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DO THEY DETERMINE DETERMINACY
PROPERTIES?**

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INTERNATIONAL MACROECONOMICS



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ABSTRACT

Estimated Interest Rate Rules: Do they Determine Determinacy Properties?

No. I demonstrate that econometric estimations of nominal interest rate rules may tell little, if anything, about an economy's determinacy properties. In particular, correct inference about the interest-rate response to inflation provides no information about determinacy. Instead, it could reveal whether optimal monetary policymaking is performed under discretion or commitment.

JEL Classification: E52 and E58

Keywords: equilibrium determinacy, estimated taylor rules, interest rate rules, monetary policy and rules vs. discretion.

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1. Introduction

Since Taylor (1993) demonstrated that U.S. monetary policy could be described by a simple relationship between the short nominal interest rate, inflation and the output gap, numerous papers have empirically analyzed interest-rate setting across countries and periods through such policy rules. With the widespread adoption of interest-rate operating procedures by central banks, the Sargent and Wallace (1975) warning of an indeterminate price level under an interest-rate peg is relevant, and empirics on how interest rates respond to various macroeconomic aggregates could therefore in principle be informative about an economy's determinacy properties.¹

One of the most famous empirical excursions into interest-rate setting, and the associated implications for determinacy, involves U.S. monetary policy in recent decades. Taylor (1999), Clarida *et al.* (2000) and Lubik and Schorfheide (2004), *inter alia*, have presented evidence suggesting that monetary policy in the pre-Volcker era (1960–1979) did not adhere to the “Taylor principle.” On the other hand, policy during the Volcker-Greenspan period (1980–1998) did. This implies that in the former period, the nominal interest rate rose less than one for one with inflation in the long run, thus causing the real interest rate to fall with inflation, and *vice versa* for the latter period.²

Combined with the logic of New-Keynesian models of monetary policymaking, which have recently become mainstream in theoretical and practical monetary policy analyses (see, Galí, 2008, Walsh, 2003, and Woodford, 2003), these results have potential implications for inference about the stability properties of the U.S. economy. To ensure determinacy in these models, and thereby exclude the potential for arbitrary fluctuations in a rational expectations equilibrium, monetary policy must be properly specified.³ In a Taylor rule where inflation is the only argument, a necessary and sufficient condition is that the rule satisfies the Taylor principle, or, alternatively, is “active” in the language of Leeper (1991).⁴ Clarida *et al.* (2000) and Lubik and Schorfheide (2004) therefore argue

¹Indeed, McCallum (1981) suggested that feedback from a macroeconomic aggregate (in his example, the nominal money supply) onto the nominal interest rate could ensure a determinate price level.

²Orphanides (2001) and Perez (2001) question this structural change through estimations based on real-time data.

³From a welfare perspective, Schmitt-Grohé and Uribe (2007) argue that securing determinacy is the most crucial role to be played by monetary policy.

⁴Formally, an active Taylor rule in the New-Keynesian framework has a coefficient on inflation larger than one, implying that any expectations-driven increase in inflation results in contractive policy as the real interest rate increases. This depresses output and, through the inflation adjustment mechanism, also inflation. The self-fulfilling nature of higher inflation expectations is then avoided, and equilibrium is determinate. In versions of the Taylor rule where the lagged interest rate appears, determinacy is secured if the cumulative response to inflation is larger than one (this is indeed the Taylor principle). I use the terms “active” and “Taylor principle” interchangeably this paper.

that they may explain the high and volatile inflation in the pre-Volcker era as a result of a “passive” monetary policy inducing indeterminacy, and the lower and more stable inflation during the “active” Volcker-Greenspan tenures as consistent with the U.S. economy being in a determinate equilibrium void of extraneously generated fluctuations.⁵

The purpose of this paper is to show that estimated coefficients of interest rate policy rules may not reveal anything about the stability properties of an economy. To meet this end, I construct artificial data from stochastic simulations of a simple New-Keynesian model that shares all the basic properties of the models used to argue that such inference is possible. Specifically, the model has the feature that *if* the central bank’s decision making is modeled as adherence to a passive Taylor-type rule, equilibrium will be indeterminate. My analysis departs, however, from most empirical literature, in the sense that I model monetary policy as the outcome of *optimization* by the central bank. In theoretical terminology, this corresponds to a scenario that Svensson (2003) characterizes as a central bank adhering to a “targeting rule;” see also Rogoff (1985) and Walsh (2003, Chap. 8). (A bank modeled as adhering to a—potentially arbitrary—interest-rate rule, is said to be following an “instrument rule.”)

I have argued elsewhere (Jensen, 2002b) that under a modeling strategy involving optimization, the first-order conditions for optimal policy, essentially the targeting rules, become determinants of the equilibrium dynamics, and being efficiency conditions they help bring about determinacy (this is also one of the messages of Giannoni and Woodford, 2002). The intuition is that optimization circumvents a problem plaguing instrument rules. They are subject to a *reverse Lucas critique*, in the sense that any structural shift in private sector behavior—say, a sunspot-driven increase in inflation—is not triggering any change in policy behavior. Modeling monetary policy by an instrument rule in empirical analyses thus seems, from a methodologically point of view, completely at odds with the great care normally put into securing that private-sector behavior is micro founded and immune to the Lucas (1976) critique.⁶ It is usually not even discussed whether representing monetary policy by a Taylor rule is an appropriate modeling strategy.⁷ To be certain, Lubik and

⁵Bullard and Singh (2008) estimate Taylor-type rules for the U.S., Germany/Euro Area and Japan for different periods, and make inference about world determinacy properties based on the multi-country New-Keynesian model of Clarida *et al.* (2002).

⁶As Svensson (2003) puts it: “Macroeconomics long ago stopped modeling private economic agents as following mechanical rules for consumption, saving, production and investment decisions; instead they are now normally modeled as optimizing agents (...) It is long overdue to acknowledge that modern central banks are (...) optimizing to at least the same extent as private economic agents” (p. 467).

⁷Using the argument that it is observed in the data is missing the point. It corresponds to model private-sector consumption behavior by an “old” Keynesian consumption function, because data exhibits a positive relationship between consumption and income.

Schorfheide (2004) simply write “The central bank follows a nominal interest rate rule by adjusting its instrument to deviations of inflation and output from their respective target levels.” (p. 193).

Focusing on optimization, the artificial data I generate is always from a determinate equilibrium where the central bank does not adhere to an instrument rule. Nevertheless, the data may reveal Taylor-type rule relationships.⁸ Importantly, whether they reveal active or passive Taylor rules, has by construction nothing to do with the model economy’s stability properties. It only reveals that *in* equilibrium, inflation and the nominal interest rate may move together more or less strongly. The determinants of determinacy, the underlying first-order conditions for optimal policy, are not identified in such single-equation Taylor-rule estimations. Various experiments then seek to uncover what is crucial for finding active or passive Taylor rules in the data.

In particular, as the New-Keynesian model emphasizes forward-looking expectations, it follows from Kydland and Prescott (1977) that optimal policy suffers a time-consistency problem. I therefore compute optimal policies under both commitment and time-consistent, discretionary policymaking, and find that the associated empirical differences are substantial. Under commitment policies, an econometrician believing in the presence of Taylor-rule behavior would be fooled to believe in indeterminacy, as a passive Taylor rule would be estimated. Nevertheless, the economy is determinate and in the best possible equilibrium. In the particular model, commitment policies are characterized by history dependence (Woodford, 2003), and inflation is successfully stabilized by credible promises of future contractive policy. In equilibrium, the correlation between the current nominal interest rate and inflation may be very low (and for some parameterizations even negative). Finding passive Taylor rules could within the New-Keynesian framework may thus be indicative of the presence of a policy commitment. Ironically, what is usually seen as an indication of successful monetary policy, an active Taylor rule, is mostly appearing in the simulations under discretionary policymaking, i.e., under policymaking lacking credibility.⁹

Other recent literature has also questioned whether one can infer anything about determinacy properties from estimated coefficients of interest rate rules. For example, Cochrane (2007) argues that the coefficients of a rule cannot be properly identified. In a simple example with an active Taylor-type rule, he shows that the true coefficient on inflation will

⁸Minford *et al.* (2002) exemplify how a money supply rule is observationally equivalent to a Taylor-rule policy, further emphasizing that what data shows need not reflect what the central bank actually does.

⁹I am not arguing that the U.S. experience thereby suggests that monetary policy was characterized by commitment in the pre-Volcker era and no commitment afterwards. I merely point out that a passive Taylor rule *can* arise in data from a determinate equilibrium under optimal policymaking with commitment.

not be a determinant of the economy's dynamics, as its only role is to secure determinacy; i.e., to establish the unstable root(s) which will pin down the unique equilibrium. Also, Beyer and Farmer (2007) question Lubik and Schorfheide's (2004) method of distinguishing between economic dynamics under indeterminacy and determinacy. Specifically, they exemplify how a stochastic second-order difference equation under determinacy is observationally equivalent to a first-order difference equation under indeterminacy driven by sunspot shocks (Cochrane, 2007, makes a related point). Hence, they argue, a more thorough examination of the economic dynamics under passive and active policies, may not add further to inference about determinacy properties. Finally, from a theoretical angle, Davig and Leeper (2007) show that a passive Taylor-type rule need in itself not be an indication of indeterminacy, if private agents assign sufficient probability to the event that policy will shift to an active stance in the future.

In all of this related literature, the maintained assumption is that monetary policy decisions *are* made according to a mechanical rule. I, on the other hand, emphasize that if policy is performed at a level of sophistication matching that of private-sector decision making, then one may find empirical regularities resembling Taylor-type rule behavior. The coefficients, however, are functions of the deep parameters of the economy (including preference parameters of the central bank), and their size will *not*, even when perfectly identified, provide clues about the economy's stability properties.¹⁰

The remainder of the paper is structured as follows. Section 2 presents theoretically, within the simplest possible model, how equilibrium relationships between the nominal interest rate and inflation can look active or passive depending on the structural characteristics of the economy and depending on the assumption made about the type of optimization performed (commitment or discretion). Section 3 then presents the extended and more empirically oriented version of the model, as well as the empirical exercises on its data. Section 4 concludes, and the Appendix contains various derivations.

¹⁰It should be noted that the results are cast within one (albeit popular) class of model. Other classes, e.g., do not necessarily prescribe active Taylor rules as needed for determinacy. E.g., models with money in the production function, models with loans constraints, limited participation models, or models of the fiscal theory of the price level determination. On determinacy requirements in such models, see, e.g., Benhabib *et al.* (2001), Carlstrom and Fuerst (2002), Christiano and Gust (1999) and Woodford (1996). However, my point that identified Taylor-rule coefficients are uninformative should carry through irrespective of the particular critical coefficient values securing determinacy in an instrument-rule framework.

2. Optimal monetary policymaking and Taylor-type rules

2.1. The simple New-Keynesian model

To get a grasp of the main properties of the “empirical” results, this section presents some theoretical results in the most basic text-book version of the closed-economy New-Keynesian model. The micro-founded model is a log linearization of an imperfect competition model with sticky prices; see Galí (2008), Walsh (2003) or Woodford (2003) for expositions. Defining x_t as the output gap, the consumption-Euler equation leads to the following IS curve:

$$x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) + \mu_t, \quad \sigma > 0, \quad (1)$$

The (short) nominal interest rate, i_t , is the monetary policy instrument. The inflation rate is π_t (the log difference of prices between $t - 1$ and t). E_t is the rational expectations operator conditional upon all information up to, and including, period t . The shock μ_t includes interest-insensitive spending and expected (log) changes in the natural rate of output. Aggregate supply is modeled by a New-Keynesian Phillips Curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \varepsilon_t, \quad \kappa > 0, \quad 0 < \beta < 1, \quad (2)$$

which can be derived from a variety of optimizing supply-side models. The term ε_t is a shock, often labelled a “cost-push” shock, cf. Clarida *et al.* (1999), and it involves any variation in π_t unexplained by the output gap and expected future inflation (for example variations in firms’ mark-up). It is assumed that $\varepsilon_t = \rho \varepsilon_{t-1} + \xi_t$, where $0 \leq \rho < 1$ and ξ_t is a white-noise innovation.

In this model, the Sargent and Wallace (1975) indeterminacy result applies, as a pure interest rate peg leads to an indeterminate equilibrium; cf. Woodford (2003).¹¹ To understand this, consider a sunspot-driven increase in inflation expectations. With a constant nominal interest rate, the real interest rate falls. This stimulates demand and the output gap. Through the interaction of the IS and Phillips curves, this increases current inflation more than the increase in expected inflation. The increase in expected inflation therefore initiates an increase in output and inflation, which is followed by the variables’ gradual return to steady state. As the increase in inflation expectations is of arbitrary size, one

¹¹In this sticky-price model, the issue under consideration is that of *real* determinacy (inflation and output gap determinacy) as opposed to nominal determinacy (price level determinacy) in the original Sargent and Wallace analysis.

cannot pin down a unique non-explosive rational expectations equilibrium.

At this point researchers often close the model with a specification of nominal interest rate setting taking the form of a Taylor-type rule, e.g.,

$$i_t = b_0\pi_t + a_0x_t. \quad (3)$$

The resulting system (1), (2) and (3) thus incorporates monetary policymaking as taking the form of adherence to an instrument rule. Accordingly, I have in Jensen (2002b) denoted the associated rational expectations equilibrium an *instrument rule equilibrium*—an “IRE.” As is well known from the literature, the IRE will be determinate only if certain restrictions on a_0 , $b_0 \geq 0$ are satisfied. Specifically, the rule should follow a generalized Taylor principle, meaning that it should feature sufficiently aggressive responses towards both inflation and output; see, e.g., Bullard and Mitra (2002) or Woodford (2003, Chapter 4). With $a_0 = 0$, a necessary and sufficient condition is $b_0 > 1$; i.e., the rule should be active.¹²

I instead close the model by assuming that monetary policy is conducted with the aim of stabilizing inflation and the output gap (around zero target values). This reflects, in my opinion, closer what are the *modi operandi* of real-life central banks, as these typically conduct policy with the aim of attaining various values for goal variables specified in their legal mandates. The choice of inflation and output gap as the goal variables for the central bank follows a long tradition in macroeconomic modeling, and is in accordance with the main mandates of inflation-targeting central banks (cf. Svensson, 2003). The preferences of the central bank are thus captured by the loss function

$$L = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} [\lambda x_s^2 + \pi_s^2], \quad \lambda > 0, \quad (4)$$

indeed reflecting that variations in inflation and the output gap are disliked. Under the model (1)-(2), L may represent a second-order approximation to the (negative of the) utility of a representative agent in the economy (Woodford, 2003). In that case, the preference parameter λ is a function of the structural parameters of the model, but for sake of generality [e.g., allowing (4) to represent a delegated loss function], I treat λ as independent. An optimizing central bank is assumed to determine policy with the aim of

¹²In estimations based on both inflation and output gap, a positive value of a_0 thus implies that an instrument rule can result in determinacy for $b_0 < 1$. However, Clarida *et al.* (2000) show numerically that the change in the required magnitude of b_0 is very small such that the focus on active versus passive Taylor rules is justified.

minimizing L subject to (1) and (2). The relevant optimality conditions (in this simple case, just one) will add to the dynamic system, and the associated rational expectations equilibrium is denoted an *optimizing central bank equilibrium*—an “OCBE.”¹³

2.2. Optimal monetary policy under discretion

Under discretionary policymaking, the central bank solves

$$\min_{\{i_s\}_{s=t}^{\infty}} \mathbb{E}_t \left[\sum_{s=t}^{\infty} \beta^{s-t} (\lambda x_s^2 + \pi_s^2) \right], \quad \text{s.t. (1) and (2),} \quad (5)$$

where minimization is performed on a period-by-period basis. The resulting necessary first-order condition is

$$\lambda x_t + \kappa \pi_t = 0. \quad (6)$$

Used in (2), one obtains the expectational difference equation for inflation:

$$\pi_t = \beta (1 + \kappa^2/\lambda)^{-1} \mathbb{E}_t \pi_{t+1} + (1 + \kappa^2/\lambda)^{-1} \varepsilon_t. \quad (7)$$

Since π_t is free and the eigenvalue $(1 + \kappa^2/\lambda)/\beta$ is strictly greater than one, it follows from Blanchard and Kahn (1980) that π_t has a unique, non-explosive rational expectations solution. Using this together with (6), the unique solutions for π_t and x_t can be written:

$$\pi_t = \frac{\lambda}{\kappa^2 + \lambda(1 - \beta\rho)} \varepsilon_t, \quad x_t = -\frac{\kappa}{\kappa^2 + \lambda(1 - \beta\rho)} \varepsilon_t. \quad (8)$$

Hence, the OCBE exhibits real determinacy. I.e., the rational expectations solutions for output gap and inflation are unique functions of the fundamentals of the economy, here just represented by the predetermined variable ε_t (as the central bank completely neutralizes the effects of the shock μ_t).

Intuitively, consider the situation where a sunspot-driven increase in inflation expectations was occurring. Absent any policy reaction, current inflation and the output gap would increase. A central bank operating with the aim of price and output gap stability, however, would immediately raise the interest rate in order to bring inflation and output towards their targets. In consequence, the imagined increase in inflation and the output gap cannot be part of a bounded rational expectations equilibrium. The first-order condi-

¹³In Jensen (2002b), I used the label “targeting rule equilibrium” as the envisioned central bank behavior corresponds to that under what Svensson (2003) calls a “general” targeting rule. Since he also operates with the concept of a “specific” targeting rule (see also Svensson and Woodford, 2005), I prefer here to use a label that explicitly reveals that optimal central bank behavior is crucial.

tion (6) precludes this, as the condition shows that the central bank acts so as to equate the marginal loss in terms of output gap and inflation to zero. In this simple model, it implies that in equilibrium any co-movement between the output gap and inflation is *negative*. Consider then a candidate equilibrium in which the relationship between inflation and the output gap is given by $x_t = (\omega/\kappa) \pi_t$, where ω is some parameter. Inserting this relationship into (2) yields the difference equation $\pi_t = \beta(1 - \omega)^{-1} \mathbb{E}_t \pi_{t+1} + (1 - \omega)^{-1} \varepsilon_t$. This leads to infinitely many solutions for π_t only if $\beta(1 - \omega)^{-1}$ is numerically greater than one. A necessary condition for this is $\omega > 0$. I.e., under indeterminacy, the co-movement between inflation and the output gap is positive. This, however, contradicts the first-order condition guiding optimal monetary policy; hence, determinacy prevails in an OCBE.

To derive the nominal interest rate that will be observed in an OCBE, lead π_t and x_t as given by (8), and take period t expectations to get

$$\mathbb{E}_t \pi_{t+1} = \frac{\lambda \rho}{\kappa^2 + \lambda(1 - \beta\rho)} \varepsilon_t, \quad \mathbb{E}_t x_{t+1} = -\frac{\kappa \rho}{\kappa^2 + \lambda(1 - \beta\rho)} \varepsilon_t,$$

which together with x_t inserted into (1), yields

$$i_t = \frac{\kappa(1 - \rho) + \sigma \lambda \rho}{\sigma [\kappa^2 + \lambda(1 - \beta\rho)]} \varepsilon_t + \frac{1}{\sigma} \mu_t. \quad (9)$$

If one replaced (3) by (9) and examined the associated IRE, one would find indeterminacy, as (9) is a function of exogenous variables only; cf. Svensson and Woodford (2005). However, (9) is not the only way of expressing the OCBE value of the nominal interest rate. As stressed by Giannoni and Woodford (2002) for a general class of models, one can write the nominal rate as infinitely many equilibrium relationships with predetermined and other endogenous variables. In this model one has:

$$i_t = \sum_{i=0}^{\infty} b_i \mathbb{E}_t \pi_{t+i} - \sum_{i=0}^{\infty} a_i \mathbb{E}_t x_{t+i} + c \varepsilon_t + \frac{1}{\sigma} \mu_t, \quad (10)$$

where there is one restriction on the coefficients $\{a_i\}_{i=0}^{\infty}$, $\{b_i\}_{i=0}^{\infty}$ and c (see Appendix A):

$$\sigma \lambda \sum_{i=0}^{\infty} b_i \rho^i + \sigma \kappa \sum_{i=0}^{\infty} a_i \rho^i + c \sigma [\kappa^2 + \lambda(1 - \beta\rho)] = \kappa(1 - \rho) + \sigma \lambda \rho. \quad (11)$$

As a result, one can empirically only uniquely identify one of the coefficients, and nothing in theory points to anyone being more appropriate than the other. This suggests that empirically examining equilibrium relationships between the nominal interest rate and

macroeconomic variables may not yield an appropriate answer as to whether determinacy prevails or not. One can represent the determinate OCBE as equilibrium relations between the nominal interest rate, which will lead to an indeterminate IRE [as in the case of (9)]. However, one can also represent it as relations between the interest rate and endogenous variables that would render the associated IRE determinate. To see this, note that as there is only one restriction on this representation, (11), one can only express the nominal interest rate as a unique function of one endogenous variable. One of these representations is the one presented by Clarida *et al.* (1999). It expresses the nominal interest rate as a function of the one-period ahead inflation expectations, and thus depicts a forward-looking Taylor rule where there is no response towards the output gap. This corresponds to a case where $a_i = 0$, all i , $b_0 = 0$, $b_i = 0$, all $i > 1$, $c = 0$, and where b_1 is determined by (11). This results in the following expression:

$$i_t = \left[1 + \frac{\kappa(1-\rho)}{\sigma\lambda\rho} \right] \mathbb{E}_t \pi_{t+1} + \frac{1}{\sigma} \mu_t. \quad (12)$$

Note that the coefficient on expected inflation is greater than one, implying that any increase in expected inflation is associated with an increase in the real interest rate. The nominal interest rate is thus in conformity with the Taylor principle. An IRE based on (1), (2) and (12) is accordingly determinate, unless, however, $\kappa(1-\rho)/(\sigma\lambda\rho)$ is not too high; see Bullard and Mitra (2002), Woodford (2003, Chap. 4) (Appendix B of Jensen (2002b) provides the condition for the present model). This qualification reflects that an interest rate rule formulated in terms of expected inflation can be “too active,” and make the economy vulnerable to expectations-driven fluctuations.¹⁴ Hence, if one econometrically identified the coefficient on expected inflation in (12) as not being too high, and if one believed that the central bank operated under an instrument rule setting, one would conclude that the economy is determinate.¹⁵

However, if one assumed that the central bank followed a Taylor-type rule based on current inflation, results and inference change dramatically. In that case, all parameters

¹⁴E.g., an arbitrary increase in $\mathbb{E}_t \pi_{t+1}$ can be self-fulfilling, as it implies a huge fall in demand and output and, hence, current inflation, because of the strong increase in the real interest rate. The economy will then “zig zag” back to steady state (e.g., $\mathbb{E}_t \pi_{t+2}$ would be below steady state, which is consistent with $\mathbb{E}_t \pi_{t+1}$ being above, as the real interest rate in $t+1$ would fall strongly thereby pushing up output and inflation above steady state in $t+1$).

¹⁵Numerical simulations presented by Clarida *et al.*, 2000, show that the critical value of b_1 above which indeterminacy prevails is empirically unrealistically large.

$\{a_i\}_{i=0}^\infty$, $\{b_i\}_{i=0}^\infty$ and c in (10) except b_0 are zero, and (11) implies

$$i_t = \left[\rho + \frac{\kappa(1-\rho)}{\sigma\lambda} \right] \pi_t + \frac{1}{\sigma} \mu_t. \quad (13)$$

As evident, the coefficient on π_t can be greater or smaller than one, which implies that an IRE associated with this interest rule would be indeterminate for some parameter constellations (those where b_0 is lower than one), and determinate for others. Hence, correct econometric identification of b_0 could lead to the erroneous inference that the economy is in an indeterminate regime.¹⁶

As an aside, note also that differences in estimated coefficients over time and countries, need not reflect differences in preferences for inflation stabilization. Even though λ is a determinant of both b_1 and b_0 , other structural parameters play a role for their size under optimal policymaking. Also, note that (12) and (13) could, if interpreted as instrument rules, mislead observers to think that the output gap is of no importance to the central bank. This emphasizes that estimated Taylor-type rules do not necessarily reveal much about the preferences of an optimizing central bank. Neither in terms of how much weight is attributed to a given aggregate, nor which aggregates are important at all (see also Svensson, 1997).

2.3. Optimal monetary policy under commitment

I consider now the case where an optimizing central bank has the ability to credibly commit to a policy plan for the entire future. The assumption of discretionary optimization is therefore abandoned in favor of an assumption that the bank acts in accordance with commitment under a “timeless perspective” (Woodford, 1999). It turns out that the resulting equilibrium relationships between the nominal interest rate and endogenous variables even stronger demonstrate that inference about determinacy properties from these can be erroneous. Solving (5) under commitment, leads to the following relevant first-order condition (see, e.g., Walsh, 2003, Chap. 11):

$$\pi_t + \frac{\lambda}{\kappa} (x_t - x_{t-1}) = 0, \quad (14)$$

¹⁶Also, note that one can express i_t as a function of the one-period ahead expectation of the output gap. It immediately follows by (6) and (12) that the interest rate expression in this format would become $i_t = -[\lambda/\kappa + (1-\rho)/(\sigma\rho)] E_t x_{t+1} + (1/\sigma) \mu_t$. An IRE under this interest rule is always indeterminate; see Jensen (2002b, Appendix C).

which along with (2) characterize the OCBE as a second-order difference equation in x_t ,

$$\mathbf{E}_t x_{t+1} = (1 + \beta^{-1} (1 + \kappa^2/\lambda)) x_t - \beta^{-1} x_{t-1} + (\beta^{-1} \kappa/\lambda) \varepsilon_t.$$

It has real and positive roots

$$\frac{1 + \beta^{-1} (1 + \kappa^2/\lambda) \pm \sqrt{(1 + \beta^{-1} (1 + \kappa^2/\lambda))^2 - 4\beta^{-1}}}{2},$$

with one root smaller than one and the other above one. The equation thus identifies a *unique*, non-explosive rational expectations equilibrium solution for x_t . The solution is easily recovered by the method of undetermined coefficients. Combining this solution with (14), the unique solution for π_t follows. The OCBE under policy commitment can be written as

$$\begin{aligned} x_t &= \chi x_{t-1} - \varphi \varepsilon_t, & \pi_t &= \frac{\lambda}{\kappa} (1 - \chi) x_{t-1} + \frac{\lambda}{\kappa} \varphi \varepsilon_t, & (15) \\ 0 < \chi &\equiv \frac{1 + \beta^{-1} (1 + \kappa^2/\lambda) - \sqrt{(1 + \beta^{-1} (1 + \kappa^2/\lambda))^2 - 4\beta^{-1}}}{2} < 1, \\ \varphi &\equiv \frac{\kappa \chi}{\lambda (1 - \chi \beta \rho)} > 0. \end{aligned}$$

Note the difference of the solution with the one under discretion. In contrast with (8), the solution under commitment includes the lagged value of the output gap. This reflects the optimality of “history-dependent” policy, or “inertial policy,” as stressed by Woodford (1999, 2003). In this model, such behavior induces a more favorable inflation-output gap trade-off in the following sense. Consider the case where the economy is hit by a (positive) cost-push shock. By committing to continue the associated contractive policy into the future, the central bank lowers inflation expectations, which dampens current inflation and thus helps to stabilize the shock.

As under discretion, one can characterize the path of the nominal interest rate as function of the predetermined variables, here x_{t-1} , ε_t and μ_t . Lead π_t and x_t as given by (15), apply the value of x_t , and take period t expectations to get

$$\mathbf{E}_t \pi_{t+1} = \frac{\lambda}{\kappa} (1 - \chi) \chi x_{t-1} - \frac{\lambda}{\kappa} \varphi (1 - \chi - \rho) \varepsilon_t, \quad \mathbf{E}_t x_{t+1} = \chi^2 x_{t-1} - \varphi (\chi + \rho) \varepsilon_t,$$

which together with x_t inserted into (1) yields

$$i_t = -\frac{\chi}{\sigma} (1 - \chi) \left(1 - \frac{\lambda\sigma}{\kappa}\right) x_{t-1} + \frac{\varphi}{\sigma} (1 - \chi - \rho) \left(1 - \frac{\lambda\sigma}{\kappa}\right) \varepsilon_t + \frac{1}{\sigma} \mu_t. \quad (16)$$

Paralleling the discussion of (9), which describes the interest rate in case of discretion, an instrument rule characterized by (16), may result in an indeterminate IRE (unless the coefficient on x_{t-1} is positive and sufficiently large; cf. Jensen, 2002b). This reflects once more that identification of a passive interest rate rule may lead to inference of indeterminacy, even though the economy is in a determinate OCBE. As under discretion, the unique OCBE value of the nominal interest rate can be represented by infinitely many equilibrium relationships between the predetermined variables and current or expected leads of the other endogenous variable. In contrast with discretion, two restrictions on the relationships must be satisfied since there are two predetermined variables (apart from μ_t , which does not play a role; cf. above). One gets

$$i_t = \sum_{i=0}^{\infty} \bar{b}_i \mathbf{E}_t \pi_{t+i} - \sum_{i=0}^{\infty} \bar{a}_i \mathbf{E}_t x_{t+i} + \bar{c} \varepsilon_t - d x_{t-1} + \frac{1}{\sigma} \mu_t, \quad (17)$$

with two restrictions on the coefficients $\{\bar{a}_i\}_{i=0}^{\infty}$, $\{\bar{b}_i\}_{i=0}^{\infty}$, \bar{c} and d (see Appendix B):

$$\sum_{i=0}^{\infty} \bar{b}_i \frac{\lambda}{\kappa} (1 - \chi) \chi^i - \sum_{i=0}^{\infty} \bar{a}_i \chi^{i+1} - d = -\frac{\chi}{\sigma} (1 - \chi) \left(1 - \frac{\lambda\sigma}{\kappa}\right), \quad (18)$$

$$-\sum_{i=0}^{\infty} \bar{b}_i \frac{\lambda}{\kappa} \varphi \left[(1 - \chi) \sum_{j=0}^{i-1} \chi^{i-j-1} \rho^j - \rho^i \right] + \varphi \sum_{i=0}^{\infty} \bar{a}_i \sum_{j=0}^{i-1} \chi^{i-j} \rho^j + \bar{c} = \frac{\varphi}{\sigma} (1 - \chi - \rho) \left(1 - \frac{\lambda\sigma}{\kappa}\right). \quad (19)$$

As the purpose of this exercise is to show that estimated equilibrium relationships tell little about determinacy, consider the representation which most closely corresponds to (12); i.e., the often presented equilibrium representation under discretion. This would now be a case of $i_t = \bar{b}_1 \mathbf{E}_t \pi_{t+1} + \bar{a}_0 x_t + (1/\sigma) \mu_t$, and where \bar{b}_1 and \bar{a}_0 are identified by (18) and (19). The following expression emerges:

$$i_t = \left[1 - \frac{\kappa}{\lambda\sigma}\right] \mathbf{E}_t \pi_{t+1} + \frac{1}{\sigma} \mu_t. \quad (20)$$

Hence, $\bar{a}_0 = 0$, $\bar{b}_1 < 1$ and one cannot even rule out that $\bar{b}_1 < 0$. That is, *in equilibrium*—a determinate equilibrium—there may be a *negative* relationship between the nominal interest rate and inflation expectations, and it is *always* the case that the Taylor rule is

passive.¹⁷ The reason is the history dependence of policy, where a contraction in period t is expected to be followed by a future contractive stance. As this reduces inflation expectations, equilibrium policy is contractive in period t , i.e., the real interest rate increases, when the nominal interest rate either increases (in which case the correlation between i_t and $E_t\pi_{t+1}$ is negative), or decreases less than inflation expectations do (in which case the correlation between i_t and $E_t\pi_{t+1}$ is positive but less than one).

Note also that since there are restrictions on the relationship between coefficients under optimal policymaking, one cannot hope to independently identify coefficients in estimations containing several variables.¹⁸ As evident by the previous example, $\bar{a}_0 = 0$ prevails in an estimation. This not only could lead to the erroneous inference that the central bank does not care about output gap fluctuations, but it also shows that there is a limit as to how many coefficients one can identify in policy rules. E.g., in the case of discretion, one would not be able to identify coefficients on inflation and output gap independently. From the model, only a linear combination of the coefficients will be identifiable. Needless to say, the adverse implications for inference about indeterminacy will be strong.

To exemplify in “practice” how estimations yield results that give false impressions about the economy’s stability properties, I now move on to simulations on an extended model that allow for independent identification of coefficients on both inflation and output gap; the coefficients usually estimated in empirical applications.

3. Estimated Taylor-type rules under optimal policymaking

3.1. An extended New-Keynesian model as a data generator

The model used in the previous section is too simple to portray an empirically realistic scenario for monetary policymaking. In this section, the model is therefore amended in a number of empirically relevant directions. First, it is assumed that the output gap is predetermined one period ahead, and inflation is predetermined two periods ahead. In consequence, monetary policy affects the output gap with a one-period lag, and inflation with a two-period lag. This seems to be broadly consistent with empirical regularities,

¹⁷In the case where λ is formulated as a function of the model’s structural parameters, one has $\lambda = \kappa/\eta$ where $\eta > 1$ is the elasticity of substitution between goods. The coefficient on expected inflation in (20) is then $1 - \eta/\sigma$, which for plausible parameter values are negative.

¹⁸When Clarida *et al.* (2000) present their Taylor-type rule to be estimated, containing a host of leads of inflation and output gap, they state that “Approximate (...) forms of this rule are optimal for a central bank that has a quadratic loss function in deviations of inflation and output from their respective targets” (p. 150-1). They do not mention that optimal behavior necessarily restricts the relationship between coefficients.

which show that effects of monetary policy are first present in output (after some time) and then later in the inflation rate; see Walsh (2003, Chapter 1) and the references provided there. From the perspective of the conventional Taylor rule relating the current interest rate to the current inflation rate and output gap, this implies that endogeneity problems are avoided, and identification of coefficients are secured through OLS estimation.¹⁹ Secondly, I introduce endogenous persistence in both the output gap and inflation equations. Again, this is mainly empirically motivated; it is easy to reject that output and inflation do not depend on their past values. See, e.g., Fuhrer (2000) on demand persistence and Galí and Gertler (1999) on inflation persistence in U.S. data.

With these alterations, the equations for the output gap and inflation become

$$x_{t+1} = \theta x_t + (1 - \theta) \mathbb{E}_t x_{t+2} - \sigma (\mathbb{E}_t i_{t+1} - \mathbb{E}_t \pi_{t+2}) + g'_{t+1}, \quad 0 \leq \theta < 1, \quad (21)$$

$$\pi_{t+2} = \phi \pi_{t+1} + (1 - \phi) \beta \mathbb{E}_t \pi_{t+3} + \kappa \mathbb{E}_t x_{t+2} + \varepsilon_{t+2}, \quad 0 \leq \phi < 1, \quad (22)$$

with

$$g'_{t+1} \equiv \theta y_t^n + g_{t+1} - y_{t+1}^n + (1 - \theta) \mathbb{E}_t y_{t+2}^n,$$

and where $g_{t+1} = \gamma_g g_t + \xi_{t+1}^g$, $0 \leq \gamma_g < 1$, is a demand shock process, and $y_{t+1}^n = \gamma_y y_t^n + \xi_{t+1}^y$, $0 \leq \gamma_y < 1$, is the evolution of the stochastic natural rate of output (thus capturing technology shocks). The innovations ξ_{t+1}^g and ξ_{t+1}^y (and ξ_{t+1}) are i.i.d. and white noise. The parameters θ and ϕ , respectively, quantify the degree of endogenous persistence in demand and inflation. Furthermore, remark that the described transmission lags of monetary policy are achieved as demand in period $t + 1$ is decided in period t (through the period t expectation of the real interest rate in period $t + 1$), and prices for period $t + 2$ are set in period t (through the period t expectation of the inflation rate in period $t + 3$ and the expectation of the period $t + 2$ output gap). An interest-rate decision in period t thus cannot affect demand in period t , but only demand in period $t + 1$ (given, of course, that the decision has implications for the period $t + 1$ real interest rate) and thus inflation in period $t + 2$.

In this more elaborate version, the model cannot be solved in closed form. Instead, I follow standard practice and express the model in state-space form and adopt conventional numerical solution algorithms for deriving optimal policies under discretion and commitment. See, e.g., Backus and Driffill (1986) or Söderlind (1999) on these methods, and Svensson (2000) and Jensen (2002a) for applications. These algorithms generally perform

¹⁹This is also the identification assumption used by Rotemberg and Woodford (1997), and does not appear overtly restrictive when the data frequency is assumed to be quarterly.

very well, but for the current model, convergence under discretion requires that the control variable enters the loss function. The loss function is therefore amended to

$$\tilde{L} = E_t \sum_{s=t}^{\infty} \beta^{s-t} [\lambda x_s^2 + \pi_s^2 + v i_s^2], \quad v > 0, \quad (23)$$

which allows for a (in the simulations small) loss from instrument variability *per se*.²⁰ In state-space form, the model is written as

$$\begin{bmatrix} \mathbf{X}_{t+1} \\ E_t \boldsymbol{\chi}_{t+1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} \mathbf{X}_t \\ \boldsymbol{\chi}_t \end{bmatrix} + \mathbf{B}^2 E_t i_{t+2} + \mathbf{B}^1 E_t i_{t+1} + \begin{bmatrix} \boldsymbol{\xi}_{t+1} \\ \mathbf{0}_{3 \times 1} \end{bmatrix}, \quad (24)$$

where \mathbf{A} is a 9×9 matrix, \mathbf{B}^2 and \mathbf{B}^1 are 9×1 vectors, $\mathbf{X}_t \equiv [\tilde{g}_t \quad y_t^n \quad \varepsilon_t \quad x_t \quad \pi_t \quad E_t \pi_{t+1}]'$ is the vector of the predetermined variables, $\boldsymbol{\chi}_t \equiv [E_t x_{t+1} \quad E_t x_{t+2} \quad E_t \pi_{t+2}]'$ is the vector of the forward-looking variables, and $\boldsymbol{\xi}_{t+1} \equiv [\xi_{t+1}^g \quad \xi_{t+1}^y \quad \xi_{t+1} \quad \xi_{t+1}^g - \xi_{t+1}^y \quad \xi_{t+1} \quad (\phi + \rho) \xi_{t+1}]'$ is the vector of innovations. Details on \mathbf{A} , \mathbf{B}^2 and \mathbf{B}^1 , as well as the solution procedures, are relegated to the GAUSS solution routines that are available upon request.

The model is, as mentioned, solved under the assumption of either commitment or discretionary policymaking. From each case, one recovers expressions for the dynamic evolution of the economy of the form

$$\begin{bmatrix} \mathbf{X}_{t+1} \\ E_t \boldsymbol{\chi}_{t+1} \end{bmatrix} = \boldsymbol{\Omega} \begin{bmatrix} \mathbf{X}_t \\ \boldsymbol{\chi}_t \end{bmatrix} + \begin{bmatrix} \boldsymbol{\xi}_{t+1} \\ \mathbf{0}_{3 \times 1} \end{bmatrix}, \quad t > 1, \quad (25)$$

with \mathbf{X}_1 given and $\boldsymbol{\chi}_1 = \mathbf{C} \mathbf{X}_1$, where $\boldsymbol{\Omega}$ and \mathbf{C} are matrices found by the solution algorithms (differing, of course, between discretion and commitment). It is important to note that, by implication of the solution algorithms, (25) represents a case where *only* fundamentals matter for the dynamics of the economy (endogenous variables are a function of predetermined variables only). Hence, the OCBE under discretion and commitment are determinate *by construction*. Under both forms of policy, 9000 periods of data are generated by stochastic simulations of (25). From these simulations, time-series data for i_t , x_t , π_t , $E_t \pi_{t+1}$, $E_t \pi_{t+2}$ are extracted for subsequent estimations of Taylor-type interest-rate rules.

As these estimations are meant to serve as illustrations of what can, and, in particular,

²⁰A similar convergence problem is reported by Svensson (2000), who therefore adds a small loss of interest rate *changes* to the loss function. This, however, introduces a rather high degree of interest rate smoothing in the optimal solution, which is also the case the current model (the solution for the interest rate will feature a response coefficient on i_{t-1} of around 0.3 even with a negligible weight on interest rate changes). I therefore refrain from including an interest-rate smoothing objective.

$\theta = 0.5,$	$\phi = 0.3,$	$\sigma = 0.25,$	$\kappa = 0.1,$
$\lambda = 0.5,$	$v = 0.05,$	$\beta = 0.99,$	
$\sigma_g = 0.02,$	$\sigma_y = 0.007,$	$\sigma_\varepsilon = 0.02,$	
$\gamma_g = 0.25,$	$\gamma_y = 0.98,$	$\rho = 0.00.$	

Table 1: Benchmark configuration for simulations

what cannot, be inferred from these, the model given by (21)-(22) has not been calibrated so as to match business cycle properties accurately. The model is interpreted as quarterly, with inflation and the nominal interest rate measured at annual rates, and the adopted parameter values have merely been chosen so as not to be grossly inconsistent with existing estimates of demand and inflation equations, and so as not to give unreasonable unconditional standard deviations of the output gap and inflation under either form of policy. The parameter values, which to a large extent are similar to those in Jensen (2002a), are summarized in Table 1, where σ_g , σ_y , and σ_ε , respectively, represent the standard deviations of the innovations ξ_{t+1}^g , ξ_{t+1}^y , and ξ_{t+1} (under this parameterization, the standard deviations of inflation and the output gap are between 2 and 2.5 percent).

3.2. Estimation of Taylor-type rules: discretion vs. commitment

The estimated Taylor-type rules are of the general form:

$$i_t = b_0\pi_t + b_1\mathbb{E}_t\pi_{t+1} + b_2\mathbb{E}_t\pi_{t+2} + a_0x_t + \alpha i_{t-1} + \epsilon_t, \quad (26)$$

where ϵ_t is an error term. Each estimation uses only a subset of the right-hand side variables.²¹ Estimation results for the benchmark parameterization when the central bank optimizes under discretion are reported in the upper half of Table 2. It is seen that a simple Taylor-type rule of the form $i_t = b_0\pi_t + a_0x_t$ is identified in the data, although the central bank is not adhering to one. Also, note that the table shows that the estimated coefficients are such that an associated IRE would be indeterminate. Hence, if one infers that the central bank has been following a Taylor rule, one would conclude that the economy is vulnerable to sunspot fluctuations, even though the economy has in fact been in a determinate OCBE in 9000 periods. Estimating a Taylor rule depending upon the one-period ahead inflation expectations gives the same result, although the estimated coefficients are “closer” to render the associated IRE determinate. In the case of a Taylor rule

²¹All estimations are OLS regressions except when right-hand side variables involve $\mathbb{E}_t\pi_{t+2}$, which is endogenous (all other variables are predetermined). In that case, IV estimation is used with $\mathbb{E}_t\pi_{t+1}$ as instrument variable.

Variables in estimated interest rate rule (26) ^a					Property of associated IRE ^b
π_t	$E_t\pi_{t+1}$	$E_t\pi_{t+2}$	x_t	i_{t-1}	
————— Discretion data —————					
0.27*	—	—	2.22*	—	I
0.29*	—	—	2.33*	-0.09*	I
—	0.65*	—	2.19*	—	I
—	0.92*	—	2.33*	-0.12*	I
—	—	1.55*	2.08*	—	D
—	—	2.15*	2.17*	-0.11*	D
————— Commitment data —————					
0.02	—	—	0.85*	—	I
0.05	—	—	0.74*	0.47*	I
—	-0.42*	—	0.84*	—	I
—	0.03	—	0.74*	0.47*	I
—	—	-1.15*	0.81*	—	I
—	—	0.10	0.74*	0.47*	I

^a “*” denotes significance at the 5% level

^b “I” denotes indeterminacy of the associated IRE; “D” denotes determinacy

Table 2: Coefficients of estimated interest rate rules. Benchmark parameters

depending upon the two-period ahead inflation expectations (the time horizon at which policy can affect inflation), estimates suggest that policy has been conducted according to an “active” Taylor rule (even though it has not).²²

Turning to the case where optimal policy is conducted under commitment, the lower half of Table 2 shows estimation results which never portray a Taylor-type rule behavior. In fact, the signs on inflation measures are often negative (consistent with what was shown to be possible in Section 2.3 for the simple model). The associated IRE are always indeterminate, and an economist believing that policy has been conducted within an instrument-rule framework would in all likelihood call for a revision in policymaking advising that the central bank should respond more aggressively towards inflation. This would be unwarranted, as the economy is in the optimal, and determinate, equilibrium.²³ This illustrates that identification of interest rate relationships in data reflecting a behavior not in conformity with the Taylor principle, need not be a reflection of disastrous monetary policymaking. In fact, in a New-Keynesian economy, it might as well reflect the performance of a central bank acting optimal and with the ability to commit.

²²In all cases, the lagged interest rate is significant but of small magnitude.

²³Note that the coefficient on the lagged interest rate is now of considerably magnitude; not because the central bank has an objective to “smooth” interest rates *per se*, but because of the history-dependent policy it follows under commitment.

Variables in estimated interest rate rule (26) ^a					Property of associated IRE ^b
π_t	$E_t\pi_{t+1}$	$E_t\pi_{t+2}$	x_t	i_{t-1}	
Discretion data					
0.30*	—	—	1.31*	—	I
0.33*	—	—	1.39*	-0.09*	I
—	0.66*	—	1.25*	—	I
—	1.08*	—	1.39*	-0.20*	I
—	—	1.72*	1.04*	—	D
—	—	2.53*	1.05*	-0.15*	D
Commitment data					
0.02*	—	—	0.50*	—	I
0.04*	—	—	0.36*	0.66*	I
—	-0.34*	—	0.50*	—	I
—	0.11*	—	0.36*	0.67*	I
—	—	-1.05*	0.48*	—	I
—	—	0.41*	0.36*	0.69*	D

^a“*” denotes significance at the 5% level

^b“I” denotes indeterminacy of the associated IRE; “D” denotes determinacy

Table 3: Coefficients of estimated interest rate rules.
Deviation from benchmark: A more “conservative” central banker, $\lambda = 0.1$

3.3. Estimation of Taylor-type rules: further experiments

The preceding subsection showed that the estimated coefficients on inflation measures depend heavily on the type of policy regime. To further emphasize how the coefficients are sensitive to different economic structures under optimizing central bank behavior, this subsection presents results for two economies that differ from the benchmark in terms of either policy preferences or inflation persistence.

Firstly, Table 3 reports results for the case where the central bank is assumed to be more inflation averse, or, “conservative” in the terminology of Rogoff (1985). The preference parameter λ is reduced to 0.1; i.e., output gap stabilization is now five times less important to the central bank relative to inflation stabilization. As one would expect, the estimated parameters on the output gap becomes lower. The estimated parameters on the inflation measures, on the other hand, increase only little. The inferences about determinacy properties one would make, compared to the benchmark, are almost the same. Believing in interest-rate setting as a result of Taylor-rule policymaking, one would conclude the economy was in an indeterminate equilibrium from all estimations except those involving two-period ahead inflation expectations. The only difference from the benchmark case is that commitment policies now satisfy the Taylor principle in the regression with this inflation measure and the lagged interest rate. Hence, even with a rather conserva-

Variables in estimated interest rate rule (26) ^a					Property of associated IRE ^b
π_t	$E_t\pi_{t+1}$	$E_t\pi_{t+2}$	x_t	i_{t-1}	
————— Discretion data —————					
1.22*	—	—	2.11*	—	D
1.20*	—	—	2.08*	0.03*	D
—	2.07*	—	2.07*	—	D
—	2.23*	—	2.17*	-0.08*	D
—	—	2.79*	1.91*	—	D
—	—	2.96*	1.97*	-0.09*	D
————— Commitment data —————					
0.31*	—	—	0.75*	—	I
0.24*	—	—	0.66*	0.51*	I
—	0.46*	—	0.74*	—	I
—	0.42*	—	0.66*	0.52*	I
—	—	0.73*	0.74*	—	NE
—	—	0.66*	0.65*	0.54*	D

^a “*” denotes significance at the 5% level

^b “I” denotes indeterminacy of the associated IRE; “D” denotes determinacy; “NE” denotes that no (bounded) equilibrium exists

Table 4: Coefficients of estimated interest rate rules.

Deviation from benchmark: More endogenous inflation persistence, $\phi = 0.5$

tive central banker, Taylor-type regressions may often give the impression of a “passive” monetary policy.

Secondly, Table 4 reports estimations of Taylor-type rules in an economy with more endogenous inflation persistence relative to the benchmark. The parameter ϕ is raised to 0.5, such that lagged inflation and expected future inflation are equally important for inflation determination. This changes inference about determinacy properties a lot. In all estimations based on data from discretionary policymaking, one would conclude that the economy was in a determinate equilibrium. All estimated coefficients on inflation measures increase. The reason is that with more inflation persistence, any increase in inflation has longer-lasting effects, and an optimizing central bank will therefore fight such increases more vigorously. This demonstrates that, apart from being uninformative about determinacy properties, differences in estimated values in Taylor-type rules need not have anything to do with different preferences for inflation stabilization. Under commitment, data also generate higher estimated values on inflation measures—in some cases, however, one would still conclude that the economy was in an indeterminate equilibrium under the hypothesis that policy was performed under an instrument-rule framework. Note also that one of the specifications under commitment gives the impression that the economy is

characterized by unbounded equilibria, even though it is not.²⁴

4. Concluding remarks

In this paper, I have investigated the issue of equilibrium determinacy within a simple model of the New-Keynesian variety. In particular, I examined if empirical analyses of nominal interest-rate setting provide lessons on whether an economy is in a determinate or an indeterminate equilibrium. This is motivated by recent literature which asserts that such lessons indeed can be drawn from data, implying that identification of “bad policies” (those leading to indeterminacy), indicates the presence of, or potential for, extraneous macroeconomic fluctuations. As these fluctuations are generally undesirable; such lessons obviously have profound policy implications.

I argue, however, that such empirical analyses may be misleading in terms of the information about determinacy they provide. When the underlying assumption about monetary policy behavior is that the central bank sets the interest rate with the aim of minimizing a loss function (i.e., the central bank is assumed to be an optimizing entity), then there is always determinacy in the simple New-Keynesian model. This may, as is well known, not be the case if the bank is adhering to an instrument rule, like a Taylor-type rule, where restrictions on the response coefficients are needed to secure determinacy. The distinction between optimization-based behavior and instrument-rule behavior is therefore crucial. Under the former type of behavior it can be seen analytically that *in* a determinate equilibrium, the relationships between the nominal interest rate and various macroeconomic variables will in some instances convey properties of an indeterminate equilibrium if the central bank is incorrectly assumed to follow an instrument rule.

In an empirically plausible extension of the simple New-Keynesian model, I extract artificial data from optimization-based policy under discretion and commitment. The data are then used for estimation of interest rate rules. In accordance with the theoretical insights, the estimated coefficients will in some instances give the false impression of a policy conducive for indeterminacy. This is particularly prevalent under commitment policies, where the Taylor principle rarely holds (and the estimated interest rate reaction to inflation may even be negative). Nevertheless, the economy is in a determinate equilibrium.

The analysis therefore emphasizes that empirical relationships between the nominal interest rate and macroeconomic variables reveal little about an economy’s stability prop-

²⁴With the parameterization’s higher focus on lagged variables, this mirrors Bullard and Mitra’s (2002) finding that lagged variables in a related model introduce the possibility of explosive equilibria for some versions of Taylor rules.

erties if one is willing to accept that the central bank optimizes—just as all private agents in the economy are assumed to.

Appendix

A. Derivation of (11)

In order for the nominal interest rate to be consistent with the OCBE values of current and expected future variables, it follows by (10) and (9) that the following relation must be satisfied:

$$\sum_{i=0}^{\infty} b_i \mathbf{E}_t \pi_{t+i} - \sum_{i=0}^{\infty} a_i \mathbf{E}_t x_{t+i} + c \varepsilon_t = \frac{\kappa(1-\rho) + \sigma\lambda\rho}{\sigma[\kappa^2 + \lambda(1-\beta\rho)]} \varepsilon_t. \quad (\text{A.1})$$

One can then substitute in the OCBE values of the contemporaneous and expected future variables [i.e., use (8)], in order to rewrite (A.1) as

$$\sum_{i=0}^{\infty} b_i \rho^i \frac{\lambda}{\kappa^2 + \lambda(1-\beta\rho)} \varepsilon_t + \sum_{i=0}^{\infty} a_i \rho^i \frac{\kappa}{\kappa^2 + \lambda(1-\beta\rho)} \varepsilon_t + c \varepsilon_t = \frac{\kappa(1-\rho) + \sigma\lambda\rho}{\sigma[\kappa^2 + \lambda(1-\beta\rho)]} \varepsilon_t.$$

If this is to hold for every realization of ε_t , it follows that the coefficients $\{a_i\}_{i=0}^{\infty}$, $\{b_i\}_{i=0}^{\infty}$ and c must satisfy (11).

B. Derivation of (18) and (19)

In order for the nominal interest rate to be consistent with the OCBE values of current and expected future variables, it follows by (17) and (16) that the following relation must be satisfied:

$$\begin{aligned} & \sum_{i=0}^{\infty} \bar{b}_i \mathbf{E}_t \pi_{t+i} - \sum_{i=0}^{\infty} \bar{a}_i \mathbf{E}_t x_{t+i} + \bar{c} \varepsilon_t - dx_{t-1} \\ &= -\frac{\chi}{\sigma} (1-\chi) \left(1 - \frac{\lambda\sigma}{\kappa}\right) x_{t-1} + \frac{\varphi}{\sigma} (1-\chi-\rho) \left(1 - \frac{\lambda\sigma}{\kappa}\right) \varepsilon_t. \end{aligned} \quad (\text{B.1})$$

One can then successively forward the OCBE expressions for inflation and the output gap, (15), in order to find

$$\begin{aligned} \mathbf{E}_t \pi_{t+i} &= \frac{\lambda}{\kappa} (1-\chi) \chi^i x_{t-1} - \frac{\lambda}{\kappa} \varphi \left((1-\chi) \sum_{j=0}^{i-1} \chi^{i-j-1} \rho^j - \rho^i \right) \varepsilon_t, \\ \mathbf{E}_t x_{t+i} &= \chi^{i+1} x_{t-1} - \varphi \sum_{j=0}^{i-1} \chi^{i-j} \rho^j \varepsilon_t, \end{aligned}$$

which inserted into (B.1) yields

$$\begin{aligned}
& \sum_{i=0}^{\infty} \bar{b}_i \left[\frac{\lambda}{\kappa} (1 - \chi) \chi^i x_{t-1} - \frac{\lambda}{\kappa} \varphi \left((1 - \chi) \sum_{j=0}^{i-1} \chi^{i-j-1} \rho^j - \rho^i \right) \varepsilon_t \right] \\
& - \sum_{i=0}^{\infty} \bar{a}_i \left[\chi^{i+1} x_{t-1} - \varphi \sum_{j=0}^{i-1} \chi^{i-j} \rho^j \varepsilon_t \right] + \bar{c} \varepsilon_t - dx_{t-1} \\
= & -\frac{\chi}{\sigma} (1 - \chi) \left(1 - \frac{\lambda \sigma}{\kappa} \right) x_{t-1} + \frac{\varphi}{\sigma} (1 - \chi - \rho) \left(1 - \frac{\lambda \sigma}{\kappa} \right) \varepsilon_t.
\end{aligned}$$

If this is to hold for every value of x_{t-1} and ε_t , it follows that the coefficients $\{\bar{a}_i\}_{i=0}^{\infty}$, $\{\bar{b}_i\}_{i=0}^{\infty}$, \bar{c} and d must satisfy (18) and (19).

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