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## ABSTRACT

# Can the Fiscal Theory of the price level explain UK inflation in the 1970s?

We investigate whether the Fiscal Theory of the Price Level can deliver a reasonable explanation for UK inflation in the 1970s, a period in which the government greatly increased public spending without raising taxes and monetary policy was accommodative. The model is tested for its implied cointegration between inflation and government spending and for its dynamics by using the method of indirect inference, under which the model's simulated behaviour is compared with the inflation time-series process. We find that the model is accepted in both respects.

JEL Classification: E31, E37, E62 and E65
Keywords: bootstrap simulation, fiscal theory of the price level, indirect inference, uk inflation and wald statistic

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# 1 Introduction

In 1972 the UK government floated the pound while pursuing highly expansionary fiscal policies whose aim was to reduce rising unemployment. To control inflation the government introduced statutory wage and price controls. Monetary policy was given no targets for either the money supply or inflation; interest rates were held at low rates with the aim of accommodating growth and falling unemployment. From this brief description of policies it would seem that the fiscal policy was non-Ricardian (that is, it was not limited by concerns with solvency) and that monetary policy was not setting any limits to inflation - in the language of Leeper (1991) fiscal policy was 'active' and monetary policy was 'passive'. Since wage/price controls would inevitably break down faced with the inflationary effects of such policies, this period appears to be a prime candidate for the Fiscal Theory of the Price Level. Furthermore, there was no basis for the belief that it would come to an end with alternative policies: both Conservative and Labour parties won elections in the 1970s and both pursued essentially the same policies. While Margaret Thatcher won the Conservative leadership in 1975 and also the election in 1979, there was no basis to assume that the monetarist policies she advocated would ever occur. Only after her election and her actual implementation of them, was this a reasonable assumption. So it appears that in the period 1972-79 there was a policy regime prevailing which was expected to continue.

Our aim in this paper is to investigate empirically whether the Fiscal Theory of the Price Level (FTPL) applies to this period, using both cointegration tests for the trends and the method of Indirect Inference for the dynamics. Under FTPL the price level or inflation is determined by the need for fiscal solvency to be imposed; thus one determines it by finding what value is necessary for the government's intertemporal budget constraint to hold at the market value of outstanding debt. Given this determinate price level, money supply growth, interest rates and output are determined recursively as the values required by the rest of the model to permit this price level .

Indirect Inference can be used to evaluate a model like this one by checking whether the model's simulated dynamic behaviour is consistent with the data. The data under Indirect Inference is described by some time-series equation. The model's simulated behaviour implies a range of time-series behaviour depending on the shocks hitting it; this range can be described by the parameters of the same time-series equation that fits the data. We can derive the implied statistical joint distribution for the parameters of this equation and test whether parameters of the time-series equation from the data lie jointly within this distribution at some confidence level. In what follows we review the UK policy background and the FTPL in section 2; in section 3 we set up the model of FTPL; in section 4 we discuss the data and test the trends for cointegration; in section 5 we set out our Indirect Inference testing procedures for the dynamics; in section 6 we show the results; section 7 concludes.

# 2 UK policy background and the FTPL

From WWII until its breakdown in 1970 the Bretton Woods system governed the UK exchange rate and hence its monetary policy. While exchange controls gave some moderate freedom to manage interest rates away from foreign rates without the policy being overwhelmed by capital movements, such freedom was mainly only for the short term; the setting of interest rates was dominated in the longer term by the need to control the balance of payments sufficiently to hold the sterling exchange rate. Pegging the exchange rate implied that the price level was also pegged to the foreign price level. Through this mechanism monetary policy ensured price level determinacy. Fiscal policy was therefore disciplined by the inability to shift the price level from this trajectory and also by the consequent fixing of the home interest rate to the foreign level. While this discipline could in principle be overthrown by fiscal policy forcing a series of devaluations, the evidence suggests that this did not happen; there were just two devaluations during the whole post-war period up to 1970, in 1949 and 1967. On both occasions a Labour government viewed the devaluation as a one-off change permitting a brief period of monetary and fiscal ease, to be followed by a return to the previous regime.

However, after the collapse of Bretton Woods, the UK moved in a series of steps to a floating exchange rate. Initially sterling was fixed to continental currencies through a European exchange rate system known as 'the snake in the tunnel', designed to hold rates within a general range (the tunnel) and if possible even closer (the snake). Sterling proved difficult to keep within these ranges, and was in practice kept within a range against the dollar and an 'effective' (currency basket) rate. Finally it was formally floated in June 1972.

UK monetary policy was not given a new nominal target to replace the exchange rate. Instead the Conservative government of Edward Heath assigned the determination of inflation to wage and price controls. A statutory 'incomes policy' was introduced in late 1972. After the 1974 election the incoming Labour government set up a 'voluntary incomes policy', buttressed by food subsidies and cuts in indirect tax rates. Fiscal policy was expansionary until 1975 and monetary policy was accommodative, with interest rates kept low to encourage falling unemployment. In 1976 the Labour government invited the IMF to stabilise the falling sterling exchange rate; the IMF terms included the setting of targets for Domestic Credit Expansion. These were largely met by a form of control on deposits (the 'corset') which forced banks to reduce deposits in favour of other forms of liability. But by 1978 these restraints had effectively been abandoned and prices and incomes controls reinstated in the context of a pre-election fiscal and monetary expansion - see Goodhart (1989), Minford (1993), Nelson (2001), Nelson and Nikolov (2003), Goodhart (2003), Allsopp et al (2006), DiCecio and Nelson (2007), and Meenagh et al (2008a) for further discussions of the UK policy environment for this and other post-war UK periods.

Our description of policy suggests that the role of nominal anchor for inflation will have been played during the 1970s by fiscal policy, if only because monetary policy was not given this task and was purely accommodative. The FTPL has been set out and developed in Leeper (1991), Sims (1994, 1997), Woodford (1996, 1998a, 1998b, 2001), Schmitt-Grohe and Uribe (2000) and Cochrane (2000, 2001) - see also comments by McCallum (2001,2003), McCallum and Nelson (2005), Buiter (1999, 2002), Arce (2004, 2005) and Niepelt (2004); and for surveys Kocherlakota and Phelan (1999), Carlstrom and Fuerst (2000) and Christiano and Fitzgerald (2000). Empirical tests have been proposed by Canzoneri, Cumby and Diba (2001), Bohn (1998), Cochrane (1999) and Woodford (1999), Davig et al (2007), Davig and Leeper (2006), Bihan and Creel (2006) and Benassy (2008). For the FTPL in an international framework see Woodford (1996), Sims (1997, 1999), Dupor (2000), Bergin (2000), Canzoneri, Cumby and Diba (2001), Andres, Ballabriga and Valles (2000) and Daniel (2001), Thams (2007) and Bajo-Rubio et al (2007).

In particular Loyo (1999) argues that Brazilian policy in the late 1970s and early 1980s was non-Ricardian and that the FTPL provides a persuasive explanation for Brazil's high inflation during that time. Sims (2001) makes a similar attempt to assess the consequences of dollarization in Mexico. Meanwhile in the context of the FTPL, Corsetti and Mackowiak (2001) relate the occurrence of currency devaluations to the existence of fiscal imbalances. The work of Tanner and Ramos (2003) also finds evidence of fiscal dominance for the case of Brazil for some important periods. Cochrane (1998, 2000) argues that an FTPL with a statistically exogenous surplus process explains the dynamics of U.S. inflation in the 1970s. This appears to be similar to what we see in the UK during the 1970s: the government made the reduction of unemployment the target for fiscal policy. Thus its aim was to use the necessary size of fiscal deficit to drive unemployment down to an acceptable target and to prevent monetary policy from frustrating this by making money supply growth endogenously accommodate the implied money demand. No thought was given to the consequences of these deficits for current or future taxes; the policy was in short "old Keynesian" in its entirety.

With fiscal policy of this type, the financial markets - forced to price the resulting supplies of government bonds - will take a view about future inflation and set interest rates and bond prices accordingly. It will set bond prices so that the government's solvency is assured ex post (i.e. in equilibrium); thus it will be ensuring that buyers of the bonds are paying a fair price. Future inflation is expected because if the bonds were priced at excessive value then consumers would have wealth to spend, in that their bonds would be worth more than their future tax liabilities; this would generate excess demand which would drive up inflation. However this mechanism would only come into play out of equilibrium; we would not observe it because markets anticipate it and so drive interest rates and expected inflation up in advance; inflation follows because of the standard Phillips Curve mechanism by which workers and firms raise inflation in line with expected inflation.

Working within the context of a full DSGE model, Davig et al (2007) and Davig and Leeper (2006) examine regime switches between fiscal and monetary policy for U.S. data. They define the Ricardian policy regime as an 'active' monetary policy coupled with a 'passive' fiscal policy - the policy mix implicit in the literature on the Taylor principle. In contrast, the non-Ricardian regime is a policy where there is fiscal policy dominance, and an accommodative or 'passive' monetary policy - this is the combination associated with the fiscal theory of the price level (Davig et al 2007). They model regime change as an on-going process and show that as long as agents are allowed to place probability on both kinds of regimes happening, and if active fiscal policy were expected to occur next period, then tax changes would have wealth effects and lead to non-Ricardian outcomes. Another attempt to locate regime switching is due to Favero and Monacelli (2005). They investigate U.S. data for the period of 1960-2002 and conclude that U.S. fiscal policy has shifted between active and passive monetary regimes. Other work on regime changes includes Daniel (2003) and Weil (2003) who consider one-time changes in fiscal regime with fixed monetary policy behaviour; Davig (2004, 2005), Leeper and Zha (2003) and Lubik and Schorfheide (2004) on the other hand deal with on-going regime change cases - see also Kim (2003) and Davig and Leeper (2009a, 2009b).

There is much empirical work mainly on European economies following the lead of Canzoneri, Cumby and Diba (2001); this work looks for time-series evidence of tax or spending rates reacting endogenously to government deficits in a Ricardian manner (so stabilising the growth of debt to ensure solvency with existing monetary policy). It is argued that under non-Ricardian policy they will be exogenous and orthogonal to deficits. Generally, this evidence suggests the dominance of a Ricardian policy regime, with occasional episodes of a non-Ricardian regime - thus for European economies, see Afonso (2002) for an EU-15 countries panel over 1970-2001; Janssen et al (2002) for the UK over 300 years; Creel et al (2005) for France; Alstadheim (2005) for Norway; Semmler and Zhang (2004) for France and Germany; Bihan and Creel (2006) for France, Germany, Italy, the UK and the US data using impulse response functions from VARs; Sabate et al (2006) for Spain; Thams (2007) using Bayesian methods for Germany and Spain; Bajo-Rubio et al (2007) following Bohn (1998) for 11 EU countries 1970-2005 using cointegration analysis and Granger causality tests.

Thus the FTPL can be regarded as a particular policy regime - one of 'active' or 'dominant' fiscal policy - within a sequence of different policy regimes. As Cochrane (1998) has observed "the fiscal theory of the price level per se has no testable implications for the time series of debt, surplus and price level. The government intertemporal budget constraint holds in equilibrium for both Ricardian and non-Ricardian regimes. Whether the FTPL holds or not for a sample period and how it affects the economy's behaviour requires one to specify the policy regime sequence and the DSGE model within which the sequence applies; to test for the FTPL then requires one to check this specification against the data". We now proceed to explain the model we will use to do this here on UK inflation data for the 1970s.

## 3 Applying the FTPL

We assume that the UK finances its deficit by issuing nominal perpetuities, each paying one pound per period and whose present value is therefore $\frac{1}{R_t}$where $R_t$ is the long-term rate of interest. We use perpetuities here rather than the usual one-period bond because of the preponderance of long-term bonds in the UK debt issue: the average maturity of UK debt at this time was approximately 10 years. All bonds at this time were nominal (indexed bonds were not issued until 1981).

The government budget constraint can then be written as

(1) $\frac{B_{t+1}}{R_t} = G_t - T_t + B_t + \frac{B_t}{R_t}$

where $G_t$ is government spending in money terms, $T_t$ is government taxation in money terms, $B_t$ is the number of perpetuities issued. Note that when perpetuities are assumed the debt interest in period t is $B_t$ while the stock of debt at the start of period t has the value during the period of $\frac{B_t}{R_t}$; end-period debt therefore has the value $\frac{B_{t+1}}{R_t}$. Note too the the perpetuity interest rate is by construction expected to remain constant into the future.

We can derive the implied value of current bonds outstanding by substituting forwards for future bonds outstanding:

(2) $\frac{B_t}{R_t} = E_t \sum_{i=0}^{\infty} (T_{t+i} - G_{t+i}) \frac{1}{(1+R_t)^{i+1}}$

We assume that at each time period there is an expected 'permanent' tax and spending share, $t_t$ and $g_t$, so that $E_t T_{t+i} = t_t E_t P_{t+i} y_{t+i}$ and $E_t G_{t+i} = g_t E_t P_{t+i} y_{t+i}$

This can be simplified (see Appendix A) to:

(3) $\frac{B_t}{R_t P_t y_t} = \frac{t_t - g_t}{(1+\pi_t+\gamma)(r_t^* - \gamma)}$

where $R_t = r_t^* + \pi_t$ (respectively the long-term or 'permanent' real interest rate and long-term or 'permanent' inflation rate), $\gamma_t$ is the 'permanent' growth rate of real $GDP$. All these expected permanent variables are by construction expected to be constant in the future at today's level. Permanent growth in this period we assume to be constant so that output is a random walk with constant drift equal to $\gamma$.

In the case of inflation we impose on the model the simplifying assumption that it is a random walk, so that future expected inflation is equal to current inflation and so is also permanent inflation. Notice that in the rest of the model we have equations for output and real interest rates, in the IS and Phillips Curves; but these cannot determine inflation as well. Hence if inflation had some dynamic time-path other than the random walk we would have to determine it exogenously; we choose the random walk for simplicity, on the basis that the off-equilibrium wealth effect would operate so powerfully on excess demand that it would drive inflation at once to its permanent value.

The pricing condition on bonds in equation (3) thus sets their value as a ratio to money $GDP$ equal to the primary surplus as a share of $GDP$ divided by the gap between the real rate of interest and the rate of growth and one plus inflation and growth of real $GDP$. Suppose now the government reduces the present value of future primary surpluses. At an unchanged real value of the debt this would be a "non-Ricardian" fiscal policy move. According to the FTPL prices will adjust to reduce the real value of the debt to ensure the government budget constraint holds and thus the solvency condition is met. This is to be compared with the 'normal' Ricardian situation, in which fiscal surpluses are endogenous. Fiscal shocks today lead to adjustments in future surpluses, while the price level remains unaffected.

Since the pricing equation sets the ratio of debt value to $GDP$ equal to a function of permanent variables, it follows that this ratio $b_t$ follows a random walk [1] such that:

(4) $b_t = \frac{B_t}{R_t P_t y_t} = E_t b_{t+1}$ and (5) $\Delta b_t = \eta_t$, an *i.i.d.*process.

This in turn allows us to solve for the inflation shock as a function of other shocks (especially shocks to government tax and spending). With the number of government bonds issued $B_t$ pre-determined (issued last period) and therefore known at $t-1$, equation (3) could be written as follows (taking logs and letting $\log x_t^{ue} = \log x_t - E_{t-1} \log x_t$, the unexpected change in $\log x_t$)

(6) $\log b_t^{ue} = -\log R_t^{ue} - \log P_t^{ue} - \log y_t^{ue}$ [LHS of equation (3)]

$= \log{(t_t - g_t)}^{ue} - \log{(1 + \pi_t + \gamma)}^{ue} - \log(r_t^* - \gamma)^{ue}$ [RHS of equation (3)]

With all the variables in the equation defined to follow a random walk, we can rewrite the above expression as (note that for small $\gamma$ $\log{(1 + \pi_t + \gamma)}^{ue} \approx \pi_t^{ue} = \log P_t^{ue}$)

(7) $-\Delta \log{(\pi_t + r_t^*)} = \Delta \log(t_t - g_t) - \log(r_t^* - \gamma)^{ue} + \log y_t^{ue}$

Using a first-order Taylor Series expansion around the sample means we can obtain a solution for $\Delta\pi_t$ as a function of change in government expenditure and tax rates

(8) $\Delta\pi_t = \kappa \times (\Delta g_t - \Delta t_t) + \eta_t$

where $\kappa = \frac{\overline{\pi} + \overline{r^*}}{\overline{t} - \overline{g}}$ is $\overline{\pi}, \overline{r^*}, \overline{t}$ and $\overline{g}$ are mean values of the corresponding variables. And $\eta_t$ is the structural error which captures the other effects on the process of $\Delta\pi_t$.

This equation states that the unexpected inflation term ($\Delta\pi_t$) comes from those expenditure and tax shocks ($\Delta g_t$ and $\Delta t_t$) plus the error $\eta_t$ that embraces surprise terms in permanent real interest rates and in output. Notice that while this equation is identically equal to the actual data on the change in inflation, the separate processes for $\Delta g_t, \Delta t_t, \eta_t$ impose a structure on the model's behaviour which is absent from the unrestricted time-series process governing $\Delta\pi_t$ in the data.

We can now complete the DSGE model as in Meenagh et al (2008a), by adding a forward-looking

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[1] A 'permanent' variable $\overline{x}_t$ is by definition a variable expected not to change in the future so that
$E_t \ \overline{x}_{t+1} = \overline{x}_t$

Thus $\overline{x}_{t+1} = \overline{x}_t + \epsilon_{t+1}$
where $\epsilon_{t+1}$ is an iid error making the process a random walk.

IS curve, derived in the usual way from the household Euler equations and the goods market-clearing condition, and a New Classical Phillips Curve:

(9) $(y_t - y_t^*) = -\alpha(r_t - r_t^*) + \zeta E_t(y_{t+1} - y_{t+1}^*) + v_t$

(10) $\pi_t = E_{t-1}\pi_t + \delta(y_t - y_t^*) + u_t$

Since inflation follows a random walk from the FTPL solution above (so that expected inflation is simply lagged inflation), we can now establish from these two equations that:

(11) $y_t = y_t^* + \frac{1}{\delta}(\Delta\pi_t - u_t)$

(12) $r_t = r_t^* - \frac{1}{\alpha\delta}(\Delta\pi_t - u_t) + \frac{1}{\alpha}v_t - \frac{\zeta}{\alpha\delta}E_t u_{t+1}$

Thus both output and real interest rates are stationary processes around their natural rates. Both $u_t$ and $v_t$ may be serially correlated.

We may note that if we were to substitute a New Keynesian supply curve for the New Keynesian one above as in Meenagh et al (2008a), the solutions for output and real interest rates would alter. Clearly there is no way of distinguishing between these various Phillips Curves in terms of the inflation equation under FTPL; however, there is some possibility of distinguishing between them in terms of output and exchange rate behaviour, though we do not do this here.

# 4 Data, estimation and testing

We first estimate a univariate process for UK 1970s inflation rate. The data for inflation is defined as the Consumer Price Level (CPI) deflator, $\frac{\text{Nominal Total Consumption (NTC)}}{\text{Real Total Consumption (RTC)}}$ - UK Office for National Statistics (ONS) databank. The mean and standard deviation for our sample period (quarterly rates of change, in fractions per quarter) are 3% and 1% respectively. It is non-stationary; both the Augmented Dickey Fuller (ADF) (1979, 1981) and Phillips-Perron (PP) (1988) tests[2] confirm that it is I(1) - Table 1.

We now go on to estimate the best fitting *ARMA* for the inflation first difference. We use a parsimonious criterion: starting with *ARMA* (0,0), we raise the order of the *AR* and *MA* each by one, and apply an *F*-test to test the validity of the lower order restriction. Both the additional coefficients of *AR* and *MA* are insignificant, suggesting that the best fitting *ARIMA* for UK inflation is *ARIMA* (0,1,0), i.e. a pure random walk - Tables 2 & 3.

Both $g$ and $t$ are seasonal variables - Figure 1 (ONS databank)- thus we include a constant and three seasonal dummies in the estimation to control for seasonality. The permanent variables $g$ and $t$ are that part of the spending and tax share of *GDP* that is expected to continue indefinitely in the future, in other words, the 'trend' values. However, actual $g$ and $t$ are not necessarily always at their trend values

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[2] Applicable to all non-stationary tests - MacKinnon's critical values for rejection of hypothesis of a unit root. Values in parentheses are $p$-values, * indicates significance at the 1%. Number of lags in the ADF test is set upon *AIC* criterion and PP test upon Newey-West bandwidth. These tests are carried out by employing Eviews 5.

| Unit Root Tests | | |
|---|---|---|
| | Levels | First Difference |
| ADF Test Statistic | -0.771310 | -5.107606 |
| | (0.3739) | (0.0000)* |
| PP Test Statistic | -2.065831 | -7.153394 |
| | (0.4671) | (0.0000)* |

Table 1: Test for Non-Stationarity of the Inflation rate

| $F$-Test for Restriction | $F$-statistic |
|---|---|
| $ARMA(0,0) \longrightarrow ARMA(1,0)$ | 0.047600 |
| $ARMA(0,0) \longrightarrow ARMA(0,1)$ | 0.454400 |
| $ARMA(0,0) \longrightarrow ARMA(1,1)$ | 0.730228 |
| $ARMA(0,1) \longrightarrow ARMA(1,1)$ | 0.252271 |
| $ARMA(1,0) \longrightarrow ARMA(1,1)$ | 0.555781 |

Table 2: F-Tests to Find Best-Fitting ARMA

unless the variables follow random walk processes. We find that $g$ is non-stationary. Both the ADF and PP test confirm that it follows a pure random walk (Table 4) - implying that its current value is also its trend value. $t$ however becomes stationary after deseasonalising with no significant deterministic trend; hence its trend value is simply a constant. We conclude that government expenditure is the only driving force for inflation that we can observe in the data. This reflects the fact that in this period the government largely increased expenditure and held tax down in the effort to keep down unemployment. The government expenditure/$GDP$ ratio remained stable in the 1950s and 1960s but in the 1970s moved upwards significantly. At the same time, by 1975, the $PSBR$ (public sector borrowing requirement) had reached over 10 per cent of $GDP$, creating concerns about solvency: thus future primary surpluses became insufficient to pay off the debt and its principal at pre-existing government bond values. These concerns were resolved by a rise in the rate of inflation and the consequent rise in nominal interest rates, which reduced debt values to consistency with the reduced future surpluses. This is the mechanism of the fiscal theory of the price level in the set-up here. The rise in inflation is engineered in the rational expectations model by agents' raising of their inflationary expectations in line with the transversality condition on the government in its intertemporal budget constraint: unless the government explicitly defaults which we rule out as a policy unacceptable to the UK government, this condition must hold. Of course given

| | Coefficient | Std. Error |
|---|---|---|
| $AR(1)$ | -0.371112$^{+}$ | 0.867089 |
| $MA(1)$ | 0.511929$^{+}$ | 0.793075 |
| $AIC$ | -6.137809 | - |
| S.E. of regression | 0.010900 | - |
| $^{+}$F-tests insignificant | | |

Table 3: Best Fitting ARMA

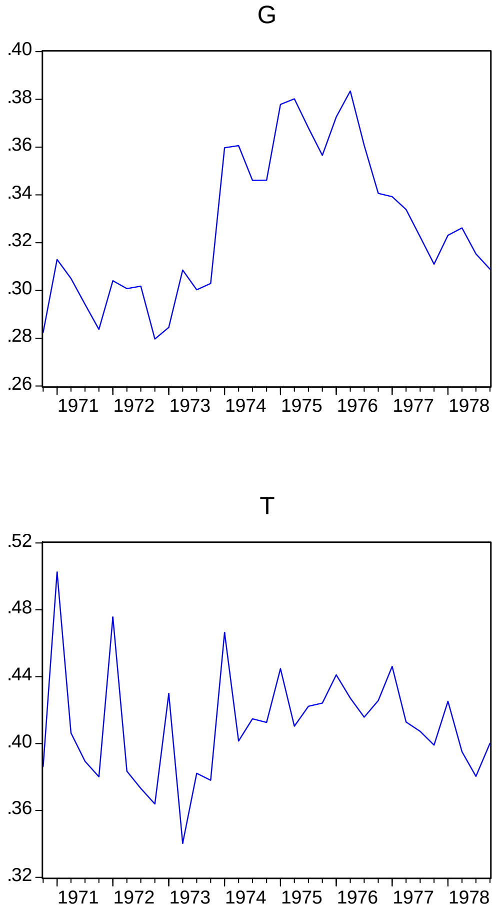

Figure 1: The government expenditure, G ($g_t$) and tax, T ($t_t$) rates

expected inflation, the Phillips Curve mechanism causes workers and firms to raise actual inflation in line with expected inflation. Because there are no dynamics in the model, inflation and expected inflation jump instantly to ensure the budget constraint holds, and output is always at its natural rate which we assume also follows a random walk in line with productivity [3].

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[3] For model convergence, the amount of government expenditure is required be less than taxation for government bonds to have a positive value. We note that since government expenditure of a capital variety is expected to produce future returns in line with real interest rates, we should deduct the trend in such spending from the trend in $g$ (derived from the data shown in the Figure 1). To implement this we assume that the average share of expenditure in the period devoted to fixed capital, health and education can be regarded as the (constant) trend in such capital spending; of course the 'capital' element in total government spending is essentially unobservable and hence our assumption is intended merely to adjust the level of the $g$ trend in an approximate way but not its movement over time which we regard as accurately capturing changes in current spending. The adjustment for these is of the order of 10% of GDP.

| Unit Root Tests | $g$ | | $t$ | |
|---|---|---|---|---|
| | Levels | First Difference | Levels | First Difference |
| ADF Test Statistic | -1.314641 | -4.887497 | -2.593144 | - |
| | (0.1705) | (0.0000)* | (0.0113) | - |
| PP Test Statistic | -1.365278 | -4.947653 | -4.042577 | - |
| | (0.1204) | (0.0000)* | (0.0002) | - |

Table 4: Test for Non-Stationarity of deseasonalised government expenditure and tax rates

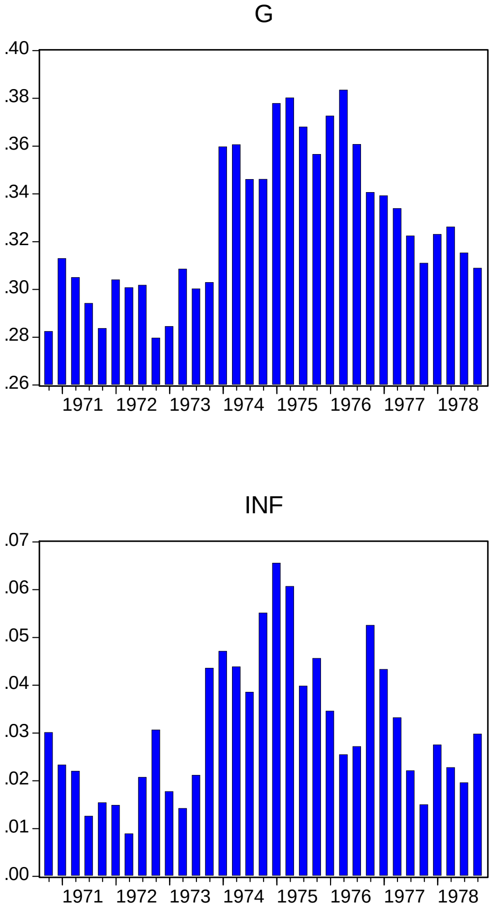

Figure 2: The patterns of government expenditure rate, G ($g$) and inflation,INF ($\pi_t$)

*An initial cointegration test*

The key issue in the fiscal theory of the price level concerns the impact of the government aggregates on the general price level. Figure 2 compares the pattern of inflation ($\pi_t$) and public spending ($g_t$): both are I(1) variables and plainly share some similarities in behaviour. The theory above implies that they should be cointegrated. Hence we begin by testing whether there is a cointegrating relationship between government spending and inflation . The relationship is tested by Johansen's (1988) methodology – Table 5. The null hypothesis of no cointegration is rejected by both trace and maximal eigenvalue statistics at the 5% level. The normalised cointegrating coefficients are also reported. The ratio of the coefficients is of the same order as the constant term $\kappa$ derived in the FTPL model. The result suggests there is a positive association between these two as suggested by the theory.

| Johansen tests:VAR=($g_t$,$\pi_t$) | Eigenvalue | Trace Stat | Max-EigenV | C.V. 95% (Trace) | C.V. 95% ($\lambda$MAX) |
|---|---|---|---|---|---|
| $r$=0 | 0.391573 | 17.68955 | 15.40321 | 15.49471 | 14.26460 |
| $r \leq 1$ | 0.071099 | 2.286344 | 2.286344 | 3.841466 | 3.841466 |
| Parameter Estimates | Coint. Vector | | | | |
| Variables | | | | | |
| $g_t$ | -1.000000 | | | | |
| $\pi_t$ | +2.469930 | | | | |

Table 5: Johansen Cointegration Tests Results

# 5 Bootstrapping and the method of indirect inference

We now replicate the stochastic environment for the FTPL model to see whether within it our estimated $ARMA$ equation for $\Delta\pi_t$ could have been generated- in other words we now test whether the dynamics of inflation match those implied by the model. This we do by bootstrapping the model above with their error processes. Meenagh et al. (2008b) explain how this procedure is derived from the method of indirect inference. This method uses an 'auxiliary model' - such as our time-series representation here - to describe the data- see Smith (1993), Gregory and Smith (1991, 1993), Gourieroux et al. (1993), Gourieroux and Monfort (1995) and Canova (2005). The method is used here to evaluate the fit of a given structural model (rather than in estimation).This is relevant as here, when we are interested in the behaviour of a structural model whose structure is rather precisely specified by the theory.

The idea of this evaluation is to create pseudo data samples - here 1000 - for inflation. We randomly draw *i.i.d.* shocks in our error processes with replacement; we then input them into their error processes and these in turn into the model to solve for the implied path of inflation over the sample period. We then run $ARMA$ regressions on all the pseudo-samples to derive the implied 95% confidence intervals for all the coefficient values found. Finally we compare the $ARMA$ coefficients estimated from the actual data to see whether they lie within these 95% confidence intervals: under the null hypothesis these values represent the sampling variation for the $ARMA$ coefficients which are generated by the model. The portmanteau Wald statistic - the 95% confidence limit for the joint distribution of the $ARMA$ parameters- is also computed. The Wald statistic is derived from the bootstrap joint distribution of the $ARMA$ parameters under the null hypothesis that the structural model holds.

Figure 3 below illustrates the method for two parameters in the auxiliary equation such as in an $ARMA$(1,1). The bootstrap distribution of these two parameters under the null are shown for two cases: one where the two parameter estimates are uncorrelated, the other where they are highly correlated (with a coefficient of 0.9). One can think of estimation via indirect inference as changing the parameters of the structural model, thus changing the implied distribution, so as to push the observed data point as far into the centre of the distribution as possible. The test however takes the structural parameters (and hence the bivariate distribution) as given and merely notes the position of the observed data point (here given as 0.1 and 0.9) in the distribution. The Wald statistic is computed as this position expressed as a percentile; thus for example 96 indicates that the observed parameter estimates lie on the 96% 'contour', ie in the 95% rejection region.

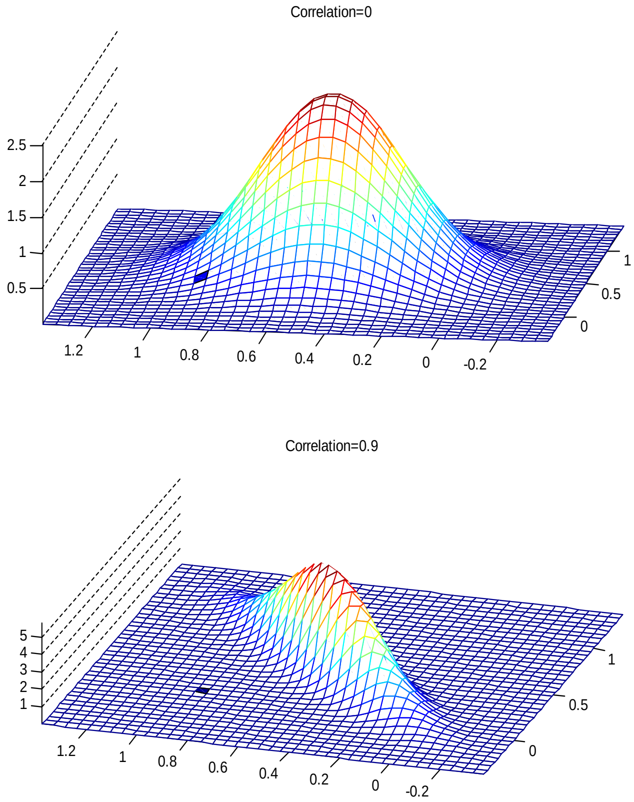

Figure 3: Bivariate Normal Distributions (0.1, 0.9 shaded) with correlation of 0 and 0.9.

# 6 Testing Results

We now use the FTPL equation above for $\Delta\pi_t$ and generate the FTPL model sampling variability by bootstrapping the random components of these $\Delta g$ and $\eta$ processes (since $t$ is stationary and its trend value is a constant, it drops out on first-differencing). We obtain 1000 pseudo-samples of $\Delta\pi_t$ then run an $ARMA$ on each of these samples to generate the distribution of the $ARMA$ parameters. The Wald statistic then tests the model at the 95% level of confidence on the basis of the complete set of $ARMA$ parameters.

## 6.1 Variance of data and bootstrap sample

We first look at the bootstrap sample variance (Table 6): the variance of $\Delta\pi_t$ in levels in the data sample is 0.000116; the 95% bounds for the bootstrap samples are 0.000088 (lower) and 0.000223 (upper) with a mean bootstrap variance of 0.000150. Thus the data variance lies inside the 95% model bounds, with the mean value fairly close to it. Thus the model replicates the data variance.

| Second Moment | Estimated value | 95% Lower Bound | 95% Upper Bound | Mean Value | IN/OUT |
|---|---|---|---|---|---|
| $Var(\Delta\pi_t)$ | 0.00011551 | 0.00008759 | 0.00022325 | 0.00014952 | IN |

Table 6: Variance of Data and Bootstraps

## 6.2 Test results for $ARMA$(1,1)

Next we use the bootstrapped samples to compare the model with the data on its dynamic aspects - here the coefficients of the $ARMA$ for $\Delta\pi_t$. Of course we have already established that $g$ is a pure random walk and that inflation is close to that too, which suggests that the model will generate similar dynamics. We run 1000 $ARMA$ regressions on the pseudo-samples to derive the implied 95% confidence intervals for both $AR$ and $MA$ coefficients. Then we compare the $ARMA$ coefficients estimated from the observed data to see whether they lie within these 95% confidence intervals. The Wald-statistic is derived from the bootstrap distribution of the $ARMA$ parameters under the null hypothesis of the model. The Wald-statistic (also called the square of the Mahalanobis distance (Meenagh 2008b)) is calculated using following formula

$(\widehat{\alpha}-\overline{\alpha})' \sum_{\alpha}^{-1} (\widehat{\alpha}-\overline{\alpha})'$

where, $\sum_{\alpha}^{-1}$ is the inverse of the variance-covariance matrix of $\widehat{\alpha}$,the ARMA parameter vector here generated by the bootstrap ($\overline{\alpha}$ is the mean of the bootstrap distribution). We compute the Mahalanobis distances and arrange the values in ascending order and get the 5% bootstrap 'multi-variant contour' - the critical percentile value for the model to be accepted as a whole.

Table 7 lists the results of this exercise. The model is accepted as a whole according to the Wald statistic. It clearly validates our hypothesis that the behaviour of inflation is explicable within the FTPL framework. All of the $ARMA$ parameters lie within the 95% bounds and full Wald statistic[4] is 18.3%.

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[4] The full Wald statistic tests the model ability to generate both dynamics (i.e. persistence) and volitility of the data.

| | Model | Estimated | 95% Confidence Interval | | IN/OUT |
|---|---|---|---|---|---|
| | | | Lower | Upper | |
| | $AR(1)$ | -0.371112 | -0.863044949 | 0.870023412 | IN |
| | $MA(1)$ | 0.511929 | -0.997119035 | 0.979783961 | IN |
| Wald statistic (ARMA alone) | 16.9% | | | | |
| Full Wald statistic (incl. variances) | 18.3% | | | | |

Table 7: Confidence Limits of first-differenced inflation process for Theoretical ARMA(1,1)

### 6.3 Sensitivity Testing

The best $ARMA$ representation was chosen as $ARMA(1,1)$ under the criterion of parsimony with the maximum order is set by one. However, if one increases the orders of both $AR$ and $MA$, the best-fitting $ARMA$ is $ARMA(3,3)$, followed by $ARMA(1,3)$ and $ARMA(2,3)$ etc. according to the $AIC$ measure. We examine these other possible auxiliary models to examine whether the main results in the previous section are robust. The complete list of our models can be found in Table 8. All detailed tests can be found in Appendix B.

| | $AR(1)$ | $AR(2)$ | $AR(3)$ | $MA(1)$ | $MA(2)$ | $MA(3)$ | $AIC$ |
|---|---|---|---|---|---|---|---|
| $ARMA(1,1)$ | -0.371112 | | | 0.511929 | | | -6.137809 |
| $ARMA(1,2)$ | 0.270202 | | | -0.295610 | -0.309869 | | -6.184849 |
| $ARMA(1,3)$ | -0.402732 | | | 0.664334 | -0.350723 | -0.790819 | -6.280456 |
| $ARMA(2,0)$ | 0.056195 | -0.360412 | | | | | -6.213213 |
| $ARMA(2,1)$ | 0.381779 | -0.366740 | | -0.392966 | | | -6.173225 |
| $ARMA(2,2)$ | 0.420718 | -0.463388 | | -0.441163 | 0.115496 | | -6.109618 |
| $ARMA(2,3)$ | -0.666351 | -0.312067 | | 0.858843 | -0.019530 | -0.597234 | -6.250935 |
| $ARMA(3,0)$ | -0.003204 | -0.373691 | -0.160067 | | | | -6.179304 |
| $ARMA(3,1)$ | -0.834652 | -0.333377 | -0.457973 | 0.952026 | | | -6.185603 |
| $ARMA(3,2)$ | -0.955016 | -0.432063 | -0.449312 | 1.094471 | 0.130641 | | -6.119408 |
| $ARMA(3,3)$ | -0.598601 | -0.482306 | 0.377436 | 1.564748 | 0.737548 | -1.813709 | -7.355737 |

Table 8: ARMA Regressions

Table 9 reports the associated Wald statistics: all lie inside the 95% bound, with only the $ARMA$ (3,3) on the 95% borderline if one ignores the information in the variances. Thus whatever representation of the inflation data one chooses the model robustly generates a distrib bbbution consistent with it.

## 7 Conclusions

We investigate whether the Fiscal Theory of the Price Level can deliver a reasonable explanation for UK inflation in the 1970s, a period in which the government greatly increased public spending without raising

| | Wald Statistic ($ARMA$ only) | Full Wald Statistic ($ARMA$ and variances) |
|---|---|---|
| $ARMA$(1,1) | 16.9% | 18.3% |
| $ARMA$(1,2) | 21.9% | 24.1% |
| $ARMA$(1,3) | 76.8% | 70.2% |
| $ARMA$(2,0) | 85.7% | 79.9% |
| $ARMA$(2,1) | 67.5% | 61.0% |
| $ARMA$(2,2) | 45.2% | 38.0% |
| $ARMA$(2,3) | 58.7% | 56.9% |
| $ARMA$(3,0) | 80.2% | 75.2% |
| $ARMA$(3,1) | 94.3% | 93.2% |
| $ARMA$(3,2) | 84.3% | 82.0% |
| $ARMA$(3,3) | 95.1% | 94.2% |

Table 9: Wald Statistics for variety of ARMA representations

taxes and monetary policy was accommodative. The model is tested for its implied cointegration between inflation and government spending and for its dynamics by using the method of indirect inference, under which the model's simulated behaviour is compared with the inflation time-series process. We find that the model is accepted in both respects.

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# AppendixA Derivation of Government budget constraint

The government budget constraint gives us

$\frac{B_{t+1}}{R_t} = G_t - T_t + B_t + \frac{B_t}{R_t}$

Where,

$G_t$ is the government spending in money terms,

$T_t$ is the government taxation in money terms,

$R_t$ is the amount of nominal interest the government must pay. The value of the bonds outstanding is $B \times \frac{1}{R}$ .

We can derive an expression for government budget constraint in the forward direction by substituting forwards for future bonds outstanding, yields

$\frac{B_t}{R_t} = \sum_{i=0}^{\infty} \left(T_{t+i} - G_{t+i}\right) \frac{1}{(1+R_t)^{i+1}}$

If $T_{t+i}$ and $G_{t+i}$ are growing with money $GDP$,

i.e. $T_{t+i} = t_t P_{t+i} y_{t+i}$

$G_{t+i} = g_t P_{t+i} y_{t+i}$

$$\begin{aligned}
\frac{B_t}{R_t} &= \sum_{i=0}^{\infty} \frac{(t_t - g_t) P_{t+i} y_{t+i}}{(1+R_t)^{1+i}} \\
&= \sum_{i=0}^{\infty} \frac{(t_t - g_t) P_t y_t (1+\gamma+\pi_t)^i}{(1+R_t)^{1+i}} \text{ (note that real output grows at rate } \gamma) \\
&= \left(t_t - g_t\right) P_t y_t \sum_{i=0}^{\infty} \frac{(1+\gamma+\pi_t)^i}{(1+R_t)^{1+i}} \\
&= \left(t_t - g_t\right) P_t y_t \sum_{i=0}^{\infty} \frac{(1+\gamma+\pi_t)^{1+i}}{(1+R_t)^{1+i}(1+\gamma+\pi_t)}
\end{aligned}$$

If $\gamma$ and $\pi_t$ are both small enough,

$\sum_{i=0}^{\infty} \frac{(1+\gamma+\pi_t)^{1+i}}{(1+R_t)^{1+i}} = \sum_{i=0}^{\infty} \left(\frac{1}{1+R_t-\gamma-\pi_t}\right)^{1+i} = \left(\frac{1}{1-\frac{1}{1+r^*_{t-}\gamma}} - 1\right) = \left(\frac{1}{r^*_t - \gamma}\right)$

Hence, $\frac{B_t}{R_t P_t y_t} = \frac{(t_t - g_t)}{((1+\gamma+\pi_t))\left(r^{*b}_t - \gamma\right)}$

## AppendixB Details of sensitivity testing

### $ARMA$(1,2)

| Model | Estimated | 95% Confidence Interval | | IN/OUT |
|---|---|---|---|---|
| | | Lower | Upper | |
| $AR(1)$ | 0.270202 | -0.858763824 | 0.861037594 | IN |
| $MA(1)$ | -0.295610 | -1.136112142 | 1.114923008 | IN |
| $MA(2)$ | -0.309869 | -0.479108628 | 0.663672466 | IN |
| Wald Statistic | 21.9% | | | |
| Full Wald Statistic | 24.1% | | | |

Table 10: Confidence Limits of change in inflation process for Theoretical ARMA(1,2)

### AppendixB.1 $ARMA$(1,3)

| Model | Estimated | 95% Confidence Interval | | IN/OUT |
|---|---|---|---|---|
| | | Lower | Upper | |
| $AR(1)$ | -0.402732 | -0.860083008 | 0.8816791 | IN |
| $MA(1)$ | 0.664334 | -1.240843978 | 1.163382205 | IN |
| $MA(2)$ | -0.350723 | -0.531607378 | 0.58973047 | IN |
| $MA(3)$ | -0.790819 | -0.735628162 | 0.708650063 | OUT |
| Wald statistic | 76.8% | | | |
| Full Wald Statistic | 70.2% | | | |

Table 11: Confidence Limits of change in inflation process for Theoretical ARMA(1,3)

### AppendixB.2 $ARMA$(2,0)

| Model | Estimated | 95% Confidence Interval | | IN/OUT |
|---|---|---|---|---|
| | | Lower | Upper | |
| $AR(1)$ | 0.056195 | -0.308134384 | 0.290690332 | IN |
| $AR(2)$ | -0.360412 | -0.315593256 | 0.252309832 | OUT |
| Wald statistic | 85.7% | | | |
| Full Wald Statistic | 79.9% | | | |

Table 12: Confidence Limits of change in inflation process for Theoretical ARMA(2,0)

## AppendixB.3 $ARMA$(2,1)

| Model | Estimated | 95% Confidence Interval | | IN/OUT |
|---|---|---|---|---|
| | | Lower | Upper | |
| $AR(1)$ | 0.381779 | -0.924444673 | 0.930405293 | IN |
| $AR(2)$ | -0.366740 | -0.366386998 | 0.283308834 | OUT |
| $MA(1)$ | -0.392966 | -0.997225582 | 0.997265422 | IN |
| Wald statistic | 67.5% | | | |
| Full Wald Statistic | 61.0% | | | |

Table 13: Confidence Limits of change in inflation process for Theoretical ARMA(2,1)

## AppendixB.4 $ARMA$(2,2)

| Model | Estimated | 95% Confidence Interval | | IN/OUT |
|---|---|---|---|---|
| | | Lower | Upper | |
| $AR(1)$ | 0.420718 | -1.101367406 | 1.03334009 | IN |
| $AR(2)$ | -0.463388 | -0.889872191 | 0.645730814 | IN |
| $MA(1)$ | -0.441163 | -1.295641054 | 1.301890517 | IN |
| $MA(2)$ | 0.115496 | -0.946594579 | 1.073720933 | IN |
| Wald statistic | 45.2% | | | |
| Full Wald Statistic | 38.0% | | | |

Table 14: Confidence Limits of change in inflation process for Theoretical ARMA(2,2)

## AppendixB.5 $ARMA$(2,3)

| Model | Estimated | 95% Confidence Interval | | IN/OUT |
|---|---|---|---|---|
| | | Lower | Upper | |
| $AR(1)$ | -0.666351 | -1.083722463 | 1.036968299 | IN |
| $AR(2)$ | -0.312067 | -0.865038378 | 0.676991618 | IN |
| $MA(1)$ | 0.858843 | -1.371771397 | 1.31779472 | IN |
| $MA(2)$ | -0.019530 | -0.95265246 | 1.444663446 | IN |
| $MA(3)$ | -0.597234 | -0.713123817 | 0.825288566 | IN |
| Wald statistic | 58.7% | | | |
| Full Wald Statistic | 56.9% | | | |

Table 15: Confidence Limits of change in inflation process for Theoretical ARMA(2,3)

## AppendixB.6 $ARMA$(3,0)

| Model | Estimated | 95% Confidence Interval | | IN/OUT |
|---|---|---|---|---|
| | | Lower | Upper | |
| $AR(1)$ | -0.003204 | -0.322151176 | 0.30682051 | IN |
| $AR(2)$ | -0.373691 | -0.326917714 | 0.257039678 | OUT |
| $AR(3)$ | -0.160067 | -0.304841952 | 0.294185268 | IN |
| Wald statistic | 80.2% | | | |
| Full Wald Statistic | 75.2% | | | |

Table 16: Confidence Limits of change in inflation process for Theoretical ARMA(3,0)

## AppendixB.7 $ARMA$(3,1)

| Model | Estimated | 95% Confidence Interval | | IN/OUT |
|---|---|---|---|---|
| | | Lower | Upper | |
| $AR(1)$ | -0.834652 | -0.911803742 | 0.933983878 | IN |
| $AR(2)$ | -0.333377 | -0.373356324 | 0.307841065 | IN |
| $AR(3)$ | -0.457973 | -0.347216206 | 0.343368036 | OUT |
| $MA(1)$ | 0.952026 | -0.997458489 | 0.997303253 | IN |
| Wald statistic | 94.3% | | | |
| Full Wald Statistic | 93.2% | | | |

Table 17: Confidence Limits of change in inflation process for Theoretical ARMA(3,1)

## AppendixB.8 $ARMA$(3,2)

| Model | Estimated | 95% Confidence Interval | | IN/OUT |
|---|---|---|---|---|
| | | Lower | Upper | |
| $AR(1)$ | -0.955016 | -1.006884812 | 1.089191207 | IN |
| $AR(2)$ | -0.432063 | -0.911678749 | 0.681650563 | IN |
| $AR(3)$ | -0.449312 | -0.366689124 | 0.375360547 | OUT |
| $MA(1)$ | 1.094471 | -1.460242841 | 1.292129158 | IN |
| $MA(2)$ | 0.130641 | -0.959008854 | 0.994998227 | IN |
| Wald Statistic | 84.3% | | | |
| Full Wald Statistic | 82.0% | | | |

Table 18: Confidence Limits of change in inflation process for Theoretical ARMA(3,2)

## AppendixB.9 $ARMA$(3,3)

| Model | Estimated | 95% Confidence Interval | | IN/OUT |
|---|---|---|---|---|
| | | Lower | Upper | |
| $AR(1)$ | -0.598601 | -0.774288545 | 0.86512598 | IN |
| $AR(2)$ | -0.482306 | -0.820791168 | 0.60810845 | IN |
| $AR(3)$ | 0.377436 | -0.733969467 | 0.726042319 | IN |
| $MA(1)$ | 1.564748 | -1.195294204 | 1.07562284 | OUT |
| $MA(2)$ | 0.737548 | -0.904225756 | 1.236254728 | IN |
| $MA(3)$ | -1.813709 | -0.968163297 | 0.982911134 | OUT |
| Wald statistic | 95.1% | | | |
| Full Wald Statistic | 94.2% | | | |

Table 19: Confidence Limits of change in inflation process for Theoretical ARMA(3,3)