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THE PROCESS BY WHICH THE DOLLAR WILL FALL: THE EFFECT OF FORWARD-LOOKING CONSUMERS

Karlygash Kuralbayeva and David Vines

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Karlygash Kuralbayeva, Paris School of Economics, Université de Paris I
David Vines, Oxford University, Australian National University and CEPR

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Centre for Economic Policy Research
53–56 Gt Sutton St, London EC1V 0DG, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

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This paper extends the analysis of the forthcoming fall in the dollar by Blanchard, Giavazzi and Sà 2005), using a model which incorporates forward-looking consumers. It provides additional underpinnings for the idea of a rapid adjustment in the value of the dollar. We analyze what will happen to the dollar value when forward-looking consumers, in anticipation of the reduction in the current account deficit, cut their consumption. We show that the real interest rate must fall as a result, and that this causes the exchange rate to fall more initially. However we also show that the interest rate does not fall greatly initially, and so that these effects are not large. But they add to pressures causing a rapid fall in the dollar.

JEL Classification: F32, F41 and G15
Keywords: imperfect substitutability, net foreign assets and valuation effects

Karlygash Kuralbayeva
Centre d'Economie de la Sorbonne
106/112 Boulevard de l'Hopital
75647 Paris Cedex 13
FRANCE

Email:
Karlygash.Kuralbayeva@univ-paris1.fr

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David Vines
Department of Economics
University of Oxford
Manor Road Building
Manor Road
OXFORD OX1 3UQ

Email:
david.vines@economics.ox.ac.uk

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www.cepr.org/pubs/new-dps/dplist.asp?authorid=100459

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‘The Process by which the Dollar will fall: the effect of Forward-Looking Consumers*

Karlygash Kuralbayeva, Paris School of Economics/University Paris I†
David Vines, Oxford University, Australian National University and CEPR‡

May 25, 2009

Abstract

This paper extends the analysis of the forthcoming fall in the dollar by Blanchard, Giavazzi and Sá (2005), using a model which incorporates forward-looking consumers. It provides additional underpinnings for the idea of a rapid adjustment in the value of the dollar. We analyze what will happen to the dollar value when forward-looking consumers, in anticipation of the reduction in the current account deficit, cut their consumption. We show that the real interest rate must fall as a result, and that this causes the exchange rate to fall more initially. However we also show that the interest rate does not fall greatly initially, and so that these effects are not large. But they add to pressures causing a rapid fall in the dollar.

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1 Introduction

The US still has a large current account deficit. The present paper considers the process of adjustment which will be required in the US economy to correct this deficit.

Prior to the current crisis, there was much agreement that the dollar must eventually fall to reverse global imbalances and many papers have estimated the size of the dollar decline needed to eliminate the current account deficit [1]. In the developing financial crisis, the dollar has not fallen, possibly because of an increase in the demand for US assets, as a precautionary defence against an increase in uncertainty. But it seems appropriate to examine the process by which this adjustment might happen.

We analyze the implications for the adjustment process resulting from consumers cutting their consumption, in anticipation of the reduction in real income which will be caused by

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†Address: Centre d'Economie de la Sorbonne 106/112 Boulevard de l'Hopital, 75647 Paris Cedex 13; email: Karlygash.Kuralbayeva@univ-paris1.fr

‡Address: Department of Economics, Manor Road Building, Manor Road, Oxford, OX1 3UQ; email: david.vines@economics.ox.ac.uk

[1]See in particular Obstfeld and Rogoff (2004 and 2005).

the adjustment. This fall will come from the depreciation of the dollar necessary to bring the adjustment about. Our aim is to describe what happens when consumers, realizing that their real income has to fall as the current account adjusts, cut their consumption. When they do this, that means that, to ensure that full employment of resources is maintained, the real interest rate must fall. This fall in the real interest rate will reduce the reward on home assets, reducing the demand for them, and so will cause a larger immediate fall in the real exchange rate. This paper thus adds to the arguments, put forward by Krugman (2007) that, when the dollar starts to fall, this fall will be rapid. But we also find that that the fall in the real interest rate will be small, relative to the fall in the exchange rate which is required to correct the current account deficit. This means that we only provide a marginal additional reason for supporting the claim by Krugman.

Our paper is builds on the paper by Blanchard, Giavazzi and Sá (2005), hereafter BGS, the analysis of which is used, in reduced form, by Krugman (2007). This approach relies on an imperfect substitutability between home and foreign assets [2]. We provide microfoundations for this analysis, and we extend their framework, by adding forward-looking consumers and endogenising the interest rate. We do this in such a way as to nest the BGS model in our framework, enabling us to make explicit comparison with the dynamics presented in that paper, and with the discussion of dynamics presented by Krugman (2007). There are other extensions of the BGS framework provided by Meredith (2007) and Sá and Viani (2007), but these been used for analysis of slightly different issues, and do not provide the nested comparison. Faruqee et al. (2005) analyze the unwinding of the current account deficit within a complex multi-country dynamic general equilibrium model, but without an explicit specification of international portfolio effects which we consider. Consumers are forward-looking in our setup, and we study the adjustment process, once it is recognized that adjustment will begin. Our analysis does not consider the implications of revisions in the expectations of myopic investors considered by Krugman (2007), once they come to recognize that a depreciation will be needed.

Our framework enables us to investigate valuation effects, which have been emphasized by a number of recent studies on current account dynamics [3]. Our model suggests that unanticipated valuation effects arising from unanticipated exchange rate fluctuations can be sizable, but that anticipated valuation effects are quantitatively smaller. Such anticipated valuation effects have two components in our model. The first of these results from expected differences in return on assets in a country's portfolio, during the process of adjustment. In addition, there are also anticipated changes in exchange rate during the process of current account adjustment, since adjustments in the exchange rate do not all happen immediately. But, as shown in the paper, the latter component of these anticipated valuation effects plays a larger role than the former one.

The paper is organized as follows. The next section discusses of the argument of the paper and intuitively outlines the key difference between the framework developed in the paper with

[2] There is a growing literature that incorporates basic principles of portfolio choice into conventional dynamic general equilibrium models (e.g., Couerdacier (2005), Kollman (2006), Tille and van Wincoop (2007), Devereux and Sutherland (2008))

[3] Empirical relevance of the valuation effects have been shown in a number of papers (e.g., Tille (2003), Higgins et al. (2005), Lane and Milesi-Ferretti (2005)).

that in the BGS setup. Section 3 sets out the model. Section 4 discusses equilibrium, and the dynamics of adjustment to an exogenous worsening of the trade deficit which requires correction. Section 5 concludes.

2 Imperfect substitutability and the possibility of slow slide

The argument for a slow slide of the dollar, coming from imperfect substitutability, comes from BGS and is as follows.

First BGS suppose that US assets and foreign assets are imperfect substitutes in international portfolios. Second, they suppose that both US residents and foreign residents have home bias in their holdings of portfolios. For simplicity, they suppose that the returns on US and foreign assets are the same in equilibrium – relative returns to holders of these assets can only differ because of gradual changes in the relative prices of these assets. Home bias then means that, in equilibrium, residents in each region desire to hold more than half of their wealth in their own home assets. That, in turn, means that if the US were to increase its external debt (which would reduce US wealth as a proportion of world wealth) then world asset demands would be directed towards foreign assets and away from US assets. As a result, the dollar would have to fall. Conversely, if there is to be a depreciation of the dollar in a move to a new equilibrium, then this will need to be accompanied by an increase in US net debt during the process of transition to equilibrium.

The has important implications for the process of current-account-correction. When the dollar falls, there will be a fall in the value of US assets as a proportion of the total value of world assets. As a result there will be excess demand for US assets. World asset-market equilibrium can only be regained, after such a fall in the dollar, if the US runs a temporary current-account deficit, so as to increase US external debt. The way for the US to do this is for the dollar not to fall initially as far as it will fall in the long run, and then to fall gradually to its final equilibrium. International asset holders will be willing to hold the available US assets, even although these assets are falling in value along the adjustment path, because of the initial excess demand for US assets caused by the initial fall in the dollar. But gradually, along the adjustment path, US debt will be rising and, as a result, because of home bias, world demand for US assets will be falling. Eventually, after a period of US deficits, the dollar will have fallen to its ultimate equilibrium level and returns on the two assets will be equalized.

The BGS model is highly simplified. There is no discussion of any possibility that aggregate demand might differ from aggregate supply. As a result, at all times along the adjustment path, the demand for US goods is assumed to be equal to the supply of US goods. This means that, along this adjustment path, the US trade deficit must exactly equal the opposite of US saving, and any correction of this deficit is met by an increase in US savings. For simplicity BGS assume a constant real interest in the model, so that interest rate changes do not play a role in bringing about this increase in savings. Instead they assume that the government 'takes measures to adjust saving as the trade balance changes – for example by reducing the fiscal deficit as the trade deficit is reduced - so as to maintain output at its natural level'. (op. cit., p. 10). But this story is incomplete, since it asks too much of fiscal policy to suppose that it acts in this way.

In the present paper we extend the analysis as follows. Unlike BGS, we abstract from any adjustment of fiscal policy. But we assume that consumption is endogenous and that consumers follow a forward-looking Euler equation. And we suppose that monetary policy is sufficiently flexible to ensure that at all times along the adjustment path, the interest rate is equal to its flex-price level equilibrium at which the demand for US goods equals the supply of US goods. This is also a simplifying assumption, but it enables us to abstract from inflation, and to continue to use a real model.

In this setup, forward-looking consumers immediately cut their expenditures, as a result of their anticipated long-run fall in real income and in the value of the dollar. If the exchange rate were to immediately fall enough to improve the trade balance as required, and if consumption were to immediately fall so as to make room for this improvement, then there would be an immediate fast fall in the dollar and no change in the interest rate. But for reasons analyzed by BGS and described above, due to imperfect substitutability between US and foreign assets, the dollar will not immediately fall all the way to this point. That is because, if it did, there would be excess demand for US assets.

We thus present an analysis of the process of current account correction which runs as follows. Forward-looking consumers immediately cut their expenditures, but the dollar does not fall all the way to its equilibrium value. This means that the current account will not improve immediately by the required amount. That in turn means that there must be a reduction in the interest rate to moderate the fall in consumption and so maintain aggregate demand equal to aggregate supply. But, in turn, that fall in the interest rate will cause the dollar to fall more immediately than it would have done in the BGS analysis (in which demand in the US was matched to supply without any change in the interest rate.)

In this paper we show how - as in the BGS analysis - the extent of the initial fall of the dollar depends on the degree of international capital mobility: the more this is the less will US debt need to increase to compensate for the effects of home bias. The new contribution is to show that the initial fall in the dollar is larger than that described by BGS, because of the fall in the interest rate. That fall means that the outcome is more likely to be a fast fall, even allowing for the fact that home and foreign assets are imperfect substitutes in portfolios. For any degree of substitutability, the initial fall in the dollar will be larger. However the effect of the fall in the interest rate is not large. That is because the required cut in the interest rate to ensure full employment of resources is quite small relative to the dynamic changes in the exchange rate along the adjustment path. This is true for all plausible values of the capital mobility parameter.

We write at a time when consumption in the US has collapsed, and savings have risen, for reasons which most attribute to a collapse in household wealth which has resulted from the subprime housing crisis. The argument of this paper suggests another reason: consumption may also have fallen because consumers have come to realize that such a fall will be necessary as the US corrects its current account position. That, in turn, suggests that part of the adjustment process studied in this paper may already be underway, and that a large movement in the dollar may follow it, when any precautionary demand for US assets comes to be moderated.

3 Model

3.1 Consumers

Our model of the household sector is familiar from Galí and Monacelli (2002), which is adapted for a two-country world. Country $J \in \{A, B\}$ is inhabited by a large number of individuals, who consume a basket of goods C, and derive utility from per capita government consumption G. Individuals' maximization problem is

$$U_J = \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\log C_{Jt} + \log G_{Jt}\right). \tag{3.1}$$

An individual chooses optimal consumption and work effort to maximize criterion (3.1) subject to the demand system and intertemporal budget constraint:

$$\sum_{s=t}^{\infty} \mathcal{E}_t(Q_{t,s}^J(C_{Jt}P_{Jt})) = \mathcal{A}_{Jt} + \sum_{s=t}^{\infty} \mathcal{E}_t Q_{t,s}^J \left(1 - \tau_{Jt}\right) Y_{Jt}, \tag{3.2}$$

where $P_{Jt}C_{Jt}$ is aggregate nominal consumption, $\mathcal{A}_{Jt}$ are nominal financial assets of a household and Y_{Jt} is output. Here τ_{Jt} is a tax rate on income. Price P_{Jt} is a price of consumption basket C_{Jt}. $Q_{t,t+1}^J$ is the stochastic discount factor which determines the price in period t to the individual of being able to carry a state-contingent amount $\mathcal{A}_{Jt+1}$ of wealth into period $t+1$. The return on financial wealth ρ_{Jt} has the following representation in terms of the stochastic discount factor:

$$\mathcal{E}_t(Q_{t,t+1}^J) = \frac{1}{(1 + \rho_{Jt})}.$$

Each individual consumes the same basket of goods. The optimization requires that the household exhaust its intertemporal budget constraint and, in addition, the household's wealth accumulation must satisfy the no Ponzi game condition:

$$\lim_{s \to \infty} \mathcal{E}_t \left(Q_{t,s}^J \mathcal{A}_{Js}\right) = 0. \tag{3.3}$$

Household optimization leads to the conventional Euler equation:

$$\beta(1 + \rho_{Jt})\mathcal{E}_t\{\frac{C_{Jt}}{C_{Jt+1}} \frac{P_{Jt}}{P_{Jt+1}}\} = 1 \tag{3.4}$$

Additionally, aggregate (nominal) asset accumulation is given by

$$\mathcal{A}_{Jt+1} = (1 + \rho_{Jt}) \left(\mathcal{A}_{Jt} + (1 - \tau_{Jt})Y_{Jt} - P_{Jt}C_{Jt}\right) \tag{3.5}$$

3.2 Exchange Rates and Prices

We define the nominal exchange rate E_t as the price of currency in country A in terms of currency in country B.We also define the real exchange rate, S_t, as the price of import in country A to the the price of home goods in country A. As the law of one price holds ($E_t = P_{BFt}/P_{AHt}$), then

$$S_t = \frac{P_{AFt}}{P_{AHt}} = \frac{P_{BHt}}{P_{BFt}}.$$

so that an increase in S_t corresponds to *real depreciation* of the exchange rate. We assume that country J consumes $1-\mu_{Jt}$ of domestically produced goods and μ_{Jt} of goods produced in the other country, so that CPI price level in country J is defined as

$$P_{Jt} = \left(\left(1-\mu_{Jt}\right) P_{JHt}^{1-\eta} + \mu_{Jt} P_{JFt}{}^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{3.6}$$

where η measures the elasticity of substitution between domestic and foreign goods.

3.3 Fiscal Constraints

The government in country J buys goods (G_{Jt}), taxes income (with tax rate τ_{Jt}) and issues nominal debt $\mathcal{B}_{Jt}$. The evolution of the nominal debt stock can be written as:

$$\mathcal{B}_{Jt+1} = (1+i_{Jt})(\mathcal{B}_{Jt} + P_{JHt}G_{Jt} - \tau_{Jt}P_{JHt}Y_t) \tag{3.7}$$

3.4 Financial Markets

We assume the two debts are not perfect substitutes and the excess return Z_t is defined as:

$$Z_t = \frac{\left(1+i_{At}\right) E_{t+1}}{\left(1+i_{Bt}\right) E_t} \tag{3.8}$$

where i_{Jt} is nominal interest rate on debt in country J.

We assume that household in each country prefers to keep its country's debt if returns are the same, i.e. there is a home bias. Let α_{Jt} is a proportion of financial wealth $\mathcal{A}_{Jt}$ that is invested in bonds $\mathcal{B}_{Jt}$. Then it follows that

$$\alpha_{At}\mathcal{A}_{At} + \left(1-\alpha_{Bt}\right)\frac{\mathcal{A}_{Bt}}{E_t} = \mathcal{B}_{At} \tag{3.9}$$

$$\left(1-\alpha_{At}\right)\mathcal{A}_{At} + \alpha_{Bt}\frac{\mathcal{A}_{Bt}}{E_t} = \frac{\mathcal{B}_{Bt}}{E_t} \tag{3.10}$$

It follows, of course, that

$$\mathcal{A}_{At} + \frac{\mathcal{A}_{Bt}}{E_t} = \mathcal{B}_{At} + \frac{\mathcal{B}_{Bt}}{E_t}$$

Total financial wealth of country J evolves as

$$\mathcal{A}_{Jt+1} = \left(1+\rho_{Jt}\right)\left(\mathcal{A}_{Jt} + \left(1-\tau_{Jt}\right)Y_{Jt}P_{JHt} - C_{Jt}P_{Jt}\right)$$

and return on wealth is

$$1+\rho_{At} = \left(1+i_{At}\right)\left(\alpha_{At} + \frac{\left(1-\alpha_{At}\right)}{Z_t}\right) \tag{3.11}$$

$$1+\rho_{Bt} = \left(1+i_{Bt}\right)\left(\alpha_{Bt} + \left(1-\alpha_{Bt}\right)Z_t\right) \tag{3.12}$$

We assume simple relationships of the following form:

$$\alpha_{At} = \alpha_A Z_t^{\phi} \tag{3.13}$$

$$\alpha_{Bt} = \alpha_B Z_t^{-\phi} \tag{3.14}$$

where parameter ϕ is interpreted as a measure of the responsiveness of capital flows to changes in excess return.

3.5 Aggregate demand

Aggregate demand comprises consumption demand coming from domestic residents, consumption demand from foreign residents and government consumption which is directed towards domestically produced goods only:

$$Y_{At} = (1-\mu_{At})\left(\frac{P_{AHt}}{P_{At}}\right)^{-\eta} C_{At} + \mu_{Bt}\left(\frac{P_{BFt}}{P_{Bt}}\right)^{-\eta} C_{Bt} + G_{At} \quad (3.15)$$

$$Y_{Bt} = \mu_{At}\left(\frac{P_{AFt}}{P_{At}}\right)^{-\eta} C_{At} + (1-\mu_{Bt})\left(\frac{P_{BHt}}{P_{Bt}}\right)^{-\eta} C_{Bt} + G_{Bt} \quad (3.16)$$

3.6 The Complete model

We now write down the final system of equations for the law of motion of the out-of-steady-state economy. We proceed in the following way. We consider symmetric steady state and log-linearize around it, so that for each variable X_t, with steady state value X, $\widehat{X}_t = \log(X_t/X)$ (details of the log-linearized system are given in the Technical Appendix of the paper). It is also convenient to introduce net debt variable: $\widehat{F}_t = \widehat{F}_{At} = \widehat{B}_{At} - \widehat{A}_{At} = -\widehat{F}_{Bt}$. Then we rewrite the model into equations of differences, by denoting gap variables with lower case letters: for any variable $x_t = \widehat{X}_{At} - \widehat{X}_{Bt}$ (derivation details are in the Technical Appendix). Please note that in log-linearization below we define $B_{Jt} = \mathcal{B}_{Jt}/P_{JHt}$ and $A_{Jt} = \mathcal{A}_{Jt}/P_{JHt}$.

3.6.1 Steady State

We assume symmetric steady state with $\mu_{At} = \mu_{Bt} = \mu, Y_A = Y_B, C_A = C_B, \alpha_A = \alpha_B = \alpha, \tau_{At} = \tau_{Bt} = \tau$ and linearize around it. We further assume that in steady state $S = 1$, so that $P_{AH} = P_A = P_{AF} = P_{BF} = P_B = P_{BH}$.

We denote the steady state ratio of stock of domestic bonds to GDP (B_A/Y_A) as B. We also assume that the proportion of government in the economy, denoted $(1-\theta)$, so that proportion of consumption in the economy is θ $(= C_J/Y_J)$. In the steady state, we can also assume that $\zeta = 1$ and $Y_A = 1$.

3.6.2 The log-linearized complete model

As a result, we obtain the following equations in differences.

The first equation defines excess returns in one country relative to the other, as a function of the difference in the real interest rate between countries (r_t) and the rate of change of the exchange rate.

$$z_t = 2r_t - (s_{t+1} - s_t) \quad (3.17)$$

The second equation enables us to study portfolio balance.

$$z_t = [2(1-\alpha)b_t - (1-2\alpha)f_t - (1-\alpha)s_t]/2\alpha\phi \quad (3.18)$$

This is an equation showing what the excess return on domestic assets must be. That return increases with the proportional increase in the relative supply of home bonds, allowing for the fact that a fraction α of these, corresponding to the proportion of wealth held in home assets in each country, will be willingly held without any change in return. Excess return increases with the proportionate increase in foreign debt, f, measured as a proportion of the initial stock of domestic bonds B. This is because a change in f redistributes wealth from the US – where the home bias is α – to abroad where home bias is also α, but now, of course, bias against holding US assets. So, the transfer of wealth from the US decreases relative demand for US assets by $(1-\alpha)f$ and increases it by αf, i.e. it causes an increase in the relative demand for US assets of $(2\alpha-1)f$. Excess return must also decrease with the depreciation of the level of the real exchange rate, s, since this reduces the value of the supply of US assets in the total value of world assets. This is scaled by the term $(1-\alpha)$, since a depreciation in the real exchange rate also decreases the value of US wealth relative to world wealth, which because of home bias will reduce relative demand by α. The effects on relative return just described will depend on that proportion of demand effects α which fall on home assets in each country and on sensitivity of demand to changes in relative returns, ϕ.

The next equation is a national income identity. This depends on that part of the proportional change in relative consumption, c, multiplied by the share of consumption in GDP, θ, after allowing for the share of consumption which is imported. It also depends on the proportional change in relative government expenditure, which is not directed to imports and thus multiplied by the share of government expenditure in GDP, $(1-\theta)$ only. National income also depends on expenditure on the real exchange rate and on the shock to aggregate demand. Variations in the real exchange rate affects relative US output through trade balance effects as follows. A proportional change in the real exchange rate s of one unit will cause a proportional increase of μ in the relative price of US goods as compared to the relative price of US consumption. That will cause - after allowing for the fact that in each country domestic consumption is only a fraction $(1-\mu)$ of total consumption, for the fact that the elasticity of demand between home and foreign goods is η, and for the fact that consumption is only a fraction θ of output - a proportionate increase in the demand for home goods of $\mu(1-\mu)\theta$. Since this effect occurs at home as well as abroad, this expression should be also multiplied by the factor 2, so $2\mu(1-\mu)\theta\eta$ shows the effect of the exchange rate.

$$y_t = (1-2\mu)\theta c_t + (1-\theta)g_t + 2\eta\mu\theta(1-\mu)s_t + \zeta_t \tag{3.19}$$

The third equation shows the evolution of foreign debt, f:

$$f_{t+1} = (1-\alpha)\, z_t + \frac{1}{\beta}\left(f_t + \frac{1-\theta}{B}g_t + \frac{\theta}{B}c_t - \frac{1}{B}y_t + \frac{\theta}{B}\mu s_t\right) \tag{3.20}$$

Equation (3.20) is current account equation, which is derived as follows. The discount factor, β, is equal to the inverse of one plus the interest rate. The first and second terms represent net debt and interest payments, including extra payments due to the changes in relative excess returns on domestic assets hold by foreign investors. That is why excess return is scaled by the share of the US portfolio in foreign assets, $(1-\alpha)$. In other words, the first term represents valuation effects (see Technical appendix for details). The next terms, $\frac{1}{\beta}\frac{1}{B}[(1-\theta)g_t + \theta c_t - y_t]$ account for the additional debt accumulated due to the excess of expenditure over output in period t, and scaled by B. And finally, the last term $\frac{1}{\beta}\frac{\theta}{B}\mu s_t$ reflects the worsening of the current account caused by terms of trade effects resulting from changes in real exchange rate, where $\mu\theta$ is the fraction of consumption of US consumers which is imported, and so captures the size of these effects.

In our simulations below we use another version of equation (3.20), those terms is thus important to analyze as well. In equation (3.20), we substitute instead of GDP its expression from (3.19):

$$f_{t+1} = (1-\alpha)\, z_t + \frac{1}{\beta}\left(f_t + 2\frac{\theta}{B}\mu c_t - \frac{\theta}{B}s_t[2\eta\mu(1-\mu) - \mu] - \frac{1}{B}\zeta_t\right) \tag{3.21}$$

This equation is interpreted as follows. Foreign debt, f, increases with the proportional change in relative consumption, c, by the amount shown - the import propensity is μ. This is then scaled by θ, which shows the ratio of consumption to relative GDP, and by B so as to obtain the effect as a proportion of GDP, measured relative to the stock of bonds. It also falls with the depreciation of the real exchange rate, s.The effect of the exchange rate on the net debt accumulation comprises two parts. The first one, $\theta\mu s_t$ discussed in details in considering equation (3.20), is the impact of exchange rate through terms of trade. The second one, $2\theta\eta(1-\mu)\mu s_t$ captures the demand effects of changes in the real exchange rate on US output.

The next equation shows the first-order condition for forward-looking consumers (the Euler equation):

$$c_t = c_{t+1} - r_t + (1-\alpha)z_t + \mu(s_{t+1} - s_t) \tag{3.22}$$

The relationship between interest rate and consumption is negative as in any other standard Euler equation. Consumption becomes more expensive when exchange rate depreciates, as some of the consumption expenditure falls on the consumption of imported goods with the share of μ, so that anticipated depreciation of the exchange rate increases consumption today relative to the consumption tomorrow. On the other hand, exchange rate depreciation increases income because of the wealth hold in the foreign assets with the share of $1-\alpha$, so that anticipated depreciation of the exchange rate allocates consumption to the future from today.

The next equation shows evolution of domestic debt, b.

$$b_{t+1} = c_{t+1} - c_t + \mu s_{t+1} - \mu s_t + (1-\alpha)z_t + \frac{1}{\beta}(b_t + \frac{1-\theta}{B}g_t - \frac{\tau}{B}y_t) \tag{3.23}$$

which states that next period government debt is equal to this period debt, plus interest rate payments on the debt and plus fiscal deficit.

If we collect all equations above, we obtain the following system of equations:

$$
\begin{aligned}
z_t &= 2r_t - (s_{t+1} - s_t) & (3.24)\\
c_t &= c_{t+1} - r_t + (1-\alpha)z_t + \mu(s_{t+1} - s_t) & (3.25)\\
0 &= 2\alpha\phi z_t - 2(1-\alpha)b_t + (1-2\alpha)f_t + (1-\alpha)s_t & (3.26)\\
f_{t+1} &= (1-\alpha)z_t + \frac{1}{\beta}\left(f_t + 2\frac{\theta}{B}\mu c_t - \frac{\theta}{B}s_t[2\eta\mu(1-\mu) - \mu] - \frac{1}{B}\zeta_t\right) & (3.27)\\
b_{t+1} &= c_{t+1} - c_t + \mu s_{t+1} - \mu s_t + (1-\alpha)z_t + \frac{1}{\beta}\left(b_t + \frac{1-\theta}{B}g_t - \frac{\tau}{B}y_t\right) & (3.28)\\
y_t &= (1-2\mu)\theta c_t + 2\theta\eta(1-\mu)\mu s_t + (1-\theta)g_t + \zeta_t & (3.29)
\end{aligned}
$$

In our model, we assume that consumption-saving behavior of the private sector adjusts to changes in the real interest rate, which are changed to keep the output fixed at the natural level. Algebraically it implies the following. In our model, the real interest rate which is consistent with output remaining fixed at its natural level can be found from the intertemporal relation (3.25); we assume that the real interest rate is determined in this way. We further abstract entirely from fiscal policy: any debt is financed by lump-sum taxes so that debt is kept at its steady state level so that $b_t = 0$ at all times and there are no changes in government expenditure so that $g_t = 0$. Now the final system of equations which describes our model is:

$$
\begin{aligned}
z_t &= 2r_t - (s_{t+1} - s_t) & (3.30)\\
c_t &= c_{t+1} - r_t + (1-\alpha)z_t + \mu(s_{t+1} - s_t) & (3.31)\\
0 &= 2\alpha\phi z_t + (1-2\alpha)f_t + (1-\alpha)s_t & (3.32)\\
f_{t+1} &= (1-\alpha)z_t + \frac{1}{\beta}\left(f_t + 2\frac{\theta}{B}\mu c_t - \frac{\theta}{B}s_t[2\eta\mu(1-\mu) - \mu] - \frac{1}{B}\zeta_t\right) & (3.33)\\
0 &= (1-2\mu)\theta c_t + 2\theta\eta(1-\mu)\mu s_t + \zeta_t & (3.34)
\end{aligned}
$$

3.7 The simpler Blanchard *et al.* model

As has been discussed, the BGS model assumes fixed real interest rate. This means that they assume that any adjustments in saving behavior in each country which are required, in order to keep resources fully employed, must be taken by fiscal policy. But how this is done is implicit in the model. We can represent this implicitly - as they do - by supposing that consumption is manipulated by fiscal policy so as to keep output fixed at its natural level, without being explicit as to how this is done. This is in contrast in our version of the model, in which the consumption behavior of the private sector adjusts to ensure this, and that the interest rate moves so as to ensure this.

As a result, if $r_t = 0$ then the outcome $y_t = 0$ cannot be brought about by interest rate changes; BGS impose this outcome. Thus the Euler equation for consumption becomes redundant in BGS case, and the BGS version of the model corresponds to the following more simple system of equations:

$$
\begin{aligned}
z_t &= -(s_{t+1} - s_t) && (3.35)\\
0 &= 2\alpha\phi z_t + (1-2\alpha) f_t + (1-\alpha) s_t && (3.36)\\
f_{t+1} &= (1-\alpha) z_t + \frac{1}{\beta}\left(f_t + 2\frac{\theta}{B}\mu c_t - \frac{\theta}{B} s_t[2\eta\mu(1-\mu) - \mu] - \frac{1}{B}\zeta_t\right) && (3.37)\\
0 &= (1-2\mu)\theta c_t + 2\theta\eta(1-\mu)\mu s_t + \zeta_t && (3.38)
\end{aligned}
$$

4 Equilibrium and Dynamics of adjustment

4.1 Calibration

We calibrate the adjustment path as follows. The values are largely taken from the BGS paper.

Our model is quarterly model, we take the real interest rate to be four percent, so that $\beta = 0.99$. We assume that at equal interest rates, both home and foreign consumers desire to hold three quarters of their wealth at home, so we set value of α equal to 0.75. We set the parameter, θ, share of consumption in the economy, equal to 0.75. The parameter ϕ, which governs the responsiveness of capital flows to relative yields, is assumed to be 1 in the benchmark calibrations. Variations of this parameter are also considered. The import propensity, μ, is assumed to be 0.1, and it is roughly equals that in the US and Europe. We set η, the elasticity of substitution between domestic and foreign goods, equal to unity. And the ratio of the stock of government bonds to GDP, B, is set to 3, which is close to the number (3.5) used in BGS.

4.2 Adjustment

In the appendix we provide detailed discussion of the stability of the system and discuss the dynamic response of the economy to the shock and produce the relevant phase diagram. The important note from this analysis is that the initial, unexpected depreciation of the exchange rate is associated with the decline in the debt (through a 'valuation effect'), which explains why the predetermined variable - the net debt - in the impulse response analysis does change immediately in response to the shock.

We now discuss simulation results. In line with BGS, we analyze the effect of decrease in ζ, that is an increase in the trade deficit. The dynamic effects of the shock are plotted in figures 1-3 for different values of ϕ, the responsiveness of the shares of the US and foreign assets in the portfolios to changes in excess return. For each value of ϕ we present the dynamic adjustment of both of our and the BGS versions of the model, plotted as straight and dashed lines. We start the discussion of these effects by looking at the baseline specification with $\phi = 1$. We also refer to country A as the US.

Our model nests the simpler BGS version of the model in a way discussed above, the dynamic adjustment of the two versions of the model are qualitatively similar, apart from the fact that there is no change in real interest rate in the BGS version. This can be seen from Figures 1-3. A comparison of the dynamic effects of a shift in the trade deficit in these two versions of the model is provided at the end of this section.

The figures show the impulse response pattern for exchange rate, net debt (as share of GDP), consumption (as share of GDP), real interest rate and excess return. The response for the interest rate is in levels, while the responses for the remaining variables are measured in percentage points. Note that in the figures the variable f has been multiplied by the value of $B = 3$ to get the response of the net of debt as a fraction of GDP, since f is measured as a proportion of the initial stock of bonds B.

Figure 1 suggests that to correct the current balance by one percent of GDP would require in the period $t = 100$ a 14 percent depreciation of the real exchange rate and an increase in debt as a fraction of GDP of 17.5 percent. If there were no debt interest rate effects the derivations in (6.76) and (6.77) show, that exchange rate had to depreciate by $s = -\zeta/[\mu\theta\{2(1-\mu)\eta + 2\mu - 1\}$ =13.3 percent, which means that the debt interest rate effects account for another 0.7 percent in our case in that period. Consumption has to fall by about 1.5 percent of GDP, while when there are no debt obligations, consumption has to fall, as suggested by $c = \zeta/\theta[\{2(1-\mu)\eta + 2\mu - 1\}]$, by 1.3 percent of GDP.

Initially, in response to the shock, the real exchange rate falls. But since this is prior to any adjustment in the stock of net debt, this depreciation causes a fall in the value of US assets as a proportion of the total value of world assets, which results in an excess demand for US assets, and so the real exchange rate falls by less than it falls in the long run. So, along the adjustment path, the exchange rate is expected to depreciate, in a way which lowers the return on US assets in such a way as to eliminate, at each point of time, the excess demand for US assets. Along the adjustment path to long-run equilibrium the external debt of the US will be rising, so that current account, while improving, remains in deficit along the path. But gradually, along the adjustment path, the value of US debt will be increasing and, because of home bias, the demand for US assets will be falling. Eventually, the dollar will fall to its ultimate equilibrium level and the value of US debt will have risen sufficiently that the returns on the two assets will be equalized. The extent of the initial fall, and the subsequent movements, in the exchange rate depend on the degree of substitutability, ϕ ,between US and foreign assets.

Figure 1 suggests that on the impact of the shock, the unexpected depreciation of the exchange rate is about 4 percent in our model. This unexpected depreciation increases the value of US holdings of foreign assets and generates an (unexpected) valuation gain, $(1-\alpha)\Delta s$ (as discussed in the technical appendix), which reduces the net debt (as share of GDP) by $(1-\alpha)$ times 4% or about 0.25*4=1%.

The simulation results (figure 1-3) also suggest that unanticipated valuation effects can be sizable (ranging from 1 to almost 10 percent of GDP changes in net external positions in response to 1 percent of GDP shock to trade deficit), while anticipated valuation effects are quantitatively smaller for plausible calibration of the model.

Forward-looking consumers realize that their real income has to fall as the current account adjusts and cut their consumption. However, in the short-run, the exchange rate does not fall to the full extent initially, for reasons just described, and so consumers must be encouraged to cut consumption by less than in the long-run, so as to ensure equilibrium in the goods market. If the initial fall in the real exchange rate is sufficiently small then consumption may actually need to rise initially.This is the case when when ϕ is equal to unity.

A temporary fall in the real interest rate is the means by which, along the adjustment path, consumption only falls gradually to its long run value.

Changes in consumption enure that the following equation holds:

$$0 = (1 - 2\mu)\theta c_t + 2\eta\mu\theta(1 - \mu)s_t + \zeta_t \tag{4.1}$$

where $(1-2\mu)\theta = 0.6, 2\eta\mu\theta(1-\mu) = 0.135$, and $\zeta_t = -1$.Using our parameterisation, when $\phi = 1$, the dollar falls by only about 4 percentage points initially, and consumption then actually needs to rise initially by about 0.76 percentage points.

Our simulations show that the fall in the real interest rate necessary to provoke the required increase in consumption is small. The reason for this is as follows. If we integrate the Euler equation for consumption we obtain:

$$c_t = c_{t+1} - r_t + (1 - \alpha)z_t + \mu(s_{t+1} - s_t) \tag{4.2}$$

After substituting the definition of excess return we obtain:

$$z_t = 2r_t - (s_{t+1} - s_t) \tag{4.3}$$

so that the equation becomes:

$$c_{t+1} + (\mu - (1 - \alpha))s_{t+1} = c_t + (\mu - (1 - \alpha))s_t + (2\alpha - 1)r_t$$

and after integration it becomes:

$$(c_{t+k} - c_t) = -(\mu - (1 - \alpha))(s_{t+k} - s_t) + (2\alpha - 1)\sum_{i=0}^{k-1} r_{t+i} \tag{4.4}$$

In our model $\mu < 1 - \alpha$, so that a depreciation in the real exchange rate increases the return on assets (because the component of them held abroad rises in value) by more than it increases the cost of consumption. As a result, an anticipated depreciation of the exchange rate allocates consumption towards the future. At $t + k = 100$:

$$c_t = -1.5 - 0.5\sum_{i=0}^{k-1} r_{t+i} - 0.15(14 - s_t) \tag{4.5}$$

where $2\alpha - 1 = 0.5$ and $\mu - (1 - \alpha) = -0.15$.

Consumption is a forward-looking variable, so it does respond not only to the change in the interest rate today, but also to the changes in the interest rate in the subsequent periods so that the change in the consumption today is affected by the integral of the changes in the interest rate along the adjustment path to the long-run equilibrium as we can see from the equation above. This explains why the interest rate falls so little in the short run relative to the change in the consumption in the same period.

The degree of responsiveness of the shares of US and foreign assets in the portfolios to the changes in the excess return, (ϕ) plays an important role in the nature of dynamic adjustment of the economy, and in particular in adjustment of exchange rate, and interest rates. To see this, we compare the dynamic effects of the shock for various values of ϕ illustrated in Figures 1-3. Comparison of these pictures yields the following observations:

First, the lower the value of the parameter ϕ, the larger the initial decline in the return on US assets must be. This means a smaller initial depreciation of the real exchange rate, so that the expected subsequent rate of decline of the dollar is faster. As a result, the lower the value of ϕ, the smaller the unanticipated valuation effect and the larger the anticipated valuation effects along the adjustment paths.At the same time, the lower the initial depreciation of the exchange rate, the greater must be the stimulus to consumption and so the larger must be the decline in the real interest rate. That is, the lower the value of ϕ, the less consumption falls immediately and so the less the degree of consumption smoothing in response to the permanent shock.

To understand the reason for this, consider two extreme cases: ($\phi \longrightarrow \infty$) and ($\phi = 0$). In the first case, exchange rate jumps to its long-run value to eliminate the shock to the trade balance, consumption falls to its long-run value and there is no change in the interest rate.To understand the second case, or in any case when (ϕ is not ∞), the move of the exchange rate in the short-run is influenced by the parameter ϕ, which can be determined from the portfolio balance equation in the intial time period:

$$2\alpha\phi z_t = -(1-\alpha)s_t \tag{4.6}$$

If $\phi = 0$, exchange rate does not change in response to the shock and along the adjustment path the economy adjusts along the portfolio balance equation:

$$0 = 2\alpha\phi z_t = -(1-\alpha)s_t + (2\alpha - 1)f_t \tag{4.7}$$

so that the exchange only moves as debt is accumulated. So in this case the change in the excess return is coming entirely from changes in the interest rate which has to fall sufficiently large to stimulate consumption and thus to move the goods market into equilibrium in the short-run. For intermediate values of ϕ the outcome lies between these two extremes.

We may now compare the dynamic effects of shift in the trade deficit in our model and in the BGS model.

First, as a part of reduction in the return on US assets comes from the interest rate reduction in our case, the exchange rate is falling less fast and as the result, the initial jump in the real exchange rate is larger and after this the adjustment of f and s is slower along the adjustment path. However, as can be seen, the effects are not large.The initial unexpected valuation effect is bigger in our case, because odf the larger initial jump in the exchange rate, so that the unexpected reduction in the debt is larger in our model compared with what observed in the BGS model.[4].

Second, the dynamic behavior of consumption is a mirror image of that of the exchange rate. The initial decline in consumption is bigger in our version of the model than that described by

[4]The technical appendix contains the detailed derivation of the balance of payments equation which has a valuation effect term. Valuation effect implies that on response to the shock, the unexpected depreciation of the dollar leads to unexpected reduction in external debt. The size of the valuation at the time of the shock is $df = -(1-\alpha)\Delta s$, where Δs is the unexpected depreciation of the exchange rate.

BGS, precisely because the real exchange rate moves more initially in our case. As a result, along the adjustment path, consumption falls more slowly to its final value in our model.

Third, the higher initial level of the debt, and the faster accumulation of the debt over time in BGS model implies that f moves in BGS model more than in our case, so that $f \mid_{BGS} >$ $f \mid_{OURS}$along the adjustment path. It also implies that, given the same ultimate equilibrium level for both variables, as debt increases, the rate of debt accumulation df/dt decreases, and this decrease in df/dt happens in BGS model earlier than in our model, so that $df/dt \mid_{BGS} <$ $df/dt \mid_{OURS}$ at some point of time before equilibrium is reached. This inequality holds, as follows from the dynamic equation for f (which is the same in two models), only if the level of exchange rate becomes larger in BGS than in our model. And this should happen much earlier the point at which $df/dt \mid_{BGS} < df/dt \mid_{OURS}$, as the level of the debt in BGS rises much faster than in our model, and an increase in the exchange rate should account for the big difference in the levels of debt between two models. This implies that dynamic path of the exchange rate in BGS model should cross the dynamic path of exchange rate in our model. The same will be true for consumption behavior.

5 Conclusion

In this paper, we revisit the Blanchard et al. (2005) argument for 'a slow slide of the dollar rather than a fast fall' using a model which incorporates forward-looking consumers. We extend their argument and provide a further argument to suggest that the fall, when it comes, will not be gradual. Our argument is as follows. Along with the initial fall in the dollar, forward-looking consumers may cut their expenditure, by recognizing that their real income will fall. However, as interest rate falls to prevent consumption falling too far initially, the dollar will fall by more initially than the fall described by the BGS, because of this initial fall in the interest rate. Subsequently, after the initial fall, on the way to a long run equilibrium, there will be a slower ultimate adjustment of debt and so a slower full adjustment of the whole system towards the final steady state, compared with that described in the BGS model.

We find that the effects of these changes are not large. This is because the fall in the rate of return on domestic assets, caused by the fall in the real interest rate, is not large, compared with the overall change in the real exchange rate required to bring about the current account correction.

This paper also provides some insights into the workings of valuation effects in the process of adjustment of asset values as adjustment takes place, in such a model with imperfect substitutability between assets. The model allows to identify unanticipated valuation effect and anticipated valuation effect. The unanticipated valuation effect arises from the unexpected depreciation of the exchange rate. The anticipated valuation effect is caused by both anticipated changes in exchange rate and by variations in anticipated excess return. We find that the unanticipated valuation effects are sizable, but that the anticipated valuation effects are quantitatively small, for a plausible calibration of the model. Our analysis also suggests that anticipated valuation effects due to the excess returns on assets plays a smaller role than the valuation effect caused by anticipated changes in exchange rate.

In this paper we explicitly abstract from any adjustment of fiscal policy. It would be possible to add a fiscal rule to the adjustment process to see the way in which this influenced the adjustment process.

References

[1] Blanchard, O., F. Giavazzi, and F. Sá, 2005. The US current account and the dollar. NBER Working paper series, No. 11137

[2] Caballero, R., Farhi, E., and P. Gourinchas, 2006. An equilibrium model of "Global imbalances" and low interest rates. NBER Working paper series, No. 11996

[3] Couerdacier, N., 2005. Do trade costs in goods market lead to home bias in equities? unpublished manuscript, LBS.

[4] Devereux, M. and A. Sutherland, 2008. Valuation effects and the dynamics of net external assets. Paper presented at International Macro-Finance, 24-25 April, IMF.

[5] Erceg, C., L. Guerrieri and C. Gust, 2005. Expansionary fiscal shocks and the US trade deficit. International Finance, 8:3, 363-397.

[6] Faruqee, H., D. Laxton, D. Muir, and P. Pesenti, 2005. Smooth landing or crash? Model-based scenarious og global current account rebalancing. NBER Working paper No. 11583.

[7] Galí, J. and T. Monacelli, 2002. Monetary Policy and exchange rate volatility in a small open economy. NBER Working paper series, No. 8905

[8] Gourinchas, P-O. and H. Rey, 2005. From world banker to world venture capitalist: US external adjustment and the exorbitant privilege. NBER working paper No. 11563.

[9] Higgins, M., T. Klitgaard and C. Tille, 2006. Borrowing without debt? Understanding the US international invsetment position. Staff Reports 271, Federal Reserve Bank of New York.

[10] Hunt, B. and A. Rebucci, 2005. The US dollar and the trade deficit: what accounts for the late 1990s? International Finance, 8:3, 399-434.

[11] Kollman, R., 2006. International portfolio equilibrium and the current account, mimeo.

[12] Kraay, A. and J. Ventura, 2005. The dot-com bubble, the Bush deficits and the US current account. NBER working paper No. 11543.

[13] Krugman, P., 2007. Will there be a Dollar crisis? Economic Policy, CEPR, CES, MSH, July 2007, 435-467.

[14] Lane, P. and G. Milesi-Ferreti, 2005. A global perspective on external positions. NBER working paper No. 11589.

[15] Mendoza, E., V. Quadrini, and J-V Rios-Rull, 2007. Financial integration, financial deepness and global imbalances. NBER working paper No. 12909.

[16] Meredith, G., 2007. Debt dynamics and global imbalances: some conventional views reconsidered, IMF working paper, WP/07/4, IMF, Washington, D.C.

[17] Obstfeld, M. and K. Rogoff, 2004. The unsustainable US current account position revisited. NBER Working paper No. 10869.

[18] Obstfeld, M. and K. Rogoff, 2005. Global Current Account imbalances and exchange rate adjustments. Brooking Papers on Economic Activity.

[19] Sá, F. and F. Viani, 2007. The macroeconomic implications of Sovereign Wealth funds, Bank of England, mimeo.

[20] Tille, C. 2003. The impact of exchange rate movements on US foreign debt. Current Issues in Economics and Finance, 9, 1-7.

[21] Tille, C. and E. van Wincoop, 2007. International capital flows. NBER working paper, No. 12856.

[22] Roubini, N. and B. Setser, 2005. Will the Bretton Woods 2 regime unravel soon? The risk of a hard landing in 2005-2006. Unpublished manuscript, New York University and Oxford University.

6 Technical Appendix

6.1 Dynamic System

6.1.1 Consumers

To derive first order conditions for household optimization problem we write Lagrangian for household in country J.

$$\begin{aligned} L &= \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\log C_{Jt} + \log G_{Jt}\right) - \lambda[\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} Q_{t,s}^J (C_{Js} P_{Js}) - \mathcal{A}_{Jt} - \\ & \sum_{s=t}^{\infty} \mathcal{E}_t Q_{t,s}^J \left((1 - \tau_{Jt}) Y_{Js} \right)] \end{aligned}$$

so that first-order conditions are:

$$0 = \frac{\partial L}{\partial C_{Js}} = \beta^{s-t} \frac{1}{C_{Jt}} - \lambda Q_{t,s}^J P_{Js} \tag{6.1}$$

$$0 = \frac{\partial L}{\partial \lambda} = \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} Q_{t,s} (C_{Js} P_{Js}) - \mathcal{A}_{Jt} - \sum_{s=t}^{\infty} \mathcal{E}_t Q_{t,s}^J \left((1 - \tau_{Jt}) \left(w_{Js} N_{Js} + \Pi_{Js} \right) \right) \tag{6.2}$$

Dividing the first FOC by itself, by taken one step ahead and taking expectations, we obtain conventional Euler equation:

$$\beta(1+\rho_{Jt})\mathcal{E}_t\{\frac{C_{Jt}}{C_{Jt+1}}\frac{P_{Jt}}{P_{Jt+1}}\} = 1 \tag{6.3}$$

where we used the condition $\mathcal{E}_t(Q^J_{t,t+1}) = 1/(1+\rho_{Jt})$.

6.1.2 Exchange Rates and Prices

By using the definition of the real exchange rate ($S_t = \frac{P_{AFt}}{P_{AHt}} = \frac{P_{BHt}}{P_{BFt}}$) and CPI price indexes in each country ($P_{Jt} = ((1-\mu_{At})\, P^{1-\eta}_{JHt} + \mu_{At} {P_{JFt}}^{1-\eta})^{\frac{1}{1-\eta}}$), we can define price ratios:

$$\frac{P_{AHt}}{P_{At}} = P_{AHt}((1-\mu_{At})\, P^{1-\eta}_{AHt} + \mu_{At}{P_{AFt}}^{1-\eta})^{-\frac{1}{1-\eta}} = ((1-\mu_{At}) + \mu_{At}{S_t}^{1-\eta})^{-\frac{1}{1-\eta}} \tag{6.4}$$

$$\frac{P_{BFt}}{P_{Bt}} = P_{BFt}((1-\mu_{Bt})\, P^{1-\eta}_{BHt} + \mu_{Bt}{P_{BFt}}^{1-\eta})^{-\frac{1}{1-\eta}} = ((1-\mu_{Bt})\, S_t^{1-\eta} + \mu_{Bt})^{-\frac{1}{1-\eta}} \tag{6.5}$$

$$\frac{P_{AFt}}{P_{At}} = P_{AFt}((1-\mu_{At})\, P^{1-\eta}_{AHt} + \mu_{At}{P_{AFt}}^{1-\eta})^{-\frac{1}{1-\eta}} = ((1-\mu_{At})\left(\frac{1}{S_t}\right)^{1-\eta} + \mu_{At})^{-\frac{1}{1-\eta}} \tag{6.6}$$

$$\frac{P_{BHt}}{P_{Bt}} = P_{BHt}((1-\mu_{Bt})\, P^{1-\eta}_{BHt} + \mu_{Bt}{P_{BFt}}^{1-\eta})^{-\frac{1}{1-\eta}} = ((1-\mu_{Bt}) + \mu_{Bt}(\frac{1}{S_t})^{1-\eta})^{-\frac{1}{1-\eta}} \tag{6.7}$$

6.1.3 Fiscal Constraints

The government in country J buys goods (G_{Jt}), taxes income (with tax rate τ_{Jt}) and issues nominal debt $\mathcal{B}_{Jt}$. The evolution of the nominal debt stock can be written as:

$$\mathcal{B}_{Jt+1} = (1+i_{Jt})(\mathcal{B}_{Jt} + P_{JHt}G_{Jt} - \tau_{Jt}P_{JHt}Y_{Jt}) \tag{6.8}$$

In log-linearization below we define $B_{Jt} = \mathcal{B}_{Jt}/P_{JHt}$

6.1.4 Financial Markets

The excess return Z_t is defined as:

$$Z_t = \frac{(1+i_{At})\, E_{t+1}}{(1+i_{Bt})\, E_t} \tag{6.9}$$

where E_t is nominal exchange rate and i_{Jt} is nominal interest rate on debt in country J.

We assume the following portfolio balance equations for each country:

$$\alpha_{At}\mathcal{A}_{At} + (1-\alpha_{Bt})\,\frac{\mathcal{A}_{Bt}}{E_t} = \mathcal{B}_{At} \tag{6.10}$$

$$(1-\alpha_{At})\,\mathcal{A}_{At} + \alpha_{Bt}\frac{\mathcal{A}_{Bt}}{E_t} = \frac{\mathcal{B}_{Bt}}{E_t} \tag{6.11}$$

It follows, of course, that

$$\mathcal{A}_{At} + \frac{\mathcal{A}_{Bt}}{E_t} = \mathcal{B}_{At} + \frac{\mathcal{B}_{Bt}}{E_t}$$

Total financial wealth of country J evolves as

$$\mathcal{A}_{Jt+1} = (1+\rho_{Jt})\left(\mathcal{A}_{Jt} + (1-\tau_{Jt})\, Y_{Jt} P_{JHt} - C_{Jt} P_{Jt}\right) \tag{6.12}$$

In log-linearization below we define $A_{Jt} = \mathcal{A}_{Jt}/P_{JHt}$.

Return on wealth is

$$1+\rho_{At} = (1+i_{At})\left(\alpha_{At} + \frac{(1-\alpha_{At})}{Z_t}\right) \tag{6.13}$$

$$1+\rho_{Bt} = (1+i_{Bt})\left(\alpha_{Bt} + (1-\alpha_{Bt})\, Z_t\right) \tag{6.14}$$

6.1.5 Aggregate demand

$$Y_{At} = (1-\mu_{At})\left(\frac{P_{AHt}}{P_{At}}\right)^{-\eta} C_{At} + \mu_{Bt}\left(\frac{P_{BFt}}{P_{Bt}}\right)^{-\eta} C_{Bt} + G_{At} \tag{6.15}$$

$$Y_{Bt} = \mu_{At}\left(\frac{P_{AFt}}{P_{At}}\right)^{-\eta} C_{At} + (1-\mu_{Bt})\left(\frac{P_{BHt}}{P_{Bt}}\right)^{-\eta} C_{Bt} + G_{Bt} \tag{6.16}$$

6.1.6 The system as a whole

In the Euler equation (6.3) we substitute expression for ρ_{At} from (6.13), and obtain (6.64). While in the definition of the excess retun we subtitite definition of the nominal exchange rate, which results in equation (6.18) below. For the rest of the model we will do the following. From Euler equations (6.3) for both countries, we find expressions for return on financial wealth (ρ_{Jt}), which can be further substituted into (6.13) and (6.14) to get expressions for nominal interest rate (i_{Jt}). Finally, these expressions can be substituted into equations (6.10), (6.12) ($J = A, B$), (6.8) ($J = A, B$), (6.15), and (6.16) to get the complete model:

$$1 = \beta(1+i_{At})\left(\alpha_A Z_t^{\phi} + \frac{\left(1-\alpha_A Z_t^{\phi}\right)}{Z_t}\right)\frac{C_{At}}{C_{At+1}}\frac{((1-\mu_{At})+\mu_{At}S_{t+1}{}^{1-\eta})^{-\frac{1}{1-\eta}}}{((1-\mu_{At})+\mu_{At}S_t{}^{1-\eta})^{-\frac{1}{1-\eta}}}\frac{P_{AHt}}{P_{AHt+1}} \quad (6.17)$$

$$Z_t = \frac{1+i_{At}}{1+i_{Bt}}\frac{((1-\mu)+\mu\left(\frac{1}{S_{t+1}}\right)^{1-\eta})^{\frac{1}{1-\eta}}}{((1-\mu)+\mu\left(\frac{1}{S_t}\right)^{1-\eta})^{\frac{1}{1-\eta}}}\frac{(\mu+(1-\mu)S_{t+1}{}^{1-\eta})^{-\frac{1}{1-\eta}}}{(\mu+(1-\mu)S_t{}^{1-\eta})^{-\frac{1}{1-\eta}}}\frac{P_{BHt+1}}{P_{BHt}}\frac{P_{AHt}}{P_{AHt+1}} \quad (6.18)$$

$$B_{At} = \alpha_{At}A_{At} + (1-\alpha_{Bt})A_{Bt}S_t \quad (6.19)$$

$$A_{At+1} = \frac{C_{At+1}}{\beta C_{At}}\frac{((1-\mu_{At})+\mu_{At}S_{t+1}{}^{1-\eta})^{\frac{1}{1-\eta}}}{((1-\mu_{At})+\mu_{At}S_t{}^{1-\eta})^{\frac{1}{1-\eta}}}\left(A_{At}+(1-\tau_{At})Y_{At}-C_{At}((1-\mu_{At})+\mu_{At}S_t{}^{1-\eta})^{\frac{1}{1-\eta}}\right) \quad (6.20)$$

$$A_{Bt+1} = \frac{C_{Bt+1}}{\beta C_{Bt}}\frac{((1-\mu_{Bt})+\mu_{Bt}\left(\frac{1}{S_{t+1}}\right)^{1-\eta})^{\frac{1}{1-\eta}}}{((1-\mu_{Bt})+\mu_{Bt}\left(\frac{1}{S_t}\right)^{1-\eta})^{\frac{1}{1-\eta}}}\left(A_{Bt}+(1-\tau_{Bt})Y_{Bt}-C_{Bt}((1-\mu_{Bt})+\mu_{Bt}(\frac{1}{S_t})^{1-\eta})^{\frac{1}{1-\eta}}\right) \quad (6.21)$$

$$B_{At+1} = \frac{C_{As+1}}{\beta\left(\alpha_{At}+\frac{(1-\alpha_{At})}{Z_t}\right)C_{At}}\frac{((1-\mu_{At})+\mu_{At}S_{t+1}{}^{1-\eta})^{\frac{1}{1-\eta}}}{((1-\mu_{At})+\mu_{At}S_t{}^{1-\eta})^{\frac{1}{1-\eta}}}(B_{At}+G_{At}-\tau_{At}Y_{At}) \quad (6.22)$$

$$B_{Bt+1} = \frac{C_{Bt+1}}{\beta C_{Bt}}\frac{((1-\mu_{Bt})+\mu_{Bt}\left(\frac{1}{S_{t+1}}\right)^{1-\eta})^{\frac{1}{1-\eta}}}{(\alpha_{Bs}+(1-\alpha_{Bs})Z_t)((1-\mu_{Bt})+\mu_{Bt}\left(\frac{1}{S_t}\right)^{1-\eta})^{\frac{1}{1-\eta}}}(B_{Bt}+G_{Bt}-\tau_{Bt}Y_{Bt}) \quad (6.23)$$

$$Y_{At} = (1-\mu_{At})((1-\mu_{At})+\mu_{At}S_t{}^{1-\eta})^{\frac{\eta}{1-\eta}}C_{At}+\mu_{Bt}((1-\mu_{Bt})S_t^{1-\eta}+\mu_{Bt})^{\frac{\eta}{1-\eta}}C_{Bt}+G_{At}+\zeta_{At} \quad (6.24)$$

$$Y_{Bt} = \mu_{At}((1-\mu_{At})\left(\frac{1}{S_t}\right)^{1-\eta}+\mu_{At})^{\frac{\eta}{1-\eta}}C_{At}+(1-\mu_{Bt})((1-\mu_{Bt})+\mu_{Bt}\left(\frac{1}{S_t}\right)^{1-\eta})^{\frac{\eta}{1-\eta}}C_{Bt}+G_{Bt}+\zeta_{Bt} \quad (6.25)$$

where ζ_{Jt} is the shock to the aggregate demand in country J.

6.1.7 Steady state and Log-linearized model

Steady state

We assume symmetric steady state with $\mu_{At} = \mu_{Bt} = \mu, Y_A = Y_B, C_A = C_B, \tau_{At} = \tau_{Bt} = \tau, i_{At} = i_{Bt}$ and linearize around it. We also assume simple relationships of the following form

$$\alpha_{At} = \alpha_A Z_t^{\phi},$$
$$\alpha_{Bt} = \alpha_B Z_t^{-\phi}.$$

and assume that $\alpha_A = \alpha_B = \alpha$.

We assume that in steady state $S = 1$, so that $P_{AH} = P_A = P_{AF} = P_{BF} = P_B = P_{BH}$.

For analysis of the dynamics of the system, it is convenient to introduce net debt $F_{Jt} = B_{Jt} - A_{Jt}$, where F is normalized by the steady state stock of domestic bonds B. We consider steady state where net debt (F_J) equals to zero, so that $B_J = A_J$.

We denote the steady state ratio of stock of domestic bonds to GDP (B_A/Y_A) as B. We also assume that the proportion of government in the economy, denoted $(1-\theta)$, so that proportion of consumption in the economy is θ $(= C_J/Y_J)$. In the steady state, we can also assume that $\zeta = 1$ and $Y_A = 1$.

Log-linearized model

We linearize around the symmetric steady state (here and everywhere below for each variable X_t with steady state value X, we use the notation $\hat{X}_t = \ln(X_t/X)$).

It is important to note that:

$$\hat{F}_t = \hat{F}_{At} = \hat{B}_{At} - \hat{A}_{At} = -\hat{F}_{Bt} \tag{6.26}$$

The log-linearized version of the system of equations (6.64)-(6.25), given (6.26), is the following:

$$\widehat{Z}_t = (i_{At} - i_{Bt}) - (\widehat{\pi}_{AHt+1} - \widehat{\pi}_{BHt+1}) - (\hat{S}_{t+1} - \hat{S}_t) \tag{6.27}$$

$$\widehat{C}_{At} = \widehat{C}_{At+1} - i_{At} + (1-\alpha)\widehat{Z}_t + \widehat{\pi}_{AHt+1} + \mu(\widehat{S}_{t+1} - \widehat{S}_t) \tag{6.28}$$

$$\hat{B}_{At} = 2\alpha\phi\widehat{Z}_t + \alpha\hat{A}_{At} + (1-\alpha)\hat{A}_{Bt} + (1-\alpha)\hat{S}_t \tag{6.29}$$

$$\hat{F}_{t+1} = (1-\alpha)\widehat{Z}_t + \frac{1}{\beta}\left(\hat{F}_t + \frac{1-\theta}{2B}\left(\hat{G}_{At} - \hat{G}_{Bt}\right) + \frac{\theta}{2B}\left(\hat{C}^R_{At} - \hat{C}^R_{Bt}\right) - \frac{1}{2B}\left(\hat{Y}_{At} - \hat{Y}_{Bt}\right) + \frac{\theta}{B}\mu\hat{S}_t\right) \tag{6.30}$$

$$\hat{B}_{At+1} = \hat{C}_{As+1} - \hat{C}_{As} + \mu\hat{S}_{t+1} - \mu\hat{S}_t + (1-\alpha)\widehat{Z}_t + \frac{1}{\beta}\left(\hat{B}_{At} + \frac{1-\theta}{B}\hat{G}_{At} - \frac{\tau Y_A}{B_A}Y_{At}\right) \tag{6.31}$$

$$\hat{B}_{Bt+1} = \hat{C}_{Bs+1} - \hat{C}_{Bs} - \mu\hat{S}_{t+1} + \mu\hat{S}_t - (1-\alpha)\widehat{Z}_t + \frac{1}{\beta}\left(\hat{B}_{Bt} + \frac{1-\theta}{B}\hat{G}_{Bt} - \frac{\tau Y_B}{B_B}Y_{Bt}\right) \tag{6.32}$$

$$\hat{Y}_{At} = (1-\mu)\theta\hat{C}_{At} + \theta\mu\hat{C}_{Bt} + 2\theta\eta(1-\mu)\mu\hat{S}_t + (1-\theta)\hat{G}_{At} + \frac{\zeta_A}{Y_A}\widehat{\zeta}_{At} \tag{6.33}$$

$$\hat{Y}_{Bt} = (1-\mu)\theta\hat{C}_{Bt} + \theta\mu\hat{C}_{At} - 2\theta\eta(1-\mu)\mu\hat{S}_t + (1-\theta)\hat{G}_{Bt} + \frac{\zeta_B}{Y_B}\widehat{\zeta}_{Bt} \tag{6.34}$$

We split this system into the system for differences and obtain the complete system of equations which determine the model's dynamics:

$$z_t = 2(i_t - \widehat{\pi}_{Ht+1}) - (s_{t+1} - s_t) \tag{6.35}$$

$$c_t = c_{t+1} - i_t + (1-\alpha)z_t + \widehat{\pi}_{Ht+1} + \mu(s_{t+1} - s_t) \tag{6.36}$$

$$0 = 2\alpha\phi z_t - 2(1-\alpha)b_t + (1-2\alpha)f_t + (1-\alpha)s_t \tag{6.37}$$

$$f_{t+1} = (1-\alpha)z_t + \frac{1}{\beta}\left(f_t + \frac{1-\theta}{B}g_t + \frac{\theta}{B}c_t - \frac{1}{B}y_t + \frac{\theta}{B}\mu s_t\right) \tag{6.38}$$

$$b_{t+1} = c_{t+1} - c_t + \mu s_{t+1} - \mu s_t + (1-\alpha)z_t + \frac{1}{\beta}\left(b_t + \frac{1-\theta}{B}g_t - \frac{\tau}{B}y_t\right) \tag{6.39}$$

$$y_t = (1-2\mu)\theta c_t + 2\theta\eta(1-\mu)\mu s_t + (1-\theta)g_t + \zeta_t \tag{6.40}$$

Let's denote real interest rate in country A relative to one in country B as $r_t = i_t - \widehat{\pi}_{Ht+1}$, so that the system above can be written in real variables only:

$$
\begin{aligned}
z_t &= 2r_t - (s_{t+1} - s_t) && (6.41)\\
c_t &= c_{t+1} - r_t + (1-\alpha)z_t + \mu(s_{t+1} - s_t) && (6.42)\\
0 &= 2\alpha\phi z_t - 2(1-\alpha)b_t + (1-2\alpha)f_t + (1-\alpha)s_t && (6.43)\\
f_{t+1} &= (1-\alpha)z_t + \frac{1}{\beta}\left(f_t + \frac{1-\theta}{B}g_t + \frac{\theta}{B}c_t - \frac{1}{B}y_t + \frac{\theta}{B}\mu s_t\right) && (6.44)\\
b_{t+1} &= c_{t+1} - c_t + \mu s_{t+1} - \mu s_t + (1-\alpha)z_t + \frac{1}{\beta}\left(b_t + \frac{1-\theta}{B}g_t - \frac{\tau}{B}y_t\right) && (6.45)\\
y_t &= (1-2\mu)\theta c_t + 2\theta\eta(1-\mu)\mu s_t + (1-\theta)g_t + \zeta_t && (6.46)
\end{aligned}
$$

Substituting the expression for the difference in the real interest rate between countries (r_t) into Euler equation, and expression for y_t in equation for f, we obtain the following set of equations which will be used for dynamic analysis in the paper:

$$
\begin{aligned}
(2\alpha-1)z_t &= -(2c_t - (1-2\mu)s_t) + (2c_{t+1} - (1-2\mu)s_{t+1}) && (6.47)\\
0 &= 2\alpha\phi z_t - 2(1-\alpha)b_t + (1-2\alpha)f_t + (1-\alpha)s_t && (6.48)\\
f_{t+1} &= (1-\alpha)z_t + \frac{1}{\beta}\left(f_t + 2\frac{\theta}{B}\mu c_t - \frac{\theta}{B}s_t[2\eta\mu(1-2\mu) - \mu] - \frac{1}{B}\zeta_t\right) && (6.49)\\
b_{t+1} &= c_{t+1} - c_t + \mu s_{t+1} - \mu s_t + (1-\alpha)z_t + \frac{1}{\beta}\left(b_t + \frac{1-\theta}{B}g_t - \frac{\tau}{B}y_t\right) && (6.50)\\
y_t &= (1-2\mu)\theta c_t + 2\theta\eta(1-\mu)\mu s_t + (1-\theta)g_t + \zeta_t && (6.51)
\end{aligned}
$$

6.2 Valuation effect

In the previous section the following equations have been defined: excess return is defined as

$$Z_t = \frac{(1+i_{At})E_{t+1}}{(1+i_{Bt})E_t} \tag{6.52}$$

Government budget constraint is:

$$\mathcal{B}_{Jt+1} = (1+i_{Jt})(\mathcal{B}_{Jt} + P_{JHt}G_{Jt} - \tau_{Jt}P_{JHt}Y_{Jt}) \tag{6.53}$$

Total financial wealth of country J evolves as

$$\mathcal{A}_{Jt+1} = (1+\rho_{Jt})(\mathcal{A}_{Jt} + (1-\tau_{Jt})Y_{Jt}P_{JHt} - C_{Jt}P_{Jt}) \tag{6.54}$$

The portfolio balance equation for country A's assets:

$$\alpha_{At}\mathcal{A}_{At} + (1-\alpha_{Bt})\frac{\mathcal{A}_{Bt}}{E_t} = \mathcal{B}_{At} \tag{6.55}$$

and return on wealth is:

$$1+\rho_{At} = (1+i_{At})\left(\alpha_{At} + \frac{(1-\alpha_{At})}{Z_t}\right) \tag{6.56}$$

Consider evolution of the net debt $\mathcal{F}_{At+1} \equiv \mathcal{B}_{Jt+1} - \mathcal{A}_{Jt+1}$ of country A by subtracting (6.12) from (6.8), and by using equations (6.55) and (6.56):

$$\begin{aligned}
\mathcal{F}_{At+1} &\equiv \mathcal{B}_{At+1} - \mathcal{A}_{At+1} \\
&= (1+i_{At})\mathcal{B}_{At} - (1+\rho_{At})\,\mathcal{A}_{At} + (1+i_{At})[P_{AHt}G_{At} - \tau_{At}P_{AHt}Y_{At}] - \\
&\quad (1+\rho_{At})[(1-\tau_{At})\,Y_{At}P_{AHt} - C_{At}P_{At}] \\
&= (1+i_{At})(1-\alpha_{Bt})\frac{\mathcal{A}_{Bt}}{E_t} - \frac{1+i_{At}}{Z_t}(1-\alpha_{At})\mathcal{A}_{At} + D(E_t) \qquad (6.57)
\end{aligned}$$

Equation (6.57) is similar to the one in BGS paper and can be interpreted in the similar way. Specifically, net debt next period is equal to next period's value of US assets held by foreign investors, minus next period's value of foreign assets held by US investors, plus next period's trade deficit. The trade deficit in the next period is equal to:

$$D(E_t) = (1+i_{At})[P_{AHt}G_{At} - \tau_{At}P_{AHt}Y_{At}] - (1+\rho_{At})[(1-\tau_{At})\,Y_{At}P_{AHt} - C_{At}P_{At}] \qquad (6.58)$$

The intuition behind the expression for the trade deficit is as follows. If, for instance, there was no change in excess return and thus $Z_t = 1$, then return on financial wealth is equal to the one on domestic bonds, and $1+\rho_{At} = 1+i_{At}$, so that the equation (6.58) becomes:

$$D(E_t) = -(1+i_{At})[P_{AHt}Y_{At} - P_{AHt}G_{At} - C_{At}P_{At}] \qquad (6.59)$$

which is trade deficit times the gross rate of return, which is trade deficit next period. Having discussed this, now it is straightforward to interpret (6.58). Let's re-write it as:

$$D(E_t) = -(1+i_{At})[\tau_{At}P_{AHt}Y_{At} - P_{AHt}G_{At}] - (1+\rho_{At})[(1-\tau_{At})\,Y_{At}P_{AHt} - C_{At}P_{At}] \qquad (6.60)$$

The first term is an excess government expenditure times the gross rate of return and the second term is an excess of consumption expenditure multiplied by its gross rate of returns.

Now we return to equation (6.57), expression for the country's A (USA) net debt next period, which now can be re-written as:

$$\begin{aligned}
\mathcal{F}_{At+1} &= (1+i_{At})(1-\alpha_{Bt})\frac{\mathcal{A}_{Bt}}{E_t} - \frac{1+i_{At}}{Z_t}(1-\alpha_{At})\mathcal{A}_{At} + D(E_t) \\
&= (i+i_{At})\{\mathcal{B}_{At} - \alpha_{At}\mathcal{A}_{At}\} - -\frac{1+i_{At}}{Z_t}(1-\alpha_{At})\mathcal{A}_{At} + D(E_t) \\
&= (1+i_{At})\mathcal{F}_{At} + (1+i_{At})(1-\alpha_{At})(1-\frac{1}{Z_t})\mathcal{A}_{At} + D(E_t) \\
&= (1+i_{At})\mathcal{F}_{At} + (1+i_{At})(1-\alpha_{At})(1-\frac{(1+i_{Bt})\,E_t}{(1+i_{At})\,E_{t+1}})(\mathcal{B}_{At} - \mathcal{F}_{At}) + D(E_t) \qquad (6.61)
\end{aligned}$$

which is similar to the one in BGS paper. The first and last terms are net debt times gross rate of return and the trade deficit next period, while the second term is valuation effect. Valuation effects imply that, when gross liabilities are denominated in domestic currency and external assets are denominated in foreign currency, depreciation of the exchange rate reduces the net debt.

Log-linearization of equation (6.61) results in the equation (3.20) in the main text:

$$f_{t+1} = (1-\alpha)\, z_t + \frac{1}{\beta}\left(f_t + \frac{1-\theta}{B} g_t + \frac{\theta}{B} c_t - \frac{1}{B} y_t + \frac{\theta}{B}\mu s_t\right) \tag{6.62}$$

where the first term is valuation effect term and the remaining terms account for the net debt times gross rate of return and the trade deficit next period values. It is important to note that $z_t = 2r_t - (s_{t+1} - s_t)$.

As discussed in the main text, the shock to the trade deficit leads to an initial, unexpected depreciation of the exchange rate, which in turn leads to an unexpected decrease in the net debt position. Given that at the initial steady state $r = r^*$, this valuation gain is equal to:

$$df = -(1-\alpha)\Delta s \tag{6.63}$$

where Δs is unexpected depreciation of the exchange rate.

6.3 Equilibrium and dynamics of adjustment

The system of equations defined by (3.30)-(3.34) can be simplified further and written as follows:

$$\begin{aligned} f_{t+1} &= \{\frac{(1-\alpha)(2\alpha-1)}{2\alpha\phi} + \frac{1}{\beta}\} f_t + \frac{1}{\beta}\frac{\theta}{B} 2\mu c_t + \{\frac{1}{\beta}\mu\frac{\theta}{B}(1-2\eta(1-\mu)) - \frac{(1-\alpha)^2}{2\alpha\phi}\} s_t - \frac{1}{\beta}\frac{1}{B}\zeta_t \\ 2c_{t+1} - (1-2\mu)s_{t+1} &= \frac{(2\alpha-1)^2}{2\alpha\phi} f_t - \{\frac{(2\alpha-1)}{2\alpha\phi}(1-\alpha) + (1-2\mu)\} s_t + 2c_t \\ 0 &= (1-2\mu)\theta c_t + 2\eta\mu\theta(1-\mu)s_t + \zeta_t \end{aligned} \tag{6.64}$$

This system generalizes the system of equations of the BGS version of the model (3.35)-(3.38), which are simplified as:

$$\begin{aligned} f_{t+1} &= \{\frac{(1-\alpha)(2\alpha-1)}{2\alpha\phi} + \frac{1}{\beta}\} f_t + \frac{1}{\beta}\frac{\theta}{B} 2\mu c_t + \{\frac{1}{\beta}\mu\frac{\theta}{B}(1-2\eta(1-\mu)) - \frac{(1-\alpha)^2}{2\alpha\phi}\} s_t - \frac{1}{\beta}\frac{1}{B}\zeta_t \\ -s_{t+1} &= \frac{(2\alpha-1)}{2\alpha\phi} f_t - \{\frac{(1-\alpha)}{2\alpha\phi} + 1\} s_t \\ 0 &= (1-2\mu)\theta c_t + 2\eta\mu\theta(1-\mu)s_t + \zeta_t \end{aligned} \tag{6.65}$$

As essentially the difference of our version of the model from the BGS version is the treatment of interest rates, the difference between system (6.64) and (6.65) is only in the second equation, which in our model is the reduced form of Euler equation.

We first find equilibrium values, which are the same for both systems and are given by the following system:

$$\{1 - \frac{(1-\alpha)(2\alpha-1)}{2\alpha\phi} - \frac{1}{\beta}\} f = \frac{1}{\beta}\frac{\theta}{B} 2\mu c + \{\frac{1}{\beta}\mu\frac{\theta}{B}(1-2\eta(1-\mu)) - \frac{(1-\alpha)^2}{2\alpha\phi}\} s - \frac{1}{\beta}\frac{1}{B}\zeta \tag{6.66}$$

$$(2\alpha-1) f = (1-\alpha) s \tag{6.67}$$

$$0 = (1-2\mu)\theta c + 2\eta\mu\theta(1-\mu)s + \zeta \tag{6.68}$$

6.3.1 Stability analysis

To discuss the stability of the systems, we consider first the BGS system. We may substitute for consumption in equations for f and s in (6.65), and after conversion to continuous time this may be written as a dynamic system in just two variables, f and s. We have:

$$\frac{df}{dt} = \{\frac{(1-\alpha)(2\alpha-1)}{2\alpha\phi} + \frac{1}{\beta} - 1\}f + \{\frac{1}{\beta}\mu\frac{\theta}{B}(1-2\eta(1-\mu)) - \frac{(1-\alpha)^2}{2\alpha\phi} - \frac{4\mu^2\theta\eta(1-\mu)}{\beta B(1-2\mu)}\}s - \frac{1}{\beta}\frac{1}{B}\frac{1}{1-2\mu}\zeta$$

$$\frac{ds}{dt} = -\frac{(2\alpha-1)}{2\alpha\phi}f + \frac{(1-\alpha)}{2\alpha\phi}s \tag{6.69}$$

Since f is predetermined and the exchange rate is a jump variable, stability requires one stable and one unstable root. The condition for this is:

$$\frac{2\alpha-1}{1-\alpha} > \{\frac{(1-\alpha)(2\alpha-1)}{2\alpha\phi} + \frac{1}{\beta} - 1\}/\{\frac{1}{\beta}\mu\frac{\theta}{B}(2\eta(1-\mu)-1) + \frac{(1-\alpha)^2}{2\alpha\phi} + \frac{4\mu^2\theta\eta(1-\mu)}{\beta B(1-2\mu)}\} \tag{6.70}$$

The equilibrium and dynamics of the system are presented in Figure 1 where the EB ('long-run external balance') line, along which $df/dt = 0$ and the PB ('long-run portfolio balance') lines, along which $ds/dt = 0$ are plotted. The lines EB and PB correspond to the following equations respectively:

$$f\{\frac{(1-\alpha)(2\alpha-1)}{2\alpha\phi} + \frac{1}{\beta} - 1\} = \{\frac{1}{\beta}\mu\frac{\theta}{B}(2\eta(1-\mu)-1) + \frac{(1-\alpha)^2}{2\alpha\phi} + \frac{4\mu^2\theta\eta(1-\mu)}{\beta B(1-2\mu)}\}s + \frac{1}{\beta}\frac{1}{B}\frac{1}{1-2\mu}\zeta \tag{6.71}$$

$$(2\alpha-1)f = (1-\alpha)s \tag{6.72}$$

Saddle path stability requires that the PB line is to be steeper than the EB line. In that case there exists a saddle path (SS). Following an increase in f, the exchange rate depreciates and the current account moves into surplus so that f returns to an equilibrium.

The slope of the PB line can be understood as follows. An increase in f (Δf) reduces demand for US assets because of home bias in both regions, and in the long-run in which $ds/dt = 0$, the real exchange rate to fall by an amount:

$$\Delta s = \frac{2\alpha-1}{1-\alpha}\Delta f \tag{6.73}$$

to restore portfolio equilibrium.

The slope of the EB line is found as follows. On one hand, an increase in net debt increases the interest obligations on debt, including interest rate payments due to the increase in the excess return on US assets. This deteriorates the current account by $[(\frac{1}{\beta}-1) + \frac{(1-\alpha)(2\alpha-1)}{2\alpha\phi}]\Delta f$

On the other hand, depreciation of the exchange rate has effects on the current account: through trade balance, and through the relative returns on the assets. The impact on the current account through the trade balance comprises the following effects. An expenditure switching effect: both US and foreign consumers increase consumption of US goods, which improves the trade balance, and is captured by $2\frac{1}{\beta B}\theta\eta(1-\mu)\mu\Delta s$ (scaled by domestic bonds B and discount factor β); a valuation effect: an increase in the price of imports for US consumers deteriorates the trade balance by $\frac{1}{\beta B}\theta\mu\Delta s$; a real income effect: exchange rate depreciation reduces real income and thus consumption and improves trade balance by $\frac{1}{\beta B}\frac{4\mu^2\theta\eta(1-\mu)}{(1-2\mu)}\Delta s$.

The effect on the current account through changes in the relative returns on the assets is a denomination effect: following depreciation of the exchange rate, less value of US assets in the total value of world assets, yields lower excess return on US assets in foreign portfolios, which improves the current account by $\frac{(1-\alpha)^2}{2\alpha\phi}\Delta s$.

In order that $df/dt = 0$, the effects of Δs and Δf must cancel each other, so that

$$[\frac{(1-\alpha)(2\alpha-1)}{2\alpha\phi}+\frac{1}{\beta}-1]\Delta f+[\frac{1}{\beta}\mu\frac{\theta}{B}(1-2\eta(1-\mu))-\frac{4\mu^2\theta\eta(1-\mu)}{\beta B(1-2\mu)}-\frac{(1-\alpha)^2}{2\alpha\phi}]\Delta s=0 \quad (6.74)$$

The slope of the PB line comes from this equation. If the PB line is steeper than the EB line, we can use (6.73) and (6.74) so obtain (6.70).

So far, we have studied stability condition for the BGS model. Stability for our model requires us to allow for the fact that our system (6.64) is a system of three variables, f, s and c, where c is a jump variable, which through Euler equation affects real interest rate r_t, which in turn affects excess return z_t and all these effects feed into the model. By contrast in the BGS model, c is linked to s through a static third equation, which holds as a result of implicit, unmodelled fiscal actions, and so there is no effect on r.

We now analyze the stability of our full model. In our analysis of the BGS model, we reduced it to a system of two dynamic variables, f_t and s_t. But our full model shown in system (3.1) contains three dynamic equations for f_t; s_t and c_t, where c_t as well as s is a jump variable. Unlike in the BGS model, c_t is now determined by the interest rate in a dynamic Euler equation; the real interest rate r_t must in turn give a value for consumption such that $y_t = 0$ in the third equation of system (3.1). This real interest rate r_t, affects the excess return z_t and these effects influence the rest of the model. (By contrast in the BGS model, c_t is linked to s_t through the requirement that $y_t = 0$, but this happens as a result of implicit unmodelled fiscal actions, and so there is no effect on r_t.)

Nevertheless, after substituting out for the jump in consumption, our system can also be reduced to two dynamic equations for f and s, namely

$$\frac{df}{dt}=\{\frac{(1-\alpha)(2\alpha-1)}{2\alpha\phi}+\frac{1}{\beta}-1\}f+\{\frac{1}{\beta}\mu\frac{\theta}{B}(1-2\eta(1-\mu))-\frac{(1-\alpha)^2}{2\alpha\phi}-\frac{4\mu^2\theta\eta(1-\mu)}{\beta B(1-2\mu)}\}s-\frac{1}{\beta}\frac{1}{B}\frac{1}{1-2\mu}\zeta$$

$$\frac{ds}{dt}=\frac{(2\alpha-1)}{A}[-\frac{(2\alpha-1)}{2\alpha\phi}f+\frac{(1-\alpha)}{2\alpha\phi}s], \quad (6.75)$$

where $A = \frac{1}{1-2\mu}[1+4\mu^2(1-\eta)+4\mu(\eta-1)]$, and where consumption is such that $y = 0$ in the third equation of system (6.64). The only difference between systems (6.69) and (6.75) is in the second equation, in which the right hand side is multiplied by a term less than 1. As a result of the similarity of these equations the equilibrium and dynamics of our system can be also represented as in Figure 1, with the same EB and PB lines.

6.3.2 Adjustment: phase diagram analysis

Given our choice of parameters (discussed in calibration section above), the stability condition is satisfied in what follows. In that case, we now can discuss the dynamic and long-run effects of the shift in trade deficit, the shock analyzed in the BSG paper as well. We will perform simulation analysis of the model to this shock below and will thus study dynamic adjustment of the economy to the shock. It is convenient, however, to begin with an analysis of the economy to the shock with help of the phase diagram.

We consider one and for all fall in ζ, which corresponds to the permanent increase of the trade deficit. The figure 1b shows the effects of a reduction in ζ for both BGS and our model. In both cases, the line EB moves up to the EB1. The outcome is that the economy jumps from point A to point B on the saddle path in the BGS case, and then converges over time along the saddle path, from B to a new equilibrium C. In our case, the initial equilibrium is the same, at point A. After fall in ζ, the economy jumps from A to D, and then converges over time along to saddle path from D to the same long-run equilibrium C.

The full steady state outcomes consistent with the move from A to C are:

$$s = -\frac{\frac{1}{B}\frac{1}{1-2\mu}}{[-\frac{1}{B}\frac{\theta\mu}{1-2\mu}\{1-2\mu-2(1-\mu)\eta\}-(1-\beta)\frac{1-\alpha}{2\alpha-1}]}\zeta \tag{6.76}$$

$$f = -\frac{1-\alpha}{2\alpha-1}\frac{\frac{1}{B}\frac{1}{1-2\mu}}{[-\frac{1}{B}\frac{\theta\mu}{1-2\mu}\{1-2\mu-2(1-\mu)\eta\}-(1-\beta)\frac{1-\alpha}{2\alpha-1}]}\zeta \tag{6.77}$$

$$c = [\frac{2\eta\mu(1-\mu)}{(1-2\mu)}\frac{\frac{1}{B}\frac{1}{1-2\mu}}{[-\frac{1}{B}\frac{\theta\mu}{1-2\mu}\{1-2\mu-2(1-\mu)\eta\}-(1-\beta)\frac{1-\alpha}{2\alpha-1}]}-\frac{1}{\theta(1-2\mu)}]\zeta \tag{6.78}$$

We see from these equations the formal solution for the move from A to C, and the outcome for C which is necessary to make this possible. It is clear that when domestic financial wealth is entirely invested into domestic assets only and thus there are no external liabilities, i.e. when $\alpha = 1$, the outcome collapses to a simple one in which $s = -\zeta/[\mu\theta\{2(1-\mu)\eta + 2\mu - 1\}]$, $c = \zeta/\theta[\{2(1-\mu)\eta + 2\mu - 1\}]$ and $f = 0$. It is home bias which is driving the movements in f, and the extent to which the movements in s has to be more than $-\zeta/[\mu\theta\{2(1-\mu)\eta + 2\mu - 1\}]$. So these values of s and c show what depreciation is needed (and reduction in consumption) to achieve immediate trade balance, given no changes in external debt.

Comparing the adjustment to the shock in two models, it is worth pointing out the following differences in the responses. First, it is clear from the system (6.75) that the slope of the saddle path is flatter than in the BGS case (that can be confirmed formally). The reason is that if, say, f is below equilibrium, this will cause the exchange rate to fall less than all the way to equilibrium. This means that consumption must fall by less than its full fall – and it may even rise. That in turn means that the real interest rate will fall. As a result part of the reduction in the return on US assets comes from this interest rate reduction; so that exchange rate will be falling less fast. As a result the initial jump in the real exchange rate is larger and after this the adjustment of both f and s is slower over time. Second, in both cases, the horizontal component of the jump of the economy from the initial steady state to the point on the saddle path represents the size of the valuation gain. The section above contains detailed derivation of the balance of payments equation which has a valuation effect term. Valuation effect implies that on response to the shock, the unexpected depreciation of the dollar leads to unexpected reduction in external debt. Denoting Δs the unexpected depreciation of the exchange rate, the size of the valuation effect at the time of the shock equals to:

$$df = -(1 - \alpha)\Delta s \tag{6.79}$$

As the valuation gain is proportional to the depreciation, the valuation gain is larger in our model than in the BGS model.

7 Figures

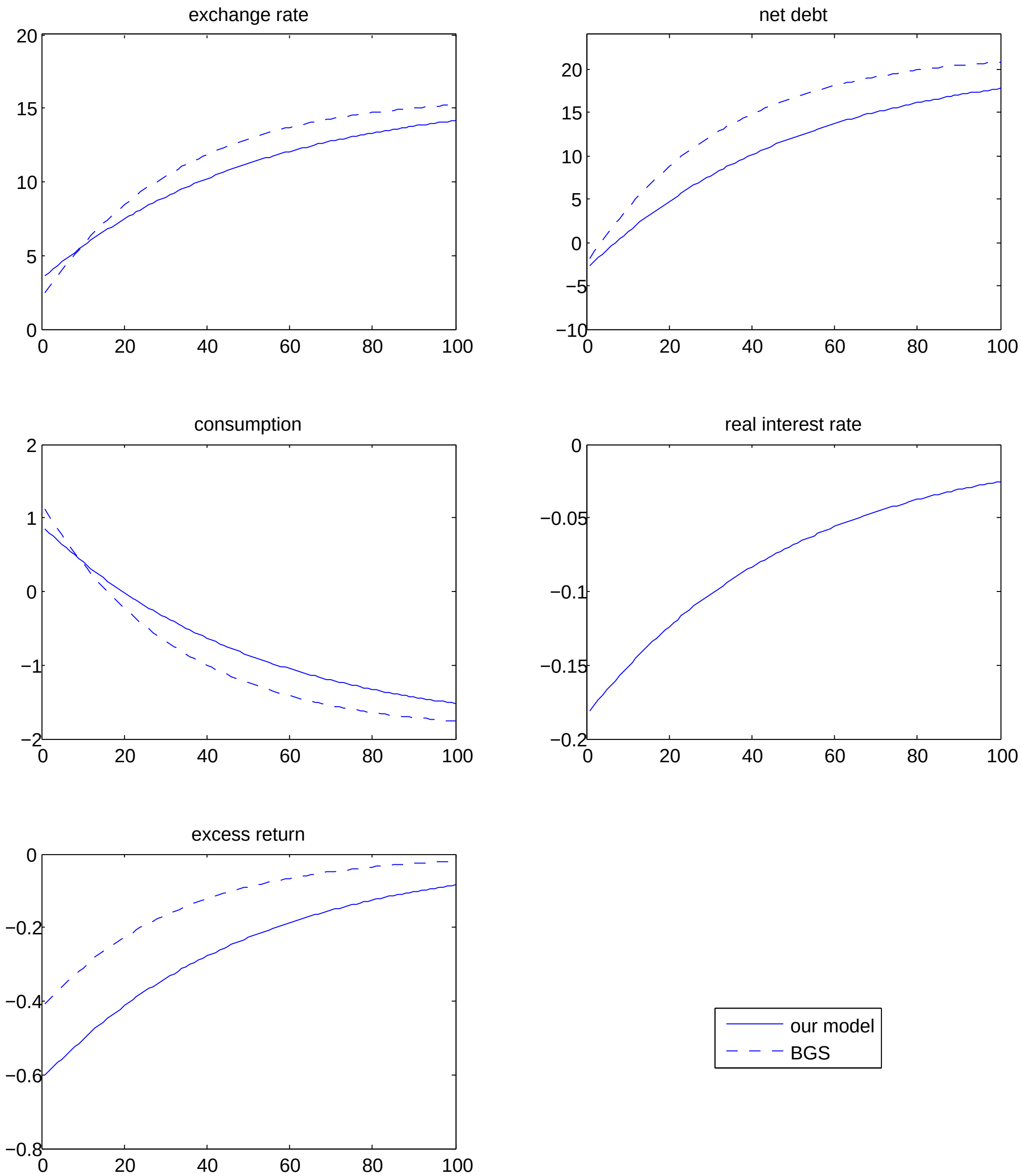

Figure 1: $\phi = 1$

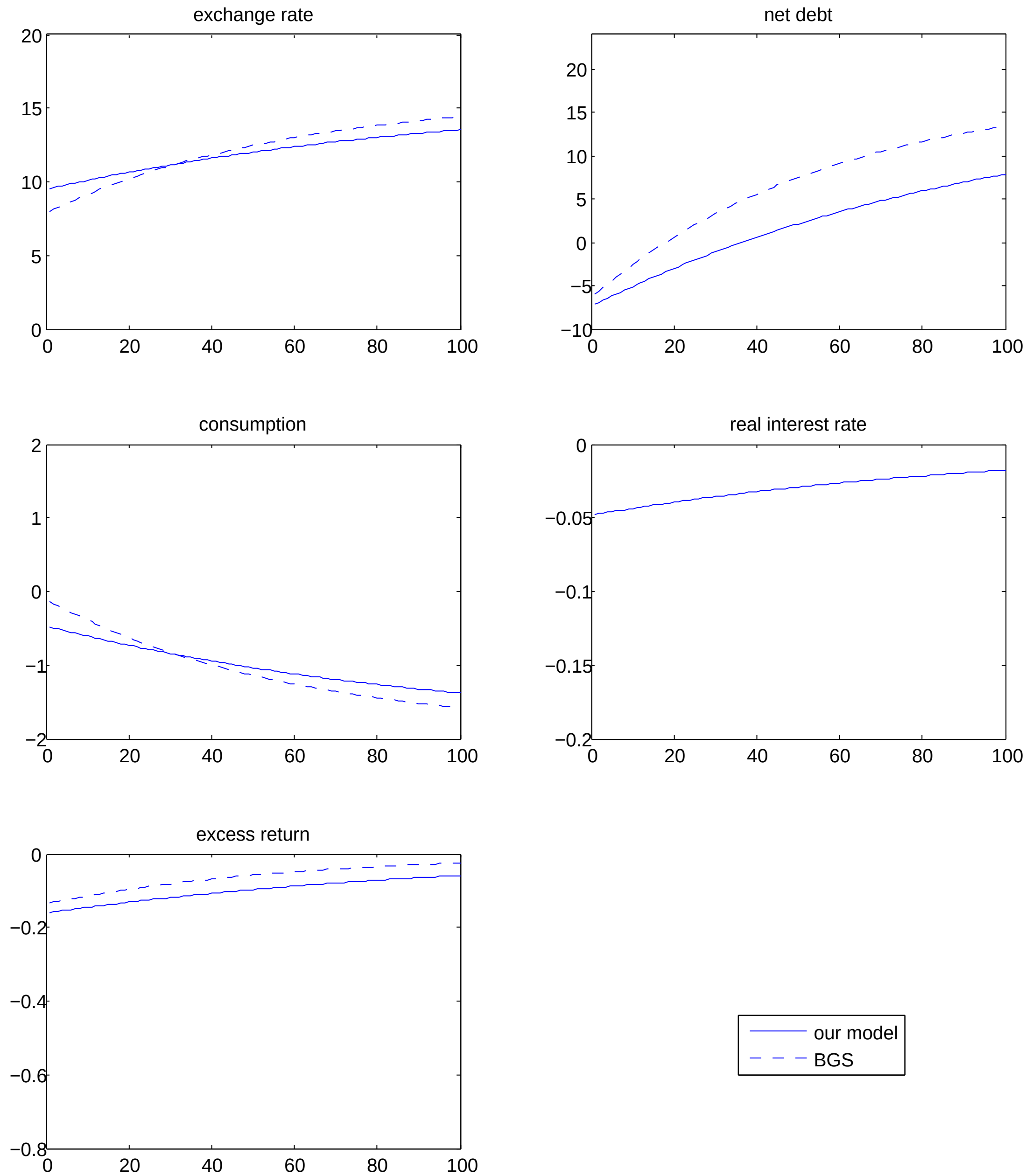

Figure 2: $\phi = 10$

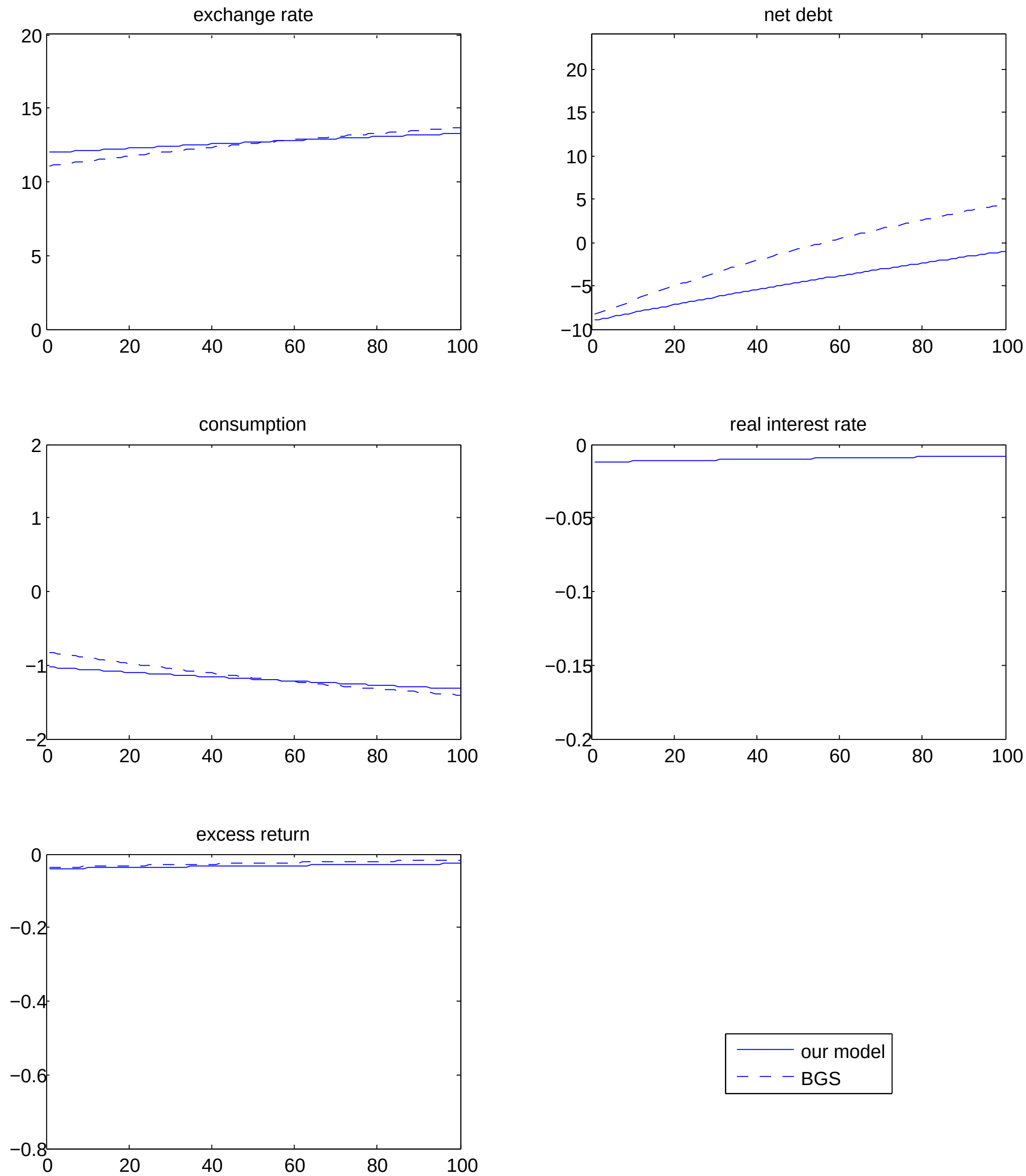

Figure 3: $\phi = 50$

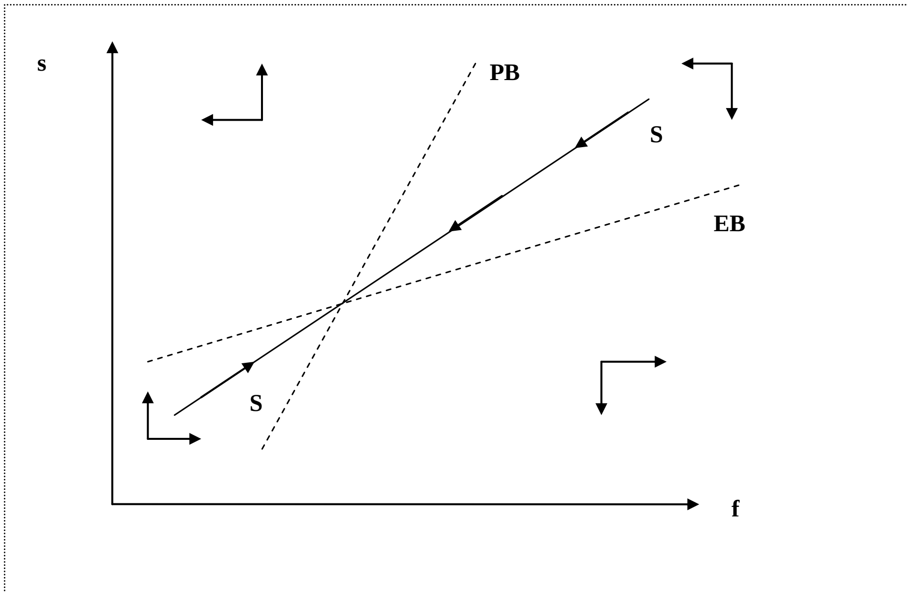

Figure 1a: Phase diagram, exchange rate and the net debt position

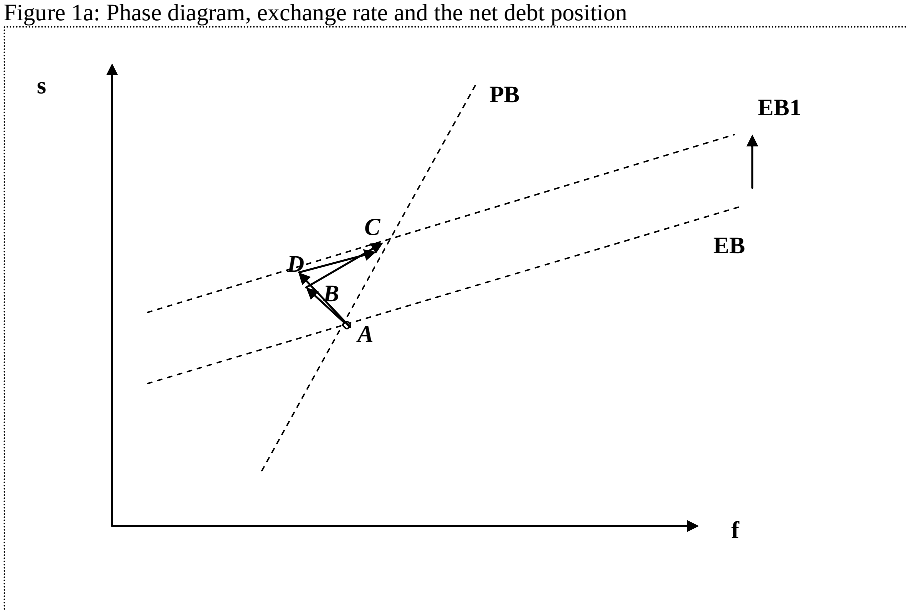

Figure 1b: Response to a trade deficit shock

Figure 4: Saddle path, adjustment of the exchange rate and the net debt position