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ABSTRACT

Is There a Fiscal Free Lunch in a Liquidity Trap?*

This paper uses a DSGE model to examine the effects of an expansion in government spending in a liquidity trap. The spending multiplier can be much larger than in the normal situation if the liquidity trap is very prolonged, and the budgetary costs minimal. But given this "fiscal free lunch," it is unclear why policymakers would want to limit the size of fiscal expansion. Our paper addresses this question in a model environment where the duration of the liquidity trap is determined endogenously, and depends on the size of the fiscal stimulus. We show that even if the multiplier is high for small increases in government spending, it may decrease substantially at higher spending levels; thus, it is crucial to distinguish between the average and marginal multiplier.

JEL Classification: E52 and E58
Keywords: DSGE model, fiscal policy, liquidity trap, monetary policy and zero bound constraint

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1. Introduction

During the past two decades, a voluminous empirical literature has attempted to gauge the effects of fiscal policy shocks. This literature has been instrumental in identifying the channels through which fiscal policy affects the economy, and, in principle, would seem a natural guidepost for policymakers seeking to assess how alternative fiscal policy actions could mitigate the current global recession.

However, it is unclear whether estimates of the effects of fiscal policy from this empirical literature – which focuses almost exclusively on the postwar period – should be regarded as applicable under conditions of a recession-induced liquidity trap.[1] Keynes (1933, 1936) argued in support of aggressive fiscal expansion during the Great Depression on the grounds that the fiscal multiplier was likely to be much larger during a severe economic downturn than in normal times, and the burden of financing it correspondingly lighter. His logic underlying a larger multiplier in a liquidity trap was formalized in subsequent IS-LM analysis, with a liquidity trap corresponding to a flat LM curve.

In this paper, we use a New-Keynesian DSGE modeling framework to examine the implications of an increase in government spending for output and the government budget when monetary policy faces a liquidity trap. A key advantage of the DSGE framework is that it allows explicit consideration of how the conduct of monetary policy – and, in particular, the zero bound constraint on nominal interest rates – affects the multiplier.

We begin by showing that the government spending multiplier can be amplified substantially in the presence of a prolonged liquidity trap. This corroborates previous analysis by Eggertson (2008) and Davig and Leeper (2009), which shows that government spending can have outsized effects when monetary policy reacts passively by allowing real interest rates to fall, and recent work by Christiano, Eichenbaum and Rebelo (2009) in a model with endogenous capital accumulation. While our baseline model is a variant of the Smets-Wouters (2007) model, we show that the spending multiplier is even larger in versions that embed hand-to-mouth agents (as in Galí,

[1] The bulk of research suggests a government spending multiplier in the range of 0.5 to slightly above unity. One strand of the literature – originating with Barro (1981, 1990) – has estimated the multiplier by examining the response of output to changes in military spending. This approach tends to yield multipliers in the range of 0.5-0.8, including in more recent work by Ramey (2009) and Hall (2009); however, as emphasized by Hall, the estimates hinge critically on the relationship between output and spending during WWII and the Korean War, and may be somewhat downward-biased due to the "command-economy" features prevalent in WWII, and because taxes were raised markedly during the Korean War. An alternative approach involves identifying the government spending multiplier using a structural VAR – as in Blanchard and Perotti (2002), and Gali, Lopez-Salido, and Valles (2007). These studies report a government spending multiplier of unity or somewhat higher (after 1-2 years), though the cross-county evidence of Perotti (2007) and Mountford and Uhlig (2008) is suggestive of a lower multiplier.

López-Salido, and Vallés 2007) and financial frictions (as in Bernanke, Gertler, and Gilchrist 1999, and Christiano, Motto, and Rostagno 2007). Moreover, an increase in government spending against the backdrop of a deep liquidity trap exerts much less upward pressure on public debt than under normal circumstances, reflecting that the larger output response translates into much higher tax revenues.

At first blush, these results seem highly supportive of Keynes' argument for fiscal expansion in response to a recession-induced liquidity trap – the benefits are extremely high, and the budgetary expense to achieve it very low. But this raises the important question of why policymakers would want to limit the magnitude of fiscal expansion, and thus pass up on what appears to be a "fiscal free lunch."

Our paper addresses this question by showing that the spending multiplier in a liquidity trap decreases with the level of government spending. The novel feature of our approach to allow the economy's exit from a liquidity trap – and return to conventional monetary policy – to be determined *endogenously,* with the consequence that the multiplier depends on the size of the fiscal response. Quite intuitively, a large fiscal response pushes the economy out of a recession-induced liquidity trap more quickly. Because the multiplier is smaller upon exiting the liquidity trap – reflecting that monetary policy reacts by raising real interest rates – the marginal impact of a given-sized increase in government spending on output decreases with the magnitude of the spending hike. This dependence of the government spending multiplier on the scale of fiscal expansion evidently contrasts to a standard linear framework in which the multiplier is invariant to the size of the spending shock.

The qualitative implication that the multiplier declines in the level of spending suggests an important rationale for limiting the size of fiscal spending packages in a liquidity trap: in particular, even if the multiplier is high for small increments to government spending, it may be relatively low at higher spending levels. Clearly, it seems crucial to characterize the behavior of the *marginal* spending multiplier to make informed judgements about the appropriate scale of fiscal intervention in a liquidity trap. Accordingly, a major focus of our paper consists of a quantitative characterization of how the government spending multiplier varies with the level of spending in an array of nested DSGE models.

Section 2 analyzes the effects of government spending shocks in a simple three equation New Keynesian model in which policy rates are constrained by the zero lower bound. Similar to previous research (e.g., Eggertson 2008), the liquidity trap is generated by an adverse taste shock that sharply depresses the potential real interest rate. A key result of our analysis is that the government spending multiplier – measured as the contemporaneous impact on output of a very small increment in government spending – is a step function in the level of government spending. If the level of spending is sufficiently small, higher government spending does not affect the economy's exit date from the liquidity trap, and the multiplier is constant at a value that is higher than in a normal situation in which monetary policy would raise real interest rates. However, as spending rises to higher levels, the economy emerges from the liquidity trap more quickly, and the marginal multiplier drops. The multiplier continues to drop discretely as government spending rises further – reflecting a progressive shortening of the liquidity trap – until spending is high enough to keep the economy from falling into a liquidity trap. Beyond this level of spending, the marginal multiplier levels out at a value equal to that under normal conditions in which policy rates are unconstrained.

The simple New Keynesian model is a very convenient tool for illustrating the salient role of inflation expectations in determining the marginal multiplier. If prices are fairly responsive to marginal cost – as implied by relatively short-lived price contracts – the multiplier is extremely high for small increments to government spending, but drops quickly at higher spending levels. Thus, the large multipliers that apply to small fiscal expansion should not be inferred to carry over to much larger fiscal expansions, and it is crucial to take account of the endogeneity of the multiplier precisely under those conditions in which the marginal multiplier is very high. By contrast, the multiplier function is much flatter under a flatter Phillips Curve slope, and even at low spending levels isn't dramatically different than in normal times.

The simple model is also convenient tool for assessing other empirically relevant factors that may affect the multiplier, including implementation lags in spending. We show that implementation lags may dampen the multiplier significantly under some circumstances, and may even cause it to be negative against the backdrop of a long-lived liquidity trap. Thus, echoing Friedman (1953), the efficacy of fiscal policy in macroeconomic stabilization – even in a liquidity trap – can be hampered by "long and variable lags."

These considerations garnered from the stylized model prove useful in interpreting the behavior of the government spending multiplier in more empirically-realistic models. In Section 3, we investigate a model that is very similar to the estimated models of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). Given the relevance of initial conditions that determine the duration and depth of the liquidity trap for the spending multiplier, we analyze the multiplier against the backdrop of a "severe recession scenario" that attempts to capture some of the features of the recession-induced liquidity trap experienced by the United States and other countries during the recent financial crisis. This scenario is constructed by a sequence of adverse consumption demand shocks that depress output by 10 percent relative to baseline, and that generates a liquidity trap of 8 quarters. The choice of an 8 quarter liquidity trap is motivated by the implied paths for policy rates derived from options data.

Under our baseline scenario, the government spending multiplier – associated with a one percentage point of GDP spending boost – is close to unity after four quarters. The peak multiplier is roughly twice as large as under normal conditions in which zero bound considerations do not affect policy. The larger multiplier translates into a smaller rise in government debt. Against the backdrop of an even deeper recession in which the liquidity trap would last 12 quarters, the government spending multiplier exceeds 2, and causes government debt to decline.

Our benchmark calibration implies a fairly flat Phillips Curve by imposing price and wage contract durations at the higher end of empirical estimates: for example, the effective duration of price contracts is ten quarters.[2] Given that the amplification of the multiplier in a liquidity trap depends crucially on the expected inflation response, the multiplier can be much higher for shorter contract durations. But under the latter conditions, the marginal multiplier declines very abruptly; for example, with four quarter contracts the marginal multiplier is nearly 10 for a very small increase in government spending, but drops to 1.3 when government spending is boosted more than 0.3 percent of baseline GDP. Thus, the marginal multiplier for spending levels of 1-2 percent of GDP is only modestly higher even for much shorter-lived contracts than under our benchmark calibration.

Another key insight from the model with both sticky prices and wages, is that both wages and prices need to be responsive to changes in government spending in order for expansions in government spending to be associated with high fiscal multipliers. If wages are sufficiently sticky, then the fiscal multiplier will not be much enhanced by a higher slope of the Phillips curve because marginal costs will only be moderately affected by the fiscal stimulus in the first place. So in more empirically realistic models, we need both wages and prices to be responsive to changes in government spending in order for expected inflation to move substantially.

[2] Throughout the paper, we will for simplicity map a given slope of the Phillips curve into average contract duration under the common assumption that marginal costs are identical across firms. However, empirical work by e.g. Galí, Gertler, and López-Salido (2001) and Altig, Christiano, Eichenbaum and Lindé (2005) has shown that if e.g. capital is instead assumed to be specific to each firm (which is surely more plausible from an empirical viewpoint), so that marginal costs are increasing with the level of production, then a given slope of the Phillips curve can be consistent with a considerably lower degree of price stickiness under plausible assumptions about capital utilization costs and the elasticity of firm demand. Consequently, even the longest contract durations we consider in this paper do not imply a lower slope of the Phillips than commonly estimated on US data.

More generally, we are skeptical about the possibility of an extremely high multiplier even for small increments to government spending. In particular, shorter-lived contract durations imply that persistently negative output gaps generate enormous downward pressure on inflation. But a salient feature of the recent worldwide recession is that core inflation - and expectations of inflation – have remained resilient in response to the large negative output gaps that have emerged in both the United States and other industrial countries.

Section 4 analyzes an augmented model that also incorporates rule-of-thumb households and financial frictions. The inclusion of rule-of-thumb agents is appealing insofar as Galí, López-Salido, and Vallés (2007) have shown that it can account for the positive response of private consumption to a government spending shock documented in structural VAR studies such as by Blanchard and Perotti (2002) and Perotti (2007).[3] Given the boost in private consumption, this model implies a peak multiplier above unity even under normal conditions in which real policy rates respond. In a liquidity trap, the stimulus to private demand coming from the rule-of-thumb agents is augmented further because real rates decline. The multiplier – in a liquidity trap and in normal times – is quite sensitive to the assumed share of rule-of-thumb households in the economy. With a 50 percent share, the multiplier rises to 2 when the liquidity trap lasts 8 quarters, though this share would seem at the upper end of a reasonable range. Even so, the marginal multiplier is only 1.3 for spending increases beyond 1 percent of GDP. The multiplier is somewhat lower under plausible assumptions about implementation lags, and when government spending must be financed by a distortionary labor tax.[4]

Taken together, our results suggest a somewhat nuanced view of the role of fiscal policy in a liquidity trap. For an economy facing a protracted recession and for which monetary policy seems likely to be constrained by the zero bound for a very prolonged period – roughly 2 years or more – there is a strong argument for increasing government spending on a temporary basis. Consistent with the views originally espoused by Keynes, this temporary boost can have much larger effects than under usual conditions, and comes at relatively low cost to the Treasury. But even under such conditions, it is important to focus on components of spending that can be increased fairly

[3] As discussed by the recent paper by Leeper, Walker and Yang (2009) and in Ramey (2009), identified VARs can produce misleading results if some of the fiscal expansion is anticipated. Accordingly, Fisher and Ryan (2009) identify government spending shocks with statistical innovations to the accumulated excess returns of large U.S. military contractors, and find that positive spending shocks are associated with an output multiplier above unity and increases in hours and consumption.

[4] Uhlig (2009) emphasizes that highly persistent (or permanent) increases in the level of government spending tend depress output significantly at horizons beyond a couple of years if the higher spending must be financed by a hike in the labor income tax rate. The short-run multiplier is also damped significantly if the labor income tax rate responds rapidly to government debt.

quickly, e.g., front-loading military equipment purchases; the multiplier may not be especially large if implementation lags are substantial, and can even be negative. For shorter-lived liquidity traps of less than two years, the multiplier is larger than under 'normal conditions' for small increases in spending, but drops relatively quickly at higher spending levels. Thus, larger spending programs may suffer from sharply diminishing returns, and may boost government debt significantly.

2. A stylized New Keynesian model

As in Eggertsson and Woodford (2003), we begin by analyzing the effects of fiscal shocks in a standard log-linearized version of the New Keynesian model that imposes a zero bound constraint on interest rates. We use this model to identify key factors that affect the size of the government spending multiplier. Our framework allows exit from the liquidity trap to be determined endogenously, rather than fixed arbitrarily, an innovation that is crucial in showing how the multiplier varies with the level of fiscal spending.

2.1. The Model

The key equations of the model are:

$$x_t = x_{t+1|t} - \hat{\sigma}(i_t - \pi_{t+1|t} - r_t^{pot}), \tag{1}$$

$$\pi_t = \beta\pi_{t+1|t} + \kappa_p x_t, \tag{2}$$

$$i_t = \max\left(-i, \gamma_\pi \pi_t + \gamma_x x_t\right), \tag{3}$$

$$r_t^{pot} = \frac{1}{\hat{\sigma}}\left(1 - \frac{1}{\phi_{mc}}\right)\left[g_y(g_t - g_{t+1}) + (1-g_y)\nu_c(\nu_t - \nu_{t+1})\right] \tag{4}$$

where $\hat{\sigma}$, κ_p, and ϕ_{mc} are composite parameters defined as:

$$\hat{\sigma} = \sigma(1-g_y)(1-\nu_c) \tag{5}$$

$$\kappa_p = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}\phi_{mc} \tag{6}$$

$$\phi_{mc} = \frac{\chi}{1-\alpha} + \frac{1}{\hat{\sigma}} + \frac{\alpha}{1-\alpha} \tag{7}$$

and where x_t is the output gap, π_t is the inflation rate, i_t is the short-term nominal interest rate, and r_t^{pot} is the potential (or "natural") real interest rate. All variables are measured as percent or percentage point deviations from their steady state level.[5]

Equation (1) parsimoniously expresses the IS curve in terms of the output and real interest rate gaps. Thus, the output gap x_t depends inversely on the deviation of the real interest rate $(i_t - \pi_{t+1|t})$ from its potential rate r_t^{pot}, as well as directly on the expected output gap in the following period. The parameter $\hat{\sigma}$ determines the sensitivity of the output gap to the real interest rate; as indicated by (5), it depends on the household's intertemporal elasticity of substitution in consumption σ, the steady state government spending share of output g_y, and a (small) adjustment factor which scales the consumption taste shock ν_c. The price-setting equation (2) specifies current inflation to depend on expected inflation and the output gap, where the sensitivity to the latter is determined by the composite parameter κ_p. Given the Calvo-Yun contract structure, equation (6) implies that κ_p varies directly with the sensitivity of marginal cost to the output gap ϕ_{mc}, and inversely with the mean contract duration $(\frac{1}{1-\xi_p})$. The marginal cost sensitivity equals the sum of the absolute value of the slopes of the labor supply and labor demand schedules that would prevail under flexible prices: accordingly, as seen in equation (7), ϕ_{mc} varies inversely with the Frisch elasticity of labor supply $\frac{1}{\chi}$, the composite parameter $\hat{\sigma}$ determining the interest-sensitivity of aggregate demand, and the labor share in production $(1 - \alpha)$.

Equation (4) indicates that the potential real interest rate is driven by two (exogenous) shocks, including a consumption taste shock ν_t and government spending shock g_t. Abstracting from possible differences in stochastic structure, these shocks affect the potential real interest rate in an identical manner, reflecting that each shock (if positive) raises the marginal utility of consumption associated with any given output level. This can easily be seen by expressing the marginal utility of consumption λ_{ct} in log-linearized form:

$$\lambda_{ct} = -\frac{1}{\hat{\sigma}} c_t + \frac{\nu_c(1-g_y)}{\hat{\sigma}} \nu_t = \frac{1}{\hat{\sigma}} \left[\frac{(g_y g_t - y_t)}{1-g_y} + \nu_c(1-g_y)\nu_t \right] \tag{8}$$

where c_t is consumption, y_t output, and g_t government spending, and the expression following the second equality uses the resource constraint to solve out for consumption. Given that the potential real interest rate depends inversely on the expected growth rate of λ_{ct}, it also varies inversely with the expected growth rate of ν_t and g_t.

[5] We use the notation $y_{t+j|t}$ to denote the conditional expectation of a variable y at period $t + j$ based on information available at t, i.e., $y_{t+j|t} = \mathrm{E}_t y_{t+j}$.

While we assume that the consumption taste shock ν_t follows a simple AR(1) process:

$$\nu_t = (1 - \rho_\nu)\nu_{t-1} + \varepsilon_{\nu,t}, \tag{9}$$

we allow the government spending shock to follow an AR(2) to capture the possibility of implementation lags in spending. It is convenient to express g_t in "error-correction" form as:

$$g_t - g_{t-1} = \rho_{g_1}(g_{t-1} - g_{t-2}) - \rho_{g_2} g_{t-1} + \varepsilon_{g,t}, \tag{10}$$

The log-linearized equation for the stock of government debt is given by:

$$b_t = (1 + r)b_{t-1} + g_y g_t - \tau_L(l_t + \zeta_t) - \tau_t, \tag{11}$$

where b_t is end-of-period government debt (as a share of baseline GDP), l_t is labor hours, ζ_t is the real wage, and τ_t is a lump-sum tax (as a share of baseline GDP). The government derives tax revenue from a fixed tax on labor income τ_L, and from the time-varying lump-sum tax τ_t. The tax rate τ_L is set so that government spending is financed exclusively by the distortionary labor tax in the steady state (with the government debt stock zero in steady state). Lump-sum taxes adjust according to the reaction function:

$$\tau_t = \varphi_\tau \tau_{t-1} + \varphi_b b_{t-1}, \tag{12}$$

Given that agents are Ricardian and that only lump-sum taxes adjust dynamically, the fiscal rule only affects the evolution of the stock of debt and lump-sum taxes (with no effect on other macro variables). In the larger model considered in Sections 4-6, we will consider the implications of rules in which distortionary taxes adjust dynamically, in which case the fiscal rule can have significant effects on output, inflation, and interest rates.

2.2. Effects of a Front-Loaded Rise in Government Spending

We now use this simple model to analyze the effects of discretionary changes in government spending against the backdrop of a recession-induced liquidity trap. A liquidity trap is interpreted as a situation in which monetary policy would like to reduce interest rates further, but is unable to do so because of the zero bound constraint.

Our benchmark calibration is quite standard. We set the discount factor $\beta = 0.995$, and the steady state (net) inflation $\pi = .005$; this implies a steady state interest rate of $i = .01$ at a quarterly rate (or four percent at an annualized rate). We set the intertemporal substitution elasticity $\sigma = 1$ (i.e. assume logarithmic period utility function), the capital share parameter $\alpha = 0.3$, the inverse Frisch elasticity of labor supply $\chi = 2.5$, the government share of steady state output $g_y = 0.2$, and the scale parameter on the consumption taste shock $\nu_c = 0.01$. We set $\xi_p = 0.80$, which is consistent with an effective price contract duration of five quarters. But we conduct extensive sensitivity analysis on this parameter, consistent with our goal of showing how estimates of the fiscal multiplier hinge crucially on the slope of the Phillips Curve. To foreshadow our results, certain implications of our analysis – together with recent empirical evidence – point to considerably longer contract durations as more plausible.[6]

The effects of fiscal policy in a liquidity trap depend crucially on initial conditions, which in our framework means the shock(s) that causes the economy to enter a liquidity trap. We assume that the liquidity trap is generated by an adverse taste shock v_t that sharply depresses the potential real interest rate, r_t^{pot}, as seen in Figure 1. The shock is calibrated so that r_t^{pot} has a persistence of 0.9 ($\rho_\nu = 0.1$), and drops about 9 percent relative to its steady state level.

This shock pushes the economy into a severe recession, and causes the nominal interest rate to remain at the lower bound of zero for a prolonged period. To highlight the channels which account for these effects, it is convenient to begin by assuming that monetary policy would completely stabilize output and inflation in the absence of a zero bound constraint (although we relax this assumption below). Such a policy is achieved by setting the coefficients in the monetary reaction function (γ_π and/or γ_x) to be arbitrarily large. Under this policy, the nominal interest rate i_t simply tracks r_t^{pot} provided that the implied nominal rate is non-negative (i.e., $i_t = r_t^{pot}$, recalling that both variables are measured as percentage point deviations from baseline); moreover, output equals potential, and inflation remains at its target level. The concurrence of the nominal and potential real interest rate is apparent in Figure 1 for all periods subsequent to period T_n, which is the first period in which r_t^{pot} exceeds $-i = -1$ percent (the figure shows the annualized interest rate, so -4 percent). However, because $r_t^{pot} < -i$ prior to T_n, equations (1)-(3) imply that the nominal interest rate must equal its lower bound of $-i$. The shock is scaled so that the liquidity trap lasts for $T_n = 8$ quarters under our benchmark calibration.

To assess the implications of the taste shock v_t for output and inflation, it is useful to solve the

[6] For instance, the estimation results in Altig, Christiano, Eichenbaum and Lindé (2005) and Smets and Wouters (2003) imply that prices are reoptimized every 10 quarters under the assumption of homogenous capital markets. Smets and Wouters (2007) obtain a higher slope of the Phillips Curve using U.S. data, but their estimate depends on allowing markup shocks to follow a highly correlated ARMA(1,1) process. Adolfson et al. (2005) show that the estimated slope of the Phillips curve is strongly dependent on the assumed process of the markup shocks, and that a specification with white noise markup shocks and a lower slope of the Phillips Curve better fits the data. This finding is supported in a revised version of the ACEL paper, see Altig et al. (2009).

IS curve forward as:

$$x_t = -\hat{\sigma} \sum_{j=0}^{T_n - 1} (-i - r^{pot}_{t+j|t}) + \hat{\sigma} \sum_{j=1}^{T_n} \pi_{t+j|t} + x_{T_n|t}, \tag{13}$$

The output gap at any date $t < T_n$ depends on three terms. First, it depends on the *cumulative* gap between the nominal interest rate $-i$ and the potential real interest rate over the interval in which the economy remains in a liquidity trap (with a sensitivity given by $\hat{\sigma}$). Second, it depends on cumulative expected inflation over the liquidity trap (or equivalently, the log change in the price level $log(P_{Tn}) - log(P_t)$). Finally, it depends on the output gap x_{T_n} when the economy exits the liquidity trap; this term drops under the assumption that monetary policy completely stabilizes the economy ($x_{T_n} = \pi_{Tn} = 0$).

The first term of (13) is proportional to the cumulative "interest rate gap" $\sum_{j=0}^{T_n-1}(-i - r^{pot}_{t+j})$ and can be interpreted as indicating how shocks to the potential real interest rate would affect the output gap *if expected inflation remained constant.* Referring to Figure 1, it is essentially the "area" between the nominal interest rate and potential real interest rate over the period in which the economy remains in a liquidity trap. The second term of (13) captures how the effects on the output gap of shocks to the potential real rate are amplified through induced changes in expected inflation ($\sum_{j=1}^{T_n} \pi_{t+j}$).

This expected inflation channel can strongly reinforce the effects of movements in the potential real rate on the output gap. This can easily be seen by solving (1)-(2) forward to express inflation in terms of current and future interest rate gaps:

$$\pi_t = -\hat{\sigma}\kappa_p \sum_{j=0}^{T_n-1} \psi(j)(-i - r^{pot}_j), \tag{14}$$

The weighting function $\psi(j)$ is given by:

$$\psi(j) = \lambda_1 \psi(j-1) + \lambda_2^j, \tag{15}$$

with the initial condition $\psi(0) = 1$, and where λ_1 and λ_2 are determined as:

$$\lambda_1 + \lambda_2 = 1 + \beta + \hat{\sigma}\kappa_p, \tag{16}$$

$$\lambda_1 \lambda_2 = \beta, \tag{17}$$

Given that $\kappa_p > 0$, the coefficients $\psi(j)$ premultiplying the interest rate gap grow exponentially with the duration of the liquidity trap T_n. Moreover, the contour is extremely sensitive to κ_p, as illustrated in Figure 2 for several values of κ_p associated with price contraction durations ranging from four to ten quarters. An immediate implication is that even small interest rate gaps – if expected to be sufficiently persistent – can exert potentially large effects on expected inflation.

The dashed line in the upper panels of Figure 3 shows the effects of the taste shock on the potential real rate, which simply replicates Figure 1. The shock induces output to fall roughly 6 percent below potential. As suggested by the first term of equation (13), output would fall below potential even if expected inflation remained unchanged. In this case, although the real interest rate would decline in lockstep with the nominal interest rate (i.e., by $i = 4$ percent), it would not fall as much as the potential real interest rate, resulting in a negative output gap. But roughly 2/3 of the output gap in Figure 3 is attributable to the large decline in expected inflation that is evident in the figure (recalling that agents have perfect foresight, so that the path of expected inflation can be inferred from the path of realized inflation). The decline in expected inflation pushes up real ex ante interest rates sharply over the duration of the liquidity trap, and thus markedly amplifies the gap between the real interest rate and potential real interest rate relative to the case of unchanged inflation expectations.

The recession-induced liquidity trap generates a substantial rise in the government debt/GDP ratio. Government revenue attributable to the labor income tax falls drastically in response to lower labor demand, and an associated fall in wages. For our parameterization of the lump-sum tax rule, the debt stock remains high for a prolonged period, even after the economy recovers.

We next consider the effect of a rise in government spending against this backdrop. It is convenient to assume initially that the government spending shock follows the same AR(1) process with a persistence of 0.9 as the taste shock (so $\rho_{g2} = \rho_v = 0.1$, and $\rho_{g1} = 0$).

The solid lines in Figure 3 show the *combined* effects of a one percentage point of (steady state) GDP rise in government spending and the negative taste shock v_t. Given that the shock processes have the same persistence, higher government spending shifts upward the time path of the potential real interest rate r_t^{pot} proportionately, so that the net effect of the two shocks is shown by the solid line in the upper left panel. In addition to stimulating aggregate demand directly by raising r_t^{pot}, the higher government spending also raises expected inflation, and hence lowers ex ante real interest rates.

The impact government spending multiplier, shown by the solid line, is around 2.1. The multiplier is simply equal to the difference between the output gap responses (for both shocks vs. v_t alone) plus the response of potential output to the government spending shock alone. The latter is labeled "no liquidity trap" in the figure, reflecting that this potential output response is equivalent to the government spending multiplier when the economy is not in a liquidity trap (under the assumed monetary policy rule). Clearly, the spending multiplier is several times larger in the liquidity trap. The outsized government spending multiplier in the liquidity trap induces a large rise in labor income (which rises even more than output as the higher spending boosts marginal costs). As a consequence, government revenue from labor taxes rise enough that the government debt/GDP actually declines relative to its path in the case of no fiscal stimulus.

One key factor explaining the large multiplier is that higher government spending boosts expected inflation substantially relative to its path in the absence of fiscal stimulus. This reduces real interest rates sharply, which induces a "crowding in" rather than the familiar crowding out of private consumption. The importance of expected inflation in amplifying the effects of the government spending shock is anticipated in our discussion of the effects of the taste shock. In that case, we noted that the output decline due to a given-sized fall in r_t^{pot} would be much smaller if expected inflation remained unchanged. Given that higher government spending simply reverses some of the decline in r_t^{pot} due to the taste shock, a symmetric argument implies that the government spending multiplier should be much lower when expected inflation is less responsive to variations in the potential real rate.

The sensitivity of the government spending multiplier to the slope of the Phillips Curve – a key determinant of expected inflation – can be inferred from Figure 4. This figure repeats the simulations shown in Figure 3 except for imposing a much higher mean contract duration of 10 quarters (rather than 5 quarters as in Figure 3). The lower Phillips Curve slope vastly reduces the contraction in the output gap in response to the taste shock alone (from -9 percent in Figure 3 to -4.5 percent), and correspondingly, implies a much smaller positive impact of the government spending shock (with the multiplier falling from above 2 to less than unity). Given that the potential real rate is unchanged across calibrations, it is clear from the expression for the output gap given by equation (13) that the difference in the output responses in the two figures is attributable exclusively to differences in the behavior of expected inflation. In particular, referring to equation (14) and Figure 2, the longer contract duration greatly reduces the sensitivity of inflation to the path of r_t^{pot} over the duration of the liquidity trap.

A second key factor accounting for the large multiplier is the duration of the liquidity trap of 8 quarters. Again referring to equation (13), a deep recession that lengthens the duration of the liquidity trap has a larger negative effect on output in part because it increases the cumulative interest rate gap. This is reinforced because a longer trap amplifies the contractionary effect on expected inflation, as expectations of future weakness in inflation and output snowball into much larger near-term effects. Insofar as higher government spending reverses some of this effect on the potential real rate, the multiplier is larger when the trap lasts longer.

The dependence of the government spending multiplier on the duration of the liquidity trap is illustrated in Figure 5. The solid line shows the effects of the "immediate" implementation case just considered above in which the new government spending is timed to coincide exactly with the emergence of the taste shock. The dashed line shows a case in which the new government spending is announced four quarters after the occurrence of the taste shock, while the dash-dotted line shows the case of a six quarter delay. One interpretation of the latter two cases is that the fiscal response to the adverse shock takes some time to materialize, and when it does, some component is unanticipated; under this interpretation, the plots show the effect of the innovation in spending (equal to one percentage point of baseline GDP). But insofar as there is no endogenous persistence in the model, a second interpretation of the latter cases is that they show the effects of an immediate rise in government spending against the backdrop of a less severe liquidity trap that lasts 4 and 2 quarters, respectively. Thus, the case with four quarter delay yields exactly the same effects as if government spending responded immediately to a negative preference shock that was scaled to reduce r_t^{pot} by the value it assumes in period 4 in Figure 1.

The government spending multiplier falls from about 2.1 when the liquidity trap lasts 8 quarters to around 1/2 when the trap lasts only four quarters (i.e., a four quarter delay). With a less persistent liquidity trap, expected inflation falls by less in response to the (smaller) adverse taste shock; and correspondingly – as shown in Figure 5 – expected inflation rises much less in response to the fiscal stimulus. This translates into a smaller fall in short-term real interest rates during the period in which the economy remains in a liquidity trap, while real rates rise more at longer horizons (because government spending is considerably higher when the economy leaves the trap). The multiplier is even smaller in the case in which the liquidity trap lasts only 2 quarters.[7]

A third key factor influencing the fiscal multiplier is the size of the government spending shock. We have thus far considered how the effects of a given-sized rise in government spending on output vary with "initial conditions," which in our model are summarized by the path of the potential real interest rate. But the size of the shock is also highly relevant, as policymakers must assess the efficacy of fiscal actions of varying magnitude against the backdrop of a given set of initial

[7] These results are helpful in understanding why Cogan et al. (2009) find a fairly small multiplier – as most of their experiments fix the policy rate for only four quarters – while others such as Eggertson (2008) and Christiano, Eichenbaum, and Rebelo (2009) suggest a much higher multiplier.

conditions such as those illustrated in Figure 1.

A distinction between the average and marginal multiplier arises because the level of government spending may influence the potential real interest rate enough to affect when the economy exits the liquidity trap. The one percentage point rise in government spending considered above was small enough to leave the duration of the liquidity trap unaffected at 8 quarters. However, given that higher government spending raises the potential real interest rate in a linear fashion, the exit date from the liquidity trap varies inversely with the size of the stimulus plan. An important feature of our solution procedure is that it allows the exit from the liquidity trap to be endogenously determined.

Figure 6 plots the marginal fiscal multiplier associated with different levels of government spending. Under our benchmark calibration with 5 quarter price contracts – the solid line – the multiplier associated with government spending shocks ranging from 0 to 1.2 percent of baseline GDP is slightly above 2.1, consistent with Figure 3. But for government spending above 1.2 percent, the multiplier drops to about 1.6, and drops further for higher levels of spending. As spending rises to higher levels, the economy exits the liquidity trap more quickly, so that additional government spending puts more upward pressure on real interest rates. For a large enough level of spending, the economy never enters into a liquidity trap at all, and the multiplier is simply equal to its value under usual conditions (the 10 percent rise in government spending shown in Figure 6 has a marginal multiplier of about 0.4 and is associated with a liquidity trap of only 2 quarter duration as indicated by the ticks on the upper top axis of each of the panels which should be read from right to left). The figure also shows the marginal multiplier for alternative price contract durations of 10 quarters and 4 quarters. Notably, while the marginal multiplier can be very high in the case of 4 quarter contracts for low levels of spending, it drops precipitously for higher spending levels. In particular this is true if the underlying duration of the liquidity trap is as long as 12 quarters, as in this case the marginal spending multiplier can be as high as 15 in the case of 4 quarter contracts.[8]

The lower panel of Figure 6 shows the implications for government debt. Low levels of government spending are associated with a very high multiplier, and can even precipitate a fall in the stock of government debt (as seen in Figure 3). However, because the marginal multiplier drops quickly with higher levels of spending, government debt eventually starts to rise, although not in Figure 6 because the marginal impact of government debt is still negative when the duration of the

[8] Notice that we can think about the marginal multipliers depicted in Figure 6 for negative spending levels as what would happen if government spending was increased on the margin in a liquidity trap with a longer duration without any fiscal stimulus.

liquidity trap exceeds 2 quarters as in the lower panel of Figure 6.

To gain further understanding why the marginal multipliers in Figure 6 decrease in a step-wise fashion, Figure 7 plots the nominal interest rate and the potential real interest rate for different sized increases in government spending (baseline - i.e. no increase, 0.5 and 5 percent increase). The vertical solid black lines show the interest rate gaps, i.e. the difference between the nominal interest rate and the potential real interest rate in eqs. (13) and (14). The response of the interest rate gap to government spending provides the key to understanding why the marginal multiplier is a step function. More specifically, it is clear from the upper panel of Figure 7 that for small increments in government spending, g_t, the derivatives of the interest rate gaps will not be affected unless the duration of the liquidity trap is affected. Consequently, the spending multiplier is constant for increases in g_t that are small enough so as not to affect the duration of the liquidity trap. But when the increase in g_t is large enough for the duration of the liquidity trap to be affected, then the marginal multiplier will be affected in a step-wise fashion as some of the interest rate gaps are excluded due to the earlier exit from the liquidity trap. This feature is visualized in the lower panel of Figure 7. In the lower level, we compute and depict the derivatives of the interest rate gaps for two different spending levels, 0.5 percent and 5 percent. The lower spending level of 0.5 percent of GDP does not cause the economy to exit earlier from the liquidity trap; hence, marginal increases in g_t affect interest rate gaps for all 8 quarters and the marginal multiplier is constant. However, a spending increase of 5 percent shortens the duration of the liquidity trap by 2 quarters, and the marginal multiplier drops. An additional increase g_t from 5 to, say, 5.1 percent, does not affect the interest rate gaps in periods 0-5 and the marginal multiplier again stays constant until the increase from 5 percent is large enough to trigger an exit from the liquidity trap after four quarters. This feature of the solution makes the marginal multiplier follow a step-function in a liquidity trap with endogenous exit. However, it is important to understand that the considerations above do not explain why the changes in the marginal multipliers depicted in Figure 6 are larger for longer liquidity trap durations. From the upper panel in Figure 6, we see that the marginal output multiplier drops from about 15 to below 10 when the duration of the liquidity trap changes from 12 to 11 quarters for 4 quarter contracts. The steps are much small for shorter liquidity trap durations. The intuition behind the larger steps are the effects stemming from large movements in expected inflation according to equation (14), i.e. the $\psi(j)$ weights plotted in Figure 2. The exponential evolution of these weights imply that the steps will be larger for longer lived liquidity traps.

2.3. Effects of Implementation Lags

We next consider the implications of lags between the announcement of higher fiscal spending and its implementation. In particular, we assume that the government announces a new stimulus plan immediately in response to the adverse preference shock, but that it takes some time for spending to peak. To capture such time delays, we assume that government spending follows an AR(2) as in (10) that implies some persistence in the growth rate of government spending (even though the level is stationary due to the error correction term).

The solid lines in Figure 8 show the effects of a phased-in rise in government spending that peaks after eight quarters (achieved by setting $\rho_{g1} = .90$ and $\rho_{g2} = 0.025$) against the backdrop of the same adverse preference shock considered previously (again depicted by the dashed lines). Given the implementation lag, the higher spending depresses r_t^{pot} over the entire period in which the economy is in the liquidity trap, while leaving the duration of the trap unchanged at 8 quarters. As seen by equation (4), the expectation that government spending will grow in the future depresses the potential real interest rate r_t^{pot} by encouraging saving. Interestingly, the multiplier is significantly *negative*, reflecting that aggregate demand is weaker over the entire period in which the economy is in the liquidity trap. Unsurprisingly, the negative multiplier is associated with a larger deterioration of the fiscal balance, and consequent boost in the government debt/GDP ratio.

These results show that implementation lags can potentially have quite substantial implications for the multiplier, possibly turning it negative if the delay is substantial enough. Even in the less extreme case – as for the six quarter lag – implementation lags may have very pronounced consequences for the marginal multiplier by pushing the economy out of the liquidity trap more quickly. Given that fiscal action is delayed to the point where the economy is already emerging from recession, the multiplier decreases very quickly as the additional spending is implemented.

Taken together, our results suggest that rather than consider a "spending" multiplier, it is important to analyze how the multiplier may differ for alternative types of government spending. Projects that can be implemented very quickly when the economy is deep in recession may have a very high marginal multiplier that declines slowly with the size of the fiscal expansion. On the other hand, projects that can only be implemented with substantial delay may have a much smaller multiplier that declines quickly with the level of spending.

2.4. Alternative Policy Rules

Our assumption that monetary policy completely stabilizes the economy is convenient to illustrate the points discussed above, since the key factors we have highlighted as affecting the multiplier remain pivotal under alternative monetary policy specifications. Nevertheless, as suggested by the literature, expectations about the monetary policy rule that will be followed after the economy leaves the liquidity trap can have substantive implications for the effects of shocks (including fiscal) while the economy is in the trap.

The government spending multiplier may be amplified considerably under a Taylor rule that reacts much less aggressively to inflation and the output gap (than in our previous analysis). Such monetary rules tend to allow inflation and the output gap to fall well below baseline in response to the negative aggregate demand shock considered above, even after the economy exits the liquidity trap. This is illustrated in Figure 9, where the dashed line shows the effects of the same taste shock v_t as considered earlier in Figure 3. Under the less aggressive rule, an identical sized negative taste shock has larger impact on the output gap and inflation in the baseline simulation relative to the aggressive policy rule used so far because expectations are less well anchored. This is obvious by comparing the dashed lines in Figure 3 (aggressive rule) with the corresponding ones in Figure 9 (less aggressive rule). However, the switch in policy does not in this case prolong the duration of the liquidity trap, which remains at $T_f = 8$ quarters in the absence of fiscal action. The larger decline in output and inflation in response to a given-sized fall in r_t^{pot} under the less aggressive rule gives more scope for stimulative fiscal policy to boost output and inflation; as seen in the figure, the multiplier peaks at roughly 3.

3. An Empirically-Validated New Keynesian Model with Capital

In this section, we present a fully-fledged model with endogenous capital accumulation. Our objectives are to assess whether the factors identified as playing a major role in influencing the multiplier in the simple New Keynesian model continue to be important in a more empirically realistic framework, as well as to provide a more reasonable quantitative assessment of the multiplier.

Our model can be regarded as a slightly simplified version of the model developed and estimated by Christiano, Eichenbaum and Evans (2005), and Smets and Wouters (2003, 2007). Christiano, Eichenbaum and Evans (2005) show that their model can account well for the dynamic effects of a monetary policy innovation during the post-war period. Smets and Wouters (2003, 2007) consider a much broader set of shocks, and estimate their model using Bayesian methods. They argue that it is able to fit many key features of U.S. business cycles.

3.1. The Model

As outlined below, our benchmark model incorporates nominal rigidities by assuming that labor and product markets exhibit monopolistic competition, and that wages and prices are determined by staggered nominal contracts of random duration (following Calvo (1983) and Yun (1996)). The model includes an array of real rigidities, including habit persistence in consumption, and costs of changing the rate of investment. Monetary policy follows a Taylor rule, and fiscal policy specifies that taxes respond to government debt.

3.1.1. Firms and Price Setting

Final Goods Production As in Chari, Kehoe, and McGrattan (2000), we assume that there is a single final output good Y_t that is produced using a continuum of differentiated intermediate goods $Y_t(f)$. The technology for transforming these intermediate goods into the final output good is constant returns to scale, and is of the Dixit-Stiglitz form:

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{1}{1+\theta_p}} \, df \right]^{1+\theta_p} \tag{18}$$

where $\theta_p > 0$.

Firms that produce the final output good are perfectly competitive in both product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of the output index Y_t, taking as given the price $P_t(f)$ of each intermediate good $Y_t(f)$. Moreover, final goods producers sell units of the final output good at a price P_t that is equal to the marginal cost of production:

$$P_t = \left[\int_0^1 P_t(f)^{\frac{-1}{\theta_p}} \, df \right]^{-\theta_p} \tag{19}$$

It is natural to interpret P_t as the aggregate price index.

Intermediate Goods Production A continuum of intermediate goods $Y_t(f)$ for $f \in [0, 1]$ is produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces a demand function for its output good that varies inversely with its output price $P_t(f)$, and directly with aggregate demand Y_t :

$$Y_t(f) = \left[\frac{P_t(f)}{P_t} \right]^{\frac{-(1+\theta_p)}{\theta_p}} Y_t \tag{20}$$

Each intermediate goods producer utilizes capital services $K_t(f)$ and a labor index $L_t(f)$ (defined below) to produce its respective output good. The form of the production function is Cobb-Douglas:

$$Y_t(f) = K_t(f)^{\alpha} L_t(f)^{1-\alpha} \tag{21}$$

Firms face perfectly competitive factor markets for hiring capital and the labor index. Thus, each firm chooses $K_t(f)$ and $L_t(f)$, taking as given both the rental price of capital R_{Kt} and the aggregate wage index W_t (defined below). Firms can costlessly adjust either factor of production. Thus, the standard static first-order conditions for cost minimization imply that all firms have identical marginal cost per unit of output.

We assume that the prices of the intermediate goods are determined by Calvo-Yun style staggered nominal contracts. In each period, each firm f faces a constant probability, $1 - \xi_p$, of being able to reoptimize its price $P_t(f)$. The probability that any firm receives a signal to reset its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, we follow Christiano, Eichenbaum and Evans (2005) and assume that it simply adjusts its price by a weighted combination of the lagged and steady state rate of inflation (i.e., $P_t(f) = \pi_{t-1}^{\iota_p} \pi^{1-\iota_p} P_{t-1}(f)$ for the non-optimizing firms). When ι_p is set close to unity, this formulation introduces structural inertia into the inflation process.

3.1.2. Households and Wage Setting

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the production sector; that is, goods-producing firms regard each household's labor services $N_t(h)$, $h \in [0,1]$, as an imperfect substitute for the labor services of other households. It is convenient to assume that a representative labor aggregator (or "employment agency") combines households' labor hours in the same proportions as firms would choose. Thus, the aggregator's demand for each household's labor is equal to the sum of firms' demands. The labor index L_t has the Dixit-Stiglitz form:

$$L_t = \left[\int_0^1 N_t(h)^{\frac{1}{1+\theta_w}} \, dh \right]^{1+\theta_w} \tag{22}$$

where $\theta_w > 0$. The aggregator minimizes the cost of producing a given amount of the aggregate labor index, taking each household's wage rate $W_t(h)$ as given, and then sells units of the labor index to the production sector at their unit cost W_t:

$$W_t = \left[\int_0^1 W_t(h)^{\frac{-1}{\theta_w}} \, dh \right]^{-\theta_w} \tag{23}$$

It is natural to interpret W_t as the aggregate wage index. The aggregator's demand for the labor hours of household h – or equivalently, the total demand for this household's labor by all goods-producing firms – is given by

$$N_t(h) = \left[\frac{W_t(h)}{W_t} \right]^{-\frac{1+\theta_w}{\theta_w}} L_t \tag{24}$$

The utility functional of a typical member of household h is

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \{ \frac{1}{1-\sigma} C_{t+j}(h) - \varkappa C_{t+j-1} - \nu_c \nu_t \}^{1-\sigma} + \frac{\chi_0}{1-\chi} (1 - N_{t+j}(h))^{1-\chi} \} \tag{25}$$

where the discount factor β satisfies $0 < \beta < 1$. The period utility function depends on household h's current consumption $C_t(h)$, as well as aggregate per capita consumption in the previous period C_{t-1}. This formulation allows the possibility of external habit persistence in consumption spending as in Smets and Wouters (2003). As in the simple model considered in the previous section, a positive taste shock ν_t raises the marginal utility of consumption associated with any given consumption level. The period utility function also depends on current leisure $1 - N_t(h)$.

Household h's budget constraint in period t states that its expenditure on goods and net purchases of financial assets must equal its disposable income:

$$\begin{aligned} & P_t C_t(h) + P_t I_t(h) + \frac{1}{2} \psi_I P_t \frac{(I_t(h) - I_{t-1}(h))^2}{I_{t-1}(h)} + \\ & P_{Bt} B_{Gt+1} - B_{Gt} + \int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h) \\ = \; & (1 - \tau_{Nt}) W_t(h) N_t(h) + (1 - \tau_{Kt}) R_{Kt} K_t(h) + \tau_{Kt} P_t \delta K_t(h) + \Gamma_t(h) - T_t(h) \end{aligned} \tag{26}$$

Thus, the household purchases the final output good (at a price of P_t), which it chooses either to consume $C_t(h)$ or invest $I_t(h)$ in physical capital. The total cost of investment to each household h is assumed to depend on how rapidly the household changes its rate of investment (as well as on the purchase price). Our specification of such investment adjustment costs as depending on the square of the change in the household's gross investment rate follows Christiano, Eichenbaum, and Evans (2005). Investment in physical capital augments the household's (end-of-period) capital stock $K_{t+1}(h)$ according to a linear transition law of the form:

$$K_{t+1}(h) = (1 - \delta) K_t(h) + I_t(h) \tag{27}$$

In addition to accumulating physical capital, households may augment their financial assets through increasing their government bond holdings ($P_{Bt}B_{Gt+1} - B_{Gt}$), and through the net acquisition of state-contingent bonds. We assume that agents can engage in frictionless trading of a complete set of contingent claims. The term $\int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h)$ represents net purchases of state-contingent domestic bonds, with $\xi_{t,t+1}$ denoting the state price, and $B_{D,t+1}(h)$ the quantity of such claims purchased at time t. Each member of household h earns after tax labor income $(1-\tau_{Nt})\, W_t(h)\, N_t(h)$, after-tax capital rental income of $(1-\tau_{Kt})R_{Kt}K_t(h)$, and a depreciation allowance of $\tau_{Kt}P_t\delta K_t(h)$. Each member also receives an aliquot share $\Gamma_t(h)$ of the profits of all firms, and pays a lump-sum tax of $T_t(h)$ (this may be regarded as taxes net of any transfers).

In every period t, each member of household h maximizes the utility functional (25) with respect to its consumption, investment, (end-of-period) capital stock, money balances, and holdings of contingent claims, subject to its labor demand function (24), budget constraint (26), and transition equation for capital (27). Households also set nominal wages in Calvo-style staggered contracts that are generally similar to the price contracts described above. Thus, the probability that a household receives a signal to reoptimize its wage contract in a given period is denoted by $1-\xi_w$; as in the case of price contracts, this probability is independent of the date at which the household last reset its wage. In addition, we specify a dynamic indexation scheme for the adjustment of the wages of those households that do not get a signal to reoptimize, i.e., $W_t(h) = \omega_{t-1}^{\iota_w}\omega^{1-\iota_w}W_{t-1}(h)$, where ω_{t-1} is the gross nominal wage inflation in period $t-1$ and $\omega = \pi g_z$ is the steady state rate of change in the nominal wage (gross price inflation times steady state gross productivity growth). Dynamic indexation of this form introduces some element of structural persistence into the wage-setting process.

3.1.3. Fiscal and Monetary Policy and the Aggregate Resource Constraint

Government purchases G_t are assumed to be set as a share of steady state output, so that $g_{y,t} = \frac{G_t}{Y}$ follows an exogenous stochastic process given by eq. (10). These purchases are assumed neither to affect the utility of households, nor to serve as an input into goods production. Government expenditures are assumed to be financed by a combination of labor taxes, taxes on capital income, and lump sum taxes. However, the government does not need to balance its budget each period, and issues nominal debt to finance budget deficits according to

$$P_{B,t}B_{G,t+1} - B_{G,t} = P_tG_t - T_t - \tau_{N,t}W_tL_t - \tau_{K,t}\left(R_{Kt} - \delta P_t\right)K_t. \tag{28}$$

In eq. (28), all quantity variables are aggregated across households, so that $B_{G,t}$ is the aggregate stock of government bonds, K_t is the aggregate capital stock, and $T_t = (\int_0^1 T_t(h)\,dh)$ are aggregate lump-sum taxes. Throughout the analysis, we will assume that capital taxes τ_{Kt} are given by an exogenous stochastic process with mean τ_K. However lump-sum taxes adjust endogenously in our benchmark specification. The tax rate reaction has the same basic form as in Section 2, but also allows taxes to respond to the gross budget deficit (i.e. the first difference of the debt/trend output ratio):

$$\tau_t - \tau = \varphi_\tau (\tau_t - \tau) + \varphi_b (b_{G,t} - b_G) + \varphi_d (b_{G,t} - b_{G,t-1}). \tag{29}$$

For sensitivity analysis, we also consider a case in which the distortionary tax rate on labor income adjusts according to eq.(29), in which case $\tau_{N,t}$ replaces τ_t. Some simple econometric analysis suggest that these specifications fit the US post-1980 evidence quite well if φ_b and φ_d are set to small values.

Monetary policy is assumed to be given by a policy rule similar to eq. (3) except allowing for a smoothing coefficient γ_i:

$$i_t = \max(-i, (1-\gamma_i)(\gamma_\pi \pi_t + \gamma_x x_t) + \gamma_i i_{t-1}) \tag{30}$$

We set $\gamma_i = 0.7$, $\gamma_\pi = 3$ and $\gamma_x = 0.25$ based on the estimation results reported by Erceg, Guerrieri and Gust (2006) for the 1983:1-2003:4 period.[9]

Finally, total output of the service sector is subject to the resource constraint:

$$Y_t = C_t + I_t + G_t + \psi_{I,t} \tag{31}$$

where $\psi_{I,t}$ is the adjustment cost on investment aggregated across all households (from eq, 26, $\psi_{I,t} \equiv \frac{1}{2}\psi_I \frac{(I_t(h) - I_{t-1}(h))^2}{I_{t-1}(h)}$).

3.1.4. Solution and Calibration

To analyze the behavior of the model, we log-linearize the model's equations around the non-stochastic steady state. Nominal variables, such as the contract price and wage, are rendered stationary by suitable transformations. To solve the unconstrained version of the model, we compute the reduced-form solution of the model for a given set of parameters using the numerical

[9] Some simple regression analysis for the sample period 1993:4-2008:3 supports the estimation results in Erceg, Guerrieri and Gust (2006) and suggest that our benchmark parameterization is in line with historical correlations. Our own analysis suggest that the federal funds rate has become somewhat more responsive to movements in the output gap and inflation in the more recent years.

algorithm of Anderson and Moore (1985), which provides an efficient implementation of the solution method proposed by Blanchard and Kahn (1980). When we solve the model subject to the non-linear policy rule (30) for the nominal interest rate, we use the techniques described in Lindé and Svensson (2009). An important feature of the Lindé and Svensson algorithm is that the duration of the liquidity trap is endogenous, and is affected by the size of the fiscal impetus.

The model is calibrated at a quarterly frequency. Thus, we assume that the discount factor $\beta = .995$, consistent with a steady-state annualized real interest rate $\overline{r}$ of about 2 percent. We assume that the subutility function over consumption is logarithmic, so that $\sigma = 1$, while we set the parameter determining the degree of habit persistence in consumption $\varkappa = 0.6$ (similar to the empirical estimate of Smets and Wouters 2003). The parameter χ, which determines the curvature of the subutility function over leisure, is set equal to 2.5. The implied Frisch elasticity of labor supply of 0.4 is well within the range of most estimates from the empirical labor supply literature (see e.g. Domeij and Flodén, 2006). The scaling parameter χ_0 is set so that employment comprises one-third of the household's time endowment.

The capital share parameter α is set to 0.35, consistent with the observed labor share in the United States. The quarterly depreciation rate of the capital stock $\delta = 0.025$, implying an annual depreciation rate of about 10 percent. We set the cost of adjusting investment parameter $\psi_I = 3$, which is somewhat smaller than the value estimated by Christiano, Eichenbaum, and Evans (2005) using a limited information approach; however, the analysis of Erceg, Guerrieri, and Gust (2006) suggests that a lower value may be better able to capture the unconditional volatility of investment.

We maintain the assumption of a relatively flat Phillips curve by setting the price contract duration parameter $\xi_p = 0.9$. As in Christiano, Eichenbaum and Evans (2005), we also allow for a fair amount of intrinsic persistence by setting the price indexation parameter $\iota_p = 0.9$. It bears emphasizing that our choice of ξ_p does not necessarily imply an average price contract duration of 10 quarters. Altig et al. (2005) show that even a model with a low slope of the Phillips curve can be consistent with frequent price reoptimization. The structure of our model is essentially identical to theirs, and for a moderate markup of $\theta_p = .10$ (i.e., a 10 percent markup) our choice of ξ_p (and implied slope of the Phillips curve of about 0.012) is consistent with about $4-5$ quarters between reoptimization of price contracts under the assumption that capital is firm-specific. Hence, our choice of ξ_p accords with empirical evidence on the Phillips curve slope e.g. Altig et al. (2005) and Smets and Wouters (2003).

Given strategic complementarities in wage-setting across households, the wage markup influences the slope of the wage Phillips curve. Our choices of a wage markup of $\theta_W = 1/3$ and a wage contract duration parameter of $\xi_w = 0.85$ imply that wage inflation is about as responsive to the wage markup as price inflation is to the price markup; thus, given we impose the same degree of indexation ($\iota_w = 0.9$),.the wage and price Phillips curves are very similar.

The parameters pertaining to fiscal policy are set as follows. The share of government spending of total expenditure is set equal to 20 percent. The steady state capital income tax rate, τ_K, is set to 0.2 while the lump sum tax revenue to GDP ratio is set to 0.02. The government debt to GDP ratio is 0.5, close to the total estimated public debt to output ratio at end-2009. The government's intertemporal budget constraint implies that labor income tax rate τ_N equals 0.27 in steady state.[10] The parameters in the fiscal policy rule in eq. (29) are set to $\varphi_\tau = 1$, $\varphi_b = 0.05$ and $\varphi_d = 0.10$, noting that the deficit is interpreted as the change in the gross debt (as share of trend output). This is not a very aggressive tax rule, and the coefficients are in line with the historical correlations between total tax revenues, government debt, and the deficit.[11] The choice of these parameters only matters for equilibrium allocations in variants of the model with distortionary taxes or Keynesian households.

Our choice of benchmark parameters is also motivated from their implications for the current downturn in the economy. In Figure 10, we plot the evolution of some key macroeconomic variables from the third quarter in 1987 to the second quarter of 2009.[12] The solid vertical line shows the third quarter of 2008. The intensification of the financial crisis in the fall of 2008 triggered a severe contraction in output relative to potential, where potential output is measured alternatively by the CBO estimate, and by the trend component of HP-filtered output. Over the same period, core consumer price inflation (which strips out the relatively volatile food and energy components) only declined by a modest amount. Nominal wage growth fell somewhat more, but remained positive through the first half of 2009.

We next assess the ability of our benchmark calibration to account for these stylized facts. Toward that end, Figure 11 simulates the effects of a sharp fall in demand for consumption goods due to a negative taste shock ν_t.[13] The inflation rate and nominal interest rates are shown in levels

[10] It should be emphasized that the results are not much affected if we consider a steady state where the government debt to output ratio is instead set to zero, as the log-linearized version of eq. (28) implies that the real interest rate has relatively modest direct effects on the evolution of government debt.

[11] We collected data on total nominal tax revenues as share of trend nominal GDP, and estimated (29) with OLS. Imposing the coefficients we are using only results in a fall in R^2 from 0.97 to 0.95 relative to the best fitting OLS estimates.

[12] Inflation and nominal wage growth are measured by the CPI (excluding food and energy) and compensation per hour in non-farm business sector, respectively. We choose to work with the core CPI because our model is not suited to capture the behavior of the food and energy sectors.

[13] More specifically, starting from a steady state, the underlying shock to demand is -51.7 percent and the

(with the steady state inflation rate equal to 2 percent, the nominal interest rate 4 percent, and wage inflation 4 percent). The output gap (deviation of output from potential under flexible prices) and output (as deviation from trend) have zero means in steady state. The parameterization described above is labeled 'benchmark' in the figure. For the benchmark calibration, we report the effects of the fall in demand when the nominal interest rate is unconstrained by the zero lower bound (labeled 'Unconstrained') and when the policy rate is constrained by the zero lower bound (ZLB). Consistent with the findings in the previous section, the effects on aggregate quantities and prices are considerably larger when the policy rate is constrained by the zero lower bound. Figure 11 also reports the effects of a fall in consumption demand for an alternative parameterization when prices and wages are more flexible, labeled 'flex p and w.' In this latter case, we set $\xi_p = 0.75$ and $\xi_w = 0.77$, which under economy wide homogeneous markets for capital and labor corresponds to about a 4 quarter duration of price and wage contracts.[14] For the latter parameterization, we see that prices and wages fall dramatically from their steady state values of 2 and 4 percent to about -5 percent for both variables. Finally, we also report results when policy is less aggressive, which we implement by reducing the parameters γ_π and γ_x in eq. (30) from 3 and 0.25 to 1.5 and 0.125, respectively. As can be seen from the 'loose rule' case in Figure 11, the alternative parameterization of the policy rule leads to a substantially larger drop in inflation and more gradual adjustment of the fed funds rate relative to our benchmark calibration, especially in the short run.

It is also evident from Figure 11 that a sharp drop in prices and wages are clearly at odds with the data during the recent crisis. The four cross-dots in Figure 11 depict the actual evolution of the U.S. economy during $2008Q3 - 2009Q2$, and are derived from the four last sample points in Figure 10 (the output gap is computed as the difference between actual and the CBO potential output in the upper left panel, whereas output is the difference between actual and trend output in the lower left panel). From the figure, it is clear that the negative consumption demand shocks under our benchmark parameterization allow the model to do quite well in accounting for the actual behavior of the US economy over this period. Conversely, under the alternative calibrations with more flexible prices and wages, our model implies a much sharper decline in consumer price inflation and wage inflation in response to a shock that depresses output by a similar magnitude than has been observed thus far during the recession.

persistence of the shock is assumed to be 0.9. In the model with financial frictions in the following section, we experimented with a combination of a negative consumption demand and net worth shocks; however, we found that the results for the pure consumption demand shock did at least as well in accounting for the actual path of key macroeconomic variables depicted in Figure 10.

[14] Given the large size of the consumption demand shock, we found that the model with the binding ZLB constraint could not be solved when the wage contract duration parameter ξ_w was below 0.77.

For the fed funds rate, the dashed-dotted line from 2009Q3 and onwards are a projection of the fed funds rate path based on overnight index swap (OIS) rates as of September 7, 2009. These projections are available 1-24 months and 36 months ahead. The projected path of the funds rate only begins to rise noticeably above zero after about two years. Moreover, the basic contour of the OIS path evident in the figure remained similar through most of the course of 2009. Although time-varying risk premia preclude interpreting the projected path from OIS rates directly as measuring private sector expectations of the funds rate, this evidence provides some comfort that the implication of an eight quarter liquidity trap in our baseline is not unreasonable.[15] However, as there is considerable uncertainty on this dimension, we investigate the sensitivity of our results to the duration of the liquidity trap.

3.2. Dynamic Effects of Fiscal policy Expansions

We now study the effects of increases in government expenditures in this model. Clearly, given the nonlinearity associated with the zero bound constraint, the characteristics of the baseline path discussed above – including the implication of an eight quarter liquidity trap – matter a great deal for the government spending multiplier. To this baseline scenario, we then add a government spending shock. Specifically, we assume that the fiscal expansion occurs in the same period as the negative consumption demand shock hits the economy. For the benchmark scenario depicted in Figure 11, this means that the fiscal expansion occurs in period 0, i.e. even before the economy enters into the liquidity trap. As indicated in the previous analysis, this assumption of a front-loaded response will tend to produce larger fiscal multipliers; subsequently, we examine the role of implementation lags in the stimulus package.

In Figure 12, we report the effects of a front-loaded increase in government expenditures equal to 1 percent of steady state output that has the same persistence as the underlying negative consumption demand shock. The impulse response functions are computed as the difference between the scenario with fiscal stimulus and negative consumption demand shocks and the baseline scenario with negative consumption demand shocks only (depicted in Figure 11). The fiscal expansion is assumed to be financed by lump-sum taxes that respond endogenously to government debt and the budget deficit as described in Subsection 3.1.4 (though given that all agents are Ricardian, the parameters of the fiscal rule have no impact on output, inflation, or interest rates).

[15] Central banks such as the Sveriges Riksbank that are effectively constrained by the zero lower bound constraint and publish interest rate forecasts of their policy rates indicate that they will start raising interest rates within 2 years.

As in the stylized model analyzed in Section 2, the fiscal policy expansion implies much larger effects on output and the output gap relative to a normal situation in which policy is not subject to the zero bound constraint, as the fiscal expansion induces real interest rates to fall when the ZLB is binding. The spending multiplier, defined as dY_{t+j}/dG_t, is slightly above unity in the short-run. The initial increase in the nominal interest rate path reflects the fact that the fiscal expansion (occurring in period 0) delays the economy's entry into a liquidity trap by one quarter. An interesting feature is that government debt (as share of actual output) does not increase to the same extent in the ZLB case relative to a normal situation due to the amplified fiscal multiplier.[16]

Figure 12 also shows impulse response functions to the same government spending shock against the backdrop of a longer-lived liquidity trap of 10 quarters. In particular, the underlying consumption demand shock is about 7 percent larger, which generates a deeper fall in output, inflation, and interest rates. The effects of the government spending hike are considerably larger in this case, which mainly reflects that a larger rise in expected inflation causes a deeper decline in real interest rates.

The upper left panel of Figure 13 depicts marginal and average multipliers for different degrees of fiscal spending. The spending multipliers are computed as the average increase in output (relative to trend) for the first 4 quarters divided by the increase in government spending to trend output in the first period. We consider a four quarter average of output in order to account for possible hump-shaped dynamics of output to fiscal stimulus. As was the case for the stylized model analyzed in Section 2, the marginal multipliers follows a step function where they are constant as long as the incremental increase in government spending does not affect the duration of the liquidity trap. The upper left panel shows that increases in government expenditures less than about 0.5 percent of trend output do not affect the duration of the liquidity trap, and the marginal multiplier is therefore identical to the average multiplier. Larger increases in government expenditures will affect the duration of the liquidity trap and therefore shrink the marginal and average multiplier to the point where the marginal multiplier equals the impulse response function for output in normal times depicted in Figure 12. However, in our benchmark parameterization of the model, an increase in government spending of over 7 percent is required to preclude the economy from ever entering

[16] Notice that the tax-rule (29) responds to government debt as share of annualized trend nominal output ratio, $b_{G,t} = \frac{B_{Gt}}{4P_tY}$. To compute government debt as share of actual output, we notice that $b_{G,t} = \frac{B_{Gt}}{4P_tY}\frac{Y_t}{Y_t} = \frac{B_{Gt}}{4P_tY_t}\frac{Y_t}{Y} = \tilde{b}_{G,t}\frac{Y_t}{Y}$ where we have defined $\tilde{b}_{G,t} \equiv \frac{B_{Gt}}{4P_tY_t}$. This implies that $db_{G,t} = d\tilde{b}_{G,t} + b_G\frac{dY_t}{Y}$, or equivalently, $\widehat{\tilde{b}}_{G,t} = \hat{b}_{G,t} - b_G\hat{y}_t$ which is how we adjust the government debt to trend output ratio with the percentage change in actual output to compute government debt relative to actual output. Notice that if the steady state government debt to output ratio b_G is set to zero, the distinction between government debt to trend or actual output is irrelevant.

a liquidity trap. The right upper panel shows the total and marginal responses of the government deficit to actual output, computed as the four quarter difference in government debt. As can be seen, modest increases in government expenditures induces the budget deficit as share of output to fall during the first year, as the fiscal spending multiplier is particularly high for modest increases in fiscal spending which does not affect the duration of the liquidity trap. When the size of fiscal stimulus in increased beyond about one percent of baseline GDP, the government deficit begins to increase, reflecting that the stimulus pushes the exit from the liquidity trap one quarter earlier. However, consistent with the results in Figure 12, the average government budget deficit is roughly zero for an increase in government spending of about one percent of GDP.

The lower left panel of Figure 13 compares the marginal multipliers for alternative parameterizations of the model; for ease of comparison, marginal multipliers for the benchmark calibration of the model are depicted by the solid lines. First, we report the multipliers when the response coefficients on inflation and the output gap are reduced from 3 and 0.25 to 1.5 and 0.125, respectively (though the smoothing coefficient is kept fixed at 0.7). The results in the lower left panel show that the marginal multiplier for small spending increments is roughly double when the central bank adheres to a more accomodative policy rule, reflecting that inflation expectations rise considerably more. We also show the marginal multipliers when the slope of the Phillips curve is increased from 0.012 to 0.085 (i.e. ξ_p is lowered from 0.90 to 0.75). Interestingly, the multiplier is not much affected under this alternative with more flexible prices, provided that wage adjustment remains sluggish as in our benchmark calibration. In a second alternative, we report marginal multipliers when both prices and nominal wages are less sticky (assuming same parameters as in the 'flex p and w' case in Figure 11). In this case, the fiscal multiplier is very much augmented, especially for low spending levels of less than 0.25 percent of GDP. Thus, in a sticky price and wage model framework, both more flexible prices and nominal wages are needed in order for expected inflation to move substantially to an increase in government spending. Finally, the lower right panel shows the associated impact on the budget deficit. A liquidity trap offers something akin to a free lunch for modest amounts of increases in government spending when wages and prices are fairly flexible or monetary policy accommodates the fiscal expansion. However, for more sizable increases in government expenditures, the budget deficit expands even when wages and prices are more flexible and policy is accomodative.

To examine the role of the conduct of monetary policy, Figure 14 reports the effects for alternative assumptions of the policy rule. In order to be able to make a cleaner comparison of the role of the policy rules, we renormalize the underlying consumption demand shocks so that each policy rule is associated with a baseline liquidity trap duration of 8 quarters. The solid line in Figure 14 shows the benchmark parameterization of the policy rule, i.e. the same impulses as already discussed in Figure 12. The first alternative parameterization (labeled 'ZLB no smoothing)' is to drop interest rate smoothing, i.e. we set $\gamma_i = 0$ but keep γ_π and γ_x unchanged. In the second case (labeled 'ZLB agg. rule' for 'aggressive' rule), we assume complete stabilization of the output gap by setting $\gamma_\pi = 100$ and $\gamma_x = 500$ and $\gamma_i = 0$. Finally, we work with a less responsive policy rule (labeled 'ZLB loose rule') where we shrink γ_π and γ_x by a factor of two (i.e. set $\gamma_\pi = 1.5$ and $\gamma_x = 0.125$). From the results in Figure 14, we see that the multipliers are reduced for a more aggressive policy rules and enhanced for more accomodative policy rules. Again, the main economic force behind this is that when the policy rule is less responsive to the movements in the output gap and inflation, expected inflation reacts more. This drives down the real interest rate, which stimulates aggregate demand. Consequently, accommodative (aggressive) monetary policy can substantially enhance (reduce) the fiscal spending multiplier, even in the case where the responsiveness of inflation to movements in marginal costs is quite low. More specifically, as shown in previous work, e.g., Eggertson and Woodford (2003), future promises regarding the aggressiveness of policy after exiting the liquidity trap can matter substantially for shaping the size of the fiscal spending multiplier today.

Finally, Figure 15 examines the sensitivity of the results to alternative assumptions about how the fiscal stimulus is financed and how quickly it can be implemented. First, we explore how sensitive the results are to dropping the assumption of a front-loaded expansion in period 0 in favor of a more gradual rise in government expenditures. Second, we examine how the results are affected by replacing the benchmark assumption of financing with lump-sum taxes with the alternative of distortionary labor-income taxes. The fiscal spending multipliers clearly are reduced in either case. In particular, the fiscal spending multiplier is damped markedly when the fiscal stimulus is affected by substantial implementation lags (the dotted line): the multipliers are close to zero initially and negative in the medium term. The reduction of the multipliers reflects that the higher spending reduces the potential real interest rate in the short-run, as the promise to increase future public spending encourages households to save. The assumption of financing with distortionary taxes instead of lump-sum taxes also tends to reduce the fiscal multiplier through its negative effect on labor supply, unless the multipliers are so large that the government debt to trend output is roughly constant. It is worth emphasizing that the parameters in the policy function for the labor-income tax-rate tend to make the tax rate path fairly unresponsive to the increase in government debt; if they were more responsive to debt or the deficit, the multiplier would drop even more. [17]

In summary, while the quantitative results differ somewhat compared with the stylized model studied in Section 2, the qualitative aspects are very similar. Thus, the analysis in this more empirically realistic model supports our beliefs that the more favorable multipliers of fiscal policy stimulus packages in a liquidity trap hinges crucially on the liquidity trap duration, sensitivity of expected inflation, implementation and financing, and, finally, to the stance of monetary policy.

4. Robustness analysis: The Empirical Model Augmented with Financial Frictions and Keynesian Households

In this section, we investigate the sensitivity of our results to the inclusion of financial frictions and Keynesian households. One important shortcoming of the model outlined in Section 3 is that private consumption does not increase much when government spending rise, which is at odds with the empirical evidence provided by e.g. Galí, López-Salido and Vallés (2007) using identified vector autoregressions. To examine the robustness of the results in an environment where the sensitivity of private consumption with respect to increases in fiscal spending is higher, we follow Galí, López-Salido and Vallés (2007) and Erceg, Guerrieri and Gust (2006) and assume that a fraction of the households in the economy simply consume their current after-tax income. In addition, frictions in financial markets have been suggested to be an important source of propagation mechanism in the current crisis. Accordingly, we augment the model with financial frictions following the basic framework of Bernanke, Gertler and Gilchrist (1999).

4.1. Key Model Equations and Calibration

This model is identical to the model described in Section 3, with two key exceptions.

First, a fraction s_{kh} of the population of the households are assumed to simply consume their current after-tax income each period:

$$P_t C_t(h) = (1 - \tau_{Nt}) W_t(h) N_t(h) - T_t.$$

The Keynesian households are assumed to set their wage to be the average wage of the optimizing households. Since Keynesian households face the same labor demand schedule as the optimizing

[17] Although not reported, we have also studied a case where the fiscal stimulus package is financed by capital income taxes, and this financing alternative is associated with considerable more negative multipliers for actual output in comparison to the ones reported in Figure 15.

households, each Keynesian household works the same number of hours as the average optimizing household. We set the population share of the Keynesian households to optimizing households, s_{kh}, to 1/2, which implies that the Keynesian households share of total consumption is about 1/3.

Second, we incorporate a Bernanke, Gertler and Gilchrist (1999) type of financial accelerator mechanism. In particular, the optimizing households are assumed to supply labor to the homogenous market for labor, and entrepreneurs supply capital to homogeneous factor markets. The optimizing households produce new capital by combining investment goods with used capital purchased from the entrepreneurs. Entrepreneurs then purchase this new capital, using a combination of their own net worth and loans from banks. Idiosyncratic productivity shocks to the entrepreneurs and asymmetric information (costly state verification) introduces financial frictions between the borrowers (i.e. the entrepreneurs) and the banks (i.e. the households). The only departure from BGG, is that we follow Christiano, Motto and Rostagno (2007) by assuming that the debt contract between the entrepreneurs and the bank is written in nominal terms, so that the return received by households from the banks is nominally non-state contingent. We adopt the calibration of the parameters pertaining to the financial accelerator mechanism to the values chosen by BGG. In particular, we set the monitoring cost, μ, expressed as a proportion of the entrepreneurs total gross revenues to 0.12. The variance of the idiosyncratic productivity shocks hitting the entrepreneurs is set 0.28. The annualized steady state default rate of the entrepreneurs is set to 0.03/4, which corresponds to a quarterly default rate of about 0.75 percent.

4.2. Dynamic Effects of Fiscal Policy Expansions

We now study the effects of expansions in government expenditures in this environment with financial frictions and Keynesian households. The parameterization of the fiscal expansion and tax rules are identical to the setup in Subsection 3.2. Figure 16 reports the effects of the fall in demand for consumption goods in the full model for key variables. As in the model without financial frictions and Keynesian households, the benchmark calibration of the model best mimics the evolution of the US economy for this period, although all model versions suggest that actual output should have recovered faster relative to what the data currently suggest. The only difference is that the size of the underlying consumption shock is set to a slightly lower value (0.91 of the shock size for the benchmark case in Subsection 3.2) and that the ZLB constraint in this model binds for periods $2-9$ instead of periods $1-8$ as was the case for the benchmark model in Subsection 3.2, although the zero lower bound is nearly reached in period 1. Thus, the model with financial frictions and

Keynesian households offer an environment whereby a smaller sized consumption demand shock give rise to liquidity traps of equal length as in the workhorse CEE/Smets-Wouters style model.[18]

Figure 17 shows the effects of a front-loaded increase in government expenditures of 1 percent to trend GDP, corresponding to the simulation in Figure 12. We report results for the benchmark parameterization of the model where monetary policy is constrained by the zero lower bound (labeled 'ZLB Full Model'). In addition, we also report results for the case when policy is not constrained by the zero lower bound (labeled 'Normal Full Model'). For our benchmark calibration, the impact multiplier is slightly higher than unity in normal times, and almost 2 in a liquidity trap, despite the fact that the fiscal expansion shortens the duration of the liquidity trap by 1 quarter.

Figure 17 also shows the spending multiplier for the benchmark calibration of the workhorse model (labeled 'ZLB CEE/SW') analyzed in the previous Subsection 3.2. Clearly, the fiscal spending multiplier in a liquidity trap is strongly enhanced by the introduction of financial frictions and Keynesian households. The key reason why the spending multiplier is higher in the augmented model is that the fiscal stimulus induces a larger increase in the potential real interest rate path relative to the CEE/SW model (as seen in Figure 17). Since the duration of the liquidity trap and the parameterization of the models are identical in all other respects, it is really the extra reduction in the cumulated gap between the actual and potential real interest discussed in detail in Section 2.2 which accounts for the higher multiplier in the extended model. In normal times, when monetary policy is unconstrained by the zero lower bound, the introduction of these extra features do not matter to the same extent, reflecting that monetary policy simply reacts more to the larger rise in the potential real interest rate. This can be seen by comparing the effects of the fiscal stimulus in normal times in Figure 17 with the effects in normal times in Figure 12.

To make a tentative assessment which one of the added features, financial frictions or Keynesian households, are most important for the enhanced fiscal multipliers for a given sized duration of the liquidity trap, we compute the impulses to an expansion in government spending for the workhorse model augmented with financial frictions only. The results with this model specification is reported in Figure 17 (labeled 'ZLB No K.H.'). Although the financial frictions enhance the fiscal multiplier somewhat relative to the model without financial frictions, the bulk of the difference in the spending multiplier between the workhorse model and the full model is driven by the inclusion of Keynesian households, and the assumption that they account for half of all households. However, it should be stressed that the model specification with Keynesian households but without financial frictions

[18] It can be shown that the stronger propagation of the negative consumption shock in this augmented framework is primarily driven by the financial accelerator mechanism.

requires a substantially larger underlying drop in consumption demand in order to end up in a liquidity trap with a duration of 8 quarters in the first place. This implies that if the size of the underlying demand shock were held constant for all model specifications, then the duration of the liquidity trap would be longest for the model specification with financial frictions and consequently also the fiscal multipliers. Hence, the assessment of the relative importance of the various frictions crucially depends on whether the duration of the liquidity trap is taken to be given or if we take the size of the underlying consumption demand shock as given.

The upper left panel of Figure 18 reports marginal and average fiscal multipliers as a function of the size of the increment to government spending. Since the duration of the liquidity trap is shortened by 1 quarter when government spending is increased by 1 percent of trend output, it is not surprising that the marginal multiplier can be higher than 2 for a small enough increase in government spending that does not affect the duration of the liquidity trap. For an increase in government spending of slightly less than 0.25 percent of trend output, the spending multiplier is as high as about 2.8. But for spending increases above 2 percent of trend output, the marginal multiplier is less than unity when the duration of the liquidity trap without fiscal stimulus is 8 quarters. The marginal multiplier when the increase in spending is so large that the economy never hits the zero bound constraint equals the impulse response function in normal times reported in Figure 17 (i.e. the first 4-quarter average of the multiplier). The larger marginal fiscal multipliers implies that the impact on the budget balance – shown in the upper right panel – is more favorable than in the CEE/SW model considered in the previous section.

The lower left panel of Figure 18 reports how the marginal multipliers are affected for alternative parameterizations of the model. As in Figure 13, we report results for two alternatives, including a more accomodative policy stance (which lowers γ_π to 1.5 and γ_x to 0.125), for more flexible prices and wages. [19] As can be seen from the figure, more accomodative policy can make marginal multipliers extremely high for small increases in government spending due to strong effects on expected inflation and thereby the future real interest rate path. However, the marginal multipliers fall very rapidly for additional increments to spending. The zero lower bound duration in the lower panel refers to the calibration of the model where policy is less aggressive, and the duration ticks in this case start from 7 quarters because the multiplier in the model with more loose policy is so high that the economy exits from the liquidity trap one period earlier even for the smallest

[19] In the case with more flexible prices and wages, we lower ξ_p and ξ_w from 0.9 and 0.85 to 0.75 and 0.80, respectively. For computational reasons, ξ_w cannot be lowered further.

incremental increase in government spending. The model specification with more flexible prices and wages is associated with sizeable increases in fiscal spending multipliers, although the increase is not as dramatic as in the accomodative policy case. [20] As in Figure 13, the results for budget deficit in the lower right panel in Figure 18 mirrors the marginal multipliers. For modest increases in government spending, a fiscal expansion may actually finance itself.

Finally, Figure 19 reports the sensitivity of the results when financing must be achieved with distortionary taxes instead of lump-sum transfers, and when the fiscal policy intervention is affected by implementation lags. Consistent with our results for the simpler model in Section 3, financing with distortionary labor income taxes instead of lump-sum taxes clearly dampens the fiscal multipliers even in a liquidity trap. Even so, for a fiscal policy expansion with peak effect after 8 quarters, the spending multiplier is still above unity for about a year in this model, which is considerably higher relative to the results reported for the CEE/Smets-Wouters model in Figure 15 where the spending multiplier was reduced to less than 0.3.

5. Conclusions

Taken together, our results suggest a somewhat nuanced view of the role of fiscal policy in a liquidity trap. For an economy facing a protracted recession and for which monetary policy seems likely to be constrained by the zero bound for a very prolonged period – roughly 2 years or more – there is a strong argument for increasing government spending on a temporary basis. Consistent with the views originally espoused by Keynes, this temporary boost can have much larger effects than under usual conditions, and comes at relatively low cost to the Treasury. And the fiscal multipliers can be even more enhanced if monetary policy is accommodative and allows expected inflation to rise substantially in the short- and medium-term. For shorter-lived liquidity traps of less than two years, the multiplier is larger than under 'normal conditions' for small increases in spending, but drops relatively quickly at higher spending levels. Thus, larger spending programs may suffer from sharply diminishing returns, and may boost government debt significantly.

We have throughout the paper assumed that no alternative measures are available to the central bank in a liquidity trap. In practice, the Federal Reserve and other central banks have deployed a number of policy tools after policy rates declined to nearly zero in the wake of the financial

[20] Although not reported, it can also be shown that in a sticky price and wage framework, reducing only the degree of price stickiness does not enhance the multipliers much, because less sticky prices will not by itself get expected inflation to move for increases in government spending unless nominal wages move, confirming the results in the second panel in Figure 13.

crisis. One such tool is forward guidance, i.e. communicating that economic conditions are likely to warrant the policy rate to be kept at zero for an extended period. To the extent that such statements extends the horizon for which households and firms expect the policy rate to remain at zero, this will stimulate aggregate demand. Apart from providing forward guidance, many central banks have used the asset side of their balance sheet to support credit markets by providing liquidity and purchase of long-term securities. Although the models considered in this paper does not allow for an assessment of the effectiveness of such actions, our analysis highlights the importance of analyzing the effects of such actions *jointly* with the fiscal stimulus packages in order to properly assess their marginal impact.

There are also are number of interesting issues that we leave for future research. For instance, we have intentionally focused on a positive analysis of government expansions, and not studied normative issues such as assessing the conditions under which a government spending hike is welfare-enhancing. Christiano, Eichenbaum and Rebelo (2009) and Nakata (2009) argue that an increase in government consumption might be welfare-enhancing in a liquidity trap. It would be interesting to explore the welfare implications of fiscal stimulus when allowing for endogenous exit. Moreover, this paper has focused on government consumption spending exclusively as the tool of fiscal policy; clearly, it would be interesting to extend our analysis by considering alternative fiscal measures such as tax cuts and targeted transfers.[21]

[21] Eggertsson (2009) argues that tax cuts aimed at stimulating aggregate demand rather than aggregate supply are preferable in a liquidity trap (e.g. sales taxes and implementing an investment tax credit).

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Figure 1: Negative Aggregate Demand Shock

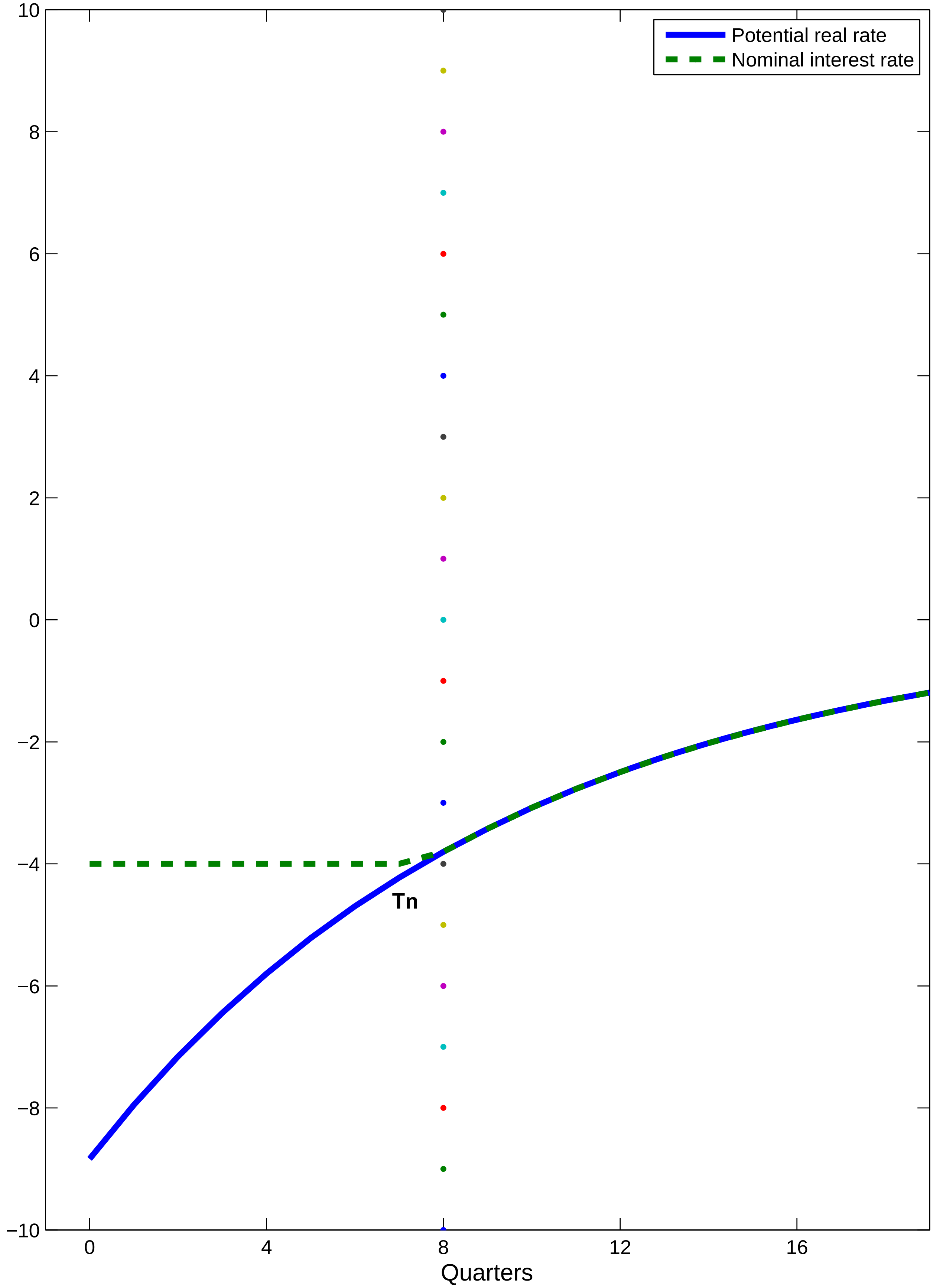

Figure 2. Weights on Leads of the Interest Rate Gap in Inflation Equation

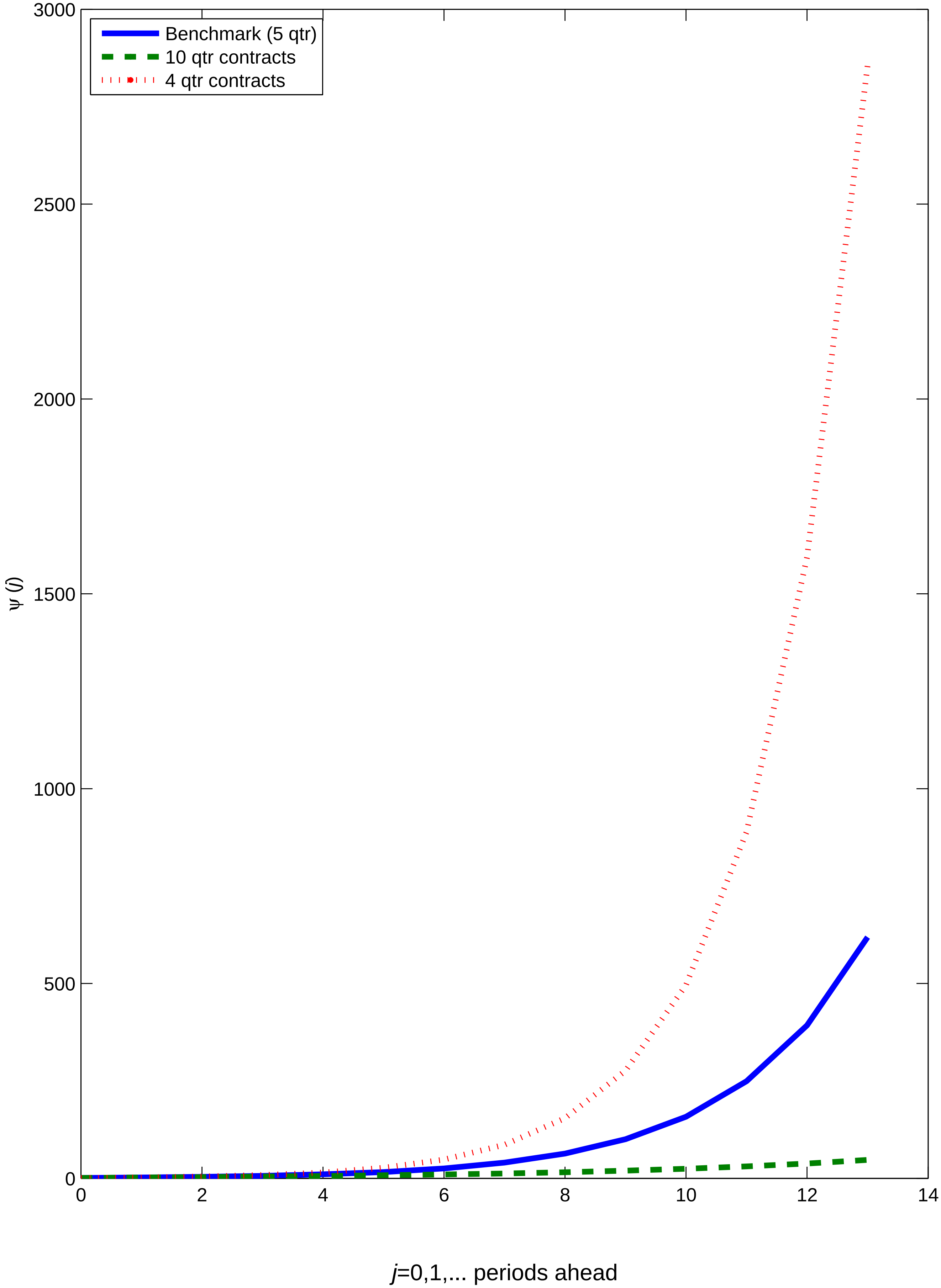

Figure 3: Immediate Rise in Government Spending

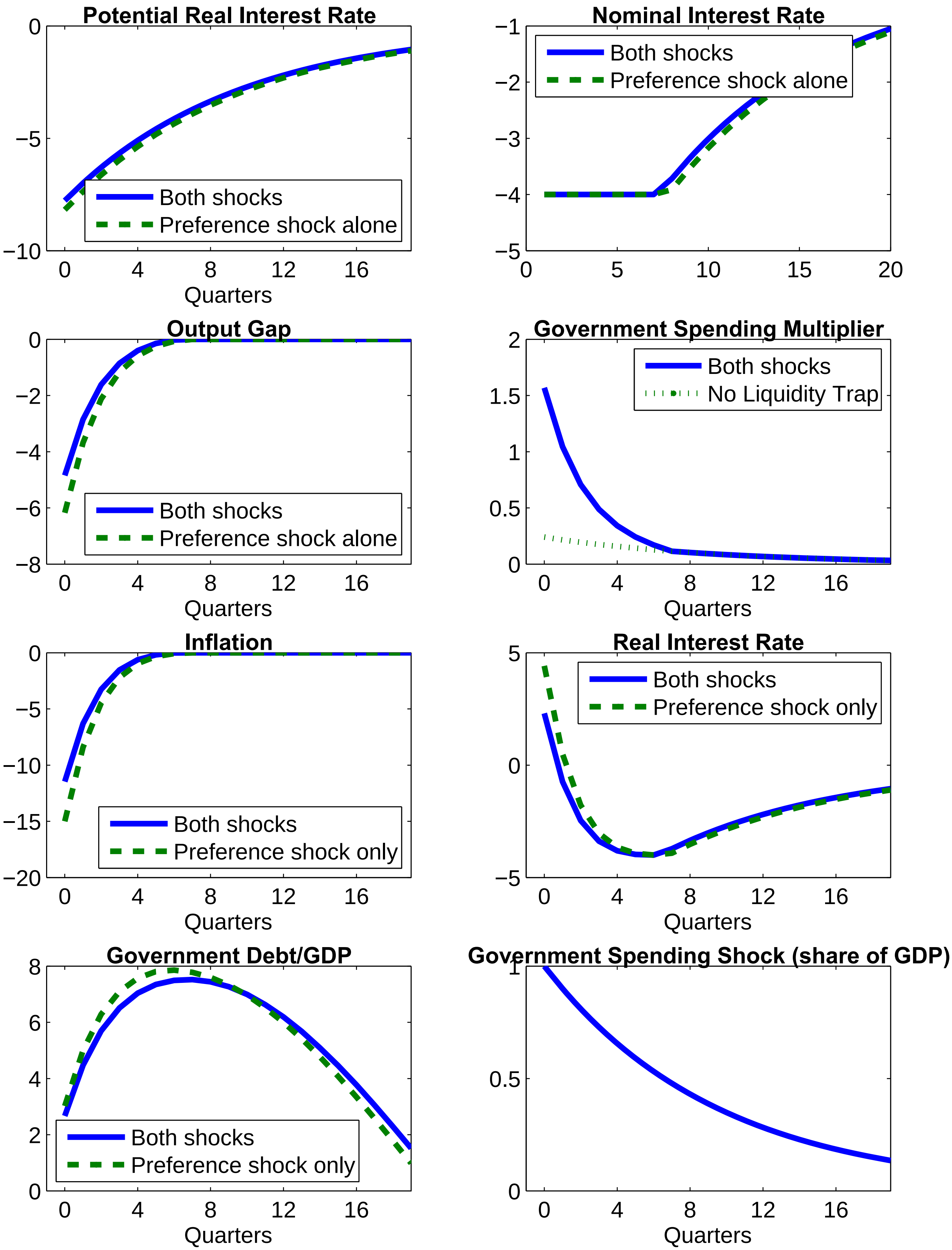

Figure 4: Immediate Rise in Government Spending (flatter Phillips Curve)

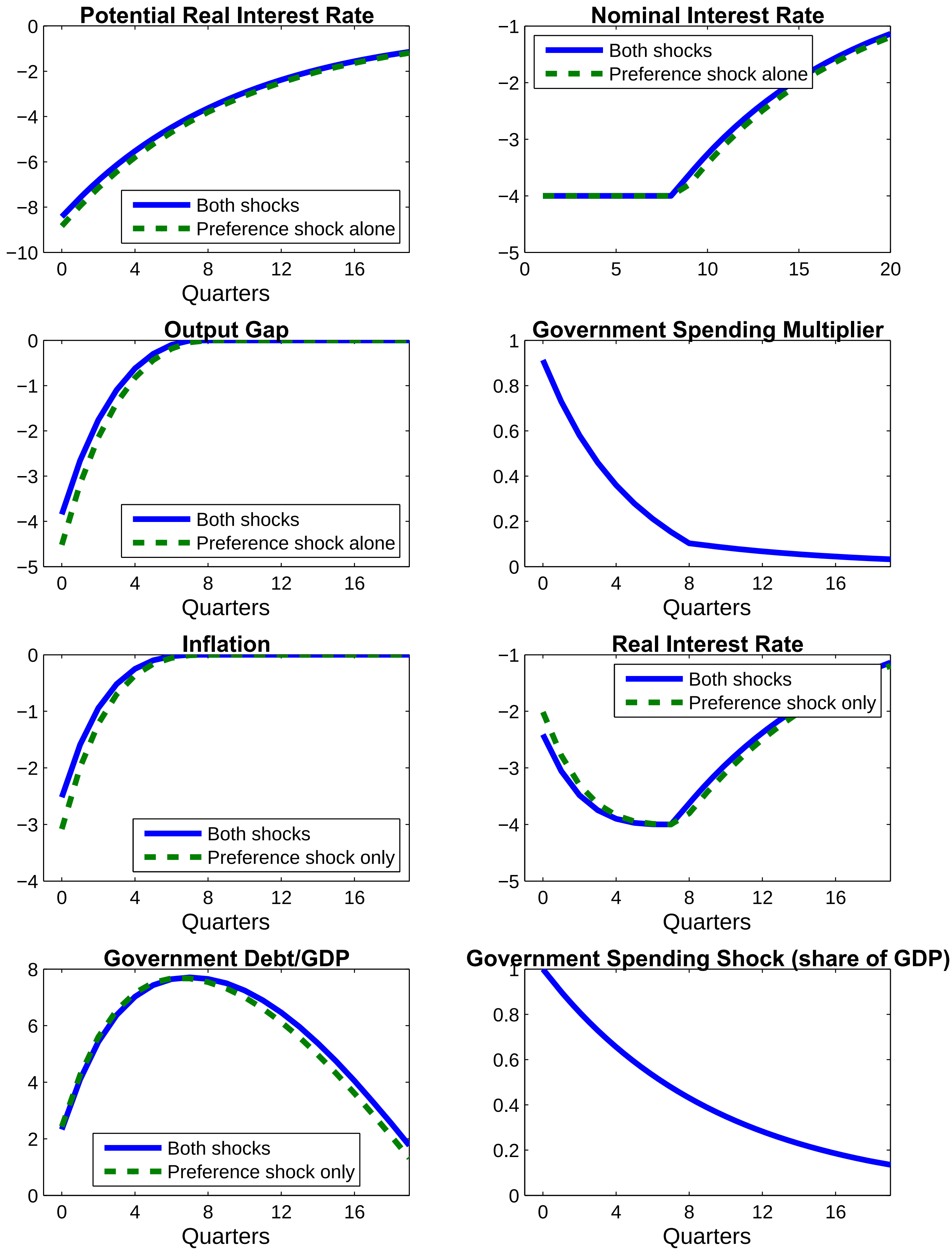

Figure 5: The Consequences of a Delayed Rise in Govt Spending

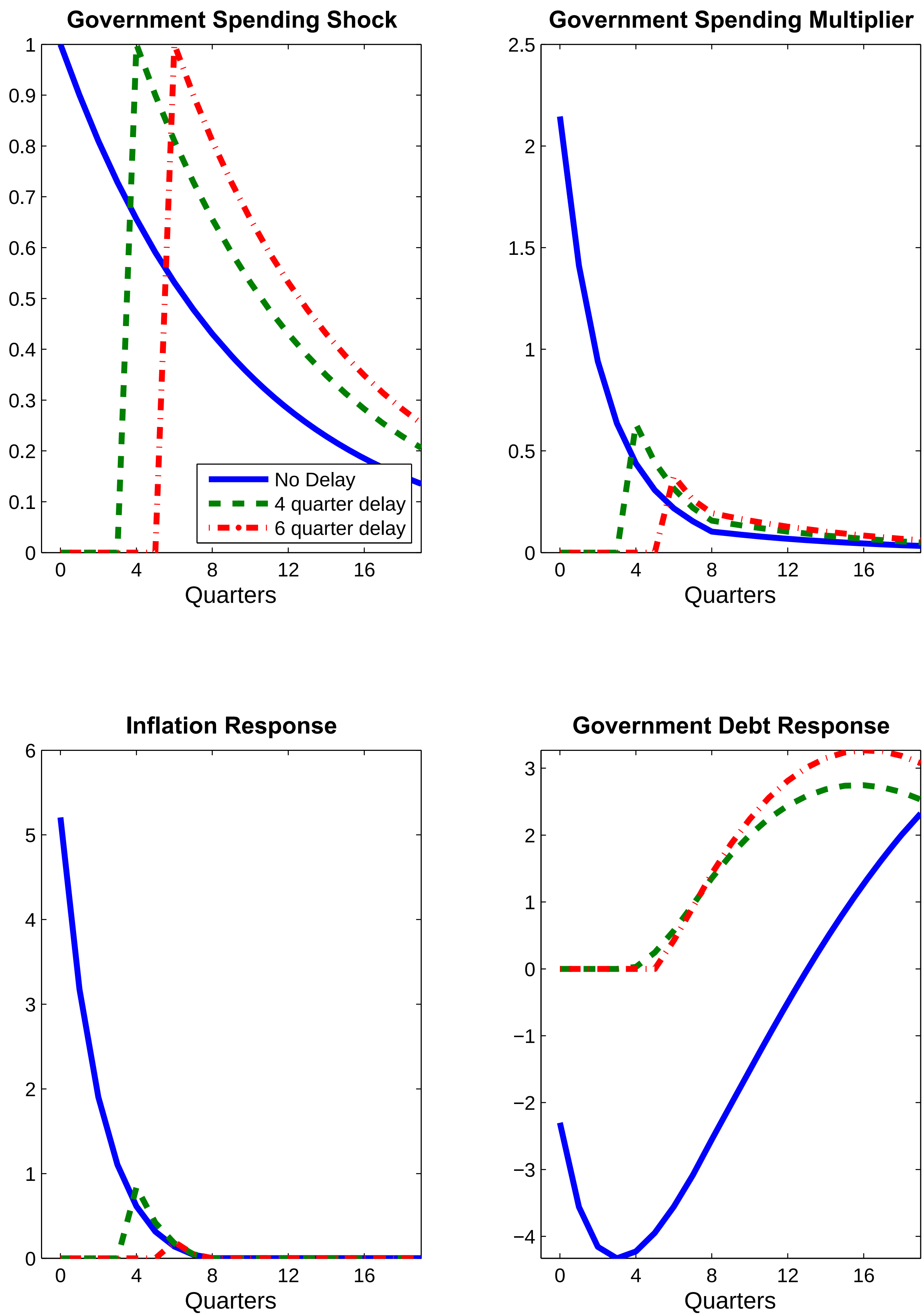

Figure 6: Marginal Government Spending Multipliers in Stylized Model

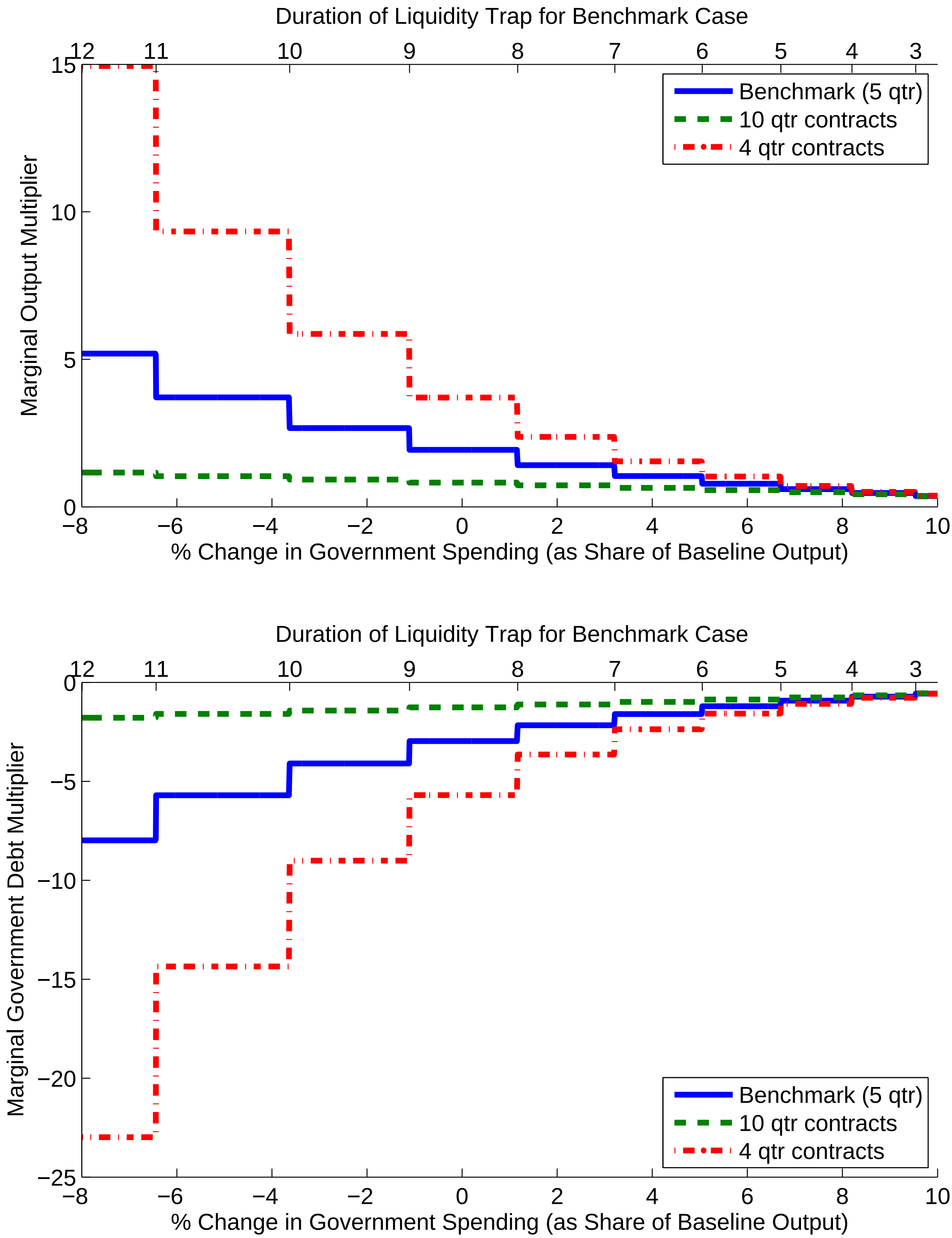

Figure 7: Effects of Government Spending on Interest Rate Gaps

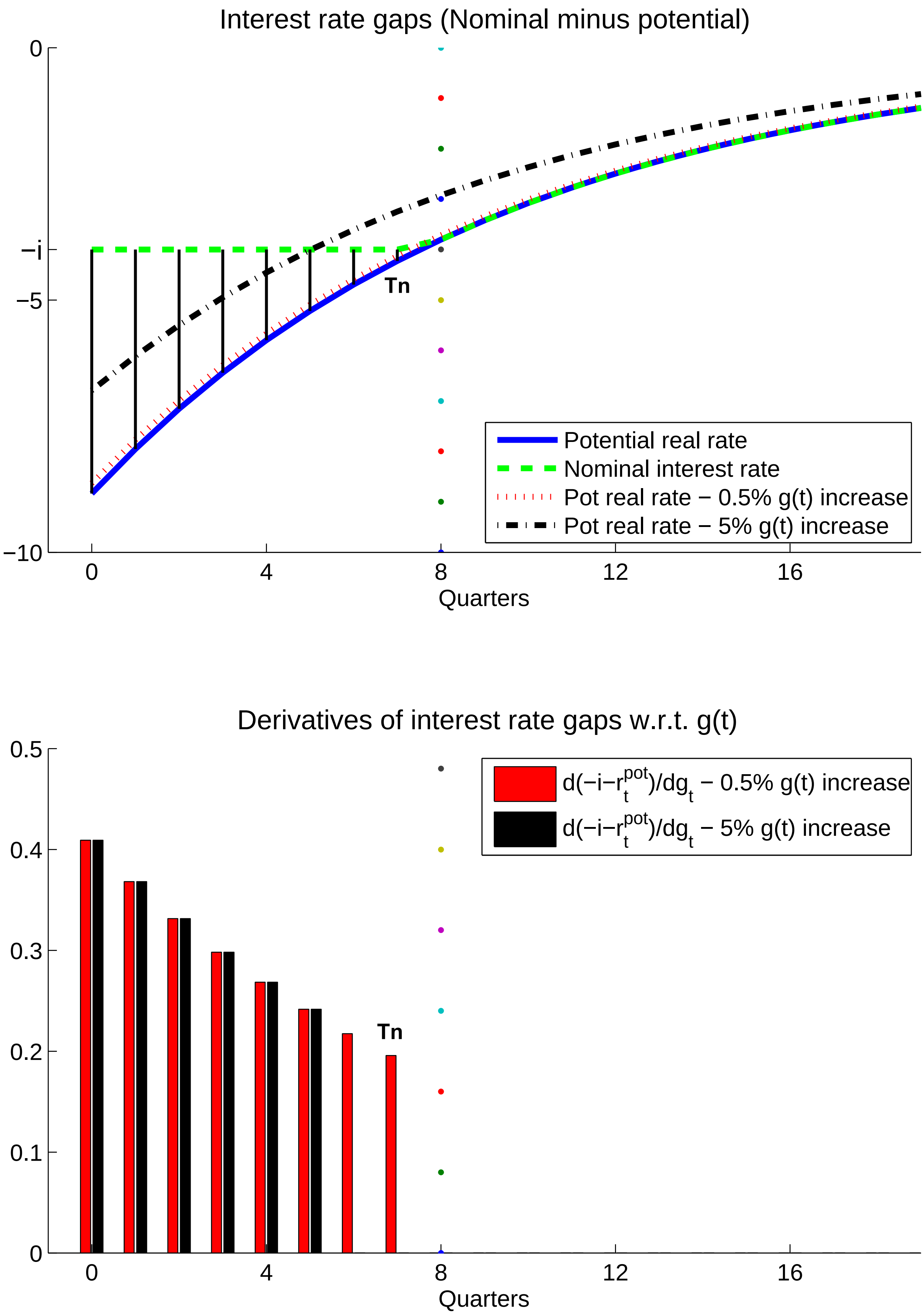

Figure 8: Government Spending Peaks after Eight Quarters

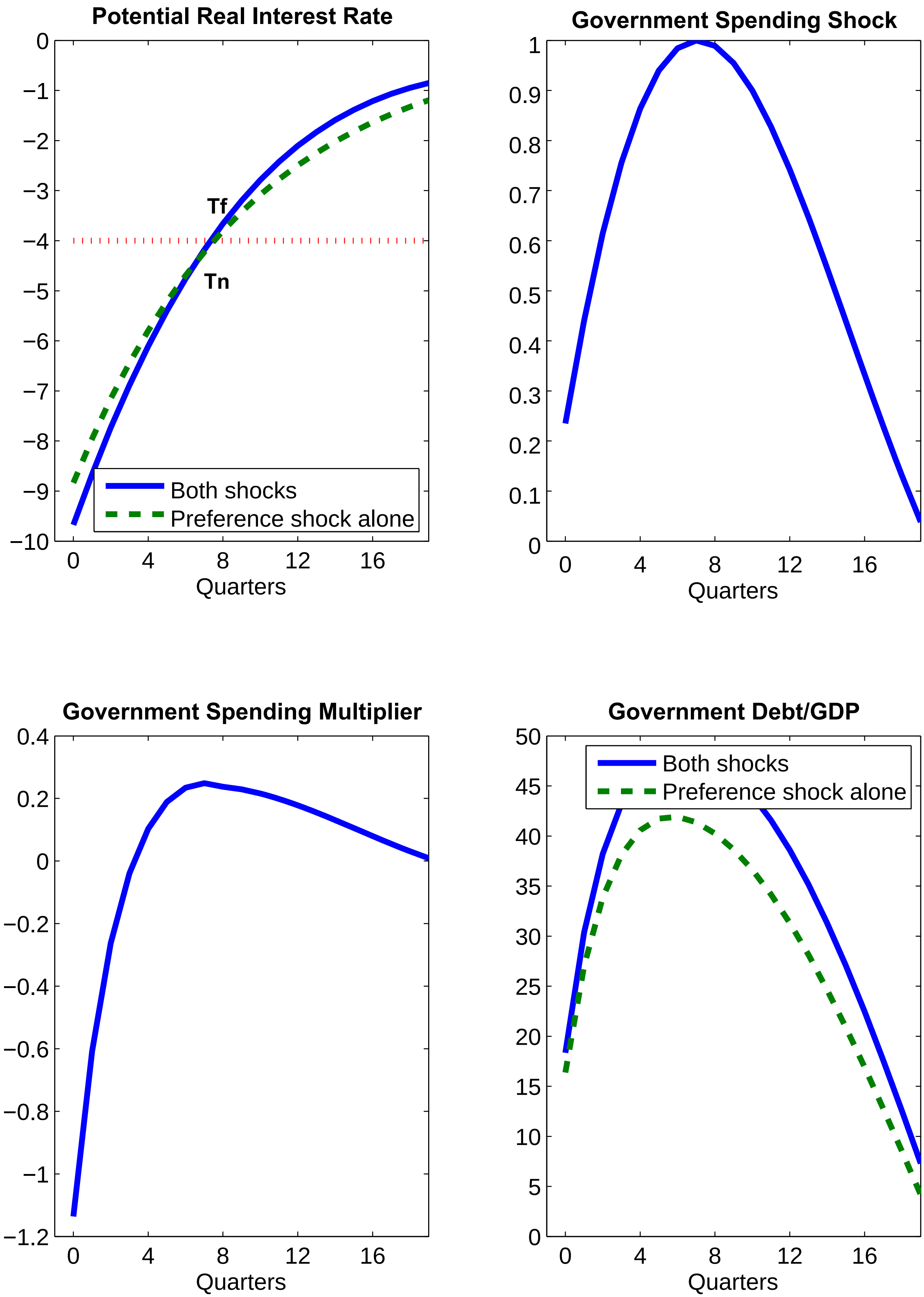

Figure 9: Immediate Rise in Government Spending (Less Aggressive Rule)

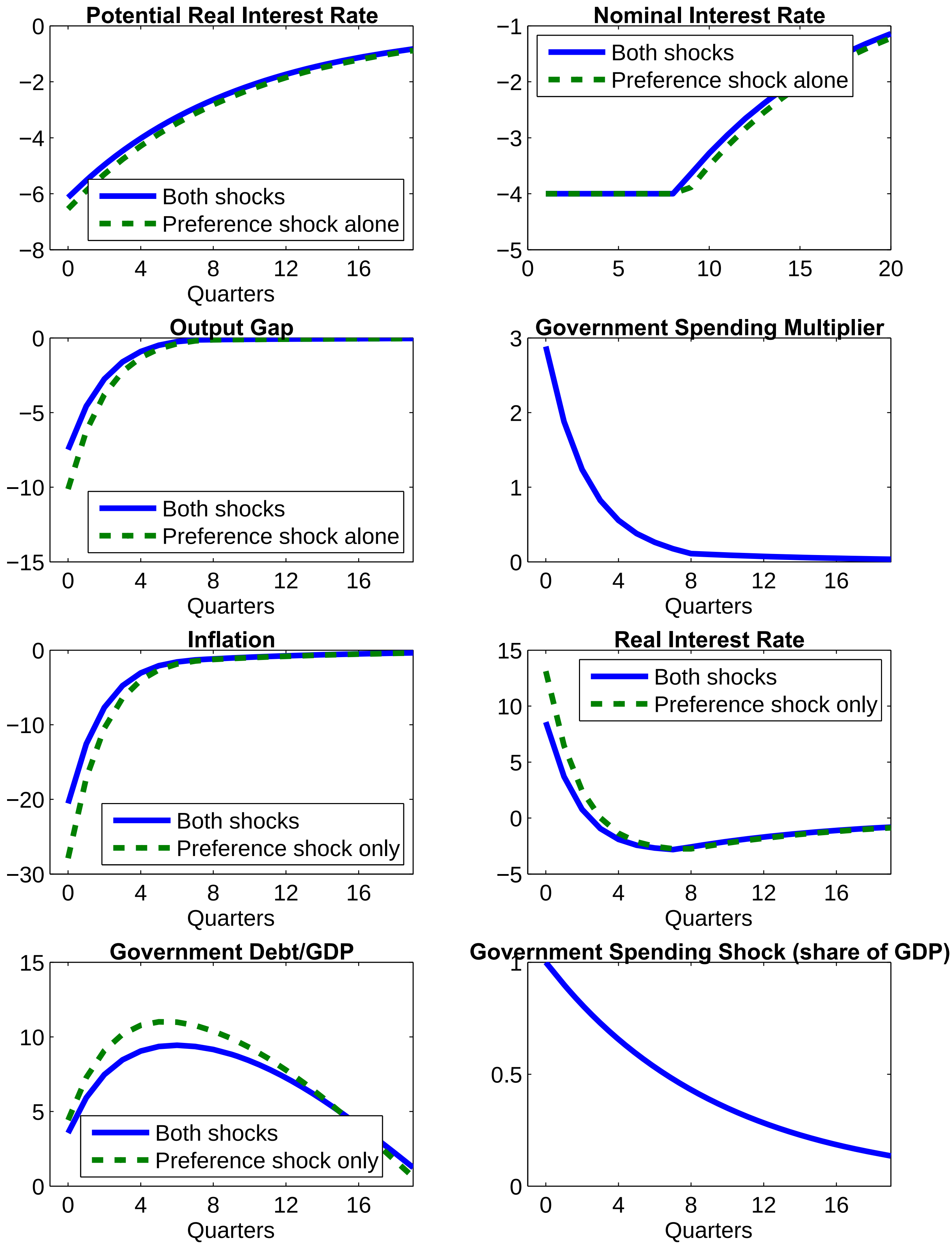

Figure 10: Evolution of Key Macrovariables 1987Q3−2009Q2
Solid Vertical Line Represents Third Quarter in 2008

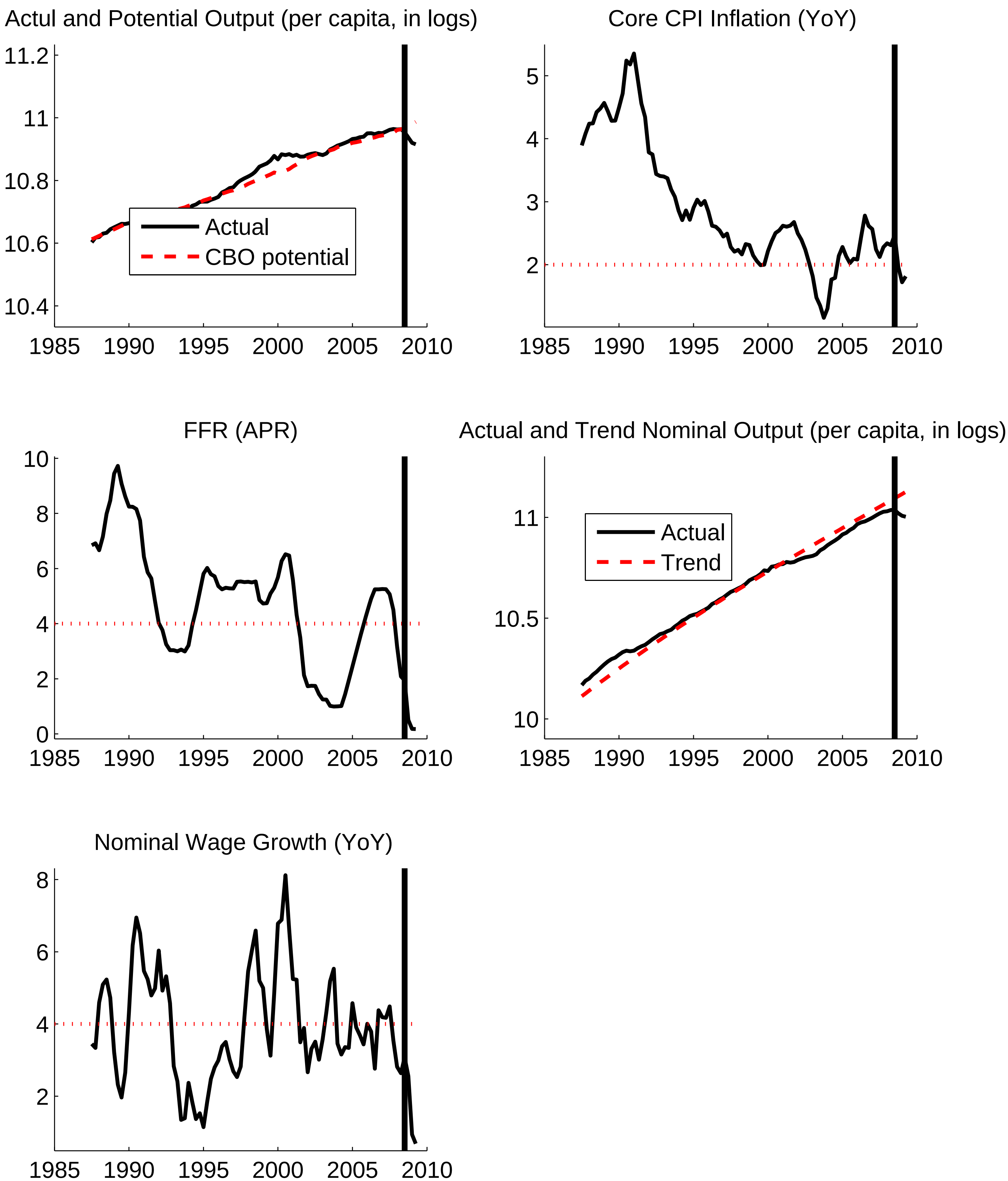

Figure 11: The Demand Driven Recession in the Model for Alternative Parameterizations and the Evolution of US Economy 2008Q3-2009Q2

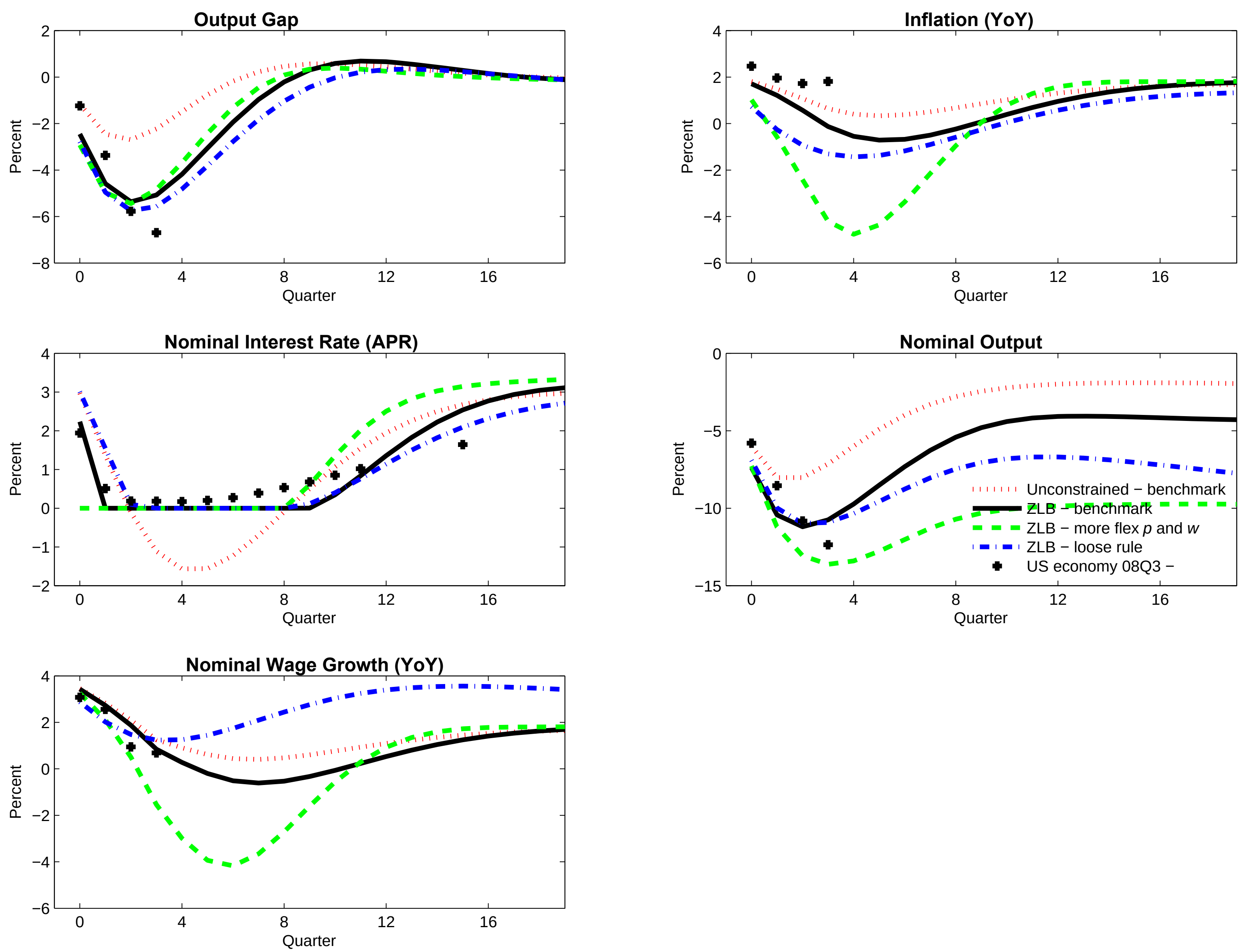

Figure 12: Responses to a Front-loaded Increase in Government Spending in Normal Times and in a Liquidity Trap in the CEE-SW Model with Capital

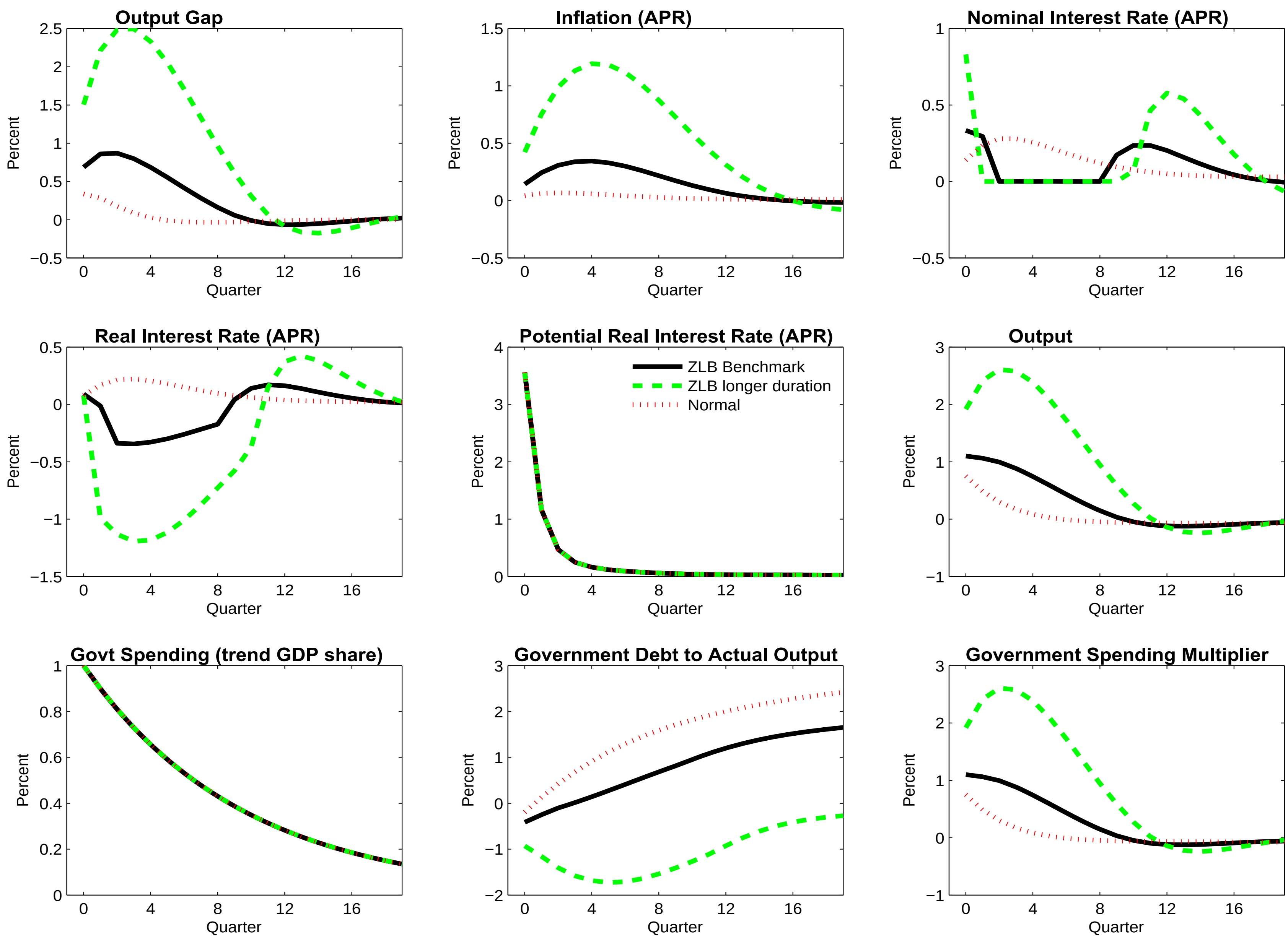

Figure 13: Average and Marginal Multipliers in the CEE-SW Model With Capital and their Sensitivity to the Degree of Price and Wage Stickiness

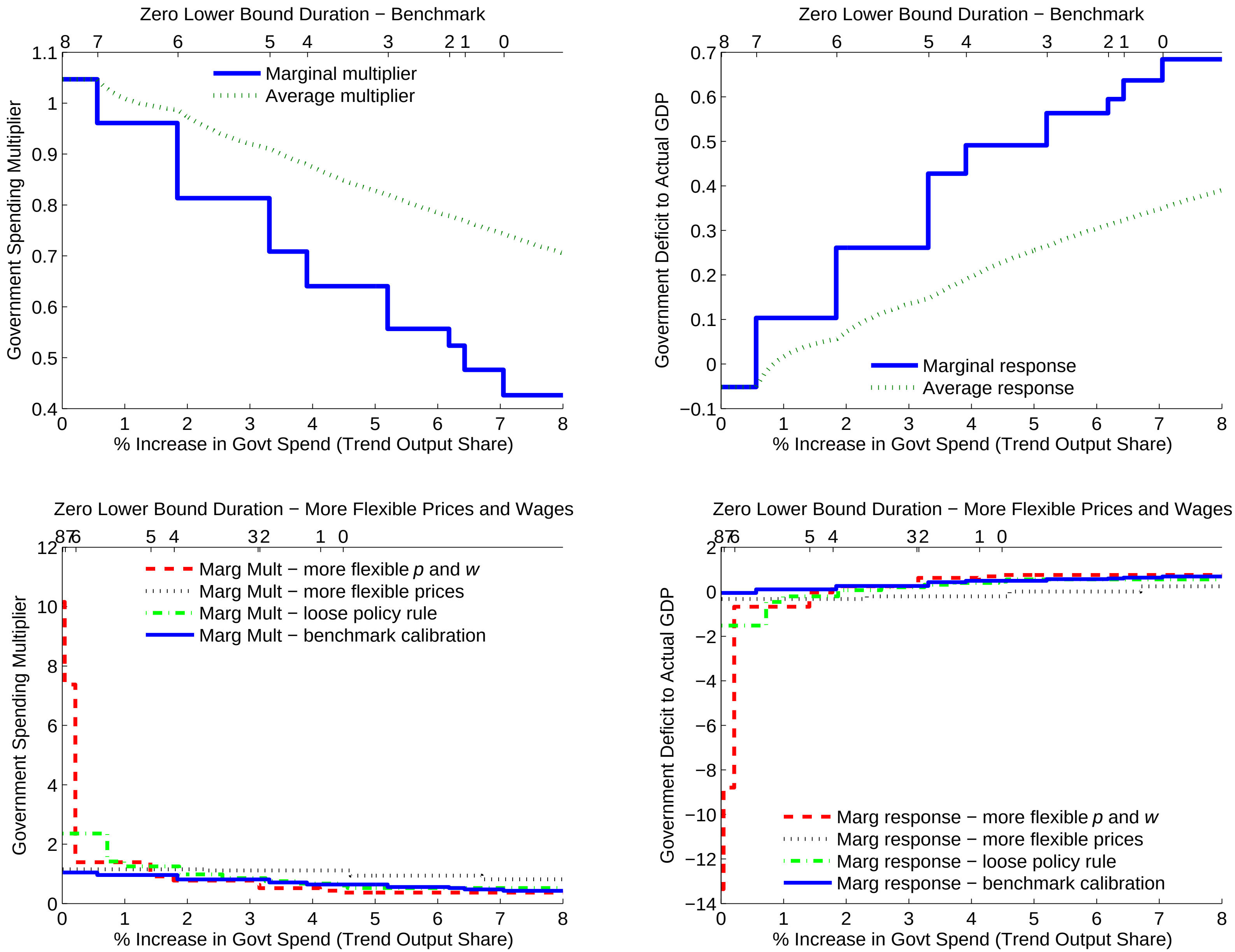

Figure 14: Responses to a Front-loaded Increase in Government Spending for Alternative Policy Rules in a Liquidity Trap in the CEE-SW Model with Capital

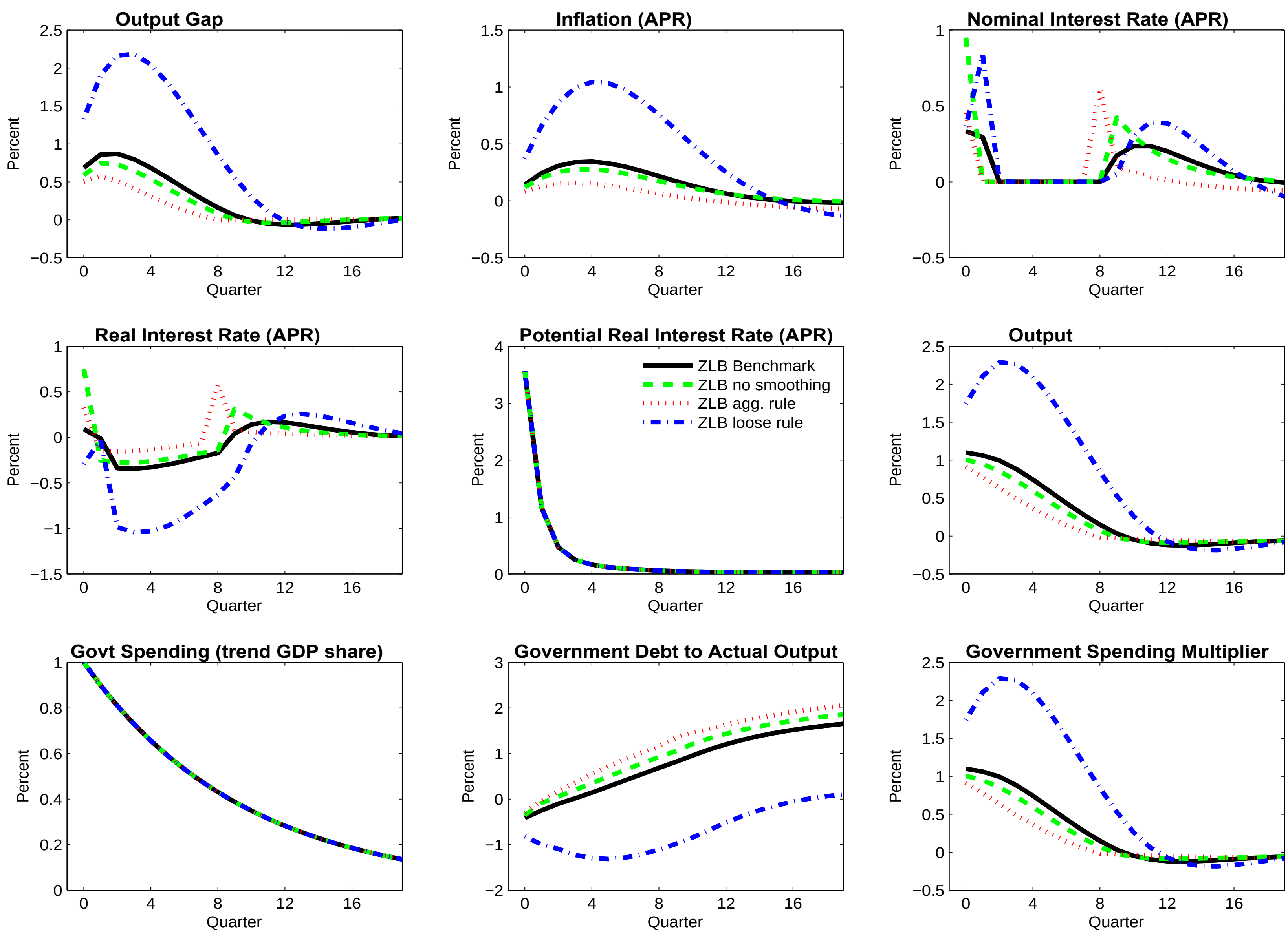

Figure 15: Responses to a Alternative Implementation and Financing of Government Spending Increase in the CEE-SW Model with Capital

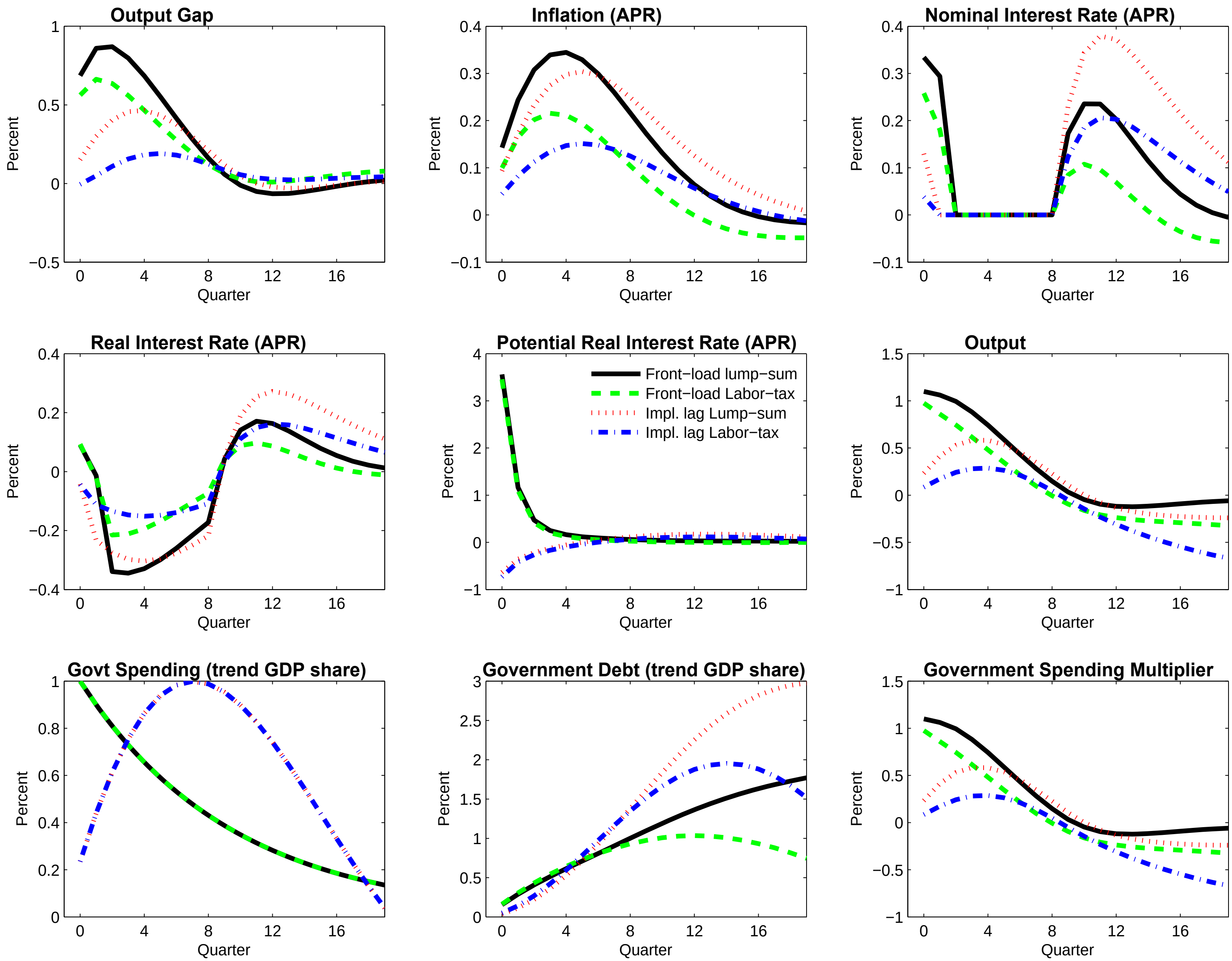

Figure 16: The Demand Driven Recession in the Full Model for Alternative Parameterizations and the Evolution of the US Economy 2008Q3-2009Q2

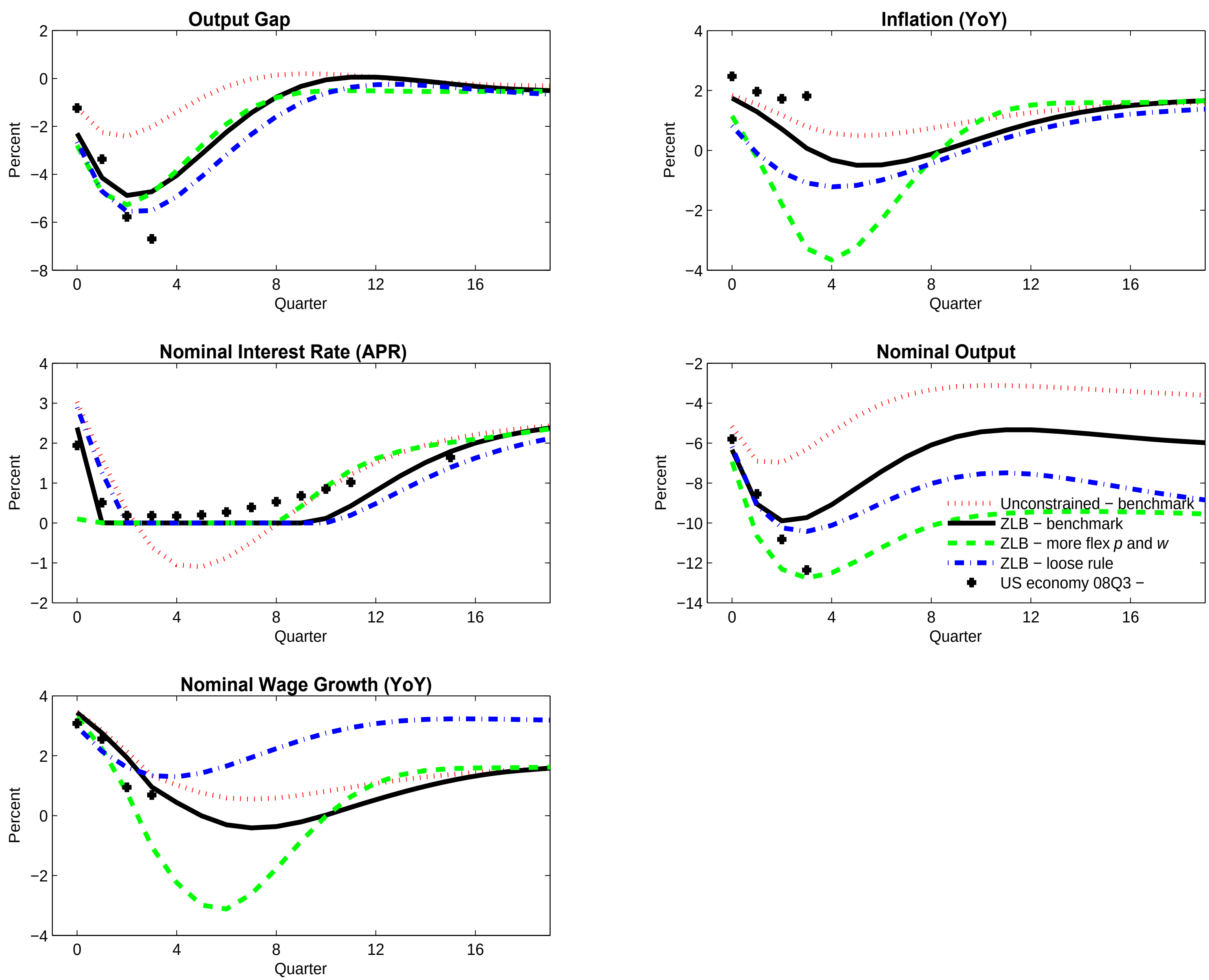

Figure 17: Responses to a Front-loaded Increase in Government Spending in Normal Times and in a Liquidity Trap in Model with Capital, Financial Frictions and Keynesian Agents

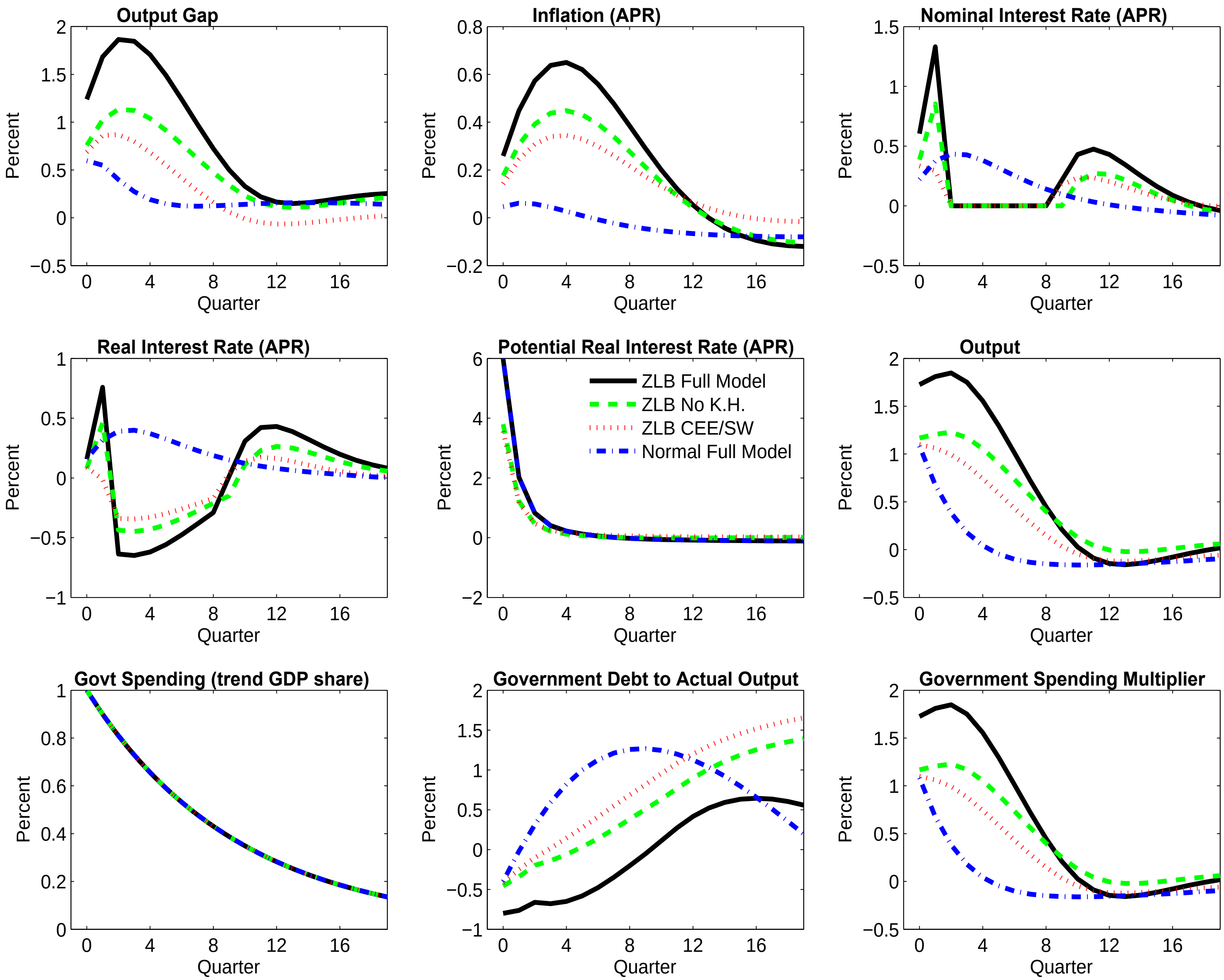

Figure 18: Responses to a Front-loaded Increase in Government Spending in Normal Times and in a Liquidity Trap in Model with Capital, Financial Frictions and Keynesian Agents

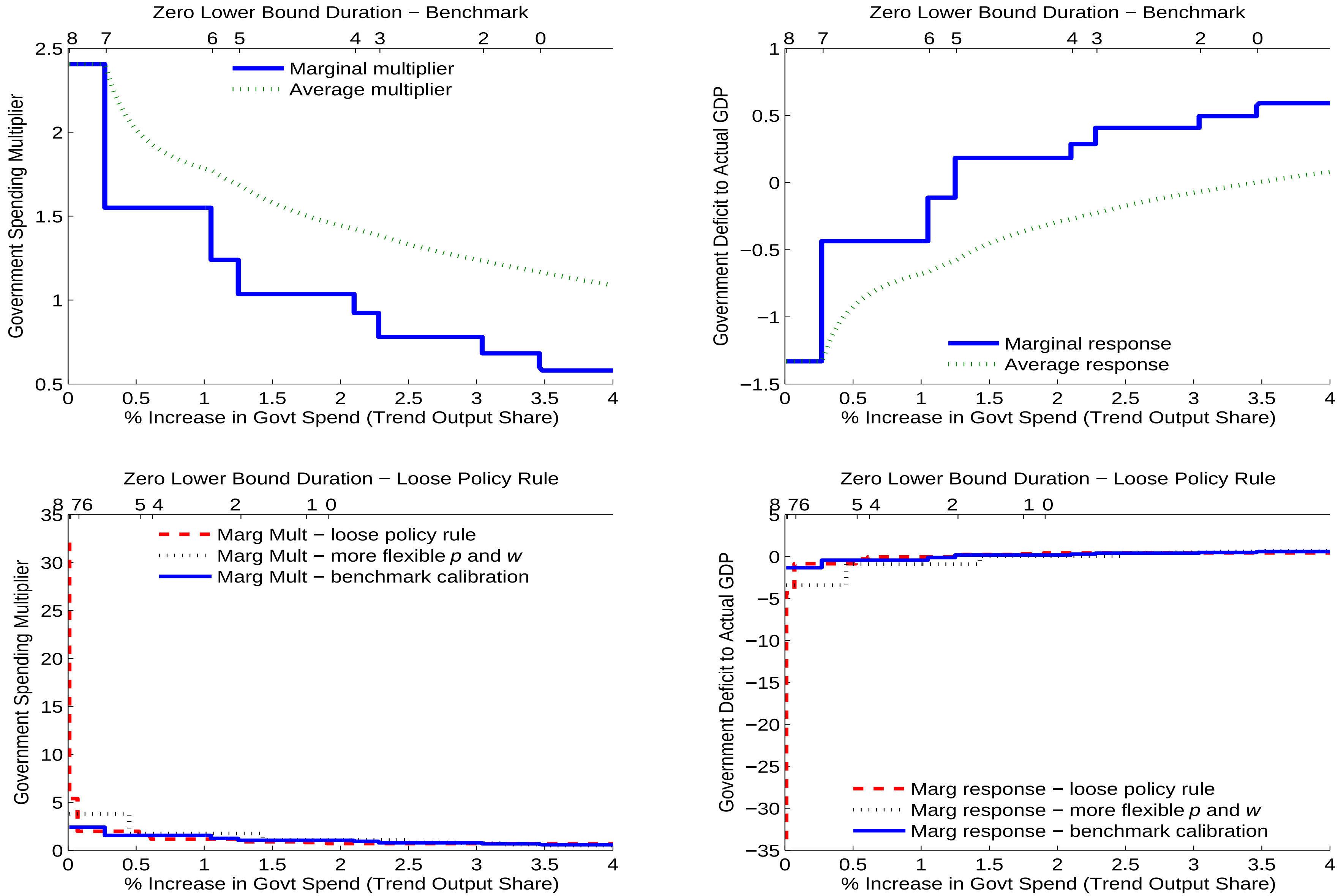

Figure 19: Responses to a Front-loaded Increase in Government Spending in Normal Times and in a Liquidity Trap in Model with Capital, Financial Frictions and Keynesian Agents