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TESTING THE MONETARY POLICY RULE IN THE US: A RECONSIDERATION OF THE FED'S BEHAVIOUR

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ABSTRACT

Testing the Monetary Policy Rule in the US: a Reconsideration of the Fed's Behaviour

We calibrate a standard New Keynesian model with three alternative representations of monetary policy- an optimal timeless rule, a Taylor rule and another with interest rate smoothing- with the aim of testing which if any can match the data according to the method of indirect inference. We find that the only model version that fails to be strongly rejected is the optimal timeless rule. Furthermore this version can also account for the widespread finding of apparent 'Taylor rules' and 'interest rate smoothing' in the data, even though neither represents the true monetary policy.

JEL Classification: E12, E17, E42, E52 and E58
Keywords: bootstrap simulation, indirect inference, monetary policy, New Keynesian model, Taylor-type rules, the 'target rule', VAR and Wald statistic

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1. Introduction

In this paper our aim is to uncover the principles according to which the Board of Governors of the US Federal Reserve System (the Fed) conducted monetary policy since the early 1980s. We do so in a novel way by asking which such principles can, when combined with a widely-accepted macro model, replicate the dynamic behaviour of the US economy during the sample period. By 'principles' we mean either an explicit rule the Fed follows (such as an interest-rate setting rule) or some other economic relationship that it aims to ensure occurs (such as a fixed exchange rate or as here an optimality condition).

The main context for this work is the influential paper by Taylor (1993), who-building on earlier work by Henderson and McKibbin (1993a, 1993b) which argued for the efficacy of interest rate rules- suggested that the Fed actually had been for some time systematically pursuing a particular interest rate rule, reacting directly to two 'gaps', one between inflation and its target rate, the other between output and its natural rate. Such a 'Taylor rule' was subsequently adopted widely in New Keynesian models to represent the behaviour of monetary policy (e.g., Rotemberg and Woodford (1997, 1998), Clarida, Gali and Gertler (1999, 2000), Rudebusch (2001), English, Nelson and Sack (2002)).

However, Minford, Perugini and Srinivasan (2001, 2002) and Cochrane (2007) have shown that a Taylor rule is not identified. Estimates of such a 'rule' may emerge from the data when the Fed is following quite other monetary policies; this is because a variety of relationships within the economy can imply a relationship between interest rate, inflation and output (gap) which mimics a Taylor rule. In the presence of such an identification problem, direct estimation of Taylor rules on the data does not establish whether the Fed was actually pursuing them or not. Some other way of testing hypotheses about monetary policy must be found. The one proposed here is to set up competing structural models which differ solely according to the monetary policies being followed, and to distinguish between these models according to the ability to replicate the dynamic behaviour of the data. Thus for example if one were to accept just one of these models and reject the rest, it would be reasonable to argue that this model succeeds because in it not only the rest of the economy but also monetary policy is well-specified. Of course other less decisive empirical outcomes of the tests are entirely possible.

The rest of this paper is organised as follows: section 2 reviews the work estimating monetary policy rules and the critique of it in terms of identification; section 3 outlines the micro-foundations of the simple New Keynesian model with which hypothetical rules in this paper are tested against the real data; section 4 explains the testing method and the results; section 5 discusses how the model can explain the apparent existence of Taylor Rules in the data; section 6 concludes.

2. Taylor Rules, Estimation and Identification

Taylor (1993) suggested that, at least for the post-1982 periods during which Alan Greenspan was chairman of the Fed, the Federal funds rate could be well described by the simple equation (with quarterly data) as:

$$i_t^A = \pi_t^A + 0.5x_t + 0.5(\pi_t^A - 0.02) + 0.02 + \xi_t \qquad [2.1]$$

where x_t is for the percentage deviation of real GDP from trend, π_t^A is the annual averaged rate of inflation over four quarters, with both the target of inflation and growth rate of the real GDP (with trend) set at 2 percents.

Equation [2.1] is the original 'Taylor rule'. However, a number of variants have also been proposed; for example, a Taylor rule where policy inertia is assumed could take the form as in Clarida, Gali and Gertler (1999) as

$$i_t^A = (1-\rho)[\alpha + \gamma_\pi(\pi_t^A - \pi^*) + \gamma_X x_t] + \rho i_{t-1}^A + \xi_t \qquad [2.2]$$

with ρ showing the degree of 'interest rate smoothing'. Others have involved lagging or leading the inflation and output gap terms- Rotemberg and Woodford (1997, 1998), Clarida, Gali and Gertler (1999, 2000), Rudebusch (2001), English, Nelson and Sack (2002).

Rules of these types are generally found to fit the actual data well in regression analysis, either via single-equation regression by GLS as in Rotemberg and Woodford (1997, 1998), Clarida, Gali and Gertler (1999, 2000) and Giannoni and Woodford (2005), or via full-model estimation by Maximum Likelihood as in Rotemberg and Woodford (1997, 1998), Smets and Wouters (2003), as well as Ireland (2007). However, besides the usual difficulties encountered in applied work (e.g., Carare and Tchaidze (2005) and Castelnuovo (2003)), these estimates face an identification problem pointed out in Minford, Perugini and Srinivasan (2001, 2002) and Cochrane (2007)- see also Minford (2008) which we use in what follows.

Lack of identification occurs when an equation could be confused with a linear combination of other equations in the model. Thus DSGE models give rise to the same correlations between interest rate and inflation as the Taylor rule, even if the Fed is doing something quite different, such as targeting the money supply. For example, Minford, Perugini and Srinivasan (2001, 2002) show this in a DSGE model with Fischer wage contracts (see also Gillman, Le and Minford (2007)).

In effect, unless the econometrician knows from other sources of information that the central bank is pursuing a Taylor rule, he cannot be sure he is estimating a Taylor rule when he specifies a Taylor rule equation because under other possible monetary policy rules a similar relationship to the Taylor rule is implied. Of course by specifying a Taylor rule he will successfully retrieve the coefficients of the 'rule'; but he cannot know that these describe the true rule the central bank is following.

To illustrate the point, we may consider a popular DSGE model but with a money supply rule instead of a Taylor rule:

(IS curve): $y_t = \gamma E_{t-1} y_{t+1} - \phi r_t + v_t$

(Phillips curve): $\pi_t = \zeta (y_t - y^*) + \nu E_{t-1}\pi_{t+1} + (1-\nu)\pi_{t-1} + u_t$

(Money supply target): $\Delta m_t = m + \mu_t$

(Money demand): $m_t - p_t = \psi_1 E_{t-1} y_{t+1} - \psi_2 R_t + \varepsilon_t$

(Fisher identity): $R_t = r_t + E_{t-1}\pi_{t+1}$

This model implies a Taylor-type relationship that looks like:

$$R_t = r^* + \pi^* + \gamma\chi^{-1}(\pi_t - \pi^*) + \psi_1\chi^{-1}(y_t - y^*) + w_t,$$

where $\chi = \psi_2\gamma - \psi_1\phi$, and the error term, w_t, is both correlated with inflation and output and autocorrelated; it contains the current money supply/demand and aggregate demand shocks and also various lagged values (the change in lagged expected future inflation, interest rate, the output gap, the money demand shock, and the aggregate demand shock). This particular Taylor-type relation was created with a combination of equations—the solution of the money demand and supply curves for interest rate, the Fisher identity, and the IS curve for expected future output[1]. But other Taylor-type relationships could be created with combinations of other equations, including the solution equations, generated by the model. They will all exhibit autocorrelation and contemporaneous correlation with output and inflation, clearly of different sorts depending on the combinations used.

All the above applies to identifying a single equation being estimated; thus one cannot distinguish a Taylor rule equation from the equations implied by the model and alternative rules when one just estimates that equation. One could attempt to apply further restrictions- e.g., on the error process- but such are hard to justify- e.g., the error in a Taylor rule ('monetary judgement' based on extraneous factors) can be autocorrelated (because those factors may be persistent).

However, when a 'monetary rule' is chosen for inclusion in a complete DSGE model, then the model imposes over-identifying restrictions through the rational expectations terms which involve in principle all the model's parameters. Thus a model with a

[1]From the money demand and money supply equation, $\psi_2\Delta R_t = \pi_t - m + \psi_1\Delta E_{t-1}y_{t+1} + \Delta\varepsilon_t - \mu_t$. Substitute for $E_{t-1}y_{t+1}$ from the IS curve and then inside that for real interest rates from the Fisher identity giving $\psi_2\Delta R_t = \pi_t - m + \psi_1(\frac{1}{\gamma})\{\phi(\Delta R_t - \Delta E_{t-1}\pi_{t+1}) + \Delta y_t - \Delta v_t\} + \Delta\varepsilon_t - \mu_t$; then, rearrange this as $(\psi_2 - \frac{\psi_1\phi}{\gamma})\Delta(R_t - R^*) = (\pi_t - m) - \frac{\psi_1\phi}{\gamma}\Delta E_{t-1}\pi_{t+1} + \frac{\psi_1}{\gamma}\Delta(y_t - y^*) - \frac{\psi_1}{\gamma}\Delta v_t + \Delta\varepsilon_t - \mu_t$, where the constants R^* and y^* have been subtracted from R_t and y_t respectively, exploiting the fact that when differenced they disappear. Finally, $R_t = r^* + \pi^* + \gamma\chi^{-1}(\pi_t - \pi^*) + \psi_1\chi^{-1}(y_t - y^*) + \{(R_{t-1} - R^*) - \psi_1\phi\chi^{-1}\Delta E_{t-1}\pi_{t+1} - \psi_1\chi^{-1}(y_{t-1} - y^*) - \psi_1\chi^{-1}\Delta v_t + \gamma\chi^{-1}\Delta\varepsilon_t - \gamma\chi^{-1}\mu_t\}$, where we have used the steady state property that $R^* = r^* + \pi^*$ and $m = \pi^*$.

particular rule is in general over-identified so that estimation by full information methods of that particular model as specified is possible (as in Rotemberg and Woodford (1997, 1998), Smets and Wouters (2003), Ontaski and Williams (2004) and Ireland (2007)). One way of putting this is that there are more structural parameters than reduced form parameters. Another is to note that the reduced form will change if the structural description of monetary policy changes- a point first made by Lucas (1976) in his 'critique' of conventional optimal policy optimization at that time, and some illustrations of how reduced forms will change for a model like the one in this paper (see Meenagh et al. (2008)). So if the econometrician posits a Taylor rule then he will retrieve its coefficients and those of the rest of the model under the assumption that it is the true structural monetary rule. He could then compare the coefficients for a model where he assumes some other rule. He can distinguish between the two models via their different reduced forms and hence their different fits to the data. Thus it is possible to identify the different rules of monetary policy behavior via full information estimation.

However, the identification problem does not go away, even when a model is over-identified in this way. The problem is that the Taylor rule included in such a model could be mimicking the joint behaviour of the rest of the model and some other (true) monetary rule (e.g., a money supply rule). Then including it in the model is capturing an approximation to this other monetary behaviour; it could fit the data to some reasonable approximation, therefore, because it is such an approximation. But until we have also estimated the possible alternative monetary rules with the same model, we cannot be sure that the chosen monetary rule is correctly specified. We need to check whether there is a better model which with its over-identifying restrictions may fit the data more precisely. In other words, the econometrician has not successfully identified the true rule until he has also estimated the rival models in order to distinguish them from the one he has assumed.

The identification problem thus shows itself here as the possibility that there is an alternative (true) model that could generate the Taylor rule features in the chosen model. In choosing to specify this model with a Taylor rule because there seems to be such a relationship in the data one would be estimating a mis-specified model, which nevertheless may provide an approximation to the data.

However, this points the way to a possible way forward. One may specify a complete DSGE model with alternative monetary rules and use the over-identifying restrictions to determine their differing behaviour. One may then test which of them is accepted by the data. Thus one may be able to select the 'true' model out of those that are merely approximating it. This is the approach taken here.

3. A Simple New Keynesian Model for Inflation, Output Gap and Nominal Interest Rate Determinations

We follow a common practice among New Keynesian authors of reducing a full DSGE model to a three-equation framework consisting of an 'IS' curve derived from the optimization problem of the representative household, a Phillips curve derived from the price-setting optimization problem of the representative firm, and a monetary policy rule (Clarida, Gali and Gertler (1999, 2000), Rotemberg and Woodford (1997, 1998), Walsh (2000)).

We adopt a standard New Keynesian DSGE set-up with representative agents as follows:

The Households

In each period, representative households are assumed to consume a composite of differentiated goods, which are produced by monopolistically competitive firms that make up of a continuum of measure 1. In particular, let the composite consumption that enters the utility function in each period be:

$$C_t = [\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj]^{\frac{\theta-1}{\theta}} \qquad [3.1]$$

where $\theta(>1)$ represents the price elasticity of demand for individual good j. Then the cost minimization process of the representative household implies the demand for individual good j could be shown as:

$$c_{jt} = (\frac{p_{jt}}{P_t})^{-\theta} C_t \qquad [3.2]$$

where p_{jt} is the price of individual good j and P_t is the general price level in period t[2].

For simplicity, assume that the representative agents care only about leisure and the level of composite consumption such that the life-time utility function to be maximized takes the form:

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i [\frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta}] \qquad [3.3]$$

where σ indicates the inverse of the intertemporal elasticity of substitution of consumption, whereas η shows the inverse of the elasticity of labour supply[3].

Suppose further that the representative agents own the firm and at the same time work as employees, the budget constraint (in real terms) they face in period t could be written as:

$$C_t + \frac{M_{t+1}}{P_t} + \frac{B_{t+1}}{P_t} = \frac{W_t}{P_t} N_t + \frac{M_t}{P_t} + (1+i_t)\frac{B_t}{P_t} + \Pi_t \qquad [3.4]$$

where M_t and B_t are respectively the initial stocks of money and nominal bond in each period, W_t is the nominal wage income, Π_t denotes the profit from running the firm and i_t defines the nominal rate of interest. Notice that the bond market is introduced such that the nominal interest rate is given a role. Note also (as will be seen from the firms' problem below) that the only factor that assumed to be input into the production process is labour, hence the only disposal income of representative households as employees is the wage income.

By assuming that consumptions can only be carried out by using cash (i.e., $P_t C_t = M_t$), the utility maximization problem of representative households can be described with the Lagrangean function as:

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[2] Details of this could be found in Walsh (2000, pp.232).

[3] The utility function here is deliberately assumed to be the same as in Woodford and Rotemberg (1998) as well as in Nistico (2007) such that the utility-based micro-founded quadratic social welfare loss function suggested by those authors are also applicable in this context; In contrast to Walsh (2000) where MIU is assumed, this model retains the role of money by taking the CIA approach, i.e., money does not directly enter the utility function, but only acts as a tool that eases transactions.

$$\underset{C_t, N_t, M_{t+1}, B_{t+1}}{Max} L_0 = E_0 \sum_{t=0}^{\infty} \beta^t \{ [\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\eta}}{1+\eta}]$$

$$- \lambda_t [C_t + \frac{M_{t+1}}{P_t} + \frac{B_{t+1}}{P_t} - \frac{W_t}{P_t} N_t - \frac{M_t}{P_t} - (1+i_t) \frac{B_t}{P_t} - \Pi_t]$$

$$- \mu_t [C_t - \frac{M_t}{P_t}]\} \qquad [3.5]$$

That is, representative households choose, in each period, the levels of consumption and labour input, as well as the end-of-period money stock and bond stock, such that the discounted life-time utility is maximized, subject to the budget constraint and the cash-in-advance constraint. The first order conditions of the problem are given as follows:

F.O.C.s:

C_t: $\quad C_t^{-\sigma} = (\lambda_t + \mu_t)$

N_t: $\quad \chi N_t^{\eta} = \lambda_t \frac{W_t}{P_t}$

M_{t+1}: $\quad \lambda_t (1 + \pi_{t+1}) = \beta(\lambda_{t+1} + \mu_{t+1})$

B_{t+1}: $\quad \lambda_t (1 + \pi_{t+1}) = \lambda_{t+1} \beta (1 + i_{t+1})$

By substituting around the F.O.C.s, the two important relationships about the household problem could be obtained:

$$C_t^{-\sigma} = \beta(1+i_t) E_t \frac{P_t}{P_{t+1}} C_{t+1}^{-\sigma} \qquad [3.6]$$

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} (1+i_t) = \frac{W_t}{P_t} \qquad [3.7]$$

In particular, [3.6] is the well-known ‘Euler’s equation’ which shows the optimal substitution between intertemporal consumptions that is commonly found in literature, whereas equation [3.7], equating the (discounted) real wage to the ratio of marginal utility of working to the marginal utility of consumption, indicates the implied optimal intratemporal marginal rate of substitution.

By Log-linearising equation [3.6] around zero-inflation steady state, it yields the linearised version of [3.6] as:

$$\tilde{c}_t = E_t\tilde{c}_{t+1} - (\frac{1}{\sigma})(\tilde{i}_t - E_t\pi_{t+1}) \qquad [3.6]'$$

where '~' denotes 'percentage deviation from steady state'[4].

Since no physical capital (and therefore no investment) is assumed for the economy, log-linearising the market clearing condition Y_t=C_t+G_t implies:

$$\tilde{c}_t = \tilde{y}_t + \ln(1 - \frac{G_t}{Y_t}) - \ln\frac{C}{Y} \qquad [3.8]$$

Combining equations [3.6]' and [3.8], it gives the 'IS' curve commonly found in the New Keynesian literature (See e.g., Clarida, Gali and Gertler (1999, 2000), Rotemberg and Woodford (1997, 1998), Walsh (2000)) as:

$$x_t = E_t x_{t+1} - (\frac{1}{\sigma})(\tilde{i}_t - E_t\pi_{t+1}) + v_t \qquad [3.9]$$

where $x_t \equiv \tilde{y}_t - \tilde{y}_t^f$, $v_t = (E_t\tilde{y}_{t+1}^f - \tilde{y}_t^f) + (E_t\hat{g}_{t+1} - \hat{g}_t)$ with $\hat{g}_t \equiv \ln(1 - \frac{G_t}{Y_t})$[5].

Notice in equation [3.9] that, 'output gap' x_t is defined as the difference between actual output and the output that would prevail if nominal flexibility were assumed, in contrast to the gap between actual and 'potential' outputs as traditionally defined. Also, v_t , which is interpreted as the 'demand shock', is a combination of disturbances that captures the effects of shocks to both technology and the exogenously determined government expenditure.

The 'IS' curve [3.9] describes the 'demand side' of the economy. As will be seen, it constitutes one of the structural equations of the simple New Keynesian model presented in this paper.

The Firms

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[4] In particular, $\tilde{i}_t \equiv i_t - i$.

[5] In Walsh (2000, pp. 244) where Y_t=C_t is assumed, it has: $v_t = E_t\tilde{y}_{t+1}^f - \tilde{y}_t^f$.

The supply side of the economy is characterised by a number of firms owned by the representative agents themselves. In particular, firms in the economy are assumed to operate under a monopolistically competitive environment with production function as:

$$y_{jt} = A_t N_{jt} \qquad [3.10]$$

where 'j' denotes the j[th] firm in the economy; A_t is the technology at time t, with $\log A_t = \xi \log A_{t-1} + z_t$, where z_t indicates shocks to productivity and is assumed to be independently identically distributed (i.i.d.). Note that equation [3.10] has assumed no physical capital will be used for production; and, the same technology, A_t, is employed across different industries.

Nominal (price) stickiness is introduced into the economy in the spirit of Calvo (1983). That is, for any given period t, only a fraction, $1-\omega$, of firms in the economy are able to reset prices to optimum, whereas the rest ω portion have to keep their prices unchanged due to the menu cost[6].

By implication of equation [3.2], the demand curve faced by each individual firm j takes the form:

$$y_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} Y_t \qquad [3.11]$$

Hence, in each period, firms producing differentiated goods but processing identical pricing strategy would set individual prices p_{jt}, subject to production constraint [3.10], the Calvo contract price-resetting probability $1-\omega$, as well as the demand constraint [3.11], such that (the discounted) real profits to firms are maximised.

Let φ denotes the real marginal cost to individual firms' production. The cost minimization process would imply:

$$\varphi_t = \frac{W_t / P_t}{A_t} \qquad [3.12]$$

[6] For simplicity, nominal wages in the labour market are presumed to be fully flexible.

It follows that the profit maximization problem of individual firm j can be described as:

$$\underset{p_{jt}}{Max}\, E_t \sum_{i=0}^{\infty} \omega^i \beta^i V_{i,t+i} [(\frac{p_{jt}}{P_{t+i}}) y_{j,t+i} - \varphi_{t+i} y_{j,t+i}] \qquad [3.13]$$

where $V_{i,t+i}$ is a discount factor, indicating the ratio of marginal utilities of consumption between periods.

Using the demand curve [3.11] to substitute away $y_{j,t+i}$, equation [3.13] can be rewritten as:

$$\underset{p_{jt}}{Max}\, E_t \sum_{i=0}^{\infty} \omega^i \beta^i V_{i,t+i} [(\frac{p_{jt}}{P_{t+i}})^{1-\theta} - \varphi_{t+i} (\frac{p_{jt}}{P_{t+i}})^{-\theta}] Y_{t+i} \qquad [3.13]'$$

The first order condition of [3.13]' with respect to individual price p_{jt} implies:

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i V_{i,t+i} Y_{t+i} [(1-\theta)(\frac{p_{jt}}{P_{t+i}}) + \theta \varphi_{t+i}] \frac{1}{p_{jt}} (\frac{p_{jt}}{P_{t+i}})^{-\theta} = 0 \qquad [3.14]$$

Log-linearising equation [3.14] around zero inflation steady state, it yields the optimal reset price for individual firm at time t as:

$$\tilde{p}^*_{jt} = (1-\omega\beta) \sum_{i=0}^{\infty} \omega^i \beta^i (E_t \tilde{\varphi}_{t+i} + E_t \tilde{P}_{t+i}) \qquad [3.15]$$

Given the 'Calvo contract', the general price level in each period can be written as the weighted average of the up-to-date reset prices and the old prices, with the weights being the reset probability, $1-\omega$, and its opposite, ω, respectively[7]. That is:

$$P_t = (1-\omega) p^*_{jt} + \omega P_{t-1} \qquad [3.16]$$

Log-linearization of equation [3.16] implies:

$$\pi_t = (1-\omega) \tilde{p}^*_{jt} + (\omega - 1) \tilde{P}_{t-1} \qquad [3.17]$$

Combining [3.15] and [3.17], it gives:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\omega)(1-\omega\beta)}{\omega} \tilde{\varphi}_t \qquad [3.18]^8$$

[7] Note that individual firms have exactly the same pricing strategy p^*_{jt} (or equivalently, $\tilde{p}^*_{jt}$).

Or more conveniently:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{\varphi}_t \qquad [3.18]'$$

where $\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}$.

Equation [3.18]' is the standard forward-looking *New Keynesian Phillips curve* that commonly derived in literature when the 'Calvo contract' is imposed (e.g., Walsh (2000), Clarida, Gali and Gertler (2001) and Holmberg (2006)). In particular, it states that inflation can be caused by expectations of the next period inflation, as well as the current deviations of real marginal cost from its steady state level.

In fact, equation [3.18]' can be further transformed such that inflation is related to the output gap as traditionally understood. This is done by log-linearising the real marginal cost equation [3.12] and the labour supply curve [3.7] (which was implied by the household's problem) and combining the results. After some tedious algebra, it can be shown that:

$$\tilde{\varphi}_t = (\eta + \sigma \frac{Y}{C})(\tilde{y}_t - \tilde{y}_t^f) = (\eta + \sigma \frac{Y}{C}) x_t \qquad [3.19]^9$$

In the spirit of Clarida, Gali and Gertler (2002), suppose further that the labour market is *not* perfectly competitive such that the mark-up of wages over the intratemporal substitution between consumption and labour is subject to stochastic shocks, equation [3.19] becomes:

$$\tilde{\varphi}_t = (\eta + \sigma \frac{Y}{C}) x_t + u_t^w \qquad [3.19]'$$

where u_t^w is interpreted as any stochastic disturbance that would cause deviations of such wage mark-ups. It follows that the New Keynesian Phillips curve [3.18]' can be rewritten as:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma x_t + u_t \qquad [3.20]$$

where $\gamma = \kappa(\eta + \sigma \frac{Y}{C})$, $u_t = k u_t^w$, and $\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}$.

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[8] Under rational expectations, equation [3.15] could be conveniently written as: $\tilde{p}_{jt}^* = \frac{(1-\omega\beta)}{(1-\omega\beta B^{-1})}(\tilde{\varphi}_t + \tilde{P}_t)$

[9] This result is obtained when the market-clearing condition is assumed to be $Y_t=C_t+G_t$; Had it been defined that $Y_t=C_t$, as in Walsh (2000), it would imply $\tilde{\varphi}_t = (\eta + \sigma) x_t$.

Equation [3.20] is the New Keynesian Phillips curve that relates inflation to the 'output gap' as traditionally understood, with u_t interpreted as the 'supply shock'[10]. It summarises the aggregate relationships of the 'supply side', and is therefore effectively the aggregate supply curve of the economy.

Monetary Policy

To close the model, most existing literature where the 'three-equation' New Keynesian framework is considered has employed an exogenously-specified Taylor-type rule that is similar to [2.2] as the monetary policy, such that a model for inflation, output gap and interest rate determinations is complete. We retain these exogenous rules for comparison. But as a natural alternative, this section starts with the situation in which the monetary authority is assumed to behave optimally- such that an 'optimal rule' would prevail.

Following Rotemberg and Woodford (1998) and Nistico (2007), let the 'social welfare loss' be defined as 'the loss in units of consumption as a percentage of steady-state output'. That is:

$$SWL_t \equiv \frac{U - U_t}{MU_c \cdot Y}$$

Under the Calvo (1983) pricing mechanism and utility function [3.3], Rotemberg and Woodford (1998) showed the social welfare loss function can approximately be expressed in terms of variance of inflation and output through second order Taylor expansion. Explicitly as in Rotemberg and Woodford (1998):

$$SWL_t = \frac{\psi}{2}[\alpha x_t^2 + \pi_t{}^2] \qquad [3.21]$$ [11]

where ψ is some measure of stickiness, α indicates the relative weight that central banks put on loss from output variations against that from inflation deviations[12].

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[10] Note, however, that under the particular setup in this paper, the fundamental driving force of such 'supply shock' is the stochastic disturbances to the wage mark-ups.

[11] Note it has implicated assumed that the steady state inflation is zero; also, the social welfare loss function [3.21] is *not* ad hoc but indeed micro-founded. The same expression is also derived by Nistico (2007), who assumed the Rotemberg (1982) pricing mechanism. In particular, Nistico (2007) showed that the relative weight α is equal to the ratio of the slope of the Phillips curve to the price elasticity of demand, i.e., $\alpha = \gamma/\theta$.

[12] Note that the value of ψ would affect the measurement of social welfare loss, but *not the form* of the implied optimal response based on that social welfare loss function, as it will be cancelled out in the F.O.Cs.

Assuming optimal behaviour and commitment by the monetary authority, the social planner's problem in each period involves minimizing the social welfare loss [3.21] subject to the Phillips curve [3.20] (i.e., the 'supply side' of the economy). In terms of a Lagrangean equation as shown in McCallum and Nelson (2001), the planner's problem can be written as:

$$\underset{\pi_{t+i}, x_{t+i}}{Min}\ L_t = E_t \sum_{i=0}^{\infty} \beta^i \{ \frac{\psi}{2} (\pi_{t+i}^2 + \alpha x_{t+i}^2) + \lambda_{t+i} (\gamma x_{t+i} + \beta \pi_{t+i+1} + u_{t+i} - \pi_{t+i}) \} \qquad [3.22]$$

Hence, in each period, the social planner chooses the level of inflation and output gap, such that the intertemporal discounted social welfare loss is minimized.

Suppose the problem above starts from time '1', the first order conditions with respect to π_t's and x_t's are:

$$\pi_1: \quad \psi \pi_1 - \lambda_1 = 0 \qquad \text{(the initial period)} \qquad [3.23]$$

$$\pi_t: \quad E_1(\psi \pi_t + \lambda_{t-1} - \lambda_t) = 0 \qquad t=2,3,\ldots\ldots \qquad [3.24]$$

$$y_t: \quad E_1(\psi \alpha x_t + \gamma \lambda_t) = 0 \qquad t=1,2,3,\ldots\ldots. \qquad [3.25]$$

For optimal policy with commitment, we follow the 'timeless perspective' advocated by McCallum and Nelson (2001), as well as Woodford (1999), in which the potential problem of time inconsistency is avoided through 'ignoring any conditions prevailing at the regime's inception' (McCallum and Nelson, 2001, pp.4). The optimal response under 'timeless perspective' is derived by combining equations [3.24] and [3.25], while the initial period condition [3.23] is neglected. The optimal policy is found to take the form:

$$\pi_t = -\frac{\alpha}{\gamma}(x_t - x_{t-1}) \qquad [3.26]^{13}$$

Equation [3.26] is what McCallum and Nelson (2001) called the 'target rule'. It states that the optimal policy that would intertemporally minimize the social welfare loss is such that the rate of inflation in each period is kept as a constant fraction of the first difference of output gap.

[13] Exactly the same results can be found in Clarida, Gali and Gertler (1999, pp.41), as well as McCallum and Nelson (2001, pp.7).

Note that equation [3.26] is the relationship that is achieved if the monetary authority has been able to behave optimally. We will assume that the implementation of monetary policy is subject to 'policy shocks' so that the required relationship [3.26] does not always hold, and so we obtain:

$$\pi_t = -\frac{\alpha}{\gamma}(x_t - x_{t-1}) + \xi_t \qquad [3.26]'$$

where ξ_t is interpreted as the 'policy shock' that causes distortions to the optimal condition [3.26].

The Simplified Model

So far, the 'IS' curve [3.9], the New Keynesian Phillips curve [3.20], as well as the optimal response [3.26]' constitute a fully micro-founded New Keynesian model for inflation, interest rate and output gap determinations, where the monetary authority is assumed to behave optimally subject to a policy shock. We treat the disturbances as representing omitted variables and following an AR(1) process. The log-linearised model can then be summarised as:

$$x_t = E_t x_{t+1} - (\frac{1}{\sigma})(\tilde{i}_t - E_t \pi_{t+1}) + v_t \qquad [3.9]$$

$$\pi_t = \beta E_t \pi_{t+1} + \gamma x_t + \kappa u_t^w \qquad [3.20]$$

$$\pi_t = -\frac{\alpha}{\gamma}(x_t - x_{t-1}) + \xi_t \qquad [3.26]'$$

with $v_t = \rho_v v_{t-1} + \varepsilon_t^v$, $u_t^w = \rho_{u^w} u_{t-1}^w + \varepsilon_t^{u^w}$, $\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t^\xi$,

where $\gamma = \kappa(\eta + \sigma\frac{Y}{C})$, $\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}$

and $\varepsilon_t^\xi \sim N(0, \sigma^\xi)$, $\varepsilon_t^{u^w} \sim N(0, \sigma^{u^w})$, $\varepsilon_t^v \sim N(0, \sigma^v)$.

While monetary policy in the baseline model above is governed by the optimal response [3.26]', notice that equation [3.26]' can be easily replaced with other exogenous rules (e.g., Taylor-type rules), such that models fitted with alternative policy regimes are readily comparable.

4. Identification of Monetary Policy Rules with Tests

4.1. The Testing Strategy

This paper aims at testing three typical rules that are usually modelled as the prevailing monetary policy in the US. They are the 'target rule' [3.26]', the original Taylor rule [2.1] and the Taylor rule with 'interest rate smoothing' [2.2], respectively. In particular, since the meaning of a hypothetical rule that is not backed with a structural model is ambiguous, the rules to be tested are all fitted into the baseline model outlined in the previous section such that three different pseudo economies can be constructed as follows:

Table 4.1: models to be tested

Model one	('IS'+PP+ the 'target rule')	
'IS' curve	$x_t = E_t x_{t+1} - (\frac{1}{\sigma})(\tilde{i}_t - E_t \pi_{t+1}) + v_t$	$v_t = \rho_v v_{t-1} + \varepsilon_t^v$
PP curve	$\pi_t = \beta E_t \pi_{t+1} + \gamma x_t + \kappa u_t^w$	$u_t^w = \rho_{u^w} u_{t-1}^w + \varepsilon_t^{u^w}$
Policy rule	$\pi_t = -\frac{\alpha}{\gamma}(x_t - x_{t-1}) + \xi_t$	$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t^\xi$
Model two	('IS'+PP+ the original Taylor rule)	
'IS' curve	$x_t = E_t x_{t+1} - (\frac{1}{\sigma})(\tilde{i}_t - E_t \pi_{t+1}) + v_t$	$v_t = \rho_v v_{t-1} + \varepsilon_t^v$
PP curve	$\pi_t = \beta E_t \pi_{t+1} + \gamma x_t + \kappa u_t^w$	$u_t^w = \rho_{u^w} u_{t-1}^w + \varepsilon_t^{u^w}$
Policy rule	$i_t^A = \pi_t^A + 0.5 x_t + 0.5(\pi_t^A - 0.02) + 0.02 + \xi_t$	
The transformed policy rule	$\tilde{i}_t = 1.5\pi_t + 0.125 x_t + \xi_t^{'}$ [14]	$\xi_t^{'} = \rho_\xi \xi_{t-1}^{'} + \varepsilon_t^\xi$

[14] Note that both the rates of interest and inflation are transformed to quarter rates; all constants in the transformed rule are dropped because the tests will use data that are in deviations from their sample means, as will be revealed in the 'Data' section. Also, recall that $\tilde{i}_t \equiv i_t - i = i_t - (\frac{1}{\beta} - 1)$ if zero-inflation steady state is assumed.

Model three	('IS'+PP+ Taylor rule with 'interest rate smoothing')	
'IS' curve	$x_t = E_t x_{t+1} - (\frac{1}{\sigma})(\tilde{i}_t - E_t \pi_{t+1}) + v_t$	$v_t = \rho_v v_{t-1} + \varepsilon_t^v$
PP curve	$\pi_t = \beta E_t \pi_{t+1} + \gamma x_t + \kappa u_t^w$	$u_t^w = \rho_{u^w} u_{t-1}^w + \varepsilon_t^{u^w}$
Policy rule	$i_t^A = (1-\rho)[\alpha + \gamma_\pi (\pi_t^A - \pi^*) + \gamma_X x_t] + \rho i_{t-1}^A + \xi_t$	
The transformed policy rule	$\tilde{i}_t = (1-\rho)[\gamma_\pi \pi_t + \gamma_x' x_t] + \rho \tilde{i}_{t-1} + \xi_t'$ [15]	$\xi_t' = \rho_\xi \xi_{t-1}' + \varepsilon_t^\xi$

As table 4.1 shows, the three models only differ in the ways in which monetary policies are implemented. Hence, by comparing their capacity to fit the real data, one should be able to tell which rule, when included in a simple New Keynesian model, provides the best explanation for the observed 'stylized facts' and therefore the most appropriate description of the existing monetary policy, since it is only the policy rules that differ.

4.2. Methodology—Testing the Models Using Indirect Inference

To evaluate each model's fitness to the actual data, this paper adopts the approach that is proposed in Minford, Theodoridis and Meenagh (2007), where models are tested using *indirect inference*[16].

Specifically, when a theoretical model is tested against the actual data, an *auxiliary model* that is completely independent of the theoretical one is employed to produce descriptors of the data against which the performance of the theory can be evaluated indirectly. Such descriptors can be either the estimated parameters of the auxiliary model or functions of these. While these are treated as the 'reality', the theoretical model being evaluated is simulated to find its implied values for them.

Indirect inference has been widely used in the estimation of structural models (e.g., Smith (1993), Gregory and Smith (1991, 1993), Gourieroux et al. (1993), Gourieroux and Monfort (1995) and Canova (2005)). Here we make a different use of indirect

[15] See footnote 14.

[16] For more applications of this approach, see Meenagh, Minford and Wickens (2008) and Le, et al. (2008, 2009).

inference as our aim is to evaluate an already estimated or calibrated structural model. The common element is the use of an auxiliary time series model. In estimation the parameters of the structural model are chosen such that when this model is simulated it generates estimates of the auxiliary model similar to those obtained from the actual data. The optimal choices of parameters for the structural model are those that minimise the distance between a given function of the two sets of estimated coefficients of the auxiliary model. Common choices of this function are the actual coefficients, the scores or the impulse response functions. In model evaluation the parameters of the structural model are taken as given. The aim is to compare the performance of the auxiliary model estimated on simulated data derived from the given estimates of a structural model - which is taken as a true model of the economy, the null hypothesis - with the performance of the auxiliary model when estimated from the actual data. If the structural model is correct then its predictions about the impulse responses, moments and time series properties of the data should statistically match those based on the actual data. The comparison is based on the distributions of the two sets of parameter estimates of the auxiliary model, or of functions of these estimates.

In other words, the testing procedure involves first constructing the errors derived from the previously estimated structural model and the actual data. These errors are then bootstrapped and used to generate for each bootstrap new data based on the structural model. An auxiliary time series model is then fitted to each set of data and the sampling distribution of the coefficients of the auxiliary time series model is obtained from these estimates of the auxiliary model. A Wald statistic is computed to determine whether functions of the parameters of the time series model estimated on the actual data lie in some confidence interval implied by this sampling distribution.

Following Minford, Theodoridis and Meenagh (2007), this paper takes a VAR(1) for the three macro variables (interest rate, output gap and inflation) as the appropriate auxiliary model and treats as the descriptors of the data the VAR coefficients and the variances of the three variables. The Wald statistic is computed from these[17]. This

[17] Note that the VAR impulse response functions, the co-variances, as well as the auto/cross correlations of the left-hand-side variables will all be implicitly examined when the VAR coefficient matrix is considered, since the formers are functions of the latter.

tests whether the observed dynamics and volatility of the chosen variables are explained by the simulated joint distribution of the corresponding parameters at a given confidence level. The Wald statistic is given by:

$$(\Phi - \overline{\Phi})'\Sigma_{(\Phi\Phi)}{}^{-1}(\Phi - \overline{\Phi}) \qquad [4.1]$$

where Φ is the vector of VAR estimates of the concerned parameters yielded in each simulation, with $\overline{\Phi}$ and $\Sigma_{(\Phi\Phi)}$ representing the corresponding sample means and variance-covariance matrix of these estimates calculated across simulations, respectively. The whole test procedure can be illustrated diagrammatically in Figure 4.1 as follows:

Figure 4.1: the principle of testing using indirect inference

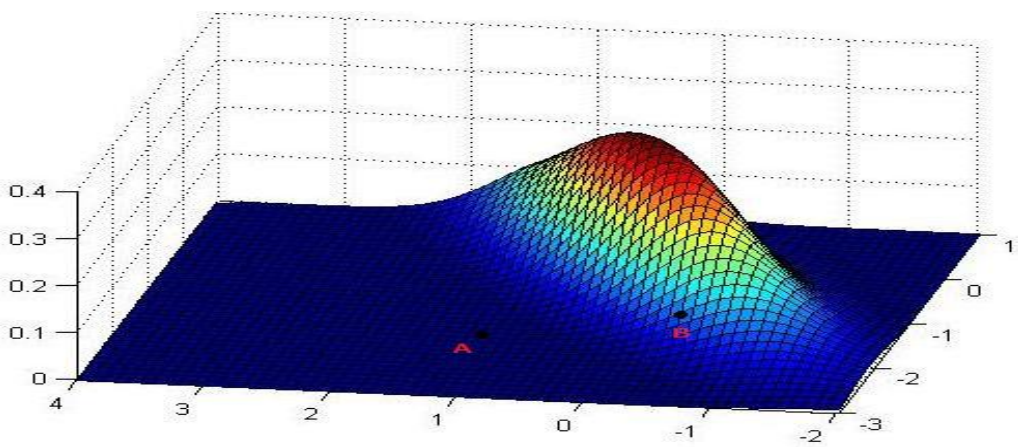
Panel A:

Model(s) to be tested
↓ (Bootstrap simulations)

Actual data		Simulated data
↓		↓
VAR representation		VAR representation
↓		↓
The VAR inference (the 'reality')	*vs.*	Distribution(s) of the VAR inference

Wald statistic

Panel B:



While the first panel in Figure 4.1 summarises the main steps of the methodology described in the past two paragraphs, the 'mountain-shaped' diagram replicated from

Meenagh, Minford and Wickens (2008, pp.8) in panel B gives an example of how the 'reality' is compared to model predictions using the Wald test when only two parameters of the auxiliary model are concerned: let either of the spots in panel B indicate the real-data-based estimates of the two concerned parameters and the 'mountain' represent their corresponding joint distribution generated from model simulations; when the real-data-based estimates are given at point A, the theoretical model in hand will fail to provide a sensible explanation for the real world, since what the model predicts is too 'far away' from what the 'reality' suggests; by contrast, if the real-data-based estimates are given at point B, which, according to the diagram, means the 'reality' is captured by the model-implied joint distribution of the corresponding parameters, the hypothesis that 'the real data are generated by the model under discussion' will be completely possible, although how likely that will be the case is dependent on what is reported for the Wald-statistic[18].

While one may refer to Minford, Theodoridis and Meenagh (2007) and Meenagh, Minford and Wickens (2008), as well as Le, et al. (2008, 2009), where a 'three-step' test procedure is explained, for more details, it is worth emphasising that the simulated joint distribution of the concerned VAR parameters mentioned above is a bootstrapped distribution, generated from the innovations implied by the data and the theoretical model[19]. This has the advantage over asymptotic distribution estimates that it uses the actual errors implied by the model and allows for any small sample effects. Under the null hypothesis that the theoretical model is true, the model-implied innovations will provide us with a sample of 'true' shocks with which the genuine distributions of innovations can be bootstrapped. In particular, to preserve any contemporaneous correlation between the innovations, the bootstraps are drawn as time vectors.

4.3. Data and Calibration

Data

[18] Note that in this particular example, only two parameters are considered and they are both assumed to be normally-distributed. Yet, the principle of the Wald test would not be changed for more general cases, where more parameters, which may follow various kinds of distribution, are concerned.

[19] By bootstrapping the innovations to Taylor-type rules, we mean those from the transformed equations.

For testing the prevailing monetary policy in the US, this paper employs the *quarterly* data published by the Federal Reserve Bank of St. Louis from 1982Q2 to 2007Q4, when most of the period are covered by the 'Greenspan era', during which the US economy is thought to have been governed by one identical monetary regime and most discussions about the Fed's behaviour are concerned[20].

Regarding the three endogenous variables involved, $\tilde{i}_t$ is defined as the deviation of current interest rate from the steady-state value, output gap x_t is defined as the percentage deviation of current real output from its HP trend, whereas π_t indicates the quarterly inflation rate measured as the log difference between the current CPI and the one captured in the last quarter[21]. For simplicity, the tests use data that are in deviations from means[22]. In particular, a linear trend is taken out of the interest rate series such that stationarity is ensured. Figure 4.2 to figure 4.4 below plot each of these series in deviation forms; the relevant unit root test results are also presented in table 4.2.

Figure 4.2: $\tilde{i}_t$

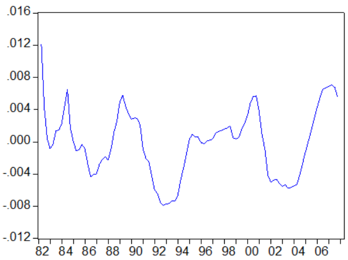


Figure 4.3: x_t

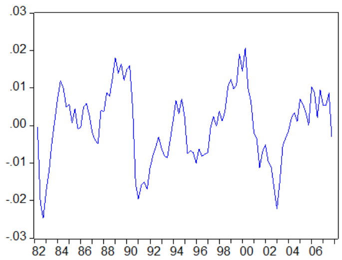


Figure 4.4: π_t

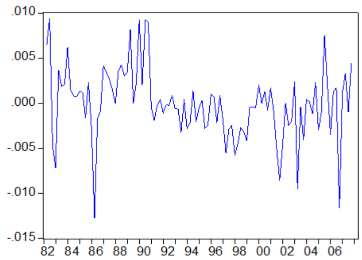


Table 4.2 Unit Root Tests for Stationarity

Time series	5% critical value	ADF test statistics	p-values*
$\tilde{i}_t$	-1.94	-2.81	0.0053
x_t	-1.94	-2.95	0.0035
π_t	-1.94	-3.60	0.0004

Note: '*' denotes the Mackinnon (1996) one-sided p-values at 5% level of significance; H_0: the time series has a unit root.

[20] Data base of Federal Bank of St. Louis: http://research.stlouisfed.org/fred2/
[21] Notice that the annual rates of interest proposed by the Fred are purposely adjusted into quarterly rates such that the frequencies of all the time series are kept consistently on quarterly basis.
[22] Nevertheless, the time series of output gap used is in level, as its sample mean is not significantly different from zero.

Note that since all the data used are in deviation from mean, a VAR(1) representation of them would contain no constant but only nine parameters in the autoregressive coefficient matrix. Also, the use of such data requires dropping the constants in any equation of the models as well. This explains why the two transformed Taylor rules involved in model two and three have no constant at all.

Calibration

The values of parameters chosen for the tests are those commonly calibrated and accepted for the US economy in literature. These parameters and their values are listed in table 4.3 as follows:

Table 4.3 Calibration of Parameters

Parameters	Definitions	Calibrated values	
β	time discount factor	0.99	
σ	inverse of elasticity of intertemporal consumption	2	
η	inverse of elasticity of labour	3	
ω	Calvo contract price non-adjusting probability	0.53	
G/Y	steady-state government expenditure to output ratio	0.23	
Y/C	steady-state output to consumption ratio	1/0.77	(implied)
κ	$\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}$	0.42	(implied)
γ	$\gamma = \kappa(\eta + \sigma\frac{Y}{C})$	2.36	(implied)
θ	price elasticity of demand	6	
$\alpha/\gamma \equiv \theta^{-1}$	(see footnote 11)	1/6	(implied)
ρ	degree of interest rate smoothness	0.76	
γ_π	interest rate response to inflation	1.44	
$\gamma_x^{'}$	interest rate response to output gap	0.14	
ρ_v	autoregressive coefficient of demand disturbance	0.91	(sample estimate)
ρ_{u^w}	autoregressive coefficient of supply disturbance	0.82	(sample estimate)
ρ_ξ	autoregressive coefficient of policy disturbance: model one	0.35	(sample estimate)

ρ_ξ	autoregressive coefficient of policy disturbance: model two	0.37	(sample estimate)
ρ_ξ	autoregressive coefficient of policy disturbance: model three	0.31	(sample estimate)

As table 4.3 shows, the quarterly time discount rate is calibrated as 0.99, implying an approximately 1% quarterly (or equivalently 4% annual) rate of interest in steady state. σ and η are set to as high as 2 and 3 respectively as in Carlstrom and Fuerst (2008), who emphasized on the values' consistency with the inelasticity of both intertemporal consumption decision and labour supply shown by the US data. The Calvo price stickiness of 0.53 and the price elasticity of demand of 6 are both taken from Kuester, Muller and Stolting (2007). Note that these values accordingly imply a contract length of more than two and a half quarters[23], while the constant mark-up of price to nominal marginal cost is 1.2. The implied steady-state output-consumption ratio of 1/0.77 is calculated based on the steady-state ratio of government expenditure over output of 0.23 calibrated in Foley and Taylor (2004). Regarding the Taylor rule tested in model three, again, calibration follows those in Carlstrom and Fuerst (2008), where the interest rate's response to a unit change in inflation and output gap are 1.44 and 0.14 respectively, with the degree of 'smoothness' of 0.76. The last five lines in table 4.2 also report the autoregressive coefficients of the structural disturbances implied by the models, which are all sample estimates based on the real data[24]. Notice that both of the demand and supply shocks are shown to be highly persistent, in contrast to the policy shocks reflected in all the three models.

4.4. Evaluating the Models' Performances—the Test Results

The test results and the corresponding evaluations for the three models proposed are presented in turn in this subsection, where the simulated 95% lower bounds and upper bounds for the concerned parameters, their real-data-based counterparts, as well as the relevant Wald statistics, are considered[25]. Since there are three endogenous variables, namely, interest rate, output gap, and the rate of inflation, in the VAR(1)

[23] To be accurate, $2\varpi^{-1} - 1 \approx 2.77$.
[24] These estimates are all significant at 5% significance level.
[25] Note that the relevant diagrams for the VAR impulse response functions and cross-correlations between variables are also plotted with lower bounds and upper bounds in appendix.

representation, twelve components are involved in calculation of the Wald statistics; these are the nine VAR coefficients and the three variances of the L.H.S. variables[26]. The detailed results for each model are as follows:

Model one ('IS'+PP+the 'target rule')

Table 4.4 below summarises the test results regarding the dynamic properties of model one:

Table 4.4: Individual VAR coefficients and the 'directed' Wald statistic

VAR(1) Coefficients	95% lower bound	95% upper bound	Values estimated with real data	In/Out
β_{11}	0.6454	0.9420	0.8017	In
β_{12}	- 0.0844	0.0439	0.0834	<u>**Out**</u>
β_{13}	- 0.1774	0.0991	0.0112	In
β_{21}	- 0.2589	0.2578	- 0.2711	<u>**Out**</u>
β_{22}	0.6685	0.9105	0.9009	In
β_{23}	- 0.4037	0.1871	- 0.1090	In
β_{31}	- 0.1821	0.1595	-0.0187	In
β_{32}	- 0.0434	0.1361	0.1428	<u>**Out**</u>
β_{33}	0.1010	0.4976	0.2552	In
'Directed' Wald statistic (for dynamics)			98.2%	

According to table 4.4, three out of the nine real-data-based estimates of the VAR coefficients that reflect the actual dynamics are found to lie outside their corresponding 95% bounds implied by the theoretical model. Specifically, the response of interest rate to the lagged output gap and the response of output gap to the lagged interest rate, as well as the response of inflation to the lagged output gap, are all shown to be more aggressive than what the theoretical model would predict. In

[26] Note that the VAR(1) representation is assumed to take the form:

$$\begin{bmatrix} \tilde{i}_t \\ x_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} \begin{bmatrix} \tilde{i}_{t-1} \\ x_{t-1} \\ \pi_{t-1} \end{bmatrix} + \Sigma_t$$

particular, the interest rate's response to the lagged output gap in reality is more than twice as great as what could be generated from model simulations. Overall, the 'directed' Wald statistic is reported as 98.2%; this indicates the model's success in capturing the actual dynamics at the 99% confidence level, although it clearly fails at the more conventional 95% level. Clearly, all the DSGE models here have problems fitting the data closely; our main purpose is to rank them and to see if one of them stands out as relatively acceptable.

Turing to the other aspect of the concerned 'stylized facts', table 4.5 below shows the extent to which the observed volatilities of real data are explained by the theoretical model:

Table 4.5: Volatilities of the endogenous variables and the 'directed' Wald statistic

Volatilities of the endogenous variables	95% lower bound	95% upper bound	Values calculated with real data	In/Out
$\mathrm{var}(\tilde{i})$	0.0102	0.0450	0.0171	In
$\mathrm{var}(x)$	0.0411	0.1601	0.0951	In
$\mathrm{var}(\pi)$	0.0094	0.0206	0.0153	In
'Directed' Wald statistic (for volatilities)			10.4%	

Note: Values reported in table 4.5 are magnified by 1000 times as their original values.

As table 4.5 shows, while the variances of the three considered endogenous variables calculated with the real data are all within the model-implied 95% bounds, the 'directed' Wald statistic is reported as 10.4%. That is, at the confidence level of 95%, the observed volatilities are not only individually, but also jointly explained by the theoretical model- with such a low Wald statistic, they are very close to the joint means of the variances.

Note that, by using the 'directed' Wald, we have been examining the theoretical model's *partial* capacities in explaining either the dynamics or the volatilities of the actual data. To evaluate the model's *overall* fitness to the real world, we consider both the dynamics and the volatilities *simultaneously*, for which we use the 'full' Wald statistic. This is reported in table 4.6 as 96.5%; hence the null hypothesis that the theoretical model explains both the actual dynamics and volatilities is easily accepted at the 99% confidence level and only *marginally* rejected at 95%.

Table 4.6: the 'full' Wald statistic

The concerned model properties	'Full' Wald statistic
Dynamics + Volatilities	96.5%

To summarise, model one does not only provide a rough explanation for the actual dynamics, but also precisely captures the volatilities shown by the real data; its overall fitness in explaining the data is fairly good, as DSGE models go and we may consider as a reasonable approximation to the real-world economy.

Model two ('IS'+PP+the original Taylor rule)

Leaving the economic environment (i.e., the 'IS' curve and the Phillips curve) unchanged, model two replaces the optimal 'target rule' assumed in model one with the original Taylor rule, widely regarded as a good description of the Fed's monetary policy from 1982 to at least the early 1990s. The test results for the dynamic behaviour of the model are reported in table 4.7 as follow:

Table 4.7: Individual VAR coefficients and the 'directed' Wald statistic

VAR(1) Coefficients	95% lower bound	95% upper bound	Values estimated with real data	In/Out
β_{11}	0.6139	1.1165	0.8017	In
β_{12}	- 0.0743	0.2385	0.0834	In
β_{13}	- 0.3098	0.2977	0.0112	In
β_{21}	- 0.1571	0.3175	- 0.2711	**Out**
β_{22}	0.6112	0.8960	0.9009	**Out**
β_{23}	- 0.4316	0.1654	- 0.1090	In
β_{31}	- 0.1055	0.6202	-0.0187	In
β_{32}	- 0.1457	0.1983	0.1428	In
β_{33}	-0.0043	0.6596	0.2552	In
'Directed' Wald statistic (for dynamics)			100%	

As revealed in table 4.7, while most of the real-data-based estimates of the VAR coefficients are individually captured by the 95% bounds implied by model simulations, the output gap's responses to the lagged interest rate and to its own lagged value are found to exceed their corresponding lower bound and upper bound, respectively. Overall, the 'directed' Wald statistic is reported as 100%, which means there is no hope *at all* for the theoretical model to generate a joint distribution of the VAR coefficients that simultaneously explains the ones observed in reality. The theoretical model thus is totally rejected by the Wald test for the dynamics.

Yet the model can still explain most of the data volatilities, as shown in table 4.8. It generates excessive interest rate variance, but reasonably matches series the variances of the output gap and inflation. The 'directed' Wald statistic for the variances is 91.5%, comfortably accepted therefore at 95%.

Table 4.8: Volatilities of the endogenous variables and the 'directed' Wald statistic

Volatilities of the endogenous variables	95% lower bound	95% upper bound	Values calculated with real data	In/Out
$\mathrm{var}(\tilde{i})$	0.0604	0.2790	0.0171	**Out**
$\mathrm{var}(x)$	0.0400	0.1527	0.0951	In
$\mathrm{var}(\pi)$	0.0475	0.1672	0.0153	In
'Directed' Wald statistic (for volatilities)			91.5%	

Note: Values reported in table 4.8 are magnified by 1000 times as their original values.

Lastly, table 4.9 shows the 'full' Wald statistic as 100%. This is hardly surprising since it fails so badly to capture the dynamics in the data.

Table 4.9: the 'full' Wald statistic

The concerned model properties	'Full' Wald statistic
Dynamics + Volatilities	100%

So far, it is clear that model two, where the original Taylor rule is set as the fundamental monetary policy, has only *partially* captured the characteristics shown by the actual data; unless the discussions are focused *exclusively* on the 'size' of the economy's fluctuations, such a model is not to be taken as a realistic description of the prevailing economic reality.

Model three ('IS'+PP+Taylor rule with 'interest rate smoothing')

In this last model, a calibrated Taylor type rule whose specification reflects the feature of 'interest rate smoothing' is assumed to be the underlying policy reaction function. In effect, the rate of interest implied by such a rule is a weighted average of what was set in the last period and what would be required had the original Taylor rule been put in place, with the weights being the degree of 'policy inertia' and its complement, respectively. While Taylor-type rules in which interest rates are 'smoothed' are commonly claimed to be supported by empirical evidence as the prevailing monetary policies (e.g., Clarida, Gali and Gertler (1999, 2000), Rotemberg and Woodford (1997, 1998)), the test results regarding model three's performance are revealed as follows:

Table 4.10: Individual VAR coefficients and the 'directed' Wald statistic

VAR(1) Coefficients	95% lower bound	95% upper bound	Values estimated with real data	In/Out
β_{11}	0.7228	0.9470	0.8017	In
β_{12}	- 0.0168	0.1287	0.0834	In
β_{13}	- 0.0029	0.1553	0.0112	In
β_{21}	- 0.1424	0.2095	- 0.2711	**Out**
β_{22}	0.6551	0.8971	0.9009	**Out**
β_{23}	- 0.2840	-0.0046	- 0.1090	In
β_{31}	- 0.1668	0.4706	-0.0187	In
β_{32}	- 0.1260	0.2655	0.1428	In
β_{33}	0.0830	0.5427	0.2552	In
'Directed' Wald statistic (for dynamics)			99.9%	

Table 4.10 above summarises how the actual dynamics are explained by the theoretical model. Again, except for the output gap's responses to the lagged interest rate and to its own lagged value, all dynamic relationships shown by the real data are individually captured by the simulated 95% bounds. Yet, the 'directed' Wald statistic reported is as high as 99.9%, indicating the theoretical model can hardly be used for

explaining the observed dynamics, as the set of real-data-based estimates of the VAR coefficients is not captured by the corresponding joint distribution generated from model simulations, even at a 99% confidence level[27].

Turning to the volatilities of the endogenous variables, table 4.11 shows the theoretical model has merely correctly mimicked the performance of the output gap, but evoked too much variance for both the interest rate and inflation; the ‘directed’ Wald statistic is reported as 99.4%, which implies the model in hand is not a proper explanation for the observed volatilities, either.

Table 4.11: Volatilities of the endogenous variables and the ‘directed’ Wald statistic

Volatilities of the endogenous variables	95% lower bound	95% upper bound	Values calculated with real data	In/Out
$\mathrm{var}(\tilde{i})$	0.0229	0.1174	0.0171	**Out**
$\mathrm{var}(x)$	0.0380	0.1430	0.0951	In
$\mathrm{var}(\pi)$	0.0532	0.1158	0.0153	**Out**
‘Directed’ Wald statistic (for volatilities)			99.4%	

Note: Values reported in table 4.11 are magnified by 1000 times as their original values.

In fact, the poor explanatory power of model three is not only detected by the ‘directed’ Wald, but also captured by the ‘full’ Wald when the model’s overall fitness is considered: note that the ‘full’ Wald statistic reported in table 4.12 is 99.9%, which is another way of saying it is hardly possible for the model to have generated data that simultaneously fit the dynamics and volatilities observed in reality.

Table 4.12: the ‘full’ Wald statistic

The concerned model properties	‘Full’ Wald statistic
Dynamics + Volatilities	99.9%

Thus model three, where a Taylor rule with ‘interest rate smoothing’ is in operation, cannot be considered to be a good proxy for the real-world economy.

[27] Note that the test results in this case are rather similar to their counterparts suggested for model two.

5. Reconsidering the Prevailing Monetary Policy Rule in Light of the Test Results

5.1 The truly-fitting monetary policy rule in the US

While the performances of the three hypothetical NK models are evaluated in the last section, recall that these models only differ in the ways in which monetary policies are set. Hence, by ranking the models in terms of their 'closeness' to the real world, one will in effect be considering whether the observed data are more likely to have been generated with the optimal 'target rule' or the original Taylor rule, or with a Taylor-type rule where the interest rate is 'smoothed'[28]. For ranking the models' performances, the test results revealed in section 4.4 are summarised as follows:

Table 5.1: Summary of the test results

NK models	'Directed' Wald statistics (for dynamics)	'Directed' Wald statistics (for volatilities)	' Full ' Wald statistics
Model one	98.2%	10.4%	96.5%
Model two	100%	91.5%	100%
Model three	99.9%	99.4%	99.9%

Given the test results reproduced in table 5.1, comparison by columns immediately shows the first model, which is combined with the optimal 'target rule', is generally superior to its rivals in fitting US data, as it consistently yields the lowest Wald statistics. More importantly, this model is the only one capable of explaining the dynamics and volatility of the data not only separately but also jointly. By contrast, in the cases where Taylor-type rules are incorporated into exactly the same economic environments, model two is only able to capture the scale of the economy's volatility, whereas model three is completely rejected by the data in all dimensions.

[28] That is, the 'true' monetary policy rule is identified as a part of the 'true' model *in a relative sense*.

5.2 Taylor rules as statistical relationships

The above suggests that the widespread success reported in single-equation regressions of Taylor rules on US data could simply represent some sort of *statistical* relationship emerging from the model with the optimal 'target rule'. To examine this possibility, we treat the optimal 'target rule' model again as the true model, the null hypothesis and ask whether the existence of empirical Taylor rules would be consistent with it.

Suppose an arbitrarily specified Taylor-type regression is estimated to infer the potential 'Taylor rule' of the US economy. For simplicity, let the Taylor-type regression take the form:

$$\tilde{i}_t = \gamma_\pi \pi_t + \gamma_x x_t + \rho \tilde{i}_{t-1} + \xi_t \qquad [5.1]$$

where variables have their usual meanings. Equation [5.1] can be estimated either using OLS if we assume the basic requirements for an OLS estimator are fulfilled, or via the IV approach to allow for possible correlations between the explanatory variables and the error term. The OLS and IV estimates based on the US data from 1982Q2 to 2007Q4 are summarised in table 5.2 below[29]:

Table 5.2: estimates of Taylor-type regression [5.1]

	γ_π	γ_x	ρ	Adjusted R^2
OLS estimates	0.0453	0.0922	0.8233	0.92
IV estimates	0.0376	0.1003	0.8017	0.90

Now, use the technique of 'indirect inference' to test if the observed 'Taylor rule' can be explained by model one based on the data simulated for the same periods[30]. The test results are revealed as follows:

[29] For the IV approach, here we take the lagged inflation and lagged output gap as instruments for their corresponding current values, respectively.

[30] Note: a) While one may expect the estimates of γ_π reported in table 5.2 be greater than one such that the 'Taylor principle' would be found, note that most existing literature has *treated* the interest rate series that is I(1) as a stationary series (See Carare and Tchaidze (2005), pp.17, footnote 17), whereas stationarity is obtained here by de-trending the data; Indeed, the 'Taylor principle' would be retrieved if the original I(1) interest rate series were used for estimation. b) In terms of the methodology, the Taylor-type regression [5.1] is now taken as the auxiliary model and the real-data-based estimates reported in table 5.2 are seen as the 'reality' in this case.

Table 5.3: Individual Taylor rule coefficients and the 'directed' Wald statistic

Panel A: Test for the OLS estimates

Taylor rule coefficients	95% lower bound	95% upper bound	Values calculated with real data	In/Out
γ_π	0.0514	0.3436	0.0453	**Out**
γ_x	-0.0702	0.0650	0.0922	**Out**
ρ	0.6330	0.9198	0.8233	In
'Directed' Wald statistic (for Taylor rule coefficients)			97.1%	

Panel B: Test for the IV estimates

Taylor rule coefficients	95% lower bound	95% upper bound	Values calculated with real data	In/Out
γ_π	-0.8867	0.3062	0.0376	In
γ_x	-0.1072	0.0514	0.1003	**Out**
ρ	0.6454	0.9420	0.8017	In
'Directed' Wald statistic (for Taylor rule coefficients)			97.8%	

According to table 5.3, although the real-data-based estimates of the 'Taylor rule' coefficients are not all individually captured by the model-implied 95% bounds, they are indeed explained as a set by the joint distribution of their simulation-based counterparts at the 99% confidence level, since the 'directed' Wald statistics are reported as 97.1% and 97.8% in panel A and panel B, respectively, indicating that it is statistically possible for model one to imply the 'Taylor rules' observed from both OLS and IV estimations as shown in table 5.2.

These results illustrate the identification problem with which we began this paper: a Taylor-type relation that has a good fit to the data may well be generated by a model where there is no *structural* Taylor rule at all[31]. Hence, any estimated or calibrated Taylor-type rule, no matter how well it might predict the actual movements of the nominal interest rate, is not by itself evidence that monetary policy follows this rule.

[31] Note that the adjusted R^2's reported in table 5.2 are as high as 0.92 for the OLS estimates and 0.90 for the IV estimates.

Note that table 5.4 below also summarises the Wald statistics when the Optimal Target Rule model is used to explain several popular variants of the Taylor rule estimated with OLS. According to the reported Wald statistics, the real-data-based estimates of these Taylor rules are all well captured by the model. The model is thus *robust* in generating essentially the whole range of Taylor rules that have been estimated on US data.

Table 5.4: Model one in explaining different Taylor-type rules (by OLS)

Taylor-type regressions	Adjusted R^2	'Directed' Wald statistic (for Taylor rule coefficients)
$\tilde{i}_t = \gamma_\pi \pi_t + \gamma_x x_t + \xi_t$ $\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t$	0.89	92.9%
$\tilde{i}_t = \gamma_\pi \pi_{t-1} + \gamma_x x_{t-1} + \xi_t$	0.40	87.0%
$\tilde{i}_t = \rho \tilde{i}_{t-1} + \gamma_\pi \pi_{t-1} + \gamma_x x_{t-1} + \xi_t$	0.90	97.9%

5.3 The 'interest rate smoothing' illusion: an implication

Another issue on which the test results in this paper and the analysis from the previous subsection sheds light is related to 'interest rate smoothing'. In an early paper Clarida, Gali and Gertler (1999) claimed that a 'puzzle' regarding the central banks' behaviour was yet to be solved, as the optimal 'target rule' which could be generally derived from a standard NK model as the optimal policy response to changes of macro variables would imply once-and-for-all adjustments of the nominal interest rate, whereas empirical 'evidence' from typical Taylor-type regressions estimated with the real data usually displayed a high degree of 'interest rate smoothing', in which case the sluggishness of interest rate variations could not be rationalized in terms of optimal behaviours.

While various authors explain such a discrepancy either at a theoretical level (e.g., Rotemberg and Woodford (1997, 1998), Woodford (1999, 2003a, 2003b)) or at an empirical level (e.g., Sack and Wieland (2000), Rudebusch (2001)), the tests in this paper support the optimal 'target rule' but reject the Taylor rule with 'interest rate smoothing'- implying the Fed has been responding to economic changes optimally without deliberately smoothing the interest rate. It is the persistence in the shocks themselves that induced the appearance of inertia in interest rate setting. Furthermore we show that one would find regressions of 'interest-smoothing Taylor rules' successfully fit the data even though this was being produced by the optimal 'target rule' model. Hence we would argue that these optimal responses by policymakers have been incorrectly interpreted as 'policy inertia' due to these misleading regressions.

6. Conclusion

In this paper we have attempted to identify the principles governing US monetary policy since the early 1980s. The 'Taylor rule' is widely regarded as a good description of these principles. Yet there is an identification problem plaguing efforts to estimate it: other relationships implied by the DSGE model in which it is embedded could imply a relationship that mimicked a Taylor rule. To get around this problem we have set up three models, each with the same New Keynesian structure but differing only in their monetary rules. The three different rules are an optimal target rule, a standard Taylor rule and another with 'interest rate smoothing'. We show, using statistical inference based on the method of indirect inference, that only the optimal target rule can replicate both the dynamics and the volatilities of the data. We also show that if the optimal target rule model was operating it would have produced data in which regressions of an interest-rate-smoothed Taylor rule would have been found. In short, the policy of the Fed in this period appears to have been approximately optimal and the fact that its behaviour looks like a Taylor rule with interest-rate smoothing is a statistical artefact.

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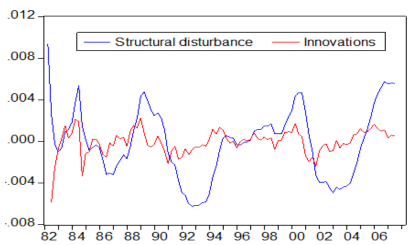
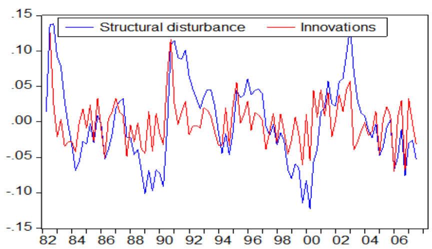
Appendix: supplementary diagnoses of the models' performances

Figure A.1: Charts of structural disturbances and innovations

Demand disturbance and innovations

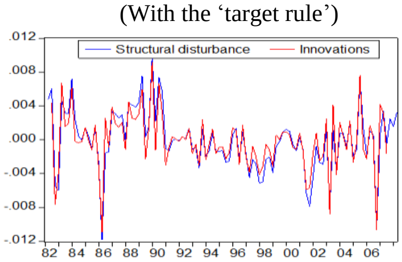


Supply disturbance and innovations

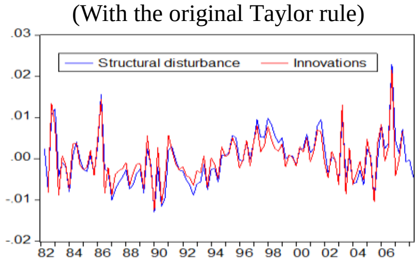


Policy disturbance and innovations

(With the 'target rule')



(With the original Taylor rule)



(With Taylor rule with 'interest rate smoothing')

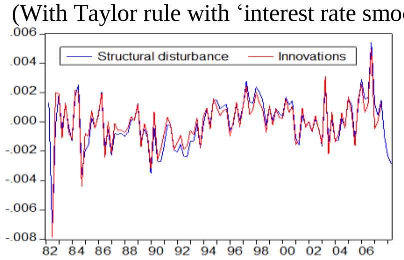


Table A.1: Estimates of relevant inference of the structural errors and innovations

Errors	AR(1) coefficients	t-statistics	Standard deviations of the innovations
Demand disturbance	0.91 [0.0359]	25.25	0.0012
Supply disturbance	0.82 [0.0581]	14.09	0.0336
Policy disturbance ('target rule')	0.35 [0.0917]	3.84	0.0034
Policy disturbance (Original Taylor rule)	0.37 [0.0916]	4.06	0.0056
Policy disturbance (Taylor rule with 'IRS'[32])	0.31 [0.0948]	3.26	0.0016

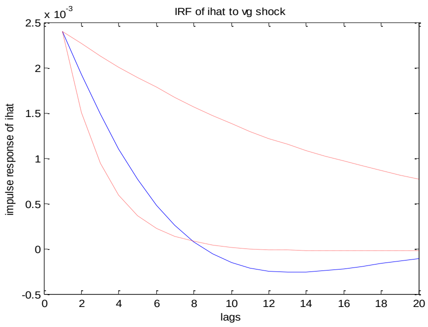
Note: Values in [ ] are the standard deviations of the estimates of AR(1) coefficients.

Behaviour of VAR Impulse Response Functions:

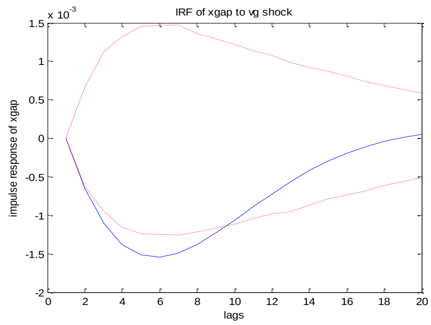
Model one (IS+PP+the 'target rule')

1a. VAR Impulse responses to demand shock (with 95% bounds):

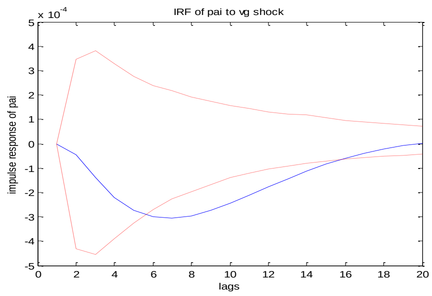
$\tilde{i}_t$



x_t

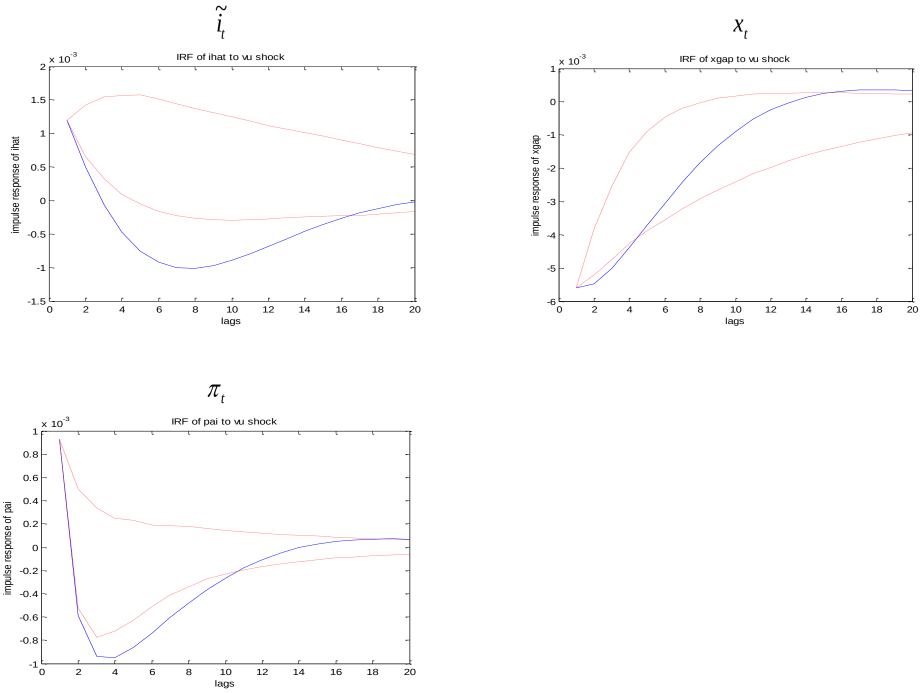


π_t

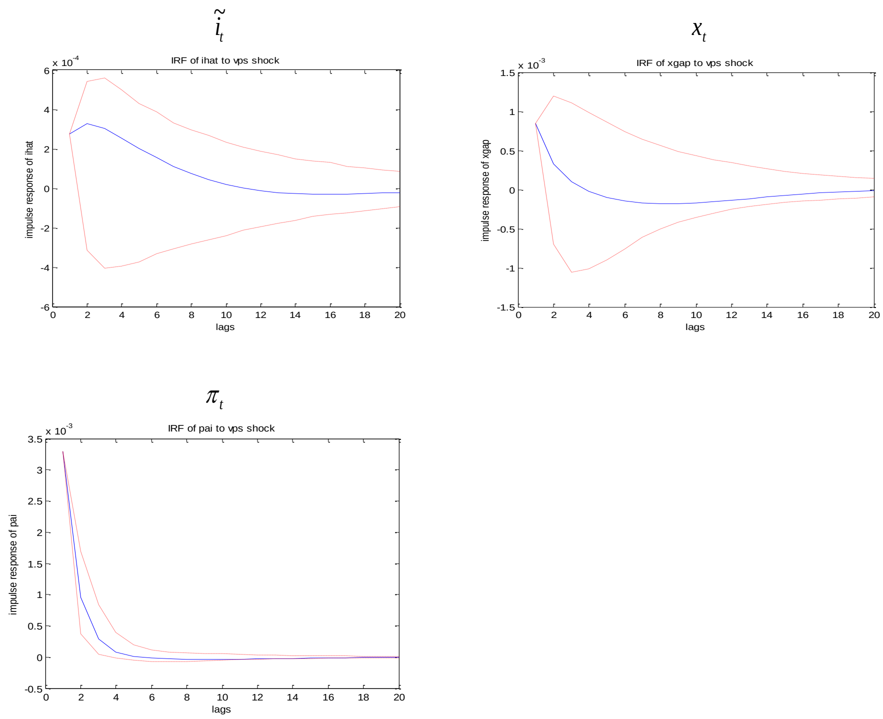


32 IRF: interest rate smoothing.

1b. VAR Impulse responses to supply shock (with 95% bounds):



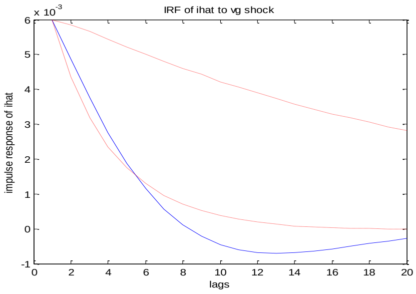
1c. VAR Impulse responses to policy shock (with 95% bounds):



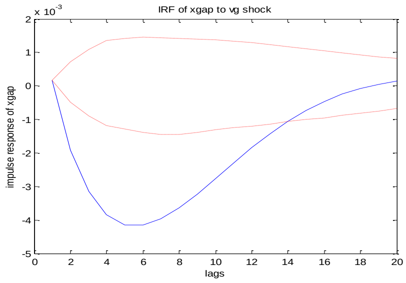
Model two (IS+PP+the original Taylor rule)

2a. VAR Impulse responses to demand shock (with 95% bounds):

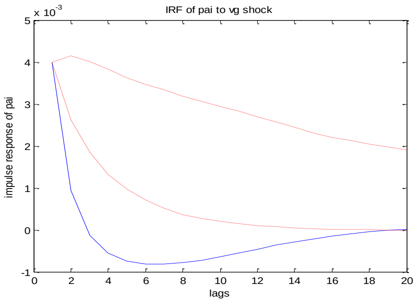
$\tilde{i}_t$



x_t

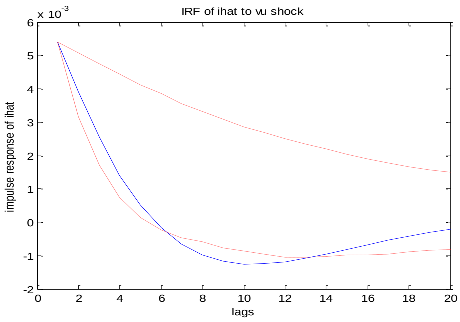


π_t

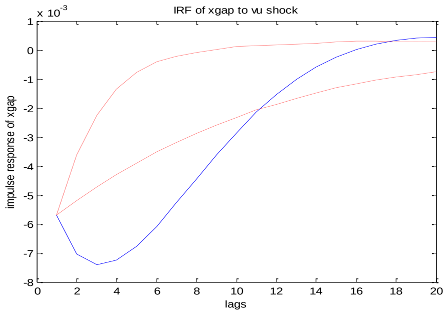


2b. VAR Impulse responses to supply shock (with 95% bounds):

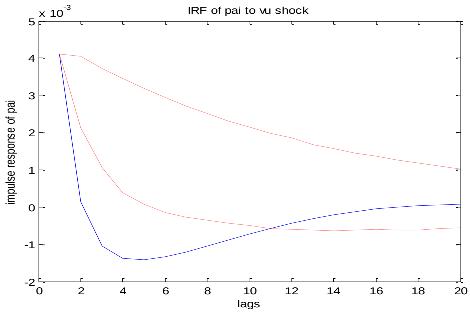
$\tilde{i}_t$



x_t

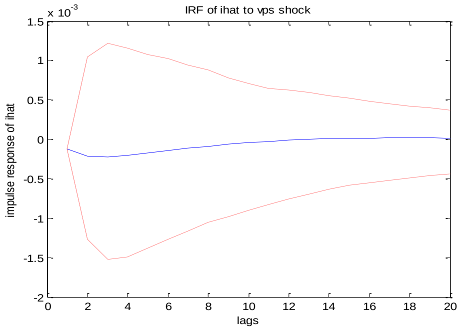


π_t

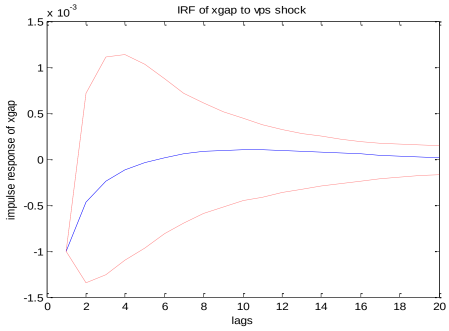


2c. VAR Impulse responses to policy shock (with 95% bounds):

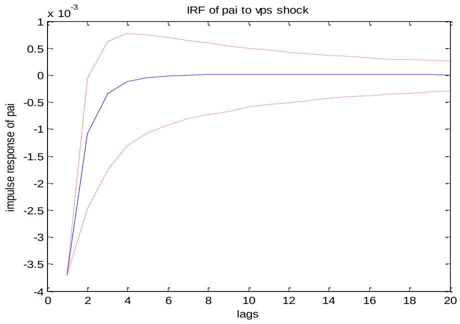
$\tilde{i}_t$



x_t



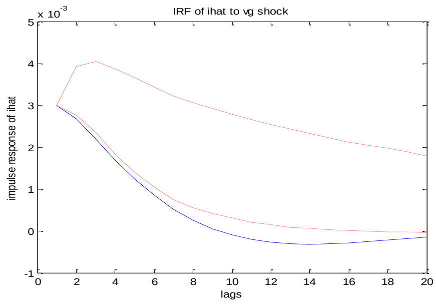
π_t



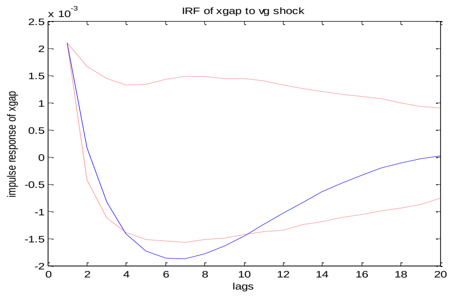
Model three (IS+PP+ Taylor rule with 'interest rate smoothing')

3a. VAR Impulse responses to demand shock (with 95% bounds):

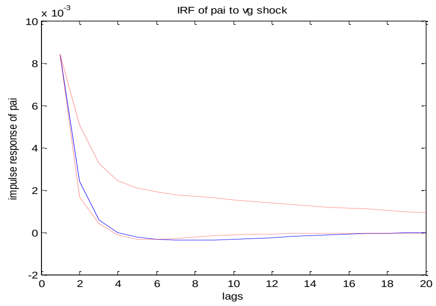
$\tilde{i}_t$



x_t

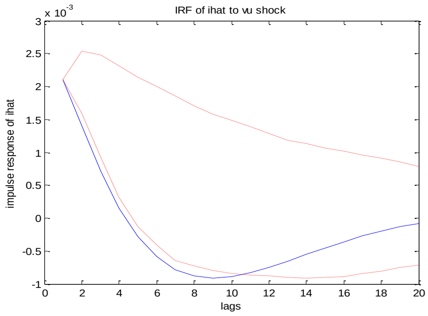


π_t

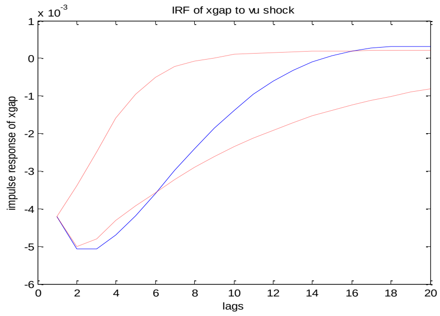


3b. VAR Impulse responses to supply shock (with 95% bounds):

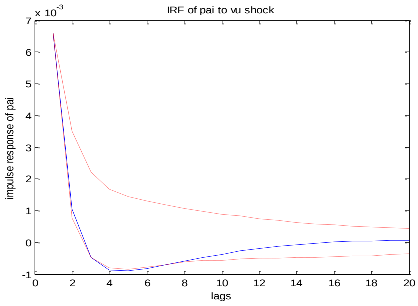
$\tilde{i}_t$



x_t

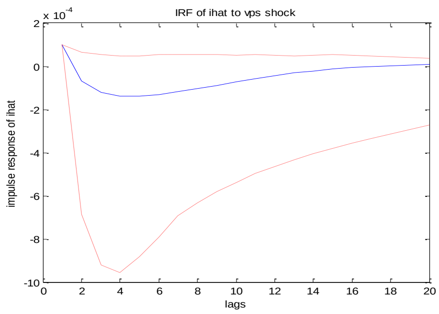


π_t

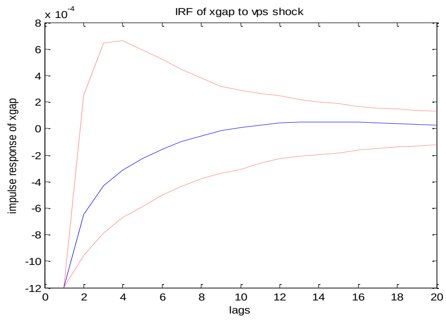


3c. VAR Impulse responses to policy shock (with 95% bounds):

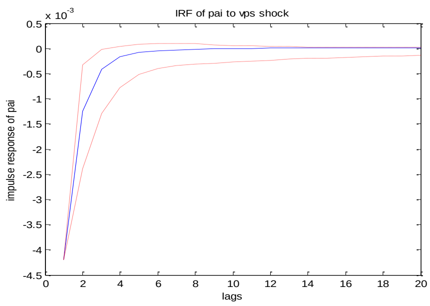
$\tilde{i}_t$



x_t



π_t

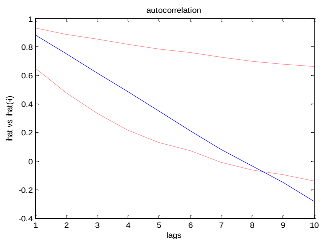


<u>Cross correlations between variables:</u>

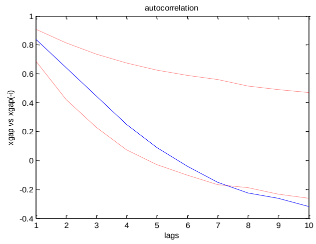
Model one (IS+PP+the 'target rule')

1a. Autocorrelations (with 95% bounds):

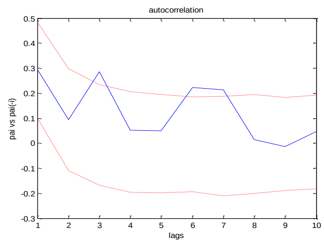
$\tilde{i}_t$ vs $\tilde{i}_{t-i}$



x_t vs x_{t-i}

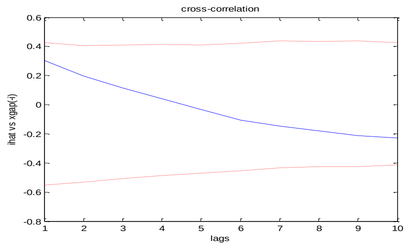


π_t vs π_{t-i}

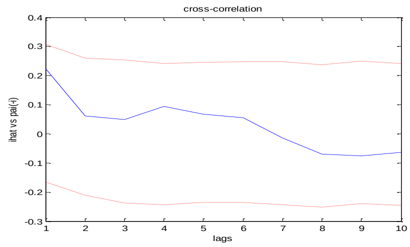


1b. Cross Correlations (with 95% bounds):

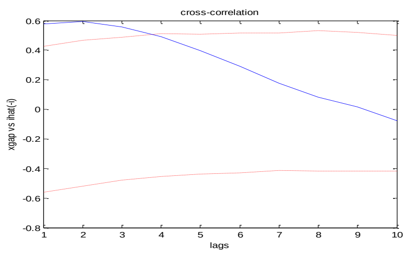
$\tilde{i}_t$ vs x_{t-i}



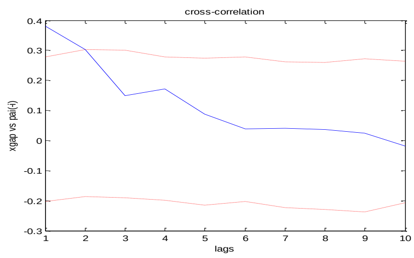
$\tilde{i}_t$ vs π_{t-i}



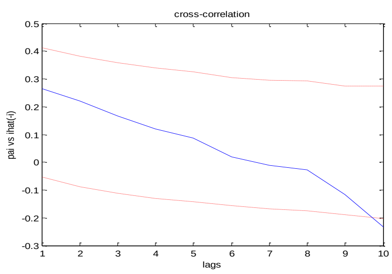
x_t vs $\tilde{i}_{t-i}$



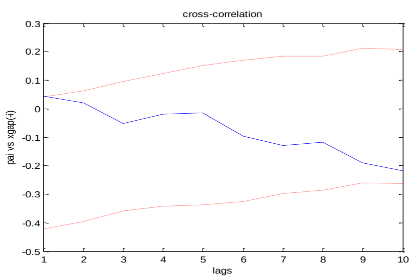
x_t vs π_{t-i}



π_t vs $\tilde{i}_{t-i}$

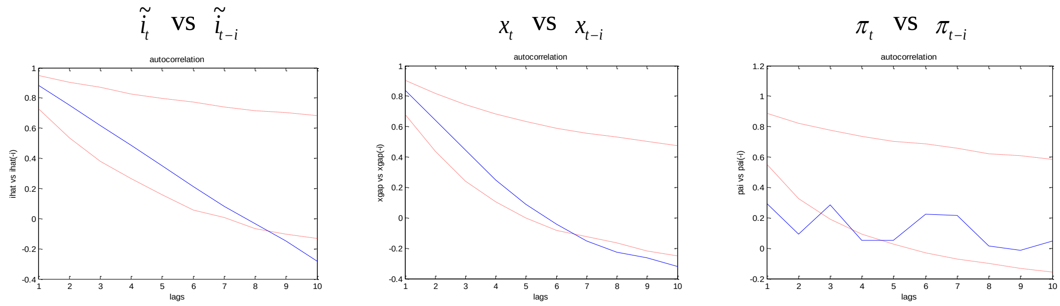


π_t vs x_{t-i}

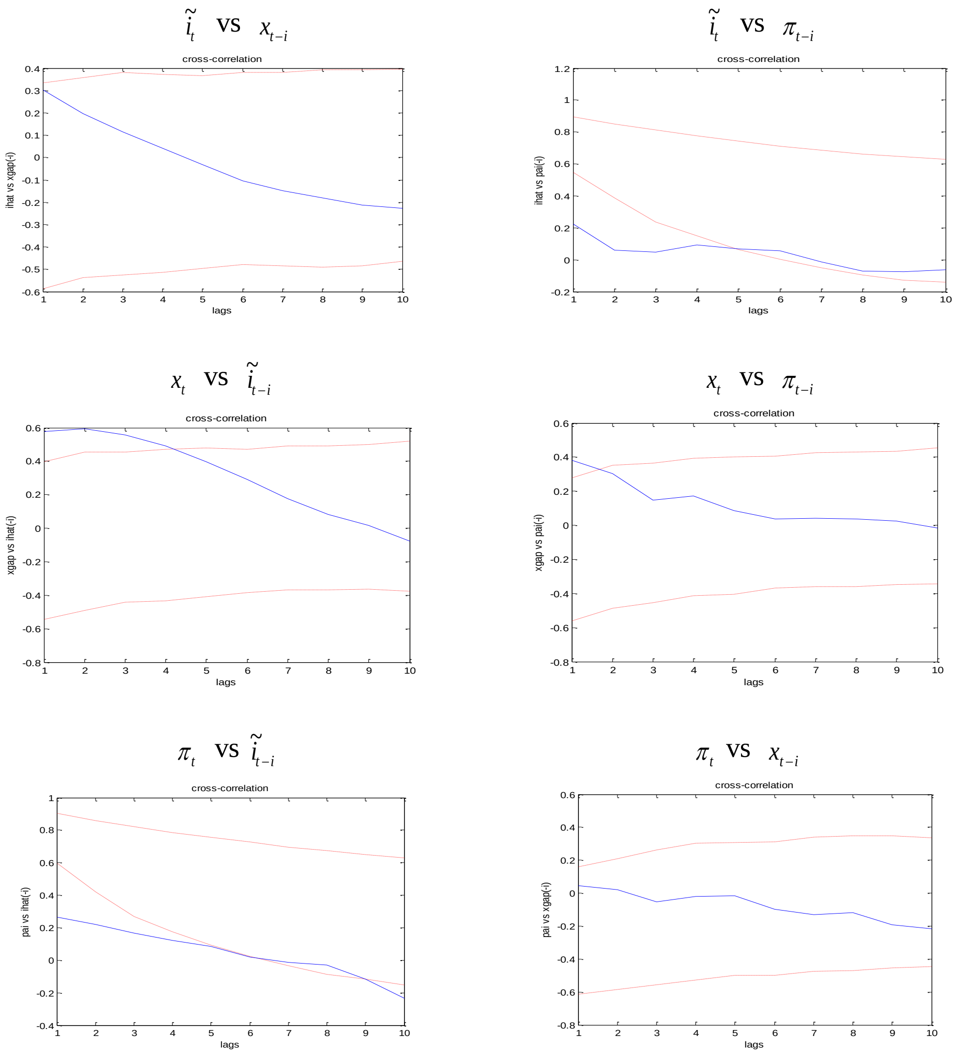


Model two (IS+PP+the original Taylor rule)

2a. Autocorrelations (with 95% bounds):



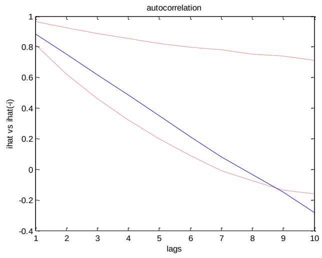
2b. Cross Correlations (with 95% bounds):



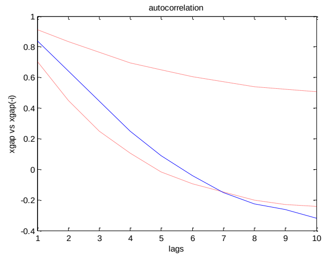
Model three (IS+PP+ Taylor rule with 'interest rate smoothing')

3a. Autocorrelations (with 95% bounds):

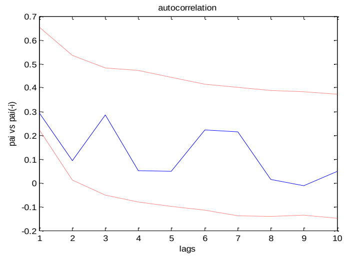
$\tilde{i}_t$ vs $\tilde{i}_{t-i}$



x_t vs x_{t-i}

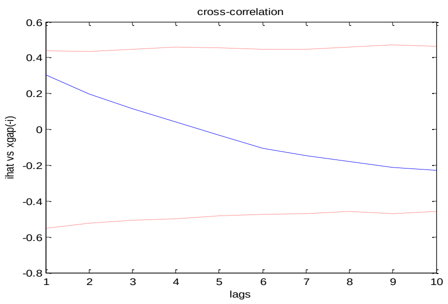


π_t vs π_{t-i}

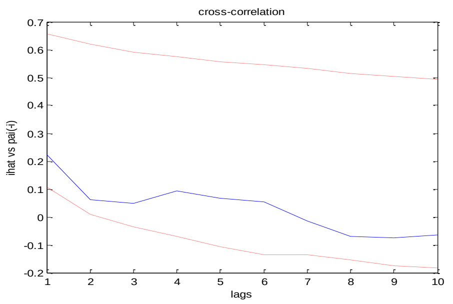


3b. Cross Correlations (with 95% bounds):

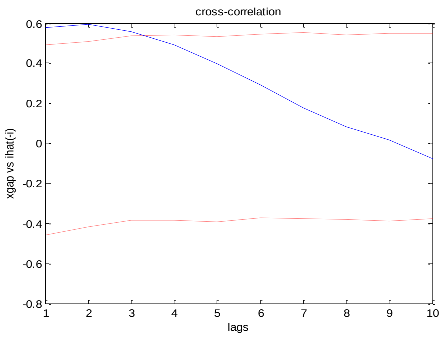
$\tilde{i}_t$ vs x_{t-i}



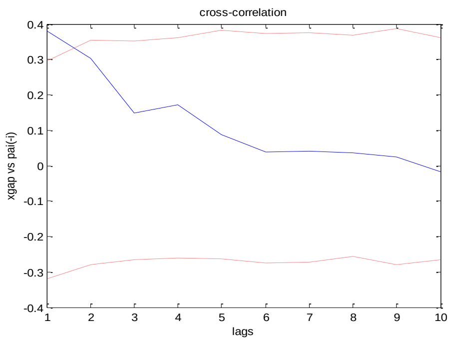
$\tilde{i}_t$ vs π_{t-i}



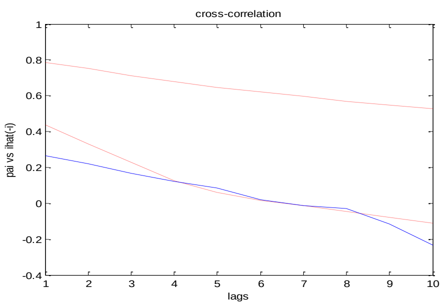
x_t vs $\tilde{i}_{t-i}$



x_t vs π_{t-i}



π_t vs $\tilde{i}_{t-i}$



π_t vs x_{t-i}