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ABSTRACT

Inequality and Aggregate Savings in the Neoclassical Growth Model

Within the context of the neoclassical growth model I investigate the implications of (initial) endowment inequality when the rich have a higher marginal savings rate than the poor. More unequal societies grow faster in the transition process, and therefore exhibit a higher speed of convergence. Furthermore, there is divergence in consumption and lifetime wealth if the rich exhibit a higher intertemporal elasticity of substitution.

Unlike the Solow-Stiglitz model, the steady state is always unique although the consumption function is concave.

JEL Classification: D30, O10 and O40
Keywords: concave consumption function, growth, income distribution and marginal propensity to consume

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1 Introduction

Rich people save more. But does this fact imply that inequality will increase forever over time? Moreover, how does such savings behavior affect the growth path? Tackling these questions is a priori a complex task: On the one hand, inequality affects capital accumulation when the marginal propensities to consume (MPC) differ. On the other hand, inequality itself changes through the accumulation process because savings rates differ and the factor prices change. It is the purpose of this paper to analyze this relationship within the context (of the neoclassical growth model) with perfect and complete markets.

Theoretical reasoning that savings propensities increase with wealth date back at least to Fisher (1930) and Keynes (1936). Carroll and Kimball (1996) show that when agents are subject to uninsurable risks or liquidity constraints the consumption function is concave expect for special cases. The empirical relevance of increasing MPC is unquestioned. Looking at household data, it is a well established fact that rich people save more - not only on average but also at the margin - out of wealth or permanent income, see the paper by Dynan, Skinner, and Zeldes (2004) and their references.[1]

What are the macroeconomic effects of increasing MPC? To make our point as simple as possible, we do our analysis under full certainty but where the different MPC arise due to non-homothetic preferences. To the best of my knowledge there is so far no study of inequality and growth when consumers save optimally and the

[1] Perhaps unsurpringly, the empirical picture is less clear on the aggregate level. The differences between studies are due to different data sets and different approaches to tackle the endogeneity problems. Although Schmidt-Hebbel and Servén (2000) and Li and Zou (2004) could not find a robust effect of inequality on saving, Cook (1995) and Smith (2001) found a positive effect of inequality on (private) saving rates. The well-known studies of Barro (2000) and Forbes (2000) obtain a positive inequality growth relationship, at least for rich countries. This is also consistent with the view that inequality raises savings.

resulting consumption function is concave. Stiglitz (1969) studied the dynamics of distribution when savings are exogenously given linear function of wealth. Chatterjee (1994), Caselli and Ventura (2001) and Bertola, Foellmi, and Zweimueller (2006, Chap. 3) study the same question in a Ramsey model where the linear savings rule is a result of dynamic optimization. The impact of concave consumption functions for the evolution of inequality and growth was previously studied by Bourguignon (1981) and Schlicht (1975) in the context of the Solow-Stiglitz model with exogenous savings propensities. Bourguignon (1981) shows that multiple steady states may emerge which can be pareto-ranked. In this paper, convex savings are the result of a dynamic optimization with intertemporally separable preferences. Surprisingly, the analysis is much simplified: the steady state equilibrium is unique and independent of the initial distribution. This result implies that more unequal societies must exhibit a higher speed of convergence because they grow faster in the transition process.

A related important strand of the literature is the work by Becker (1980), Lucas and Stokey (1984) and, more recently, Sorger (2002). They study conditions when the long run distribution of wealth is non-degenerate in steady state. Furthermore, Bliss (2004) analyzes a general class of preferences to study whether convergence occurs in the accumulation process. However, the focus of these papers is not to analyze the impact of inequality on growth.

The paper is structured as follows. Section 2 presents the model. Both the competitive equilibrium and the social planner's solution are analyzed. Section 3 then presents a numerical simulation and in the final section 4 the differences to the Bourguignon's model are discussed.

2 The model

2.1 Set-up

Preferences All consumers have the same intertemporal additive preferences and the same discount rate. The time horizon is infinite. Hence, the intertemporal utility function is given by

$$U_i = \int_0^\infty e^{-\rho t} u(c_i(t))dt \tag{1}$$

where $c_i(t)$ denotes consumption of individual i at date t. We assume that (i) $u(\cdot)$ is twice continuously differentiable above some (subsistence) level $\bar{c} \geq 0$. (ii) We take the usual assumption that $u' > 0 > u''$, i.e. marginal utility is declining but the individual is non-satiated (at least over the relevant range). (iii) Further we assume $\lim_{c \to \bar{c}} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$. Assumptions (i) and (ii) imply that the elasticity of substitution is positive for all $c > \bar{c}$:

$$-\frac{u'(c)}{u''(c)c} > 0 \text{ for } c > \bar{c} \geq 0.$$

Individual factor endowments We assume that - at date 0 - household i is endowed with l_i units of labor, which is assumed to be constant over time, and $k_i(0)$ units of capital. We restrict the inequality in the way that all households are viable, i.e., each household can afford to consume more than $\bar{c}$ in every period of time. We will come back to this assumption below. The number of households is constant. Hence total amount of labor L is also constant and we normalize it to one. Hence, the total amount of labor and capital in the economy is given by

$$\begin{aligned} K &\equiv \int_{\mathcal{N}} k_i(t) dP_i \\ 1 &\equiv \int_{\mathcal{N}} l_i dP_i \end{aligned}$$

where $\mathcal{N}$ denotes the set of families and dP_i the size of family i.

Technology and competitive factor rewards The inputs labor and capital are used to produce a homogenous output good Y which can be both used for consumption and investment. Production takes place with a standard neoclassical production function $F(\cdot,\cdot)$ with constant returns to scale and diminishing marginal products. The production function shall be twice continuously differentiable in its arguments. There is no technological progress, i.e., we focus on transitional dynamics only.[2]

$$Y(t) = F(K(t), 1) \equiv f(K(t))$$

The factors are rewarded their marginal products, hence the interest rate and the wage rate are given by

$$\begin{aligned} r(t) &= f'(K(t)) \\ w(t) &= f(K(t)) - K(t)f'(K(t)) \end{aligned} \tag{2}$$

and are uniquely determined by the current capital stock $K(t)$.

2.2 The social planner's problem

Before turning to the market equilibrium it is useful to consider the social planner's problem. The planner assigns welfare weights ω_i to the individuals which are pinned down by the (initial) distribution of k_i and l_i in the decentralized optimum analyzed in the next section.[3] Setting up the current value Hamiltonian with $\{c_i(t)\}$ as control and $K(t)$ as state variable

$$H = \int_{\mathcal{N}} \omega_i u(c_i(t)) dP_i + \lambda(t)\dot{K}(t)$$

[2] As is well known, with positive growth we get steady states only if the intertemporal elasticity of substitution is constant, i.e., utility is CRRA.

[3] In a decentralized equilibrium, consumption depends monotonically on lifetime resources - which in turn are determined by the initial distribution of k_i and l_i . Hence for each distribution of lifetime resources there is a distribution of welfare weights ω_i such as to mimic the decentralized solution.

subject to the capital accumulation constraint (the output good can be used both for consumption and investment)

$$\dot{K}(t) = f(K(t)) - \int_{\mathcal{N}} c_i(t) dP_i \tag{3}$$

leads to the first order conditions

$$\omega_i u'(c_i(t)) - \lambda(t) = 0 \tag{4}$$

and

$$\rho\lambda(t) - \dot{\lambda}(t) = \lambda(t) f'(K(t)). \tag{5}$$

We may disregard the Kuhn-Tucker conditions because of the Inada conditions and the distributional assumptions. The first order conditions (4) and (5) and the capital accumulation equation (3) give the standard pair of differential equations, we omit time indices,

$$\begin{aligned} \frac{\dot{\lambda}}{\lambda} &= \rho - f'(K) \\ \dot{K} &= f(K) - \int_{\mathcal{N}} c(\omega_i, \lambda) dP_i \end{aligned} \tag{6}$$

where $c(\omega_i, \lambda)$ is implicitly defined by $\omega_i u'(c_i) = \lambda$. Figure 1 depicts equations (6) with K on the horizontal and λ on the vertical axis. The $\dot{\lambda} = 0$ locus is vertical at $f'(K) = \rho$, and the $\dot{K} = 0$ locus is monotonically decreasing as $c(\omega_i, \lambda)$ is decreasing in λ. The system has a unique saddle path with negative slope. Hence the policy function $\lambda(K)$ is uniquely determined.

Figure 1

2.3 The decentralized equilibrium

Markets are perfect and complete. We assume that each household is able to consume more than $\bar{c}$. All individuals face the same factor prices, thus the household's income is given by $w(t)l_i + r(t)k_i(t)$. The evolution of individual wealth then reads $\dot{k}_i(t) = w(t)l_i + r(t)k_i(t) - c_i(t)$. Imposing the transversality condition we get the intertemporal budget constraint. The utility maximization problem of the consumer reads

$$\max_{\{c_i(t)\}} \int_0^\infty e^{-\rho t} u(c_i(t))dt \ \text{ s.t. } \int_0^\infty e^{-R(t)} c_i(t)dt \leq k_i(0) + \int_0^\infty e^{-R(t)} w(t) l_i dt$$

where $R(t) = \int_0^t r(s)ds$. The first order condition reads

$$e^{-\rho t} u'(c_i(t)) - \mu_i e^{-R(t)} = 0 \tag{7}$$

where μ_i denotes the marginal utility of wealth. Our assumptions on the production function imply that $R(t)$ is differentiable. Hence, we may differentiate (7) with respect to time and get the familiar Euler equation

$$\dot{c}_i(t) = -\frac{u'(c_i(t))}{u''(c_i(t))} \left(r(t) - \rho \right). \tag{8}$$

It is easy to see that the FOC of the decentralized equilibrium are equivalent to those of the social planner's problem. Differentiating (4) with respect to time, we get $\dot{c}_i = \dot{\lambda} u'(c_i) / \left(\lambda u''(c_i) \right)$. Using (6) to replace $\dot{\lambda}/\lambda$, immediately leaves us with the Euler equation (8). The resource constraint is clearly the same in both cases. Hence, the decentralized equilibrium is unique and pareto efficient.

Aggregating (8) we obtain the equation of motion for aggregate consumption C

$$\dot{C}(t) = (r(t) - \rho) \int_{\mathcal{N}} -\frac{u'(c_i(t))}{u''(c_i(t))} dP_i. \tag{9}$$

Equation (9) already allows us to determine how inequality affects consumption growth and the savings rate.

Lemma 1 *Iff* $c_i(0) > c_j(0)$, *then* $c_i(t) \geq c_j(t)$ $\forall t$.

Proof. The first order condition (7) may be rewritten $u'(c_i(t)) = e^{-R(t) + \rho t} u'(c_i(0))$. This implies that $c_i(t)$ is monotonic in $c_i(0)$. ■

Lemma 2 *Individual consumption is monotonically increasing in wealth* $k_i(0)+\int_0^\infty e^{-R(t)}w(t)l_i dt$.

Proof. Assume to the contrary that a poorer agent's consumption today is higher than that of a richer agent. Lemma 1 implies that the poor's consumption will not be lower than of the rich in the future. As the rich agent's intertemporal budget constraint is satisfied with equality, the poor would violate his budget constraint. ∎

Lemma 3 *If* $\dot{c}$ *is convex in* c*,* $c(t)$ *must be convex in* $c(0)$*, i.e.,* $\partial^2 c(t)/\partial c(0)^2 > 0$.

Proof. We use $u'(c(t)) = e^{-R(t)+\rho t}u'(c(0))$ (individual indices are omitted). Differentiate this equation with respect to $c(t)$ and $c(0)$ to get

$$\frac{dc(t)}{dc(0)} = \frac{\phi(c(t))}{\phi(c(0))}.$$

where we defined $-u''(x)/u'(x) \equiv \phi(x)$ and replaced $e^{-R(t)+\rho t}$ by $u'(c(t))/u'(c(0))$.To determine $d^2c(t)/dc(0)^2$, we take the derivative with respect to $c(0)$

$$\begin{aligned}\frac{d^2c(t)}{dc(0)^2} &= \frac{\phi'(c(t))\frac{\phi(c(t))}{\phi(c(0))}\phi(c(0)) - \phi(c(t))\phi'(c(0))}{[\phi(c(0))]^2} \\ &= \frac{\phi(c(t))}{[\phi(c(0))]^2}\left[\phi'(c(t)) - \phi'(c(0))\right].\end{aligned}$$

Hence, $d^2c(t)/dc(0)^2 > 0$ iff $\phi'(c(t)) > \phi'(c(0))$. This holds true if $\phi(.)$ is convex and $c(t) > c(0)$. ∎

Lemma 4 *If* $-u'(c)/u''(c)$ *is convex and the economy is growing (*$r(t) > \rho$*), consumption is a concave function of wealth.*

Proof. Define $a_i(0) \equiv k_i(0)+\int_0^\infty e^{-R(t)}w(t)l_i dt$. We differentiate the intertemporal budget with respect to wealth and get (individual indices i omitted)

$$\begin{aligned}\frac{dc(0)}{da(0)} &= \left[\int_0^\infty e^{-R(t)}\frac{\partial c(t)}{\partial c(0)}dt\right]^{-1} \\ \frac{d^2c(0)}{da(0)^2} &= -\left(\frac{dc(0)}{da(0)}\right)^{-1}\int_0^\infty e^{-R(t)}\frac{\partial^2 c(t)}{\partial c(0)^2}dt\end{aligned}$$

By Lemma 3 we know that $\dot{c}$ is convex iff $-u'(c)/u''(c)$ is convex. By Lemma 2 $dc(0)/da(0) > 0$, this implies $d^2c(0)/da(0)^2 < 0$. ■

Lemma 4 gives the condition such that the rich have a higher marginal savings rate than the poor. Obviously, under these conditions, aggregate savings are higher in a more unequal society when the conditions in Lemma 4 hold.

Proposition 1 *If $-u'(c)/u''(c)$ is convex and the economy is growing, more unequal societies (aggregate wealth held constant) have a higher savings rate and grow faster.*

Proof. According to Lemma 4, a regressive transfer in wealth decreases aggregate consumption. A second order stochastic dominance shift in the c_i- distribution increases $\int_{\mathcal{N}} -\frac{u'(c_i(t))}{u''(c_i(t))} dP_i$ when $-u'(c)/u''(c)$ is convex. Hence, consumption growth in (9) is higher with a more unequal wealth distribution. ■

As a corollary note that $-u'(c)/u''(c)$ being concave would imply that more unequal societies save less. In addition, note that the results reverse if we consider a shrinking economy where $r(t) < \rho$ and hence $c(t) < c(0)$, see the proof of Lemma 3. Finally, savings are independent of distribution when $-u'(c)/u''(c)$ is linear. This is the well known result that income distribution has no effect on accumulation when preferences take the HARA (hyperbolic risk aversion) form.

At the same time we are able to draw conclusions on the evolution of the consumption and the wealth distribution.

Proposition 2 *Consumption and wealth inequality increases (decreases) in a growing economy if the elasticity of substitution $-u'(c)/u''(c)c$ increases (decreases) in c.*

Proof. From (8) we see that the growth rate of individual consumption $\dot{c}_i/c_i$ increases in c_i when $-u'(c)/u''(c)c$ increases (decreases) in c. Wealth inequality moves pari passu with consumption inequality since consumption is monotone in wealth. ■

Comparing Propositions 1 and 2, the condition on the evolution of inequality is not directly related to the concavity of the consumption function. The concavity of the consumption function is a statement on *marginal* propensities to save. Instead, the evolution of the wealth and consumption inequality is governed by differences in saving rates, i.e. the *average* propensities to save. To take an example, when the consumption function is linear but exhibits a positive axis intercept due to subsistence consumption inequality will widen over time. This is the case with Stone-Geary utility $u(c) = \ln(c - \bar{c})$, for example. Intuitively, the subsistence consumption level forces a poor individual to save only little today which implies that the subsequent growth rate of wealth and consumption is lower for the poor.

2.3.1 Steady State

As there is no technical progress, the economy will be in steady state when C, Y, and K are constant. Setting $\dot{\lambda} = 0$ and $\dot{K} = 0$ in (6) yields us the steady state value of the interest rate and the consumption level

$$
\begin{aligned}
r^* &= f'(K^*) = \rho \\
C^* &= f(K^*).
\end{aligned}
$$

Hence, we see that the steady state capital stock is unique and independent of the distribution.[4] This is a sharp difference to Bourguignon's (1981) result. In a model with optimizing agents the macroeconomic analysis of increasing marginal savings on individual level turns out to be much simpler. Since individual consumption increases - although the growth rate may differ because of the varying intertemporal rate of substitution - if the interest rate exceeds the rate of time preference, there must be a unique stationary steady state. For any (separable) utility function (1) it is optimal to choose a constant consumption flow only if $r = \rho$.

A further difference to the Bourguignon-Solow model concerns welfare. Bourguignon's (1981) analysis of the Solow model with convex savings suggests that the

[4] Although the aggregate values of K and C are unique, the individual c_i- and k_i-distribution is indeterminate and is governed by the initial distribution (see Sorger, 2002).

poor might indirectly gain from redistribution. He showed that inegalitarian steady states may occur where consumption of the rich and the poor is higher than in an egalitarian steady state. More inequality raises savings and investment and therefore wages as the economy produces more capital intensive. This mechanism is the reason why the consumption levels of the poor and the rich are higher in the inegalitarian steady state than in the egalitarian one. Hence, the inegalitarian steady state is "pareto-dominant". (Of course such a comparison is not possible since there are no utility functions in the Solow model and the transitional process would have to be taken into account). In the Ramsey model, however, the equilibrium allocation is always pareto optimal. The well known reason lies in the fact markets are perfect and complete. Out of steady state, however, the welfare level of agent i increases, if inequality at date 0 is higher because of a regressive transfer between other agents. The reason is that the economy grows faster. Obviously, this is not a Pareto improvement since the agent, whom is taken wealth of in the regressive transfer, is worse off.

2.4 Speed of convergence

We saw that all economies converge to the same steady state but unequal economies grow faster in the transitional process. To bring these two results together we must follow that more unequal societies exhibit a higher speed of convergence towards the steady state. To calculate the speed of convergence $\dot{K}(t)/\left(K(t)-K^{*}\right)$ we linearize the economy around its steady state

$$\frac{\dot{C}(t)}{C(t)-C^{*}} \cong \frac{\dot{K}(t)}{K(t)-K^{*}} \cong \mu \equiv \frac{1}{2}\left[\rho-\sqrt{\rho^{2}-4f''(K^{*})\int_{\mathcal{N}}\frac{u'(c_i^{*})}{u''(c_i^{*})}dP_i}\right]. \qquad (10)$$

The derivation of equation (10) is shown in the appendix. The following proposition proves our intuition.

Proposition 3 *More unequal societies exhibit a higher speed of convergence.*

Proof. An increase in income dispersion increases $\int_{\mathcal{N}} -\frac{u'(c_i(t))}{u''(c_i(t))} dP_i$ and increases the absolute value of μ. ■

Along the same lines, we get the expressions for the evolution of aggregate consumption and capital stock around the steady state

$$\begin{aligned} \frac{C(t) - C^*}{C^*} &\cong \int_{\mathcal{N}} -\frac{u'(c_i^*)}{u''(c_i^*) c_i^*} \frac{c_i^*}{C^*} dP_i \frac{f''(K^*)}{\mu} e^{\mu t} \frac{K(0) - K^*}{K^*} \\ \frac{K(t) - K^*}{K^*} &\cong e^{\mu t} \frac{K(0) - K^*}{K^*}. \end{aligned}$$

This analysis was restricted to a neighborhood of the steady state. In particular, the consumption inequality is evaluated at its steady state level. Hence, the linearization does not allow for "feedback" effects of income distribution on growth and vice versa. To study the dynamics outside of steady state we therefore have to refer to numerical simulations which is done in section 3.

3 Numerical exercise

To study the quantitative effects involved we perform a simple quantitative exercise. Let marginal utility be given by $u'(c) = (c^\gamma - 1)^{-\sigma}$ where a consumption of unity may be interpreted as the subsistence level and $\gamma > 1$. It is easy to show that the resulting consumption function is concave in wealth when the interest rate exceeds the rate of time preference. Furthermore, the elasticity of substitution $-u'(c)/u''(c)c$ is increasing in consumption. The preference parameters are chosen as $\rho = 0.02$, $\sigma = 2$, and $\gamma = 0.01$. The new parameter γ determines the concavity of the consumption function. The MPC will react more strongly to changes in wealth the higher γ is. The aggregate production function takes a Cobb-Douglas form, $Y = K^\alpha$. The capital share is given by $\alpha = 0.33$. Hence the steady states values of capital and consumption are given by $K^* = (\alpha/\rho)^{1/(1-\alpha)} = 65.6$ and $C^* = (K^*)^\alpha = 3.94$.

To simplify things further we assume that there are only two groups in the population: β poor and $1 - \beta$ rich agents. According to Wolff (1998), the top 20% in the

US population own about 80% of financial wealth. To match the (financial) wealth distribution, let $\beta = 0.8$ be the group size of the poor and we choose the following individual wealth levels at date 0: $k_P(0) = 10$ and $k_R(0) = 110$. Hence, with this specification, the richest 20% own 73% of aggregate wealth. The aggregate capital stock equals $K(0) = 30$ or around 45% of its steady state value. The only free parameter left is the distribution of wage incomes (labor endowments). In the low inequality simulation we chose $l_P = 0.8$ (a poor individual earns 80% of average wage income) whereas in the high inequality case we set $l_P = 0.5$.

Table 1, *Figure* 2

How well can this simple model with perfect markets account for differences in savings rates? The difficulties in estimating cross country relationships between inequality and savings rates notwithstanding, Smith (2001) estimated that an increase in the Gini coefficient by one standard deviation or 10 percentage points results is associated with a 1.5% increase in the country's savings rate. In Table 1 we see that, with the values of the parameters chosen, an increase in the consumption Gini by 10 percentage points the savings rate increases by 0.6 - 1 percentage points,[5] with higher marginal effects for higher levels of inequality. Although no elements of uncertainty are present, the model is able to generate reasonable quantitative effects. Further, the simulation shows the evolution of inequality and in particular the influence of higher savings rates of the rich. The positive subsistence consumption $\bar{c} = 1$ forces the poor to choose a flat consumption path (see Figure 2) which results in a slow accumulation of assets. For the high inequality specification the poor's assets in steady state are even lower than at the starting date (see Table 1, last column).

[5]Note that we evaluate the savings rates at the starting point of the transition process. Obviously, as the economy moves closer to the steady state the savings rates decline and equal zero in steady state.

4 Conclusion

We analyzed the macroeconomic implications of increasing marginal savings propensities. With optimal savings and *infinite* horizons the equilibrium sequences of interest rates and wages are unique and pareto-efficient. If savings are convex, more inequality leads to a higher speed of convergence. This holds true although inequality affects accumulation in the transition path with a general utility function $u(c)$.These results stand in a sharp contrast to Bourguignon's findings when assuming exogenous convex savings behavior. Intuitively, the extreme differences in the outcomes are analogous to the comparison of the Ramsey and the OLG model: The Solow-Stiglitz model with exogenous savings can be rationalized by an OLG economy with (warm glow) bequests. In OLG models, multiple steady states may emerge as new generations enter and agents have finite horizons. In this paper, horizons are infinite which precludes multiple equilibria. However, this conjecture needs to be explored.

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5 Appendix

To derive the speed of convergence, we take a first-order Taylor approximation around the steady state (where $f'(K) = \rho$). For the evolution of individual consumption (8) we get

$$\begin{aligned}\dot{c}_i &\cong \frac{\partial \dot{c}_i}{\partial c_i}\left[c_i - c_i^*\right] + \frac{\partial \dot{c}_i}{\partial K}\left[K - K^*\right] \\ &= f''(K)\frac{u'(c_i)}{u''(c_i)}\left[K - K^*\right].\end{aligned}$$

By aggregation we get the evolution of aggregate consumption (note that $\dot{C} = \overset{\cdot}{C - C^*}$)

$$\overset{\cdot}{C - C^*} \cong f''(K)\int_{\mathcal{N}}\frac{u'(c_i)}{u''(c_i)}dP_i\left[K - K^*\right]. \tag{A1}$$

In the same way we approximate the capital accumulation equation $\dot{K} = f(K) - C$,

$$\overset{\cdot}{K - K^*} \cong \rho\left[K - K^*\right] - \left[C - C^*\right]. \tag{A2}$$

As (A1) and (A2) are linear in C and K, the growth rates of $[C - C^*]$ and $[K - K^*]$ coincide. The solution of this log linearized system is (10).

Table 1: Calibration

	High inequality	Low inequality	Representative Agent
Initial values			
Labor endowment l_P	0.5	0.8	1
Asset endowment $k_P(0)$	10	10	30
Asset endowment $k_R(0)$	110	110	30
Consumption of the poor $c_P(0)$	1.367	1.912	2.652
Consumption of the rich $c_R(0)$	7.320	5.430	2.652
Aggregate consumption $C(0)$	2.557	2.615	2.652
Consumption GINI	37.2	21.5	0
Savings rate	16.8%	14.9%	13.7%
Steady State			
Consumption of the poor c_P*	1.512	2.421	3.940
Consumption of the rich c_R*	13.799	10.018	3.940
Aggregate consumption C*	3.940	3.940	3.940
Assets of the poor k_P*	9.70	16.16	64.74
Assets of the rich k_R*	289.38	263.44	64.74

Figure 1: Phase Diagram

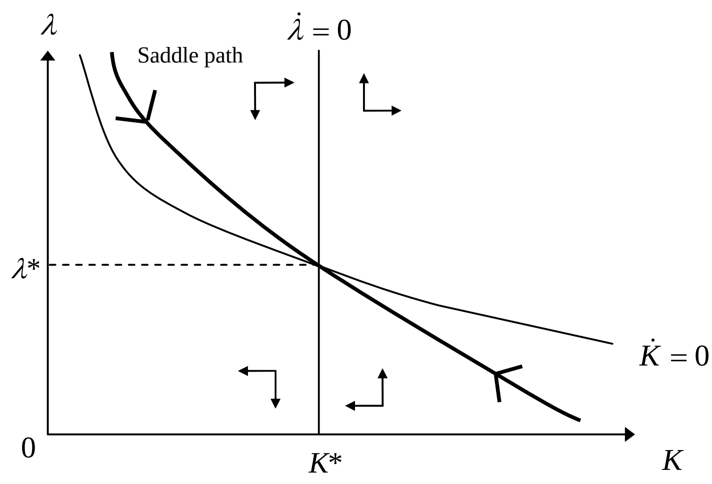

Figure 2: Dynamics of individual variables for low initial inequality