolio selection, and, in particular, to further develop theories of portfolio choice and asset pricing in which investors hold only a subset of the universe of available investable assets.

A Appendix: Proofs of the Propositions

Proof of Proposition 1

This proposition is a special case of Proposition 2, and so we provide the proof only for Proposition 2.

Proof of Proposition 2

The solution to the inner minimization problem of (8)–(9) is

$$\mu_n = \hat{\mu}_n - \text{sign}(\pi_n)\sigma_{\hat{\mu}_n}\alpha_n, \quad n = 1, \ldots, N, \tag{A1}$$

which, when substituted back into the original problem, gives

$$\max_{\pi} \left\{ \pi^\top \hat{\boldsymbol{\mu}} - \sum_{n=1}^{N} \text{sign}(\pi_n)\pi_n\sigma_{\hat{\mu}_n}\alpha_n - \frac{\gamma}{2}\pi^\top\Sigma\pi \right\}. \tag{A2}$$

Given our specification of the N stock return processes, it must be the case that $\pi_2 = \cdots = \pi_N$. We will denote this common value by $\tilde{\pi}_U$. From the first order condition, if the optimal weights $\pi \neq 0$, they must satisfy

$$\pi = \begin{bmatrix} \pi_F \\ \pi_U \end{bmatrix} = \begin{bmatrix} \pi_F \\ \tilde{\pi}_U \\ \vdots \\ \tilde{\pi}_U \end{bmatrix} = \frac{1}{\gamma}\Sigma^{-1} \begin{bmatrix} \hat{\mu} - \text{sign}(\pi_F)\sigma_{\hat{\mu}}\alpha_F \\ \hat{\mu} - \text{sign}(\tilde{\pi}_U)\sigma_{\hat{\mu}}\alpha_U \\ \vdots \\ \hat{\mu} - \text{sign}(\tilde{\pi}_U)\sigma_{\hat{\mu}}\alpha_U \end{bmatrix}. \tag{A3}$$

Because all risky assets have the same expected return, $\hat{\mu}$, the above N-dimensional condition simplifies to the following two-dimensional one:

$$\begin{bmatrix} \pi_F \\ \tilde{\pi}_U \end{bmatrix} = \frac{1}{\gamma}\widetilde{\Sigma}^{-1} \begin{bmatrix} \hat{\mu} - \text{sign}(\pi_F)\sigma_{\hat{\mu}}\alpha_F \\ (N-1)\left(\hat{\mu} - \text{sign}(\tilde{\pi}_U)\sigma_{\hat{\mu}}\alpha_U\right) \end{bmatrix}, \tag{A4}$$

where

$$\widetilde{\Sigma} = \begin{bmatrix} \sigma^2 & (N-1)\rho\sigma^2 \\ (N-1)\rho\sigma^2 & (N-1)[1+(N-2)\rho]\sigma^2 \end{bmatrix}. \tag{A5}$$

To solve (A4) we need to consider several possible cases depending on the sign of π_F and $\tilde{\pi}_U$.

Case I. Suppose $\pi_F > 0$ and $\tilde{\pi}_U > 0$, then, from (A4)

$$\begin{aligned} \begin{bmatrix} \pi_F \\ \tilde{\pi}_U \end{bmatrix} &= \frac{1}{\gamma}\widetilde{\Sigma}^{-1} \begin{bmatrix} \hat{\mu} - \sigma_{\hat{\mu}}\alpha_F \\ (N-1)(\hat{\mu} - \sigma_{\hat{\mu}}\alpha_U) \end{bmatrix} \\ &= \frac{1}{\gamma\sigma^2(1-\rho)(1+(N-1)\rho)} \begin{bmatrix} \hat{\mu}(1-\rho) - \sigma_{\hat{\mu}}((1+(N-2)\rho)\alpha_F - (N-1)\rho\alpha_U) \\ \hat{\mu}(1-\rho) - \sigma_{\hat{\mu}}(\alpha_U - \rho\alpha_F) \end{bmatrix}. \end{aligned} \tag{A6}$$

Because $\alpha_F < \alpha_U$, $\alpha_U - \rho\alpha_F > \alpha_F - \rho\alpha_U > (1+(N-2)\rho)\alpha_F - (N-1)\rho\alpha_U = \alpha_F - \rho\alpha_U + (N-2)\rho(\alpha_F - \alpha_U)$, and hence $\pi_F > \tilde{\pi}_U$ in (A6). Moreover, $\pi_F > 0$ and $\tilde{\pi}_U > 0$ if and only $\tilde{\pi}_U > 0$, which happens if $\frac{\hat{\mu}}{\sigma_{\hat{\mu}}} > \alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F)$. This proves Case I.

Cases II and III. Suppose that $0 \leq \frac{\hat{\mu}}{\sigma_{\hat{\mu}}} \leq \alpha_F + \frac{1}{1-\rho}(\alpha_U - \alpha_F)$. We start by first proving three intermediate results: (i) it is impossible to have $\pi_U < 0$ and $\pi_F \geq 0$; (ii) it is impossible to have $\pi_U > 0$ and $\pi_F < 0$; (iii) it is impossible to have $\pi_F < 0$ and $\tilde{\pi}_U < 0$.

(i) Suppose that $(\pi_F, \tilde{\pi}_U)$ is the optimal portfolio weight vector with $\tilde{\pi}_U < 0$. Then there exists a v such that $\hat{\mu} - \sigma_{\hat{\mu}} v \in [\hat{\mu} - \sigma_{\hat{\mu}}\alpha_F, \hat{\mu} + \sigma_{\hat{\mu}}\alpha_F]$ and

$$\begin{aligned}\begin{bmatrix} \pi_F \\ \tilde{\pi}_U \end{bmatrix} &= \frac{1}{\gamma}\widetilde{\Sigma}^{-1}\begin{bmatrix} \hat{\mu} - \sigma_{\hat{\mu}} v \\ (N-1)(\hat{\mu} + \sigma_{\hat{\mu}}\alpha_U) \end{bmatrix} \\ &= \frac{1}{\gamma\sigma^2(1-\rho)(1+(N-1)\rho)}\begin{bmatrix} \hat{\mu}(1-\rho) - \sigma_{\hat{\mu}}(v(1+(N-2)\rho) + (N-1)\rho\alpha_U) \\ \hat{\mu}(1-\rho) + \sigma_{\hat{\mu}}(\alpha_U + \rho v) \end{bmatrix}.\end{aligned}$$

However, $\hat{\mu}(1-\rho) + \sigma_{\hat{\mu}}(\alpha_U + \rho v) \geq \hat{\mu}(1-\rho) + \sigma_{\hat{\mu}}(\alpha_U - \rho\alpha_F) \geq 0$. This condition would imply $\pi_U > 0$, a contradiction.

(ii) Suppose next that $\pi_F < 0$, $\pi_U > 0$. Then

$$\begin{bmatrix} \pi_F \\ \tilde{\pi}_U \end{bmatrix} = \frac{1}{\gamma\sigma^2(1-\rho)(1+(N-1)\rho)}\begin{bmatrix} \hat{\mu}(1-\rho) + \sigma_{\hat{\mu}}(\alpha_F(1+(N-2)\rho) + (N-1)\rho\alpha_U) \\ \hat{\mu}(1-\rho) - \sigma_{\hat{\mu}}(\alpha_U + \rho\alpha_F) \end{bmatrix}.$$

Because $\hat{\mu}(1-\rho) + \sigma_{\hat{\mu}}(\alpha_F(1+(N-2)\rho) + (N-1)\rho\alpha_U) \geq \hat{\mu}(1-\rho) + \sigma_{\hat{\mu}}(\alpha_U - \rho\alpha_F) \geq 0$, we have a contradiction.

(iii) Suppose finally that $\pi_F < 0$, $\pi_U < 0$. Then, from (A4)

$$\begin{aligned}\begin{bmatrix} \pi_F \\ \tilde{\pi}_U \end{bmatrix} &= \frac{1}{\gamma}\widetilde{\Sigma}^{-1}\begin{bmatrix} \hat{\mu} + \sigma_{\hat{\mu}}\alpha_F \\ (N-1)(\hat{\mu} + \sigma_{\hat{\mu}}\alpha_U) \end{bmatrix} \qquad \text{(A7)} \\ &= \frac{1}{\gamma\sigma^2(1-\rho)(1+(N-1)\rho)}\begin{bmatrix} \hat{\mu}(1-\rho) + \sigma_{\hat{\mu}}((1+(N-2)\rho)\alpha_F + (N-1)\rho\alpha_U) \\ \hat{\mu}(1-\rho) + \sigma_{\hat{\mu}}(\alpha_U + \rho\alpha_F) \end{bmatrix}.\end{aligned}$$

Because $\hat{\mu} > 0$, $\pi_F > 0$ and $\tilde{\pi}_U > 0$ in (A7), we have a contradiction.

We can now prove Cases II and III. From the intermediate results derived in (i), (ii), and (iii) above, the only possibilities are either (a) $\tilde{\pi}_U = 0$ and $\pi_F \gtreqless 0$, or (b) $\tilde{\pi}_U > 0$ and $\pi_F = 0$. In (a), the optimal π_F is the quantity x solving the utility maximization problem

$$\max_x \min_{\mu \in \Gamma_F} \mu x - \frac{\gamma}{2}x^2\sigma^2 \quad \text{where} \quad \Gamma_F = \left\{\mu : \frac{(\mu - \hat{\mu})^2}{\sigma_{\hat{\mu}}} \leq \alpha_F\right\}. \qquad \text{(A8)}$$

And, in (b), the optimal $\tilde{\pi}_U$ is the quantity x solving

$$\max_x \min_{\mu \in \Gamma_U} \mu x - \frac{\gamma}{2} x^2 \sigma^2 (1 + (N-2)\rho) \quad \text{where} \quad \Gamma_U = \left\{ \mu : (\mu - \hat{\mu})^2 / \sigma_{\hat{\mu}} \leq \alpha_U \right\}. \tag{A9}$$

Because $\alpha_F < \alpha_U$, $\Gamma_F \subset \Gamma_U$. Moreover, $\sigma^2(1 + (N-2)\rho) > \sigma^2$, and hence

$$\max_\pi \min_{\mu \in \Gamma_F} \mu x - \frac{\gamma}{2} x^2 \sigma^2 > \max_x \min_{\mu \in \Gamma_U} \mu x - \frac{\gamma}{2} x^2 \sigma^2 (1 + (N-2)\rho). \tag{A10}$$

Thus, (b) cannot be optimal. Hence the optimal portfolio weight must correspond to (a). Because $\hat{\mu} > 0$, $\pi_F < 0$ cannot be a solution of (A8). This leaves only two cases, corresponding to Cases II and III in the proposition. In Case II, $\pi_F > 0$, which happens if $\hat{\mu} - \sigma_{\hat{\mu}} \alpha_F > 0$, or, equivalently $\alpha_F < \frac{\hat{\mu}}{\sigma_{\hat{\mu}}}$. In this case, the optimal portfolio is

$$\pi_F = \frac{1}{\gamma \sigma^2} (\hat{\mu} - \sigma_{\hat{\mu}} \alpha_F) \tag{A11}$$

$$\tilde{\pi}_U = 0. \tag{A12}$$

In Case III, $0 < \frac{\hat{\mu}}{\sigma_{\hat{\mu}}} \leq \alpha_F$, and so the optimal portfolio is

$$\pi_F = 0 \tag{A13}$$

$$\tilde{\pi}_U = 0. \tag{A14}$$

This completes the proof of the proposition. ∎

Proof of Corollary 1

Let $\Delta = \alpha_U - \alpha_F$. Cases II and III are immediate from the definition of relative weights. Case I follows from Proposition 2, by defining:

$$\omega_F = \frac{\pi_F}{\pi_F + \pi_U^\top \mathbf{1}_{N-1}} \tag{A15}$$

$$\omega_U = \frac{\pi_U}{\pi_F + \pi_U^\top \mathbf{1}_{N-1}}. \tag{A16}$$

Note that the quantity $\hat{\mu} - \sigma_{\hat{\mu}} \alpha_F - \Delta \sigma_{\hat{\mu}} \left(1 - \frac{1}{N}\right) > 0$, because by the condition that defines Case I, $\frac{\hat{\mu}}{\sigma_{\hat{\mu}}} > \alpha_F + \frac{1}{1-\rho}\Delta > \alpha_F + \Delta > \alpha_F + \left(1 - \frac{1}{N}\right)\Delta$. ∎

Proof of Proposition 3

The limits in (32) and (34) follow immediately from the portfolio weights (20) and (21). ∎

Proof of Proposition 4

The solution to the inner minimization problem of the maxmin problem (8)–(9) is the same as in (A1), which, when substituted back into the original problem, gives the problem in (A2). Note that assets in each subclass will have identical portfolio weight. The case of one asset class is trivial. The optimal portfolio must satisfy

$$\max_{\pi} \pi \left(\hat{\mu} - \text{sign}(\pi)\sigma_{\hat{\mu}}\alpha_F\right) - \frac{\gamma}{2}\pi^2\sigma^2. \tag{A17}$$

The solution π to this problem is

$$\pi = \begin{cases} \frac{1}{\gamma\sigma^2}N_1(\hat{\mu} - \sigma_{\hat{\mu}}\alpha_1) & \text{if } \hat{\mu} > \sigma_{\hat{\mu}}\alpha_1 \\ 0 & \text{if } \hat{\mu} \leq \sigma_{\hat{\mu}}\alpha_1 \end{cases}.$$

We now prove the proposition by induction. That is, assuming that the claim of the proposition is true for the case of N assets with $M-1$ exclusive asset classes with $M \geq 2$, we show that the claim is also true for the case of N assets with M exclusive asset classes.

Let $\pi_{\{1,2,\ldots,m\}}$ denote the $m \times 1$ vector of portfolio weights in asset classes $1, 2, \ldots, m$, where $m = 1, \ldots M$. In general, the maxmin problem has the solution of the form:

$$\pi_{\{1,\ldots,M\}} = \frac{1}{\gamma}\widetilde{\Sigma}_M^{-1} \begin{bmatrix} N_1(\hat{\mu} - \sigma_{\hat{\mu}}v_1) \\ N_2(\hat{\mu} - \sigma_{\hat{\mu}}v_2) \\ \vdots \\ N_M(\hat{\mu} - \sigma_{\hat{\mu}}v_M) \end{bmatrix}, \tag{A18}$$

where

$$\widetilde{\Sigma}_M = \sigma^2 \begin{bmatrix} N_1(1+(N_1-1)\rho) & N_1N_2\rho & \ldots & N_1N_M\rho \\ N_2N_1\rho & N_2(1+(N_2-1)\rho) & \ldots & N_2N_M\rho \\ \vdots & \vdots & \ddots & \vdots \\ N_MN_1\rho & N_MN_2\rho & \ldots & N_M(1+(N_M-1)\rho) \end{bmatrix}, \tag{A19}$$

with appropriate $v_i \in [-\alpha_i, \alpha_i]$, $i = 1, \ldots, M$, to be determined. To solve for $\pi_{\{1,\ldots,M\}}$, rewrite (A18) as

$$\gamma\widetilde{\Sigma}_M\pi_{\{1,\ldots,M\}} = \begin{bmatrix} N_1(\hat{\mu} - \sigma_{\hat{\mu}}v_1) \\ N_2(\hat{\mu} - \sigma_{\hat{\mu}}v_2) \\ \vdots \\ N_M(\hat{\mu} - \sigma_{\hat{\mu}}v_M) \end{bmatrix}. \tag{A20}$$

Using the expression for $\widetilde{\Sigma}_M$ in (A19), we have

$$\begin{aligned}
\gamma\sigma^2\left((1-\rho)\pi_1+\rho\sum_{j=1}^{M}N_j\pi_j\right) &= \hat{\mu}-\sigma_{\hat{\mu}}v_1\\
\gamma\sigma^2\left((1-\rho)\pi_2+\rho\sum_{j=1}^{M}N_j\pi_j\right) &= \hat{\mu}-\sigma_{\hat{\mu}}v_2\\
&\vdots\\
\gamma\sigma^2\left((1-\rho)\pi_M+\rho\sum_{j=1}^{M}N_j\pi_j\right) &= \hat{\mu}-\sigma_{\hat{\mu}}v_M.
\end{aligned}$$

Subtraction yields

$$\pi_i-\pi_1=\frac{1}{\gamma(1-\rho)\sigma^2}\sigma_{\hat{\mu}}(v_1-v_i) \tag{A21}$$

Thus

$$\pi_i=\pi_1+\frac{1}{\gamma(1-\rho)\sigma^2}\sigma_{\hat{\mu}}(v_1-v_i)$$

Substituting this expression for π_i back into the first equation of the system of equations (A20) and solving the resulting equations yields an expression for π_1. Then it can be verified that the component π_i of $\pi_{\{1,\ldots,M\}}$ can be expressed as

$$\pi_i=\frac{\hat{\mu}(1-\rho)-\sigma_{\hat{\mu}}\left((1-\rho)v_i+\rho\sum_{j\neq i}^{M}N_j(v_i-v_j)\right)}{\gamma\sigma^2(1-\rho)\left(1+\rho(N-1)\right)},\quad i=1,\ldots,M. \tag{A22}$$

We will make use of this fact in the proof.

First consider the case where $\hat{\mu}>s(M)$. Then the optimal portfolio weights are:

$$\pi_{\{1,\ldots,M\}}=\frac{1}{\gamma}\widetilde{\Sigma}_M^{-1}\begin{bmatrix}N_1(\hat{\mu}-\sigma_{\hat{\mu}}\alpha_1)\\N_2(\hat{\mu}-\sigma_{\hat{\mu}}\alpha_2)\\\vdots\\N_M(\hat{\mu}-\sigma_{\hat{\mu}}\alpha_M)\end{bmatrix}, \tag{A23}$$

provided that $\pi_i>0$ for all $i=1,\ldots,M$. We verify that this is indeed the case. By (A22)

$$\pi_i=\frac{\hat{\mu}(1-\rho)-\sigma_{\hat{\mu}}\left((1-\rho)\alpha_i+\rho\sum_{j\neq i}^{M}N_j(\alpha_i-\alpha_j)\right)}{\gamma\sigma^2(1-\rho)\left(1+\rho(N-1)\right)},\quad i=1,\ldots,M. \tag{A24}$$

Notice that $\pi_1\geq\pi_2\geq\ldots\geq\pi_M$. Since $\hat{\mu}>s(M)$, $\pi_M>0$.

Next consider the case where $s(M-1) < \hat{\mu} \leq s(M)$. We conjecture that the optimal portfolio weights are given by

$$\pi_{\{1,\ldots,M-1\}} = \frac{1}{\gamma}\widetilde{\Sigma}_{M-1}^{-1}\begin{bmatrix} N_1(\hat{\mu}-\sigma_{\hat{\mu}}\alpha_1) \\ \vdots \\ N_{M-1}(\hat{\mu}-\sigma_{\hat{\mu}}\alpha_{M-1}) \end{bmatrix}, \tag{A25}$$

$$\pi_M = 0. \tag{A26}$$

Suppose to the contrary that the optimal $\pi_{\{1,\ldots,M\}}$ in (A18) is such that $\pi_M \neq 0$. There are four subcases to consider. (i) Consider first the subcase where $\pi_M < 0$. Then in (A18), $v_M = -\alpha_M$. Since $\alpha_i \leq \alpha_j$ for $i \leq j$, (A22) implies that $\pi_M > 0$, a contradiction. (ii) Next consider the subcase where $\pi_M > 0$, and there is at least one $j < M$ such that $\pi_j < 0$. Then for the largest j such that $\pi_j < 0$, (A22) again implies that $\pi_j > 0$, a contradiction. (iii) In the third subcase, $\pi_M > 0$, and there is at least one $j < M$ such that $\pi_j = 0$. Fixing π_j at zero, the original maxmin problem reduces to a problem with N assets with $M-1$ classes. By assumption, the claim of the proposition is true for a problem with N assets with $M-1$ classes. But $\pi_M > 0$ is inconsistent with the claim of the proposition for $M-1$ asset classes, a contradiction. (iv) In the fourth subcase, which is the only remaining possibility, all $\pi_i > 0$, $i = 1, \ldots, M$, which implies that in (A18) $v_i = \alpha_i$. But then (A22) and $\hat{\mu} \leq s(M)$ would imply $\pi_M < 0$, a contradiction. The four subcases together imply that $\pi_M \neq 0$ is impossible. Thus, for the optimal portfolio weight vector, (A26) holds. Fixing π_M at zero, the original maxmin problem reduces to a problem with N assets with $M-1$ classes. Then, by the claim of the proposition, (A25) holds.

Next consider the case where $s(m) < \hat{\mu} \leq s(m+1)$, $m = 1, \ldots, M-1$. In this case, $\hat{\mu} \leq \mu(M)$ as well. By what is shown above, $\pi_M = 0$, and the problem can therefore be reduced to the case of N assets with $M-1$ class. Thus, the claim of the proposition holds. ∎

Proof of Proposition 5

The relative portfolio weights are

$$\omega_{F} = \frac{\pi_{F}}{\pi_{F} + \pi_{U}^{\top}\mathbf{1}_{N-1}}, \quad \omega_{U} = \frac{\pi_{U}}{\pi_{F} + \pi_{U}^{\top}\mathbf{1}_{N-1}}.$$

Then the variance of the portfolio of all risky assets is $(\omega_{F}, \omega_{U})\Sigma(\omega_{F}, \omega_{U})^{\top}$. As in the proof of Proposition 2, define

$$\widetilde{\Sigma} = \begin{bmatrix} \sigma^2 & (N-1)\rho\sigma^2 \\ (N-1)\rho\sigma^2 & (N-1)[1+(N-2)\rho]\sigma^2 \end{bmatrix}.$$

Then the variance of the portfolio, as $N \to \infty$, is given by,

$$\rho\sigma^2 + \sigma_{\hat{\mu}}^2 \frac{(\alpha_U - \alpha_F)^2}{(\hat{\mu} - \sigma_{\hat{\mu}}\alpha_U)^2} \frac{\rho^2}{(1-\rho)^2}(1-\rho)\sigma^2 = \text{cov} + \frac{\sigma_{\hat{\mu}}^2 \sigma^2 \rho^2 (\alpha_U - \alpha_F)^2}{(1-\rho)(\hat{\mu} - \sigma_{\hat{\mu}}\alpha_U)^2}.$$

For Case II, by Corollary 1, $\omega_F = 1$ and $\omega_U = \mathbf{0}_{N-1}$. Thus the variance is σ^2. For Case III, the variance is zero. ∎

Proof of Proposition 6

The result is obtained by taking derivatives of the relative weights in Corollary 1. ∎

Proof of Proposition 7

The result is obtained by taking derivatives with respect to σ of the weights in Equations (32) and (34), keeping cov constant. ∎

Proof of Proposition 8

We need to derive the equilibrium of the economy. We study the case where all agents are symmetric. Let π_{i0} denote agent i's portfolio weight on the riskless asset. Because agents in the economy are symmetric, it must be the case that $\pi_{i0} = \pi_0$ for all i. In equilibrium, because the riskless asset is in zero net supply, π_0 must be equal to zero. Therefore, in equilibrium we must have that for each agent, $\pi_F + \pi_U^\top \mathbf{1}_{N-1} = 1$. Now we consider the three cases in Proposition 2.

Case I. The equilibrium condition implies that

$$1 = \frac{N\hat{\mu} - \sigma_{\hat{\mu}}[(N-1)\alpha_U + \alpha_F]}{\gamma(1+\rho(N-1))\sigma^2} = \frac{\hat{\mu} - \sigma_{\hat{\mu}}\alpha_U}{\gamma(1/N + \rho(1-1/N))\sigma^2} + \frac{\sigma_{\hat{\mu}}(\alpha_U - \alpha_F)/N}{\gamma(1/N + \rho(1-1/N))\sigma^2}. \tag{A27}$$

Solving the above equation for $\hat{\mu}$ leads to the expression of the equilibrium expected return in the proposition.

Case II. The equilibrium condition in this case implies that if $\hat{\mu} > \sigma_{\hat{\mu}}\alpha_F$, then, for each investor,

$$1 = \frac{\hat{\mu} - \sigma_{\hat{\mu}}\alpha_F}{\gamma\sigma^2}. \tag{A28}$$

Solving this equation for $\hat{\mu}$ then leads to the expression for the equilibrium return in the proposition.

Case III. In this case, all the portfolio weights are strictly negative. This is in contradiction to the condition for equilibrium: $1 = \pi_F + (N-1)\pi_U$. Thus, this case cannot arise in equilibrium. ∎

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