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THE EVOLUTION OF PAPER MONEY

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ABSTRACT

The Evolution of Paper Money

This paper tells the story of how paper money evolved as a result of lending by banks. While lending commodity money requires holding large reserves of commodity money to ensure liquidity, issuing convertible paper money reduces these costs significantly. The paper also examines the possibility of issuing inconvertible notes and shows that while they further reduce the cost of borrowing they also have adverse effects on the stability of the banking system. As a result, governments often intervened, either outlawing the issuance of such notes, or monopolizing them for themselves by issuing fiat money. The paper examines the process of creation of paper money, but also sheds light on more general issues, like the relation between money and financial intermediation.

JEL Classification: E4, E5, N1 and N2
Keywords: banks, convertibility, fiat money, financial intermediation, liquidity and paper money

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The Evolution of Paper Money

1. Introduction

This paper tells a story on how paper money was created. This story is told by a model, or rather by a series of models, which describe how banks began to operate, and how they began to issue paper money to facilitate financial operations, first convertible notes and then non-convertible notes. We then try to explain why bank-issued inconvertible paper money was not a stable equilibrium and why it was monopolized by governments, who issued fiat money. This is not a historical paper, but it is based on the historical observation that paper money was issued first by private banks and only later by governments. This paper tries to explain this past development, but we hope it can be relevant for the present as well, since it analyzes how financial institutions and liquidity are related and more broadly how financial intermediation and money are related.

We begin with a simple OLG model where some individuals have early income in first period of life and others have late income in second period of life. Since all derive utility from consumption in both periods of life, the early lend to the late to enable them to consume when young. But income of the late in second period of life is uncertain, so they might default. It is further assumed that each early agent can lend to a single borrower, so that risk cannot be diversified by individuals. Hence, private loans are risky. In addition to the consumption good, which is perishable, there is sliver in the economy, which is durable. Hence, in addition to lending to late agents, which pays a positive but risky return, the early young also hold silver, which has zero but safe return.

The paper then introduces a new type of agents, individuals who have accumulated large amounts of silver. They can lend to many borrowers and thus diversify the risk. These individuals become money lenders, or bankers. We assume that bankers live forever and that they are risk neutral, in order to simplify the analysis. We show that these bankers lend the silver they own to the late young, and thus increase the supply of loans, reducing the interest rate. Bankers can reduce the interest rate because their investment is not risky, as they can fully diversify the risk. But at this stage of the analysis we also introduce the need for liquidity in the model, as we assume that trading is done only in a specific time in the middle of the period, which is called the 'trading session.' As a result bankers must lend silver before the trading session and thus have to hold it from the previous period, which is costly.

In the next stage of the analysis we show that bankers can reduce the cost of holding silver reserves by issuing paper money. They first issue convertible notes. In this case they still have to hold a certain amount of reserves, since we assume that some of the transactions must be performed by silver and not by paper money. Since bankers hold a smaller amount of reserves, they have lower costs of lending. This explains why convertible notes are adopted by all bankers, once they appear and demonstrate how they reduce costs.

Once bankers realize that they can reduce costs by holding fewer reserves, they look for a way to reduce these costs further on. This can be achieved by issuing non-convertible notes. In the next stage of the analysis we show that indeed inconvertible notes reduce costs even more and thus lower the interest rate. Furthermore, issuing non-convertible notes does not lead to inflation, if there is enough silver in the economy, since a bank that inflates the money it prints is punished as the public does not hold this bank's money and holds silver instead. Hence, banks cannot earn from inflation when silver is competing against their money. But if the quantity of silver is limited, people will hold paper money even if there is inflation and as a result banks might be tempted to inflate their own notes. We show that this is indeed the case only if the banking sector is not fully competitive and there is a leading bank. This bank will issue greater and greater amounts of money and will lead to inflation. This finding explains why governments monopolize the issuance of paper money to themselves. They do it both in order to increase stability of the monetary system, and also because they want to keep the inflation tax revenues to themselves. Thus our models show the incentives to create paper money by banks, first convertible and then non-convertible, and then they show the possible instability involved in such banking, especially if specie is in shortage and if there is a leading bank. Then fiat money might be issued by the government.

There are several studies who deal with the evolution of money from commodity money to paper money or to fiat money.[1] Most of these studies focus on the transition to fiat money, which is issued by the government. These studies usually claim that fiat money is superior to commodity money but the transition cannot take place merely by market forces. Such claims appear in Kiyotaki and Wright (1989), Dowd (2001) and Selgin (2003). Ritter (1995), Kim (2001) and Araujo and Camargo (2006) argue that the circulation of fiat money requires a credible commitment by the government to refrain from over-issuing and from overusing inflation tax. Selgin (2003) disagrees and presents as evidence many episodes of hyper-inflation of government issued fiat money.

Interestingly, there is little attention in the literature to the role of banks in the evolution of paper money, which is the main subject of this paper. Selgin (2003) argues that bank notes are superior to commodity money, because they enable trade with two groups of agents: those who accept notes and those who accept the backing commodity. He, Huang and Wright (2005) present a model where risk of theft leads customers to deposit their money at safe banks and use a claim on that money as a medium of

[1] There are many papers which study the evolution from barter to commodity money. See Jones (1976), Sargent and Wallace (1983), Oh (1989), Kiyotaki and Wright (1989), Marimon et al. (1990), Wright (1995), Iwai (1996, 2001), and Luo (1999).

exchange. But this model applies to bank liabilities like checking accounts or traveler checks and not to bank notes that can be stolen as well.

Sargent and Velde (2002) provide a different view on the emergence of paper money by claiming that it followed the emergence of token money. Tokens were coins made of cheap metal which circulated with market value significantly higher then their intrinsic metallic value. Their main use was as small denomination money, “small change,” as precious metals were not suitable for small change. Through a process of trial and error, governments found that they can satisfy the demand for small change by issuing tokens and committing to convert them into real money. The success of tokens paved the way to pure fiat money. According to this approach, bank notes are simply another form of convertible token money, regardless of their role in banking activity.

Our paper claims that banking activity is central in understanding the development of new forms of money. We show that banks have incentives to issue convertible money as it reduces their cost of reserves, and we show that they may go further and issue inconvertible money under some conditions. Indeed historical evidence shows that banks were the first to issue convertible notes, and in few episodes of free banking they also began to reduce the convertibility of notes. Hence, our model considers banking activity as a major driver in the evolution of money.

The paper is organized as follows. Section 2 presents the basic economy with private lending and borrowing without banks. Section 3 adds banks, who lend silver only. Section 4 introduces convertible notes and section 5 shows banks’ incentive to issue inconvertible notes. Section 6 shows that if there is shortage of silver and a leading bank, price instability might emerge and the government might step in. Section 7 presents a few historical episodes that support our analysis. Section 8 concludes.

2. The Basic Economy: Commodity Money Only

In this section we introduce the basic setup of the model and derive the demand for money and the monetary equilibrium in an economy without banks. Consider an Over-Lapping Generations economy of two-period lived agents[2], with fixed population. There are two goods in the economy. One is a physical non-durable good, which is used for consumption. It therefore has to be consumed in the same period of production. The second good is durable and is called silver. Silver is used as money as it can circulate among agents and over time without depreciating. We assume for simplicity that one unit of silver and one unit of the consumption good are interchangeable in production and thus the relative price of the two goods is 1. Hence, silver serves in our model as commodity money.

Production is by labor only, and the model abstracts from physical capital. Agents come in two types. Half in each generation produce an amount y of the consumption good when young and zero of it when old. These agents are called "Early" and are denoted E. The other half are called "Late" and are denoted L. They do not produce when young but only when old. Their productivity when old is random. Each old late produces y with probability q, but with probability $1 - q$ she produces a much smaller amount, dy, with $d << 1$. The size of each of the groups in each generation is normalized to 1, so that the total size of each generation is 2 and the size of the total population is 4.

All agents derive utility from consumption in the two periods of life, and the utility function is assumed to be logarithmic:

[2] The overlapping-generation framework has been used extensively in monetary models since Samuelson (1958). Brock and Scheinkman (1980) suggest viewing each period as a short period (say, a week). So, one week A is a seller and B is a buyer, and the other week A is the buyer and B the seller. This interpretation differs than OLG models of consumption and savings where the periods represent working and retirement years.

(1) $$U_t^X = \log\left(c_{t,1}^X\right) + \beta E_t \log\left(c_{t,2}^X\right),$$

where $c_{t,1}$ is consumption of an agent from generation t when young and $c_{t,2}$ is consumption when old. The index X relates to type of agent, E or L.

Since agents face large variation of income over their lifetime, they want to smooth consumption by trading with one another. We assume that the ability to contract is imperfect. Note that in this economy the early agents (E) are the lenders and the late (L) are the borrowers. If L earns y she returns the debt in full, but if she earns dy, she is declared bankrupt and defaults on the debt. Hence, lending to L is risky. The credit market imperfection we impose is that lenders E cannot diversify lending and each one lends to one agent only. Hence, private lenders are exposed to default risk. Since silver is durable and its value is not risky, it serves as an additional asset that transfers income over time. It pays no interest but its future value is perfectly safe, compared to lending.

We next examine the optimal decisions of the various agents when young. An early young earns y in period t, consumes $c_{t,1}^E$ and saves by lending l_t and by holding an amount s_t of silver. In the next period of life he receives the fruits of his savings and consumes. Lending is risky as he receives $l_t(1+r_t)$ in period t+1 if the borrower repays, with probability q, and 0 if the borrower defaults. The real interest rate r_t is determined in period t. Silver is not risky but it does not earn interest. The early young agent maximizes the following lifetime expected utility:

(2) $$U_t^E = \log\left(y - l_t - s_t\right) + q\beta \log\left[s_t + l_t\left(1+r_t\right)\right] + \left(1-q\right)\beta \log\left(s_t\right).$$

Since the maximum is not a corner solution, we get two first order conditions. The first one is with respect to lending:

$$\frac{-1}{y - l_t - s_t} + q\beta \frac{1+r_t}{s_t + l_t\left(1+r_t\right)} = 0.$$

The second first order condition is with respect to silver holdings:

$$\frac{-1}{y-l_t-s_t}+q\beta\frac{1}{s_t+l_t(1+r_t)}+(1-q)\beta\frac{1}{s_t}=0.$$

From the first order conditions we derive the following supply of lending:

(3) $$l_t=\frac{\beta y}{1+\beta}\cdot\frac{q(1+r_t)-1}{(1+r_t)-1},$$

and the demand for silver:

(4) $$s_t=\frac{\beta y}{1+\beta}\cdot\frac{(1+r_t)(1-q)}{(1+r_t)-1}.$$

As expected, the supply of lending is positively related to the interest rate, while the demand for silver is negatively related to this rate. We next turn to describe the demand for borrowing by the late young.

A late young consumes in the first period of life what she borrows and in the second period of life consumes *y* minus the repaid debt if she succeeds or *dy* if not. Hence, a late young maximizes the following expected lifetime utility:

(5) $$U_t^L=\log(b_t)+q\beta\log[y-b_t(1+r_t)]+(1-q)\beta\log(dy),$$

where b_t denotes borrowing by late young. The FOC is:

$$\frac{1}{b_t}-q\beta\frac{1+r_t}{y-b_t(1+r_t)}=0.$$

Hence, borrowing by late young is:

(6) $$b_t=\frac{y}{(1+\beta q)(1+r_t)}.$$

The equilibrium condition in the market for private funds is:

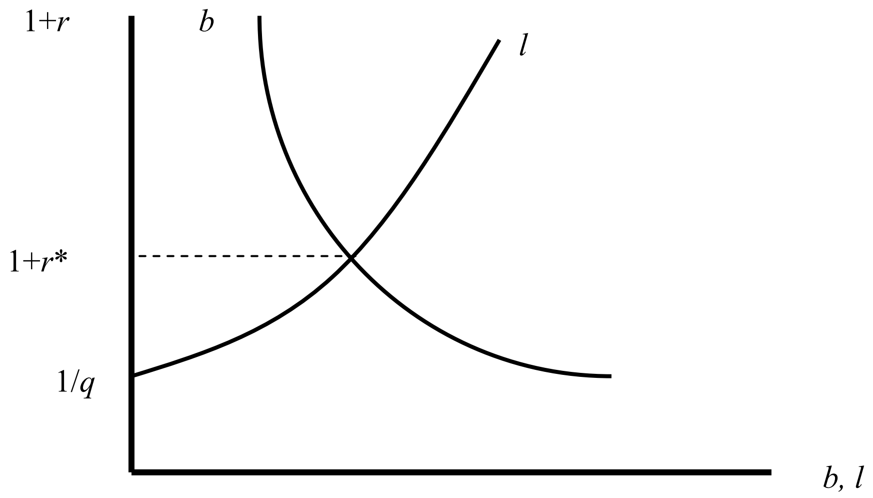
(7) $$b_t=l_t.$$

Applying this condition to (3) and (6) implies that the equilibrium rate of interest is constant over time and the economy is thus in a steady state. The following equilibrium condition determines the equilibrium interest rate r^*:

(8) $$\frac{\beta}{1+\beta}\cdot(1+\beta q)[q(1+r^*)-1]+\frac{1}{1+r^*}=1.$$

This equilibrium interest rate is also described in the following diagram.

Figure 1: Lending and Borrowing and the Equilibrium Rate of Interest



Note, that when the probability of debt payment q increases, the interest rate is reduced, which is intuitive. From the equilibrium real interest rate (8) and from equation (4) we get that the equilibrium amount of silver used as money is determined by:

(9) $$1-\left(\frac{\beta}{1+\beta}-\frac{s}{y}\right)(1+\beta q)=\frac{\beta}{1+\beta}\frac{1-q}{s}y.$$

This equilibrium condition shows that as q increases the ratio of real balances to income declines, namely as bonds expected pay rises and risk declines, people hold less money.

In the basic model in this section the assets are only commodity money and loans. The advantage of commodity money over loans is being riskless, while loans are risky. But loans have a positive rate of return, while the return from money is zero. The model demonstrates one of the main motivations for the emergence of commodity money in ancient economies, where financial or insurance markets were practically absent and lenders were fully exposed to the risk of borrowers default. When the risk was high, namely when q was low, which was the case many times, people saved mainly money, as can be seen in this model as well. But the problem of risk was not something that could not be handled, since it needed mainly a financial intermediary that could diversify the many idiosyncratic risks. Such a financial intermediary had to be someone who has a large amount of wealth in order to be able to perform such diversification. This gave rise to a new institution, a bank. In the next section we describe how banks stepped in and how money received a new role, being used by banks to perform their own lending.

3. Introducing Banks to the Economy

In this section we add a third type of agents, bankers. More precisely, we model a banker as a moneylender, which was the early form of banking. Thus a banker lends to late young agents. The early young do not deposit at banks, as we assume that such activity is too costly for them.[3] The banker is wealthy, so he can supply loans to many borrowers and thus can fully diversify the risks. We assume that unlike the rest of the population, bankers are infinitely lived. There are many bankers, so that they are perfectly competitive. Each banker has a large amount of wealth held in silver. Note that a banker's consumption is deterministic, due to his ability to diversify risk. Assume that bankers derive utility from consumption of size c_t in period t, and they are risk neutral. Namely, utility of a banker at time 0 is:

$$(10) \qquad U_0^B = \sum_{i=0}^{\infty} \frac{c_t}{(1+\rho)^t}.$$

[3] The assumption of no bank deposits clarifies the exposition of the model and is maintained throughout the paper. It is also consistent with what is known about the early days of banking, when interest bearing deposits were scarce. In the appendix we discuss the introduction of interest bearing deposits into the model and show that all the important results of the model remain unchanged.

The subjective discount rate of bankers, ρ, is not necessarily equal to that of the other agents, namely $(1+\rho)\beta$ can be equal to 1 or differ than 1.

Clearly a banker does not lend the consumption good, as early agents do, since he does not produce any consumption good. The banker can lend only silver, which is the durable medium of exchange. We next add an important assumption that introduces liquidity to our model. Assume that all trade in goods is done in a specific time in the middle of the period, which is called "the trading session." Hence, people who want to purchase with silver must have it prior to the "trading session." As a result, the bank must lend silver to the late young *before* the trading session begins. This assumption, therefore, introduces the need for liquidity in the model.

The bank lends silver to the late young at the beginning of the period, before the trading session, and receives the paid debts from the late old only after the trading session, during which they sell their goods for silver[4]. Hence, new loans are extended before old loans are repaid, creating a liquidity problem. To overcome it, the bank must carry reserves of silver at the end of each period in order to be able to lend at the beginning of the next period. This will be the second role of money in this economy, to enable banks to lend. Note that direct lending from early young to late young can be done in goods, as it involves only writing a contract of future delivery of goods in exchange for a current delivery of goods. Yet, the bank can lend only silver and since it cannot produce silver it must obtain it in advance at the end of each period. Hence, at the beginning of each period silver is held by bankers and early old agents only. It is then borrowed from the bank by the late young, exchanged for goods during the trading session, and at the end of the period it returns to the bank by late old agents who repay their loans, or carried by the early young agents to the next period.

[4] Dubey & Geanakoplos (2003) also present a monetary model of exchange where money is borrowed from the bank prior to trade, exchanged with goods during the trade, and then repaid to the bank.

The need to hold reserves in advance yields a reserve constraint on the banker that imposes a liquidity cost. Denote lending by the bank to late young in period t by L_t, the interest rate by i_t and the reserves of the bank held from period t-1 to t by R_{t-1}. Then:

(11) $$R_{t-1} \geq L_t.$$

Note that the banker does not need silver to purchase his own consumption, because late old can repay their loans to the banker in consumption goods as well. Equation (11) is actually a liquidity constraint resulting from the assumption that banking operations and trade occur in different points in time.

After the trading session silver enters the bank from late old returning their bank loans. Recall that some of the loans are repaid in consumption goods and consumed by the banker. Hence the new silver reserves are equal for all $t \geq 1$ to:[5]

(12) $$R_t = R_{t-1} - L_t + L_{t-1}(1+i_{t-1})q - c_t.$$

Note that bankers get the average repayment rate q, because they fully diversify the risk. Note also that they cannot re-lend the returned loans from t – 1 in period t but only in period t + 1, since these are returned after the trading session. This will be reflected in the required rate of return below.

From (12) we get that consumption is equal to:

(13) $$c_t = R_{t-1} + L_{t-1}(1+i_{t-1})q - L_t - R_t.$$

Substituting (13) in (10) we get that the banker maximizes:

(14) $$\sum_{i=0}^{\infty} \frac{L_{t-1}(1+i_{t-1})q - L_t + R_{t-1} - R_t}{(1+\rho)^t},$$

under the constraint: $L_t \leq R_{t-1}$ for all $t \geq 0$, with R_{-1} given by history. Solution of this maximization program leads to the following result: $L_t = R_{t-1}$ as long as lending is

[5] In period 0 the returned loans are zero as $L_{-1} = 0$. Also in the first period loans L_0 are below R_{-1} and the rest of the reserves are used for lending in period 1.

profitable, namely as long as $1+i_t \geq (1+\rho)^2/q$. If $1+i_t < (1+\rho)^2/q$ there is no lending and $L_t = 0$. Hence, the supply of lending by banks is infinitely elastic, up to the total amount of silver they hold, at:

(15) $$1+i_t = \frac{(1+\rho)^2}{q}.$$

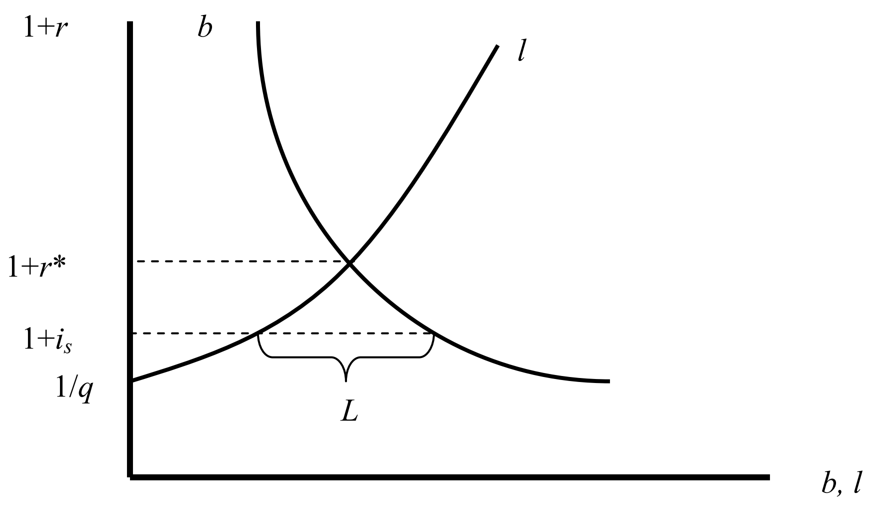
Namely, bankers require a rate of return that should compensate them for two periods of delayed consumption, since lending in t requires the banker to hold silver already in t-1 and the loan is repaid in t+1. Denote this required interest rate by i_s, namely the interest rate of lending silver.

Note first, that for the young L agents borrowing from the bank is the same as borrowing from other agents, since the terms of borrowing are the same. Hence, the interest rate on bank loans must be equal in equilibrium to the interest rate on direct lending if there is lending by banks at all: $r = i_s$. We therefore add the infinitely elastic supply of bank loans as a horizontal line and the equilibrium is described in Figure 2 below, where the curves b and l are the same as in Figure 1. It follows that banks are operating only if:

(16) $$r^* > i_s.$$

If this condition is satisfied, the economy is at a steady state with a constant interest rate, constant consumption levels for early and young, and also constant lending and reserves for banks. Due to the linearity of banks' utility we can look at aggregate banking and their overall lending is L, which is described in Figure 2.

Figure 2: Lending, Borrowing, and Bank Lending without Paper Notes



Note that Figure 2 assumes that the initial amount of reserves R_{-1} is sufficiently large, namely it is greater than $2L$.[6] From Figure 2 it follows that condition (16) is equivalent to $L = b - l > 0$. Using equations (3) and (6) at the equilibrium real interest rate (15), we can calculate the amount of bank lending, and show that the condition for banking is given by:

(17) $$L = b - l = qy\left[\frac{1}{(1+\beta q)(1+\rho)^2} - \frac{\beta}{1+\beta}\frac{(1+\rho)^2 - 1}{(1+\rho)^2 - q}\right] > 0.$$

Hence, bank loans exist when:

$$q \le \frac{(1+\rho)^2\left[1+\beta-\beta(2\rho+\rho^2)\right]}{1+\beta+(1+\rho)^2\beta^2(2\rho+\rho^2)}.$$

This condition captures the cost-benefit aspects of this type of banking. The condition states that the bank is beneficial when bonds are risky enough so that the benefit of risk diversification outweighs the cost of holding the reserves required for the operation of the bank. The condition $i_s < r^*$ means that banks reduce the cost of lending.

[6] If not bankers abstain from consumption a few periods, save silver and then the equilibrium described in Figure 2 can be reached.

We can next calculate the overall utility of bankers. Consumption in each period is derived from (13) given (15) and (17): $c_t = L[(1+\rho)^2 - 1]$ for all $t \geq 1$ and $c_0 = R_{-1} - 2L$. Hence overall utility is given by:

$$(18) \qquad U_0^B = R_{-1} - 2L + \frac{L[(1+\rho)^2 - 1]}{\rho} = R_{-1} + L\rho.$$

Clearly banks are better off lending, since their welfare if they do not lend is equal to R_{-1}. So emergence of banking raises welfare of bankers and of borrowers, namely late agents, but it reduces welfare of early agents, since the return on loans is reduced.

We can summarize the second model in the paper as presenting a very simple form of bank lending. Banks lend silver only and it is subject to high liquidity costs due to the need to hold large silver reserves. This type of banking resembles actual historical banking in its early stages, when banks evolved from wealthy merchants who possessed large sums of gold and silver and lent it to others. Such banking was costly because banks had to hold large stocks of silver in order to be able to lend to the next customers. A better instrument of lending would have been an asset that is available at the moment of lending, without the need to hold it in advance. Indeed, such assets emerged within the banking system and were called convertible paper money (or “Bank Money”).

4. Convertible Paper Money

Convertible bank notes were the first widespread type of paper money.[7] Convertible notes provided their holders with the right to convert them back into commodity (silver) at the bank. More importantly, the notes could be transferred from one person to another with the rights vested in them. This way, the notes could serve as a medium of exchange. Several legal developments were required before convertible notes could

[7] The first issuance of bank notes was probably done in China. See Tullock (1957) for a historical review of the Chinese monetary system starting at 700 A.D.

emerge.[8] But once the legal system had sufficiently matured, convertible notes spread until they dominated the money market. This section shows why this innovation was adopted so widely. The main argument is that issuing convertible notes reduces the cost involved in reserves and with it the cost of bank lending, since banks could now issue convertible notes and lend them instead of silver.

One important characteristic of such bank notes is that they do not earn interest for obvious technical reasons. The note promises to pay an amount of silver at any time it is redeemed, and this amount is printed on the note. Therefore, it cannot pay a higher amount to those who hold it longer. Hence, such notes are equivalent to silver, the commodity money, in terms of rate of return. But bank notes are not fully equivalent to silver in terms of their qualities as media of exchange. Bank notes might not be useful when purchasing small amounts of goods, and sometimes sellers might be reluctant to receive them for payment, for example if they live far away from the bank. For these reasons banks must hold some reserves of silver.

To formalize this need for reserves assume that bank notes are not universally acceptable and a share s of transactions must be conducted by silver. More specifically, assume that people purchase a continuum of consumption goods, and a share s of these goods can be purchased by silver only. Assume also that not only each buyer, but each producer as well sells all this continuum of goods, so that she ends up the trade session with $1-s$ of the money received in notes and s in silver.[9] There are, therefore, three types of transactions in the economy: trade with private debt as mean of payment, monetary trade with silver and monetary trade with notes. Hence, the banker is required to hold some silver reserves to meet the demand for silver at the beginning of each period.

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[8] Examples for such developments were the concept of "payable to bearer" and laws of bills.

[9] It can be shown that if each producer-seller specializes in only one of the consumption goods, the demand for money would change, but the equilibrium interest rate and hence the main results of this model would remain the same.

The dynamics of goods and payments are as follows. At the beginning of period t the late young borrow an amount L_t of money from the bank and they request a share s of their loans to be in silver for monetary transactions that cannot be performed with notes. The bank cannot refuse to lend silver because agents can always borrow in notes and then convert them at the bank into silver. Hence the following liquidity constraint that binds the amount of lending by the bank must hold:

(19) $$R_{t-1} \geq sL_t.$$

The holders of money at the beginning of the period are not only the late young, but also the early old, who obtained money from sales in the previous period. Their money already comprises a share s of silver and a share 1-s of notes, since each sells both goods traded in notes and goods traded in silver. Hence, they do not exchange notes for silver at the bank.

After the trading session late old repay their bank loans. An amount equal to the bankers' consumption is paid with goods and the rest is paid with money. The share of silver in the money repayments is again s, so at the end of the period the bank holds silver reserves equal to:

(20) $$R_t = R_{t-1} - sL_t + s\left[L_{t-1}(1+i_{t-1})q - c_t\right].$$

The condition holds for all $t \geq 1$, and at period 0 we have $c_0 = R_{-1} - sL_0 - R_0$.

From (20) we can calculate the size of consumption in each period and substitute in the utility of the banker (10). We get that bankers maximize:

$$\sum_{i=0}^{\infty} \frac{L_{t-1}(1+i_{t-1})q - L_t + \dfrac{R_{t-1}}{s} - \dfrac{R_t}{s}}{(1+\rho)^t},$$

given the constraint $R_{t-1} \geq sL_t$, in every period t. The maximization of utility shows that lending is positive as long as $1+i_t \geq (1+\rho)^2/q$. Hence, the supply of loans by banks is infinitely elastic at the same interest rate as in the case of silver lending. We therefore get that the equilibrium is similar to that in Section 3 and the interest rate in the case of inconvertible notes is equal to i_s as well, and so is the amount of bank lending L, which is given by (17). Similarly, consumption of the early and late agents is the same as in the case of silver lending.

The only difference between the equilibrium in the case of convertible notes and silver lending is in the welfare of bankers. According to (20) their equilibrium consumption in periods $t > 0$ is: $L[(1+\rho)^2 - 1]$, and consumption in period 0 is: $R_{-1} - 2sL$. Hence, the welfare of bankers in the case of convertible notes is:

(21) $$U_0^B = R_{-1} - 2sL + \frac{L[(1+\rho)^2 - 1]}{\rho} = R_{-1} + L\rho + 2(1-s)L.$$

Comparing (18) to (21) explains why bankers prefer to issue convertible notes, since it increases their welfare relative to lending in silver only. The welfare gain is due to their ability to lend the same amount while holding significantly smaller reserves.

The amount of bank notes in circulation at the end of the period is denoted M_t and is equal to:

(22) $$M_t = M_{t-1} + (1-s)L - (1-s)\{L(1+\rho)^2 - c_t\} = M_{t-1}.$$

The RHS in (22) is equal to notes in circulation at the end of the previous period plus notes issued and lent at the beginning of the current period, minus the amount of notes returning the bank at the end of the period as loans are repaid. As (22) shows, money is constant over time. Since the money issued in period 0 is $(1\text{-}s)L$, this is the amount of money in the economy in all the following periods.

So far, we analyzed banking systems that operated without any form of government intervention. These systems are sometimes called "free banking" systems. There are many historical episodes of private banks that issued convertible notes. In some cases one bank dominated the market, as was the case of the Bank of England, whereas in other cases there was no single dominating bank. That was the case in the famous free banking era in Scotland and in the British colonies of North America. A unique phenomenon that evolved at some stage in these banking systems was the issuance of inconvertible money by private banks, which were the first inconvertible money. In the next section we analyze this issue in detail.

5. Inconvertible Paper Money

In a convertible money regime the bank must hold silver reserves to satisfy the demand for silver each period. Holding such reserves imposes a cost on the bank because the reserves bear no interest. However, if the bank issues inconvertible notes, he can reduce the requirement to hold reserves, because agents do not have a lawful right to convert their notes into silver. Hence, under the convertible money regime banks have strong incentive to minimize the scope of convertibility and ultimately issue inconvertible notes. This has happened in some episodes of free banking where banks issued notes with limited convertibility. We next use our model to examine such a transition.

Assume that bankers decide to issue inconvertible notes, while the rest of the economy is as described in Section 4. There are two main consequences. First, agents can no longer withdraw silver from the bank. They obtain their required silver only through the market. Second, the link between the value of bank notes and the value of silver breaks down, as the bank is not obliged to exchange silver for notes at a fixed rate. In a way issuing non convertible notes is similar to floating a currency.

Let P_t denote the price of consumption goods in terms of notes. The bank lends L_t notes before the trading session, and receives physical consumption c_t and the amount $L_{t-1}(1+i_{t-1})q - P_t c_t$ of notes as debt repayment. The big question is how does the bank determine the amount of nominal lending L_t? This determines the amount of money issued by the bank and as a result it determines the value of this money. The outstanding amount of money is determined by:

(23) $$M_t = M_{t-1} + L_t - L_{t-1}(1+i_{t-1})q + P_t c_t .$$

Assume first that the bank keeps the outstanding amount of notes fixed, to avoid inflation of its own notes. We describe below, in Section 6, what happens if the bank follows an inflationary policy. If $M_t = M_{t-1}$ equation (23) leads to the following budget constraint for the bank in periods $t \geq 1$:

(24) $$P_t c_t = L_{t-1}(1+i_{t-1})q - L_t .$$

In period 0 bankers consume all the initial amount of silver R_{-1}.

Maximizing the intertemporal utility of bankers, subject to the budget constraint (24), leads to the following result. Bankers have an infinitely elastic supply of loans at the following real interest rate:

(25) $$1 + r_{nc} = \frac{1+i_t}{1+\pi_{t+1}} = \frac{1+\rho}{q}.$$

We denote by r_{nc} the real interest rate in the case of inconvertible money, and by π_{t+1} the next period expected inflation rate. Note that the real interest rate is lower than in the case of lending silver and in the case of convertible money i_s, as the constraint of reserves is now zero.

We next turn to find the price level. We consider first the case of zero inflation: $\pi_t = 0$. When inflation is zero early young agents are indifferent between holding silver and holding banknotes. Hence, (4) represents silver and notes held by early young. Note that banks do not hold silver any longer and so are the late, young and old, and the early young. Since agents need silver for transactions, but cannot withdraw silver from the bank, they purchase it in the market from the early old. Recall that a share s of all monetary transactions has to be made with silver. Hence, equilibrium is attained when the early old possess sufficient silver to satisfy the transaction demand of all agents. Denote the real size of monetary transactions in the economy by Z_t, which includes the money purchases of early old, and the purchases of late young agents not financed by direct loans, namely b_t-l_t. We can sum these transactions together, using (6), (4) and (3) at the prevailing interest rate (25), and get:

$$(26) \qquad Z_t = \frac{\beta y}{1+\beta} \cdot \frac{(1+\rho)(1-q)-q\rho}{1+\rho-q} + \frac{qy}{(1+\beta q)(1+\rho)}$$

Hence, in each period the transaction demand for silver is equal to sZ_t. Early old satisfy the demand by holding sZ_t silver and the rest in notes denoted m_t. Hence, from (4) we get that the amount of real balances of inconvertible notes is equal to:

$$(27) \qquad \frac{m_t}{P_t} = \frac{\beta y}{1+\beta} \cdot \frac{(1+\rho)(1-q)}{1+\rho-q} - sZ.$$

Note that the price level is proportional to the quantity of notes, as implied by the quantity theory. Hence, when the bank maintains a constant stock of notes the price level is constant and the rate of inflation is indeed zero.

Note that the market between silver and notes should operate twice each period, before and after the trading session. However, if there is no inflation and silver and money are equivalent in the rate of return, and can be traded freely each time, the market after the trading session ends at the same price level as before the trading session. This is not the case if the bank is having an inflationary policy. In this case, if prices are expected to be higher in the next period, the early young will prefer to hold silver only and sell all notes after the trading session. Hence, an inflating bank will find that his notes that were lent at the beginning of the period to late young agents are returned to the bank as loan repayment by late old agents at the end of the period. Since no one carries the notes to the next period, the bank cannot earn from inflation. Hence, in our model banks have no incentive to inflate, when they compete against other media of exchange, which are more stable.

The inability of increasing the supply of inconvertible notes over time is dependent on the existence of two alternative means of exchange. One is silver, and the other is the notes issued by other banks, which are not inflationary. This analysis points that once silver is in shortage and banks are not fully competitive, there can be inflation of inconvertible notes.

6. Inflation of Inconvertible Notes

In this section we consider the case of shortage of silver. Modeling such shortage in our model is quite difficult. We can drop the assumption that the supply of silver is infinitely elastic and assume instead that there is a fixed amount of silver. In that case the price of silver is no longer 1 and it is determined in the market. Let us denote the overall quantity of silver by S and the market price of silver by Q, in terms of the physical good. It can be shown that the equilibrium value of silver, $Q\,S$, is independent of the physical amount of silver. This makes it difficult to analyze shortage of silver in this model, since any reduction in S is matched by a rise in the same proportion in the price of silver Q and there is no effect on the equilibrium as a whole. In reality of course this is not the case, since silver cannot be used for the same amount of transactions if its quantity becomes smaller and smaller, since there are returns to scale. To capture it in a very simple way we model the shortage of silver as a situation in which there is no silver at all.

Assume that there is no silver in the economy, and the only means of exchange are inconvertible notes, which are issued by banks. It can be shown that in this economy the real interest rate is the same as in Section 5, namely it is equal to r_{nc}. Let us denote by μ the rate of expansion of money issued by the bank, where μ is known in advance. Then μ is also the rate of inflation of the inconvertible notes. It can be shown that in this case the consumption of the banker is equal to:

$$c_t = \rho(b-l) + \frac{M_t}{P_t}\frac{\mu}{1+\mu}. \tag{28}$$

Money is held in this economy by the early young. We cannot use equation (4) to describe their demand for money, since that is the demand for money when the rate of inflation is zero. It can be shown that if inflation is positive the demand for money is described by:

$$\frac{M_t}{P_t} = \frac{\beta y}{1+\beta}\cdot\frac{(1+\rho)(1+\mu)(1/q-1)}{(1+\rho)(1+\mu)/q-1}, \tag{29}$$

and the quantity of private loans is:

$$l_t = \frac{\beta q y}{1+\beta}\cdot\frac{(1+\rho)(1+\mu)-1}{(1+\rho)(1+\mu)-q} \tag{30}$$

Substituting (29), (30) and (6) in (28) we get that banker's consumption is equal to:

$$c_t = \frac{\rho q y}{(1+\beta q)(1+\rho)} - \frac{\beta q \rho y}{1+\beta}\frac{(1+\rho)(1+\mu)-1}{(1+\rho)(1+\mu)-q} + \frac{\beta y}{1+\beta}\cdot\frac{(1+\rho)(1-q)}{(1+\rho)(1+\mu)-q}\mu. \tag{31}$$

It can be shown that the derivative of (31) is positive at $\mu \geq 0$, namely that creating inflation increases the welfare of bankers. Hence bankers have incentive to inflate their inconvertible notes. But this can be done only if the bank is a leading bank. Otherwise, a bank that inflates its notes, while the other banks do not inflate their notes, will find that the public will stop holding its notes and thus its welfare will be reduced significantly. Hence, inflation of inconvertible notes emerges when two conditions are met, one is a shortage of silver, and the second is that the banking sector is not in perfect competition and there is a leading bank.

When a private bank inflates its paper money, there might be public pressure on the government to intervene and stop this process. Such intervention can be in two forms. The first is legal, by legislation that forbids banks to issue inconvertible notes. The second is by issuing fiat money by the government, namely by public monopolization of the issuance of paper money. It is important to note that such monopolization by the government occurred in all countries, not only where banks issued inconvertible notes. The reason for that is probably that governments coveted the seignorage, namely the profits of issuing paper money, and preferred to do it themselves, even before private banks had the chance to issue such money.

7. Three Historical Episodes

In this section we briefly describe three historical episodes of suspension of convertibility. These three episodes show the basic forces described by our model. First, convertible notes were very common two hundred years ago, before governments issued fiat money. Second, banks had a strong incentive to suspend convertibility of their notes in order to reduce the cost of holding reserves of specie, the precious commodity. Third, once specie was in shortage, and there was a leading bank, which was not afraid that the public might use other bank's notes, there was a tendency to inflate the money.

7.1. The Boston "Land Bank" and "Silver Bank" Schemes

The economy of the Massachusetts Bay in the 17th and 18th century suffered from a chronic shortage of specie and lack of appropriate substitutes. Several attempts were made to establish a bank that would supply an alternative medium of exchange. All of these schemes, described in details by Davis (1896), suggested issuing notes that were not convertible into silver on demand, contradicting the practice of issuing convertible bank notes prevailing at that time in Europe. The proponents of private banking tried to convince the authorities that backing the notes with land through unconventional legal arrangements can act as a replacement to silver. The “Land Bank”, it was hoped, should be the cure.

The intensive debate of the time and further deterioration of the scarcity of specie have finally led to the establishment of a private “Land Bank” in 1740. The original plan of the “Land Bank” scheme was to issue inconvertible notes and lend them to the 396 founders of the bank who will put them into circulation through their own businesses. The notes were meant to be purely inconvertible and the only promise made by the founders of the bank was to accept the notes in all payments, something like a private mechanism of “legal tender”. However, Davis (1897) mentions that the original plan did not materialize, and a more concrete obligation was introduced in the notes. Namely, paying the possessor of the note its value in manufactures of the province. This gave the notes the name “Manufacture Notes”. Apparently this was not enough because the time of payment and the value of the manufactures were not stated. Hence, eventually the bills contained a specific time for redemption and a value in silver. The final wording of the bills according to Davis (1897) was:

> *“We jointly and severally promise for ourselves and partners to take this bill as lawful money at 6s. 8d. per ounce in all payments, trade and business, and for stock in our treasury at any time; and, after twenty years, to pay the same at that estimate on demand to Mr. Joseph Marion, or order, in the produce or manufactures enumerated in our scheme, for value received.”*

Note that the real value of the goods to be paid after twenty years is vague. First, the quality of silver, according to which the value of the goods will be estimated, is not specified. Second, the identity of the goods and their price in terms of silver are not determined. This was an example of a note that was not convertible on demand, and its value in maturity was not determined precisely. The structure of the note enabled the bank to operate without holding large reserves, because it was not obliged to convert on a daily basis.

The opponents of the "Land Bank" took an action and established a rival enterprise - the "Silver Bank". They issued notes convertible into silver after fifteen years, and promised to receive the notes in trade and business at specified rates for each year during the fifteen years to maturity. The notes tenor was as follows[10]:

> *"the Directors, jointly and severally Promise to pay to Isaac Winslow Merchant or Order in Boston -------- Ounces of Silver Sterling Alloy, or -------- Standard Gold, both coin'd and Troy Weight by the 31st of December 1755. Value receiv'd, &c."*

Clearly, the silver bank offered a sounder basis for its notes, but like the Land Bank the notes were not convertible on demand but only after a period of 15 years.

The notes of both banks circulated for less than two years. Many opposed their circulation, especially the Land Bank notes, but others favored them. The operation of the banks was outlawed in 1941 by an Act of Parliament, after administrative actions by the Governor trying to destroy the Land Bank did not suffice.

7.2. The Option Clause of Scotland Banks

The Scottish banking system developed during the 18th century after the establishment of the Bank of Scotland in 1695 (Checkland 1975). All banks were privately-owned and most of them issued convertible notes. The competition between banks, which sometimes even developed into violent bank wars, led to the creation of the well-known

[10] See an anonymous pamphlet published in 1744 "An account of the rise, progress, and consequences of the two late schemes commonly call'd the Land Bank or Manufactury Scheme and the Silver Scheme, in the Province of the Massachusetts Bay".

‘optional clause.’ This clause, which originated in the notes of the Bank of Scotland from 1730, allowed the issuing bank to suspend payments of specie for six months from demand, and at the end of the period pay the note holder the face value of the note plus an interest of 2.5%.

The optional clause became a popular device and during the years 1762-1765 all banks issued notes with the clause. Graham (1911) gives the example of the Dundee Bank that issued notes with option that were convertible into notes with option issued by other banks. Holders of the Dundee Bank notes had to wait 12 months until receiving payments because the option clause could be exercised twice. The Scottish banks used other methods of postponing payments as well. Some banks paid specie only at their main office, so remote holders could hardly demand specie. Others paid with long dated bills on London instead of specie, or “paid slowly or in troublesome manner” (Checkland 1975). Checkland summarizes the practice of inconvertibility of the Scottish banks in the following words (p.101):

> *“This tendency to refuse specie was a mutually confirming thing among Scottish bankers, for any company which made specie readily available might easily have a run made upon it: thus all bankers were forced to take the same hard line, and did so with great effect, confirming Scottish dependence upon paper.”*

Hamilton (1966) refers to the inconvertibility of Scottish bank notes (p. 264) in the following words:

> *“In the ten years ending 1765 there were numerous complaints all over the country at the scandal of what was virtually an inconvertible paper currency.”*

The exercise of the option clause increased the quantity of circulating notes, eventually causing depreciation of the notes by 4-5%, as mentioned in Adam Smith's *Wealth of Nations*, Graham (1911) and Checkland (1975). The period was characterized with chronic inflation, exchange crisis and specie shortage (Checkland 1975). The public strongly protested against the option clause and the inability to receive specie and called for government intervention. Eventually, in 1765 the option clause was outlawed and all notes were required to be convertible on demand. This episode therefore presents the endogenous development of money issued by private banks that was not convertible on demand, and served as a tool to minimize the stock of bank reserves. This episode also presents the subsequent government intervention that followed.

7.3. The Suspension of Payments by the Bank of England

The English banking system in the 18^{th} century composed of many banks that issued convertible money. The Bank of England, which was a private bank established in 1694, was the most important issuer in the system due to its favored legal status and special relations with the government (White 1984).

In 1797 the demand for specie strengthened as a result of fears due to the Napoleonic Wars. In face of the shortage of reserves, the Bank of England decided to suspend payments after receiving permission from the government. Consequently, all banks in England stopped payments. It is worth noting that the Scottish banks at that time suspended their payments without a government permit violating the 1695 act (White 1984).

The suspension period lasted 24 years, during which the notes of all banks were inconvertible. The result was a significant depreciation of the notes causing high inflation. Convertibility was resumed in 1821 as part of a comprehensive government reform in the money market. This was by far the longest period of privately-issued inconvertible money. Government intervention was necessary in order to restore price stability.

8. Summary and Conclusions

This paper describes the development of paper money as it emerges from the lending activity of banks, which invent and use paper money to reduce the costs of holding large reserves of silver to maintain liquidity. The paper shows, using a sequence of models, how banks emerged out of money lending, how they began to issue first convertible notes, that reduce their lending costs, and the incentive they had to move from convertible to inconvertible notes.

Historically, the episodes of banks' inconvertible paper money are not so common. The main reason for that is that at that time governments already stepped in and started to monopolize the issuance of paper money. They did it because they wanted to get the seignorage for themselves instead of leaving it to the private banks. Such monopolization occurred first by making one of the private banks a central bank, as in England, so that the paper money issued by that bank became government fiat money. The monopolization also occurred directly, when governments created central banks which took the right of issuing paper money from the private banks.

This paper describes a historical development, but we believe that it is also relevant to the present, since it describes an important link through which money and financial intermediation are related to each other.

Appendix: **Adding bank deposits**

Throughout the paper we maintained the assumption that banks do not issue interest-bearing deposits. We want to relax that assumption now to see the effect on our main results. We concentrate first on the model of convertible money which generalizes the case of silver only (when s=1).

When early young agents can save in the form of interest-bearing deposits, their optimal choice of private loans and deposits is:

(A1) $$l_t = \frac{\beta y}{(1+\beta)}\left[\frac{q-(1+i_t^D)/(1+i_t)}{1-(1+i_t^D)/(1+i_t)}\right].$$

(A2) $$D_t = \frac{\beta y}{1+\beta}\left[\frac{(1-q)}{1-(1+i_t^D)/(1+i_t)}\right].$$

where D_t denotes bank deposits paying interest i_t^D. Note that bank deposits are perfectly safe so when the interest is positive they dominate silver and bank notes in terms of rate of return. Hence, early young agents will not hold silver or bank notes at the end of the period as they are always weakly dominated by bank deposits (and strictly dominated when the interest is positive).

Hence, after the trading session early young agents deposit all their money (silver and notes) in the bank. They withdraw it with interest at the beginning of the next period, when they are old. According to our assumption, they need a share *s* of their money in silver for their silver transactions. Hence, they withdraw *s* of their deposits in the form of silver. As a result, the liquidity constraint (19) becomes:

(A3) $$R_{t-1} \geq s\left(L_t + D_{t-1}\left(1+i_{t-1}^D\right)\right).$$

We can see that the banker holds silver for the withdrawals of late young and early old agents. In the original model only late young agents withdrew silver.

The stock of silver reserves at the end of the period is now affected by the withdrawal of silver from bank deposits at the beginning of the period and deposit of silver at the bank at the end of the period. Hence, instead of (20) we get:

(A4) $$R_t = R_{t-1} - s\left[L_t + D_{t-1}\left(1+i_{t-1}^D\right)\right] + s\left[L_{t-1}(1+i_{t-1})q - c_t + D_t\right].$$

The banker maximizes:

$$\sum_{i=0}^{\infty} \frac{L_{t-1}(1+i_{t-1})q - D_{t-1}\left(1+i_{t-1}^D\right) - L_t + D_t + \frac{R_{t-1}}{s} - \frac{R_t}{s}}{(1+\rho)^t},$$

subject to (A3). The maximization yields the same interest rate on loans as in the no-deposit case, namely $(1+i_{t-1})q = (1+\rho)^2$. The deposit interest rate is zero, so adding deposits have no effect on the results.

When we solve the model for the case of inconvertible money (s=0) we get that the lending interest rate equals (1+ρ)/q-1, as was the case without deposits. The deposit interest rate is now equal to ρ, and agents deposit all their notes in the bank. The main difference now compared to the model without deposits is that inflation does not reduce the quantity of notes used in exchange because agents can fully protect themselves against inflation by depositing their notes in the bank at the end of each period. The price level is still determined by the quantity of notes that the bank issues, so moving to privately-issued inconvertible money has the same destabilizing effects on the price level as before.

References

Araujo, Luis and Camargo Braz "Information, Learning and the Stability of Fiat Money" *Journal of Monetary Economics*, vol. 53 (2006), pp. 1571-1591.

Brock W.A. and Scheinkman J.A. 1980 "Some Remarks on Monetary Policy in an Overlapping Generations Model" In Kareken John and Wallace Neil (eds.), *Models of Monetary Economies*. Federal Reserve Bank of Minneapolis, 211-32.

Checkland, S. G. *Scottish Banking A History, 1695-1973* Glasgow: Collins, 1975

Davis, A.M. 1896 "Currency Discussion in Massachusetts in the Eighteenth Century" *The Quarterly Journal of Economics*, Vol. 11, No. 1. (Oct., 1896), pp. 70-91.

Davis, A.M. 1897 "Currency Discussion in Massachusetts in the Eighteenth Century" *The Quarterly Journal of Economics*, Vol. 11, No. 2. (Jan., 1897), pp. 136-160.

Dowd, Kevin "The Emergence of Fiat Money: A Reconsideration" *Cato Journal*, Vol. 20, no. 3 (2001), pp. 467-476).

Dubey & Geanakoplos (2003) "Inside and outside fiat money, gains to trade, and IS-LM"

Graham, W. *The One Pound Note in the History of Banking in Great Britain* 2nd ed., Edinburgh: J. Thin, 1911

Hamilton, H. 1966 *The Industrial Revolution in Scotland*, Frank Cass & Co. ltd., 1966.

He Ping, Huang Lixin and Wright Randall "Money and Banking in Search Equilibrium" *International Economic Review*, vol. 46, no. 2 (May 2005), pp. 637-670.

Iwai, Katsuhito "The Bootstrap Theory of Money: a Search-Theoretic Foundation of Monetary Economics" *Structural Change and Economic Dynamics*, Vol. 7 (1996), pp. 451-477.

Iwai, Katsuhito "Evolution of Money" in *Evolution of Economic Diversity* Nicita, A. and Pagano, U., eds. Routledge, 2001.

Jones, Robert A. "The Origin and Development of Media of Exchange" *The Journal of Political Economy*, Vol. 84, No. 4, Part 1. (Aug., 1976), pp. 757-776.

Kim, YS. "Liquidity, Prices, Seigniorage, and the Transition from Barter to Fiat Money" *Macroeconomic-Dynamics* Vol. 5 no. 3 (June 2001), pp. 353-379.

Kiyotaki, Nobuhiro and Wright, Randall "On Money as a Medium of Exchange" *Journal of Political Economy*, Vol. 97. no. 4 (1989), pp. 927-954.

Luo Guo Ying "The Evolution of Money as a Medium of Exchange" *Journal of Economic Dynamics and Control*, vol. 23 (1999), pp. 415-458.

Marimon, Ramon, McGratten, Ellen and Sargent, Thomas J. “Money as a Medium of Exchange in an Economy with Artificially Intelligent Agents” *Journal of Economic Dynamics and Control*, vol. 14 (1990), pp. 329-373.

Oh, Seonghwan “A Theory of a Generally Accepted Medium of Exchange and Barter” *Journal of Monetary Economics*, Vol. 23 (1989), pp. 101-119.

Ritter, Joseph A. “The Transition from Barter to Fiat Money” *The American Economic Review*, vol. 85, no. 1 (Mar., 1995), 134-149.

Samuelson, P. A. 1958 “An exact consumption-loan model of interest with or without the social contrivance of money” *Journal of Political Economy* 66 (December): 467-82.

Sargent, Thomas J. and Velde, François R. 2002 *The big problem of small change* Princeton, N.J.: Princeton University Press, 2002.

Sargent, Thomas J. and Wallace, Neil “A Model of Commodity Money” *Journal of Monetary Economics*, Vol. 12 (1983), pp. 163-187.

Selgin, George “Adaptive Learning and the Transition to Fiat Money” *The Economic Journal*, vol. 113 (January 2003), pp. 147-165.

Smith, A. 1937 *An Inquiry into the Wealth of Nations*, Random House, Inc., 1937.

Tullock, Gordon 1957 "Paper Money – A Cycle in Cathay" *The Economic History Review*, vol. 9(3), pp. 393-407.

White, L.H. 1984 *Free banking in Britain: theory, experience, and debate, 1800-1845*, Cambridge University Press, 1984.

Wright, Randall “Search, Evolution and Money” *Journal of Economic Dynamics and Control*, vol. 19 (1995), pp. 181-206.