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AGING NATIONS AND THE FUTURE OF CITIES

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ABSTRACT

Aging Nations and the Future of Cities

We investigate whether an aging population may challenge the supremacy of large working-cities. To this end, we develop an economic geography model with two types of individuals (workers and retirees) and two sectors (local services and manufacturing). Workers produce and consume; the elderly consume only. As a result, the mobility decision of workers is driven by both the wage gap and the cost-of-living gap, unlike the elderly who react to the differences in the cost of living only. We show that the return of pre-industrial urban system dominated by rentier cities does not seem to be on the agenda. Quite the opposite, the future of large working-cities is still bright, the reason being that today's urban costs act as a strong force that prevents a large share of local services and manufacturing firms from following the rentiers in the elderly-cities, while the supply of differentiated b2c services prevent their complete separation.

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1 Introduction

Over the last decades, the weight of large cities in national economies has significantly increased. According to OECD (2006), the metropolitan areas of Brussels and Dublin account for almost half of their national GDP, while London, Paris and Tokyo account for about one third. Tokyo and New-York - the two richest cities in the world - have a higher GDP than India. Out of the 25 wealthiest cities, as measured by their GDP per capita in PPPs, 22 are set up in the United States while the others are in the European Union (London, Paris and Dublin). The ranking by labor productivity is also dominated by the same American and European cities. Recently, however, it has been argued that the supremacy of those large working-cities could well be challenged by the growth of an aging population.

The population of the main industrialized countries is indeed forecasted to age substantially. The old-age dependency ratio (the ratio people age 65 and older to people age 15 to 64) is projected to double by 2050, meaning that the European Union will move from having four persons of working age for every elderly citizen to only two (Carone and Costello, 2006). This ratio is expected to be lower in the United States, with a rise from 19% to 32%, but higher in Japan, with a rise from 25% in 2000 to 72% in 2050 (United Nations, 2003). Such a demographic change is likely to have a strong impact on the urban system for at least two reasons.

The elderly are driven by location factors that differ from those governing workers' residential choices. Indeed, workers' welfare depends on local services, land rent and wages, whereas retirees' welfare depends only upon on local services/amenities and land rent. As a consequence, a pessimistic scenario consists of a projection in which the elderly will relocate into cities endowed with natural and/or cultural amenities (Clark and Hunter, 1992). This in turn will trigger the relocation of firms toward these cities. As long as large working-cities enjoy the benefits of substantial economies of agglomeration, this will affect negatively their productivity, thus threatening the national prosperity. According to Davezies (2008), the share of the elderly is likely to reach a sufficiently high value to jeopardize existing urban systems in most developed countries. Such a scenario falls within a general debate on the future of cities.

In his "Return of the rentier city" published in the *New York Times* (October 5, 2007), Krugman argued that 21st century metropolitan areas could well resemble 17th century cities. The best description of old rentier cities is that provided by Cantillon (1755) in Chapter 5 of his *Essay on the Nature of Commerce*. His starting point lies in the idea that "the landlords who have several large estates have the means to go and live at a distance from them to enjoy agreeable society with other landowners and gentlemen of the same condition." Cantillon went on by asserting that "[f]or the service of [...] noblemen, bakers, butchers, brewers, wine merchants, manufacturers of all kinds, will be needed". Consequently, "the assemblage of several rich landowners living together in the same place suffices to form what is called a city." In such a context, the location choices made by the rentiers appear to be the main engine of urban growth. The *Wall Street Journal* (October 5, 2007) echoes Cantillon when the journalists Cassell Bryan-Low and Jeanne Whalen speculate that foreign billionaires (Middle Eastern sheiks, Russian businessmen, Indian industrialists) are behind the recent boom of London. In other words, the location of jobs would follow the location of households whose income does not come from labor. However, recent and detailed empirical evidence provided by Chen and Rosenthal (2008) suggests that the American economy is not moving along these lines, but instead toward a pattern of urban specialization involving both working- and elderly-cities: "locations with improving consumer amenities become increasingly oriented toward retirees relative to workers while locations with improving business environments experience the opposite effect". Given such contradicting viewpoints, there is a need for better theoretical foundations.

This paper aims at studying the impact of an aging population on the future of large cities. The modern approach to city formation is based on the idea that the agglomeration of activities in working-cities is triggered by a circular causation process à la Myrdal, which involves two effects that reinforce each other: (i) more workers in a city give rise to a larger demand which, therefore, leads more firms to locate there in order to better exploit scale economies; (ii) more firms in a city induce higher real wages, making this place more attractive to workers (Krugman, 1991; Fujita *et al.*,1999). The crucial point in this argument is the fact that individuals are both workers and consumers. Once we account for the presence of old people who do not work, the process of circular causation sparked by workers' location choice could well be challenged when the share of old people reaches sufficiently high values.

We show that such extreme predictions are not warranted. On the contrary, the trend uncovered by Chen and Rosenthal (2008) appears to be the most likely scenario. To reach our goal, we develop a setting that borrows ingredients from urban economics and economic geography, which allows us to study how the residential choices made by retirees may impact on the between- and within-city distribution of jobs. Specifically, our framework takes into account four main features of modern economies that are overlooked in economic geography. (i) We recognize the fact that individuals become more geographically and professionally mobile, while industries are almost free from immobile production factors. (ii) We concur with Glaeser and Kohlhase (2004) and assume that the cost of trading commodities across developed nations is negligible. (iii) In such countries, housing and commuting costs, which we call *urban costs*, account for more than one-third and sometimes as much as one-half of households' budgets (Bureau of Labor Statistics, 2003; Winqvist, 1999). Bringing (ii) and (iii) together leads us to replace the standard thought experiment of economic geography, i.e. the impact of falling trade costs for commodities on the distribution of economic activity, by the impact of an aging population on this distribution. (iv) We introduce local b2c services that are differentiated and non-tradeable (Daniels, 1993). Despite strong decreases in transport and communication costs, many b2c services are still supplied locally (e.g., health care, restaurants and movie theaters). As observed by Glaeser *et al.* (2001), the success of a city depends more than before on its role as a center of consumption, that is, on the supply of local amenities and services. It is, therefore, critical to account for the way workers distribute themselves between the manufacturing and service sectors once the elderly's residential choices are integrated into the picture, a feature that our model captures. Our setting is, therefore, very different from economic geography models à la Krugman. In particular, we consider two sectors having different characteristics (manufacturing and local services) as well as two types of consumers with different tastes (workers and retirees).

Our setting encapsulates the following major trade-off. On the one hand, consumers' residential choices depend on the location of differentiated local services, which builds a strong link between retirees and workers. On the other hand, growing urban costs caused by the agglomeration of individuals and the consumption of amenities by the elderly act as a force pushing toward the separation of the different types of consumers. That said, our main results may be summarized as follows. We first consider the benchmark case in which the elderly are equally distributed between cities and show that an aging population works against the agglomeration of manufacturing firms. This result supports the idea that an increasing share of retirees in the economy challenges the supremacy of the working-cities. However, once we allow for the elderly to choose their location, the story vastly changes. Indeed, the economy now displays two stable equilibria that behave very differently: in the former one, the mobility of the elderly does not jeopardize the existing urban hierarchy, whereas it does in the latter one. This could explain why contradicting opinions regarding the evolution of urban systems in aging nations coexist. However, we will see that the equilibrium in which the working-city remains the primate city, the other city accommodating the larger share of retirees, is the one that agrees with current empirical evidence.

In addition, our results show that the employment level in working-cities decreases because the city accommodating the larger share of retirees is a growing outlet for service firms, which makes this place more attractive to manufacturing workers. However, the gradual migration of retirees toward cities endowed with more amenities raise the level of urban costs in these cities and/or decrease their consumption of local services, thus making the working-cities more attractive to manufacturing firms. Nevertheless, this need not be true for b2c services because they benefit from large outlets in the elderly-cities. Thus, *cities tend to become more divergent in their job and* *demographic structures.* It is worth stressing, however, that during the aging process the working-cities remain the larger ones. Hence, *the return of pre-industrial urban system à la Cantillon does not seem to be on the agenda.* Quite the opposite: the future of large working-cities is still bright. This is because today's urban costs act as a strong force that prevents a large share of local services and manufacturing firms from following the rentiers in the elderly-cities, while the supply of differentiated b2c services impede their complete separation.

The remainder of the paper is organized as follows. In the next section, we present the model. In order to disentangle the different effects at work, we study in Section 3 the impact of an aging nation on the location of activities in the special, but relevant, case of a uniform distribution of old people. In Section 4, we allow for the endogenous determination of the spatial distribution of the elderly and show how the residential choices made by the elderly affect the spatial organization of the economy. In the last section, we explain why our results cast doubts on the possibility of a return of pre-industrial urban systems.

2 The model

Consider an economy formed by a continuum of individuals whose number is L, by two cities $r = H, F$, and by four goods.[1] Since the average size of cities is arbitrary, our two-city setting allows us to consider urban systems within small and large economies. The population is split between two groups of consumers, i.e. the *elderly* and the *workers* whose shares are θ and $1-\theta$, respectively. Hence, the *old-age dependency ratio* is given by $\theta/(1-\theta)$. It is both relevant and convenient to distinguish between what we call a *short-run equilibrium*, in which workers and elderly are immobile, and a *long-run equilibrium* when these agents are mobile across space.

The first good is homogenous and available as an endowment denoted $\overline{q}_0$; it can be shipped costlessly between the two cities and is chosen as the *numéraire*. The second and third goods are differentiated and made available under the form of a continuum of varieties. The second good stands for a composite of *manufactured goods* that are freely tradeable (trade costs are zero), while the third good is a bundle of *services* locally produced and non-tradeable (trade costs are prohibitive). Preferences are the same across individuals and a consumer's utility is given by:

$$U(q_0; q_i(v), v \in [0, N_i], i = 1, 2) = \sum_{i=1}^{2} \left[\alpha_i \int_0^{N_i} q_i(v) dv - \frac{(\beta_i - \gamma_i) N_i}{2(N_1 + N_2)} \int_0^{N_i} [q_i(v)]^2 dv - \frac{\gamma_i}{2(N_1 + N_2)} \left(\int_0^{N_i} q_i(v) dv \right)^2 \right] + q_0 \quad (1)$$

[1] We discuss the extension of our setting to an arbitrary number of cities in Section 4.

where $q_i(v)$ is the quantity of variety v of good $i = 1, 2$ and q_0 the quantity of the numéraire. All parameters α_i, β_i and γ_i are positive; $\gamma_i > 0$ measures the substitutability between varieties of the i-good, whereas $\beta_i - \gamma_i > 0$ expresses the intensity of love for variety.

In (1), N_1 is the total number of good 1-varieties because this good is freely tradeable, whereas N_2 must be replaced by the number of good 2-varieties supplied in the city where the consumer lives. Because consumers have a preference for variety, the second term in the right-hand side of (1) is weighted by $N_i/(N_1 + N_2)$ to capture the idea that a good supplied under a small range of varieties has less impact on the consumer well-being than a good made available as a large array of varieties.[2] This specification of individual preferences allows us to capture various substitution effects between manufactured goods and local services that make the distributions of workers and retirees interdependent as well as the size and location of the two sectors. In particular, we will see that an increase in the number of service firms in city r leads to a higher local demand for the manufactured good.

The fourth good is *land* (or housing). In order to keep matters simple, we consider the case of a one-dimensional and continuous space in which each location has a unit amount of land, while the opportunity cost of land is normalized to zero. Each city has a central business district (CBD) where manufacturing and service firms locate.[3] The two CBDs are sufficiently far apart for the two cities to be distinct. As in standard urban economics, firms do not consume land, whereas individuals consume land and commute to the local CBD where jobs and goods are supplied. For simplicity, they consume a fixed lot size normalized to one and commuting costs are linear in distance. We assume that elderly's unit commuting cost, given by $\tau > 0$, exceeds workers' unit commuting cost $t > 0$ ($\tau > t$). Admittedly, this assumption is debatable from the empirical point of view but it allows us to derive simple expressions, whereas assuming $t > \tau$ would lead to cumbersome expressions that do not affect the nature of our main results. What matters for our argument is that urban costs increase with the number of retirees.

Each variety of good i is supplied by a single firm producing under increasing returns so that N_i is also the total number of firms operating in sector i. The fixed requirement of labor needed to produce a good i-variety is denoted by $\phi_i > 0$, while the corresponding marginal requirement is set equal to zero for simplification. Consequently, the number of good 1-varieties available in the economy is proportional to the total number of individuals working in the manufacturing sector, whereas the number of good 2-varieties available in a city is proportional to the number of individuals living in this city and working in the service sector. Let s_{ir} be the *share* of workers

[2] See Tabuchi and Thisse (2006) for further details.

[3] Duranton and Puga (2004) survey the main explanations for a CBD to exist. Introducing such considerations into the model would make it much less tractable without gaining any important insight.

in sector $i = 1, 2$ and city $r = H, F$. The corresponding labor market-clearing conditions imply

$$N_1 = \frac{(1-\theta)L(s_{1H} + s_{1F})}{\phi_1} \qquad N_{2r} = \frac{s_{2r}(1-\theta)L}{\phi_2} \tag{2}$$

where N_{2r} the number of service firms located in city r, with $N_2 = N_{2H} + N_{2F}$. Note that both N_1 and N_{2r} are linear functions of L. Let $s_r = s_{1r} + s_{2r}$ be the share of workers and σ_r the share of elderly people living in city $r = H, F$, with $s_{1H} + s_{1F} = 1 - \theta$ and $\sigma_H + \sigma_F = \theta$. The *population* L_r residing in city r is then given by

$$L_r = s_r(1-\theta)L + \sigma_r\theta L \qquad r = H, F \tag{3}$$

with $L_H + L_F = L$.

At the land market equilibrium, individuals in city r are uniformly distributed over the interval $[-L_r/2, L_r/2]$. For notational simplicity, we focus on the city-RHS. Since the elderly people have the higher commuting cost, they are located in the land strip $[0, \sigma_r\theta L/2]$, while workers live in $(\sigma_r\theta L/2, \sigma_r\theta L/2 + s_r(1-\theta)L/2]$. Furthermore, because each individual consumes one unit of land, the equilibrium land rent paid by a retiree located at $0 \leq x \leq \sigma_r\theta L/2$ is given by

$$R_o^*(x) = \tau\left(\frac{\sigma_r\theta L}{2} - x\right) + t\frac{s_r(1-\theta)L}{2}$$

while a worker residing at $\sigma_r\theta L/2 < x \leq (\sigma_r\theta + s_r(1-\theta))L/2$ pays a land rent equal to

$$R^*(x) = t\left[\frac{s_r(1-\theta)L + \sigma_r\theta L}{2} - x\right]$$

with $R_o^*(\sigma_r\theta L/2) = R^*(\sigma_r\theta L/2)$. Thus, both land rents are linear functions of L and are shifted upward with the number of workers and retirees living city r.

It remains to specify the property right on land as well as the elderly' income. We close the model by assuming that *land is collectively owned by the elderly.* The income of a retiree is, therefore, given by the aggregate land rent ALR divided by the total number of elderly. It thus varies with the distribution of activities as well as with the way the retirees distribute themselves between the two cities.[4]

Since preferences and technologies are symmetric within each sector, firms sell their variety v at the same price in each city. Furthermore, as shipping good 1 is costless, manufacturing firms sell their output at the same price p_1 regardless of their customers' location. Consequently, the budget constraint of a worker residing in city r and earning wage w_{ir} in sector i is given by

$$N_1 p_1 q_{1r} + N_{2r} p_{2r} q_{2r} + t\frac{s_r(1-\theta)L + \sigma_r\theta L}{2} + q_0 = \overline{q}_0 + w_{ir}$$

[4] Alternately, land could be owned by the whole population. If individuals view the well-being of the retirees as a global public good, financing pensions by means of the aggregate land rent looks like a Henry George rule.

while the budget of an old person living in this city is

$$N_1 p_1 q_{1r} + N_{2r} p_{2r} q_{2r} + \tau \frac{\sigma_r \theta L}{2} + t \frac{s_r(1-\theta)L}{2} + q_0 = \overline{q}_0 + \frac{ALR}{\theta L}$$

where p_{2r} is the consumer price of a good 2-variety in city r and q_{ir} the individual consumption of a good i-variety by an individual living in city r. The initial endowment $\overline{q}_0$ is supposed to be sufficiently large for the equilibrium consumption of the numéraire to be positive for each individual. This assumption is made to capture the idea that consumers wish to consume all the goods available in the economy.

Set $a_i \equiv \alpha_i/\beta_i$, $b_i \equiv 1/(\beta_i - \gamma_i)$ and $c_i \equiv \gamma_i/[\beta_i(\beta_i - \gamma_i)]$ with $b_i > c_i$. It is then easy to see that the individual demand q_{ir} for a good i-variety in city r is given by

$$q_{1r} = \left(a_1 - b_1 p_1 + c_1 \frac{P_1}{N_1}\right)\left(1 + \frac{N_{2r}}{N_1}\right) \tag{4}$$

$$q_{2r} = \left(a_2 - b_2 p_{2r} + c_2 \frac{P_{2r}}{N_{2r}}\right)\left(1 + \frac{N_1}{N_{2r}}\right) \tag{5}$$

while the price indices are as follows:

$$P_1 \equiv \int_0^{N_1} p_{1r}(v)dv \qquad P_{2r} \equiv \int_0^{N_{2r}} p_{2r}(v)dv.$$

Note that P_1 is defined over the entire range of good 1-varieties because the market for good 1 is integrated, whereas P_{2r} is defined only over the range of good 2-varieties produced in city r because this good is non-tradeable. Although our demand system involves no income effect, it displays a rich pattern of substitution via the relative number of varieties. Specifically, when the number of good i-varieties available in city r increases, the individual demand for any variety of this good is shifted downward. Simultaneously, the individual demands for good j-varieties are shifted upward because good j becomes relatively more attractive. Therefore, the size and distribution of the service sector (N_{2r}) affects individual demands for the manufactured good in each city. Therefore, even though the cost of trading the manufactured good is zero, the firms producing this good benefit from the agglomeration of services. Likewise, the size of the manufacturing sector (N_1) affects the individual demand for services in each city and, therefore, the distribution of this sector. However, the distribution of manufacturing firms has no direct impact on individual demands because trading their output is costless. Therefore, one would expect manufacturing firms to be indifferent between locations. One must keep in mind, however, that manufacturing workers are also attracted by places supplying a large array of non-tradeable services. This is the linkage that may keep retirees and manufacturing workers together.

Let $Q_1(p_1) \equiv q_{1H}(p_1)L_H + q_{1F}(p_1)L_F$ be the total demand for a good 1-variety and $\Pi_{1r} \equiv p_1 Q_1(p_1) - \phi_1 w_{1r}$ the profits made by a manufacturing firm established in city r. Hence, manufacturing firms' profits depend on the distribution of the elderly as well as on the distribution and size of both sectors. Plugging (4) into Π_{1r} yields the following expression for profits:

$$\Pi_{1r} = p_1 \left(a_1 - b_1 p_1 + c_1 \frac{P_1}{N_1} \right) \frac{\sum_r N_r L_r}{N_1} - \phi_1 w_{1r}$$

where $N_r = N_1 + N_{2r}$ is the total number of varieties available in city $r = H, F$ and w_{1r} the wage a manufacturing firm pays to its workers. Each firm being negligible to the market, it treats the price index P_1 and the wage w_{1r} as parameters when choosing its own price. It is then readily verified that the equilibrium price of good 1 is equal to

$$p_1^* \equiv \frac{a_1}{2b_1 - c_1}$$

which is the same in both cities. Free entry and exit imply that profits are zero in equilibrium, meaning that the wage paid by the manufacturing firms (w_1) is the same in both cities:

$$w_1^*(s_{ir}) = \frac{(p_1^*)^2 \sum_r N_r L_r}{N_1 \phi_1} \tag{6}$$

which is a linear and increasing function of L.

Last, the profits made by a service firm set up in city r are given by

$$\Pi_{2r} = p_{2r} q_{2r}(p_{2r}) L_r - \phi_2 w_{2r}$$

where p_{2r} is the price quoted by a service firm located in r and w_{2r} the wage a service firm pays to its workers. Each service firm treats the price index P_{2r} and the wage w_{2r} as parameters so that the equilibrium price of a good 2-variety in city r is given by

$$p_2^* \equiv p_{2r}^* = \frac{a_2}{2b_2 - c_2}$$

which is the same in both cities.[5] This implies that the equilibrium wage paid by service firms established in city r is equal to

$$w_{2r}^*(s_{ir}) = \frac{(p_2^*)^2 N_r L_r}{N_{2r} \phi_2} \tag{7}$$

which is also a linear function of L.

Consequently, once workers have made their residential and occupational decisions and elderly are located, there exists a unique short-run equilibrium. Furthermore, the equilibrium prices and wages are continuous with respect to the shares (s_{ir}, σ_r). This shows that the pattern of interaction between the two types of population, the two sectors and the two cities is fairly involved.

[5] Note that our model does not capture the pro-competitive effects generated by the agglomeration of firms. Substitution effects go through the number of available varieties instead of prices.

3 Aging population and the urban system

In this section, we focus on the occupational and residential choices of workers when the distribution of elderly people is fixed. Let

$$S_i = \frac{\left[a_i - (b_i - c_i)\, p_i^*\right]^2}{b_i - c_i}$$

be the consumer surplus generated by any good i-variety at the equilibrium market price p_i^*. Note that the consumer surplus $N_1 S_1$ is the same regardless of the city in which consumers live because all good 1-varieties are available everywhere at the same price. So it does not play any role in workers' decision to migrate. This is not the case for the consumer surplus generated by good 2 because $N_{2r} S_2$ changes with the supply of varieties in city r. However, for each available variety of good 2, the surplus S_2 does not vary with the city. To highlight our main results, we make the following normalizations: $a_1 = a_2$, $b_1 = b_2$ and $c_1 = c_2$ so that $p_1^* = p_2^*$. Without loss generality, we then set $S_1 = S_2 = 1$. These assumptions do not affect qualitatively the properties of the spatial equilibria but simplifies the algebra. Note that the fixed requirements ϕ_1 and ϕ_2 are not supposed to be equal.

The welfare of an individual working in sector $i = 1, 2$ and living in city $r = H, F$ is given by her indirect utility V_{ir} evaluated at the equilibrium prices and wages: $V_{ir} = N_1 + N_{2r} + w_{ir}^*(s_{ir}) - tL_r/2$ or, equivalently,

$$V_{ir} = \frac{(s_{1H} + s_{1F})\,(1-\theta)L}{\phi_1} + \frac{s_{2r}(1-\theta)L}{\phi_2} + w_{ir}^*(s_{ir}) - t\frac{s_r(1-\theta)L + \sigma_r \theta L}{2}. \tag{8}$$

Hence, *the welfare level of an individual depends on the spatial and sectoral distribution of workers as well as on the intercity distribution of elderly*, thus showing the interdependence between job locations and the residential/occupational choices made by workers and retirees.

3.1 The long-run equilibrium: some preliminary results

A long-run equilibrium arises when no worker has an incentive to change place and/or to switch job. Formally, this means that $\overline{V}$ exists such that

$$\begin{aligned} V_{ir} &= \overline{V} \quad \text{if} \quad s_{ir}^* > 0 \\ V_{ir} &\leq \overline{V} \quad \text{if} \quad s_{ir}^* = 0. \end{aligned} \tag{9}$$

Note that Proposition 1 of Ginsburgh *et al.* (1985) implies that such an equilibrium always exists.

However, we do not know whether this equilibrium is unique, thus leading us to focus upon stable equilibria only. The stability of an equilibrium is determined with respect to the replicator dynamics, which allows us to treat the choice of a job and location in a symmetric way:

$$\dot{s_{ir}} = s_{ir}\left(V_{ir} - \sum_{i=1,2}\sum_{r=H,F} s_{ir}V_{ir}\right). \tag{10}$$

In words, workers out-migrate (resp., in-migrate) from sector i in region r when her utility V_{ir} is lower (resp., higher) than the intersectional and interregional average utility. This implies that a change in the population of industry i-workers in one city is not necessarily accompanied by an identical change in the number of firms belonging to that industry. Note also that any long-run equilibrium is a steady-state of (10), i.e., when V_{ir} is equal to the average utility for $i = 1, 2$ and $r = H, F$, whereas $s_{ir} = 0$ when working in sector i and city r yields a utility level lower than the average utility.

Consider an *interior* long-run equilibrium: $s_{ir}^* > 0$ for $i = 1, 2$ and $r = H, F$. The mobility of workers across sectors within each city ensures that all workers earn the same wage within a city ($w_{1r}^* = w_{2r}^*$). Because the manufacturing firms set the same price regardless of their customers' location, they make the same operating profits in both cities, implying that they pay the same wage w_1^* to all their workers. Consequently, factor price equalization holds at an interior long-run equilibrium. As a result, when the distribution of workers between cities is uneven, the corresponding urban cost differential (the last term in (8)) is exactly compensated by the difference in the number of non-tradeable varieties supplied in each city (the first term in (8)). In other words, *workers choose to live in a large city where they bear higher urban costs because they have access to a larger array of local services.*

Furthermore, expression (7) has a major implication: the service sector is never agglomerated in a single city r. Otherwise, the equilibrium wage in city $s \neq r$ would become arbitrarily large since $L_F \geq \sigma_s\theta L > 0$. This is due to the fact that the individual demand for services becomes arbitrarily large when $N_{2s} \to 0$. When the manufacturing sector is not agglomerated in one city, it must be that $w_1^* = w_{2r}^*$. Using (6) and (7), this equality implies that $\sum_r N_{2r}^*\phi_2 = N_1^*\phi_1$ and, therefore,

$$s_{1H}^* + s_{1F}^* = s_{2H}^* + s_{2F}^* = \frac{1}{2}. \tag{11}$$

Consequently, we have:

Lemma 1 *At any interior long-run equilibrium, the labor force is equally split between the two sectors.*

In order to reduce the number of cases to be investigated, we mainly focus on the relevant case in which the manufacturing sector is *not* fully agglomerated.

3.2 The pure effect of an aging population

In order to uncover the pure impact of an aging population on the urban structure of the economy, we consider the case in which the elderly are equally distributed between the two cities ($\sigma_r = 1/2$).[6]

Solving (9) shows that there exist *two* candidate interior long-run equilibria (up to a permutation between H and F in the configuration (13)):

$$\begin{pmatrix} s_{1H}^* & s_{1F}^* \\ s_{2H}^* & s_{2F}^* \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix} \tag{12}$$

and

$$\begin{pmatrix} s_{1H}^* & s_{1F}^* \\ s_{2H}^* & s_{2F}^* \end{pmatrix} = \begin{pmatrix} \frac{1}{4} + \frac{(2-t\phi_2)\sqrt{\Delta}}{4t\phi_1\phi_2} & \frac{1}{4} - \frac{(2-t\phi_2)\sqrt{\Delta}}{4t\phi_1\phi_2} \\ \frac{1}{4} + \frac{\sqrt{\Delta}}{4\phi_1} & \frac{1}{4} - \frac{\sqrt{\Delta}}{4\phi_1} \end{pmatrix} \tag{13}$$

where

$$\Delta \equiv \phi_1(\phi_1 + 2\phi_2) - \frac{2t\phi_1\phi_2^2}{1-\theta}. \tag{14}$$

Consequently, the total number of good i-varieties is given by $N_i = (1-\theta)L/2\phi_i$, which decreases with the share of elderly in the economy. Note also that, in both equilibria, the total population does not affect the spatial distribution of jobs.[7]

The symmetric pattern (12), which involves two cities having the same size and the same industrial structure, is a long-run equilibrium whatever the value of the commuting cost t. On the other hand, the asymmetric configuration (13) is a long-run equilibrium if and only if $\Delta > 0$ and $0 < s_{1r}^* < 1/2$ (remember that local services are never fully agglomerated). First, it is readily verified that $\Delta > 0$ if and only if $\theta < \overline{\theta}$ where

$$\overline{\theta} \equiv 1 - \frac{2\phi_2^2 t}{\phi_1 + 2\phi_2}$$

with $\overline{\theta} > 0$ if and only if $t < \phi_1/2\phi_2^2 + 1/\phi_2$. In other words, *for a large city and a small city to coexist, commuting costs must be sufficiently low.* This should not come as a surprise. Indeed, when commuting costs take on large values, urban costs in the large city are too high for all workers to stay put. It remains to determine when $0 < s_{1r}^* < 1/2$ for the asymmetric configuration to be an interior equilibrium.

It is readily verified that $s_{1r}^* < s_{2H}^* < 1/2$ holds when $t > 1/\phi_2$, so that the asymmetric configuration is always an interior equilibrium if and only if $t > 1/\phi_2$ and $\theta < \overline{\theta}$. Once $t < 1/\phi_2$,

[6] This is reminiscent of Krugman's (1991) core-periphery model in which farmers are immobile and evenly distributed between two regions.

[7] When $\theta = 0$, (13) is identical to the asymmtric equilibrium obtained by Tabuchi and Thisse (2006).

$s_{1H}^* < 1/2$ (or $s_{1F}^* > 0$) if and only if $\Delta < t^2\phi_1^2\phi_2^2/(2 - t\phi_2)^2$ or, equivalently, $\theta > \underline{\theta}$ where

$$\underline{\theta} \equiv 1 - \frac{(2 - t\phi_2)^2 \phi_2^2 t}{t^2\phi_2^3 + (1 - t\phi_2)(\phi_1 + 2\phi_2)}.$$

It is straightforward to check that $\underline{\theta} = 1$ at $t = 0$, $\underline{\theta} = 0$ at $t = 1/\phi_2$, and $\mathrm{d}\underline{\theta}/\mathrm{d}t < 0$ for $0 < t < 1/\phi_2$. Consequently, when $t < 1/\phi_2$, the asymmetric interior configuration is a long-run equilibrium if and only if $0 < \Delta < t^2\phi_1^2\phi_2^2/(2 - t\phi_2)^2$ or, equivalently, $\underline{\theta} < \theta < \overline{\theta}$. We show in appendix that this equilibrium is stable whenever it exists, while the symmetric equilibrium ceases to be stable in this interval.

Accordingly, when commuting costs are sufficiently large for $\overline{\theta} < \theta$ to hold, the economy involves full dispersion ($s_{ir}^* = 1/4$). On the other hand, when commuting costs take intermediate values, implying $\theta < \overline{\theta}$, city H, say, hosts more workers than city F. Indeed, it is readily verified that $s_H^* > 1/2 > s_F^*$. Note that city H also involves more residents since $L_H^* = \theta/2 + (1 - \theta)s_H^* > \theta/2 + (1 - \theta)s_F^* = L_F^*$.

Insert Figure 1 about here

The different domains and the corresponding equilibria are illustrated in Figure 1, while the main results are summarized in the following proposition.

Proposition 2 *Assume that the elderly are immobile and equally distributed between cities. Then,*

(i) if $\theta < \overline{\theta}$, the economy involves a large and a small city;

(ii) if $\theta \geq \overline{\theta}$, the economy involves two identical cities.

In words, when the number of retirees is sufficiently low ($\theta < \overline{\theta}$), the space-economy is asymmetric with one city attracting more than half of the labor force and population. This is because firms are able to exploit increasing returns by being partially agglomerated in the large city. Proposition 2 also enables us to shed light on the evolution of the structure of cities. In particular, it is easy to see that the economic structure of the large city varies with the level of commuting costs and the old-age dependency ratio. Indeed, as long as $t < 2/\phi_2$ holds, the large city supplies a wider array of varieties of each good than the small city, thus yielding an urban hierarchy that agrees with Christaller's (1933) central place theory. Note that this result is the outcome of a fairly involved process of interaction. Supplying a larger array of services makes the large city more attractive to *all* workers. In addition, the supply of more services shifts upward the demand for the manufactured good in the large city (see (4)). This yields higher operating profits for manufacturing firms, another force that fosters their clustering in the large city. In turn, more manufacturing firms, hence more workers, in the large city sparks a larger demand for services. Consequently, as expected, *a larger export sector boosts urban growth by attracting services.* This is only one side of the coin, however. Indeed, *a larger service sector may also lead to the expansion of the export sector* and can serve, therefore, as another engine of urban development. Note also that, when commuting costs keep decreasing, *the size of both sectors within the large city increases*, exacerbating urban asymmetry.

In contrast, when $t > 2/\phi_2$ holds (whence ϕ_1 exceeds $2\phi_2$), the larger share of manufacturing firms is set up in the small city, while the larger share of services remains in the large city. In other words, the size advantage supplied by the large city for the manufacturing sector does not compensate any more manufacturing workers for the higher urban costs they should bear there. Thus, the urban system involves two specialized cities.

Consider now our new thought experiment, i.e. *the impact of increasing the share of elderly people on the urban system and city structure.* When $\underline{\theta} < \theta < \overline{\theta}$, it is readily verified that

$$\frac{\mathrm{d}s_H^*}{\mathrm{d}\theta} = -\frac{\phi_2}{2(1-\theta)\sqrt{\Delta}} < 0.$$

Thus, when the old-age dependency ratio rises, the large city loses jobs. Furthermore, we also have:

$$\begin{aligned}
\frac{\mathrm{d}s_{1H}^*}{\mathrm{d}\theta} &= -\frac{\mathrm{d}s_{1F}^*}{\mathrm{d}\theta} = -\frac{(2-t\phi_2)\phi_2}{4(1-\theta)\sqrt{\Delta}} \\
\frac{\mathrm{d}s_{2H}^*}{\mathrm{d}\theta} &= -\frac{\mathrm{d}s_{2F}^*}{\mathrm{d}\theta} = -\frac{t\phi_2^2}{4(1-\theta)\sqrt{\Delta}} < 0.
\end{aligned}$$

In other words, *both sectors get more dispersed when the population gets older.*[8] This may be explained as follows. An aging population leads to a smaller number of workers. This reduces the demand for good 1 in higher proportion in the large city (see (4)). Hence, everything else being equal, the decrease in the supply of local services is stronger in H than in F, inducing workers to migrate toward the small city where urban costs are also lower than in the large city. Furthermore, this contraction makes local services more attractive and shifts downward the demands for the manufacturing good in city H (see (4)). This yields lower operating profits for the manufacturing firms set up in H, thus leading them to pay lower wages to their workers. This additional force fosters the migration of more workers from H to F.

Last, when the high old-age dependency ratio becomes high enough (so that $\theta \geq \overline{\theta}$), the economy involves two identical cities. Indeed, each city being populated by a large number of retirees, even the partial agglomeration of workers would make the level of urban costs very high.

Consequently, we may conclude as follows.

[8] This is obvious for the service sector. Regarding the manufacturing sector, we know that $\lambda_{1H}^* < 1/4$ when $t\phi_2 < 2$, while $\lambda_{1H}^* > 1/4$ otherwise. In both cases, the larger share of manufacturing firms decreases with θ.

Proposition 3 *Assume that the elderly are immobile and evenly distributed between cities. Then, an aging population fosters the spatial dispersion of economic activity.*

4 Retiree location and urban development

We now suppose that the elderly are free to choose the city in which they want to live, denoting by σ the *endogenous* share of elderly in city F. We also assume that city F is endowed with an amenity $\xi > 0$, which is valued only by the elderly.[9] Both workers and retirees choose simultaneously their location according to their respective inter-city utility differential. The long-run equilibrium for the elderly is such that

$$V_H^{\circ} - V_F^{o} = \xi$$

where

$$V_r^o = N_1 + N_{2r} + \frac{ALR}{\theta L} - \tau\frac{\sigma_r \theta L}{2} - t\frac{s_r(1-\theta)L}{2} \qquad r = H, F$$

is the indirect utility of a retiree living in city r. Note also that the utility differential $V_H^{\circ} - V_F^o$ is a linear function of L. An equilibrium arises at $0 < \sigma^* < 1$ when the utility differential $\Delta V^{\circ}(\sigma^*) \equiv V_H^{\circ}(\sigma^*) - V_F^{\circ}(\sigma^*) = \xi$, or at $\sigma^* = 1$ when $\Delta V^{\circ}(1) \geq 0$. An interior equilibrium is stable if and only if the slope of the indirect utility differential ΔV is strictly negative in a neighborhood of the equilibrium, i.e., $\mathrm{d}\Delta V(\sigma)/\mathrm{d}\sigma < 0$ at σ^*, whereas an agglomerated equilibrium is stable whenever it exists.

It is readily verified that the solution to $\Delta V^{\circ} = \xi$ is unique and given by

$$\sigma = \frac{1}{2} + \frac{2\xi}{\theta L \tau} + \frac{2\left(s_{2F} - s_{2H}\right)}{\theta \tau \phi_2} - \frac{t(1-\theta)(s_F - s_H)}{\theta \tau} \tag{15}$$

while $\partial \Delta V^{\circ}/\partial \sigma = -\tau\theta$ is always negative. As expected, the elderly are attracted by cities endowed with an amenity (the second right-side term in (15)) and a large number of local services (the third term in (15)). On the other hand, they are repelled by high urban costs (the fourth term in (15)).

Since the existence of amenities makes the two cities asymmetric, the two mirror equilibria (13) are replaced by two distinct equilibria. Consequently, there are now *three* candidate interior long-run equilibria (when $\xi = 0$, these candidate equilibria are identical to (12) and (13), respectively). As in Section 3, there exists a threshold $\overline{\theta}$ such that one equilibrium is stable when $\theta > \overline{\theta}$, while two equilibria are stable when $\theta < \overline{\theta}$. These two cases are discussed in the next two sub-sections. The impact of parameters θ and t on the distribution of activities is discussed in sub-section 4.3.

[9] Alternately, ξ can be interpreted as the difference in amenity endowments, i.e. $\xi \equiv \xi_F - \xi_H > 0$.

Before proceeding, the following comment is in order. We will show that the equilibrium allocation of the elderly between cities is the same regardless of the type of equilibrium and given by

$$\sigma^* = \frac{1}{2} + \frac{\xi}{\theta L(\tau - t)} \in (1/2, 1) \tag{16}$$

if

$$\theta > \theta_a \equiv \frac{2\xi}{L(\tau - t)} \tag{17}$$

while $\sigma^* = 1$ otherwise. Hence, all the elderly choose to live in city F when the old-age dependency ratio is low, the amenity level is high, or both.

Thus, more elderly choose to live in the city endowed with the amenity advantage than in the working-city, an effect that rises with the amenity level. A larger share of elderly in the global economy also increases the number $\sigma^*\theta L$ of old people living in city F but reduces their share in this city. Indeed, when workers' locations are given, an aging population leads to higher urban costs in the city hosting the majority of retirees, which triggers their relative dispersion. Simultaneously, the number of old people residing in city H increases, meaning that *the population of both cities gets older.*[10]

4.1 Low share of elderly people

When the share of the elderly people in the economy is low ($\theta < \overline{\theta}$), there exist two stable equilibria in which the primate city is either the working-city or the elderly-city.[11]

The working-city is the primate city The first equilibrium allocation of workers between sectors and cities is as follows:

$$\begin{pmatrix} s_{1H}^* & s_{1F}^* \\ s_{2H}^* & s_{2F}^* \end{pmatrix} = \begin{pmatrix} \frac{1}{4} + \frac{\theta}{1-\theta}\left(\sigma^* - \frac{1}{2}\right) + \frac{(2-t\phi_2)\sqrt{\Delta}}{4t\phi_1\phi_2} & \frac{1}{4} - \frac{\theta}{1-\theta}\left(\sigma^* - \frac{1}{2}\right) - \frac{(2-t\phi_2)\sqrt{\Delta}}{4t\phi_1\phi_2} \\ \frac{1}{4} + \frac{\sqrt{\Delta}}{4\phi_1} & \frac{1}{4} - \frac{\sqrt{\Delta}}{4\phi_1} \end{pmatrix} \tag{18}$$

where Δ is given by (14), while the term

$$\frac{\theta}{1-\theta}\left(\sigma^* - \frac{1}{2}\right) = \frac{\xi}{(1-\theta)L(\tau - t)} \tag{19}$$

shows how the elderly' location choices affect the distribution of activities.

Let θ_1 be the unique solution to $s_{1H}^* = 1/2$, so that the manufacturing sector is fully agglomerated in city H when $\theta < \theta_1$. This, together with (17), implies that manufacturing workers and retirees are completely separated for very small values of θ ($\theta < \min\{\theta_a, \theta_1\}$). This is because

[10] This result agrees with the projections made for French regions (Léon and Godefroy, 2006).

[11] Plugging the equilibria (18) and (20) into (15) yields (16). The same holds for (21) considered below.

city F has both low urban costs and few service firms. Once the old-age dependency ratio increases, both the level of urban costs and the number of service firms in city F rise, leading some manufacturing workers to move to city F. When θ exceeds θ_1, the configuration (18) becomes a stable equilibrium. In this configuration, regardless of the distribution of retirees, the manufacturing sector is dispersed between the two cities. However, city H hosts more manufacturing and service firms than city F because workers' commuting costs are lower for the reasons explained in sub-section 3.2. In other words, *manufacturing firms and retirees are not attracted by the same city*, in accordance with current demographic trends of US cities (Gabriel and Rosenthal, 2004). Since the population of city H exceeds that of city F ($L_H^* > L_F^*$), H is the working-city while F is the elderly-city. In addition, as shown by (16) and (18), amenity-improving policies in city F, more intense preferences for amenities among retirees, or both lead to a larger concentration of elderly in the elderly-city and of jobs in the working-city. This provides a simple rationale for recent empirical evidence that suggests that retirees and workers, who used to live together, do so less and less in several developed countries.

The elderly-city is the primate city The second equilibrium is as follows:

$$\begin{pmatrix} s_{1H}^* & s_{1F}^* \\ s_{2H}^* & s_{2F}^* \end{pmatrix} = \begin{pmatrix} \frac{1}{4} + \frac{\theta}{1-\theta}\left(\sigma^* - \frac{1}{2}\right) - \frac{(2-t\phi_2)\sqrt{\Delta}}{4t\phi_1\phi_2} & \frac{1}{4} - \frac{\theta}{1-\theta}\left(\sigma^* - \frac{1}{2}\right) + \frac{(2-t\phi_2)\sqrt{\Delta}}{4t\phi_1\phi_2} \\ \frac{1}{4} - \frac{\sqrt{\Delta}}{4\phi_1} & \frac{1}{4} + \frac{\sqrt{\Delta}}{4\phi_1} \end{pmatrix}. \quad (20)$$

Let θ_2 be the unique solution to $s_{1F}^* = 1/2$, so that the manufacturing sector is fully agglomerated in city F when $\theta < \theta_2$ with $\theta_2 < \theta_1$. The configuration (20) is a stable equilibrium if and only if $\theta_2 < \theta < \overline{\theta}$. Observe that (20) cannot be obtained by permuting H and F in (18), thus implying that (20) is not the mirror-image of (18) once $0 < \theta < \overline{\theta}$.

It is easy to check that city F is now the primate city ($L_F^* > L_H^*$). Because of the larger number of elderly people living in F, this city is specialized in the service sector ($s_{2F}^* > s_{2H}^*$). Manufacturing firms are not indifferent to this new distribution of elderly people because a larger array of service varieties available in city F makes this city more attractive to their workers. However, the larger share of old-age people in city F generates higher land rents so that the city specialized in manufacturing is a priori undetermined. It is readily verified that both θ_2 and s_{1F}^* reach their highest values when $\xi = 0$. Furthermore, increasing the amenity level in city F makes this city more attractive to the elderly, thus inducing the relocation of some manufacturing jobs in the working-city where urban costs are lower.[12]

It is also worth noting that the specialization of the primate city varies with ξ. First, if $\xi < \underline{\xi}$

[12]When ξ is high enough, all retirees reside in the primate city. Consequently, a change in ξ does not affect anymore the distribution of the manufacturing sector.

with

$$\underline{\xi} \equiv \frac{(1-\theta)L(\tau-t)(1-t\phi_2)\sqrt{\Delta}}{2t\phi_1\phi_2}$$

we have $s_{1F}^* > s_{2F}^* > 1/4$. In words, the elderly-city is more specialized in the manufacturing sector than in the service sector. The amenity level being low, the difference in the number of retirees living in cities H and F is not sufficiently high to generate a big gap in urban costs. Thus, *manufacturing firms and retirees are now attracted by the same city*.

Second, if $\underline{\xi} < \xi < \overline{\xi}$ with

$$\overline{\xi} \equiv \frac{(1-\theta)L(\tau-t)(2-t\phi_2)\sqrt{\Delta}}{4t\phi_1\phi_2} > \underline{\xi}$$

we have $s_{2F}^* > s_{1F}^* > 1/4$. The amenity level being higher, the primate city now hosts more elderly people (see (16)). As a result, this city is becomes more specialized in services because the market for these services is larger. Regarding manufacturing firms, its share s_{1F}^* now decreases because of the larger urban costs triggered by the larger share of elderly living in F. However, s_{1F}^* still exceeds $1/4$ because the gains resulting from a wider range of services keep offsetting the higher urban costs.

Last, if $\overline{\xi} < \xi$, we have $s_{2F}^* > 1/4 > s_{1F}^*$. Stated differently, the amenity level, hence the share of elderly people, is so high that city H now hosts the higher share of the manufacturing sector. This is because urban costs in city F are very high. On the other hand, the primate city still accommodates the larger share of services due to the larger demand that prevails in this city.

Proposition 4 *If the share of the elderly in the economy is low ($\theta < \overline{\theta}$), then there exist two stable long-run equilibria.*

(i) Under (18), the primate city is the city with the smaller share of old people and the larger share of both sectors.

(ii) Under (20), the primate city is the city with the higher share of old people and the larger share of local services.

4.2 High share of elderly people

When $\theta > \overline{\theta}$, there exists a unique stable equilibrium, which is defined as follows:

$$\begin{pmatrix} s_{1H}^* & s_{1F}^* \\ s_{2H}^* & s_{2F}^* \end{pmatrix} = \begin{pmatrix} \frac{1}{4} + \frac{\theta}{1-\theta}\left(\sigma^* - \frac{1}{2}\right) & \frac{1}{4} - \frac{\theta}{1-\theta}\left(\sigma^* - \frac{1}{2}\right) \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix} \tag{21}$$

Because city F accommodates more than half of the old population, city H hosts more manufacturing firms than city F, the gap $s_{1H}^* - s_{1F}^*$ increasing with the share of old people in the global economy. The number of service firms being the same in both cities, we may then conclude that the numbers of elderly and jobs are inversely related. Indeed, having more elderly in city F yields urban costs that are proportionally higher than in city H, thus inducing manufacturing firms to set up in this city. Furthermore, city F is specialized in services ($s^*_{2F} > s^*_{1F}$), while city H is specialized in manufacturing ($s^*_{1H} > s^*_{2H}$). Last, an aging population leads both cities to become more specialized, even though the total level of activity decreases in each city.

Note, finally, that both cities have the same population size ($L^*_H = L^*_F = 1/2$) but different industrial and age structures. The proportion of old people in city F's population is therefore higher than in city H. City F is then the elderly-city, while city H is the working-city.

In sum, we have:

Proposition 5 *If the share of the elderly in the economy is high ($\theta > \overline{\theta}$), then the elderly-city is specialized in local services while the other city is specialized in manufacturing.*

4.3 Comparative statics of city structure

In this sub-section, we study the impact of an aging population (θ) and of decreasing workers' commuting costs (t) on the spatial and occupational distribution of workers. To this end, note that

$$\frac{\mathrm{d}s^*_{1r}}{\mathrm{d}x} = \frac{\partial s^*_{1r}}{\partial \sigma}\frac{\mathrm{d}\sigma^*}{\mathrm{d}x} + \frac{\partial s^*_{1r}}{\partial x}$$

where $x = \theta$ and t. This expression tells us that the total impact of a change in x depends on the sign and magnitude of the first right-side term since the sign of the second right-side term is always unambiguous.

(i) Recall that increasing θ leads to a lower number of workers in the global economy. When $\theta > \overline{\theta}$, inspecting (21) shows that $\mathrm{d}s^*_{1H}/\mathrm{d}\theta > 0$. An aging population thus triggers a redistribution of activities that makes city H more specialized in manufacturing. Note that the spatial distribution of services is unaffected ($\mathrm{d}s^*_{2r}/\mathrm{d}\theta = 0$).

When $\theta < \overline{\theta}$, regardless of the type of equilibrium, the impact of θ on the spatial allocation of services is the same as in the case of a symmetric distribution of retirees (see sub-section 3.2). Hence, an aging population induces the dispersion of services. In contrast, an aging population has an impact on the location of manufacturing firms that varies with the type of equilibrium.

Under (18), it is easy to check that

$$\frac{\mathrm{d}s^*_{1H}}{\mathrm{d}\theta} = -\frac{\mathrm{d}s^*_{1F}}{\mathrm{d}\theta} = \frac{\xi}{(1-\theta)^2 L(\tau - t)} + \frac{\partial s^*_{1H}}{\partial \theta}$$

where $\partial s^*_{1H}/\partial\theta$ is negative and independent of ξ. Hence, an aging nation leads to the gradual dispersion of both sectors when the amenity level in city F is not high ($\mathrm{d}s^*_{1H}/\mathrm{d}\theta < 0$). When ξ

gets higher, the stronger concentration of retirees in city F makes urban costs in this city higher, thus allowing city H to accommodate an increasing share of manufacturing firms.

On the other hand, under (20), repeating the above argument shows that an aging population always decreases the share of each sector in the primate city at the benefit of the small city. In particular, the mobility of the elderly weakens the agglomeration of manufacturing workers in city F $((\partial s_{1F}^*/\partial\sigma)(\mathrm{d}\sigma^*/\mathrm{d}\theta) < 0)$. Consequently, since the growth in the number of elderly in city F is higher than in city H, urban costs in the former city rise faster than in the latter. This in turn renders the primate city less attractive.

Summarizing the above discussion, we have the following proposition.

Proposition 6 *If $\theta < \overline{\theta}$, then an aging population fosters the gradual agglomeration of workers when the amenity level in the elderly-city is sufficiently high; if $\theta > \overline{\theta}$, then an aging population fosters the gradual agglomeration of workers in the working-city.*

(ii) We now consider the impact of decreasing workers' commuting costs. When $\theta > \overline{\theta}$, inspecting (21) reveals that s_{1H}^* decreases when t falls, while s_{2H}^* is constant. Assume now that $\theta < \overline{\theta}$. Using (18) and (20), it is readily verified that lowering commuting costs always favors the gradual agglomeration of service firms in the city hosting the larger share of services. As for the manufacturing sector, when the distribution of the elderly population is fixed, we know that decreasing commuting costs favor the agglomeration of manufacturing firms, regardless of the type of equilibrium (see sub-section 3.2). However, when the elderly are free to choose where to live, the impact of commuting costs varies with the type of equilibrium that prevails.

Under (18), we have:

$$\frac{\mathrm{d}s_{1H}^*}{\mathrm{d}t} = -\frac{\mathrm{d}s_{1F}^*}{\mathrm{d}t} = \frac{\xi t}{(1-\theta)L(\tau - t)^2} + \frac{\partial s_{1H}^*}{\partial t}$$

where $\partial s_{1H}^*/\partial t$ is negative and independent of ξ. Hence, the agglomeration process in the working-city is slowed down by the arrival of retirees. More precisely, the elderly leave city F in order to enjoy a wider range of services in the working-city. As a growing share of old people reside in city H, urban costs rise in this city. Therefore, when the primate city is the city hosting the lower share of elderly, the agglomeration force triggered by decreasing commuting costs is partially offset by the new dispersion force generated by the relocation of retirees.

On the other hand, under (20), repeating the above argument shows that the mobility of the elderly increases the share of each sector in the primate city $((\partial s_{1F}^*/\partial\sigma)(\mathrm{d}\sigma^*/\mathrm{d}t) < 0)$. Indeed, since the number of retirees residing in city F decreases, urban costs in this city decline. Hence, city F attracts more workers.

To summarize,

Proposition 7 *If $\theta < \overline{\theta}$, then lowering workers' commuting costs favors the gradual agglomeration of workers in the primate city provided that the amenity level in the elderly-city is not too high; if $\theta > \overline{\theta}$, then lowering workers' commuting costs favors the dispersion of workers.*

4.4 The case of several cities

So far we have confined our analysis to the special case of two cities. It is, therefore, legitimate to ask what our results become in an urban system involving an arbitrary number of cities. When the old-age dependency ratio rises, a growing number of retirees can alleviate the burden of urban costs by getting dispersed in a larger number of amenity-cities while keeping their consumption of the manufactured good unchanged since shipping this good is costless. However, these cities would accommodate a relatively small number of local service providers. Because retirees have a love for variety in non-tradeable services, this has a negative impact on their welfare. This effect could be offset by a shift of the labor force toward the service sector. This in turn would reduce the number of varieties produced by the manufacturing sector, making this sector more attractive to workers who also have a love for variety. As a result, regardless of the number of cities in the economy, the consumption of non-tradeable differentiated services tends to link the elderly and the manufacturing workers, even though their complete separation may arise for some parameter configurations. So it seems hard to say a priori what behavior a multi-city economy will display because of the connections between the residential and occupational choices made by workers.

Yet, the analysis of the three-city case will allow us to gain some insights about the robustness of our results. Specifically, the structure of equilibria obeys, mutatis mutandis, the same principles as in the two-city case. In particular, workers are equally split between the manufacturing and service sectors, regardless of the number of cities (see (11)). In addition, individuals working in the manufacturing sector are still attracted by cities endowed with a relatively high number of local services, while they are repelled by places hosting a large share of elderly because of higher urban costs.

Consider three cities H, F and G, where the last two cities are endowed with the same amenity ξ. Following the same approach as in the two-city case, we obtain

$$\sigma^* \equiv \sigma_F^* = \sigma_G^* = \frac{1}{3} + \frac{2\xi}{3\theta L(\tau - t)} \qquad \sigma_H^* = 1 - \sigma^*.$$

When $\Delta' \equiv 3\phi_1(3\phi_1 + 8\phi_2) - 24\phi_1\phi_2^2 t/(1-\theta) > 0$, there are two other candidate-equilibria given by

$$\begin{pmatrix} \frac{1}{6} + \frac{2\theta}{1-\theta}\left(\sigma^* - \frac{1}{3}\right) + \frac{(2-t\phi_2)\sqrt{\Delta'}}{12t\phi_1\phi_2} & \frac{1}{6} - \frac{\theta}{1-\theta}\left(\sigma^* - \frac{1}{3}\right) - \frac{(2-t\phi_2)\sqrt{\Delta'}}{24t\phi_1\phi_2} & \frac{1}{6} - \frac{\theta}{1-\theta}\left(\sigma^* - \frac{1}{3}\right) - \frac{(2-t\phi_2)\sqrt{\Delta'}}{24t\phi_1\phi_2} \\ \frac{1}{4} + \frac{\sqrt{\Delta'}}{12\phi_1} & \frac{1}{8} - \frac{\sqrt{\Delta'}}{24\phi_1} & \frac{1}{8} - \frac{\sqrt{\Delta'}}{24\phi_1} \end{pmatrix}$$

and

$$\begin{pmatrix} \frac{1}{6}+\frac{2\theta}{1-\theta}\left(\sigma^*-\frac{1}{3}\right)-\frac{(2-t\phi_2)\sqrt{\Delta'}}{12t\phi_1\phi_2} & \frac{1}{6}-\frac{\theta}{1-\theta}\left(\sigma^*-\frac{1}{3}\right)+\frac{(2-t\phi_2)\sqrt{\Delta'}}{24t\phi_1\phi_2} & \frac{1}{6}-\frac{\theta}{1-\theta}\left(\sigma^*-\frac{1}{3}\right)+\frac{(2-t\phi_2)\sqrt{\Delta'}}{24t\phi_1\phi_2} \\ \frac{1}{4}-\frac{\sqrt{\Delta'}}{12\phi_1} & \frac{1}{8}+\frac{\sqrt{\Delta'}}{24\phi_1} & \frac{1}{8}+\frac{\sqrt{\Delta'}}{24\phi_1} \end{pmatrix}.$$

which are the counterparts of (18) and (20) in the two-city case. At the first equilibrium, city H has the largest share of firms and remains, therefore, the working-city. It also accommodates more than half of the total supply of local services. At the second equilibrium, city H no longer has the largest share of the manufacturing sector and has less than half of the services.

When $\Delta' < 0$, there is a unique equilibrium given by

$$\begin{pmatrix} s_{1H}^* & s_{1F}^* & s_{1G}^* \\ s_{2H}^* & s_{2F}^* & s_{2G}^* \end{pmatrix} =$$

$$\begin{pmatrix} \frac{1}{6}+\frac{2\theta}{1-\theta}\left(\sigma^*-\frac{2}{3}\right) & \frac{1}{6}+\frac{\theta}{1-\theta}\left(\sigma^*-\frac{1}{3}\right) & \frac{1}{6}+\frac{\theta}{1-\theta}\left(\sigma^*-\frac{1}{3}\right) \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}. \tag{22}$$

which may be viewed as the counterpart of (21). As in (21), local services are equally split across cities whereas city H accommodates the largest share of the manufacturing sector. The main distinctive feature is that the share of services in the working-city decreases (from 1/4 to 1/6), as argued by Davezies (2008). More generally, *the share of local services in the working-city can be shown to decrease when the number of elderly-cities rises.*

In sum, growing urban costs caused by the agglomeration of individuals act as a force pushing toward the separation of manufacturing workers and retirees. However, the existence of differentiated and non-tradeable services builds a strong link between these two groups of consumers, implying that a significant share of them will live within the same cities.

5 Back to pre-industrial urban systems?

Our results shed light on the debate as to whether the supremacy of working-cities may be challenged by an aging population. Using the case where the elderly are immobile and evenly distributed as a benchmark, we have seen that an aging nation weakens the supremacy of working-cities. In the limit, the economy is formed by two cities having the same size and economic structure. In other words, once we control for the mobility of the elderly, an aging population acts as a strong dispersion force.

When the elderly are free to choose their residence, the amenity-city always attracts the larger share of retirees. Regarding the evolution of the urban system itself, our analysis shows that making strong predictions is not easy because the multiplicity of stable long-run equilibria allows for various scenarios. For example, a strong and rapid rise in the old-age dependency may turn the elderly-city into the primate city. Yet, it seems possible to tell some plausible stories. As shown by Figure 2, larger regions tend to accommodate lower shares of old people in France, while large cities have lower old-age dependency ratios than their national economies (OECD, 2006). This fact is captured by equilibria (18) but not by (20). Furthermore, the values of commuting costs in developed economies are low from the historical standpoint (Glaeser and Kahn, 2004). Last, the old-age dependency ratio, though rising quickly in some countries, has not yet reached very high values.

Insert Figure 2 about here

All of these lead us to believe that the current situation is well approximated by the equilibrium pattern (18). This means that city H is both the primate and the working-city. As the old-age dependency ratio rises, the employment share of each sector keeps decreasing in the working-city, but remains higher than in the elderly-city. In this respect, note that, in the United States, upstate cities have lost young adults while sun-belt cities have attracted population (Chen and Rosenthal, 2008). So, if the agglomeration of manufacturing activities generates additional benefits not taken into account in our model, the global economy will incur efficiency losses. This could explain why the future growth of the old-age dependency ratio has triggered much concern in France and other countries. However, once θ exceeds the threshold $\overline{\theta}$, the economy would be described by the equilibrium pattern (21). The transition from one equilibrium to the next is continuous and, because the latter equilibrium is unique, no bifurcation arises. From now on, the working-city starts recouping a growing employment share, at least in the manufacturing sector. Alternatively, if a new amenity-city is formed, our analysis shows that the working-city retains its primacy but its inhabitants have access to a smaller number of services.

Unlike pre-industrial urban systems, working-cities can be expected to maintain their primacy while manufacturing firms and rentiers will tend to be separated but, like pre-industrial systems, the need to consume b2c services should keep the divergence between manufacturing firms and rentiers within fairly strict limits. Although we would be the last to claim that our model provides an exhaustive account of all the factors at work, it is our contention that the main forces discussed in this paper will play a major role in the evolution of urban systems in aging developed nations.

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Appendix

(i) Determination of interior equilibria Since $s_{ir}^* > 0$, we know from Lemma 1 that the following equalities

$$s_{1F} = \frac{1}{2} - s_{1H} \qquad s_{2F} = \frac{1}{2} - s_{2H}.$$

In addition, at any interior equilibrium, we have

$$\delta_i \equiv V_{iH} - V_{iF} = 0 \qquad \text{and} \qquad \delta_w \equiv w_{2H}^* - w_{2F}^* = 0.$$

Using (8), we have

$$\delta_i = (1-\theta)\left(\frac{2}{\phi_i} - t\right)\left(s_{2H} - \frac{1}{4}\right) - t(1-\theta)\left(s_{1H} - \frac{1}{4}\right) + t\theta\left(\sigma - \frac{1}{2}\right) \qquad i = 1,2$$

and

$$\delta_w = \frac{-2(1-\theta)}{t\phi_2^2}\left(s_{2H} - 1/4\right)u(s_{2H})$$

where

$$u(s_{2H}) \equiv \frac{(s_{2H} - x')(s_{2H} - x'')}{(1/2 - s_{2H})s_{2H}} \qquad \text{with} \qquad x' \equiv \frac{1}{4} - \frac{\sqrt{\Delta}}{4\phi_1} < \frac{1}{4} \quad x'' \equiv \frac{1}{4} + \frac{\sqrt{\Delta}}{4\phi_1} > \frac{1}{4}.$$

Note that $u(s_{2H}) > 0$ when $1/2 > s_{2H} > 0$ and $\theta > \overline{\theta}$, so that $s_{2H}^* = 1/4$ is the unique solution to $\delta_w = 0$ when $\theta > \overline{\theta}$. When $\underline{\theta} < \theta < \overline{\theta}$, s_{2H}^* can take three values: $1/4$, $x\prime$ and x''. Plugging each of these values in δ_1 and solving $\delta_1 = 0$ yields the corresponding values of s_{1H}^*.

(ii) Stability of interior equilibria The utility differential for workers belonging to the service sector is given by

$$\delta_2 \equiv V_{2H} - V_{2F} = N_{2H} - N_{2F} - (w_{2H}^* - w_{2F}^*) - \frac{t}{2}(L_H - L_F).$$

Regardless of $0 < s_{1r} < 1/2$, we have

$$\begin{aligned} \lim_{s_{2H}\to 0^+} \delta_2 &= +\infty \qquad \lim_{s_{2H}\to 1/2^-} \delta_2 = -\infty \\ \lim_{s_{2H}=1/4} \delta_2 &= 0 \qquad \lim_{s_{2H}=x'} \delta_2 = 0 \qquad \lim_{s_{2H}=x''} \delta_2 = 0 \end{aligned}$$

as well as

$$\left.\frac{\mathrm{d}\delta_2}{\mathrm{d}s_{2H}}\right|_{s_{2H}=1/4} < 0 \quad \text{if and only if} \quad \theta > \overline{\theta}.$$

Last, when $\theta < \overline{\theta}$, we have

$$\left.\frac{\mathrm{d}\delta_2}{\mathrm{d}s_{2H}}\right|_{s_{2H}=\{x',x''\}} < 0.$$

Accordingly, the equilibria (12) and (21) are stable when $\theta > \overline{\theta}$, while (13), (18) and (20) are the only stable equilibria when $\underline{\theta} < \theta < \overline{\theta}$.

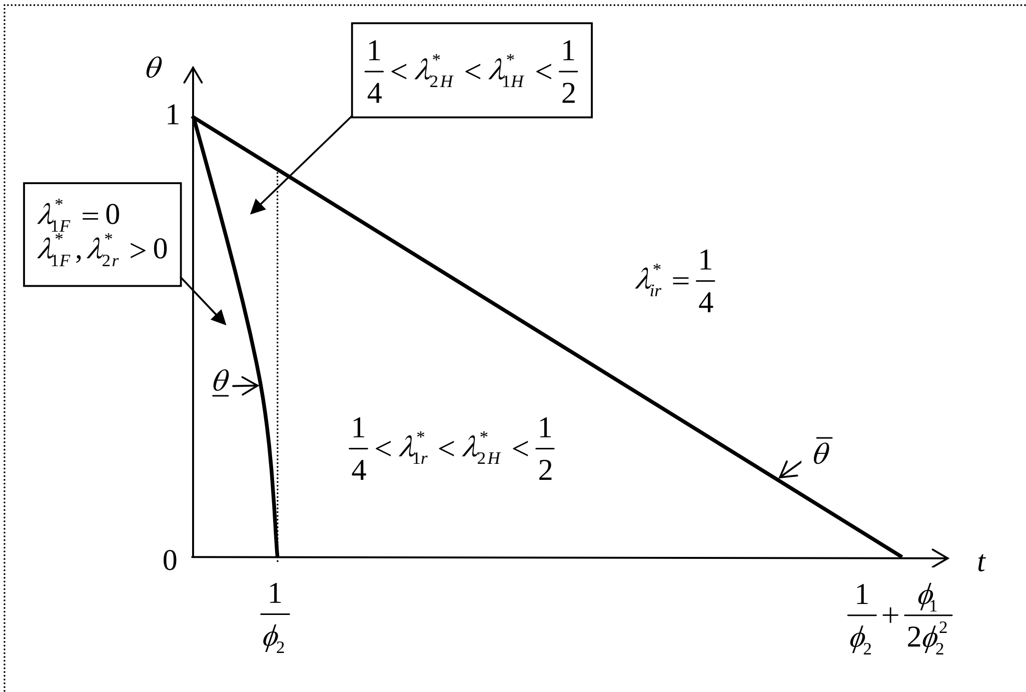

Figure 1. The set of equilibria

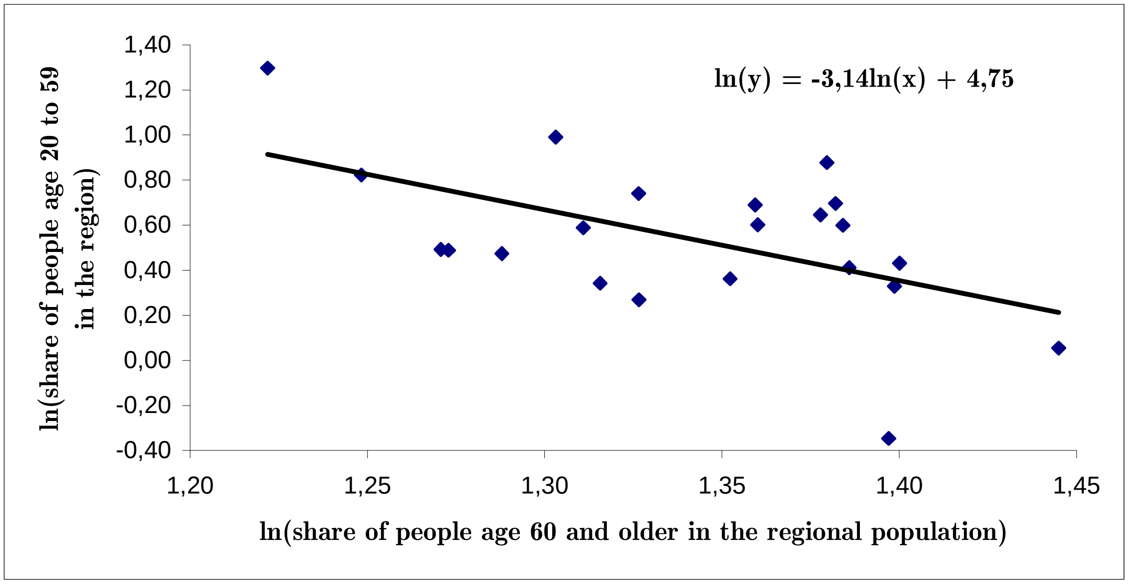

Figure 2. Spatial distribution of people age 20 to 64 and location of older people (Source: INSEE, 2004).