ISSN 0265-8003

INSTABILITY OF THE VELOCITY OF MONEY: A NEW APPROACH BASED ON THE EVOLUTIONARY SPECTRUM

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Discussion Paper No. 735
November 1992

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CEPR Discussion Paper No. 735

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ABSTRACT

Instability of the Velocity of Money: A New Approach Based on the Evolutionary Spectrum*

Money demand instability has been the subject of considerable attention in the recent literature. This paper examines the stability of velocity series for the United States and for five European countries (France, Germany, Italy, the Netherlands and the United Kingdom). A distribution-free test due to Subba Rao (1981) and based on the concept of the evolutionary spectrum is used. The test does not require the assumption that distributions are Gaussian and is able to decompose (by frequency) the sources of instability. The technique reveals structural breaks of long-term significance for only broad money velocity in the United States and narrow money velocity in the United Kingdom.

JEL classification: E41
Keywords: money demand, instability, velocity, evolutionary spectrum, structural breaks, frequency domain

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*This paper is produced as part of a CEPR research project on 'The European Monetary System: Sustainability, Operation and Long-Term Credibility' supported by the Leverhulme Trust (no. F558B). The authors are grateful to both institutions without implicating either.

Submitted 2 October 1992

NON-TECHNICAL SUMMARY

This paper investigates the instability of money demand in a new way using techniques drawn from spectral analysis.

The stability, or otherwise, of money demand is of vital importance in the formulation of monetary policy. It is often argued, for example, that in the 1980s demand was unstable due to changes in the financial markets ('financial innovation'). Such perceived instability undermined policies based on the targeting of monetary aggregates and led to the adoption of alternative targets e.g. for 'money GDP' or for the exchange rate, as in the ERM countries. Instability may not, however, affect all measures of the money supply: pessimistic conclusions concerning monetary targeting, based on perceptions of instability in the broader aggregates – as in the United Kingdom in the 1980s – need not follow if demand for the narrower monetary aggregates is more stable. In this paper we concentrate on measures of the money supply which are relatively 'narrow', in particular on M1 (as defined by the IMF). For the United Kingdom and United States we additionally use data for broader money supply definitions. Reflecting the widespread presumption that financial innovation in the 1980s was most important in the United Kingdom and the United States and was less significant in continental European countries, our data sample covers France, Germany, Italy and the Netherlands as well as the United Kingdom and the United States. Typically, two alternative approaches have been brought to bear upon the issue of instability of the money demand function. The first is a direct approach based on identifying a suitable demand for money function and examining it for structural beaks using conventional tests. There are, however, serious problems in identifying the demand for money function, both on the grounds of measurement errors in the explanatory variables, and (more importantly) of the impossibility of disentangling supply from demand effects. This criticism has provoked a second line of approach to the analysis of the stability problem based on the velocity of money which may ameliorate these difficulties. In this paper we characterize the 'demand for money' by a single statistic, the velocity of money (the ratio of nominal income to money).

The methodological purpose of the study was to test for instability using novel methods (based on spectral or frequency domain analysis). The key to the test is the concept of the 'evolutionary spectrum'. The behaviour of a time series of observations of any variable can be decomposed into components of different frequencies. The weights attached to the components at each frequency constitute the 'spectrum' of the variable. For example, a series may have important 'long-run' (low-frequency) cyclical components, imparting a wave-like behaviour over time, together with noisy 'short-run' (high-frequency) components, which disturb the underlying long-run cycle. The spectrum of this

variable will have large weights attached to the cyclical and noise components. The spectrum – the weights attached to the components at different frequencies – may itself change over time. The weight attached to the noise component, for example, may increase if the series begins to behave more erratically. The paper makes use of this notion of an 'evolutionary spectrum', in which the weights attached to the components at different frequencies can vary over time. Tests for structural breaks in the series can then be carried out by testing for shifts in the weights. The test statistics employed to detect instability have the advantage that if a structural break is identified it is possible to determine not only at what point of time the break occurs, but also whether it is the weights attached to the high- or to the low-frequency components which have shifted. This additional information could yield some insights concerning the transitory or permanent nature of the shift in the behaviour of the series. Besides yielding this additional information the technique we employ also avoids the very strong assumptions made by customary methods for identifying structural breaks. In particular, the technique requires no assumption concerning the underlying distribution of the series studied.

The spectral methods employed and the specific test used require large samples of high-frequency data. In this study we adopt as a proxy for monthly nominal GDP, therefore, monthly data over the period 1960–90 (1970–1990 for the United Kingdom) on the product of industrial production and producer prices.

The results suggest that in the six countries studied, instability in money demand was confined in the 1980s to the United Kingdom and the United States out of the sample of countries studied. This is consistent with the view that financial innovation in the continental European countries was less sweeping in character and had less impact on overall monetary stability than in the United Kingdom or the United States .

Introduction

A great deal of attention has been lavished in the literature on the demand for money function, in view of its stability being a crucial element in the formulation of monetary policy. Early empirical studies were unambiguously supportive of a stable money demand function in the industrialised countries (e.g. Kavanagh and Walters (1966) for the U.K. and Goldfeld (1973) for the U.S.). Around the mid-1970s, however, the unanimity on stability seemed to come increasingly under pressure. Existing money demand models tended to systematically underpredict money balances in the U.K., particularly for the 1971-73 period (see Artis & Lewis (1974), 1976), and Hacche (1974)). In the U.S. the mid-1970s were also characterized by prediction failure (of money demand models) but in this case the tendency was for overpredicting the money balances (a phenomenon referred to as "missing money" by Goldfeld (1976)). After 1981, the prediction failures continued in both the U.K. and the U.S. but in the latter overprediction was reversed into underprediction (see Leventakis & Brissimis (1991)). The instability of the money demand function was more pronounced for the broader money aggregates than for the narrow concept in most of the industrialised countries (see Artis & Lewis (1991)).

Typically, two alternative approaches have been brought to bear upon the issue of instability of the money demand function. The first is a direct approach based on identifying a suitable demand for money function and examining it for structural breaks using conventional tests such as the Chow test. In an influential paper, Cooley & LeRoy (1981) cast serious doubts on the possibility of the identification of a demand for money function -both on the grounds of measurement errors in the explanatory variables, and (more importantly) of the impossibility of disentangling supply and demand effects. This criticism has provoked a second line of approach to the analysis of the stability problem based on the velocity of money (see e.g. Bordo & Jonung (1987), Hamilton (1989) and Bomhoff (1991)). The velocity approach certainly makes the objections raised by Cooley & Leroy less glaring although it is debatable whether it circumvents them altogether.

In this paper, velocity (with respect to broad and narrow money) is examined for structural breaks as indirect evidence of the stability or otherwise of money demand functions. The test proposed for locating structural breaks is the CUSUM test of Subba Rao (1981) based on the concept of an "evolutionary spectrum". A major advantage of the test procedure is that it is distribution-free and so does not require the variables to be tested to be Gaussian - a requirement imposed by most other tests of structural change (such as Chow (1960), Farley Hinich & McGuire (1975), Hawkins (1977), Worsley (1979), etc.). As a matter of fact, the test can be applied to any time series which is "oscillatory" (see the definition in Section 2) and admits of an "evolutionary spectrum". The class of "oscillatory" process is sufficiently general to include most processes likely to be encountered in economics. Yet another advantage of the Subba Rao test is that, if desired, the test can indicate the frequency at which the break occurs, so that one can infer whether it is the short-run characteristics of the series or the long-run ones which are exhibiting a structural change at the point in question. The power properties of the test vis-à-vis other tests of structural change, however, have not so far been documented, though preliminary investigations by Subba Rao (1981) and Priestley (1988) are favourable. (The issue is currently under investigation by the authors.)

The plan of the paper is as follows. Section two introduces the concept of the evolutionary spectrum and details the Subba Rao test. The results of applying the test to various velocity series are reported in section three, while conclusions are presented in section four.

Evolutionary Spectrum

A number of alternative approaches to time-dependent spectra prevail in the literature (see Page (1952), Tjostheim (1976), Melard (1975) and Priestley

(1965)). The evolutionary spectrum of Priestley (1965) is a particularly attractive concept, both because it has a recognizable physical interpretation and because it encompasses most other approaches as special cases.

The concept of the evolutionary spectrum may be introduced as follows.

Definition 1: A discrete parameter time series $\{X_t\}$ is said to be oscillatory if it has the following representation:

$$X_t=\sum_{-\pi}^{\pi} \exp(itw)\ A_t(w)\ dZ(w),\ (t=0,\ \pm 1,\ \pm 2) \tag{1}$$

where for each w, the sequence $\{A_t(w)\}$ has a generalised Fourier transform, whose modulus has an absolute maximum at the origin and Z(w) is an orthogonal process on $(-\pi, \pi)$ with $E\{\ |\ dZ(w)\ |^2\} = d\mu(w)$ where $\mu(w)$ is a measure.

Definition 2: Let $\{X_t\}$ be an oscillatory process as defined above. The evolutionary spectral distribution function of $\{X_t\}$ at time t with respect to the family of sequences $F = \{\exp(itw)A_t(w)\}$ is then defined as:

$$dH_t(w) = |A_t(w)|^2 d\mu(w), -\pi \le w \le \pi \tag{2}$$

If the measure $\mu(w)$ is absolutely continuous we can write $dH_t(w) = h_t(w)dw$ where $h_t(w)$ is the evolutionary spectral density function defined throughout $(-\pi, \pi)$.

A few comments are in order here. Generally speaking, a given oscillatory process would have a number of evolutionary spectral representations corresponding to different choices of an oscillatory family F. It is therefore reasonable to select that family for which the $\{A_t(w)\}$ are in some sense "most slowly varying", i.e. having their generalized Fourier transforms highly concentrated around the origin.

The estimation of the evolutionary spectrum proceeds as follows. Suppose it is desired to estimate the evolutionary spectrum at the frequency w_0.

The data is passed through a linear filter centred at w_0, yielding an output U(t). A weighted average of $|U(t)|^2$ in the neighbourhood of the time point t is now computed to yield an estimate of the evolutionary spectrum $h_t(w)$ at frequency w_0. Thus, given observations on $\{X_t\}$, $t=0, 1, 2\ldots T$ we can write

$$U_t=\sum_{u=t}^{t-T} g_u X_{t-u}\exp[-iw_o(t-u)] \tag{3}$$

$$\hat{h}_t(w_o)=\sum_{v=t}^{t-T'} w_v |U_{t-v}|^2 \tag{4}$$

where $\{g_u\}$ and $\{w_v\}$ are suitably chosen windows. In most applications the following "double window" due to Priestley (1966) is usually deployed.

$$g_u=\begin{cases}\frac{1}{2\sqrt{h\pi}} & |u|\leq h\\ 0 & |u|>h\end{cases} \tag{5}$$

$$w_u=\begin{cases}1/T' & |u|\leq T'/2\\ 0 & |u|>T'/2\end{cases} \tag{6}$$

There is a form of Uncertainty Principle operating here (the so-called Grenander Uncertainty Principle), according to which simultaneous attainment of high degree of resolution in both the time and frequency domains is impossible. If we give equal weight to the two resolutions, then for the double window (5) - (6) the appropriate choice of h is 7 (see Chan and Tong (1975)). There are no clear-cut guidelines for the choice of the truncation point T' One would possibly need to resort to some kind of "window closing" procedure as is suggested by Jenkins and Watts (1969) for the time-independent case. The simulation study of Chan and Tong (1975) indicates that the evolutionary spectral estimates are fairly robust for T′ taking values in the range (T/4) to (T/6).

The following properties need to be noted for the evolutionary spectral estimator $h_t(w)$ based on (5) - (6)

(i) $$E\{\hat{h}_t(w)\} \doteq h_t(w)$$

(ii) $$var\{\hat{h}_t(w)\} = \frac{4h}{3T'} h_t^2(w)$$

and (iii) $$Cov\{\hat{h}_{t_1}(w_1), \hat{h}_{t_2}(w_2)\} = 0 \quad \text{if either}$$

(a) $|w_1 \pm w_2| >$ bandwidth of $|\Gamma(\Theta)|^2$ where $\Gamma(\Theta)$ is the Fourier transform of g_u or

(b) $|t_1 - t_2| >$ bandwidth of (6).

Testing for Structural Change.

Let $X_1 \ldots X_T$ be observations from a discrete parameter process $\{X_t\}$ with evolutionary spectral density $h_t(w)$. Further, suppose the evolutionary spectrum estimate $\hat{h}_t(w)$ is constructed using the "double windows" (5) and

(6), and evaluated at times $\{t_i\}$, $i=1, 2 \ldots I$ (each t_i is a member of the set 1, 2 ... T) and the set of frequencies $\{w_j\}$, $j=1, 2 \ldots J$.

Denote ln $[\hat{h}_{t_i}(w_j)]$ by Y_{ij} and $H_{t_i}(w_j)$ by h_{ij}.

From Priestley (1988) we may write

$$Y_{ij} = h_{ij} + e_{ij} \qquad (7)$$

If we now assume the our $\{t_i\}$ and $\{w_j\}$ satisfy the conditions (iii)(a) and (iii)(b) listed at the end of Section 2.1, then it can be shown that the e_{ij} are approximately i.i.d., $N(O,\sigma^2)$ with $\sigma^2 = \text{var}[\ln\hat{h}_t(w)]$. The normality and independence of the e_{ij} now enable us to invoke the CUSUM and CUSUMSQ tests of Brown *et al.* (1975), specifically, defining

$$\bar{Y}_{i.} = \frac{1}{J}\sum_{j=1}^{J} Y_{ij}, \qquad i = 1, 2 \ldots I \qquad (8)$$

we may compute the CUSUM W_r and the CUSUMSQ S_r as follows:

$$W_r = \sum_{i=1}^{r} (\bar{Y}_{i.}-k) / \frac{1}{I}\sum_{i=1}^{I} (\bar{Y}_{i.}-k)^2 \quad , \ r=1 \ldots I \qquad (9)$$

$$S_r = \sum_{i=1}^{r} (\bar{Y}_{i.}-k)^2 / \sum_{i=1}^{I} (\bar{Y}_{i.}-k)^2 \ , \quad r=1 \ldots I \qquad (10)$$

where k is a chosen reference value. (The choice of k for our problem is discussed in section 3.) The tests for structural break may now be executed in a straightforward manner as discussed in Harvey (1981).

A great advantage to this procedure is that it enables isolation of the frequency (or frequencies) at which the break occurs. (This information is useful in judging whether the structural breaks are of a transitory or persistent nature.) This may be done simply by replacing $\bar{Y}_{i.}$ in (9) and (10) by Y_{ij} (for $j=1, 2...J$) and examining the resulting CUSUMs and CUSUMSQs for each j. Of course the reference value will not be the same as in (9) or (10) and may itself vary for each frequency.

Results

The structural break test (Subba Rao (1981)) proposed to be used here is asymptotic and hence dictates the use of a high frequency velocity series, encompassing a fairly long stretch of time. The maximum frequency achievable in this case is monthly, provided one opts for industrial production as the income proxy rather than gross domestic product (data on which are not available for frequencies higher than quarterly). Frequent definitional revisions in money supply concepts in the OECD countries (which nonetheless offer the best scope for conducting studies of this kind) pose another formidable constraint. Among these countries, a fairly long and consistent series of monthly velocity (based on M1 and an industrial production index) could be constructed for France, Italy, Netherlands, West Germany, the U.K. and the U.S.A. (For France, two alternative series in this category could be constructed, based on two separate sources.) In view of the suggestion (see section 1) that instability is more pronounced in the broad money demand function, as compared to the narrow money demand function, special interest would have attached to an examination of the broad money-based velocity series. However, velocity series based on the broad money aggregate could be assembled only for the U.K. (M1 plus Quasi-money) and the U.S.A. (M3). In the interest of extracting the maximum available information from each series, the time span of analysis was allowed to be non-uniform for the different series. Imposition of uniformity in this regard would have dissipated valuable degrees of freedom. Complete details of the various velocity series are summarised in appendix I.

In the calculation of the evolutionary spectrum for each of the above series, the "double window" (5) - (6) was used. The two crucial parameters to be chosen are h and T'. The value of h was chosen to be 7 reflecting equal weightage to time, and frequency domain resolution. Various values of T' were tried out at the evolutionary spectral estimates were fairly robust for T' in the range 15 to 30. A value of T'=20 was thus selected for the final set of calculations. As discussed in section 2.2, applicability of the Subba Rao test hinges upon the chosen $\{t_i\}$, $i=1 \ldots I$ and $\{w_j\}$, $j=1 \ldots J$ satisfying conditions (iii)(a) and (iii)(b) of Section 2.1. With our choice of h, the approximate bandwidth of $|\Gamma(\Theta)|^2$ is $\pi/7$ and this yields the following choice $\{w_j\}$ for the set of 7 frequencies

$$w_j=(j\pi/20), \quad j=1, 4, 7, 10, 13, 16, 19 \qquad (11)$$

Similarly the bandwidth of (6) being equal to T' (=20), the chosen points $\{t_i\}$ must be spaced at least 20 months apart. We choose the spacing to be 24 months and the initial value t_1 as the June value of the initial year for the series in question. This enables us to check for structural breaks in a series every 2 years with the June value as the reference point.

The reference value k in (9) and (10) for calculating the CUSUMs and CUSUMSQs has to be chosen with some care. In some sense, k will be some "target level" which the $\bar{Y}_{i.}$ are supposed to track (see Woodward & Goldsmith (1964)). However, if we take k to be an average of the logarithm of the evolutionary spectrum over the $\{t_i\}$ and $\{w_j\}$ that we have selected, then W_I in (9) will be zero and this will bias the CUSUM test. We avoid this problem by selecting k to be the average of the logarithm of the evolutionary spectrum at the point of time $\{t_i\}$ and the frequency points $\{w_i\}$ given by

$$w_j = (j\pi/20), \quad j = 1, 2, 3 \ldots 19 \qquad (12)$$

(The set w_i in (12) consists of 19 points as opposed to 7 points in (11)). The set t_i continues to be the same as before.

The CUSUM test identifies structural breaks at points where W_r as defined by (9) ($r = 1, 2 \ldots I$) falls outside the pair of straight lines given by

$$W^{(1)} = \pm\left\{0.850\sqrt{I} + \frac{1.700r}{\sqrt{I}}\right\} \text{for a 10\% significance level} \qquad (13)$$

$$W^{(2)} = \pm\left\{0.948\sqrt{I} + \frac{1.896r}{\sqrt{I}}\right\} \text{ for a 5\% significance level} \qquad (14)$$

$$W^{(3)} = \pm\left\{1.143\sqrt{I} + \frac{2.286r}{\sqrt{I}}\right\} \text{ for a 1\% significance level} \qquad (15)$$

The CUSUMSQ test similarly identifies structural breaks of points where S_r as defined by (10) ($r = 1, 2 \ldots I$) falls outside the pair of straight lines

$$WW_r = \pm C_o + \frac{r}{I} \qquad (16)$$

where the values of C_o (corresponding to various values of I and different significance levels) may be recovered from Harvey (1981) p. 364.

We present details of our calculations for one specific case, viz. M1VUK (i.e. the U.K. velocity with respect to narrow money). The log of the evolutionary spectrum for this series is presented in Table 1 and the structural break tests in Table 2. Since the calculations for the other series are similar, only the main conclusions pertaining to them are displayed in Table 3. For completeness, however, we present the log of the evolutionary spectrum for the remaining series in Appendix II.

A few comments on the contents of Table 2 are in order. Since the CUSUM test failed to record any structural breaks at the 5% significance level, a 10% significance level was also tried out in this case but did not detect any breaks either. The CUSUMSQ test located the first structural break in 1983:6 (at 5% significance level), with the divergence from the reference value k (chosen along the lines of the discussion immediately preceding (12)) being reinforced at the next time point considered (1985:6) where the break is highly significant (i.e. at 1%). The movement away from

Table 1

Log of Evolutionary Spectrum: United Kingdom M1

Time \ Frequency	$\pi/20$	$4\pi/20$	$7\pi/20$	$10\pi/20$	$13\pi/20$	$16\pi/20$	$19\pi/20$	Y_t
t = 1 (1971:6)	-2.9711	-5.8385	-6.7857	-7.4956	-8.1352	-8.5457	-8.7119	-6.9262
t = 2 (1973:6)	-2.0409	-4.6214	-5.7268	-6.7449	-8.2982	-10.7917	-8.6871	-6.7016
t = 3 (1975:6)	-1.7519	-4.3371	-5.4639	-6.4851	-8.1701	-10.2416	-8.4412	-6.4130
t = 4 (1977:6)	-1.5904	-4.2121	-5.3475	-6.4046	-8.0656	-10.5265	-8.1800	-6.3324
t = 5 (1979:6)	-1.5957	-4.2058	-5.3632	-6.3895	-7.7886	-10.3394	-8.3092	-6.2702
t = 6 (1981:6)	-1.6781	-4.2775	-5.4016	-6.3787	-8.0394	-10.6849	-8.4234	-6.4120
t = 7 (1983:6)	-1.9247	-4.5175	-5.6764	-6.7194	-8.3968	-11.3095	-8.6333	-6.7397
t = 8 (1985:6)	-2.1696	-4.7813	-5.9397	-6.9660	-8.6129	-10.8390	-8.9600	-6.8955
t = 9 (1987:6)	-2.6630	-5.2443	-6.4042	-7.4775	-9.0645	-11.7419	-9.4041	-7.4285
t = 10 (1989:6)	-3.7787	-6.7866	-7.7529	-8.5674	-9.3639	-9.4271	-9.5970	-7.8962

Note: For choice of time points $\{t_i\}$ and frequency points $\{W_j\}$, see section 3 of the text.

Table 2

Location of Structural Breaks via Subba Rao Test

r	W_r	5% CUSUM Bands	10% CUSUM Bands	S_r	5% CUSUMSQ Bands	1% CUSUMSQ Bands
1 (1971:6)	-0.3261	(-3.5974 , 3.5974)	(-3.2255 , 3.2255)	0.0106	(-3.3464 , 0.05464)	(-0.4421 , 0.6421)
2 (1973:6)	-0.2366	(-4.1970 , 4.1970)	(-3.7630 , 3.7630)	0.0114	(-0.2464 , 0.6464)	(-0.3421 , 0.7421)
3 (1975:6)	0.3869	(-4.7966 , 4.7966)	(-4.3005 , 4.3005)	0.0503	(-0.1464 , 0.7464)	(-0.2421 , 0.8421)
4 (1977:6)	1.1594	(-5.3962 , 5.3962)	(-4.8380 , 4.8380)	0.1099	(-0.0464 , 0.8464)	(-0.1421 , 0.9421)
5 (1979:6)	2.0469	(-5.9958 , 5.9958)	(-5.3755 , 5.3755)	0.1888	(0.0536 , 0.9464)	(-0.0421 , 1.0421)
6 (1981:6)	2.6698	(-6.5954 , 6.5954)	(-5.9130 , 5.9130)	0.2275*	(0.1536 , 1.0464)	(0.0579 , 1.1421)
7 (1983:6)	2.6888	(-7.1950 , 7.1950)	(-6.4505 , 6.4505)	0.2276*	(0.2536 , 1.1464)	(0.1579 , 1.2421)
8 (1985:6)	2.4196	(-7.7946 , 7.7946)	(-6.9879 , 6.9879)	0.2350**	(0.3536 , 1.2464)	(0.2579 , 1.3421)
9 (1987:6)	1.1648	(-8.3942 , 8.3942)	(-7.5255 , 7.5255)	0.3923*	(0.4536 , 1.3464)	(0.3579 , 1.4421)
10 (1989:6)	-0.9674	(-8.9938 , 8.9938)	(-8.0629 , 8.0629)	1.0000	(0.5536 , 1.4464)	(0.4579 , 1.5421)

Notes: (i) The reference value k for the calculation of W_r and S_r (see equations (9) and (10)) is based on the 19 frequencies mentioned in (12). This is given by k = -6.7502

(ii) The CUSUM and CUSUMSQ bands are given by equations (13) - (16)

(iii) (*) and (**) indicate structural breaks at 5% and 1% levels of significance respectively

(iv) The CUSUMSQ test is a two tailed test and thus the values for C_a (in this case) are 0.4461 at the 5% significance level, and 0.5421 at the 1% significance level the reference value persists with 1987:6 indicating a break too (but at 5% significance level)[1].

A valuable insight into the nature of these tests has been provided by Dufour (1982) who notes that the CUSUM test is mainly aimed at detecting "systematic" breaks whereas the CUSUMSQ test is more oriented towards the recording of "haphazard rather than systematic" departures from the structure. In our context, a structural break identified by the CUSUM test would refer to a consistent movement of the log evolutionary spectrum in one direction away from the reference value k, and it could be conjectured that this would happen if the underlying series (velocity) was dominated by deterministic trends. On the other hand, the CUSUMSQ test simply totes up the deviations on either side of the reference value of the log of the evolutionary spectrum and in this case any structural break would be symptomatic of the underlying series being dominated by stochastic trends.[2]

Further information about the nature of the structural break may be inferred from a frequency-wise analysis (as mentioned in section 2.2). Such an analysis for the series

Table 3

Summary of Main Results

Series	M1VFRA1	M1VFRA2	M1VITA	M1VNETH	M1VDEU	M1VUK	M4VUK	M1VUS	M3VUS
Dates of Structural Breaks:									
(i) CUSUM (10% Significance)	-	-	-	-	-	-	-	-	-
(ii) CUSUM (5% Significance)	-	-	-	-	-	-	-	-	-
(iii) CUSUM (1% Significance)	-	-	-	-	-	-	-	-	-
(iv) CUSUMSQ (5% Significance)	-	-	1972:6	1986:6	-	1983:6 1985:6 1987:6	-	-	1972:6 to 1986.6
(v) CUSUMSQ (1% Significance)	-	-	-	-	-	1985:6	-	-	1976:6 to 1984:6
Frequency at which structural breaks occur:									
(i) CUSUMSQ (5% Significance)	-	-	19π/20	16π/20	-	16π/20. π/20	-	-	π/20, 4π/20. 7π/20, 16π/20
(ii) CUSUMSQ (1% Significance)	-	-	-	-	-	π/20	-	-	π/20. 4π/20. 7π/20

Notes: The frequency-wise breaks were examined via the CUSUMSQ test as well as the CUSUM test. However, no breaks were identified by the latter test and hence this information is omitted here

(M1VUK) indicated structural breaks (via the CUSUMSQ test) at the frequency component ($16\pi/20$) at 1983:6, corresponding to a cycle of 2.5 months, as also at the frequency component ($\pi/20$) at 1985:6 (which corresponds to a cycle of 40 months).[3]

The upshot of the preceding discussion seems to be that the series M1VUK was subject to two significant structural breaks - the one in 1983:6 being confined to the short-run characteristics of the series with the one in 1985:6 reflecting a more permanent or long-run break. Both breaks are likely to be mirroring stochastic shifts rather than deterministic or steady movements.

Passing on to Table 3, we find that our test did not discover any structural breaks in the case of the 2 velocity series for France (M1VFRA1 & M1VFRA2), the velocity series for Germany (M1VDEU), the narrow money velocity series for the U.S. (M1VUS), and the broad money velocity series for the U.K. (MQVUK). For the other series, structural breaks were identified by the CUSUMSQ test, but not by the CUSUM test, thus pointing to stochastic rather than deterministic shifts in all cases. The breaks in M1VITA and M1VNETH are at high frequencies corresponding to periods of 2.1 months and 2.5 months respectively - reflecting changes in the short-term characteristics of the series. The structural breaks in M3VUS, apart from being highly significant, are spread across a wide band of frequencies, implying changes in long-term as well as short-term characteristics of the series.

Conclusions

In this paper, the issue of instability of the demand for money function is subjected to a fresh scrutiny based on certain recent developments in spectral analysis. Such an inquiry, we felt, was warranted by the lack of unanimity on empirical evidence relating to instability. On methodological grounds, the Subba Rao (1981) test that we have used here has the merit of being distribution-free (and hence not insisting on Gaussianity of the original series) and also of being able to attribute the breaks to different frequencies.

Our conclusions throw interesting light on the instability issue. Firstly it does not seem to be as pervasive as usually supposed; secondly, even where breaks occur, these are usually in the short-term characteristics of the series and hence unlikely to be of permanent significance. The only significant breaks of a long-term importance occur in the U.S. velocity series based on M3 (M3VUS) and in the U.K. velocity series based on M1 (M1VUK). The last finding seems to accord well with the received literature (see Leventakis & Brissimis (1991) and Artis & Lewis (1991) for the U.S. and U.K. evidence respectively in the 1980s).

Endnotes

1. Note that, by definition, the last value of S_r will always lie within the CUSUMSQ bands, whatever the level of significance.

2. We emphasize the conjectual nature of this observation. Several simulations by one of the authors seem to be supportive of this conjecture.

3. Both shifts are detected by the CUSUMSQ test at the 5% significance level. In conducting the frequency-wise analysis, the choice of k for each frequency is based on the average of the log evolutionary spectrum at a fixed frequency evaluated at points t_i spaced 12 months apart (i.e. a total of 20 points in the case of M1VUK).

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Appendix I: Data Definition

Monthly data for the velocity of circulation in each country was constructed as the ratio of nominal income to the end-of-period stock of money balances. Nominal income was calculated as the product of the Index of Industrial Production and an appropriately defined price index, scaled to the value of nominal GDP in 1985. The price index chasen was, with the exception of France, the Producer Price Index; for France the Wholesale Price Index was used. Country specific details for the data are given below.

France: Two alternative series were constructed:

(i) M1VFRA1 (1960:1 to 1989:12) Source: IMF database from 1960:1 to 1969:11 and OECD database from 1969:12 to 1989:12 (360 observations);

(ii) M1VFRA2 (1960:1 to 1989:12) Source: IMF database from 1960:1 to 1978:3 and OECD database from 1978:4 to 1989:12 (360 observations);

Italy: M1VITA (1961:12 to 1990:1) (349 observations);

Netherlands: M1VNETH (1961:12 to 1990:12) (355 observations);

W.Germany: M1VDEU (1961:12 to 1990:1) (349 observations);

U.K.: Two alternative series were constructed:

(i) M1VUK (1971:6 to 1989:7) (218 observations);

(ii) MQVUK (1971:6 to 1989:11) (222 observations), based upon

M1+Quasi money balances;

U.S.A.: Two alternative series were constructed:

(i) M1VUS (1960:1 to 1990:6) (366 observations);

(ii) M3VUS (1960:1 to 1990:6) (366 observations) based upon the M3 definition.

Appendix II.1 Log of Evolutionary Spectrum: France - Definition 1

Frequency Time	$\pi/20$	$4\pi/20$	$7\pi/20$	$10\pi/20$	$13\pi/20$	$16\pi/20$	$19\pi/20$	Y_i
t = 6 (1960:6)	-1.8419	-4.5311	-5.6673	-6.3195	-7.4401	-7.6546	-7.6645	-5.8741
t = 30 (1962:6)	-1.7659	-4.3433	-5.4050	-6.4621	-7.9609	-8.9656	-8.3443	-6.1782
t = 54 (1964:6)	-1.7955	-4.3928	-5.3674	-6.3439	-7.7225	-8.3943	-8.1780	-6.0278
t = 78 (1966:6)	-1.7520	-4.3435	-5.3255	-6.2671	-7.6000	-8.2857	-8.0376	-5.9445
t = 102 (1968:6)	-1.8281	-4.3327	-5.4460	-6.4053	-7.3708	-8.0787	-8.2418	-5.9576
t = 126 (1970:6)	-1.1938	-3.8276	-4.8656	-5.8847	-6.9099	-7.9437	-7.5431	-5.4526
t = 150 (1972:6)	-1.4421	-3.9825	-5.0147	-6.0180	-7.3718	-8.3976	-7.7576	-5.7120
t = 174 (1974:6)	-0.9683	-3.6149	-4.6636	-5.6507	-7.1452	-7.6226	-7.3190	-5.2835
t = 198 (1976:6)	-1.4051	-4.0112	-5.0771	-6.0247	-7.3625	-8.2299	-7.5791	-5.6699
t = 222 (1978:6)	-1.7131	-4.2627	-5.2349	-6.2517	-7.7890	-8.4670	-7.8939	-5.9446
t = 246 (1980:6)	-1.6979	-4.2828	-5.4148	-6.3814	-7.9161	-8.8250	-8.0782	-6.0852
t = 270 (1982:6)	-1.7664	-4.3490	-5.4497	-6.4825	-7.9171	-9.1878	-8.1475	-6.1857
t = 294 (1984:6)	-1.6715	-4.2876	-5.3943	-6.3581	-8.0374	-8.9871	-7.8977	-6.0905
t = 318 (1986:6)	-2.0646	-4.6244	-5.7126	-6.6785	-8.2781	-8.8613	-8.3272	-6.3638
t = 342 (1988:6)	-1.8384	-4.4702	-5.5642	-6.5041	-8.1430	-9.0013	-8.0986	-6.2314

Notes: For choice of time points $\{t_i\}$ and frequency points $\{w_j\}$, see section 3 of the text.

Appendix II.2 Log of Evolutionary Spectrum: France - Definition 2

Frequency	Time	$\pi/20$	$4\pi/20$	$7\pi/20$	$10\pi/20$	$13\pi/20$	$16\pi/20$	$19\pi/20$	Y_i
t=6	(1960:6)	-1.8419	-4.5311	-5.6673	-6.3195	-7.4401	-7.6546	-7.6645	-5.8741
t=30	(1962:6)	-1.7659	-4.3433	-5.4050	-6.4621	-7.9609	-8.9656	-8.3443	-6.1782
t=54	(1964:6)	-1.7955	-4.3928	-5.3674	-6.3439	-7.7225	-8.3943	-8.1780	-6.0278
t=78	(1966:6)	-1.7520	-4.3435	-5.3255	-6.2671	-7.6000	-8.2857	-8.0376	-5.9445
t=102	(1968:6)	-1.8281	-4.3329	-5.4462	-6.4051	-7.3715	-8.0812	-8.2428	-5.9583
t=126	(1970:6)	-1.1984	-3.8324	-4.8683	-5.8873	-6.9230	-7.9432	-7.5440	-5.4566
t=150	(1972:6)	-1.4445	-3.9847	-5.0175	-6.0224	-7.3758	-8.4015	-7.7611	-5.7154
t=174	(1974:6)	-1.0087	-3.6762	-4.7450	-5.6142	-7.2185	-7.7212	-7.3404	-5.3320
t=198	(1976:6)	-1.4005	-4.0064	-5.0675	-6.0404	-7.3703	-8.2036	-7.5880	-5.6681
t=222	(1978:6)	-1.6630	-4.2023	-5.2860	-6.1999	-7.8098	-8.3518	-8.0388	-5.9359
t=246	(1980:6)	-1.6979	-4.2828	-5.4148	-6.3814	-7.9161	-8.8250	-8.0782	-6.0852
t=270	(1982:6)	-1.7664	-4.3490	-5.4494	-6.4825	-7.9171	-9.1878	-8.1475	-6.1857
t=294	(1984:6)	-1.6715	-4.2876	-5.3943	-6.3581	-8.0374	-8.9871	-7.8977	-6.0905
t=318	(1986:6)	-2.0646	-4.6244	-5.7126	-6.6785	-8.2781	-8.8613	-8.3272	-6.3638
t=342	(1988:6)	-1.8384	-4.4702	-5.5642	-6.5041	-8.1430	-9.0013	-8.0986	-6.2314

Note: For the choice of time points $\{t_i\}$ and frequency points $\{W_i\}$, see section 3 of the text.

Appendix II.3 Log of Evolutionary Spectrum: Netherlands

Frequency	Time	$\pi/20$	$4\pi/20$	$7\pi/20$	$10\pi/20$	$13\pi/20$	$16\pi/20$	$19\pi/20$	Y_i
t = 7	(1962:6)	-13.1165	-15.8429	-16.9194	-17.7940	-18.7492	-19.2403	-19.1328	-17.2565
t = 31	(1964:6)	-12.7706	-15.3618	-16.4517	-17.6348	-19.2364	-22.1465	-19.5707	-17.5961
t = 55	(1966:6)	-12.7054	-15.3044	-16.3951	-17.6036	-19.1597	-22.3811	-19.4690	-17.5740
t = 79	(1968:6)	-12.6813	-15.2757	-16.3863	-17.6014	-19.0406	-21.5774	-19.2643	-17.4039
t = 103	(1970:6)	-12.5997	-15.2119	-16.3194	-17.5248	-18.9175	-22.0833	-19.2688	-17.4179
t = 127	(1972:6)	-12.6835	-15.2839	-16.4086	-17.5919	-19.0949	-21.2930	-19.2907	-17.3781
t = 151	(1974:6)	-12.6608	-15.2548	-16.4019	-17.5689	-19.0463	-21.4339	-19.1778	-17.3635
t = 175	(1976:6)	-12.7359	-15.2879	-16.4116	-17.6271	-18.9562	-20.7142	-19.2361	-17.2813
t = 199	(1978:6)	-12.8788	-15.4684	-16.6127	-17.8045	-18.9858	-21.3658	-19.2730	-17.4842
t = 223	(1980:6)	-12.8962	-15.4723	-16.6451	-17.8357	-19.2660	-21.4044	-19.4929	-17.5732
t = 247	(1982:6)	-12.8129	-15.3761	-16.5793	-17.7534	-19.1562	-21.0383	-19.2877	-17.4291
t = 271	(1984:6)	-12.9751	-15.5421	-16.7117	-17.8859	-19.2692	-21.3009	-19.5462	-17.6044
t = 295	(1986:6)	-12.9902	-15.5743	-16.7647	-17.8915	-19.4118	-20.5110	-19.7241	-17.5525
t = 319	(1988:6)	-13.3133	-15.8397	-17.0532	-18.1714	-19.6845	-21.6179	-20.0195	-17.9571
t = 343	(1990:6)	-13.7435	-16.3452	-17.4530	-18.4659	-19.1521	-19.8067	-19.5439	-17.7872

Note: For choice of the time points $\{t_i\}$ and frequency points $\{W_j\}$, see section 3 of the text.

Appendix II.4 Log of Evolutionary Spectrum: Italy

Frequency	Time	$\pi/20$	$4\pi/20$	$7\pi/20$	$10\pi/20$	$13\pi/20$	$16\pi/20$	$19\pi/20$	Y_i
t = 7	(1962:6)	-4.4532	-7.1401	-8.1752	-9.2436	-9.9510	-10.6733	-10.3227	-8.5656
t = 31	(1964:6)	-3.9738	-6.5873	-7.7454	-8.8207	-10.0805	-11.3613	-10.4573	-8.4323
t = 55	(1966:6)	-4.1101	-6.7092	-7.8248	-8.8354	-10.3848	-11.2166	-10.3617	-8.4918
t = 79	(1968:6)	-4.3730	-6.9733	-8.0503	-9.0291	-10.3235	-11.2510	-10.6471	-8.6639
t = 103	(1970:6)	-4.8165	-7.3634	-8.5322	-9.4429	-10.7043	-11.4048	-10.8672	-9.0188
t = 127	(1972:6)	-5.2839	-7.8367	-8.8531	-9.8367	-11.1134	-11.7632	-11.3613	-9.4355
t = 151	(1974:6)	-4.8856	-7.5127	-8.5437	-9.3856	-10.5317	-10.9765	-10.8697	-8.9579
t = 175	(1976:6)	-4.7314	-7.3532	-8.3354	-9.2141	-10.4851	-10.6069	-10.3813	-8.7296
t = 199	(1978:6)	-4.9526	-7.5157	-8.5138	-9.3259	-10.7513	-11.0117	-10.7230	-8.9706
t = 223	(1980:6)	-4.8405	-7.4371	-8.3946	-9.1223	-10.4756	-10.5462	-10.3960	-8.7446
t = 247	(1982:6)	-4.8111	-7.4124	-8.3232	-8.9697	-10.2098	-10.2621	-9.9394	-8.5611
t = 271	(1984:6)	-4.9256	-7.5285	-8.4370	-9.0671	-10.5508	-10.5783	-10.2215	-8.7584
t = 295	(1986:6)	-5.1204	-7.6892	-8.6387	-9.2543	-10.5716	-10.6324	-10.4410	-8.9068
t = 319	(1988:6)	-5.1325	-7.7117	-8.6808	-9.3421	-10.6667	-10.7330	-10.5359	-8.9718

Note: For choice of time points $\{t_i\}$ and frequency points $\{W_j\}$, see section 3 of the text.

Appendix II.5 Log of Evolutionary Spectrum: United Kingdom M1+Quasi Money

Frequency	Time	$\pi/20$	$4\pi/20$	$7\pi/20$	$10\pi/20$	$13\pi/20$	$16\pi/20$	$19\pi/20$	Y_t
t = 1	(1971:6)	-4.1985	-7.0059	-8.0312	-8.6895	-9.6906	-9.8714	-9.8315	-8.1884
t = 25	(1973:6)	-3.7199	-6.2331	-7.3749	-8.4599	-10.0200	-12.7966	-10.3426	-8.4210
t = 49	(1975:6)	-3.6148	-6.1740	-7.3209	-8.3475	-10.0345	-12.2602	-10.2912	-8.2919
t = 73	(1977:6)	-3.2207	-5.7696	-6.9535	-7.9802	-9.6599	-12.5719	-9.7298	-7.9837
t = 97	(1979:6)	-3.1078	-5.6516	-6.8514	-7.8471	-9.3551	-11.6528	-9.8438	-7.7585
t = 121	(1981:6)	-3.4168	-5.9485	-7.1420	-8.1011	-9.7233	-12.0275	-10.0848	-8.0634
t = 145	(1983:6)	-3.7639	-6.3090	-7.5033	-8.5083	-10.1695	-13.3617	-10.3941	-8.5728
t = 169	(1985:6)	-3.8855	-6.4334	-7.6337	-8.6448	-10.2953	-13.0977	-10.5640	-8.6506
t = 193	(1987:6)	-4.2792	-6.8000	-8.0147	-9.0420	-10.6468	-13.1531	-10.9584	-8.9849
t = 217	(1989:6)	-5.1156	-7.8388	-8.8800	-9.8402	-10.7392	-11.4448	-11.1370	-9.2851

Note: For choice of time points $\{t_i\}$ and frequency points $\{W_j\}$, see section 3 of the text.

Appendix II.6 Log of Evolutionary Spectrum: United States M1

Frequency	Time	$\pi/20$	$4\pi/20$	$7\pi/20$	$10\pi/20$	$13\pi/20$	$16\pi/20$	$19\pi/20$	Y_i
t = 6	(1960:6)	-2.3401	-5.0480	-6.1019	-7.0478	-8.0115	-8.5137	-8.3545	-6.4882
t = 30	(1962:6)	-1.8728	-4.4540	-5.6395	-6.7847	-8.3590	-12.4848	-8.7019	-6.8995
t = 54	(1964:6)	-1.7383	-4.3022	-5.4944	-6.6369	-8.1978	-12.0389	-8.5508	-6.7085
t = 78	(1966:6)	-1.4945	-4.0824	-5.2741	-6.4106	-7.9816	-11.8527	-8.3277	-6.4891
t = 102	(1968:6)	-1.4711	-4.0386	-5.2297	-6.3683	-7.9150	-12.2687	-8.2855	-6.5110
t = 126	(1970:6)	-1.4843	-4.0574	-5.2455	-6.3860	-7.9354	-12.4242	-8.2935	-6.5466
t = 150	(1972:6)	-1.3625	-3.9135	-5.1002	-6.2420	-7.7578	-10.9113	-8.1221	-6.2014
t = 174	(1974:6)	-0.9768	-3.5451	-4.7356	-5.8782	-7.4180	-10.7338	-7.7923	-5.8685
t = 198	(1976:6)	-0.8508	-3.4334	-4.6211	-5.7566	-7.3132	-11.1263	-7.6730	-5.8249
t = 222	(1978:6)	-0.6086	-3.1780	-4.3691	-5.5004	-7.0549	-11.0633	-7.4158	-5.5986
t = 246	(1980:6)	-0.3591	-2.9419	-4.1392	-5.2753	-6.8247	-11.2671	-7.2021	-5.4299
t = 270	(1982:6)	-0.4603	-3.0204	-4.2199	-5.3547	-6.8975	-10.7258	-7.2590	-5.4197
t = 294	(1984:6)	-0.5309	-3.1390	-4.3317	-5.4691	-7.0333	-11.0928	-7.3903	-5.5696
t = 318	(1986:6)	-0.8983	-3.4520	-4.6570	-5.7823	-7.3318	-10.6549	-7.6681	-5.7778
t = 342	(1988:6)	-0.8883	-3.4687	-4.6726	-5.8066	-7.3743	-11.4763	-7.7113	-5.9140
t = 366	(1990:6)	-1.4446	-4.2321	-5.1637	-6.1816	-6.6913	-7.2905	-7.2390	-5.4633

Note: For choice of time points $\{t_i\}$ and frequency points $\{W_j\}$, see section 3 of the text.

Appendix II.7 Log of Evolutionary Spectrum: United States M3

Frequency	Time	$\pi/20$	$4\pi/20$	$7\pi/20$	$10\pi/20$	$13\pi/20$	$16\pi/20$	$19\pi/20$	Y_i
t = 6	(1960:6)	-3.9367	-6.6553	-7.7022	-8.6390	-9.5733	-10.0409	-9.9198	-8.0667
t = 30	(1962:6)	-3.6608	-6.2502	-7.4352	-8.5832	-10.1633	-14.4479	-10.4915	-8.7189
t = 54	(1964:6)	-3.7365	-6.3072	-7.4974	-8.6452	-10.2245	-14.1280	-10.5533	-8.7275
t = 78	(1966:6)	-3.6353	-6.2298	-7.4224	-8.5627	-10.1445	-13.9783	-10.4689	-8.6346
t = 102	(1968:6)	-3.6989	-6.2659	-7.4596	-8.6003	-10.1690	-14.2569	-10.5050	-8.7079
t = 126	(1970:6)	-3.7375	-6.3294	-7.5205	-8.6602	-10.2182	-13.8651	-10.5504	-8.6973
t = 150	(1972:6)	-3.8679	-6.4255	-7.6201	-8.7648	-10.3229	-13.7794	-10.6408	-8.7745
t = 174	(1974:6)	-3.6741	-6.2463	-7.4468	-8.5925	-10.1557	-13.4442	-13.4906	-9.0072
t = 198	(1976:6)	-3.7158	-6.3034	-7.4953	-8.6357	-10.2216	-14.2448	-10.5394	-8.7366
t = 222	(1978:6)	-3.6284	-6.1963	-7.3929	-8.5343	-10.1211	-14.3510	-10.4265	-8.6644
t = 246	(1980:6)	-3.4976	-6.0873	-7.2886	-8.4323	-10.0098	-14.6442	-10.3406	-8.6143
t = 270	(1982:6)	-3.7365	-6.2867	-7.4949	-8.6344	-10.1999	-13.7694	-10.5109	-8.6618
t = 294	(1984:6)	-3.8523	-6.4522	-7.6525	-8.7944	-10.3760	-14.3277	-10.6996	-8.8792
t = 318	(1986:6)	-0.0721	-2.6754	-3.6229	-4.3683	-5.1827	-5.4254	-5.3817	-3.8183
t = 342	(1988:6)	-0.8731	-2.8094	-3.1721	-3.1650	-4.3591	-3.8854	-3.2391	-3.0719
t = 366	(1990:6)	-4.6927	-7.4773	-8.4196	-9.4016	-9.9652	-10.4913	-10.4891	-8.7053

Note: For choice of time points $\{t_i\}$ and frequency points $\{W_j\}$, see section 3 of the text.

Appendix II.8 Log of Evolutionary Spectrum: West Germany M1

Frequency	Time	$\pi/20$	$4\pi/20$	$7\pi/20$	$10\pi/20$	$13\pi/20$	$16\pi/20$	$19\pi/20$	Y_i
t = 7	(1962:6)	-1.5161	-4.2284	-5.3052	-6.2326	-7.1730	-7.5297	-7.4798	-5.6378
t = 31	(1964:6)	-1.2679	-3.8556	-4.9645	-6.1475	-7.6259	-10.1478	-8.0495	-6.0084
t = 55	(1966:6)	-1.2899	-3.8907	-4.9977	-6.1721	-7.7786	-10.3933	-8.1135	-6.0908
t = 79	(1968:6)	-1.4373	-4.0068	-5.1101	-6.3093	-7.7799	-10.3008	-8.1644	-6.1584
t = 103	(1970:6)	-1.2959	-3.9050	-5.0005	-6.1920	-7.7106	-10.0867	-8.1089	-6.0428
t = 127	(1972:6)	-1.3472	-3.9357	-5.0309	-6.1847	-7.7962	-9.8932	-8.1298	-6.0454
t = 151	(1974:6)	-1.3305	-3.9343	-5.0319	-6.1646	-7.6971	-9.7688	-8.0863	-6.0019
t = 175	(1976:6)	-1.4714	-4.0356	-5.1200	-6.2714	-7.8185	-10.0659	-8.1901	-6.1390
t = 199	(1978:6)	-1.6108	-4.2100	-5.2921	-6.4734	-7.9865	-10.1339	-8.3872	-6.2991
t = 223	(1980:6)	-1.6998	-4.2991	-5.4015	-6.5589	-7.9427	-9.9954	-8.4992	-6.3424
t = 247	(1982:6)	-1.5459	-4.1500	-5.2170	-6.3634	-7.8567	-10.0163	-8.3253	-6.2107
t = 271	(1984:6)	-1.6638	-4.2315	-5.3153	-6.4628	-7.8701	-9.7489	-8.4173	-6.2442
t = 295	(1986:6)	-1.6163	-4.2404	-5.3163	-6.4022	-8.0675	-9.8767	-8.3936	-6.2733
t = 319	(1988:6)	-1.9538	-4.5263	-5.6036	-6.7752	-8.2725	-10.0984	-8.7097	-6.5628

Note: For choice of time points $\{t_i\}$ and frequency points $\{W_j\}$, see section 3 of the text.