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ABSTRACT

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This paper presents a rational expectations model of optimal executive compensation in a setting where managers are in a position to manipulate short-term stock prices, and managers' propensity to manipulate is uncertain. Stock-based incentives elicit not only productive effort, but also costly information manipulation. We analyze the tradeoffs involved in conditioning pay on long- versus short-term performance and characterize a second-best optimal compensation scheme. The paper shows manipulation, and investors' uncertainty about it, affects the equilibrium pay contract and the informational efficiency of asset prices. The paper derives a range of new cross-sectional comparative static results and sheds light on corporate governance regulations.

JEL Classification: D8, G30, G34, J33, J41 and K2
Keywords: corporate governance, executive compensation, long- versus short-term and manipulation uncertainty

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1 Introduction

Is improperly structured incentive compensation a key factor underlying the the current financial crisis and the recurrent corporate fraud scandals of the past decade? A recent survey reports that 98% of responding banks "believe the compensation structures were a factor underlying the crisis."[1] Regulators are similarly concerned about the impact of short-term pay, especially in the financial sector, and are actively considering policies to reform pay practices: US Treasury Secretary Geithner states that "This financial crisis had many significant causes, but executive compensation practices were a contributing factor ... Some of the decisions that contributed to this crisis occurred when people were able to earn immediate gains without their compensation reflecting the long-term risks they were taking for their companies and their shareholders ... Companies should seek to pay top executives in ways that are tightly aligned with the long-term value and soundness of the firm."[2] Meanwhile, a growing body of empirical evidence finds that the increase in stock-and option-based compensation since the late 1980s is responsible for the spate of corporate scandals exemplified by WorldCom and Enron, as well as the exponential increase in accounting restatements reported by the GAO (2002, 2006).[3] The evidence suggests that there is a downside to short-term performance-based pay: it can encourage management to focus excessively on enhancing the short term stock price, distracting the focus away from the creation of long-term value. This raises the issue of how compensation contracts can be designed to balance incentives for productive effort against incentives for wasteful manipulation of short term stock prices.

This paper provides a rational expectations model that characterizes optimal executive compensation in a setting where managers are in a position to manipulate short-term stock prices. In the model, pay that depends on the short-term stock price elicits both effort and manipulation. Long-term pay does not elicit wasteful manipulation, but has the disadvantage of imposing some additional risk on the manager. We analyze the tradeoffs involved in conditioning pay on long- versus short-term performance and characterize a second-best optimal compensation scheme. The analysis has implications for public policy regarding

[1] Survey Finds Banks Aware of Pay Flaws, *Wall Street Journal*, 30 March 2009.

[2] Statement by Treasury Secretary Tim Geithner on Compensation, June 10, 2009. http://www.ustreas.gov/press/releases/tg163.htm

[3] The empirical studies use a variety of indicators of performance report manipulation: earnings management as measured by discretionary accruals , SEC accounting enforcement actions, accounting restatements and shareholder class action litigation (Bergstresser and Philippon, 2006, Burns and Kedia, 2006, Peng and Röell, 2008a, and Johnson, Ryan and Tian, 2009).

corporate governance and financial disclosure standards and provides some guidance for reform of pay practices.

A key feature of the model is that in contrast to the existing manipulation literature, the managers' propensity to manipulate is uncertain: thus in equilibrium even fully rational investors are deceived, and shocking disparities between their beliefs based on firms' reports and the true state of affairs are an integral part of the model. If all managers always had the same propensity to exaggerate their reported performance, a rational investor would be able to back out the degree of manipulation and a correct assessment would be reflected in the share price as, for example, in the signal-jamming models of Narayanan (1985), Stein (1989) and Goldman and Slezak (2006). In such a setting, manipulation can be costly and wasteful, but it does not have any systemic impact on the efficiency of stock market price formation as the investors are fully aware of the true state of the company. In contrast, uncertainty about the degree to which reports are inflated imposes an additional source of risk on investors, resulting in informationally inefficient stock prices and less effective contracting, even in settings where the direct costs of manipulation are modest or merely involve a transfer of relatively small amount of money from the shareholders to the managers. Our model addresses the huge chasm between investors' expectations and the true state of the company that is associated with many real-world cases of corporate fraud.

The introduction of the manipulation uncertainty into the model of optimal contracting has several implications that have not received attention in the previous literature. In the presence of manipulation uncertainty, the strength of incentives is determined by the product of the sensitivity of pay to the stock price and that of the stock price to reported firm performance. Thus even if pay is very sensitive to the stock price, incentives for effort may be low if the stock price is unresponsive to reported performance - which would be the case if investors suspect that performance reports are inflated to an uncertain degree, and therefore unreliable, as pointed out in Fischer and Verrecchia's (2000) paper on managerial reporting bias. Thus the stock price sensitivity (or elasticity) of pay alone may be a misleading measure of the strength of incentives.

Regarding the optimal pay contract we show that, paradoxically, an increase in manipulation uncertainty may actually call for pay to be more sensitive to the short-term stock price. Our model thus predicts how the optimal contract varies across different firms or industries: those where the propensity to manipulate is more uncertain are more likely to use higher powered incentive contracts. Furthermore, such firms have less informative stock prices in the short run and higher *ex ante* return uncertainty in the long run. This is consistent with evidence that startup firms and high-tech, high-growth, intangible asset-intensive industries typically feature extremely stock price-sensitive pay and unusually large disparities between expected and actual performance. The analysis shows that regulations or policies that help to reduce manipulation uncertainty (such as tighter accounting standards or heavier penalties for inflated reporting) can improve contracting efficiency and make it possible to induce more effort.

Even if in the short run stock price can be manipulated, in the long run the truth will come out. Therefore it is important to allow for long term incentive pay components in the contract, and to investigate the optimal mix of long- and short-term incentives. Long term incentives align the manager's objectives with long term shareholder value, thereby mitigating the waste of valuable managerial time and resources on manipulation associated with short term incentives. In general, when long term incentives are allowed, at least part of the incentive pay is shifted from the short term to the long term, resulting in higher equilibrium effort choice and enhanced firm value. Practical examples of pay that is *de facto* sensitive to long term performance include stock and option pay plans with long vesting periods; the structuring of private equity contracts so that any excessive carried interest distributions to the general partners are "clawed back" at the end of the fund's life; and more generally, the clawback of unwarranted pay in the wake of misconduct imposed in Section 304 of the Sarbanes-Oxley Act of 2002. For example, in December 2007 the ex-CEO of United Health settled an options backdating case by agreeing to reimburse the company for $600 million worth of previously awarded bonus pay and options. Currently, an attempt to align pay more closely with long-term performance is a central feature of the Obama administration's initiative to reform how financial companies pay employees and executives.[4] Empirical work by Chi and Johnson (2009) finds that long-term incentives have a more significant impact on firm value than short term incentives. But long term incentive pay has its own downside: it introduces extra risk from longer term shocks that are outside the current manager's control into his pay. This is costly to the firm because it needs to compensate the manager for bearing the additional risk. Thus short run pay should remain part of the optimal contract if long term volatility is substantial or if the manager is very risk averse.

Our focus on the interplay of short-term and long-term incentives in the optimal pay

[4] U.S. Eyes Bank Pay Overhaul — Administration in Early Talks on Ways to Curb Compensation Across Finance, *Wall Street Journal*, 13 May 2009.

contract starts from the model of Peng and Röell (2008b), extending that model to incorporate long-term as well as short-term incentives. Part of the underlying intuition comes from the analysis of Holmström and Tirole (1993), who show how pay should depend on a mix of short- and long-term performance measures when no single measure is free of noise as an indicator effort. Other papers that address the tensions between short- and long-run incentives but take a very different approach include Bolton, Scheinkman and Xiong (2005 and 2006), who drop the assumption that the principal's goal is long-term value maximization, and explore the conflict of interest between future shareholders and short-termist current shareholders, who design a contract that encourages ramping up the short-term share price; and Axelson and Baliga (2009), who consider the possibility of renegotiation and argue that scope for manipulation that obscures short-term value can be beneficial in discouraging *ex post* efficient but *ex ante* inefficient renegotiation.

Quite independently of the issues relating to manipulation, it is useful to add a new model to the arsenal of examples of agency models of executive compensation that can be solved with relative ease. We formulate a multiplicative framework for analyzing optimal executive pay contracts: the manager's preferences are constant relative risk averse and Cobb-Douglas in wealth (consumption) and leisure, effort is modeled as a time cost and impacts firm value proportionately rather than additively, and firm value is lognormally distributed. Meanwhile, compensation schemes are limited to a loglinear form, rather than the usually posited linear form, in order to keep the analysis tractable and generate closed-form solutions. The traditional basic agency model of executive compensation has normally distributed firm value, effort has an additive impact, the manager is constant absolute risk averse and the cost of effort is independent of his wealth, and pay is linear in firm value. These features may be poor descriptors of reality and thus hard to reconcile with the data: a concern expressed, for example, by Baker and Hall (2004). Our specification is worth exploring in its own right because it has several features that may be more realistic than the usual normal-exponential model. Unlike in the traditional model, in our framework optimal incentives vary with firm size and with the manager's personal wealth. It is useful to check the robustness of model predictions to seemingly quite minor changes in modeling strategy when analyzing executive compensation contracts and drawing conclusion regarding their optimality.

Given that the pay scheme is assumed to be constant elastic, our model makes direct and easily interpreted predictions about the elasticity of pay (rather than the sensitivity of pay to firm value as in the traditional linear-normal- exponential framework), thus providing testable comparative static insights that are more closely aligned with the empirical literature. In his survey of empirical work on executive compensation, Murphy (1999) argues that in comparison with the sensitivity approach, the elasticity approach generally produces a better empirical "fit" in cross-sectional analysis of the relationship between pay and firm value and has the desirable property that it can be better compared across firms of different size.

A final feature of the model that deserves mention is the formulation of the cost of manipulation as a time cost. In the model, all managers need to spend some time to convince the stock market that their company has value: the term "manipulation" is perhaps too pejorative, since even a manager who wishes to convey the truth to investors needs to devote time to investor relations to convey the company's worth. This is time taken away from productive effort directed at enhancing the firm's true underlying value. In reality, the time constraint is one of the most important constraints faced by managers. And they do complain of the significant amount of time and attention they are forced to devote to public relations and reassuring the stock market. In Europe, prominent business leaders have pointed out that the threat of a takeover, now that corporate control is more contestable than it used to be, is having the unfortunate side effect of distracting management from running the underlying business. This time cost comes out clearly in the London Stock Exchange's *A Practical Guide to Listing.*[5]

Our interpretation of the cost of manipulation as purely a demand on managerial time differs from other approaches taken in the literature. Narayanan (1985), Stein (1989) and Bolton *et al.* (2005, 2006) view the main cost as a distortion of real investment decisions towards projects that give palpable results in the short run. Other papers such as Goldman and Slezak (2006) regard it as a money cost: expenditure on accountants, *etc.* Kedia and Philippon (2009) and Bemmelich, Kandel and Veronesi (2009) argue that overinvestment

[5] "Both the flotation process itself and the continuing obligations – particularly the vital investor relations activities ... - use up significant amounts of management time which might otherwise be directed to running the business ... It is vital that you maintain your company's profile, and stimulate interest in its shares on a continuing basis. Many listed companies, even relatively small ones, employ specialist financial public relations and investor relations advisors on a retainer basis to keep the business on the financial pages and in the minds of investors ... However, you cannot leave press or investor relations to your advisers. Top executives will commonly devote at least a couple of days a month to developing and nurturing such contacts ... This commitment will increase sharply around regular announcements ..., at the launch of a new product or strategy, or at times when the business or its profile have been hit by adverse events. This must be regarded as time well-spent ... As a publicly-quoted company, it is a core element of running your business properly and responsibly." (pp. 11, 47-48). Available at http://www.londonstockexchange.com.

and/or excessive employment are the real underlying cost of manipulation. Our approach complements these other views in an interesting and plausible way.

The paper is organized as follows: In Section 2 we describe the model setup and derive the equilibrium stock price, the manager's actions and utility, and expected firm value for any given pay contract. Section 3 considers the optimal pay contract in the benchmark case where there is no manipulation. Short term optimal pay contracts in the presence of manipulation are analysed in Section 4. The more general contract that trades off short- and long- term incentives is characterised in Section 5, together with a discussion of the empirical predictions of the model. Section 6 addresses policy implications, and Section 7 concludes. Proofs of all propositions are collected in the Appendix.

2 The model

In this section we describe and motivate the basic model and solve it for the equilibrium stock price, expected managerial utility and pay, and expected firm value, for any given managerial pay contract. These results will be drawn upon in subsequent sections to characterize executive pay contracts in a variety of settings.

2.1 Model assumptions

We analyze a multiperiod model with decisions taken at dates 0 and 1 and final payoffs established at date 2. At time 0, the manager and the firm's shareholders sign a contract *ex ante*, before the cost of manipulation is observed. The shareholders are risk neutral: their objective is to maximize expected firm value net of managerial compensation. The manager is assumed to be risk averse. Between dates 0 and 1, the manager privately finds out the cost of manipulation. He then chooses his level of productive effort as well as his degree of manipulation; the latter is modeled as a factor by which he inflates a report regarding firm value at date 1. The true underlying value of the firm at date 1 is determined by an exogenous shock and the manager's level of effort. The performance report released by the manager at this point conflates the true value of the firm at date 1 and his degree of manipulation. The stock price at time 1 is based on the manager's report. At date 2, the true long-term value of the firm, which may incorporate a further exogenous shock, is revealed. For simplicity, we set the interest rate to zero.

The true value of the company is subject to random shocks, ϵ_1 and ϵ_2, at dates 1 and 2 respectively. It is assumed that the firm's value in the short run, V_1, and in the long run,

V_2, is multiplicative in the manager's *bona fide* effort E:

$$V_1 = E \cdot V_0 \cdot \epsilon_1 \tag{1}$$
$$V_2 = V_1 \cdot \epsilon_2$$

where multiplicative value shocks ϵ_1 and ϵ_2 are independent mean-one lognormally distributed, $\varepsilon_t \equiv \ln \epsilon_t \sim N(-\frac{1}{2}\Sigma_t, \Sigma_t)$ for $t = 1, 2$.

At time 1, the manager sends a report S about the firm's true value that portrays the firm's true value V_1 as observed by him at time 1, factored up by a manipulation multiple M:

$$S = MV_1 = MEV_0\epsilon_1 \tag{2}$$

The manager is constant relative risk averse, with a constant relative risk aversion coefficient of $1 - \phi$, and a preferences that are Cobb-Douglas in leisure and money:

$$U = \frac{1}{\phi}\left[\left(\overline{L} - C_E E^{1/\alpha} - C_M M^{1/\beta}\right)^{\Psi} W\right]^{\phi} \tag{3}$$
$$\text{where } 0 \leq \ \alpha, \beta \leq 1, \ \ \Psi > 0 \ \text{ and } \ \phi < 1$$

where $\overline{L}$ is his time endowment, $E \geq 0$ is effort devoted increasing the firm's true value, $M \geq 0$ is effort devoted to upward manipulation of the firm's perceived value, and W is the manager's wealth, derived from his employment at the firm.[6] The manager's personal time cost for both effort and manipulation is taken to be a convex function parameterized by a constant scaling the time used (C_E and C_M respectively) and a convexity parameter (the exponents, α and β respectively). Note that, as β decreases and the manipulation cost function becomes more convex, the marginal time cost of manipulation ($\frac{C_M}{\beta} M^{\frac{1}{\beta}-1}$) decreases for small enough M (below $e^{-\beta}$) but increases when M is larger. This means that reducing β brings the reporting bias of different managers closer together because extreme values of M (both low and high) are discouraged.

[6] An alternative model of manipulation cost regards it as a psychological cost for the manager, that is:

$$U = \frac{1}{\phi}\left[\left(\overline{L} - C_E E\right)^{\Psi} \left(\overline{R} - C_M M\right)^{\Theta} W\right]^{\phi}.$$

where $\overline{R}$ is an endowment of "self-respect". This formulation yields very similar results to the version used in this paper.

For convenience we assume that C_E is a fixed, known cost of effort that is identical for all managers, but that the manipulation cost parameter C_M is random, with the following log normal distribution:

$$\ln C_M \equiv c_M \sim N\left(\bar{c}_M, \Omega\right) \tag{4}$$

The parameter Ω captures the investors' uncertainty regarding manager's propensity to manipulate. In practice, investors often cannot tell to what degree the information released by managers is over-optimistic: it depends on unknown factors such as the manager's ethical compass, his ability to project a (possibly unjustified) aura of success, his salesmanship in portraying the merits of an unfamiliar project, and the susceptibility of the firm's business opportunities to hype. Not all management teams are equally adept at managing investors' perceptions, and indeed a few are downright dishonest. The uncertainty is likely to be higher for young managers who have a relatively short track record. Similarly, some industries present more opportunities for misleading investors than others. The variance Ω is likely to be highest in high growth, high tech industries, industries with more intangible assets such as patents, industries where new, complex and hard to value products are introduced, and generally industries where current earnings are a poor guide to the future. Examples are IT companies that develop large but infrequent customer-specific software projects (for which the timing of revenue recognition is a major issue) and financial services firms that are able book current fee income by taking on complex risky positions whose ultimate value will only emerge in the future.

Participants in the stock market observe the report, and determine the market price as

$$P_1 = E[V|S] \tag{5}$$

where for convenience we define P_1 as the gross-of-pay stock price, that is, the gross expected value of the firm, *before executive pay is deducted*: the actual stock market capitalization would be equal to this expected gross value of the firm, minus expected executive pay.

We conjecture that P_1 takes the following log-linear price form:

$$P_1 = \pi V_0^{1-\gamma} \cdot S^{\gamma} \tag{6}$$

for some values of the parameters π and γ that are to be determined in equilibrium.

We restrict attention to a three-parameter pay contract where the manager's compensation package (parameterized by $\{\omega,\mu,\eta\}$) is constant-elastic in the firm's long- and short-term value, that is, the pay contract takes the log-linear form:

$$W = \omega P_1^{\mu} V_2^{\eta} \tag{7}$$

This multiplicative functional form is chosen as a convenient approximation that yields closed-form solutions. A rather unusual aspect of this specification is that performance at dates 1 and 2 does not enter additively into pay, but in mutually reinforcing fashion. This is not unrealistic. The pay contract can be thought of as one in which short-run performance as represented by the stock price at time 1 determines the size of a package of fixed pay, stock and options to be awarded to the manager; if the firm outperforms in the long run, this package will increase further in value. The chosen three-parameter constant-elastic functional form is tractable and flexible, and yields direct insights into the optimal elasticity of pay to long- and short-run firm performance.

Note that the manager's pay does not depend directly on his report S: such a report is presumed to be unverifiable and too complex to summarize into a form that a pay contract can be based upon. It may include predictions about market share, product quality, earnings, the business climate, the competence and health of the management team, *etc.* It is left to the impersonal judgment of stock market participants to distil this information into a summary judgment about the firm's underlying value, captured by P_1.

2.2 The manager's problem

After signing the compensation contract at date 0, the manager discovers C_M, that is, how costly it is to manipulate the performance signal. He then chooses the level of effort E that he will exert and the manipulation factor M by which he will scale up his report of the firm's value at time 1:

$$\max_{\{E,M\}} \mathrm{E}_1 \left[\frac{1}{\phi} \left\{ \left(\overline{L} - C_E E^{1/\alpha} - C_M M^{1/\beta} \right)^{\Psi} \widetilde{W} \right\}^{\phi} \right] \tag{8}$$

Substituting out P_1 and V_2 in equation (7) using equations (1) and (6), the manager's compensation can be expressed as:

$$\widetilde{W} = \omega \widetilde{P}_1^{\mu} \widetilde{V}_2^{\eta} = \omega \pi^{\mu} V_0^{\mu+\eta} M^{\gamma\mu} E^{\gamma\mu+\eta} \widetilde{\epsilon}_1^{\gamma\mu+\eta} \widetilde{\epsilon}_2^{\eta} \tag{9}$$

The manager's optimal choice of effort, manipulation and leisure is then given by:

$$E = \left(\frac{\overline{L}}{C_E}\frac{\alpha(\gamma\mu+\eta)}{\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu}\right)^{\alpha} \tag{10}$$

$$M = \left(\frac{\overline{L}}{C_M}\frac{\beta\gamma\mu}{\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu}\right)^{\beta} \tag{11}$$

$$L = \overline{L} - C_E E^{1/\alpha} - C_M M^{1/\beta} = \frac{\Psi}{\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu}\overline{L} \tag{12}$$

Note that, from Equation (10), the time cost of manipulation and the time devoted to effort are:

$$C_M M^{1/\beta} = \overline{L}\frac{\beta\gamma\mu}{\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu} \tag{13}$$

$$C_E E^{1/\alpha} = \overline{L}\frac{\alpha(\gamma\mu+\eta)}{\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu} \tag{14}$$

Observe that for the Cobb-Douglas formulation of preferences used in this paper, the choice of effort and manipulation is independent of the exogenous firm value shocks ϵ_1 and ϵ_2. We have presented the manager's time allocation decision as taking place before he becomes aware of the outcome of these random shocks, but the model can alternatively be interpreted as one where the manager observes the output shocks before he decides how much to manipulate.

2.3 The short run stock price

At time 1, the stock market observes only the report sent by the manager, not the true value of the firm. Market participants do observe the pay contract signed at time 0 and understand the manager's incentives to inflate the performance report. Thus they correctly back out the optimal level of effort exerted by the manager and form estimates of the firm's value and the level of manipulation based on the *ex-ante* distribution of the manipulation cost C_M and the shock to the firm's value ϵ_1. The rational-expectations short term stock price (gross of expected compensation) at time 1 is the conditional expectation of the firm value given the manager's report S, set down in the following proposition.

Proposition 1 *The short term gross-of-pay stock price is given by*

$$P_1 = \pi V_0^{1-\gamma} S^{\gamma} \tag{15}$$

where

$$\gamma = \frac{\Sigma_1}{\beta^2\Omega + \Sigma_1} \tag{16}$$

and

$$\begin{aligned}\pi &= \left(\frac{\overline{L}}{C_E}\frac{\alpha(\gamma\mu+\eta)}{\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu}\right)^{\alpha(1-\gamma)}\left(\frac{\overline{L}}{\overline{C}_M}\frac{\beta\gamma\mu}{\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu}\right)^{-\beta\gamma} && (17)\\ &= E^{1-\gamma}\overline{M}^{-\gamma} && (18)\end{aligned}$$

and $\overline{C}_M$ *denotes* $\exp \overline{c}_M$*.*

The short term stock price is a weighted geometric average of the investors' prior expectation of firm value, $\mathrm{E}_0[V_1]$, and the manager's report, discounted by a factor equal to the median level of manipulation $\overline{M} \equiv \left(\frac{\overline{L}}{\overline{C}_M}\frac{\beta\gamma\mu}{\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu}\right)^{\beta}$:

$$P_1 = E^{1-\gamma}\overline{M}^{-\gamma}V_0^{1-\gamma}S^{\gamma} = (V_0\ E)^{1-\gamma}\left(\frac{S}{\overline{M}}\right)^{\gamma} = (\mathrm{E}_0[V_1])^{1-\gamma}\left(\frac{S}{\overline{M}}\right)^{\gamma}. \tag{19}$$

Accounting for the possibility that the report S may be manipulated, the stock market attaches less weight γ to the report, the more uncertain it is about the degree of manipulation (M). This uncertainty, as captured by the variance of $\ln M$ is, from equation (10), given by $\beta^2\Omega$: the parameter Ω captures the dispersion in managers' time cost of manipulating their reports, while the manipulation cost convexity parameter β parameterizes the degree to which extremely low or high degrees of manipulation are penalized by wasted time. Meanwhile the stock market attaches a greater weight to the report S, the greater the fundamental uncertainty about the firm (Σ_1) (holding constant the noisiness of the report) because updating the estimate of firm value becomes more important. In summary, the sensitivity γ of the short-term expected firm value to the manager's report decreases with Ω and β and increases with Σ_1.

Based on the distribution of manipulation cost, the investors are able to back out the average degree of manipulation (captured by the median $\overline{M}$) and adjust for it accordingly in setting equilibrium prices. However, to the extent that the actual manipulation cost may be higher or lower than average, the stock price is inaccurate. Investors underestimate the degree of manipulation for the managers with a below-average manipulation cost C_M, setting a short term stock price that is too high. The manager's short term compensation is also too high, resulting in a loss of net firm value. Investors will be disappointed in the long run when stock prices eventually revert to fundamental value. For example, at the turn of the century Enron was an extreme case of a low-C_M easy-to-manipulate company, with a huge chasm between the financial reports and the true state of the company. Similarly, in the runup to the crisis of 2008, financial services firms were able to overstate the profitability of their business to an extent that was not anticipated, partly as a result of the opacity of the innovative financial transactions used to structure their deals.

2.4 Manager's expected utility and net firm value

Given any compensation contract $\{\mu, \omega, \eta\}$, the manager expects to choose optimal levels of effort and manipulation at the interim date 1, and the stock market to respond rationally to his report. Standing at date 0, before the realization of the manipulation cost or any random shocks to the firm's value, his *ex ante* expected utility is as given in the following proposition.

Proposition 2 *The manager's ex ante expected utility, before he is aware of his own propensity to manipulate, is given by:*

$$\begin{aligned}\mathbf{E}_0\left[U\right] &= \frac{1}{\phi}\left[\omega\left(\frac{\Psi}{\Psi+\alpha\left(\gamma\mu+\eta\right)+\beta\gamma\mu}\overline{L}\right)^{\Psi}V_0^{\mu+\eta}\left(\frac{\overline{L}}{C_E}\frac{\alpha\left(\gamma\mu+\eta\right)}{\Psi+\alpha\left(\gamma\mu+\eta\right)+\beta\gamma\mu}\right)^{\alpha(\mu+\eta)}\right]^{\phi} \\ &\quad\cdot\exp\left\{-\frac{\phi}{2}\left[\left(\gamma\mu+\eta\right)\Sigma_1+\eta\Sigma_2\right]+\frac{\phi^2}{2}\left[\left(\gamma\mu^2+2\gamma\mu\eta+\eta^2\right)\Sigma_1+\eta^2\Sigma_2\right]\right\} \qquad (20)\end{aligned}$$

where γ is given in Proposition 1 and ω, μ and η are the terms of the compensation contract that is signed.

The *ex ante* expected wealth of the manager is obtained by setting $\phi = 1$ and removing the term representing leisure in equation (20):

$$\begin{aligned}\mathrm{E}_0\left[W\right] &= \omega V_0^{\mu+\eta}\left(\frac{\overline{L}}{C_E}\frac{\alpha\left(\gamma\mu+\eta\right)}{\Psi+\alpha\left(\gamma\mu+\eta\right)+\beta\gamma\mu}\right)^{\alpha(\mu+\eta)}\cdot \qquad (21)\\ &\quad\cdot\exp\left\{\frac{1}{2}\left[\left(-\gamma\mu+\gamma\mu^2+2\gamma\mu\eta-\eta+\eta^2\right)\Sigma_1+\left(-\eta+\eta^2\right)\Sigma_2\right]\right\}\end{aligned}$$

The expected firm value is the gross expected value minus the expected payment to the manager.

Proposition 3 *The ex ante company expected firm value, net of executive compensation, is:*

$$\begin{aligned} \mathbf{E}_0\left[V_2 - W\right] &= V_0\left(\frac{\overline{L}}{C_E}\frac{\alpha\left(\gamma\mu+\eta\right)}{\Psi+\alpha\left(\gamma\mu+\eta\right)+\beta\gamma\mu}\right)^{\alpha} \\ &\quad -\omega V_0^{\mu+\eta}\left(\frac{\overline{L}}{C_E}\frac{\alpha\left(\gamma\mu+\eta\right)}{\Psi+\alpha\left(\gamma\mu+\eta\right)+\beta\gamma\mu}\right)^{\alpha(\mu+\eta)} \cdot \\ &\quad \cdot \exp\left\{\frac{1}{2}\left[\left(-\gamma\mu+\gamma\mu^2+2\gamma\mu\eta-\eta+\eta^2\right)\Sigma_1+\left(-\eta+\eta^2\right)\Sigma_2\right]\right\} \end{aligned} \tag{22}$$

In deriving optimal contracts we will assume that the manager has initial wealth $\widehat{W}$ and will accept an employment contract that yields utility of at least $U(\overline{W}, \overline{L})$, for exogenously determined $\overline{W} \geq \widehat{W}$. Thus his "reservation wage" is effectively $(\overline{W} - \widehat{W})$. To preserve the constant-elastic functional form for his final wealth, it will be assumed that the manager sinks any initial wealth $\widehat{W}$ that he owns into the firm, in return for a constant-elastic pay contract that gives him money-equivalent expected utility $\overline{W}$. In most of what follows we will suppress $\widehat{W}$, implicitly setting it to 0. If that is not so, the only change in interpreting the analysis is that the firm's net-of-pay expected value needs to be increased by $\widehat{W}$, the initial wealth pledged by the manager.

3 Optimal contracts in the absence of manipulation

In this section, we analyse two benchmark settings. First, we consider the first-best level of effort and pay in the absence of agency problems. We next characterize the optimal second-best contract in a setting without manipulation, that is, the setting of the classical principal-agent problem. The results will serve as a basis for comparison with the optimal contracts in the presence of manipulation that will be analyzed in Sections 5 and 6.

3.1 The first best

In the first best optimum, Pareto-efficient effort and compensation are chosen to maximize the expected wealth of the firm's owners, subject to the agent's participation constraint of reaching an expected utility of at least $U\left(\overline{W}, \overline{L}\right) \equiv \frac{1}{\phi}\left(\overline{L}^{\Psi}\overline{W}\right)^{\phi}$, his reservation utility. Of course no wasteful manipulation takes place ($M^* = 0$), the manager is paid a fixed salary W^*, and effort and compensation solve the program:

$$\max_{\{E,W\}} E \cdot V_0 - W \tag{23}$$

$$\text{subject to } \left(\overline{L} - C_E E^{\frac{1}{\alpha}}\right)^{\Psi} W \geqslant \overline{L}^{\Psi}\overline{W}$$

$$\text{and } \ E \geqslant 0.$$

Taking first-order conditions, the first-best solution for W^* and E^* is given by:

$$\frac{\left(\overline{L} - C_E E^{*\frac{1}{\alpha}}\right)^{\Psi+1}}{C_E E^{*\frac{1}{\alpha}-1}} = \frac{\Psi \overline{L}^{\Psi}\overline{W}}{V_0 \alpha} \tag{24}$$

$$W^* = \left(\frac{\overline{L}}{\overline{L} - C_E E^{*\frac{1}{\alpha}}}\right)^{\Psi} \overline{W}$$

In what follows we will focus primarily on the special case where α equals to 1, where the first-best effort, leisure and wage can be written down explicitly as:

$$E^* = \frac{\overline{L}}{C_E}\left(1 - \left(\Psi \frac{C_E \overline{W}}{V_0 \overline{L}}\right)^{\frac{1}{\Psi+1}}\right) \tag{25}$$

$$L^* = \overline{L}\left(\Psi \frac{C_E \overline{W}}{V_0 \overline{L}}\right)^{\frac{1}{\Psi+1}} \tag{26}$$

$$W^* = \overline{W}\left(\frac{1}{\Psi}\frac{V_0 \overline{L}}{C_E \overline{W}}\right)^{\frac{\Psi}{\Psi+1}} \tag{27}$$

where for the case $\alpha = 1$ the following condition is required to assure that the solution to the first order conditions for effort is within the feasible (nonnegative) range:

$$\overline{W} \leq \frac{V_0 \overline{L}}{\Psi C_E} \tag{28}$$

If not, first-best effort is zero ($E^* = 0$, $L^* = 0$) and presumably shareholders would not wish to enter into an employment relationship with the manager.

The first-best expected firm value net of executive compensation is (assuming the contributes personal wealth $\widehat{W}$ and needs to attain utility $U(\overline{W}, \overline{L})$):

$$\frac{V_0 \overline{L}}{C_E}\left[1 - \frac{\Psi+1}{\Psi} \cdot \left(\Psi \frac{C_E \overline{W}}{V_0 \overline{L}}\right)^{\frac{1}{\Psi+1}}\right] + \widehat{W} \tag{29}$$

which is nonnegative as long as

$$\overline{W} \leqslant \frac{\Psi^{\Psi}}{(\Psi+1)^{\Psi+1}} \frac{V_0\overline{L}}{C_E}\left(1+\frac{C_E\widehat{W}}{V_0\overline{L}}\right)^{\Psi+1}. \tag{30}$$

Naturally, this condition must be met if the firm's shareholders are to be willing to participate. For example, if the manager has no initial wealth to contribute($\widehat{W} = 0$) and the consumption-leisure preference parameter $\Psi = 1$, then the reservation wage $\overline{W}$ cannot exceed one-quarter of the firm's potential gross value $\frac{V_0\overline{L}}{C_E}$ that would be reached if the manager were to work full-time.

Note that there is a downward-sloping efficient locus of $\{E^*, W^*\}$ combinations mapped out by solutions with different levels of managerial utility:

$$E^* = \frac{\overline{L}}{C_E} - \frac{W^*\Psi}{V_0} \tag{31}$$

In the executive's preferences, money and leisure are complementary. While E^* decreases with $\overline{W}$, W^* increases with $\overline{W}$. As the executive's reservation utility increases, he both obtains a higher wage and puts in less effort in a first-best contract. Thus the preference formulation has the property that it is efficient for "fat cats" to put in less effort than "lean and hungry" executives.

Would the firm prefer to hire a rich or a poor executive? Suppose the firm only needs to compensate executives for their time, so that their reservation utility is the utility they would obtain with their personal wealth endowment $\widehat{W} > 0$ if they had full leisure $\overline{L}$. Observe that if the executive only needs to be paid enough to compensate him for the effort he puts into the company and no more, then $\overline{W} = \widehat{W}$. Then the net firm value is:

$$\frac{V_0\overline{L}}{C_E}\left(1-\frac{\Psi+1}{\Psi}\cdot\left(\Psi\frac{C_E\widehat{W}}{V_0\overline{L}}\right)^{\frac{1}{\Psi+1}}\right)+\widehat{W} \tag{32}$$

This is readily shown to be decreasing in $\widehat{W}$ over the range where (28) is valid for $\overline{W} = \widehat{W}$. The firm prefers to hire the poor executive rather than the rich one. Both are endowed with an equal amount of time. But the poor one is more willing to exert effort in return for extra money. Naturally, this conclusion rests on the assumption that they are equally skilled.

3.2 Second-best contract in the absence of manipulation

We now characterize the second-best optimal contract in a classical agency model setting without opportunities for manipulation, in which pay is constrained to depend only on the long-term value of the firm ($M = 0$ and $\mu \equiv 0$). As is well known in this setting, there is a tradeoff between riskbearing and incentives which leads to a below-first-best level of effort.

Applying the usual agency model that maximizes expected net firm value as expressed in equation (22) subject to the constraint that the manager's *ex ante* expected utility in equation (20) be greater than or equal to his reservation utility $U\left(\overline{W}, \overline{L}\right)$, we have the following:

$$\max_{\omega,\eta} V_0 \left(\frac{\overline{L}}{C_E} \frac{\alpha\eta}{\Psi + \alpha\eta} \right)^{\alpha} - \omega V_0^{\eta} \left(\frac{\overline{L}}{C_E} \frac{\alpha\eta}{\Psi + \alpha\eta} \right)^{\alpha\eta} \cdot \exp\left\{ \frac{1}{2} \left(-\eta + \eta^2 \right) \Sigma \right\} \tag{33}$$

subject to:

$$\omega \left(\frac{\Psi}{\Psi + \alpha\eta} \right)^{\Psi} V_0^{\eta} \left(\frac{\overline{L}}{C_E} \frac{\alpha\eta}{\Psi + \alpha\eta} \right)^{\alpha\eta} \exp\left\{ \frac{1}{2} \left(-\eta + \phi\eta^2 \right) \Sigma \right\} \geq \overline{W}$$

where $\Sigma \equiv \Sigma_1 + \Sigma_2$ is the total exogenous volatility over the long-run horizon, and for convenience utilities are expressed in "money-equivalent" form $(\phi U)^{\frac{1}{\phi}} / \overline{L}^{\Psi}$ (defined as the amount of money that together with full leisure $\overline{L}$ would yield utility U).

Substituting out for ω on the assumption that the participation constraint is binding we can simplify this to:

$$\max_{\eta} V_0 \left(\frac{\overline{L}}{C_E} \frac{\alpha\eta}{\Psi + \alpha\eta} \right)^{\alpha} - \overline{W} \cdot \left(\frac{\Psi + \alpha\eta}{\Psi} \right)^{\Psi} \exp\left\{ \frac{1}{2} (1 - \phi)\, \eta^2 \Sigma \right\} \tag{34}$$

Proposition 4 *When managerial pay depends only on the long-term value of the firm, there is no opportunity for manipulation. The optimal contract is characterized by an elasticity of pay to the long term firm value, η, given by the following first-order condition:*

$$\begin{aligned} -\frac{V_0}{\overline{W}} \left(\frac{\overline{L}}{C_E} \right)^{\alpha} &= \frac{(\Psi + \alpha\eta)^{\alpha+\Psi}\, \eta^{1-\alpha}}{\alpha^{\alpha+1} \Psi^{\Psi+1}} \left[\alpha\Psi + (\Psi + \alpha\eta)\, \eta\, (1 - \phi)\, \Sigma \right] \cdot \\ &\quad \cdot \exp\left\{ \frac{1}{2} (1 - \phi)\, \eta^2 \Sigma \right\} \end{aligned} \tag{35}$$

where $\Sigma \equiv \Sigma_1 + \Sigma_2$.

If $\alpha > 1$ there always exists a value of $\eta > 0$ that satisfies this equation; for $\alpha = 1$, there is a solution with $\eta > 0$ as long as the parameters strictly satisfy equation (28), that is, as long as first-best effort E^* would be strictly greater than zero.

If the agent were risk neutral so that $\phi = 1$ (holding constant Ψ so that the tradeoff between money and leisure remains unchanged), η would be the solution to:

$$\frac{V_0}{\overline{W}}\left(\frac{\alpha\overline{L}}{C_E}\right)^{\alpha}=\frac{(\Psi+\alpha\eta)^{\alpha+\Psi}\,\eta^{1-\alpha}}{\Psi^{\Psi}} \tag{36}$$

This level of η would elicit the first best effort level. This is readily verified by inserting effort as determined by incentives from equation (10) into expression (24) for the first best effort level E^*, yielding (36).

But if the agent is risk averse ($1 > \phi$), as in the usual formulation of the agency problem, there is a tradeoff between incentives and riskbearing which means that incentives for effort fall short of the first best level. Specifically, since the right hand side of the first order condition (35) is increasing in both $(1-\phi)$ and in η, η will have to take a lower value than that needed to elicit the first-best effort level. As a result, the optimal level of effort will be lower than the first best level.

3.3 Discussion

It is worth noting that in our model it is the elasticity of managers' wealth to firm value (percent-for-percent) rather than the sensitivity (dollar-for-dollar) that determines effort, because the model assumes Cobb-Douglas managerial preferences and multiplicative impact of effort on firm value, in contrast to the traditional specification used in the literature, which is additive in both respects. In our model the elasticity of pay to gross firm value, η, determines effort:

$$E=\left(\frac{\overline{L}}{C_E}\frac{\alpha\eta}{\Psi+\alpha\eta}\right)^{\alpha}.$$

Thus if this model captures reality, empirical measurement of the power of incentives should focus on elasticities not sensitivities, and Jensen and Murphy's (1990) finding that prior to the 1990s, CEO wealth increased by \$3.25 for every \$1000 increase in firm value, is not necessarily a sign of weak incentives. For if the manager is not independently wealthy and his average pay is low relative to the value of the firm, such a low slope for the pay-performance relationship may well entail strong incentives in terms of the elasticity. For example, a 1% increase in the value of a firm worth \$100 bn would have led to a \$3.25 mn increase in executive pay, an increase in wealth by nearly 1/3 for an executive with initial wealth \$10 mn. To illustrate why \$-for-\$ pay sensitivity is not the appropriate measure of incentives in our model, consider the case of a manager who has no wealth of his own and is compensated entirely through the appreciation of his stock: he will put in an equal amount of effort, no matter what proportion of the total stock he holds, as the elasticity of his pay to firm value (η) is one, regardless. His stake (ω) needs only be large enough to induce him to accept the job. What matters in inducing effort is the elasticity of the compensation scheme - the more elastic it is, the more effort is induced.

The elasticity approach also aligns well with the empirical literature. Murphy's (1999) survey of empirical work on executive compensation notes that models phrased directly in terms of elasticities fit better:

> "The primary advantage of the elasticity approach is that it produces a better “fit” in the sense that rates of return explain more of the cross-sectional variation of Δln(CEO Pay) than changes in shareholder value explain of Δ(CEO Pay)."

Our model focuses on elasticities directly. In particular, while the optimal pay sensitivity is independent of firm size in traditional models, firm size plays an important role in the paper. Equation (35) predicts that the larger the firm relative to the manager's reservation wage, the more performance sensitive the pay contract. The intuition is that the gain in firm value per unit of managerial time devoted to effort is proportional to V_0, while the disutility of time spent on effort is proportional to the manager's wealth. So, all else equal, a more elastic pay contract providing stronger incentives is optimal for larger firms and for managers who are poorer (low $\overline{W}$). Managers who are more able (*i.e.* those with a low cost of effort C_E) will also face stronger incentives. In practice, more skilled managers are likely to have accumulated more wealth and/or to have more attractive alternative opportunities, so that the comparative statics regarding skills and managerial wealth may to some extent offset one another. Lastly, it follows directly from adapting equation (34) that a firm would prefer, all else equal, to hire a less wealthy manager if the pay only needs to be high enough to compensate the manager for his effort and for the risk imposed on him, that is, $\overline{W} = \widehat{W}$. For the net expected pay to a manager who starts out with personal wealth $\widehat{W}$ would have to be:

$$
\begin{aligned}
& \widehat{W} \cdot \left(\frac{\Psi + \alpha\eta}{\Psi} \right)^{\Psi} \exp\left\{ \frac{1}{2} (1 - \phi) \eta^2 \Sigma \right\} - \widehat{W} \\
= \; & \widehat{W} \left[\left(\frac{\Psi + \alpha\eta}{\Psi} \right)^{\Psi} \exp\left\{ \frac{1}{2} (1 - \phi) \eta^2 \Sigma \right\} - 1 \right].
\end{aligned}
$$

This is, for any given level of incentives (η), increasing in (indeed, proportional to) $\widehat{W}$ and therefore more wealthy managers are less attractive.

The multiplicative structure of the model in this paper follows Peng and Röell (2008b). Edmans, Gabaix and Landier (2009) similarly show that in a multiplicative model, effort depends on the elasticity, not the \$-for-\$ sensitivity of pay to firm value. Their model differs in featuring a two-state distribution of firm value, a binary effort choice, and risk neutral preferences. In that model, the pay elasticity needs to be high enough to induce the single first-best effort level and therefore is constant across firms. In contrast, our model endogenises the second-best optimal effort level for risk averse management, and thus provides a rich set of insights into the determinants of optimal pay-performance elasticities.

4 Optimal short-term pay contracts with manipulation

We now turn to a setting where stock price manipulation is possible: pay is linked to the short-term stock price, and that price is based on a manipulable report by the manager. In the current section we will assume that it is impossible to tie pay to long-term performance (so that $\eta \equiv 0$), an assumption relaxed in section 5; the present model generalises the model sketched out by Peng and Röell (2008b).

Inducing effort by making pay depend on short term performance has the inevitable side effect of encouraging the manager to manipulate the stock price. Consider the optimization problem for this case. Setting $\eta \equiv 0$ so that pay is constrained to depend on short-run performance only, the pay contract $\{\omega, \mu\}$ needs to maximize net firm value as expressed in equation (22) subject to the constraint that the manager's expected utility be greater than or equal to its reservation value $U\left(\overline{W}, \overline{L}\right)$, that is:

$$\max_{\omega,\mu} V_0 \left[\frac{\overline{L}}{C_E} \frac{\alpha\gamma\mu}{\Psi + (\alpha+\beta)\gamma\mu}\right]^{\alpha} - \omega V_0^{\mu} \left[\frac{\overline{L}}{C_E} \frac{\alpha\gamma\mu}{\Psi + (\alpha+\beta)\gamma\mu}\right]^{\alpha\mu} \cdot \exp\left\{\frac{-\gamma\mu}{2}(1-\mu)\Sigma_1\right\} \tag{37}$$

subject to:

$$\omega \left[\frac{\Psi}{\Psi + (\alpha+\beta)\gamma\mu}\right]^{\Psi} V_0^{\mu} \left[\frac{\overline{L}}{C_E} \frac{\alpha\gamma\mu}{\Psi + (\alpha+\beta)\gamma\mu}\right]^{\alpha\mu} \exp\left\{-\frac{\gamma\mu}{2}(1-\phi\mu)\Sigma_1\right\} \geqslant \overline{W}$$

where γ is the sensitivity of the stock price to the manager's short-term performance report, determined by equation (16). Substituting out again for ω on the assumption that the manager's participation constraint binds, we have:

$$\max_{\mu} V_0 \left[\frac{\overline{L}}{C_E} \frac{\alpha\gamma\mu}{\Psi + (\alpha+\beta)\gamma\mu}\right]^{\alpha} - \overline{W} \left[\frac{\Psi + (\alpha+\beta)\gamma\mu}{\Psi}\right]^{\Psi} \cdot \exp\left\{\frac{\gamma\mu^2}{2}(1-\phi)\Sigma_1\right\} \tag{38}$$

Proposition 5 *When managerial pay depends only on the short-term value of the firm, the optimal contract is characterized by an elasticity of pay to the short term firm value, μ, given* *by the following first-order condition:*

$$\frac{V_0}{\overline{W}}\left(\frac{\overline{L}}{C_E}\right)^{\alpha}\alpha^{\alpha+1} = \frac{\left[\Psi+(\alpha+\beta)\,\gamma\mu\right]^{\alpha+\Psi}(\gamma\mu)^{1-\alpha}}{\Psi^{\Psi+1}}\cdot \cdot\left[(\alpha+\beta)\,\Psi+\left[\Psi+(\alpha+\beta)\,\gamma\mu\right]\mu\,(1-\phi)\,\Sigma_1\right]\cdot \cdot\exp\left\{\frac{\gamma\mu^2}{2}(1-\phi)\,\Sigma_1\right\} \tag{39}$$

This proposition implies that the possibility of manipulating leads to an optimal pay contract which calls forth lower effort E than in the case where manipulation is impossible. Moreover, the greater the uncertainty about manipulation cost, the lower the effort induced. Let us verify both these claims.

To see that the opportunity to manipulate reduces effort even when there is no uncertainty about the propensity to manipulate ($\Omega = 0$ and thus $\gamma = 1$), set $\gamma = 1$ and $\Sigma_1 = \Sigma$ in equation (39):

$$\frac{V_0}{\overline{W}}\left(\frac{\overline{L}}{C_E}\right)^{\alpha}\alpha^{\alpha+1} = \frac{\left[\Psi+(\alpha+\beta)\,\mu\right]^{\alpha+\Psi}\mu^{1-\alpha}}{\Psi^{\Psi+1}}\cdot \cdot\left[(\alpha+\beta)\,\Psi+\left[\Psi+(\alpha+\beta)\,\mu\right]\mu\,(1-\phi)\,\Sigma\right]\cdot \cdot\exp\left\{\frac{\mu^2}{2}(1-\phi)\,\Sigma\right\} \tag{40}$$

Now compare μ as determined in equation (40) with η from equation (35). The only difference between the two is the presence of terms in β in (40). Since the RHS of equation (40) is increasing in both β and μ, the optimal μ must be lower when $\beta > 0$ than when $\beta = 0$. Since by equation (10) the equilibrium level of effort is increasing in μ and decreasing in β, the effort level is necessarily lower in the setting with manipulation opportunities. Just as in the multitask model of Holmström and Milgrom (1991), the detrimental side effects tend to reduce the power of optimal incentives.

To see that the presence of manipulation cost uncertainty Ω further reduces effort, compared with the "signal jamming" situation where $\Omega = 0$ and $\gamma = 1$, observe that the RHS of condition (39) is increasing in both μ and γ, so that as Ω increases, reducing γ, μ increases. However the product $\gamma\mu$ must fall when γ falls and μ increases, because the RHS of condition (39) is increasing in μ and the composite variable $\gamma\mu$, through which γ enters the expression. Again using equation (10), the equilibrium level of effort is increasing in its determinant $\gamma\mu$, so that the effort level falls.

Intuitively, there are two distinct reasons why the optimum level of effort is lower in the presence of manipulation.

First, incentive compensation encourages effort, but brings with it the undesirable side effect of encouraging manipulation, which carries with it a resource cost in terms of available managerial time. Indeed, with a short-term pay contract ($\eta \equiv 0$), the total time needed to produce a unit of productive effort is scaled up by a factor $\frac{\alpha+\beta}{\beta}$ since by equations (10) and (11) the time wasted on manipulation is proportional to the time used for productive effort, $\frac{C_M M^{\frac{1}{\beta}}}{C_E E^{\frac{1}{\alpha}}} = \frac{\beta}{\alpha}$. The time wasted on manipulation is a source of agency cost, making it more costly to incentivize effort.

Second, the uncertainty about the manipulation cost further reduces the equilibrium level of effort. This uncertainty makes it hard to distinguish true performance from managerial hype, and therefore contaminates the value of the short-term stock price as a measure of managerial effort. Managers who do not exaggerate their performance as much as others are unable to persuade the market that they are simply modest, not lazy. This imposes an additional risk on the risk-averse managers *ex ante*, before they learn their type.

What is more surprising is that the elasticity of pay with stock price, μ, is higher when uncertainty about manipulation cost (Ω) is high. The intuitive explanation is that the stock price is becoming less responsive to managerial reports regarding firm value; in the optimal contract, the incentives are partially restored by an offsetting increase in elasticity of pay to the stock price. This suggests that for companies or industries in which the uncertainty about managerial manipulation is high, that is, it is harder to disentangle the true firm value from hype in the manager's disclosures, the optimal contract should actually be more elastic with respect to price. Thus the model predicts that options, viewed as a means of adding convexity ($\mu > 1$) to the pay scheme, might be more prevalent for such companies. This prediction is consistent with empirical studies that find that the use of options is more prevalent in high-technology, "new economy" firms, and firms in growth industries (such as computer, software, and pharmaceutical firms) and less so in utilities, "old-economy" manufacturing firms and firms in low-growth industries in general (Murphy, 1999, Core and Guay, 2001, Ittner, Lambert and Larcker, 2003). This finding contradicts the traditional intuition that in a riskier industry, the optimal contract should provide weaker incentives in order to reduce the risk imposed on the managers. Our model provides a new rationale for these empirical findings, over and above the usual suggestion that these industries feature particularly high-risk profit opportunities that managers need to be encouraged to take on by making their pay more convex.

The remaining determinants of the optimal pay contract have the same impact as in the traditional agency model of section (3.2): an increase in the fundamental volatility of the firm (Σ_1) increases γ, as the investors rely more on the signal when *ex ante* fundamental uncertainty is large. Therefore, as the RHS of condition (39) is increasing in Σ_1, γ, and μ, the optimal μ decreases with Σ_1. A higher Σ_1 also reduces $\gamma\mu$ and therefore managerial effort E; this can be verified by noting that the RHS of condition (39) is increasing in the two composite variables $\gamma\mu$ and $\frac{\Sigma_1}{\gamma} \equiv \Sigma_1 + \beta^2\Omega$. Increased risk aversion $(1-\phi)$ has a similar impact, decreasing μ and thus reducing the level of effort. Equation (39) also implies that for firms whose value is large relative to the manager's reservation wage ($\frac{V_0}{W}$ high), the optimal pay contract should be more elastic.

These comparative static results yield some further insight into what kind of settings would induce more manipulation on average, because the incentives for productive effort also induce manipulation as a side effect. Thus the model predicts that managers will devote more time and attention to presenting stock-price enhancing information in firms that are large and less risky, especially when the managers are relatively poor and less risk averse.

Lastly, to check the robustness of the striking result that increased manipulation uncertainty Ω leads to a higher pay-performance elasticity, we investigate a competing intuition for what is driving that result. We argued above that the increase in elasticity μ is needed to partially offset the reduced sensitivity γ of the stock price to reported performance. A competing and complementary view is that the volatility of the period 1 stock price is decreasing in Ω, thus reducing the risk imposed on the manager by stock-price-driven incentive pay and encouraging a more price-sensitive contract. To see this, observe that using equations (2) and (6), the first-period logarithmic return can be written as:

$$\ln \frac{P_1}{V_0 E} = \ln \pi + \gamma(\ln M + \varepsilon_1)$$

Thus the variance of the period 1 log-return is given by

$$var\left[\ln \frac{P_1}{V_0 E}\right] = \gamma^2(\Sigma_1 + \beta^2\Omega) = \frac{\Sigma_1^2}{\Sigma_1 + \beta^2\Omega} = \gamma\Sigma_1$$

substituting out for γ using equation(16). Thus the variance of the log-return is scaled down by a factor γ.

But this immediately begs the question of how the volatility of firm value can possibly be reduced below Σ_1, the variance of the fundamentals. In a weak-form efficient market volatility cannot be made to disappear, it can only be pushed into the future. The current model setup only describes a single period, in which uncertainty about the achievements of the previous manager is assumed to have already been resolved at time 0 when the new pay contract is signed. This adequately describes the situation arising when a firm newly enters into a new line of business characterised by a fresh value of Ω, but does not capture the situation of an ongoing firm.

Consider instead an alternative "relay race" model of the long-run steady state, where a firm has already settled into a line of business with associated underlying per-period fundamental risk Σ_1 and manipulation uncertainty Ω, and the volatility from period to period is equal to the inherent variance of the fundamentals Σ_1. Each period is the lifespan of a management team, who pass on their responsibilities to a new team at the end of the period as in a relay race. The price used as the basis for the final payout to the previous team simultaneously enters as V_0 in the new team's contract. Then at the date of signing a new management team's pay contract the full impact of the previous team's efforts is as yet unknown to anyone, so that a proportion $(1-\gamma)\Sigma_1$ of the stock price variance resulting from that team's efforts is still to come. This random resolution becomes public knowledge during the current management's tenure and is incorporated into the stock price between the contract-signing date 0 and date 1, the time at which final pay is determined based on the stock price at that time.

This implies that the model needs to be modified to add an uncorrelated shock of variance $(1-\gamma)\,\Sigma_1$ impacting stock return; but the model is otherwise unaffected. Then the maximization problem (37) will no longer carry a factor γ in the exponential terms, but otherwise it will be identical, and the first order condition (39) changes to:

$$\begin{aligned}\frac{V_0}{\overline{\overline{W}}}\left(\frac{\overline{L}}{C_E}\right)^{\alpha}\alpha^{\alpha+1} \quad = \quad & \frac{\left[\Psi+(\alpha+\beta)\,\gamma\mu\right]^{\alpha+\Psi}(\gamma\mu)^{1-\alpha}}{\Psi^{\Psi+1}}\cdot \\ & \cdot\left[(\alpha+\beta)\,\Psi+\left(\frac{\Psi}{\gamma}+(\alpha+\beta)\,\mu\right)\mu\,(1-\phi)\,\Sigma_1\right]\cdot \\ & \cdot\exp\left\{\frac{\mu^2}{2}\,(1-\phi)\,\Sigma_1\right\}.\end{aligned} \tag{41}$$

This equation is not monotonic in γ and so in the steady state setting it is no longer always true that greater manipulation uncertainty Ω (reducing γ) will lead to an increase in the incentive parameter μ. However, there is still a broad range of parameter values for which that is the case. For example, it is sufficient (but not necessary) that $\left[\Psi+(\alpha+\beta)\,\gamma\mu\right]^{\alpha+\Psi}\gamma^{-\alpha}$ is increasing in γ, that is, $\gamma\geq\frac{1}{\mu}\frac{\alpha}{\alpha+\beta}$, a condition that is more likely to be met if γ and μ are large, that is if Ω is not too large relative to Σ_1 and the firm is large and valuable relative to the manager's wealth, thus requiring strong incentives. Thus even in the steady state, when greater Ω has no impact on the risk exposure of the manager, manipulation uncertainty still typically calls forth a more price-elastic pay contract (an increase in μ) in order to offset the fact that the price is less responsive to the performance report. Meanwhile, the impact of an increase in manipulation uncertainty on equilibrium effort remains unambiguously negative as in the original model (this can readily be verified by recasting the RHS of equation (41) in terms of the composite variable $\gamma\mu$ and γ: it is increasing in $\gamma\mu$ and decreasing in γ when holding constant $\gamma\mu$).

5 The optimal tradeoff between short- and long-term pay

In section 4 we have considered constrained second-best optimal contracts in the presence of manipulation which are constrained so that only short-term performance is allowed to determine compensation. We now consider what happens when both short- and long-term performance can be included. We will maximize net firm value as expressed in equation (22) subject to the constraint that the manager's expected utility in equation (**??**) is greater than or equal to his reservation utility $\frac{1}{\phi}\left(\overline{L}^{\Psi}\overline{W}\right)^{\phi}$. Again eliminating the pay scaling constant ω using the participation constraint of the manager, the optimal incentive contract is the set of parameters $\{\mu, \eta\}$ that solves the optimization problem:

$$\max_{\mu,\eta} V_0\left[\frac{\overline{L}}{C_E}\frac{\alpha(\gamma\mu+\eta)}{\Psi+(\alpha+\beta)\gamma\mu+\alpha\eta}\right]^{\alpha} \tag{42}$$

$$-\overline{W}\left[\frac{\Psi+(\alpha+\beta)\gamma\mu+\alpha\eta}{\Psi}\right]^{\Psi}\cdot\exp\left\{\frac{1-\phi}{2}\left[\left(\gamma\mu^2+2\gamma\mu\eta+\eta^2\right)\Sigma_1+\eta^2\Sigma_2\right]\right\}$$

The first order conditions are:

$$\text{FOC w.r.t. } \mu \quad : \quad \frac{V_0}{\overline{\overline{W}}}\left(\frac{\overline{L}}{C_E}\right)^{\alpha}\alpha^{\alpha+1} \tag{43}$$

$$= \frac{\left[\Psi+(\alpha+\beta)\gamma\mu+\alpha\eta\right]^{\alpha+\Psi}(\gamma\mu+\eta)^{1-\alpha}}{\Psi^{\Psi}(\Psi-\beta\eta)}\cdot$$

$$\cdot\left\{(\alpha+\beta)\Psi+\left[\Psi+(\alpha+\beta)\gamma\mu+\alpha\eta\right](\mu+\eta)(1-\phi)\Sigma_1\right\}\cdot$$

$$\cdot\exp\left\{\frac{1-\phi}{2}\left[\left(\gamma\mu^2+2\gamma\mu\eta+\eta^2\right)\Sigma_1+\eta^2\Sigma_2\right]\right\}$$

$$(\text{or } \mu = 0 \text{ and } \leq \text{ replaces } = \text{ in the FOC}).$$

$$\text{FOC w.r.t. } \eta \quad : \quad \frac{V_0}{\overline{\overline{W}}}\left(\frac{\overline{L}}{C_E}\right)^{\alpha}\alpha^{\alpha+1} \tag{44}$$

$$= \frac{\left[\Psi + (\alpha+\beta)\gamma\mu + \alpha\eta\right]^{\alpha+\Psi}(\gamma\mu+\eta)^{1-\alpha}}{\Psi^{\Psi}(\Psi+\beta\gamma\mu)} \cdot$$

$$\cdot \left\{\alpha\Psi + \left[\Psi + (\alpha+\beta)\gamma\mu + \alpha\eta\right](1-\phi)\left[\gamma\mu\Sigma_1 + \eta(\Sigma_1+\Sigma_2)\right]\right\}$$

$$\cdot \exp\left\{\frac{1-\phi}{2}\left[\left(\gamma\mu^2 + 2\gamma\mu\eta + \eta^2\right)\Sigma_1 + \eta^2\Sigma_2\right]\right\}$$

We first establish some general properties of the optimal pay contract in the presence of long-term incentives.

Proposition 6 *When $\beta > 0$, the second-best optimal contract has the following properties:*

(i) the maximized net firm value is (weakly) decreasing in risk aversion ($1-\phi$), in short-term volatility Σ_1 (holding constant either Ω or γ, regardless), in long-term volatility Σ_2, and in the manipulation uncertainty Ω

(ii) the long-term incentive parameter, η, is strictly greater than zero but the short-term parameter, μ, is only weakly greater than zero;

(iii) long- and short-term incentives are substitutes in the sense that in the unconstrained optimal contract, the short-term incentive parameter μ is lower than it would be if long-term incentives were constrained to be absent ($\eta = 0$);

(iv) effort is below the first-best optimal level.

In general, the optimal contract includes long- as well as short-term incentives. Long-term incentives enable the firm to align the manager's objectives with long-term firm value, mitigating the undesirable side effect associated with short-term incentives, namely the waste of valuable time on manipulating the short-term stock price. Long-term incentives, on the other hand, have the disadvantage of bringing extra risk (captured by Σ_2) from longer-term exogenous shocks to firm value into the manager's pay. Interestingly, in the model long-run incentives may displace the short-run incentives altogether for some parameter configurations; but it is always advantageous to include at least some long-term element in pay.

The comparative statics relating the properties of the optimal pay contract to the underlying parameters is quite complex because it is not just the overall intensity of incentives that varies. The relative power of long- and short-run incentives varies as well. We will use numerical analysis to illustrate these effects. We present the results using a set of benchmark parameter values and varying some of these parameters individually to gain insight on their effects. Our results are robust to a wide range of reasonable parameter values.

We will set the effort cost parameter $\alpha = 1$ throughout. We set $V_0 = 100$ and without loss of generality, normalize $C_E = \overline{L} = \overline{W} = 1$: these four parameters are basically just one as what matters for the optimal incentive is the ratio $\frac{V_0 L}{W C_E}$. We set short run return variance Σ_1 to be 1 and long run return variance Σ_2 to be 5. The manipulation uncertainty Ω is set to 1. The value of ϕ is set to -1, which corresponds to a constant relative risk aversion coefficient $1-\phi$ of 2, a value of the coefficient of relative risk aversion generally accepted as reasonable. The coefficient on leisure in the manager's utility function Ψ is set to 1. The budget share of leisure is $\Psi/(\Psi+1)$. A low value for this ratio indicates a workaholic manager and a high value indicates a slacker. We also set the parameter β to be 1 in the benchmark case.

5.1 The impact of manipulation uncertainty

A key focus of our model is uncertainty about the manager's propensity to manipulate, captured by randomness in his manipulation cost C_M. In Figure 1, we demonstrate the impact of varying the manipulation uncertainty parameter, Ω. Panel A considers the optimal contract where both short term and long term incentive pay are allowed and Panel B considers a restricted contract with only short term incentive pay. The left graph shows the elasticities of pay to stock price, μ (and $\gamma\mu$) and η. In the middle graph, the areas labeled E, M and L represent the proportion of time the manager spends on the effort, manipulation and leisure. The right graph show the ex-ante expected gross and net firm value.

The top panel of Figure 1 shows that an increase in the uncertainty (Ω) regarding the manager's propensity to manipulate naturally leads to a shift away from short-term incentives ($\gamma\mu$ falls) towards long-term incentives (η rises); this is consistent with empirical work such as that of Bizjak, Brickley and Coles (1993), who find that asymmetry of information between managers and the stock market increases the reliance on long-term incentives relative to short-term incentives. Even so, in the case pictured, the elasticity of pay to the short-term stock price (μ) rises so that the pay contract becomes more elastic to both short- and long-run value. However, effort falls despite the increase in the elasticity of pay to both short- and long-term value, because the performance report at date 1 is less informative and so the stock price is less responsive to the report (γ falls) and the rapid drop in short term incentive for effort ($\gamma\mu$) dominates the increase in long term incentive (η). Manipulation declines even more precipitately, because unlike effort it is not subject to an offsetting push from the rising long-term incentives. When long-term incentives are not available, the case depicted in the bottom panel of Figure 1, the impact of manipulation cost uncertainty on short-term incentives is qualitatively similar but much stronger. In particular, firm value is much more sensitive to Ω, decreasing dramatically by about 45% as Ω increases (only 5% in the unconstrained contract); Similarly, the stock price elasticity of pay μ increases more markedly, to nearly 3.5 while it remains near its initial value below 0.5 in the unconstrained setting.

The effect of manipulation uncertainty on the elasticity (convexity) of optimal incentive pay can potentially reconcile Dittman and Maug's (2007) finding that the option contracts are generally a suboptimal form of compensation with the widespread use of options in practice. They calibrate a standard additive principal-agent model with constant relative risk aversion and lognormal stock prices with executive compensation data, and argue that the observed contracts cannot be rationalized by the standard model. In our model, once manipulation uncertainty is taken to account, the short-term elasticity of the optimal contract μ (and hence its convexity) increases. Even for reasonable levels of risk aversion as shown in Figure 1, options may be needed to achieve the optimal degree of convexity, especially if the manipulation uncertainty is high or the firm prefers more short-term-based pay for reasons such as high long-run fundamental uncertainty.

In terms of the effect of manipulation uncertainty on the information efficiency of the short term stock prices, P_1, we analyze the return variance for the period between time 1 and 2. Everything else equal, a more informative P_1 would deviate less from the long run fundamental value and the associated return variance would be lower. Given the following logarithmic return,

$$\begin{aligned} \ln\left(\frac{V_2}{P_1}\right) &= \ln\left(\frac{EV_0\epsilon_1\epsilon_2}{\pi V_0^{1-\gamma}(MEV_0\epsilon_1)^\gamma}\right) \\ &= \ln\frac{E^{1-\gamma}}{\pi} + (1-\gamma)\ln\epsilon_1 + \ln\epsilon_2 - \gamma\ln M, \end{aligned} \tag{45}$$

the log-return variance is:

$$\begin{aligned}
\operatorname{var}\left[\ln\left(\frac{V_2}{P_1}\right)\right] &= (1-\gamma)^2\Sigma_1+\Sigma_2+\gamma^2\beta^2\Omega && (46)\\
&= \left(\frac{\beta^2\Omega}{\Sigma_1+\beta^2\Omega}\right)^2\Sigma_1+\left(\frac{\Sigma_1}{\Sigma_1+\beta^2\Omega}\right)^2\beta^2\Omega+\Sigma_2\\
&= \frac{\beta^2\Omega\Sigma_1}{\Sigma_1+\beta^2\Omega}+\Sigma_2\\
&= (1-\gamma)\Sigma_1+\Sigma_2. && (47)
\end{aligned}$$

The first part of the return variance shows how much P_1 is "noised up" because of the uncertainty on the manager's propensity to manipulate and is increasing in the uncertainty Ω. If the uncertainty is zero, return variance would simply be Σ_2 and P_1 would capture the true state of the firm at the time. Given that firms or industries with high manipulation uncertainty are also more likely to use higher powered incentives, Equation (46) predicts that investors are more likely to be surprised in the long run by firms in industries with more intangible assets, with more use of off-balance transactions and in growth industries, where there is more manipulation uncertainty and at the same time relatively high-powered incentives μ: high tech startups, R&D intensive companies and banks and other financial services firms. Such companies are more likely to have large returns, positive or negative, and thus (when there are outsize negative returns) to trigger shareholder class action lawsuits and SEC investigations, as found by Peng and Röell (2008a) and Johnson *et al* (2009). It is interesting to note that in any particular instance, when a manager has a higher manipulation cost parameter C_M than the median $\overline{C}_M$, investors underestimate the degree of his manipulation and arrive at an inflated short run valuation P_1. But in the long run, the overvaluation will be corrected as V_2 emerges, resulting (on average) in a less-than-expected long run return $\ln\left(\frac{V_2}{P_1}\right)$.

In terms of implications for empirical work on executive compensation, in our model the short term incentive depends on $\gamma\mu$, the product of the sensitivity of pay to stock price and the sensitivity of stock price to the performance report disclosed by the manager; while empirical studies typically use the elasticity of managerial pay with respect to stock price μ alone. We have shown that in the presence of high manipulation uncertainty, the stock market is less responsive to managers' reports and therefore a higher powered incentive μ is necessary to induce optimal effort even though the product $\gamma\mu$ (and thus effort E) are actually lower. That is, a high pay sensitivity to stock price μ but low effort E could be consistent with optimal contracting if it is difficult to persuade the stock market that performance is high. Simply using μ to proxy for incentives, as done typically in empirical studies, may give rise to misleading conclusions to the extent that manipulability and thus γ differs across industries.

5.2 The manipulation cost function

Next, we consider how the equilibrium is affected by the parameter which determines the convexity of the time cost of manipulation, $\beta \in [0,1]$. The lower β, the more convex the time cost of manipulation, and therefore the closer together the reports issued by managers with different manipulation cost parameters C_M: as can be seen from equation (10), the distribution of $\ln M$ has a variance of $\beta^2\Omega$ and so convexity of the cost function reduces uncertainty about the amount of manipulation. Moreover, from equations (13) and (16), for any given pay contract $\{\omega, \mu, \eta\}$, provided that $\Sigma_1 \geq \beta^2\Omega$ (for which it is sufficient that $\Sigma_1 \geq \Omega$ given that $\beta \leq 1$), time spent on manipulation is increasing in β while effort is decreasing in β. Thus as long as the amount of manipulation uncertainty does not exceed the uncertainty regarding fundamentals, it is beneficial to increase the convexity of the manipulation function (that is, reduce β)because that leads to a reduction in both the time wasted manipulating and the unnecessary and harmful noise injected into the agency contract. In the extreme case when β is zero, the manager will not waste any time manipulating.

Figure 2 illustrates how the optimal contract, effort-manipulation-leisure choice, and firm value vary with β. When β equals one, the time cost of manipulation is linear, which results in the greatest manipulation uncertainty. This effect makes short term incentives less desirable for similar reasons as in the case of high Ω. Therefore it is more efficient to tilt the incentives away from the short term toward the long term when β takes a high value, resulting a high η but low $\gamma\mu$. In contrast, when β approaches 0, the manager will refrain from manipulating and the ideal compensation is purely short term, as this does not impose long run risk on the managers.

Interestingly, as shown in the middle panel of figure 2, under the optimal contract the time spent on manipulation can be nonmonotonic in β: as β increases, the time spent on manipulation first increases but eventually decreases, even though the time spent on effort is decreasing in β. The third panel of figure 2 shows that lowering β is value-enhancing – changing β from 1 to 0 allows the firm to nearly double its net value. Thus making the manipulation function more convex tends to enhance net firm value.

Regarding the average cost of manipulation, it is worth noting that changes in the median $\overline{C}_M$ alone have no real impact in the current model, because the time devoted to manipulation ($C_M M^{\frac{1}{\beta}}$) is independent of C_M, and that waste of time is the real resource cost involved. Of course the amount by which the manager's report is inflated is scaled in proportion to $\overline{C}_M^{-\beta}$, but investors are presumed to be aware of $\overline{C}_M$ and therefore able to correctly adjust their inferences. Therefore both the stock price's sensitivity to the signal and the elasticity of pay are independent of $\overline{C}_M$.

Thus in our model policy measures that increase the average time cost of manipulation have no real impact. But policies that increase the convexity of the manipulation cost function, discouraging extreme amounts of manipulation and encouraging agents with differing cost of manipulation to bring their reports M closer together, can be beneficial.

5.3 The role of firm size

The impact of changes in the size of the firm relative to the manager's reservation wage is illustrated in Figure 3. The expression $\frac{V_0}{W}\left(\frac{\overline{L}}{C_E}\right)^{\alpha}$ that appears in equations (43) and (44) is the ratio of the maximum value of the firm, attained if the manager devotes 100% of his time to effort, to his reservation wage. An increase in the size of the firm V_0 means that effort becomes more productive; not surprisingly, the optimal amount of effort elicited increases with firm size. Different from the monotonic relation between short term incentives and firm size in the case where the long-term incentive is unavailable, there is an interesting nonmonotonicity in the short-term incentives once long-term incentives are allowed. At the lower end of the range of firm sizes, short-term incentives increase and the amount of manipulation increases concomitantly. But the proportion of time spent on manipulation reaches a maximum at an intermediate firm size and then decreases as relatively more reliance is placed on long-term incentives. This contrasts with the comparative statics of the simpler model with short-run pay only, where both effort and manipulation were monotonically increasing in the relative firm size. Intuitively, as firm size increases effort increases and the manager's time becomes a relatively more scarce resource. There comes a point beyond which the waste of time spent on manipulation outweighs the risk advantages of short-term contracts, and thus at the higher end of the firm size scale incentives are shifted to the long run.

5.4 The effect of risk aversion and fundamental volatility

Figure 4 describes how the optimal contract, effort-manipulation-leisure choice, and firm value vary with the managers's coefficient of relative risk aversion $(1 - \phi)$. In contrast to the results we have on the restricted contract where only short term incentive pay is allowed, the incentives under the general contract display interesting and non-monotonic patterns when risk aversion varies: when the risk aversion is low enough, only long-term incentives are used: pay is not linked to the short-term stock price. However, as risk aversion increases, short-term incentives become an attractive way to manage risk and the elasticity of pay to long-term value is reduced; and as a side effect, managers devote part of their time to manipulation. As risk aversion increases further, both long- and short-term elasticities fall due to the costs of bearing risk, and both effort and manipulation decline. The figure thus illustrates that short-term incentives are not monotonic in risk aversion: from zero, they first increase as they replace long-term incentives, and then they decline alongside the long-term incentives as risk aversion increases further.

The impact of the riskiness of the company's business is illustrated in Figure 5. Focusing on the impact of the volatility of the period 1 exogenous shock (Σ_1), the impact on short-term incentives is straightforward: they decline steadily as risk increases, because the cost of compensating the manager for bearing the risk attached to the incentives increases; and there comes a point beyond which it is optimal to provide no short-term incentives $(\mu^* = 0)$. The impact of increasing risk on the long-term incentives is strikingly nonmonotonic. Initially there is a slight decline as would be natural given that the risk associated with any given η increases. But beyond that there is a range where the long-term incentives as captured by η actually increase, presumably as a substitute for the drop in short-term incentives $(\gamma\mu)$: as Σ_1 increases (holding constant Σ_2) the risk-reduction advantages of short-term compensation become less salient and the manipulation-cost-saving advantages of long-term compensation dominate. Once short-term incentives have been driven down to zero altogether, there is no further substitution effect and so increases in risk simply lead to decreases in long-term incentives.

Regarding the impact of long-term volatility Σ_2, it is readily verified (see the proof of Proposition 6) that when the additional risk from long-term incentives is low enough, it is optimal not to use short-term incentives, and $\mu^* = 0$. For the parameter ranges examined, as Σ_2 increases, incentives are loaded onto short-term performance because the extra risk from conditioning payments on long-term performance increases.

6 Policy implications

In terms of policy implications, our discussion of uncertainty about manipulation costs suggests that policies that lower Ω, such as tighter accounting standards or better internal controls through more board monitoring, would allow investors to make sharper inferences about manipulation and firm value, making possible an optimal contract that elicits more effort. Surprisingly, such policies that increase $\gamma\mu$ would also lead to more manipulation in equilibrium. Even so, net firm value is raised, stock prices are more informationally efficient and investors' welfare is improved. In contrast, policies such as the relaxation of mark-to-market requirements for the financial services sector as instituted in early 2009, may give managers more leeway in reporting and thus increase uncertainty Ω, making the stock price less efficient and harming the effectiveness of incentive compensation based on the short-term price.

Policies that lower β and make the manipulation cost function more convex would not only reduce investors' uncertainty about the degree to which the managerial signal is manipulated (M) but also reduce the waste of valuable managerial time spent on manipulation relative to productive effort, under reasonable conditions. This type of policy could be any measure that makes it more costly to make egregiously inflated reports without increasing reporting costs across the board. Examples include increased efforts to detect and penalize unusually optimistic reports, accounting standards that require managers to give a detailed justification if the reports deviate substantially from those of similar firms or an established benchmark, and extra scrutiny for accounts that exhibit signs of possible manipulation, based on measures of manipulation such as discretionary accruals or extreme stock price changes. In contrast, policies that simply change the average proportional cost of manipulation ($\overline{C}_M$) across the board may not have real impact in reducing the deadweight cost caused by manipulation.

In our analysis, extending the contract space to allow for both short term and long term incentives is welfare improving. The optimal mix between short- and long-term contracts depends on the degree of manipulation uncertainty, the convexity of the manipulation cost function, the manager's risk aversion, the short- versus long-term fundamental uncertainty, as well as firm size. Contracts that focus too much on short term incentives have the side effect of inducing unnecessary wasteful manipulation. The current call by politicians and the press for instituting incentives with long vesting periods and "clawback" clauses are a step in the right direction. However, one must keep in mind that it is not optimal to entirely eliminate short-term incentives. Short-term incentives may be part of the optimal compensation contract when managers are very risk averse or when long-term uncertainty is large, despite the necessary evil of calling forth some manipulation.

7 Conclusion

We analyze a rational expectations model of optimal executive compensation in a setting where managers are in a position to manipulate short-term stock prices, and managers' propensity to manipulate is uncertain. Stock-based incentives elicit not only productive effort, but also costly information manipulation. We derive a second-best optimal compensation scheme and analyze the tradeoffs involved in balancing effort and manipulation and in conditioning pay on long- versus short-term performance.

We show that manipulation, and investors' uncertainty about it, affects the equilibrium pay contract and the informational efficiency of asset prices. When manipulation uncertainty is high, pay can actually be more closely linked to the stock price even though the efficiency of equilibrium stock price formation is now lower, while the equilibrium level of effort is lower and the stock price is less sensitive to the manager's report. The cross-sectional implication of this result is that firms or industries that have high uncertainty regarding the propensity to manipulate, such as growth firms or industries that involves more intangible assets, are at the outset more likely to use high powered incentive contracts, such as stock options. Policies that reduce manipulation uncertainty or increase the convexity of manipulation costs would allow investors to make better inferences about the degree of manipulation and result in more efficient contracts that enhance shareholder value. However, policies that only change the average cost of manipulation may not have much real impact.

For companies with high manipulation uncertainty, our model suggests that shifting incentive pay from short term to long term based pay can help mitigate the economic waste associated with short term incentives, resulting in higher effort and improved firm value. For example, it might be optimal for these firms to use stock or options with long vesting periods, or to use "clawback" clauses that allow firms to recover unwarranted pay if long run performance falters.

Our analysis also cautions against empirical methodologies that measure incentives by simply looking at the sensitivity of pay to the contemporaneous stock price return, since the speed at which underlying performance is incorporated into the stock price can vary between industries. Seemingly powerful incentives may elicit little effort if the full impact of the effort on the stock price is delayed beyond the pay horizon. Ignoring this can lead to misleading conclusions regarding the relation between incentives provided by the contract and performance.

Lastly, we provide a tractable multiplicative model of optimal executive pay with constant relative risk averse managerial preferences that are Cobb-Douglas in wealth and leisure, and with effort modeled as an input of managerial time that impacts log normally distributed firm value proportionally. This formulation generates empirically testable predictions about the determinants of optimal pay contracts. Incentives for effort depend on the elasticity of pay to firm value rather than the "dollar-for-dollar" sensitivity, and the model makes direct predictions about the elasticity of pay to firm value, providing a convenient reference point for empirical research.

8 Appendix: Proofs of propositions

We use lower-case letters to denote the logarithms of the corresponding upper-case variables throughout. It is useful to introduce the short hand notation

$$\begin{aligned} k_E &= \ln\left(\frac{\alpha(\gamma\mu+\eta)}{\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu}\overline{L}\right) \text{ and } c_E = \ln C_E \\ k_M &= \ln\left(\frac{\beta\gamma\mu}{\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu}\overline{L}\right) \text{ and } c_M = \ln C_M \end{aligned} \tag{A1}$$

so that the effort and manipulation choices in equation (10) can be expressed as:

$$\begin{aligned} e &= \alpha(k_E - c_E) \\ m &= \beta(k_M - c_M) \end{aligned} \tag{A2}$$

Proof of Proposition 1. Equation (2) is rewritten in logarithmic form as:

$$s = v_0 + \widetilde{m} + e + \widetilde{\varepsilon}_1 \tag{A3}$$

where v_0, $\widetilde{m}$, e, and $\widetilde{\varepsilon}_1$ are the logarithms of V_0, $\widetilde{M}$, E, and $\widetilde{\epsilon}_1$. The joint distribution of $\widetilde{m}$ and $\widetilde{\varepsilon}_1$ is as follows:

$$\begin{pmatrix} m \\ \varepsilon_1 \end{pmatrix} \sim N\left(\begin{pmatrix} \beta(k_M - \overline{c}_M) \\ -\frac{1}{2}\Sigma_1 \end{pmatrix}, \begin{pmatrix} \beta^2\Omega & 0 \\ 0 & \Sigma_1 \end{pmatrix}\right) \tag{A4}$$

Then we have

$$\begin{aligned}
& \mathrm{E}\left[v_1|s\right] && \text{(A5)} \\
= & \mathrm{E}\left[v_0 + e + \widetilde{\varepsilon}_1 \left| s = v_0 + \widetilde{m} + e + \widetilde{\varepsilon}_1 \right.\right] \\
= & v_0 + \alpha\left(k_E - c_E\right) - \frac{1}{2}\Sigma_1 + \frac{\Sigma_1}{\beta^2\Omega + \Sigma_1}\left(s - \left(v_0 + \alpha\left(k_E - c_E\right) + \beta\left(k_M - \overline{c}_M\right) - \frac{1}{2}\Sigma_1\right)\right) \\
= & v_0 \frac{\beta^2\Omega}{\beta^2\Omega + \Sigma_1} + s\frac{\Sigma_1}{\beta^2\Omega + \Sigma_1} + \left[\alpha\left(k_E - c_E\right) - \frac{1}{2}\Sigma_1\right]\frac{\beta^2\Omega}{\beta^2\Omega + \Sigma_1} - \beta\left(k_M - \overline{c}_M\right)\frac{\Sigma_1}{\beta^2\Omega + \Sigma_1}
\end{aligned}$$

and

$$var\left[v_1|s\right] = \frac{\beta^2\Omega\Sigma_1}{\beta^2\Omega + \Sigma_1} \tag{A6}$$

From (5), the logarithm of the (gross-of-expected-pay) market price is given by:

$$\begin{aligned}
p_1 &= \ln P_1 = \ln \mathrm{E}\left[V_1|S\right] = \ln \mathrm{E}\left[\exp\left(v_1\right)|s\right] = \mathrm{E}\left[v_1\left|s\right.\right] + \frac{1}{2}var\left[v_1\left|s\right.\right] && \text{(A7)} \\
&= v_0 \frac{\beta^2\Omega}{\beta^2\Omega + \Sigma_1} + s\frac{\Sigma_1}{\beta^2\Omega + \Sigma_1} + \left[\alpha\left(k_E - c_E\right) - \frac{1}{2}\Sigma_1\right])\frac{\beta^2\Omega}{\beta^2\Omega + \Sigma_1} \\
&\quad -\beta\left(k_M - \overline{c}_M\right)\frac{\Sigma_1}{\beta^2\Omega + \Sigma_1} + \frac{1}{2}\frac{\beta^2\Omega\Sigma_1}{\beta^2\Omega + \Sigma_1} \\
&= v_0 \frac{\beta^2\Omega}{\beta^2\Omega + \Sigma_1} + s\frac{\Sigma_1}{\beta^2\Omega + \Sigma_1} + \alpha(k_E - c_E)\frac{\beta^2\Omega}{\beta^2\Omega + \Sigma_1} - \beta\left(k_M - \overline{c}_M\right)\frac{\Sigma_1}{\beta^2\Omega + \Sigma_1} \\
&= v_0\left(1-\gamma\right) + s\gamma + \ln\pi \\
&= \left(v_0 + e_1\right)\left(1-\gamma\right) + \left(s - \overline{m}\right)\boldsymbol{\gamma} && \text{(A8)}
\end{aligned}$$

where:

$$\begin{aligned}
\gamma &= \frac{\Sigma_1}{\beta^2\Omega + \Sigma_1} && \text{(A9)} \\
\ln\pi &= \alpha\left(1-\gamma\right)\left(k_E - c_E\right) - \beta\gamma\left(k_M - \overline{c}_M\right) && \text{(A10)}
\end{aligned}$$

■

Proof of Proposition 2. Rewriting manager's pay in equation (9) in logarithmic form and substituting out for m and e using equation (A2) and for π using equation (A10):

$$\begin{aligned}
\widetilde{w} &= \ln\omega + \mu\ln\pi + \left(\mu+\eta\right)v_0 + \gamma\mu\widetilde{m} + \left(\gamma\mu+\eta\right)e + (\gamma\mu+\eta)\widetilde{\varepsilon}_1 + \eta\widetilde{\varepsilon}_2 && \text{(A11)} \\
&= \ln\omega + \left(\mu+\eta\right)v_0 + \gamma\mu\beta\left(\overline{c}_M - \widetilde{c}_M\right) + \left(\mu+\eta\right)\alpha\left(k_E - c_E\right) + (\gamma\mu+\eta)\widetilde{\varepsilon}_1 + \eta\widetilde{\varepsilon}_2
\end{aligned}$$

and thus the expected value of the logarithm of the manager's terminal wealth is:

$$\begin{aligned}
\mathrm{E}_0[\widetilde{w}] &= \ln\omega + \left(\mu+\eta\right)\left[v_0 + \alpha\left(k_E - c_E\right)\right] && \text{(A12)} \\
&\quad -\frac{1}{2}\left[(\gamma\mu+\eta)\Sigma_1 + \eta\Sigma_2\right]
\end{aligned}$$

and its variance is

$$\operatorname{Var}(\widetilde{w}) = (\gamma\mu+\eta)^2\Sigma_1 + \eta^2\Sigma_2 + \gamma^2\mu^2\beta^2\Omega \tag{A13}$$

The logarithm of the expected utility of the manager (after finding out his effort cost C_M but before observing the firm value shocks ϵ_1 and ϵ_2) is then:

$$\begin{aligned}
\ln \mathrm{E}_0\left[\widetilde{U}\right] &= \ln \mathrm{E}_0\left[\frac{1}{\phi}\left(L^{\Psi}\widetilde{W}\right)^{\phi}\right] \qquad\qquad (A14)\\
&= -\ln\phi + \phi\Psi\ln L + \phi \mathrm{E}[\widetilde{w}] + \frac{1}{2}\phi^2 \operatorname{Var}(\widetilde{w})\\
&= -\ln\phi + \phi\Psi\ln\frac{\Psi\overline{L}}{\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu}\\
&\quad +\phi\left\{\ln\omega + (\mu+\eta)\left[v_0+\alpha\left(k_E-c_E\right)\right]\right\}\\
&\quad -\frac{\phi}{2}\left[(\gamma\mu+\eta)\Sigma_1+\eta\Sigma_2\right] + \frac{\phi^2}{2}\left[(\gamma\mu+\eta)^2\Sigma_1+\eta^2\Sigma_2+\gamma^2\mu^2\beta^2\Omega\right]
\end{aligned}$$

where the manager's choice for leisure L is substituted in from equation (12). Using the definition of γ to substitute out for Ω using $\beta^2\Omega = \Sigma_1\frac{1-\gamma}{\gamma}$, we obtain:

$$\begin{aligned}
\ln \mathrm{E}_0\left[\widetilde{U}\right] &= -\ln\phi + \phi\Psi\ln\frac{\Psi\overline{L}}{\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu} \qquad\qquad (A15)\\
&\quad +\phi\left\{\ln\omega + (\mu+\eta)\left[v_0+\alpha\left(k_E-c_E\right)\right]\right\}\\
&\quad -\frac{\phi}{2}\left[(\gamma\mu+\eta)\Sigma_1+\eta\Sigma_2\right]\\
&\quad +\frac{\phi^2}{2}\left[\left(\gamma\mu^2+2\gamma\mu\eta+\eta^2\right)\Sigma_1+\eta^2\Sigma_2\right]
\end{aligned}$$

■

Proof of Proposition 3. The expected payment to the manager is:

$$\begin{aligned}
\mathrm{E}_0\left[\widetilde{W}\right] &= \exp\left\{\mathrm{E}_0\left[\widetilde{w}\right] + \frac{1}{2}\operatorname{Var}(\widetilde{w})\right\}\\
&= -\omega V_0^{\mu+\eta}\left(\frac{\overline{L}}{C_E}\frac{\alpha(\gamma\mu+\eta)}{\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu}\right)^{\alpha(\mu+\eta)}\cdot\\
&\quad \cdot\exp\left\{\frac{1}{2}\left[\left(-\gamma\mu+\gamma\mu^2+2\gamma\mu\eta-\eta+\eta^2\right)\Sigma_1+\left(-\eta+\eta^2\right)\Sigma_2\right]\right\}
\end{aligned}$$

since the wage is lognormally distributed, and using equations (A12) and (A13).

The *ex ante* company expected value is then:

$$
\begin{aligned}
\mathrm{E}_0\left[\widetilde{V}_2-\widetilde{W}\right] &= \mathrm{E}_0\left[\widetilde{V}_2\right]-\mathrm{E}_0\left[\widetilde{W}\right] && \text{(A16)}\\
&= \mathrm{E}_0\left[V_0\cdot E\cdot\widetilde{\epsilon}_1\cdot\widetilde{\epsilon}_1\right]-\mathrm{E}_0\left[\widetilde{W}\right] \\
&= V_0\cdot E-\mathrm{E}_0\left[\widetilde{W}\right] && \text{(A17)}\\
&= V_0\left(\frac{\overline{L}}{C_E}\frac{\alpha\left(\gamma\mu+\eta\right)}{\Psi+\alpha\left(\gamma\mu+\eta\right)+\beta\gamma\mu}\right)^{\alpha} \\
&\quad -\omega V_0^{\mu+\eta}\left(\frac{\overline{L}}{C_E}\frac{\alpha\left(\gamma\mu+\eta\right)}{\Psi+\alpha\left(\gamma\mu+\eta\right)+\beta\gamma\mu}\right)^{\alpha(\mu+\eta)}\cdot \\
&\quad \cdot\exp\left\{\frac{1}{2}\left[\left(-\gamma\mu+\gamma\mu^2+2\gamma\mu\eta-\eta+\eta^2\right)\Sigma_1+\left(-\eta+\eta^2\right)\Sigma_2\right]\right\}
\end{aligned}
$$

■

Proof of Proposition 4. Immediate: first order condition for maximization problem (34).

Proof of Proposition 5. Immediate: first order condition for maximization problem (38).

Proof of Proposition 6.

(i) Inspecting the optimization problem (42), it is immediately obvious that for any given values of μ and η, the maximand increases if CRRA $(1-\phi)$ falls, long-term risk Σ_2 falls, or Σ_1 falls whilst γ remains constant (*i.e.* Ω falls in proportion to Σ_1). It is also decreasing in Ω because a fall in Ω means an increase in γ; and an increase in γ, when μ varies in such a way as to keep $\mu\gamma$ constant, entails a decrease in μ which decreases the $\gamma\mu^2$ term in the exponent, increasing the maximand. Lastly, when Σ_1 falls holding Ω constant, $\gamma\equiv\frac{\Sigma_1}{\Sigma_1+\beta^2\Omega}$ falls. But suppose that you vary μ in such a way as to keep $\gamma\mu$ unchanged, then the first term in the exponent is $(\gamma\mu)^2\,\Sigma_1/\gamma=(\gamma\mu)^2\left(\Sigma_1+\beta^2\Omega\right)$ and thus decreasing along with Σ_1, so that the entire maximand increases. All these arguments show that the maximand can be increased, even when the endogenous parameters μ and η are not adjusted in an optimal manner; naturally it would increase even more if they were.

(ii) An optimal contract cannot have just short-term but not long-term incentives. Suppose not, that is, suppose the solution has $\eta=0$ and $\mu\neq0$. The relevant FOCs are then:

$$\text{FOC w.r.t. } \mu \quad : \quad \frac{V_0}{\overline{\overline{W}}}\left(\frac{\overline{L}}{C_E}\right)^{\alpha}\alpha^{\alpha+1} \tag{A18}$$

$$= \frac{\left[\Psi+(\alpha+\beta)\gamma\mu\right]^{\alpha+\Psi}(\gamma\mu)^{1-\alpha}}{\Psi^{\Psi+1}}\cdot$$

$$\cdot\left\{(\alpha+\beta)\Psi+\left[\Psi+(\alpha+\beta)\gamma\mu\right]\mu(1-\phi)\Sigma_1\right\}\cdot$$

$$\cdot\exp\left[\frac{1-\phi}{2}\gamma\mu^2\Sigma_1\right]$$

$$\text{FOC w.r.t. } \eta \quad : \quad \frac{V_0}{\overline{\overline{W}}}\left(\frac{\overline{L}}{C_E}\right)^{\alpha}\alpha^{\alpha+1} \tag{A19}$$

$$\leq \frac{\Psi}{\Psi+\beta\gamma\mu}\frac{\left[\Psi+(\alpha+\beta)\gamma\mu\right]^{\alpha+\Psi}(\gamma\mu)^{1-\alpha}}{\Psi^{\Psi+1}}\cdot$$

$$\cdot\left\{\alpha\Psi+\left[\Psi+(\alpha+\beta)\gamma\mu\right](1-\phi)\gamma\mu\Sigma_1\right\}$$

$$\cdot\exp\left\{\frac{1-\phi}{2}\gamma\mu^2\Sigma_1\right\}$$

Clearly these two conditions cannot be met simultaneously because the RHS of the first FOC is strictly greater than that of the second one (both due to a factor β on Ψ in the FOC w.r.t. μ, and the appearance of $\gamma \leq 1$ in two extra places reducing the RHS of the FOC w.r.t. η). Contradiction.

On the other hand it is conceivable that the solution has $\eta > 0$ but $\mu = 0$ simultaneously:

$$\text{FOC w.r.t. } \mu \quad : \quad \frac{V_0}{\overline{\overline{W}}}\left(\frac{\overline{L}}{C_E}\right)^{\alpha}\alpha^{\alpha+1} \tag{A20}$$

$$\leq \frac{\left[\Psi+\alpha\eta\right]^{\alpha+\Psi}\eta^{1-\alpha}}{\Psi^{\Psi}(\Psi-\beta\eta)}\cdot$$

$$\cdot\left\{(\alpha+\beta)\Psi+\eta(\Psi+\alpha\eta)(1-\phi)\Sigma_1\right\}\cdot$$

$$\cdot\exp\left\{\frac{1-\phi}{2}\eta^2(\Sigma_1+\Sigma_2)\right\}$$

$$\text{FOC w.r.t. } \eta \quad : \quad \frac{V_0}{\overline{\overline{W}}}\left(\frac{\overline{L}}{C_E}\right)^{\alpha}\alpha^{\alpha+1} \tag{A21}$$

$$= \frac{\left[\Psi+\alpha\eta\right]^{\alpha+\Psi}\eta^{1-\alpha}}{\Psi^{\Psi}\Psi}\cdot$$

$$\cdot\left[\alpha\Psi+\eta(\Psi+\alpha\eta)(1-\phi)(\Sigma_1+\Sigma_2)\right]$$

$$\cdot\exp\left\{\frac{1-\phi}{2}\eta^2(\Sigma_1+\Sigma_2)\right\}$$

In particular such a contract may be optimal if Σ_2 is small enough relative to $(\Sigma_1 + \Sigma_2)$ so that the extra risk attributable to long-term incentives is sufficiently small:

$$\frac{\Psi}{\Psi - \beta\eta}\left\{(\alpha+\beta)\Psi + \eta(\Psi+\alpha\eta)(1-\phi)\Sigma_1\right\} > \alpha\Psi + \eta(\Psi+\alpha\eta)(1-\phi)(\Sigma_1+\Sigma_2) \quad \text{(A22)}$$

$$\text{i.e.} \quad \beta\Psi > \eta(1-\phi)\left[\Psi\Sigma_2 - \beta\eta(\Sigma_1+\Sigma_2)\right] \quad \text{(A23)}$$

Fixing $\Sigma_1 + \Sigma_2$ and thus η, this condition is satisfied if Σ_2 is small enough.

(iii) If we allow long term incentives, i.e. $\eta > 0$, then short term incentives are lower i.e. μ is reduced and a smaller proportion of time is devoted to manipulation. This is immediately obvious from the FOC with respect to μ: the RHS is increasing in both η and μ so holding all else constant an increase in the one must be offset by a decrease in the other. This also implies there will be less time spent on manipulation because

$$M = \left(\frac{\overline{L}}{C_M}\frac{\beta\gamma\mu}{\Psi+(\alpha+\beta)\gamma\mu+\alpha\eta}\right)^{\beta}$$

(iv) From equation (10), effort is determined by:

$$E = \left(\frac{\overline{L}}{C_E}\frac{\alpha(\gamma\mu+\eta)}{\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu}\right)^{\alpha}$$

and first best effort (E^*) is determined in equation (25) as

$$\frac{\left(\overline{L} - C_E E^{*\frac{1}{\alpha}}\right)^{\Psi+1}}{C_E E^{*\frac{1}{\alpha}-1}} = \frac{\overline{L}^{\Psi}\overline{W}\Psi}{V_0\alpha}$$

Since the left hand side of the above equation is decreasing in E, to show that $E \leq E^*$, we need to show that

$$\frac{\left(\overline{L} - C_E E^{\frac{1}{\alpha}}\right)^{\Psi+1}}{C_E E^{\frac{1}{\alpha}-1}} \geq \frac{\overline{L}^{\Psi}\overline{W}\Psi}{V_0\alpha}.$$

Equivalently, using equation (10) to replace effort E by its determinants, it needs to be shown that:

$$\left(\overline{L}\frac{\Psi+\beta\gamma\mu}{\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu}\right)^{\Psi+1} \geq \frac{\overline{L}^{\Psi}\overline{W}\Psi}{V_0\alpha}\frac{C_E^{\alpha}\overline{L}^{1-\alpha}\alpha^{1-\alpha}(\gamma\mu+\eta)^{1-\alpha}}{(\Psi+\alpha(\gamma\mu+\eta)+\beta\gamma\mu)^{1-\alpha}}$$

i.e.

$$\frac{V_0}{\overline{\overline{W}}}\left(\frac{\overline{L}}{C_E}\right)^{\alpha}\alpha^{\alpha+1} \geq \frac{\left[\Psi+(\alpha+\beta)\gamma\mu+\alpha\eta\right]^{\alpha+\Psi}(\gamma\mu+\eta)^{1-\alpha}}{\Psi^{\Psi}(\Psi+\beta\gamma\mu)}\left(\frac{\Psi}{\Psi+\beta\gamma\mu}\right)^{\Psi}\alpha\Psi$$

But the expression on the RHS satisfies:

$$
\begin{aligned}
& \frac{\left[\Psi+(\alpha+\beta)\gamma\mu+\alpha\eta\right]^{\alpha+\Psi}(\gamma\mu+\eta)^{1-\alpha}}{\Psi^{\Psi}(\Psi+\beta\gamma\mu)}\left(\frac{\Psi}{\Psi+\beta\gamma\mu}\right)^{\Psi}\alpha\Psi \\
\leq\; & \frac{\left[\Psi+(\alpha+\beta)\gamma\mu+\alpha\eta\right]^{\alpha+\Psi}(\gamma\mu+\eta)^{1-\alpha}}{\Psi^{\Psi}(\Psi+\beta\gamma\mu)} \\
& \cdot\underbrace{\left\{\alpha\Psi+\left[\Psi+(\alpha+\beta)\gamma\mu+\alpha\eta\right](1-\phi)\left[\gamma\mu\Sigma_1+\eta\left(\Sigma_1+\Sigma_2\right)\right]\right\}}_{>\alpha\Psi} \\
& \cdot\underbrace{\exp\left\{\frac{1-\phi}{2}\left[\left(\gamma\mu^2+2\gamma\mu\eta+\eta^2\right)\Sigma_1+\eta^2\Sigma_2\right]\right\}}_{>1} \\
=\; & \frac{V_0}{\overline{W}}\left(\frac{\overline{L}}{C_E}\right)^{\alpha}\alpha^{\alpha+1}
\end{aligned}
$$

where the last equality follows from equation (44), the FOC with respect to η. This proves the claim that effort is below first-best.

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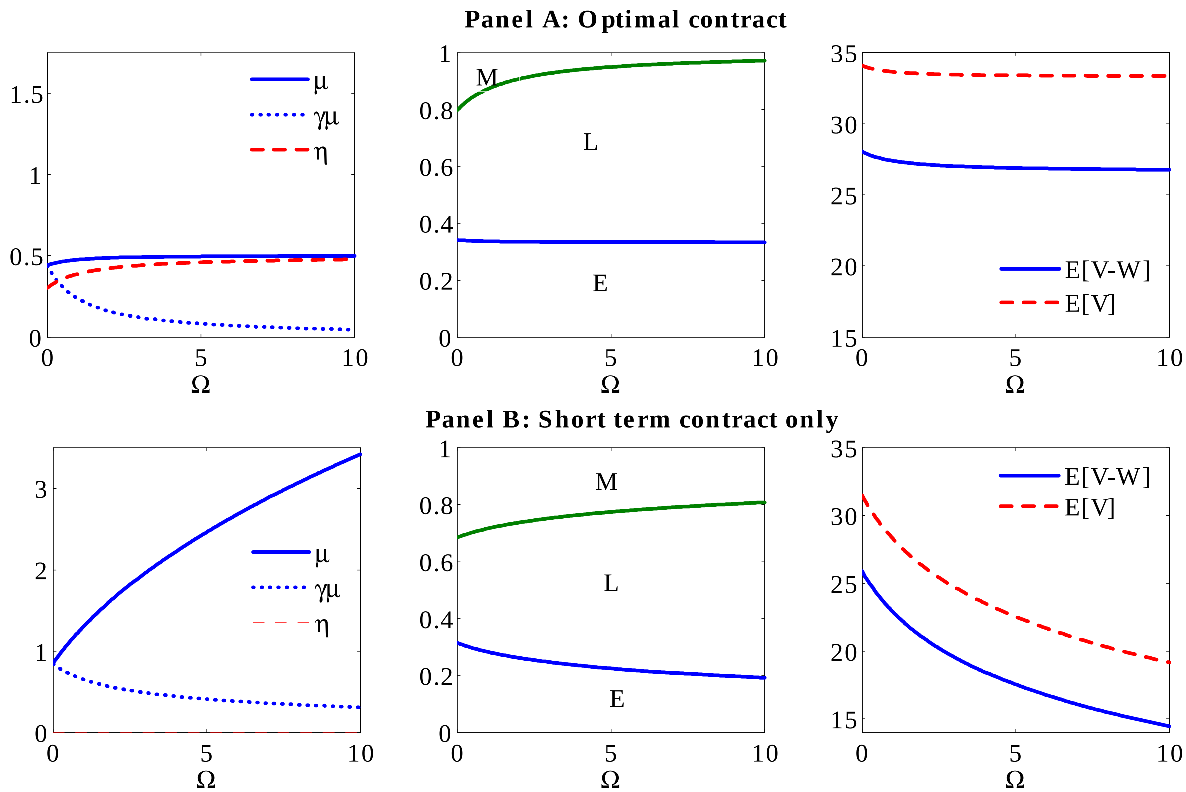

Figure 1: Optimal contract as a function of manipulation uncertainty.

This figure illustrates how the optimal contract, effort-manipulation-leisure choice, and firm value vary with the uncertainty about manipulation, Ω. μ and η are the short term and long term elasticity of pay to gross firm value, respectively. γ is the elasticity of the stock price to the manager's report. The areas labeled E, M, and L represent the proportion of time the manager spends on effort, manipulation and leisure. $E[V]$ is the ex-ante expected gross firm value and $E[V-W]$ is the ex-ante expected net firm value after substracting expected managerial compensation. The parameter values used in the simulation are: $C_E = 1$, $\overline{L} = 1$, $\overline{W} = 1$, $V_0 = 100$, $\alpha = \beta = 1$, $\Sigma_1 = 1, \Sigma_2 = 5$, $\phi = -1$, and $\Psi = 1$.

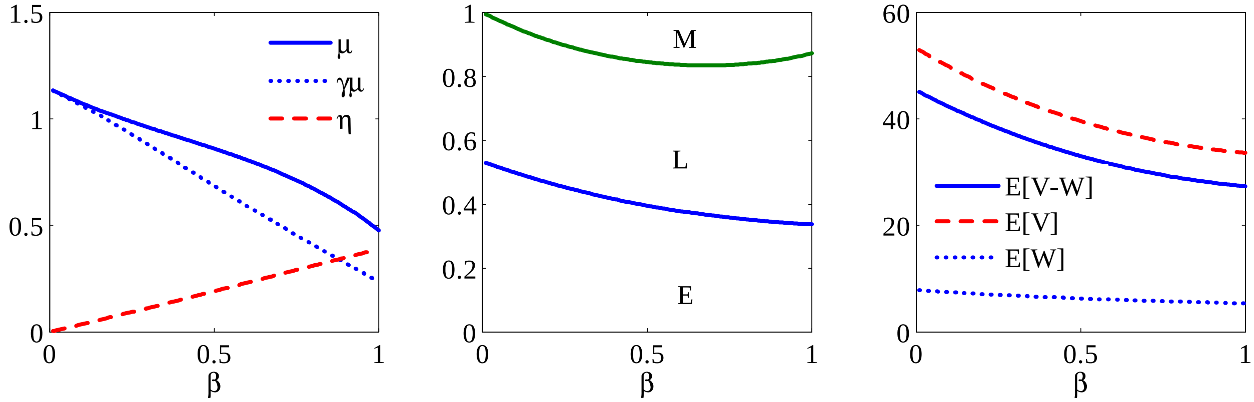

Figure 2: Optimal contract as a function of β.

This figure describes how the optimal contract, effort-manipulation-leisure choice, and firm value vary with β. μ and η are the short term and long term elasticity of pay to gross firm value, respectively. γ is the elasticity of the stock price to the manager's report. The left hand graph has two vertical axes, with the left one for η and the right one for $\gamma\mu$. The areas labeled E, M, and L represent the proportion of time the manager spends on effort, manipulation and leisure. $E[V]$ is the ex-ante expected gross firm value and $E[V-W]$ is the ex-ante expected net firm value after subtracting expected managerial compensation. The parameter values used in the simulation are: $C_E = 1$, $\overline{L} = 1$, $\overline{W} = 1$, $V_0 = 100, \alpha = 1$, $\phi = -1, \Sigma_1 = 1$, $\Sigma_2 = 5$, $\Omega = 1$, and $\Psi = 1$.

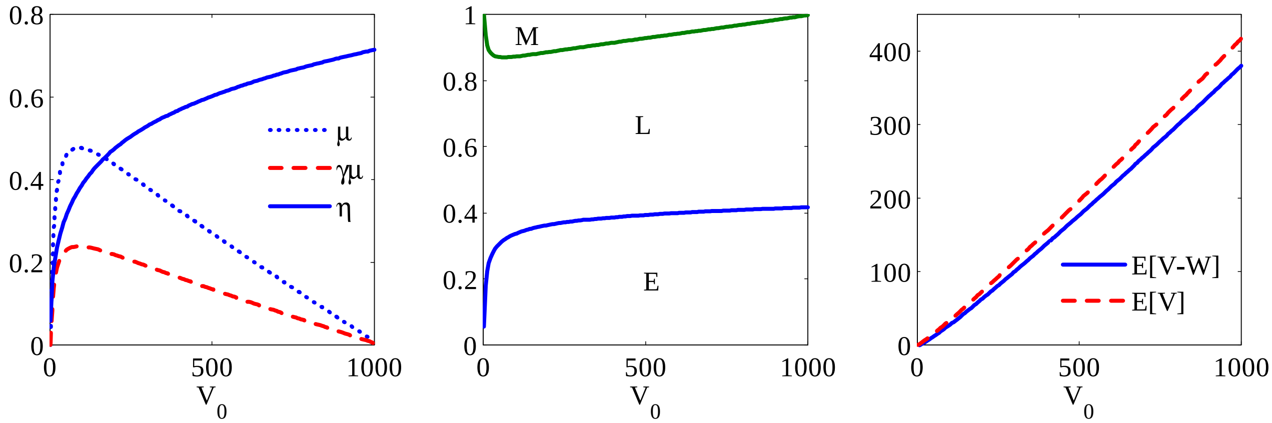

Figure 3: Optimal contract as a function of initial firm value.

This figure describes how the optimal contract, effort-manipulation-leisure choice, and firm value vary with V_0, the initial firm value. μ and η are the short term and long term elasticity of pay to gross firm value, respectively. γ is the elasticity of the stock price to the manager's report. The areas labeled E, M, and L represent the proportion of time the manager spends on effort, manipulation and leisure. $E[V]$ is the ex-ante expected gross firm value and $E[V-W]$ is the ex-ante expected net firm value after subtracting expected managerial compensation. The parameter values used in the simulation are: $C_E = 1$, $\overline{L} = 1$, $\overline{W} = 1$, $\alpha = \beta = 1$, $\Sigma_1 = 1, \Sigma_2 = 5, \Omega = 1$, $\phi = -1$, $\Psi = 1$, and thus $\gamma \equiv 0.5$ throughout.

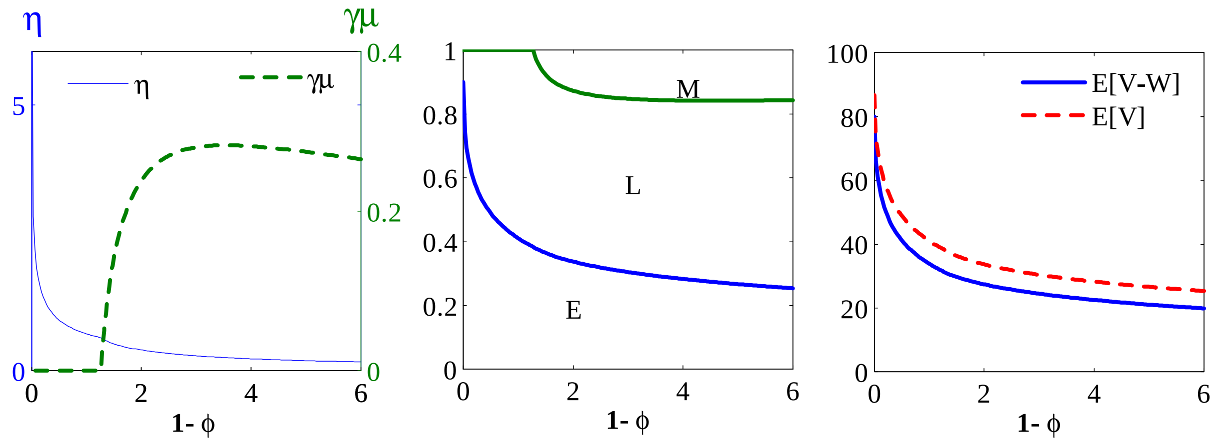

Figure 4: Optimal contract as a function of manager's relative risk aversion.

This figure describes how the optimal contract, effort-manipulation-leisure choice, and firm value vary with $1-\phi$, the relative risk aversion coefficient of the manager. μ and η are the short term and long term elasticity of pay to gross firm value, respectively. γ is the elasticity of the stock price to the manager's report. The left hand graph has two vertical axes, with the left one for η and the right one for $\gamma\mu$. The areas labeled E, M, and L represent the proportion of time the manager spends on effort, manipulation and leisure. $E[V]$ is the ex-ante expected gross firm value and $E[V-W]$ is the ex-ante expected net firm value after subtracting expected managerial compensation. The parameter values used in the simulation are: $C_E = 1$, $\overline{L} = 1$, $\overline{W} = 1$, $V_0 = 100, \alpha = \beta = 1$, $\Sigma_1 = 1, \Sigma_2 = 5$, $\Omega = 1$, $\Psi = 1$, and thus $\gamma \equiv 0.5$ throughout.

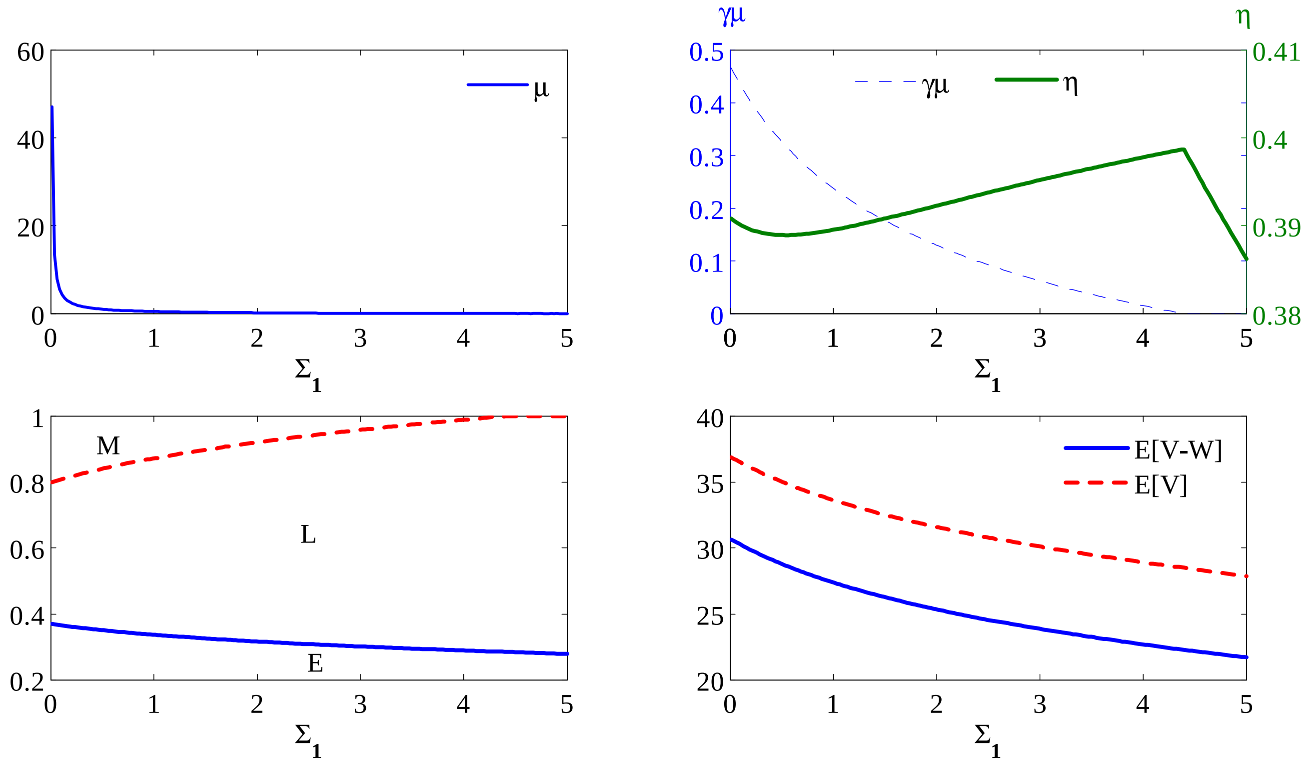

Figure 5: Optimal contract as a function of short term uncertainty about firm fundamentals.

This figure illustrates how the optimal contract, effort-manipulation-leisure choice, and firm value vary with Σ_1, the short term uncertainty about firm fundamentals. μ and η are the short term and long term elasticity of pay to gross firm value, respectively. γ is the elasticity of the stock price to the manager's report. The upper righthand graph has two vertical axes, with the left one for $\gamma\mu$ and the right one for η. The areas labeled E, M, and L represent the proportion of time the manager spends on effort, manipulation and leisure. $E[V]$ is the ex-ante expected gross firm value and $E[V-W]$ is the ex-ante expected net firm value after subtracting expected managerial compensation. The parameter values used in the simulation are: $C_E = 1$, $\overline{L} = 1$, $\overline{W} = 1$, $V_0 = 100$, $\alpha = \beta = 1$, $\Sigma_2 = 5$, $\Omega = 1$, $\phi = -1$,and $\Psi = 1$.