DISCUSSION PAPER SERIES

No. 7395

THE PRODUCT-MIX AUCTION: A NEW AUCTION DESIGN FOR DIFFERENTIATED GOODS

Paul Klemperer

FINANCIAL ECONOMICS, INDUSTRIAL ORGANIZATION and PUBLIC POLICY

Centre for Economic Policy Research

www.cepr.org

Available online at: **www.cepr.org/pubs/dps/DP7395.asp**

ISSN 0265-8003

THE PRODUCT-MIX AUCTION: A NEW AUCTION DESIGN FOR DIFFERENTIATED GOODS

Paul Klemperer, Nuffield College, University of Oxford and CEPR

Discussion Paper No. 7395
August 2009
Revised December 2009

Centre for Economic Policy Research
53–56 Gt Sutton St, London EC1V 0DG, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 7395

August 2009
Revised December 2009

ABSTRACT

The Product-Mix Auction: a New Auction Design for Differentiated Goods

I describe a new static (sealed-bid) auction for differentiated goods - the "Product-Mix Auction". Bidders bid on multiple assets simultaneously, and bidtakers choose supply functions across assets. The auction yields greater efficiency, revenue, information, and trade than running multiple separate auctions. It is also often simpler to use and understand, and less vulnerable to collusion, than a simultaneous multiple round auction. I designed it after the 2007 Northern Rock bank-run to help the Bank of England fight the credit crunch; in 2008 the U.S. Treasury planned using a related design to buy "toxic assets"; it may be used to purchase electricity.

JEL Classification: D44 and E58
Keywords: central banking, multi-object auction, Product-Mix Auction, simultaneous ascending auction, TARP, term auction and treasury auction

Paul Klemperer
Nuffield College
University of Oxford
Oxford
OX1 1NF
UK

Email:
paul.klemperer@economics.ox.ac.uk

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=103328

Submitted 29 July 2009
Revised 16 December 2009

Journal of the European Economic Association, 2010, 8, forthcoming

An earlier version of this paper was published as CEPR DP7395, “A New Auction for Substitutes: Central-Bank Liquidity Auctions, "Toxic Asset" Auctions, and Variable Product-Mix Auctions."

I have advised the Bank of England and the U.S. Treasury and have been consulted by other Central Banks, government agencies, etc., about these issues. I thank the relevant officials for help, but the views here are my own and do not represent those of any organisation. I am particularly grateful to Jeremy Bulow and Daniel Marszalec for their help in advising the Bank of England, and I also especially thank Elizabeth Baldwin, Eric Budish, Vince Crawford, Aytek Erdil, Meg Meyer, Marco Pagnozzi, Rakesh Vohra, the editor and anonymous referees, and many other friends and colleagues for helpful advice.

1. Introduction

How should goods that both seller(s) and buyers view as imperfect substitutes be sold, especially when multi-round auctions are impractical?

This was the Bank of England's problem in autumn 2007 as the credit crunch began.[1] The Bank urgently wanted to supply liquidity to banks, and was therefore willing to accept a wider-than-usual range of collateral, but it wanted a correspondingly higher interest rate against any weaker collateral it took. A similar problem was the U.S. Treasury's autumn 2008 Troubled Asset Recovery Program (TARP) plan to spend up to $700 billion buying "toxic assets" from among 25,000 closely-related but distinct sub-prime mortgage-backed securities.

Because financial markets move fast, in both cases it was highly desirable that any auction take place at a single instant. In a multi-stage auction bidders who had entered the highest bids early on might change their minds about wanting to be winners before the auction closed,[2] and the financial markets might themselves be influenced by the evolution of the auction, which magnifies the difficulties of bidding and invites manipulation.[3]

An equivalent problem is that of a firm choosing its "product mix": it can supply multiple varieties of a product (at different costs), but with a total capacity constraint, to customers with different preferences between those product varieties, and where transaction costs or other time pressures make multiple-round auctions

[1] The crisis began in early August 2007, and a bank run led to Northern Rock's collapse in mid-September. Immediately subsequently, the Bank of England ran four auctions to supply additional liquidity to banks -- but received no bids in any of them (for reasons that are not the subject of this note). The Bank then consulted me, and I got valuable assistance from Jeremy Bulow and Daniel Marszalec. Starting later in 2007, the Bank ran additional simple (more successful) auctions while consulting on the ideas discussed below.

[2] Some evidence is that most bids in standard Treasury auctions are made in the last few minutes, and a large fraction in the last few seconds. For a multi-round auction to have any merit, untopped bids cannot be withdrawn without incurring penalties.

[3] The Bank of England insisted on a single stage auction. Ausubel and Cramton (2008) argued a multi-stage auction was feasible for the U.S. Treasury.

infeasible.[4] The different varieties of a product could include different points of delivery, different warranties, or different restrictive covenants on use.

This paper outlines a solution to all these problems – the Product Mix Auction. I first developed it for the Bank of England,[5] and later made a similar proposal to the U.S. Treasury (which would probably have used a related design if it had not abandoned its plans to buy toxic assets).[6] A regulator is currently considering a proposal to use my Product Mix Auction for selling two close-substitute “types” of electricity.

My design is straightforward in concept – each bidder can make one or more bids, and *each* bid contains a *set* of mutually exclusive offers. Each offer specifies a price (or, in the Bank of England’s auction, an interest-rate) for a quantity of a specific "variety". The auctioneer looks at all the bids and then selects a price for each "variety". From each bid offered by each bidder, the auctioneer accepts (only) the offer that gives the bidder the greatest surplus at the selected prices, or no offer if all the offers would give the bidder negative surplus. All accepted offers for a variety pay the same (uniform) price for that variety.

---

[4] That is, the Bank of England can be thought of as a "firm" whose “product” is loans, which come in different “varieties” corresponding to the different collaterals they are made against, and the total supply of which may be constrained. The Bank's "customers" are its counterparties, and the "prices” they bid are interest rates.

[5] See note 1. The Bank has held consultations on the proposal, and used some simple versions, but the continuing unsettled state of the financial markets has meant repeated delays. I do *not* give full details of the Bank of England's objectives, constraints, and concerns here, and not all the issues I discuss are relevant to that Bank -- the solution I describe contains more features than needed for most Central Banking applications.

[6] After I proposed my solution to the Bank of England, I learned that Paul Milgrom was independently pursuing related ideas. He and I therefore made a joint proposal to the U.S. Treasury, together with Jeremy Bulow and Jon Levin in September-October 2008. Other consultants, too, proposed a static (sealed-bid) design, and the Treasury was planning to run a first set of sealed-bid auctions, each for a related group of assets, when it suddenly abandoned its plans to buy subprime assets (in November 2008). Note, however, Larry Ausubel and Peter Cramton (who played an important role in demonstrating the value of using auctions for TARP, see e.g., Ausubel et al., 2008) had proposed running dynamic auctions, and the possibility of doing this at a later stage was still being explored.

Milgrom (2009) shows how to represent a wide range of bidders' preferences very elegantly while at the same time restricting to substitute preferences. He also shows his highly-efficient linear-programming approach yields an integer allocation when demands and constraints are integer -- this could be important in some applications, although not in a context such as that of the Bank of England, for which my proposal seems more straightforward and transparent.

The idea is that the menu of mutually-exclusive sets of offers allows each bidder to approximate a demand function, so bidders can, in effect, decide how much of each variety to buy *after* seeing the prices chosen. Meanwhile the auctioneer can look at demand *before* choosing the prices; allowing it to choose the prices ex-post creates no problem here, because it allocates each bidder precisely what that bidder would have chosen for itself given those prices.[7] Importantly, offers for each variety provide a competitive discipline on the offers for the other varieties, because they are all being auctioned simultaneously.

Compare this with the "standard" approach of running a separate auction for each different “variety”. In this case, outcomes are erratic and inefficient, because the auctioneer has to choose how much of each variety to offer before learning bidders' preferences, while bidders have to guess how much to bid for in each auction without knowing what the price-differences between varieties will turn out to be; the wrong bidders may win, and those who do win may be inefficiently allocated across varieties.[8] Furthermore, each individual auction is much more sensitive to market power, to manipulation, and to informational asymmetries, than if all offers compete directly with each other in a single auction. The auctioneer’s revenues are correspondingly generally lower.[9] All these problems also reduce the auctions’ value as a source of information. They may also reduce participation, which can create "second-round" feedback effects furthering magnifying the problems.[10]

---

[7] That is, it chooses prices like a Walrasian auctioneer who is equating bidders' demand with the bid-taker's supply in a decentralized process (in which the privately-held information needed to determine the allocation is directly revealed by the choices of those who hold it).
The result assumes the conditions for “truthful” bidding are satisfied – see below.

[8] Milgrom (2004, p.11) gives an entertaining account of some of the inefficiencies of running separate auctions.

[9] Thus, for example, if the U.S. Treasury had simply predetermined the amount of each type of security to purchase, ignoring the information about demand for the large number of closely-related securities, competition would have been inadequate. There were perhaps 300 likely sellers, but the largest 10 held of the order of two-thirds of the total volume, and ownership of many individual securities was far more highly concentrated.

[10] The feedback effects by which low participation reduces liquidity, which further reduces participation and liquidity, etc., are much more important when there are multiple agents on both sides of the market -- see Klemperer (2008).

Another common approach is to set fixed price supplements for "superior" varieties, and then auction all units as if they are otherwise homogenous. This can sometimes work well, but such an auction cannot take any account of the auctioneer's preferences about the proportions of different varieties transacted.[11] Furthermore, the auctioneer suffers from adverse selection.[12]

The question, of course, is whether my alternative approach can actually be implemented, and -- crucially -- whether it can be done in a way that is simple and robust, and easy for bidders to understand, so that they are happy to participate. I now show this is feasible.

Section 2 provides a simple graphical illustration of my Product-Mix Auction for the two-good case. Section 3 discusses extensions. In particular, it is easy to include multiple buyers and multiple sellers, and "swappers" who may be on either, or both, sides of the market. Section 4 observes that the design is essentially a "proxy" implementation of a "two-sided" simultaneous multiple-round auction (SMRA) -- but because it is static, it is simpler and cheaper and less susceptible to collusion and other abuses of market power than is a standard dynamic SMRA. Section 5 concludes.

2. A Simple Two-Variety Example

The application this auction was originally designed for provides a simple illustration. A single seller, the Bank of England (henceforth "the Bank") auctioned just two "goods", namely a loan of funds secured against strong collateral, and a loan of funds secured against weak collateral. For simplicity I refer to the two

[11] Moreover, a Central Bank might not want to signal its view of appropriate price-differentials for different collaterals to the market in advance of the auction.
[12] If, for example, the U.S. Treasury had simply developed a "reference price" for each asset, the bidders would have sold it large quantities of the assets whose reference prices were set too high -- and mistakes would have been inevitable, since the government had so much less information than the sellers.

goods as "strong" and "weak".[13] In this context, a per-unit price is an interest-rate. The rules of the auction are as follows:

1. Each bidder can make any number of bids. *Each bid* specifies a *single* quantity and an offer of a per-unit price for *each* variety. The offers in each bid are mutually exclusive.

2. The auctioneer looks at all the bids and chooses a minimum “cut-off" price for each variety – I will describe later in this section how it uses the construction illustrated in Figures 1a, 1b, and 2 to determine these minimum prices uniquely, for any given set of bids, and given its own preferences.

3. The auctioneer accepts all offers that exceed the minimum price for the corresponding variety, *except* that it accepts at most one offer from each bid. If both price-offers in any bid exceed the minimum price for the corresponding variety, the auctioneer accepts the offer that maximizes the bidder’s surplus, as measured by the offer’s distance above the minimum price.[14]

4. All accepted offers pay the minimum price for the corresponding variety – that is, there is “uniform pricing” for each variety.[15]

Thus, for example, one bidder might make three separate bids: a bid for £375 million at {5.95% for (funds secured against) weak OR 5.7% for (funds secured against) strong}; a bid for an additional £500 million at {5.75% for weak OR 5.5% for strong}; and a bid for a further £300 million at {5.7% for weak OR 0% for strong}. Note that since offers at a price of zero are never selected, the last bid is equivalent to a traditional bid on only a single collateral.[16]

An example of the universe of all the bids submitted by all the bidders is illustrated in Figure 1a. The prices (i.e., interest rates) for weak and strong are plotted vertically and horizontally, respectively; each dot in the chart represents an “either/or” bid. The number by each dot is the quantity of the bid (in £millions).

[13] We assume (as did the Bank) that there is no adverse selection problem regarding collateral. For the case in which bidders have private information regarding the value of the collateral they offer, see Manelli and Vincent (1995).

[14] See note 20 for how the auctioneer breaks ties, and rations offers that equal the minimum price. Klemperer (2008) discusses alternative rules for choosing between offers that both exceed the minimum prices.

[15] Klemperer (2008, Appendix 2) discusses using discriminatory instead of uniform pricing.

[16] A bidder can, of course, restrict each of its bids to a single variety. Note also that a bidder who wants to guarantee winning a fixed total quantity can do so by making a bid at an arbitrarily large price for its preferred variety, and at an appropriate discount from this price for the other variety. I discuss other kinds of bids that bidders can be permitted to make in the next section.

The three bids made by the bidder described above are the enlarged dots highlighted in bold.

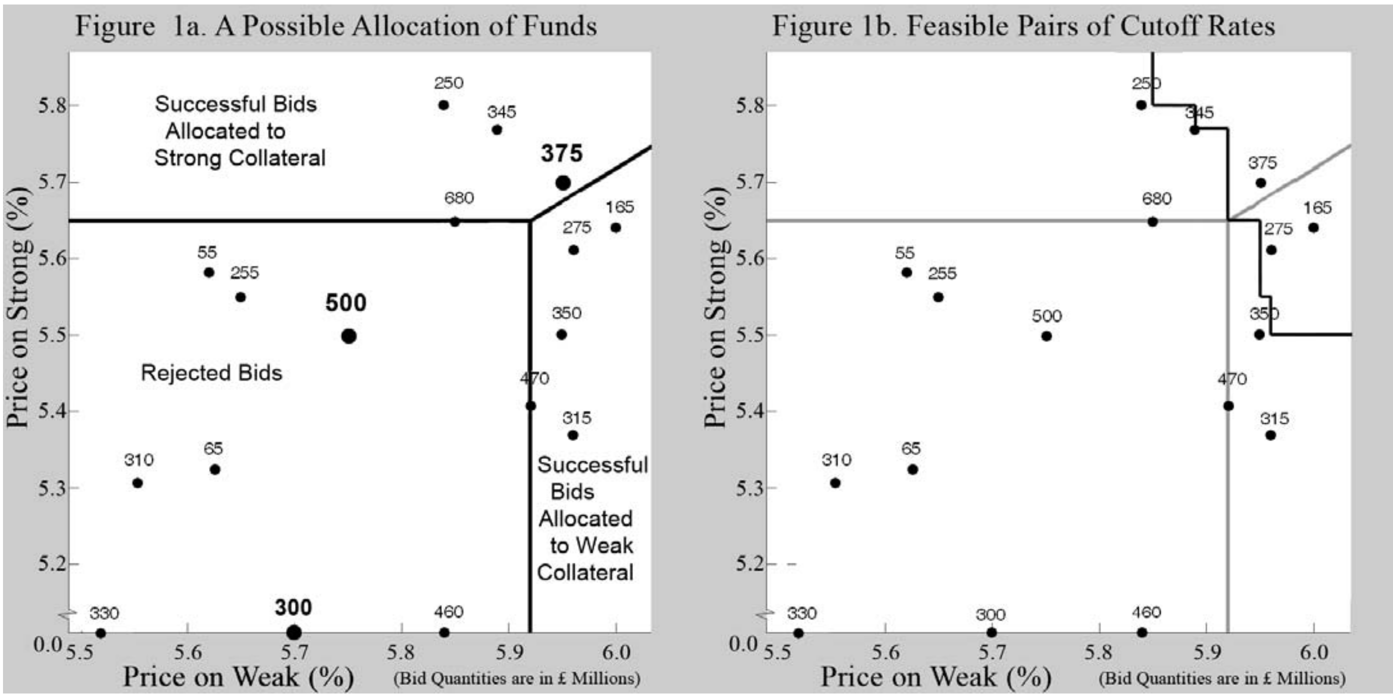

Figure 1: An Example of Bids in the Bank of England's Auction.

The cut-off prices and the winning bids are determined by the Bank's objectives. If, for example, the Bank wants to lend £2.5 billion, and there are a total of £5.5 billion in bids, then it must choose £3 billion in bids to reject.

Any possible set of rejected bids must lie in a rectangle with a vertex at the origin. Figure 1a shows one possible rectangle of rejected bids, bounded by the vertical line at 5.92% and the horizontal line at 5.65%. If the Bank were to reject this rectangle of bids, then all the accepted bids -- those outside the rectangle --

would pay the cut-off prices given by the boundaries: 5.92% for weak, and 5.65% for strong.

Bids to the north-east of the rectangle (i.e, those which could be accepted for either variety) are allocated to the variety for which the price is further below the offer. So bids that are both north of the rectangle, and north-west of the diagonal 45° line drawn up from the upper-right corner of the rectangle, receive strong, and the other accepted bids receive weak.

Of course, there are many possible rectangles that contain the correct volume of bids to reject. On any 45° line on the plane, there is generically exactly one point that is the upper-right corner of such a rectangle.[17] It is easy to see that the set of all these points forms the stepped downward–sloping line shown in Figure 1b.[18] This stepped line is therefore the set of feasible pairs of cut-off prices that accept exactly the correct volume of bids.

Every point on Figure 1b's stepped line (i.e., every possible price pair) implies both a price-difference and (by summing the accepted bids below the corresponding 45° line) a proportion of sales that are weak. As the price-difference is increased, the proportion of weak sales decreases. Using this information we can construct the downward-sloping "demand curve" in Figure 2.

[17] Moving north-east along any 45° line represents increasing all prices while maintaining a constant difference between them. Because the marginal bid(s) is usually rationed, there is usually a single critical point that rejects the correct volume of bids. But if exactly £3 billion of bids can be rejected by rejecting entire bids, there will be an interval of points between the last rejected and the first accepted bid. As a tie-breaking rule, I choose the most south-westerly of these points, though other selection rules are possible.

[18] The initial vertical segment is at the highest price for weak such that enough can be accepted on weak when none is accepted on strong. (This price is the weak price of the bid for 680.) The initial segment continues down until the highest price bid for strong is reached. (This price is the strong price of the bid for 250.) At this point some strong replaces some weak in the accepted set, and there is a horizontal segment until we reach the next price bid for weak (the weak price of the bid for 345). At this point more strong replaces weak in the accepted set, and there is another vertical segment, etc.

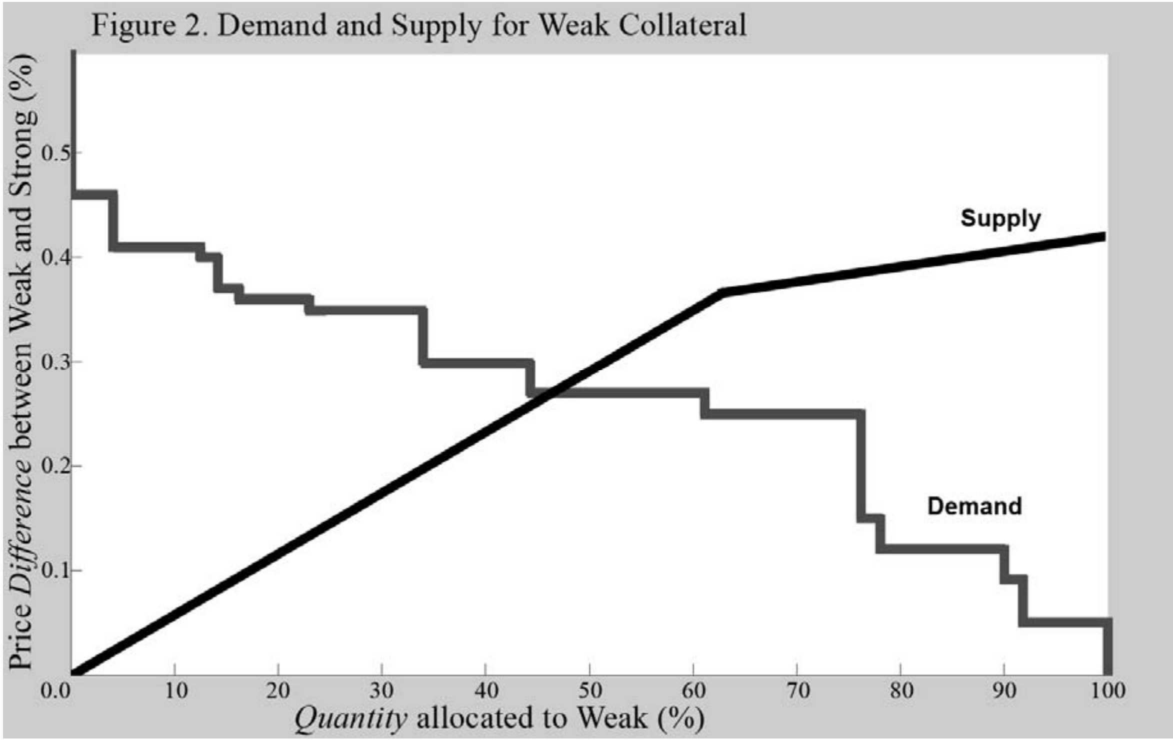

Figure 2: Equilibrium in the Bank of England's Auction.

If it wished, the auctioneer (the Bank) could give itself discretion to choose any point on the "demand curve" (equivalently, any feasible rectangle in Figures 1a, 1b) after seeing the bids.

In fact, the Bank prefers to precommit to a rule that will determine its choice. That is, the Bank chooses a "supply curve" or "supply schedule" such as the upward-sloping line in Figure 2 so the proportion allocated to weak increases with the price-difference.[19]

[19] Choosing a horizontal supply curve would fix the price differential between weak and strong. (Equivalently, the rectangle of rejected bids would be constrained to be a fixed amount wider than it was high.) Choosing a vertical supply curve would fix the proportions of the two goods.

The proposal for the U.S. TARP to spend a predetermined amount on each asset corresponds to choosing the multi-dimensional equivalent of a vertical supply curve. The proposal to employ a

The point of intersection between the Bank's supply curve and the "demand curve" constructed from the bids determines the price differential and the percentage of weak sold in the auction. With the supply curve illustrated, the price difference is 0.27% and the proportion of weak is 45% -- corresponding to the outcome shown in Figure 1a.[20]

This procedure ensures that bidders whose bids reflect their true preferences[21] receive precisely the quantities that they would have chosen for themselves if they had known the auction prices in advance. So unless a bidder thinks its own bids will affect the auction prices, its best strategy is to bid "truthfully"; if bidders all do this, and the Bank's supply curve also reflects its true preferences, the auction outcome is the competitive equilibrium.[22]

The design generalises easily:

"reference price" for each asset corresponds to choosing a horizontal supply curve. As I noted above, both these suggestions were flawed. Choosing an upward-sloping supply curve maintains the advantage of the reference-price approach, while limiting the costs of mispricing,

The optimal choice of supply-curve slope involves issues akin to those discussed in Poole (1970), Weitzman (1974), Klemperer and Meyer (1986), etc. (Maintaining the reserve power to alter the supply curve after seeing the bids protects against collusion, etc., see Klemperer and Meyer (1989), McAdams (2007), etc.)

[20] By determining the proportion of weak, Figure 2 also determines what fractions of any bids on the borders of the rectangle are filled, and the allocation between goods of any bids on the 45° line.

[21] For example, the bidder I described above is indifferent between receiving any of (i) nothing (ii) £375 million of weak at 5.95%, and (iii) £375 million of strong at 5.7%, etc.

We do not require pure "private value" preferences, but the condition that bids perfectly reflect true preferences at all prices is inconsistent with most forms of "common values" in which bidders would update their preferences in response to observing the information revealed by others' bids about others' valuations.

If common-value issues are important, we can extend our mechanism to account for these: take bids as above, and report the "interim" auction prices that would result if the auctioneer's supply were scaled up by some pre-determined multiple (e.g., 1.25). Then allow bidders to revise the prices of any bid that would win at the interim prices, except that the price on the variety that the bid would win cannot be reduced below that variety's interim price. Multiple such stages can be used, and/or more information can be reported at each stage, before final prices and allocations are determined -- we offered such an option to the U.S. Treasury, though it was not our main recommendation.

[22] Since of the order of 40 commercial banks, building societies, etc., bid in the Bank of England's auctions, it is unlikely that any one of them could much affect the prices. I assume the Bank's supply curve is upward sloping, so it is clear from the construction that, given our tie-breaking rules (see note 17), if there are multiple competitive equilibria the outcome is the unique one that is lowest in both prices.

3. Easy Extensions

3.1 Multiple buyers and multiple sellers

It is easy to include additional potential sellers (i.e., additional lenders of funds, in our example). Simply add any additional seller(s)' maximum supply to the total that the auctioneer sells, but allow them to participate in the auction as usual. If a potential seller wins nothing in the auction, the auctioneer has sold the seller's supply for it. If the potential seller wins its total supply back, there is no change in its position.

3.2 "Swappers" who might want to be on either side of the market

Exactly the same approach permits a trader to be on either, or both, sides of the market. If, for example, letting the auctioneer offer its current holdings of weak, a bidder in the auction wins the same amount of strong, it has simply swapped goods (paying the difference in the market-clearing price).

3.3 Variable total quantity

Making the total quantity sold (as well as the proportions allocated to the different varieties) depend upon the prices is easy; the auctioneer can simply repeat the procedure of Section 2 to solve for the prices corresponding to every possible total quantity, and choose its preferred outcome.

For example, the Bank might precommit to the total quantity being a particular increasing function of the price of strong. It would then solve for the strong price that would result from every possible total quantity – this will be a weakly decreasing function. The unique intersection of the two functions then yields the equilibrium total quantity, and hence all the prices and quantities.

3.4 Other easy extensions

Several other extensions are also easy. For example, bidders can be allowed to ask for different amounts of the different goods in a bid. Or a bidder can specify that a total quantity constraint applies across a group of bids. And there can, of course, be more than two goods, with a cut-off price for each, and a bid rejected only if *all* its offers were below the corresponding cut-off prices.

Bidders can express more complex preferences by using several bids in combination: for example, a bidder might be interested in £100 million weak at up to 7%, and £80 million strong at up to 5%. However, even if the prices of funds are high, the bidder wants an absolute minimum of £40 million. This can be implemented by making all of the following four bids, if negative bids are permitted:

1. £40 million of {weak at maximum permitted bid OR strong at maximum permitted bid *less* 2%}.
2. £100 million of weak at 7%.[23]
3. £80 million of strong at 5%.
4. *minus* £40 million of {weak at 7% OR strong at 5%}.

The point is that the fourth bid kicks in at exactly the same point where one of the second and third bids are accepted, and this negative bid then exactly cancels the first bid for £40 million "at any price" (since 2% = 7% - 5%).

4. Further Extensions, and the Relationship to the Simultaneous Multiple Round Auction

To see further potential extensions, observe that my auction is equivalent to a static (sealed-bid) implementation of a simplified version of a "two-sided" simultaneous multiple round auction (SMRA). (By "two-sided" I mean that sellers as well as buyers can make offers – see below.)

Begin by considering the special case in which the auctioneer has predetermined the quantity of each variety it wishes to offer, and the bids in my auction represent bidders' true preferences. Then the outcome will be exactly the

[23] Formally, this bid includes "OR of strong at 0%", and the next bid includes "OR of weak at 0%".

same as the limit as bid increments tend to zero of a standard SMRA if each bidder bids at every step to maximize its profits at the current prices given those preferences,[24] since both mechanisms simply select the competitive-equilibrium price vector.[25]

It is not much harder to understand the general case in which the auctioneer offers a more general supply curve relating the proportions of the different varieties sold to the price differences. We now think of the auctioneer as acting *both* as the bid-taker selling the maximum possible quantity of both varieties, *and* as an additional buyer bidding to buy units back to achieve a point on its supply curve. That is, in our example in which the Bank auctions £2.5 billion, we consider an SMRA which supplies £2.5 billion weak *and* £2.5 billion strong, and we think of the Bank as an additional bidder who has an inelastic total demand for £2.5 billion and who bids in exactly the same way as any other bidder.[26]

More generally, if there are other sellers (or "swappers") we consider an SMRA in which their potential sales (or "swaps") are added to those offered in the auction, and think of these participants as bidding for positive amounts like any other bidders.

---

[24] In a SMRA the bidders take turns to make bids in many ascending auctions that are run simultaneously (e.g, 55% of 2.5 billion = 1.375 billion auctions for a single £1 of strong, and 45% of 2.5 billion = 1.125 billion auctions for a single £1 of weak). When it is a bidder's turn, it can make any new bids it wishes that beats any existing winning bid by at least the bidding increment. (It cannot top up or withdraw any of its own existing bids, but it would anyway not want to do so in our case.) This continues until no one wants to submit any new bids. (See, e.g., Milgrom, 2000, Binmore and Klemperer, 2000, and Klemperer, 2004, for more details, and applications, of the SMRA.)

Bidding as described is rational behaviour if the number of bidders is large *and* they do not update their preferences in response to observing others' bidding -- that is, they have "private values".

[25] All the types of bids discussed in the previous sections reflect preferences such that goods are mutual substitutes in demand for all bidders, and I assume the auctioneer also has such preferences (i.e., the Bank's supply curve is upward sloping), so if there are multiple competitive equilibria, there is a unique one in which all prices are lowest and both mechanisms select it (see note 22 and Crawford and Knoer 1981, Kelso and Crawford 1982, Gul and Stacchetti 1999, and Milgrom 2000).

[26] That is, whenever it is the Bank's turn to bid in the SMRA, it makes the minimum bids to both restore its quantity of winning bids to £2.5 billion and win the quantity of each variety that puts it back on its supply curve, given the current price-difference. It can always do this (to within one bid increment) since the weak-minus-strong price difference can only be more (less) than when it previously bid if its weak (strong) bids have all been topped, so it can increase the quantity of strong (weak) it repurchases relative to its previous bids, as it will wish to do in this case.

So my procedure is equivalent to one in which bidders submit their preferences, and the auctioneer and other (potential) sellers submit their supply curves, and a computer then calculates the equilibrium of a SMRA. (Note that though the way we described the auctioneer's supply function may have obscured this, our procedure is symmetric between buyers and sellers.[27])

The only difference between my procedure and a (two-sided) SMRA with "proxies" (bidding rules), is that I have limited the preferences that bidders' proxies can express.

In principle I could allow bidders to specify preferences corresponding to *any* proxy bidding rule, subject to computational issues. These issues are not very challenging in our simple example (or in the Bank of England's actual problem). However, some constraints on the permissible forms of bids are desirable. For example, I am cautious about allowing bids that express preferences under which varieties are complements.[28]

Importantly, exercising market power is much harder than in a standard SMRA, precisely because my procedure does not allow bidders to express preferences that depend on others' bids. In particular, coordinated demand reduction (whether or not supported by explicit collusion) and predatory behaviour may be almost impossible. In a standard dynamic SMRA, by contrast, bidders can learn from the bidding when such strategies are likely to be profitable, and how they can be implemented – in an SMRA, bidders can make bids that signal threats

[27] It is not quite symmetric if the auctioneer doesn't precommit to its supply curve, but if bidders behave competitively their bids are unaffected by this. Klemperer (2008) discusses implications of an auctioneer (such as the Bank of England) being a single player and having non-profit objectives.

[28] See Klemperer (2008, Appendix 3) for extensions to the permitted forms of bidding discussed above. The difficulty with complements is the standard one that without "substitutes preferences" there might be multiple unrankable competitive equilibria, or competitive equilibrium might not exist (see note 25 and the references cited there), and proxy bidding in an SMRA can lead to different outcomes depending upon the order in which bidders take turns to bid. (If an increase in the price of X means a bidder wants less Y, it might never have made an earlier bid for Y if the order of play had resulted in prices evolving differently.) In independent work, Milgrom (2009) explores how to restrict bidders to expressing "substitutes preferences".

and offers to other bidders, and can easily punish those who fail to cooperate with them.[29]

Finally, the parallel with standard sealed-bid auctions makes my mechanism more familiar and natural than the SMRA to counterparties. In contexts like that of the Bank of England, my procedure is much simpler to understand.

5. Conclusion

I have described a simple-to-use, sealed-bid, auction that allows bidders to bid on multiple differentiated assets simultaneously, and bid-takers to choose supply functions across assets. It can be used in environments in which a simultaneous multiple-round auction (SMRA) is infeasible because of transaction costs, or the time required to run it. The design also seems more familiar and natural than the SMRA to bidders in many potential applications, and makes it harder for bidders to collude or exercise market power in other ways.
Relative to running separate auctions for separate goods, it yields better "matching" between suppliers and demanders, reduced market power, greater volume and liquidity, and therefore also improved efficiency, revenue, and quality of information. Its potential applications therefore extend well beyond the financial contexts for which I developed it.

---

[29] In a standard SMRA, a bidder can follow "collusive" strategies such as "I will bid for (only) half the lots if my competitor does also, but I will bid for more lots if my competitor does", see, e.g., Klemperer (2002, 2004), but in our procedure the bidder has no way to respond to others' bids.

Of course, a bidder who represents a significant fraction of total demand will bid less than its true demand in *any* procedure that charges it constant per-unit prices. (In our procedure, it will submit multiple bids, with all but its first bid at prices below its true values, to allow for the effect of demanding incremental units on the price it pays for its inframarginal units.) But it is much easier for a bidder to (ab)use its market power in this way in an SRMA in which it can learn from the bidding when such a strategy is likely to be profitable.

A multi-round procedure (either an SMRA, or an extension of our procedure – see note 21) may perhaps be desirable if there are important "common-value" components to bidders' valuations, although it may discourage entry of bidders who feel less able than their rivals to use the information learned between rounds.

References

Ausubel, Lawrence, and Peter Cramton (2008). "A Troubled Asset Reverse Auction." mimeo, University of Maryland.

Ausubel, Lawrence, Peter Cramton, Emel Filiz-Ozbay, Nathaniel Higgins, Erkut Ozbay, and Andrew Stocking (2008). "Common-Value Auctions with Liquidity Needs: An Experimental Test of a Troubled Assets Reverse Auction." mimeo, University of Maryland.

Binmore, Ken and Paul Klemperer (2002). "The Biggest Auction Ever: the Sale of the British 3G Telecom Licenses." *Economic Journal*, 112, C74-C96.

Crawford, Vincent P., and Elsie Marie Knoer (1981). "Job Matching with Heterogeneous Firms and Workers." *Econometrica*, 49, 437-450.

Gul, Faruk, and Ennio Stacchetti (1999). "Walrasian Equilibrium with Gross Substitutes." *Journal of Economic Theory*, 87, 95-124.

Kelso, Alexander S. Jr., and Vincent P. Crawford (1982). "Job Matching, Coalition Formation, and Gross Substitutes." *Econometrica*, 50, 1483-1504.

Klemperer, Paul (2002). "What Really Matters in Auction Design." *Journal of Economic Perspectives*, 16, 169-189.

Klemperer, Paul (2004). *Auctions: Theory and Practice*, Princeton University Press, Princeton, US.

Klemperer, Paul (2008). "A New Auction for Substitutes: Central Bank Liquidity Auctions, the U.S. TARP, and Variable Product-Mix Auctions." mimeo, Oxford University.

Klemperer, Paul and Margaret Meyer (1986). "Price Competition vs. Quantity Competition: The Role of Uncertainty."*Rand Journal of Economics*, 17, 618-638.

Klemperer, Paul and Margaret Meyer (1989). "Supply Function Equilibria in Oligopoly under Uncertainty," *Econometrica*, 57, 1243-1277.

McAdams, David (2007). "Uniform-Price Auctions with Adjustable Supply." *Economics Letters*, 95, 48-53.

Manelli, Alejandro M. and Daniel Vincent (1995). "Optimal Procurement Mechanisms." *Econometrica*, 63, 591-620.

Milgrom, Paul (2000). “Putting Auction Theory to Work: The Simultaneous Ascending Auction.” *Journal of Political Economy*, 108, 245-272.

Milgrom, Paul (2004). *Putting Auction Theory to Work*. Cambridge, UK: Cambridge University Press.

Milgrom, Paul (2009). “Assignment Messages and Exchanges.” *American Economic Journal: Microeconomics*,1, 95-113.

Poole, William (1970). “Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model.” *Quarterly Journal of Economics*, 84, 197-216.

Weitzman, Martin (1974). "Prices vs. Quantities." *Review of Economic Studies*, 41, 477-491.