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ABSTRACT

The Macroeconomics of Money Market Freezes

This paper develops a tractable general equilibrium model in which money markets provide structural funding to some banks. When bank default risk becomes significant, retail deposit insurance creates an asymmetry between banks that operate in savings-rich regions, which can remain financed at cheap risk-free rates, and in savings-poor regions, which have to pay either large spreads in money markets or high rates for the scarce regional savings. We show that this asymmetry can cause a severe distortion of the aggregate allocation of credit. When interdependencies across borrowers are large (e.g., via demand externalities), output and welfare losses are also large and can be dramatically reduced by an aggressive subsidization of money market borrowing. The analysis offers some insights on the rationale for responding to a money markets freeze with full-allotment fixed-rate lending policies by central banks or the extension of government guarantees on non-deposit liabilities.

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1 Introduction

During the last decades, banks have increasingly turned to money markets not just to offset *temporary* liquidity deficits and surpluses, but also *structural* ones. Wholesale funding coming from money markets allowed banks with low amounts of core deposit funding to engage in high amounts of retail lending. The asymmetry between deposit-rich and deposit-poor banks remained inconsequential until the perceived risk of bank failure (which is extremely small in normal times) soared during the current financial crisis. In this paper we argue that, in the presence of retail deposit insurance, this asymmetry is very important and can explain the freezing of money market trade and severe distortions in the allocation of credit during a crisis.¹

Deposit insurance has played a clear stabilizing role in the current crisis, where massive withdrawals of retail deposits have been mostly avoided by reassuring depositors about the ample coverage of the guarantees. So deposit insurance has facilitated the continued funding of banks with a base of core deposits wide enough to sustain their basic investment activities, i.e. in a net lending position in wholesale markets. The same has not been true for banks in a net borrowing position in those markets.

At the beginning of the 2007-2009 crisis, banks' money market liabilities (as well as non-deposit liabilities in general) were not covered by deposit insurance or any other type of explicit government guarantee.² With the dramatic rise in the perceived risk of bank failure, banks with insufficient core deposit funding had to pay spreads that were much higher than usual for the refinancing their non-deposit liabilities, which placed them in a very asymmetric position vis-a-vis banks with sufficient core deposit funding. Insofar as the retail lending of banks to households and firms remains a largely local and relationship intensive activity, this

¹Deposit insurance has received lots of attention in the discussion of other banking crises, but much less in the current one. The lack of deposit insurance was behind the dimension and destructive power of retail bank panics in the Great Depression and in many other US banking crises that occurred before its introduction in 1934 (see Gorton, 2009). On the other hand, moral hazard problems exacerbated by the inadequate pricing of deposit insurance and the lack of supplementary prudential regulation have been claimed to be central in the inception of the US savings and loans crisis at the end of the 1980s and other bank crises elsewhere (see Demirguc-Kunt and Detragiache, 2002).

²Before the crisis, this aspect was believed to have the advantage of introducing market discipline.

asymmetry translated into an uneven access to credit for some classes of retail borrowers, in effect producing a credit crunch for borrowers dependent on banks routinely funded in wholesale markets. Importantly, in some regions the majority of banks systematically relied on wholesale funding, so the credit crunch became a problem for entire regions, and because of tight regional interdependencies (e.g. via trade) for the whole economy.³

Putting together insights from the microeconomic literature on banking and the macroeconomic literature on credit, this paper proposes a tractable general equilibrium model with banks that captures the structural funding role of money markets in a world in which the distribution of savings across regions is not uniform. The model allows for a qualitative as well as quantitative evaluation of the implications of a “solvency crisis” (a rise in the risk of bank failure) and the asymmetries that arise from the fact that, unlike retail deposits, money market liabilities are not typically insured. It also allows for an evaluation of policy responses that aim to correct the implications of the asymmetries for the allocation of credit across banks and their final borrowers (special discount window facilities, public programmes to subsidize or directly buy money market instruments, or the extension of outright government guarantees to bank liabilities).

In the model, agents are risk-neutral and investment opportunities take the form of intertemporal production through a (commonly accessible) neoclassical technology that transforms capital and labor at some initial date into units of the numeraire at some final date. We focus on a bank-based perfectly competitive financial system where the retail activities of banks are characterized by regional specialization: banks collect savings from regional households and extend loans to regional firms. Labor is not mobile and regions have heterogeneous endowments of savings, which are channeled to the regional banks, mostly in the form of government-insured retail deposits. Banks use money markets to borrow from and lend to each other, so banks from savings-poor regions tend to be net borrowers, while banks from savings-rich regions tend to be net lenders. Hence the key role of money markets is to

³Reflecting the banking side of the savings-investment imbalances built up over the last decades, the national banking systems of the US and many European countries (with the notable exception of Germany) were in a net borrowing position vis-a-vis institutions and individuals from the rest of the world (most notably, China, Japan, and the oil exporting countries).

reallocate savings across regions.⁴

We model the crisis as a shock to the solvency of the firms in some regions which is negative enough to compromise the solvency of the corresponding regional banks. In the pre-crisis situation, when the risk of bank failure is negligible, the difference between borrowing and lending banks is immaterial to the allocation of total savings across regions. In contrast, when the risk of bank failure becomes significant, formerly-lending banks can continue financed at the relatively cheap risk-free rate paid on insured deposits, while formerly-borrowing banks must either pay a high spread on that rate in money markets or pay a relatively expensive autarkic deposit rate to the regional savers. The result is an asymmetric allocation of total savings across regions.

The model shows that, under realistic values of the relevant parameters, spreads of around 200 basis points (resulting from a bank failure probability of roughly 2%) can be associated with reductions in money market volumes of more than 75% (with full collapse when the probability of bank failure reaches 3%) and cause a large reallocation of capital across regions. This reallocation has a significant impact on the output, wages, returns to savings, and aggregate welfare (which we measure as the aggregate expected final net worth of all the agents in the economy, net of the distortionary cost of taxation) of the affected regions.⁵

An important determinant of the size of output and wealth losses at the aggregate level is the extent to which the value of investment opportunities in a region is related to the level of economic activity in other regions. We capture such possible linkages by allowing the total factor productivity of the technology used in each region to depend on the levels of activity in other regions: productivity in all regions may fall because the level of activity in the savings-poor regions falls. In this context, the output in savings-rich regions may fall in spite of the fact that they end up with more funding. In our central scenario with

⁴We could refer to capital markets more generally. We focus on money markets since the typical maturity of their instruments is more consistent with the short-run horizon of our analysis. But it will become obvious that the logic of our argument could also be applied to banks that attempt to cover their refinancing needs by placing longer-term debt among investors from other regions.

⁵We parameterize the model economy so as to exhibit, prior to the crisis, equilibrium risk-free rates, capital-to-output ratios, labor shares, expected probabilities of default, losses given default, etc. in line with those observed in the data (see Section 6).

interdependencies, aggregate GDP falls by roughly 5% and welfare by 1.75%.⁶

In this setup, we examine the impact of policies that, without removing retail deposit insurance and without changing its essentially flat pricing, can effectively ameliorate the asymmetry across borrowing and lending regions. Specifically, we consider policies that are tantamount to making the government (or central bank) accept or absorb non-fully compensated counterparty risk from the borrowing banks. This can be achieved if, for example, the central bank acts as a market-maker, accepting deposits from banks at a given rate and lending to other banks at the same rate plus a reduced or zero spread—like in the fixed-rate full-allotment refinancing and discount window facilities offered by major central banks during the current crisis.⁷ Equivalently, the central bank or government could insure money market liabilities (charging below-market insurance premia or fees) or the government could lend directly to the firms that operate in savings-poor regions (or to any firm willing to accept its financing terms).

We show that, if the distortionary cost of taxation is zero, the policy within this class that maximizes aggregate welfare (net worth net of tax costs) involves full insurance of money market liabilities or, equivalently, full-subsidization of the market spread or full-allotment central bank lending at the risk-free rate.⁸ In other words, such a policy would aim to restore the (symmetric) allocation of capital across lending and borrowing regions that occurs when counterparty risk is (virtually) zero. In our parameterization of the model, this policy may

⁶Conservatively, we parameterize the degree of interdependence across regions to a level that produces just a mild recession in savings-rich regions.

⁷Our analysis suggests that effectively correcting the distortions associated with the widening of market spreads may require the central bank to follow a full-allotment policy at the relevant lending rate, since what matters for the final (competitive) pricing of bank loans is the marginal funding cost of banks. The provision of cheap funding in large but eventually “rationed” amounts might not affect loan rates and hence be tantamount to giving a pure rent to the receiving banks.

⁸In terms of our model, the risk-free rate is the relevant marginal funding rate for banks with a funding surplus. In practice this might correspond to the rate paid by central banks on their standing deposit facilities. In the jargon of the European Central Bank (ECB), the disparity between the rates of the overnight standing lending and deposit facilities is called the *interest rate corridor*, but probably the size of such a corridor is an upper bound to the empirical counterpart of difference between the rate paid by borrowing and lending banks in our model. The reason for this is that currently central banks such as the Federal Reserve or the ECB conduct most of their lending to banks through periodic auctions under a fixed-rate full-allotment rule (like the Fed’s Term Auction Facility) and the rate used in these auctions is probably the closest counterpart of the rate paid by borrowing banks in the model.

reduce aggregate output and welfare losses resulting from the rise in the risk of bank failure by almost 50%.⁹

The macroeconomic literature has paid some attention to banks but almost no attention to money markets. Papers such as Romer (1985) and Van den Heuvel (2008) introduce intermediaries in otherwise standard general equilibrium frameworks. Papers in the tradition started by Bernanke and Gertler (1987) and Williamson (1987) study the role of banks in the transmission of monetary (and non-monetary) shocks to the real economy. Contributions such as Holmstrom and Tirole (1997) and Bolton and Freixas (2006) have merged ingredients taken from modern financial intermediation and corporate finance theories with the focus on aggregate economic activity. Recent work in this field is more quantitatively oriented and includes Chen (2001), Van den Heuvel (2007), Repullo and Suarez (2008), and Meh and Moran (2009).

In the banking literature, most academic papers on money markets focus almost exclusively on the interbank markets and their role in providing banks the opportunity to smooth out transitory funding gaps or surpluses; see Allen and Gale (2007), and Freixas and Rochet (2008) for excellent overviews and detailed references. Following Diamond and Dybvig (1983), many papers highlight the liquidity role of bank deposits, the room for panic associated with the maturity transformation function of banks, the room for contagion due to interbank linkages, and the additional frictions brought about by asymmetries of information and agency problems. Recent contributions with a focus on financial crises and possible policy responses include Acharya, Gromb, and Yorulmazer (2008), Allen, Carletti, and Gale (2008), Huang and Ratnovski (2008), and Heider, Hoerova, and Holthausen (2009).

Within this tradition, Freixas and Jorge (2008) share with us the emphasis on the effects of interbank market frictions on the allocation of credit. These authors study the impact of monetary policy in a model in which asymmetric information in the interbank market can

⁹When the distortionary cost of taxation is strictly positive, the full restoration of the symmetric outcome is no longer optimal; the optimal policy involves free but partial insurance of money market liabilities, or full insurance in exchange for a small premium or fee, or partial subsidization of the market spread or full-allotment lending by the central bank at a rate above the risk-free rate. However, for realistic parameterizations of the model, the optimal subsidization of the spread reaches 90% or more, even when the social cost of 1 dollar of taxes is as high as 1.5 dollars.

produce credit rationing among the borrowing banks and, through them, among the bank-dependent final borrowers. With a different approach (and no reference to money markets), Bebchuk and Goldstein (2008) obtain a credit market freeze as a self-fulfilling outcome in a context where firms are (like in our model) both interdependent and dependent on the funding from specific banks. They show that, when macroeconomic fundamentals are weak, banks may not lend to their firms of reference simply because they (correctly) anticipate that other firms will not receive funding from their own banks.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3, we analyze the (partial) equilibrium in each region (i.e., the decisions of households, firms, and banks) for given values of the variables determined at the interregional level. In Section 4 we characterize the general interregional equilibrium, crucially associated with the operation of the money market. In Section 5 we discuss the welfare losses associated with the asymmetries that arise when counterparty risk becomes significant, and we evaluate the effects of policies oriented to reduce these asymmetries. Section 6 describes the baseline parameterization of the model and the quantitative results. Section 7 concludes. The Appendix contains a first section that discusses the economy without deposit insurance and a second section with a detailed definition of all the variables reported in the quantitative part.

2 The Model

Consider a perfect competition model with two dates ($t = 0, 1$) and a continuum of measure one of regions indexed by $j \in [0, 1]$. In each region there are a representative household, a continuum of measure one of firms (each owned by an entrepreneur), and a representative bank. There is also a government that operates across regions. All agents are risk-neutral. There is a single good in each period which is the numeraire and can be used for both consumption and investment (i.e., as physical capital).

2.1 Households

The representative household in region j has some exogenous initial savings S_j and inelastically supplies one unit of labor in the regional labor market, where the regional wage rate is w_j . The household receives its salary income at $t = 0$ and its objective is to maximize its net worth at $t = 1$. For households, the deposits and the equity issued by the regional bank are the only means of transferring wealth from $t = 0$ to $t = 1$. Deposits pay an interest rate r_{dj} and are fully insured (principal plus interest) by the government—for the purposes of comparison in the Appendix we consider the economy without deposit insurance. Bank equity will be described below.

For simplicity, we assume that there is a fraction π of savings-rich regions with $S_j = S_H$ and a fraction $1 - \pi$ of savings-poor regions with $S_j = S_L$, where $S_L < S_H$. It is convenient to refer to the aggregate exogenous savings as $\bar{S} \equiv \pi S_H + 1 - \pi S_L$.

2.2 Firms

Each firm $i \in [0, 1]$ in a given region j operates a constant return to scale technology that allows it to transform the capital k_i and labor n_i employed at $t = 0$ into

$$z_{ij}[AF(k_i, n_i) + (1 - \delta)k_i] + (1 - z_{ij})(1 - \lambda)k_i \quad (1)$$

units of the consumption good at $t = 1$, where $z_{ij} \in \{0, 1\}$ is a binary random variable realized at $t = 1$ that indicates whether the firm's production process succeeds or fails, A is an aggregate productivity factor (on which we further elaborate below), and δ and λ are the rates at which capital depreciates when the firm succeeds and when it fails, respectively. For simplicity we adopt a standard Cobb-Douglas specification with

$$F(k_i, n_i) = k_i^\alpha n_i^{1-\alpha}, \quad (2)$$

where $\alpha \in (0, 1)$.

Firms fail to produce output on top of k_i whenever $z_{ij} = 0$. To capture different degrees of dependence among firms' *failure* in a simple way, we assume that all firms in a region fail simultaneously with probability ε , while, otherwise, z_{ij} is independently and identically

distributed across firms with $\Pr[z_{ij} = 0] = p > 0$. Hence, by the law of large numbers, the distribution of the fraction of failing firms in region j is as follows:

$$x_j = \begin{cases} p & \text{with prob. } 1 - \varepsilon, \\ 1 & \text{with prob. } \varepsilon. \end{cases} \quad (3)$$

So ε can be interpreted as a measure of the exposure to a negative regional shock which makes all firms fail, while p determines the failure rate in “normal times.” We assume that regional shocks are independently distributed so that the ex post fraction of regions whose firms fail all at once will be ε , by the law of large numbers.

Each firm i is owned and managed by a penniless *entrepreneur* who is interested in maximizing his expected net worth at $t = 1$. The firm uses bank loans to pay in advance for the capital k_{ij} and labor n_{ij} utilized at $t = 0$. So the loan required by firm i at $t = 0$ has size $l_{ij} = k_{ij} + w_j n_{ij}$. In exchange for l_{ij} , the firm promises a repayment R_{ij} to its banks at $t = 1$ and the effective payment to banks at $t = 1$ is determined as in a standard debt contract: the bank receives R_{ij} if the firm does not fail and $\min\{R_{ij}, (1 - \lambda)k_{ij}\}$ when the firm fails. The variables in the tuple $(k_{ij}, n_{ij}, l_{ij}, R_{ij})$ are set by a contract signed at $t = 0$. Since banks, as specified below, are perfectly competitive, this contract will maximize the positive part of the firm’s future expected profit (net of debt repayments), subject to the representative bank’s participation constraint.

2.3 Banks

The representative bank in region j is owned and managed by a coalition of regional households for whom contributing equity capital e_j to the bank at $t = 0$ implies incurring a utility cost ϕe_j at $t = 1$, as well as becoming the bank’s residual claimants at that date, under standard limited liability provisions. We think of ϕ as a reduced-form for the *excess cost* of equity financing vis-à-vis deposit financing, which is typically attributed to the higher issuance costs, disadvantageous tax treatment, lower liquidity, and higher monitoring needs of equity financing. The bank maximizes its shareholders’ expected net worth at $t = 1$, net of the utility cost ϕe_j .

In addition to equity financing, the bank can also attract deposits d_j , remunerated at a

rate r_{dj} , from the regional households. And the bank can use its financial resources to make loans $l_j = \int_0^1 l_{ij} di$ to the regional firms. In addition, all banks have access to an interregional money market (MM) where they can lend to and borrow from each other. Thus, the bank's balance sheet constraint imposes

$$l_j + a_j = d_j + e_j, \tag{4}$$

where a_j denotes its net lending position in the MM. Net *lenders* have $a_j > 0$ and net *borrowers* have $a_j < 0$.

2.4 The money market

Realistically, we assume that MM liabilities take the form of unsecured debt that is junior to deposit liabilities. In practical terms this means that MM lenders are junior to the deposit insurance guarantor (the government) if the bank fails. We will calibrate the model so that (in equilibrium) a bank fails when the negative regional shock is realized (and all the firms in the region fail), so MM lending will be perceived by the lenders as having a probability of default ε .

In principle, MM defaults might put MM lenders at risk of insolvency. However, to simplify the discussion, we are going to assume that, for prudential reasons, regulation requires MM lenders to hold regionally-diversified portfolios of MM loans. This eliminates the possibility of contagion (failure induced by MM exposures) and implies that all MM lending will require the same *expected* net return r .¹⁰ The value of this reference *risk-free* rate will be endogenously determined in equilibrium.

To compensate for default risk and the losses incurred by MM lenders upon default, the *contractual* interest rate on bank-to-bank MM borrowing may have to include some positive endogenous *spread* s over r . To sharpen the discussion, we will focus on parameterizations in which MM lenders recover zero when a bank defaults, in which case, it makes sense to

¹⁰Notice that limited liability distortions might induce strange preferences for risk among the MM lenders who have a positive probability of failing. The imposed diversification of MM portfolios makes them "safe" and avoids considering this complication.

assume that the spread s will be the same for all MM borrowers, independently of the exact investment decisions undertaken by each of them.¹¹

2.5 Prudential regulation and the government

We assume that prudential regulation, in addition to obliging banks to hold well diversified portfolios of MM loans, also obliges them to diversify their portfolios of regional loans across firms. For realism, we also assume that regulation establishes a minimum capital requirement of the form $e_j \geq \gamma l_j$.

The government in this economy insures retail deposits and may get involved in borrowing and lending in the MM at $t = 0$.¹² As for the latter, it can place government debt in an amount $B \geq 0$ among MM lenders at the risk-free rate r and lend this amount to borrowing banks at the rate $r + s$. Other forms of intervention in MM, such as offering guarantees on MM instruments or subsidizing the spreads can be shown to be equivalent to some form of direct involvement of the government (or the central bank) in borrowing and lending. Hence, we will not consider them separately.

At $t = 1$, the government raises taxes T from households in order to cover its net financial obligations, including those associated with deposit insurance, with its MM interventions, or with any guarantees offered to MM lenders. To capture possible unmodeled distortions caused by taxation, we assume that T has a social cost of $(1 + \eta)T$ at $t = 1$, where $\eta \geq 0$ is the so-called *marginal cost of public funds*.

Whenever relevant, the government will make its decision on B or any other policy variable, in order to maximize aggregate social welfare at $t = 1$, that is, the aggregate net worth of households, firms, and banks of all regions at $t = 1$, net of the distortionary cost of taxation.

¹¹When, as discussed below, the government becomes the relevant MM lender, we will keep the assumption that all borrowers are charged the same flat spread.

¹²Recall that the economy without deposit insurance is briefly discussed in the Appendix.

2.6 Interregional feedback

We capture regional interdependence by making the productivity factor A that appears in (1) a function of all regions' levels of activity.¹³ Since labor is inelastically supplied and equal to one in each region, the aggregate level of activity in each region j can be summarized by the total amount of physical capital invested in that region, $k_j \equiv \int_0^1 k_{ij} di$. We postulate that the levels of activities of the various regions contribute to A through a constant elasticity of substitution (CES) aggregator of the form:

$$A = \left[\int_0^1 k_j^\rho dj \right]^{\frac{\tau}{\rho}}, \quad (5)$$

where $\rho \leq 1$ is the elasticity parameter and $\tau < 1 - \alpha$ is a returns-to-scale parameter. Intuitively, ρ measures the importance of evenly distributing across regions the total aggregate capital invested in the economy, $K \equiv \int_0^1 k_j dj$, while τ measures how productivity would increase with the level of investment across all regions.

It turns out that, in this economy, the equilibrium amount of aggregate capital, K , is always equal to the aggregate initial savings, \bar{S} , which are fixed, so the returns-to-scale parameter τ does not play an important role. In contrast, the parameter ρ is key to the importance of interregional feedback. For instance, if $\rho = 1$, the various k_j enter as perfect substitutes and $K = \bar{S}$ implies $A = \bar{S}^\tau$, irrespectively of the interregional allocation of capital. However, for $\rho < 1$, the various k_j are imperfect substitutes and, hence, making the distribution of capital across regions more uneven reduces aggregate productivity.¹⁴ The analysis below will show that $\rho < 1$ is not necessary to produce either MM freezes or the asymmetries in the allocation of \bar{S} across regions provoked by them, but it will confirm that $\rho < 1$ is key to amplify (and spread across all regions) the aggregate losses due to those freezes.

¹³This is a reduced-form approximation than can be justified by the type of technological externalities (returns to specialization, learning-by-doing, technological spill-overs, etc.) first considered by the literature on endogenous growth (Romer, 1986; Lucas, 1989). It can also capture other causes of interdependency such as the demand externalities that arise when there is monopolistic competition in a intermediate goods sector with differentiated products (Blanchard and Kiyotaki, 1987). These externalities are at the core of the new open economy macroeconomics started by Obstfeld and Rogoff (1995).

¹⁴In the polar case with $\rho \rightarrow \infty$, the associated Leontief complementarities would imply that A equals the minimal k_j observed across all regions.

3 Partial equilibrium analysis

The goal of this section is to characterize the (partial) equilibrium in savings-rich and savings-poor regions for given values of the risk-free rate r , the spread s , and the productivity term A , since these three variables will be determined at the interregional (general) equilibrium level. The contents is organized in two subsections. In subsection 3.1 we describe the contracts through which firms and banks in each region arrange their relationships. The optimization problems that determine the equilibrium optimal contracts embed the optimal production decisions of firms and the optimal financing decisions of firms and banks. In subsection 3.2 we analyze the solution of these problems together with equilibrium conditions in the regional labor markets, thereby determining the net lending or borrowing positions with which the representative banks of each region go the interbank market for given values of r , s , and A .

3.1 The firm-bank contracts

Consider a given region j . All firms in the region are ex ante equal and operate under constant returns to scale. The requirement that the representative regional bank holds a well diversified portfolio of regional loans implies that it should finance a continuum of firms, so that firm-idiosyncratic risk is fully diversified. We can assume, without loss of generality, that all firms $i \in [0, 1]$ in the region are funded under exactly the same contract (k, n, l, R) , where we remove the firm and regional subscripts i and j , for brevity.

To simplify the discussion, we restrict ourselves to parameter combinations for which the following two conditions hold.

A1 The capital requirement γ is low enough to guarantee that, under the equilibrium arrangement, when all firms in a region fail ($x_j = 1$), the corresponding regional bank goes bankrupt.

A2 In savings-poor regions, regional deposit liabilities are large enough for the recoveries of MM lenders to be zero when the regional bank goes bankrupt.

Assumption A1 is necessary for us to talk meaningfully about counterparty risk in money markets, while A2 considerably simplifies the determination of the resulting spreads. Neither

assumption turns out to be restrictive in the calibration of the model in the quantitative section below.¹⁵

Under A1 and A2, whenever the marginal MM lenders are banks—rather than the government—the spread s charged on MM funds will satisfy the relationship:

$$(1 - \varepsilon)(1 + r + s) = 1 + r, \quad (6)$$

where $1 + r$ is, by definition, the lenders’ required expected rate of return per unit of MM lending—as well as the rate at which the government issues its debt, whenever relevant. From (6), the *laissez-faire* spread can be written as follows:

$$s = \frac{\varepsilon}{1 - \varepsilon}(1 + r), \quad (7)$$

which does not depend on specific decisions of the borrowing bank.

If the government decides to lend to banks at a spread below the one determined by (7), it will have to opt between fully replacing bank-to-bank lending (by attending all the demand for funding at its below-market rate) or somehow rationing banks’ access to its “cheap” funding. It will become evident, however, that if the government rations such cheap lending to banks, its activity is most likely to just generate (intramarginal) rents among the banks that benefit from the cheap funding, while having no impact on the banks’ marginal funding costs. In this case, the marginal lending terms offered by banks to the firms in their regions will not be affected and the effects on investment, wages and output in the corresponding regions will be null. Hence, both here and in Section 5, we will just consider government interventions involving no rationing.

When the representative regional bank is a *MM borrower* ($a < 0$), the problem that determines (k, n, l, R) can be formally stated as follows:

$$\begin{aligned} \max_{(k, n, l, R)} & \quad (1 - \varepsilon)(1 - p)[AF(k, n) + (1 - \delta)k - R] \\ \text{s.t.} & \quad (1 - \varepsilon)\{(1 - p)R + p(1 - \lambda)k - [(1 + r_d)d + (1 + r + s)(l - d - e)]\} \geq (1 + r_d + \phi)e \\ & \quad k + wn = l \\ & \quad e \geq \gamma l \\ & \quad l - d - e > 0. \end{aligned} \quad (8)$$

¹⁵From the derivations below, one could check that a simple sufficient condition for A1 is $\gamma < \lambda$, and for A2 is $(1 - \mu) > (1 - \lambda + \gamma)(1 - \pi)$ as long as deposit interest rates remain positive.

The following paragraphs justify the various elements of this problem.

The objective function establishes that the contract under which the bank provides funding to each firm maximizes each entrepreneur's expected profit at $t = 1$. This profit is only positive if the firm does not default, which occurs with probability $(1 - \varepsilon)(1 - p)$; otherwise, the entrepreneur, who is protected by limited liability, receives zero. Since both physical capital and labor are pre-paid at $t = 0$ using the loan l , the profit when the firm does not fail equals the firm's output, $F(k, n)$, plus the depreciated capital, $(1 - \delta)k$, minus the previously agreed repayment to the bank, R .

The first constraint in (8) is the bank's participation constraint. It establishes that bank owners' expected payoff at $t = 1$ under the equilibrium contract must compensate them for the opportunity cost of the equity capital e invested at $t = 0$. This condition can be justified by the fact that entry in the banking sector is free: any coalition of households can create a new bank, endow it with equity capital, attract deposits and MM funds, and compete with the existing banks in financing each firm (improving on the terms of the contract offered to the firm).

Bank owners' expected payoffs reflect that the default rate x on bank loans has the two point support described in (3), and that the bank ends up with negative net worth (and hence goes bankrupt) when $x = 1$ (assumption A1). Since banks owners are protected by limited liability, the distribution of the bank owners' payoff is:

$$\begin{cases} (1 - p)R + p(1 - \lambda)k - [(1 + r_d)d + (1 + r + s)(l - d - e)], & \text{with prob. } 1 - \varepsilon, \\ 0, & \text{with prob. } \varepsilon, \end{cases} \quad (9)$$

where the term $p(1 - \lambda)k$ reflects the expected value of the recoveries on defaulted loans when $x = p$, and $(1 + r_d)d$ and $(1 + r + s)(l - d - e)$ are for liabilities vis-a-vis the regional depositors and the MM lenders, respectively. From here, the expected payoffs that appear in the left hand side of the participation constraint can be trivially computed. The right hand side reflects that the bank owners must be compensated for the financial opportunity cost of e (which they could invest as regional deposits at rate r_d) as well as the excess cost of equity financing ϕe .

The second constraint in (8) reflects the firm's financing constraint: the bank loan l must

allow it to pay for its capital k and its workers' payroll wn , in advance. The third constraint reflects the existence of a minimum capital requirement γ . In the last constraint, we ensure, for consistency, that the regional bank is a MM borrower. We impose $a < 0$ with strict inequality for analytical convenience—we will refer to the special case of banks with $a = 0$ as a limit case with *autarkic* banks that do not trade in the MM.

When the representative regional bank is a *MM lender* ($a > 0$), the only significant difference in the problem that defines the contract (k, n, l, R) signed between firms and the bank is in the term reflecting the bank's *net* liabilities at $t = 1$, which are now the *difference* between deposit liabilities, $(1 + r_d)d$, and the gross return of MM lending, $(1 + r)(d + e - l)$ (which equals its expected value by the assumptions that MM lending is perfectly diversified and regional default risk is diversifiable). Hence, the problem can be stated as follows:

$$\begin{aligned}
& \max_{(k,n,l,R)} && (1 - \varepsilon)(1 - p)[AF(k, n) + (1 - \delta)k - R] \\
\text{s.t.} & && (1 - \varepsilon)\{(1-p)R + p(1-\lambda)k - [(1+r_d)d - (1+r)(d + e - l)]\} \geq (1+r_d+\phi)e \\
& && k + wn = l \\
& && e \geq \gamma l, \\
& && d + e - l > 0
\end{aligned} \tag{10}$$

where, of course, we have also modified the fourth constraint to now guarantee $a > 0$ —again the autarkic case will be treated as a limit case.

Problems (8) and (10) have a similar structure, which allows us to discuss the cases of borrowing banks and lending banks very much in parallel. In both cases, the loan repayment R enters only in the objective function (which is decreasing in it) and in the bank's participation constraint, so it will always be optimal to reduce R so as to make this constraint hold with equality. The following lemma builds on further exploring the choices of d and e that could help reduce R for given values of k and n . The result implies a dramatic simplification of the corresponding optimization problems:

Lemma 1 *Equilibrium with $a < 0$ and $d > 0$ requires $r_d = r + s$; equilibrium with $a > 0$ and $d > 0$ requires $r_d = r$. In both cases, the minimum capital requirement will be binding, i.e., the corresponding bank will choose $e = \gamma l$.*

Proof of Lemma 1 For $a < 0$ (respectively, $a > 0$) the fourth constraint in problem (8)

(resp. (10)) can be ignored. But then d only enters linearly in the bank's participation constraint, which is binding. Clearly, for values of r_d different from $r + s$ (resp. r), the optimal choice of d , aimed to reduce R , would be either $d = 0$ or $d \rightarrow \infty$, both of which would be incompatible with an equilibrium with $d > 0$. ■

Particularizing (8) and (10) to the deposit rates described in Lemma 1 and after imposing the minimum capital requirement constraint with equality, the substitution of the second and third constraints into the bank's (binding) participation constraints allow us to rewrite them compactly as follows:

$$(1 - \varepsilon)[(1 - p)R + p(1 - \lambda)k] = c(r + \xi s)(k + wn), \quad (11)$$

where $\xi \in \{0, 1\}$ is an indicator variable that takes value 1 for a borrowing bank and value 0 for a lending bank, and

$$c(r + \xi s) = (1 + r + s\xi)[1 - \varepsilon(1 - \gamma)] + \gamma\phi \quad (12)$$

is the *gross marginal cost of funds* to the corresponding class of bank.

The first term in (12) reflects that the bank funding costs are directly connected to the bank's *marginal funding rate*, $r + \xi s$, which is the interest rate *promised* by the bank to its marginal financiers (depositors or MM lenders) in each case. The term $-\varepsilon(1 - \gamma)$ accounts precisely for the fact that, by limited liability, this rate is not *effectively* paid to depositors and MM lenders when the bank fails. Finally, the term $\gamma\phi$ accounts for the excess cost of equity financing. It turns out that, in terms of (11), the only difference between MM borrowers and MM lenders is that, if the spread s is positive, the former pay a larger marginal funding rate and thereby suffer a larger marginal cost of funds.

In the Appendix, we show that the crucial difference between the gross marginal cost of funds of lending and borrowing banks (reflected in the impact of ξ in (12)) is due to combining deposit insurance with the existence of a positive probability of bank failure ($\varepsilon > 0$) and hence a positive spread s . There we show that, if deposits were not insured, a positive probability of bank failure would not create asymmetries between lending and borrowing banks since depositors would not, in that case, be effectively different from MM lenders in demanding compensation for their losses in case of failure.

Once problems (8) and (10) are reduced to maximizing the respective objective functions subject to (11), substituting this constraint in the objective function allows us to reduce the firms-bank problem in each class of region to a similar unconstrained optimization problem:

$$\max_{(k,n)} (1 - \varepsilon) \{ (1 - p)AF(k, n) + [1 - (1 - p)\delta - p\lambda]k \} - c(r + \xi s)(k + wn), \quad (13)$$

where the second term within the curly brackets reflects the expected value of the physical capital recovered (by either the entrepreneurs or their banks) at $t = 1$.

Hence, firm-bank contracts will determine the production plans (k, n) that, from the point of view of each firm-bank coalition, maximize the firm's expected gross output plus recovered physical capital minus the cost of funding the firms' inputs. From an optimal production plan, the remaining elements of the original contract tuple (k, n, l, R) can be recursively obtained using $l = k + wn$ to find l and (11) to find R .

3.2 Partial equilibrium

We now further analyze problem (13) with the final goal of characterizing the equilibrium in *borrowing* regions and *lending* regions for given values of the risk-free rate r , the spread s , and the productivity factor A , since these three variables are, in principle, determined at the interregional level.

It follows from the homogeneity of degree one of the production function $F(k, n)$, as well as the linearity in k and n of the cost terms that appear in (13) that, by Euler's theorem, the various cost terms will exhaust the expected gross returns obtained by the firm at $t = 1$ (inclusive of recovered capital), so the expected profits of entrepreneurs in an interior optimum will be zero. The first order conditions for an interior optimum are:

$$(1 - \varepsilon)[(1 - p)AF_k(k, n) + 1 - (1 - p)\delta - p\lambda] = c(r + \xi s) \quad (14)$$

and

$$(1 - \varepsilon)(1 - p)AF_n(k, n) = c(r + \xi s)w. \quad (15)$$

Conditions (14) and (15) have the standard interpretation: the expected value of the marginal product of each factor is equalized to its gross marginal *funding* cost. Recall that both capital,

with a unit price of one at $t = 0$, and labor, with a unit cost of w at $t = 0$, are paid in advance using bank loans. So these cost terms appear multiplied by $c(r + \xi s)$ above, reflecting the marginal cost of transferring funds from $t = 0$ to $t = 1$ for the corresponding bank.

By bringing the physical capital recovery terms to the right hand side and substituting in (12), condition (14) can be rewritten in more conventional terms:

$$(1 - \varepsilon)(1 - p)AF_k(k, n) = (1 - \varepsilon)[(1 - p)\delta + p\lambda] + [1 - \varepsilon(1 - \gamma)](r + s\xi) + \gamma(\varepsilon + \phi) \quad (16)$$

where the right hand side represents the *users' cost of capital* in this setup.

Of course, describing the equilibrium that emerges in each region for given values of the variables determined at the interregional level also involves taking into account the relevant region-level market clearing conditions. In addition to the conditions already subsumed in the constraints of the representative firm-bank problem, the relevant condition here is $n = 1$, since the representative household has an inelastic supply of labor equal to 1. Imposing $n = 1$ directly in (14) and solving for k yields:

$$k = g(r + \xi s, A) \equiv \left[\frac{(1 - \varepsilon)(1 - p)\alpha A}{(1 - \varepsilon)[(1 - p)\delta + p\lambda] + [1 - \varepsilon(1 - \gamma)](r + s\xi) + \gamma(\varepsilon + \phi)} \right]^{\frac{1}{1-\alpha}}. \quad (17)$$

Now, using (15), we can recursively find the labor-market clearing wage

$$w = \frac{(1 - \varepsilon)(1 - p)(1 - \alpha)A}{c(r + \xi s)} [g(r + \xi s, A)]^\alpha, \quad (18)$$

and, following standard accounting conventions, the expected output (or expected GDP) in the region can be recursively written as

$$y = (1 - \varepsilon)(1 - p)A[g(r + \xi s, A)]^\alpha. \quad (19)$$

By analyzing the impact of $\xi \in \{0, 1\}$ on these expressions, we can state the following result:

Lemma 2 *In equilibria with $s > 0$, borrowing regions are characterized by lower investment k , lower wages w , and lower expected output y than lending regions. The induced asymmetries are larger for larger s .*

Proof of Lemma 2 The results follow directly from the impact of ξ on (17), (18), and (19) when $s > 0$, and the dependency of k , w , and y with respect to s when $\xi = 1$. ■

4 General equilibrium

This section deals with equilibrium at the interregional (or general) level, as defined next.

Definition 1 *An equilibrium is a tuple of the form $((r_H, r_L), (k_H, k_L), B)$ that describes banks' marginal funding rate in savings-rich and savings-poor regions, r_H and r_L , the capital allocated to savings-rich and savings-poor regions, k_H and k_L , and the size of the government's money market interventions, B , such that:*

C1 *Households, firms, and banks make optimal decisions given r_H and r_L , and the value of A associated with the allocation of k_H and k_L to savings-rich and savings-poor regions, respectively.*

C2 *The government satisfies its budget constraint.*

C3 *All markets clear at $t = 0$.*

Notice that we are now referring to banks' marginal funding rates as r_H and r_L rather than $r + \xi s$, with $\xi = 0$ for lending regions and $\xi = 1$ for borrowing regions, because we want to encompass the situation in which banks end up making no trade MM, i.e., in financial autarky. With operative MM, the pair (r_H, r_L) entails an implicit description of the risk free rate $r = r_H$ and the spread $s = r_L - r_H$ in the exact terms used in Section 3.

Next we will discuss the implications of each of the equilibrium conditions in turn.

Optimization (C1) Guaranteeing optimization requires that $k_H = g(r_H, A)$ and $k_L = g(r_L, A)$, where the function g is as defined in (17), and the value of the productivity term A , consistently with (5), satisfies

$$A = [\pi k_H^\rho + (1 - \pi)k_L^\rho]^{\frac{1}{\rho}}. \quad (20)$$

When banks from savings-rich regions directly lend to banks from savings-poor regions (without further government intervention), optimization also requires that $r = r_H$ and $s = r_L - r_H$ satisfy (7). In all cases with operative MM, we must additionally check that MM positions

have the presumed sign, $a_H > 0 > a_L$, since the limit case with $a_H = a_L = 0$ would correspond to financial autarky.

The following lemma discusses the determination of (k_H, k_L) for given values of (r_H, r_L) after taking into account the complexity associated with the regional interdependencies that operate through the productivity term A .

Lemma 3 *Each pair (r_H, r_L) identifies a unique candidate allocation of capital across regions which can be described by some functions, $k_H(r_H, r_L)$ and $k_L(r_H, r_L)$, strictly decreasing in both arguments, that satisfy $k_H(r_H, r_L) \geq k_L(r_H, r_L)$, with equality only if $r_H = r_L$.*

Proof of Lemma 3 From (17) and (20), it turns out that, for given values of (r_H, r_L) , the pair (k_H, k_L) must satisfy the subsystem of equations:

$$k_H = g(r_H, 1)[\pi k_H^\rho + (1 - \pi)k_L^\rho]^{\frac{\tau}{\rho(1-\alpha)}}, \quad (21)$$

and

$$k_L = g(r_L, 1)[\pi k_H^\rho + (1 - \pi)k_L^\rho]^{\frac{\tau}{\rho(1-\alpha)}}. \quad (22)$$

Equations (21) and (22) can be interpreted as implicitly defined “reaction functions” describing how k_H and k_L , respectively, “react” to each other. With $\tau < 1 - \alpha$ and $\rho < 1$, both reaction functions are increasing and concave in their corresponding argument and intersect twice: at the trivial (and unstable) solution $(k_H, k_L) = (0, 0)$, which cannot be an equilibrium allocation, and at a unique (and stable) solution with $k_H > 0$ and $k_L > 0$. Increasing r_H and r_L , respectively, moves each of these reaction functions inwards, implying that $k_H(r_H, r_L)$ and $k_L(r_H, r_L)$ are strictly decreasing in both arguments. The fact that $k_H(r_H, r_L) \geq k_L(r_H, r_L)$ with equality only if $r_H = r_L$ follows immediately from inspection of (21) and (22).■

The government’s budget constraint (C2) Satisfying the government’s budget constraint simply requires that the taxes T charged on households at $t = 1$ are those required to cover the government’s net financial obligations at that date. The first component of these

obligations, denoted T_1 , are the outstanding deposit insurance liabilities:

$$T_1 = (1 - \varepsilon)\{\pi[(1 + r_H)(d_H - a_H) - (1 - \lambda)k_H] + (1 - \pi)[(1 + r_L)d_L - (1 - \lambda)k_L]\}, \quad (23)$$

where d_j denotes the deposits of each bank from a region of class $j = H, L$ and $a_H > 0$ is the MM lending of each bank from savings-rich regions. The terms in k_H , k_L , and a_H account for what the deposit guarantor can recover out of the assets of defaulted banks from each class of region (recall that default risk is diversifiable across regions and hence lending banks obtain with certainty the gross *expected* returns of their MM assets).

The second component of the government's obligations, denoted T_2 , are the net losses at $t = 1$ due to money market interventions at $t = 0$. Since the government borrows B from some banks at rate $r_H = r$ and lends B to some banks at rate $r_L = r + s$, we have

$$T_2 = [(1 - \varepsilon)(1 + r + s) - (1 + r)]B, \quad (24)$$

which is strictly positive if, given r , the spread s is below the value that satisfies (7).

Market clearing (C3) As for market clearing, the firm-bank optimization problems described in subsection 3.1 already took care of guaranteeing that the regional loan markets clear, $l_j = k_j + w_j n_j$, and the partial equilibrium analysis of subsection 3.2 already took care of labor market clearing, $n_j = 1$. As for the clearing of regional deposit and equity markets, notice that households will be indifferent between investing their savings, $S_j + w_j$, in deposits or in equity (notice that savings include the wages w_j paid in advance by firms at $t = 0$). However, banks will demand $e_j = \gamma l_j$, just complying with the capital requirement, so equilibrium deposits can be residually computed as $d_j = S_j + w_j - \gamma l_j$, which we can assume to be positive under any realistic parameterization. The desired net position of each region's representative bank in money markets can then be written down by combining the previous expressions with (4),

$$a_j = (d_j + e_j) - l_j = (S_j + w_j) - (k_j + w_j) = S_j - k_j, \quad (25)$$

where we have used parenthesis to help identify the correspondence between the terms after the first and second equalities.

Finally, since the government's intervention as both a borrower and a lender nets out, the condition for money market clearing is simply:

$$\pi a_H + (1 - \pi)a_L = 0. \quad (26)$$

Now, combining (25) and (26), and taking into account the dependence of k_H and k_L with respect to r_H and r_L (Lemma 3), we get the following combined equilibrium condition:

$$\pi k_H(r_H, r_L) + (1 - \pi)k_L(r_H, r_L) = \bar{S}, \quad (27)$$

which embeds the market clearing condition for the interregional goods' market at $t = 0$.¹⁶

4.1 Laissez-faire equilibrium

The following result summarizes the discussion on equilibrium for the case in which MM operate without government intervention ($B = 0$).

Proposition 1 *A laissez-faire equilibrium with operative MM exists (and is unique) if the pair (r_H^*, r_L^*) that solves (7) and (27), also satisfies $k_H(r_H^*, r_L^*) < S_H$. Otherwise, the (unique) laissez-faire equilibrium is autarkic, with marginal funding rates r_j^a that satisfy $k_j(r_H^a, r_L^a) = S_j$ for $j = H, L$.*

Proof of Proposition 1 Condition (7) is required for MM to operate without government intervention and (27) combines firm-bank optimization and market clearing as explained above. The condition requiring $k_H(r_H^*, r_L^*) < S_H$ (which, under (27), also implies $k_L(r_H^*, r_L^*) > S_L$) is necessary for banks from savings-rich (savings-poor) regions to actually act as non-trivial lenders (borrowers) in MM. If the solution to (7) and (27) does not satisfy this condition and banks' implied roles in MM are reversed, the roles of r_H^* and r_L^* in (7) should also be reversed, in which case (r_H^*, r_L^*) would no longer be a candidate equilibrium. The candidate equilibrium in this case involves financial autarky, with $a_H = a_L = 0$, and the conditions $k_j(r_H^a, r_L^a) = S_j$ for $j = H, L$, that determine (r_H^a, r_L^a) , are implied by (27). The uniqueness of equilibrium can be established from standard arguments using the fact that

¹⁶This is an instance of Walras' Law.

the functions $k_j(r_H, r_L)$ are strictly decreasing in their two arguments (Lemma 3), going to zero for sufficiently large (r_H, r_L) and to infinity for sufficiently small (r_H, r_L) . ■

Notice that the candidate equilibrium rates defined in Proposition 1 satisfy $r_H^a < r_H^* \leq r_L^* < r_L^a$ when MM are operative, and $r_H^* \leq r_H^a < r_L^a \leq r_L^*$ otherwise. Moreover, by (27), the case of $r_H^* = r_L^*$ (only) occurs if $\varepsilon = 0$, so we can state the following corollary.

Corollary 1 *If $\varepsilon = 0$, the laissez-faire equilibrium involves an operative MM with $r_H^* = r_L^*$ and a symmetric allocation of capital across regions, $k_H = k_L = \bar{S}$. With $\varepsilon > 0$, the laissez-faire equilibrium is not symmetric.*

4.2 Government-supported equilibria

Equilibria with government-supported MM involve some subsidization of MM borrowing (or the undertaking of non-fully compensated counterparty risk) by the government. Formally, these equilibria can be described as situations in which banks' marginal funding rates are given by a pair $(r_H^B, r_L^B) = (r^B, r^B + s^B)$ that satisfies

$$\pi k_H(r_H^B, r_L^B) + (1 - \pi)k_L(r_H^B, r_L^B) = \bar{S}$$

and $k_H(r_H^B, r_L^B) < S_H$ (as in a laissez-faire equilibrium with operative MM), but, instead of satisfying (7), has

$$s^B < \frac{\varepsilon}{1 - \varepsilon}(1 + r^B), \quad (28)$$

so that the expected gross return from lending to banks is lower than the gross return on government debt, $1 + r^B$. In this case, the government becomes the only counterparty of banks in the MM, so we must have

$$B = \pi[S_H - k_H(r_H^B, r_L^B)] = (1 - \pi)[k_L(r_H^B, r_L^B) - S_L]. \quad (29)$$

The following proposition, whose intuitive proof is embedded in the explanation below, characterizes this class of equilibria.

Proposition 2 *With $\varepsilon > 0$, there is a continuum of equilibria with government-supported MM, which can be indexed by $s^B \in [0, \bar{s})$, with $\bar{s} = \min\{r_L^* - r_H^*, r_L^a - r_H^a\}$ and B strictly decreasing in s^B . With $\varepsilon = 0$, no government supported equilibrium exists.*

This result says that the government can induce as many equilibria as different subsidized MM spreads it can sustain. The lower bound to subsidization is determined by the laissez-faire “spread” \bar{s} (which is given by the proper spread $r_L^* - r_H^*$, if laissez-faire MM are operative, and by the difference between the autarkic rates, $r_L^a - r_H^a$, if they are not). Obviously with $s^B > \bar{s}$ banks will demand funding from other banks rather than the government. Symmetrically, the upper limit to subsidization is determined by the fact that s^B cannot be negative, since this will open up an arbitrage opportunity for all banks (that would then like to simultaneously borrow from and lend to the government in unlimited amounts). Since with $\varepsilon = 0$, we have $\bar{s} = 0$, no equilibrium with government-supported MM exists.

The last result in this section states a corollary on the symmetry of the capital allocations induced by the government-supported equilibria:

Corollary 2 *If $\varepsilon > 0$, the maximum-subsidization government-supported equilibrium involves $s^B = 0$ and a symmetric allocation of capital across regions, $k_H = k_L = \bar{S}$. Any other government-supported equilibrium is not symmetric.*

5 Policy analysis

We measure aggregate social welfare as the aggregate expected net worth of all agents at $t = 1$, net of the distortionary cost of taxation.¹⁷ Agents include households, who own the banks, entrepreneurs who own the firms, and the government. However, firms expected profits are zero in equilibrium and the government budget constraint implies that the government simply breaks even. So eventually aggregate welfare can be computed as households’ aggregate expected net worth at $t = 1$ net of the excess cost of equity financing and the full cost of taxes. Households’ net worth is made up of the gross returns of their deposit and equity investments in the regional banks. However, taking into account that in equilibrium households must be

¹⁷This measure does not assign any special benefit to the existence of deposit insurance, which is exogenously imposed in the baseline version of the model. So the model is not intended to provide a comprehensive account of the costs and benefits of deposit insurance. Rather, we focus on the distortions associated with the asymmetries that arise across savings-rich and savings-poor regions when bank default risk becomes significant.

indifferent between investing in deposits or in bank equity, aggregate welfare can be measured as:

$$W = \pi(1 + r_H)(S_H + w_H) + (1 - \pi)(1 + r_L)(S_L + w_L) - (1 + \eta)T, \quad (30)$$

since households indifference implies that their expected net worth (net of the excess cost of equity financing) must be $(1 + r_j)(S_j + w_j)$ (with $j = H$ in savings-rich regions and $j = L$ in savings-poor regions), as if all their savings, $S_j + w_j$, were invested in (fully insured) deposits at the rate r_j .¹⁸ The tax term $T = T_1 + T_2$ is defined as implied by equations (23) and (24), and η accounts for the distortionary cost of taxation.

Not surprisingly, (30) can be combined with equilibrium conditions and various adding up constraints in order to obtain an alternative expression for W which is closer to its fundamental determinants:

$$W = \bar{S} + [\pi\text{NDP}_H + (1 - \pi)\text{NDP}_L] - \phi\gamma[\bar{S} + \pi w_H + (1 - \pi)w_L] - \eta T, \quad (31)$$

where

$$\text{NDP}_j = (1 - \varepsilon)\{(1 - p)[AF(k_j, 1) - \delta k_j] - p\lambda k_j\} - \varepsilon\lambda k_j$$

is the *net* domestic product (NDP) of a region that devotes k_j units of physical capital to production. Intuitively, aggregate net worth consists of the initial exogenous savings \bar{S} plus the NDP generated across regions minus the aggregate excess costs of equity financing minus the distortionary cost of taxation. In the writing of the equity costs term we have used the fact that, in equilibrium, banks' equity financing is $e_j = \gamma l_j = \gamma(k_j + w_j)$ and $\pi k_H + (1 - \pi)k_L = \bar{S}$. As for taxes, notice that only the distortionary cost appears in (31), since the other effects of the tax itself either get offset with the direct or indirect subsidies that households receive from the government (via deposit insurance and MM interventions) or show up through the allocation of capital across regions.

Equation (31) will be used below for the quantitative assessment of welfare in the various *laissez-faire* and government-supported equilibria of the model under some realistic param-

¹⁸Equity yields a larger gross expected return than deposits so as to exactly compensate for the excess cost of equity financing ϕ , which then does not explicitly appear in (30).

terizations. In analytical terms, we can only obtain clear-cut results for the case in which the distortionary cost of taxation is zero ($\eta = 0$).

Proposition 3 *If $\eta = 0$, the welfare measure W is maximized with $k_H = k_L = \bar{S}$. For $\varepsilon = 0$, this (symmetric) allocation coincides with the laissez-faire allocation. Otherwise, it coincides with the maximum-subsidization ($s^B = 0$) government-supported allocation.*

Proof of Proposition 3 Consider first the case with $\phi = \eta = 0$. (31) particularizes to $W = \bar{S} + [\pi\text{NDP}_H + (1 - \pi)\text{NDP}_L]$, where \bar{S} is constant and the terms NDP_j only depend on k_j . Hence, the socially optimal allocations can be found by maximizing $\pi\text{NDP}_H + (1 - \pi)\text{NDP}_L$ subject to the resource constraint $\pi k_H + (1 - \pi)k_L = \bar{S}$, which yields the symmetric allocation as the unique solution. From here, the remaining parts of the proposition follow directly from Corollaries 1 and 2, respectively.

In the case with $\phi > 0$ and $\eta = 0$, maximizing welfare is equivalent to maximizing $G(k_H, k_L) \equiv \pi(\text{NDP}_H - \phi\gamma w_H) + (1 - \pi)(\text{NDP}_L - \phi\gamma w_L)$ (where w_H and w_L are written as functions of (k_H, k_L)) subject to $\pi k_H + (1 - \pi)k_L = \bar{S}$. When the term A is a constant, the symmetry of the solution is an implication of the fact that each term $\text{NDP}_j - \phi\gamma w_j$ in $G(\cdot)$ is the same strictly concave function of the corresponding k_j , and it then follows from Jensen's inequality that $G(\bar{S}, \bar{S}) > G(k_H, k_L)$ for any $(k_H, k_L) \neq (\bar{S}, \bar{S})$ with $\pi k_H + (1 - \pi)k_L = \bar{S}$. When A is defined as in (20), the advantages of the symmetric allocation are even stronger, but the terms $\text{NDP}_j - \phi\gamma w_j$ are affected by both k_H and k_L , and the strategy of the proof requires some adaptations whose discussion we skip for brevity. ■

When η is positive, the analysis gets complicated by the tax terms in (31). These terms can be written so as to only have the pair (k_H, k_L) as endogenous variables affecting them, and a problem similar to that mentioned in the proof of Proposition 3 could be used to characterize the candidate optimal allocations of capital across regions. One can then use reverse engineering to find out whether such an allocation can be achieved in the laissez-fair equilibrium or in one of the government-supported equilibria. Of course, the symmetric allocation remains optimal for $\varepsilon = 0$, but the analytical approach does not yield clear-cut results for $\varepsilon > 0$.

In the quantitative section below, we show that, for the explored parameterizations, the symmetric allocation is not welfare-maximizing when both η and ε are positive. However, we find that, even for a marginal cost of public funds as high as $\eta = 0.5$, the optimal allocation is actually very close to the symmetric one, and coincides with a government-supported equilibrium with a very large amount of subsidization (with a spread s^B equal to about one-tenth of the laissez-fair spread \bar{s}).

All in all, the analytical results here and the quantitative results below provide a rationale for a sizeable absorption of counterparty risk by the government (or the central bank) in response to a solvency crisis that raises the laissez-faire spreads from negligible to significant. The welfare-maximizing policy among the class of policies analyzed above is sizeable in terms of both the required amount of direct lending (or lending supported through public guarantees) and the difference between the required policy spread and the laissez-faire spread.

In light of the results shown in the Appendix for the economy without deposit insurance, these policy implications can be seen as an instance of the Second Best Theorem. An adequate response to the distortions created by deposit insurance (which are important with $\varepsilon > 0$) may involve—if deposit insurance is not to be removed—extending the insurance to MM liabilities as well, or an equivalent absorption of counterparty risk by the government.

6 Quantitative analysis

In this section we explore the quantitative implications of the model by considering the laissez-faire and the optimal government-supported equilibria that emerge under a number of either realistic or theoretically interesting parametric scenarios. In each of the exercises, summarized by the tables of results that appear below, we consider a pre-crisis scenario in which the probability of bank failure is zero ($\varepsilon = 0$) and a post-crisis scenario in which this probability becomes positive ($\varepsilon > 0$). We assume that the single period considered in the model corresponds to a calendar year. As a reference post-crisis scenario we choose $\varepsilon = 2\%$ since this produces a MM spread of about 200 basis points (bp), in line with the spreads observed in the US and European interbank markets in September and October of 2008.

Table 1 shows the values of the parameters in the baseline scenario and reports the equilibrium values of a number of macroeconomic and financial variables taken as calibration targets. In Tables 2-4 we analyze the impact of some critical parameters on the magnitude of the effects of the crisis assuming no government action (a *laissez-faire* equilibrium), while in Table 5 we evaluate the effects of policy interventions that induce government-supported equilibria like those discussed in Section 5.

6.1 Calibration

Many of the parameters values in Panel A of Table 1 are tightly linked to the target variables in Panel B and can be chosen according to macroeconomic convention. The first two rows in Panel A set the parameters that determine the allocation of savings across regions. We choose to have just half of the regions of each type ($\pi = 0.5$) so as to focus all the exogenous asymmetry between region types on their different endowment of initial savings. We choose the savings asymmetry parameter, $\mu \equiv \pi S_H / \bar{S}$, to be equal to 60%, in order to have a sufficiently large money market—with a pre-crisis size equal to 10% of the capital invested in the economy or, equivalently, equal to around 30% of aggregate pre-crisis GDP (for a target capital-to-output ratio of around 3). Given the values of all the remaining parameters, we compute the total amount of exogenous savings \bar{S} (not reported in Table 1), so as to get a pre-crisis risk-free interest rate, r_H , of 4%.

We choose the capital elasticity parameter α to target an aggregate (and regional) labor share of 70%. The returns-to-scale parameter τ of our CES specification for A (recall equation (20)) is fixed at 0.5 so as to keep the overall returns to capital, $\alpha + \tau$, below one. Given our way to fix \bar{S} , however, this choice only affects the scale of the baseline economy but none of the interest rates and other relative magnitudes (including rates of variation) that we report below. The interdependency parameter ρ is set at a level that makes the arrival of the solvency crisis in the baseline scenario produce just a mild recession in the savings-rich regions (as shown in Table 3, less than a 1% fall in their GDP when ε jumps from 0% to 2%).

The depreciation rates in case of success, δ , and in case of failure, λ , are chosen so

Table 1: Baseline parameterization

Panel A. Parameter values

Parameters			Baseline values
Savings:	Measure of savings-rich regions	π	0.5
	Savings asymmetry	$\mu \equiv \pi S_H / \bar{S}$	0.6
Technology:	Capital elasticity parameter in F	α	0.3
	Returns-to-scale parameter in A	τ	0.5
	Interdependency parameter in A	ρ	-4
	Depreciation rate if success	δ	4.5%
	Depreciation rate if failure	λ	35%
Default risk:	Probability of bank failure	ε	0%-2%
	Probability of idiosyncratic firm failure	p	3%
Frictions:	Capital requirement	γ	8%
	Excess cost of equity financing	ϕ	6%
	Marginal cost of public funds	η	0%

Panel B. Calibration targets

Variables		Values
Macroeconomic:	Pre-crisis risk-free rate	4%
	Capital/GDP ratio	3
	Labor share	70%
Financial:	MM spread	0-200bp
	Loan default probability	3%-5%
	Loan loss-given-default	45%

as to produce, respectively, an aggregate capital-to-output ratio of around 3 (which is a conventional choice in macroeconomic calibrations) and a loss-given-default on loans to firms of around 45% (which is the LGD assumed in the foundation internal-ratings-based approach of Basel II for unrated corporate loans of one-year maturity). The probability of idiosyncratic firm failure, p , is chosen to produce a loan default rate of 3% in the pre-crisis scenario. This choice implies a loan default rate of slightly above 5% in the crisis scenario.

The capital requirement γ is set at 8%, to match the standard requirement for commercial and industrial loans of Basel I (which is very close to the one required by the standardized approach of Basel II to unrated corporate loans of one-year maturity). We make the excess cost of equity financing ϕ equal to 6%, which coincides with the historical equity premium and, more consistently with our risk-neutrality assumptions, with estimates of the excess cost of equity financing based on net tax disadvantages of equity financing (vis-a-vis debt financing). Finally, since we have no prior on the size of the distortionary costs of taxation, we first consider a value of the marginal cost of public funds equal to 0% but then, when discussing the possible policy response (Table 5), we compare the results with the case in which it reaches a possibly exaggerated 50% (implying a social cost of 1.5 dollars per 1 dollar of taxes).

6.2 The effects of counterparty risk

Table 2 shows the effects of counterparty risk, by considering several crisis scenarios with subsequently larger values of ε (1%, 2%, and 3%). The first rows summarize the effects on banks' marginal funding rates and the operation of the money market. The differences between the marginal funding rates faced by lending and borrowing banks increase with ε (reflecting an increase in the spreads if MM remain in operation) and MM volumes shrink dramatically. In fact, MM volume fully disappears for $\varepsilon = 3\%$, as regions fall into financial autarky. Of course, the initial size of the MM is closely related to the choice of the savings imbalance parameter μ , whose effect on the results is further discussed below with reference to Table 4.

As counterparty risk increases, the asymmetry in the allocation of capital across regions

Table 2: The effects of counterparty risk

Panel A reports key variables for scenarios with different bank default probabilities ε . Other parameters are as in Table 1. Panel B reports percentage changes relative to the baseline scenario with $\varepsilon = 0\%$ for some other key variables. All variables are defined in the Appendix.

Panel A. Variables in levels

		Probability of bank failure (ε)			
		0%	1%	2%	3%
Banks' funding rate (%)	H	4.00	3.35	2.63	2.14
	L	4.00	4.40	4.73	4.86
MM / baseline GDP (%)	Aggr.	30.32	18.85	7.12	0.00
Loan rate (%)	H	6.06	5.47	4.81	4.38
	L	6.06	6.58	7.03	7.27
DI costs / baseline GDP (%)	H	0.00	1.63	3.36	5.09
	L	0.00	1.10	2.50	4.00
	Aggr.	0.00	1.36	2.93	4.55

Panel B. Changes relative to baseline scenario in %

		Probability of bank failure (ε)		
		1%	2%	3%
Capital	H	7.56	15.30	20.00
	L	-7.56	-15.30	-20.00
GDP	H	0.47	-0.58	-2.30
	L	-3.99	-9.37	-13.49
	Aggr.	-1.76	-4.97	-7.89
Pre-tax workers' wealth	H	1.40	1.27	0.46
	L	-3.10	-7.67	-11.04
	Aggr.	-0.85	-3.20	-5.29
Pre-tax savers' wealth	H	-0.62	-1.31	-1.79
	L	0.38	0.70	0.83
	Aggr.	-0.22	-0.51	-0.74
Post-tax total wealth (= Welfare)	H	-0.67	-1.66	-2.57
	L	-0.71	-1.89	-2.98
	Aggr.	-0.69	-1.76	-2.74

increases. The channel of transmission for this is the cost of the loans that the banks in each region can offer to the firms operating in them. The differences in loan rates across regions open up in proportion to the opening up of the differentials in their banks' marginal funding rates.

The change in the allocation of capital has significant implications for output and wages (and, hence, workers' pre-tax wealth) in each region and overall. Output falls dramatically in the savings-poor regions and first mildly increases and then mildly decreases in the savings-rich region (because of the negative feedback effects). The overall output losses are sizeable (and increase more than proportionally with ε): the fall in GDP reaches roughly 5% with $\varepsilon = 2\%$.

Workers in savings-rich regions benefit from the higher wages associated with a larger capital-to-labor ratio in their region, while workers in savings-poor regions lose out dramatically due to the lower wages associated with a smaller capital-to-labor ratio in their region. In the aggregate workers' wealth falls reflecting the cost of the inefficiencies and negative feedback effects associated with the reallocation of capital.

The increase in counterparty risk also negatively affects the overall value at $t = 1$ of households' exogenous savings, which we call "savers' wealth" in the tables to distinguish it from "workers' wealth". Here, the big losers are the savers in savings-rich regions, since their banks end up investing more in their own regions, where the marginal return to capital ends up being relatively low because of the larger level of investment.¹⁹ For the same reason, the winners are the savers in savings-poor regions, since their banks end up investing more in their own regions, where the marginal return to capital ends up being relatively high.

In welfare terms, after adding up the wealth effects experienced by households as both workers and savers and after subtracting the cost of taxation, the rise in counterparty risk implies net losses in all regions, which overall reach 1.75% with $\varepsilon = 2\%$ (and again rise more than proportionally with ε).

¹⁹Recall that in terms of fundamentals regions just differ in their endowments of exogenous savings and, hence, the "discrimination" across them that operates via the MM spreads is not really justified in terms of the underlying return to capital.

Just to complete the comments on variables that appear in the table, notice that with $\varepsilon = 2\%$ deposit insurance liabilities (*DI costs*) amount to almost 3% of pre-crisis GDP and increase more than proportionally with ε . To understand this, recall that both principal and interest are insured (so equilibrium deposit rates affect the amount of liabilities of the deposit guarantor when a bank fails) and that MM assets in a failed bank contribute positively to the guarantor's recoveries in case of failure. This latter feature explains why DI liabilities in the savings-rich regions rise when the MM positions of the banks in these regions shrink.

6.3 The role of feedback

As we anticipated, regional interdependence, as captured by the form of A in (5) and measured by ρ is important for the size of the output and welfare losses implied by the rise in counterparty risk. This importance is illustrated in Table 3, which compares the effects of increasing ε from 0% to 2% across three “economies” that differ in the value of ρ . Recall that $\rho < 1$ implies some degree of imperfect substitutability between the levels of activity in the various regions (as measured by the capital invested in them). We compare the baseline economy, in which we have $\rho = -4$ (baseline feedback), with economies with $\rho = -3$ (lower feedback) and $\rho = 1$ (no feedback). Recall that with $\varepsilon = 0$ the feedback parameter is not relevant, since capital is always symmetrically distributed across regions, so the three compared economies share the same pre-crisis situation.

The results are self-explanatory. Most variables in Panel A show little differences in the impact of the crisis across the three economies. There are also little differences in terms of the final reallocation of capital across regions. The big differences appear in the variables that are associated with the size and cross-regional distribution of the output and net worth losses. The message is very clear: feedback effects do not play a role in generating high spreads, low MM volumes, and asymmetries in the allocation of capital across regions, but they play a clear role in explaining the propagation (across regions) and amplification (at all levels) of the effects of these changes. These feedback effects will also affect the size of the gains that can be obtained with an optimal policy intervention (see Table 5 below).

Table 3: The role of feedback

Panel A reports key variables for scenarios with different degrees of feedback as measured by ρ . The “no feedback” scenario corresponds to the economy in which A is constant. In the baseline scenario with $\varepsilon = 0\%$, feedback is irrelevant because the distribution of capital is symmetric. Other parameters are as in Table 1. Panel B reports percentage changes relative to the corresponding baseline scenario with $\varepsilon = 0\%$ for some other key variables. All variables are defined in the Appendix.

Panel A. Variables in levels

		Baseline	$\varepsilon = 2\%$		
			$\rho = -4$	$\rho = -3$	No feedback
Banks' funding rate (%)	H	4.00	2.63	2.69	2.89
	L	4.00	4.73	4.78	4.99
MM / baseline GDP (%)	Aggr.	30.32	7.12	7.24	7.69
Loan rate (%)	H	6.06	4.81	4.86	5.08
	L	6.06	7.03	7.08	7.30
DI costs / baseline GDP (%)	H	0.00	3.36	3.36	3.40
	L	0.00	2.50	2.50	2.53
	Aggr.	0.00	2.93	2.93	2.97

Panel B. Changes relative to baseline scenario in %

		$\rho = -4$	$\varepsilon = 2\%$	
			$\rho = -3$	No feedback
Capital	H	15.30	15.22	14.94
	L	-15.30	-15.22	-14.94
GDP	H	-0.58	-0.06	2.18
	L	-9.37	-8.85	-6.64
	Aggr.	-4.97	-4.46	-2.23
Pre-tax workers' wealth	H	1.27	1.80	4.08
	L	-7.67	-7.15	-4.90
	Aggr.	-3.20	-2.67	-0.41
Pre-tax savers' wealth	H	-1.31	-1.26	-1.05
	L	0.70	0.75	0.97
	Aggr.	-0.51	-0.46	-0.24
Post-tax total wealth (= Welfare)	H	-1.66	-1.54	-1.01
	L	-1.89	-1.73	-1.09
	Aggr.	-1.75	-1.62	-1.04

6.4 The role of savings imbalances

Table 4 explores the role of savings imbalances. In addition to the baseline economy, in which savings-rich regions have 60% of average savings ($\mu = 0.60$), we consider two additional economies in which this number is 55% (less savings imbalances) and 65% (more savings imbalances) respectively. In the economy with $\mu = 0.55$, the crisis implies a jump to autarky, which explains why many variables differ more across the $\mu = 0.55$ and $\mu = 0.60$ scenarios than across the $\mu = 0.60$ and $\mu = 0.65$ scenarios (many of whose aggregate variables change by exactly the same amount because the relevant marginal funding rates and costs are equal and change equally with the crisis).

Interestingly, the quicker fall into autarky with $\mu = 0.55$ (which is associated with the fact that the savings-poor regions are in this case are less “poor”) acts as a protection against the crisis since the marginal funding rates that prevail in autarky in this case are less asymmetric than those that would be associated with an active MM (which would be the ones seen in the cases with $\mu = 0.60$ and $\mu = 0.65$). Consequently, with $\mu = 0.55$, both the aggregate fall in GDP (1.5% instead of almost 5%) and the aggregate fall in welfare (1.32% rather than 1.76%) are smaller than in the other two economies (where they are, actually, identical). So the cost of the crisis increase in a somewhat non-differentiable manner when the underlying savings imbalances are sufficiently large.

6.5 Policy analysis

We now turn to the discussion of the effect of government (or central bank) policies directed to correct the distortions associated with a money market freeze. As explained in Section 5, we consider policies in which the government gets involved in direct borrowing and lending vis-à-vis banks, applying subsidized spreads to borrowers.

In Table 5, we compare the outcomes of the baseline economy after it is hit by the baseline solvency shock (ε) across three different policy scenarios: (i) the laissez-faire scenario, where no policy is deployed, that has already shown in previous tables, (ii) the maximum-subsidization scenario, where the government executes the $s^B = 0$ policy which maximizes

Table 4: The role of savings imbalances

Panel A reports key variables for scenarios with different savings imbalance parameter μ . $\Delta(\text{MM}/\text{baseline GDP})$ refers to a change in MM/baseline GDP relative to a baseline with the given μ , but $\varepsilon = 0\%$. Other parameters are as in Table 1. Panel B reports percentage changes relative to the scenario with the given μ but $\varepsilon = 0\%$ for some other key variables. All variables are defined in the Appendix.

Panel A. Variables in levels

		Baseline	$\varepsilon = 2\%$		
			$\mu = 0.55$	$\mu = 0.60$	$\mu = 0.65$
Banks' funding rate (%)	H	4.00	3.07	2.63	2.63
	L	4.00	4.45	4.73	4.73
$\Delta \text{MM} / \text{baseline GDP}$ (%)	Aggr.	0.00	-15.16	-23.20	-23.20
Loan rate (%)	H	6.06	5.27	4.81	4.81
	L	6.06	6.73	7.03	7.03
DI costs / baseline GDP (%)	H	0.00	3.29	3.36	3.36
	L	0.00	2.92	2.50	1.86
	Aggr.	0.00	3.11	2.93	2.61

Panel B. Changes relative to baseline scenario in %

			$\varepsilon = 2\%$		
			$\mu = 0.55$	$\mu = 0.60$	$\mu = 0.65$
Capital	H	10.00	15.30	15.30	
	L	-10.00	-15.30	-15.30	
GDP	H	-0.39	-0.58	-0.58	
	L	-6.21	-9.37	-9.37	
	Aggr.	-3.30	-4.97	-4.97	
Pre-tax workers' wealth	H	1.46	1.27	1.27	
	L	-4.46	-7.67	-7.67	
	Aggr.	-1.50	-3.20	-3.20	
Pre-tax savers' wealth	H	-0.89	-1.31	-1.31	
	L	0.43	0.70	0.70	
	Aggr.	-0.30	-0.51	-0.61	
Post-tax total wealth (= Welfare)	H	-1.29	-1.66	-1.64	
	L	-1.36	-1.89	-1.95	
	Aggr.	-1.32	-1.76	-1.76	

our welfare measure if the distortionary cost of taxes η is zero (Proposition 3), and (iii) the welfare-maximizing response for the rather extreme case with $\eta = 50\%$.

It turns out that in the latter case, in spite of the high distortionary cost of taxation, the welfare-maximizing policy is very close to the maximum subsidization policy (leaving the spread in less than one tenth of its laissez-faire value). This is because welfare gains are large in relation to the cost of subsidization (and the distortionary cost of the associated taxes). Notice also that the costs of subsidizing the lending to banks (which are labeled as “policy costs” in the table) are fairly small (0.63% of pre-crisis GDP when $\eta = 0$) when compared with the deposit insurance liabilities (2.5% of pre-crisis GDP), which, additionally, are significantly reduced by the policy (down from their level of 2.9% of pre-crisis GDP in the laissez-faire scenario).

The policy interventions shown in Table 5 are very effective in reducing the asymmetries associated with the laissez-faire equilibrium, and have a sizeable positive impact on GDP and welfare. Without distortionary costs of taxation, the maximum-subsidization policy reduces the GDP loss from roughly 5% to 2%, and the aggregate welfare loss from 1.75% to roughly 1%. With distortionary taxation, the impact on the GDP loss is very similar, but the impact on the welfare loss is smaller, since the welfare loss includes the (high) cost of the policy in terms of tax distortions.

To reinforce the message, Figure 1 compares the GDP and welfare variations associated with different levels of counterparty risk across two policy scenarios: the laissez-faire scenario and the maximum subsidization scenario (which corresponds to the policy that maximizes our welfare measure for $\eta = 0$).

Table 5: Policy analysis

Panel A reports key variables for scenarios with different degrees of subsidization. The baseline scenario corresponds to the economy with $\varepsilon = 0\%$, where subsidization is unnecessary. Other parameters are as in Table 1. Panel B reports percentage changes relative to the scenario with $\varepsilon = 0\%$ for some other key variables. All variables are defined in the Appendix.

Panel A. Variables in levels

		Baseline	$\varepsilon = 2\%$		
			Laissez faire	Policy response	
				$\eta = 0\%$	$\eta = 50\%$
Banks' funding rate (%)	H	4.00	2.63	3.82	3.77
	L	4.00	4.73	3.82	3.87
MM / baseline GDP (%)	Aggr.	30.32	7.12	30.32	29.20
Loan rate (%)	H	6.06	4.81	6.06	6.01
	L	6.06	7.03	6.06	6.12
DI costs / baseline GDP (%)	H	0.00	3.36	3.13	3.14
	L	0.00	2.50	1.87	1.90
	Aggr.	0.00	2.93	2.50	2.52
Policy costs / baseline GDP (%)	Aggr.	0.00	0.00	0.63	0.58
Taxes w. policy / baseline GDP (%)	Aggr.	0.00	2.93	3.13	3.10

Panel B. Changes relative to baseline scenario in %

		Laissez faire	$\varepsilon = 2\%$	
			Policy response	
			$\eta = 0\%$	$\eta = 50\%$
Capital	H	15.30	0.00	0.74
	L	-15.30	0.00	-0.74
GDP	H	-0.58	-2.00	-1.79
	L	-9.36	-2.00	-2.22
	Aggr.	-4.97	-2.00	-2.01
Pre-tax workers' wealth	H	1.27	-0.17	0.04
	L	-7.67	-0.17	-0.40
	Aggr.	-3.20	-0.17	-0.18
Pre-tax savers' wealth	H	-1.31	-0.17	-0.22
	L	0.70	-0.17	-0.12
	Aggr.	-0.51	-0.17	-0.18
Post-tax total wealth (= Welfare)	H	-1.66	-1.01	-1.43
	L	-1.89	-0.95	-1.34
	Aggr.	-1.76	-0.99	-1.39

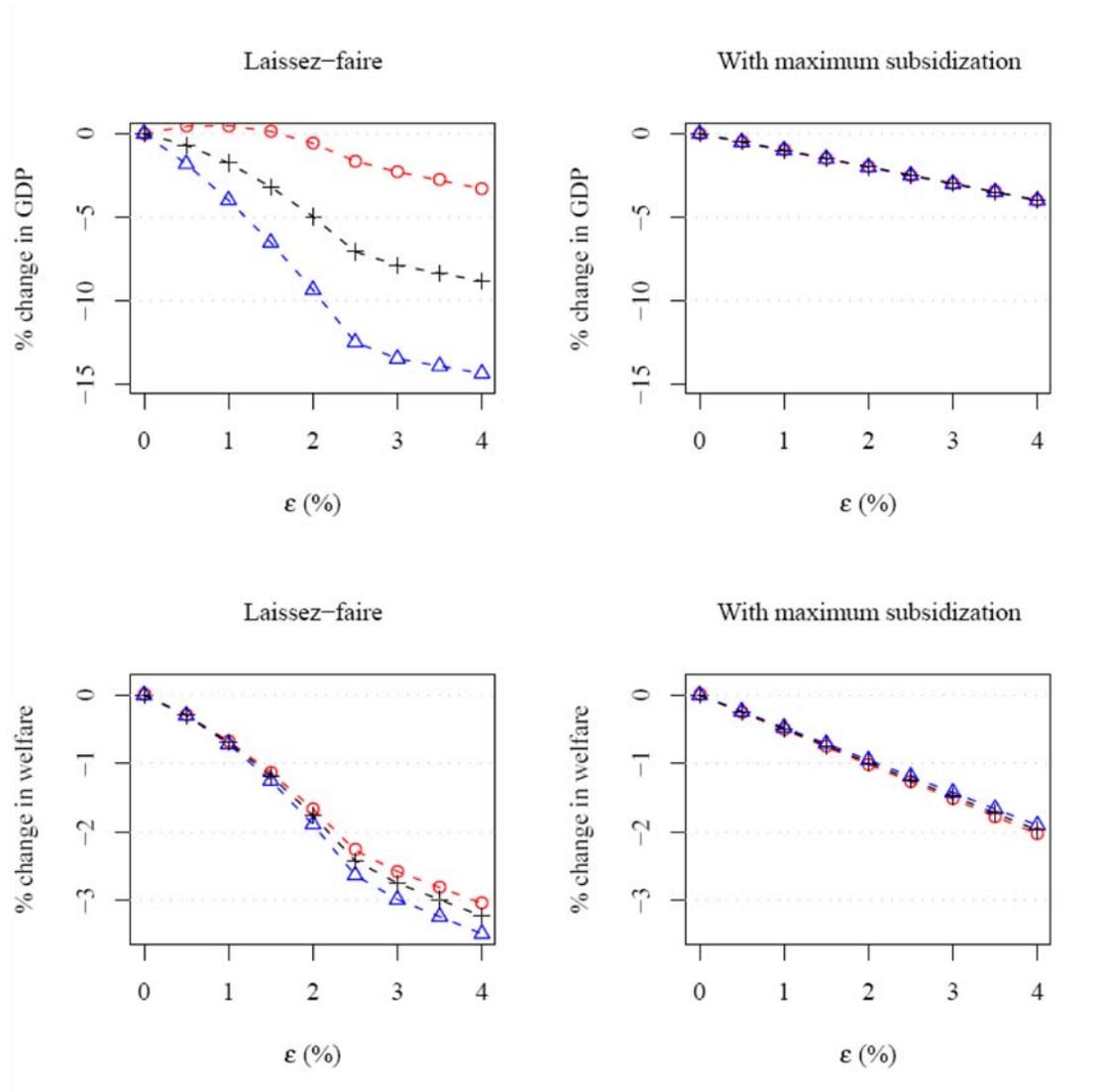


Figure 1: Effect of subsidization policies on output and welfare

The figure shows the effects of the maximum subsidization policies on output changes and welfare changes for different ε . \circ denotes changes in savings-rich regions (H), \triangle denotes changes in savings-poor regions (L), $+$ denotes aggregate changes. Welfare is calculated for $\eta = 0$.

7 Conclusions

We have developed a tractable general equilibrium model that captures the role of money markets in providing structural funding to some banks. We have shown that, in the presence

of deposit insurance, a relatively modest rise in the risk of bank failure can make these markets freeze, causing severe distortions to the aggregate allocation of credit. The model highlights a channel for the transmission and amplification of a shock to the solvency of financial intermediaries which is not directly related to the illiquidity of bank assets or to the contagion of insolvency risk via interbank positions. Rather, this channel is related to the asymmetric effect of the shock on intermediaries which, due to differences in the relative scarcity of savings in their local retail markets, act as either net lenders or net borrowers in money markets. While the former can remain funded in the margin by cheap insured deposits, the latter have to either pay high spreads in the money market or high deposit rates in their local retail market.

The model allows us to evaluate the impact of a solvency crisis and the associated reallocation of funds across final borrowers on standard macroeconomic variables such as interest rates, wages, and GDP, as well as on other aggregates especially relevant in a crisis context, such as deposit insurance liabilities. When counterparty risk increases, money market spreads widen and volumes shrink, producing a reallocation of funds from the retail borrowers which depend on former borrowing banks to those which rely on former lending banks. We find that, when the interdependence between the final users of bank funding is large, the output and welfare losses associated with the freeze are large.

The analytical and quantitative results of the paper suggest that the uncompensated absorption of counterparty risk by the government (or the central bank) can dramatically reduce the output and welfare losses of a solvency crisis. We find that the policy in this class that minimizes the welfare loss is sizeable; both in terms of the involved amount of lending which is directly undertaken or guaranteed by the government and the degree of subsidization of the spreads paid by the borrowing banks. The intuition here is that such policies can offset the asymmetries originated by the fact that some banks can be funded by abundant insured deposits while others cannot. In this sense, the analysis offers some insights on the rationale for the full-allotment fixed-rate lending policies and public guarantees programs orchestrated by central banks and governments in response to the current financial crisis.

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Appendix

A The economy without deposit insurance

In this section we analyze how firm-bank relationships will be conducted in an economy without deposit insurance (DI). Building on the notation and logic of analysis of Sections 3 and 4, we show that in such an economy the existence of a positive probability of bank failure (and positive spreads in MM) does not generate asymmetries in banks' laissez-faire marginal cost of funds and, hence, in the laissez-faire allocation of capital across regions with different endowments of savings. This also implies that the (unique) laissez-faire equilibrium involves operative MM and an efficient allocation of capital across regions, leaving no room for the type of policies discussed in Section 5 for the case in which, with DI, the laissez-faire allocation of capital was asymmetric. We show these results in four steps.

1. *In the absence of DI, whenever MM are active (or the government borrows at rate r), deposits should render an expected rate of return also equal to r .* This can be proven by contradiction. If deposits' expected rate of return were lower than r in any region, a bank in that region could make profits by specializing in taking deposits in that region and lending in MM (or to the government) at rate r , which is incompatible with equilibrium in the regional deposit market. Similarly, if deposits' expected rate of return were higher than r in any region, all banks in that region would find it more profitable to obtain their funding from money markets, which is also incompatible with equilibrium in the regional deposit market.

2. *In the absence of DI, the gross marginal cost of funds of a borrowing bank is $1+r+\gamma\phi$.* Following the same logic applied in subsection 3.1 to the analysis of firm-bank contracts in the economy with DI, the problem that determines the contract (k, n, l, R) when the representative bank is a MM borrower ($a < 0$) can be stated as follows:

$$\begin{aligned}
 & \max_{(k, n, l, R)} (1 - \varepsilon)(1 - p)[AF(k, n) + (1 - \delta)k - R] \\
 \text{s.t.} \quad & (1 - \varepsilon)\{(1-p)R + p(1-\lambda)k - [(1+r_d)d + (1+r+s)(l - d - e)]\} \geq (1+r+\phi)e \\
 & (1 - \varepsilon)(1+r_d)d + \varepsilon(1-\lambda)k \geq (1+r)d \\
 & k + wn = l \\
 & e \geq \gamma l, \\
 & l - d - e > 0
 \end{aligned} \tag{32}$$

There are three differences between problems (8) and (32). First, the required return on equity that appears in the right hand side of the first constraint in (32) is $1 + r + \phi$ rather than $1 + r_d + \phi$, since the expected return that equity holders could obtain as depositors is now r rather than the "promised" rate r_d . Second, this promised rate r_d is now endogenously

determined as part of the solution to this problem—what the competitive bank takes as given is the required expected return r . Third, depositors' participation constraint is added as a second constraint in (32), accounting for the fact that the payment of $(1 + r_d)d$ when the bank is solvent and the appropriation of the recoveries from the portfolio of loans $(1 - \lambda)k$ when the bank is insolvent must guarantee such an expected return r to the depositors. To understand the recovery term, recall that deposits are assumed to be senior to MM lenders and A2 is assumed to hold.

For reasons similar to those explained when solving the same type of problem in the economy with DI, it turns out that both the bank's and the depositors' participation constraints will be binding in the optimum. Moreover, if the second is used to substitute for $(1 + r_d)d$ in the first, and we use the fact that the spread s charged on MM funds satisfies $(1 - \varepsilon)(1 + r + s) = 1 + r$, the terms in d drop out, and some rearrangement allows us to write the resulting combined constraint as

$$(1 - \varepsilon)(1 - p)R + [(1 - \varepsilon)p + \varepsilon](1 - \lambda)k - (1 + r)(l - e) = (1 + r + \phi)e, \quad (33)$$

which, by the logic explained in the proof of Lemma 1, implies that the capital requirement will be binding. From here, the substitution of l for $k + wn$ using the third constraint in the original contract problem, allows us to reformulate it as

$$\max_{(k,n)} (1 - \varepsilon)(1 - p)[AF(k, n) + (1 - \delta)k] + [(1 - \varepsilon)p + \varepsilon]\lambda k - (1 + r + \phi\gamma)(k + wn), \quad (34)$$

where the term $\phi\gamma$ shows the only “distortion” due to the financing of the inputs k and n through banks (which is due to the excess cost of equity ϕ and the existence of the capital requirement γ).

3. *In the absence of DI, the gross marginal cost of funds of a lending bank is also $1 + r + \gamma\phi$.* When the representative bank is a MM lender ($a > 0$), the contract problem is as follows:

$$\begin{aligned} \max_{(k,n,l,R)} & (1 - \varepsilon)(1 - p)[AF(k, n) + (1 - \delta)k - R] \\ \text{s.t.} & (1 - \varepsilon)\{(1 - p)R + p(1 - \lambda)k - [(1 + r_d)d - (1 + r)(d + e - l)]\} \geq (1 + r + \phi)e \\ & (1 - \varepsilon)(1 + r_d)d + \varepsilon(1 - \lambda)k + \varepsilon(1 + r)(d + e - l) \geq (1 + r)d \\ & k + wn = l \\ & e \geq \gamma l, \\ & d + e - l > 0 \end{aligned} \quad (35)$$

The differences between problems (10) and (35) are qualitatively the same as those between (8) and (32) explained above. Notice that depositors' participation constraint (the second constraint) includes the expected returns of the bank's MM lending, $(1 + r)(d + e - l)$, as part of what depositors' recover if the bank fails.

As in the case of the borrowing bank, we can write down the first two constraints with equality, use the second to substitute for $(1 + r_d)d$ in the first, and use the MM pricing condition $(1 - \varepsilon)(1 + r + s) = 1 + r$ to find an expression for the combined constraint in which the terms in d drop out, making it identical to (33). So for the same reasons mentioned above, the contract problem when the representative regional bank is a lender can also be reduced to (34), which means that the gross marginal cost of funds of a lending bank coincides with that of a borrowing bank.

4. *In the economy without DI, the (unique) laissez-faire equilibrium involves operative MM and an efficient allocation of capital across regions, leaving no room for the type of policies discussed in Section 5.* In the previous points we have shown that, assuming operative MM, the problem that determines the production plans of firms in each region will be the same in all regions, irrespectively of the position of the representative bank in the MM (i.e., of whether the region is endowed with a large or a small amount of exogenous savings), so a laissez-faire equilibrium with operative MM must be necessarily symmetric in the allocation of capital. But then, reproducing the logic of Section 4, the situation with $k_H = k_L = \bar{S}$ and $a_H = S_H - \bar{S} > 0$ and $a_L = S_L - \bar{S} < 0$, clearly constitutes a laissez-faire equilibrium with operative MM irrespectively of the value of ε (compare this result with Proposition 1 and Corollary 1, for the economy with DI). Moreover, this is the unique laissez-faire equilibrium, since an alternative autarkic allocation would involve a divergence in the marginal cost of funds across banks in savings-rich and savings-poor regions that would open up profitable trading opportunities (for a bank that specializes in taking deposits in one of the savings-rich regions and lends the funds to banks in savings-poor regions), which is incompatible with equilibrium. Finally, notice that, in the logic of Proposition 2 and Corollary 2, the fact that banks' laissez-faire marginal costs of funds are equal across regions (i) leaves no room for policies based on reducing the (now inexistent) asymmetries by subsidizing MM spreads and (ii) eliminates the potential welfare gains associated with these policies.

B Variable definitions

Name	Definition*
Pre-crisis risk-free rate	r_H
Capital/GDP ratio	$(k_H + k_L)/y$
Labor share	$[\pi(1 + r_H)w_H + (1 - \pi)(1 + r_L)w_L]/y$
MM spread	$s = r_H - r_L$
Loan default probability	$1 - (1 - \varepsilon)(1 - p)$
Loan loss-given-default	$1 - [(1 - \lambda)k_j]/(k_j + w_j)$

Name	Definition*
Bank's funding rate (%)	H** r_H L r_L
MM / GDP	Aggr. $\pi a_H/y$
Loan rate	H $[(1 - \delta)k_H + F(k_H, n_H)] / (w_H + k_H) - 1$ L $[(1 - \delta)k_L + F(k_L, n_L)] / (w_L + k_L) - 1$
Deposit Insurance (DI) costs / GDP	H $DI_H/y_H = \varepsilon[(1 + r_H)(d_H - a_H) - (1 - \lambda)k]/y_H$ L $DI_L/y_L = \varepsilon[(1 + r_L)d_L - (1 - \lambda)k]/y_L$ Aggr. T_1/y
Policy costs / GDP	Aggr. T_2/y
Taxes w. policy / GDP	Aggr. $T/y = (T_1 + T_2)/y$

* All variables and parameters are defined either in the main text or elsewhere in this Appendix.

** H refers to savings-rich regions, L to savings-poor regions, and Aggr. to aggregate variables.

Name		Definition*
Capital	H**	k_H
	L	k_L
GDP	H	$y_H = (1 - \varepsilon)(1 - p)F(k_H, n_H)$
	L	$y_L = (1 - \varepsilon)(1 - p)F(k_L, n_L)$
	Aggr.	$y = \pi y_H + (1 - \pi)y_L$
Pre-tax workers' wealth	H	$(1 + r_H)w_H$
	L	$(1 + r_L)w_L$
	Aggr.	$\pi(1 + r_H)w_H + (1 - \pi)(1 + r_L)w_L$
Pre-tax savers' wealth	H	$(1 + r_H)S_H$
	L	$(1 + r_L)S_L$
	Aggr.	$\pi(1 + r_H)S_H + (1 - \pi)(1 + r_L)S_L$
Post-tax total wealth	H	$W_H = (1 + r_H)(S_H + w_H) - (1 + \eta)(DI_H + T_2)$
	L	$W_L = (1 + r_L)(S_L + w_L) - (1 + \eta)(DI_L + T_2)$
	Aggr.	$W = \pi W_H + (1 - \pi)W_L$

* All variables and parameters are defined either in the main text or elsewhere in this Appendix.

** H refers to savings-rich regions, L to savings-poor regions, and Aggr. to aggregate variables.