DISCUSSION PAPER SERIES

No. 7279

EXISTENCE OF PURE STRATEGIES NASH EQUILIBRIA IN SOCIAL INTERACTION GAMES WITH DYADIC EXTERNALITIES

Michel Le Breton and Shlomo Weber

PUBLIC POLICY

Centre for Economic Policy Research

www.cepr.org

Available online at: www.cepr.org/pubs/dps/DP7279.asp

ISSN 0265-8003

EXISTENCE OF PURE STRATEGIES NASH EQUILIBRIA IN SOCIAL INTERACTION GAMES WITH DYADIC EXTERNALITIES

Michel Le Breton, Toulouse School of Economics, GREMAQ and IDEI
Shlomo Weber, Southern Methodist University and CEPR

Discussion Paper No. 7279
April 2009

Centre for Economic Policy Research
53–56 Gt Sutton St, London EC1V 0DG, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 7279

April 2009

ABSTRACT

Existence of Pure Strategies Nash Equilibria in Social Interaction Games with Dyadic Externalities

In this note we introduce a general class of games where the payoff of every player are affected by her intrinsic taste for available strategic choices; intensity of her dyadic social interactions of with others in the peer group; and conformity effect. We show, that if the dyadic social influences are symmetric and the conformity effect is identical for all players, every game in our class admits a Nash equilibrium in pure strategies. Our proof relies on the fact that our game is potential (Rosenthal (1973), Monderer and Shapley (1996)). We also illustrate the universality of our result through a large spectrum of applications in economics, political science and sociology.

JEL Classification: C72 and D85
Keywords: conformity effect, dyadic externalities, nash equilibria, potential games and social interactions

Michel Le Breton
School of Economics
Manufacture des Tabacs, Accueil
MF404
21 allée de Brienne
31000 Toulouse
FRANCE

Email: lebreton@cict.fr

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=159295

Shlomo Weber
Department of Economics
Richard B. Johnson Center for
Economic Studies
Southern Methodist University
Dallas, Texas 75275-0496
USA

Email: sweber@smu.edu

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=161025

Submitted 16 April 2009

The paper has been presented at the New Economic School, the Central Economics and Mathematics Institute, Moscow, and SMU.We wish to thank Fuad Aleskerov, Vladimir Danilov, Hideo Konishi, Nikolai Kukushkin, and Victor Polterovitch for useful comments.

1 Introduction

The dependence of individuals' utility on the actions of others in the same peer group has been recognized in various studies of social interactions and social networks.[1] Since the peer influence with a social network may depend on the nature of the individuals' relationship, it is important to examine, both theoretically and empirically, the strength of bilateral influence, or *dyadic social externality*, imposed on each other by the corresponding pair (dyad) of players. As shown below, the class of games with dyadic social externalities contains many important models in economics, political science and sociology. From the empirical perspective, the dyadic setting has been utilized for empirical studies of various conflicts that produced dyadic influence matrices.[2]

In this note we introduce a general class of normal form games describing a social environment where a player's payoff for any given action is determined by intrinsic preference for the action (*taste component*); dyadic externalities exerted by peers (*dyadic component*); number of other players choosing this player's action (*conformity component*). We show that, in presence of symmetric dyadic social externalities and conformity effect, our game admits a pure strategies Nash equilibrium. To prove our existence result, we demonstrate that our game has a *potential* (Rosenthal (1973), Monderer and Shapley (1996)), whose maximum over the set of all strategy profiles yields a Nash equilibrium of our game.[3]

In the next section we introduce the model and state our result. In Sections 3 and 4 we offer various applications of our model.

[1]See, e.g., Becker (1974), Schelling (1978), Glaeser, Sacerdote and Scheinkman (1996), Blume (1993), Akerlof (1997), Durlauf (1999,2003), Manski (2000), Blume and Durlauf (2001, 2003), Brock and Durlauf (2001, 2002), Brock and Durlauf (2002), Glaeser and Scheinkman (2003), Galeotti at al. (2006), Jackson (2008)).

[2]See Bueno de Mesquita (1975), Axelrod and Bennett (1993), Fafchamps and Gubert (2007).

[3]It is interesting to point out that the problem of maximizing the potential is NP-hard even in the case of two feasible actions for each player AND in absence of two out of the three components of the payoff functions above. In fact, this problem is equivalent to the celebrated MAX-CUT problem in combinatorial optimization.

2 Social Interactions Games

Consider a class of games G with a finite set of players (society) $N = \{1, 2, ..., n\}$, which consists of M exogenous peer groups P_m, where $N = \bigcup_{m=1}^{M} P_m$. We denote by P^i the peer group that contains player i. Each player i has a set of feasible actions (pure strategies) X_i. Let $X = \bigcup_{i=1}^{n} X_i$. For simplicity, we assume that all X_i (and, thus, X) are compact subsets of the Euclidean space $\Re^T, T \geq 1$. Players' choices $x_i \in X_i$ generate the $n-$dimensional strategies profile $\mathbf{x} = (x_1, x_2, ..., x_n)$, which yields the partition of N into clusters of players choosing the same strategies. We denote this partition by $\pi(\mathbf{x})$ and let $S^i(\mathbf{x})$ be the cluster in $\pi(\mathbf{x})$ which contains player i, as well as all other players j for whom $x_j = x_i$.

The payoff $U_i(\mathbf{x})$ of player i is the sum of three terms:

$$U_i(\mathbf{x}) = V_i(x_i) + \sum_{j \in P^i} W_i^j(x_i, x_j) + H\left(x_i, |S^i(\mathbf{x})|\right), \qquad (*)$$

where $|Z|$ stands for the cardinality of the set Z. The first term describes the intrinsic taste of player i for her chosen action x_i. The second term represents the bilateral social interactions of player i with the members of her peer group. The last term captures a facet of social influence represented by the number of players who have chosen the same action. We impose the following assumptions:

Assumption A.1: Upper Semi-continuity. The following functions are upper semi-continuous: $V_i(\cdot) : X^i \to \Re$ for all $i \in N$, $W_i^j(\cdot, \cdot) : X^i \times X^j \to \Re$ for all $i, j \in N$ and $H(\cdot, r) : X \to \Re$ for all $r, 1 \leq r \leq n$.

Assumption A.2: Symmetry. $W_i^j(x_i, x_j) = W_j^i(x_j, x_i)$ for every $i, j \in N$, every $x_i \in X_i$ and every $x_j \in X_j$. Let also $W_i^i(x, x) = 0$ for every $i \in N$ and every $x \in X_i$.

Assumption A.3: Conformity. $H(x, \cdot)$ is increasing for all $x \in X$.

Assumption A.1 is of a technical nature. Assumption A.2 implies the symmetry of dyadic influence between any two players i and j. Assumption A.3 represents the *conformity effect*. Our main result is:

Theorem: Under A1, A2, A3, every game in G admits a Nash equilibrium in pure strategies.

The proof of the theorem, which is relegated to the Appendix, utilizes the ingenious idea of Rosenthal (1973) to show that our game of social interactions is a *potential game* studied in Monderer and Shapley (1996) (see also Kukushkin (2007)).

Note that Assumption A1 is vacuous when X is a finite set. Also in that case, we can dispense with the conformity assumption A3, and, moreover, allow for "congestion effects" when individuals' utility is adversely affected by the number of others making the same choice (Milchtaich (1996), Konishi, Le Breton, Weber (1997b)).

Corollary: If the set X is finite, then under A2, every game in G admits a Nash equilibrium in pure strategies.

3 Linear-Quadratic Games

We now turn to the examination of environments covered by our result and begin with a class of social interaction games where the taste and dyadic components in (*) consist of linear and quadratic terms.

3.1 Network Games

Akerlof (1997) considered a *status model* where strategic choices of all players represent a unidimensional interval and utility of individual depends on the difference between individual's own status and the status of others within the society:

$$U_i(\mathbf{x}) = -ax_i^2 + bx_i - d\sum_{j\neq i}(x_j - x_i),$$

where a, b, c, d are positive constants and x is a status-producing variable. Akerlof (1997) also considered a *conformity model*, where individuals minimize the social distance from their peers:

$$U_i(\mathbf{x}) = -ax_i^2 + bx_i + c - d\sum_{j\neq i}|x_j - x_i|.$$

Ballester, Calvó-Armengol and Zenou (2006) studied the contribution game with the following specification of utilities:

$$U(\mathbf{x}) = -ax_i^2 + bx_i^2 + \sum_{j\neq i}\sigma_{ij}x_ix_j,$$

where, $a, b > 0$, x_i is i's contribution and the dyadic social influences are captured by the cross-derivatives $\frac{\partial^2 U_i}{\partial x_i \partial x_j} = \sigma_{ij} = \sigma_{ji}$, $i \neq j$. When $\sigma_{ij} > 0$, the contributions by i and j are strategic complements, whereas when $\sigma_{ij} < 0$, these two efforts are strategic substitutes. A variant of this model has been examined by Corbo, Calvó-Armengol and Parkes (2007), where $U(\mathbf{x}) = -\dfrac{x_i^2}{2} + x_i + \theta\sum_{j\neq i}\sigma_{ij}x_ix_j$, where θ can be either negative or positive and $\sigma_{ij} = \sigma_{ij} \in \{0, 1\}$ for all $i \neq j$. Again, the case where $\theta < 0$ can be considered as a variant of the contribution game with free ride incentives. A particular case of this specification is considered by Glaeser, Sacerdote and Scheinkman (2003) in their exploration of a *social multiplier*, where the value σ_{ij} is equal to $1/(|P_m| - 1)$ whenever different individuals i, j belong to the same peer group P_m, and zero, otherwise.

Notice that one can expand the unidimensional framework described above. Indeed, let X_i be a compact subset of the $T-$dimensional Euclidean space $\Re^T, T > 1$ and define the utility functions as follows:

$$U(\mathbf{x}) = \langle \alpha_i, x_i \rangle - \sigma_i\frac{\|x_i\|^2}{2} + \sum_{j\neq i}\sigma_{ij}\langle x_i, x_j \rangle,$$

where $\|\cdot\|$ and $\langle\cdot,\cdot\rangle$ denote, respectively, the Euclidean norm and the Euclidean scalar product of vectors in $\Re^T$. If for every $i \in N$, the set of pure strategies X_i is a product set, then the analysis of this game can be undertaken along single dimensions as the utility function is separable across the components. However, if X_i is not a product set, then the multidimensional setting cannot be decomposed into several unidimensional components.

3.2 Neighborhood and Local Interactions

Glaeser, Sacerdote and Scheinkman (2003) also examine a continuous set of actions, where player i's utility is given by:

$$h_i x_i - E\left\{\sum_{j=1,j\neq i}^{n} J_{ij}\left(x_i - x_j\right)^2\right\} + \epsilon_i\left(x_i\right), \quad \text{or,} \quad h_i x_i + 2E\left\{\sum_{j=1,j\neq i}^{n} J_{ij} x_i x_j\right\} + \epsilon_i\left(x_i\right).$$

This specification can be decomposed into a private component $h_i x_i + \epsilon_i\left(x_i\right)$ and the interaction effect $E\left\{\sum_{j=1,j\neq i}^{N} J_{ij} x_i x_j\right\}$. The private component can be further decomposed into its mean, $h_i x_i$, and a 0-mean stochastic deviation $\epsilon_i\left(x_i\right)$. The terms J_{ij} measure the advantages of conformity: when all J_{ij} are positive, there is an incentive to conform. Our theorem applies when a realization of the random variable ϵ_i is a common knowledge. When the realization is a private information of player i, the analysis requires an alternative approach which we will briefly discuss in the next section.

4 The Binary Setting

In many social environments, players' strategies are reduced to acceptance or rejection of the action under consideration:[4] quit smoking (Harris and Lopez-Varcarcel (2006)), become a member of the club (Dixit (2003)), participate in criminal activities (Glaeser, Sacerdote and Scheinkman (1996)), join an industrial alliance (Axelrod et al. (1995)), participate in a riot or a strike (Schelling (1978), Chwe (1999), Granovetter (1978)),

[4]See Granovetter (1978) and Schelling (1973) for additional illustrations of this setting.

choose a side in international conflict (Altfeld and Bueno de Mesquita (1979), Axelrod and Bennett (1993)), purchase a house in a specific residential area (Schelling (1969)), display a national flag on the Independence Day (Chwe (2006)).

4.1 Critical mass and threshold models of collective action

A celebrated example of the analysis of the social influence channels in the dichotomic setting has been offered by Shelling (1978). In his *threshold model of collective action*, the participation of an individual in an action depends on the fraction of the population engaged in the action. This model is a special case of (*) without the second component, where, moreover, $H(Y, r) = r$ and $H(N, r) = 0$ for every positive integer r. Indeed, each player i who must chose between participation (Y) and non-participation (N) is represented by a critical mass t_i of the number of individuals of players choosing (Y) in order for i to endorse her own participation. That is, for every i, α_i is the smallest integer such that $V_i(N) - V_i(Y) \leq t_i$.

In his pioneering paper Granovetter (1978) has pointed out that the influence any given person has on one's decision may also depend on the nature of the pair's relationship. Thus, one needs to reintroduce the second (social interaction) term in (*) to capture heterogeneity of players' dyadic social externalities.

4.2 A Theory of International Alliances

In their strategic analysis of international alignments,[5] Axelrod and Bennett (1993) (AB - henceforth) developed the so-called *landscape theory* based on n actors (nations), an n-dimensional vector s whose coordinates represent the size (or importance) of the nations, an $n \times n$ matrix p, whose entry p_{ij} (positive or negative) represents the *propensity* of nations i and j to work together. An outcome (configuration) consists of a partition π

[5]See also the pioneering work of Bueno de Mesquita (1975, 1981) on systematic polarity.

of all nations into two mutually exclusive blocks, S and T. AB define the *frustration* $F_i(\pi)$ of country $i \in S$ from π, $F_i(\pi) = \sum_{j \in T} s_j p_{ij}$, and the *energy*, $E(\pi)$, of the configuration π by: $E(\pi) = \sum_{i \in N} s_i F_i$. AB then examine stable configurations that yield a local minimum of energy over all possible two-bloc alliances.[6] AB focus on the case where all the entries p_{ij} are symmetric and negative, and define the " distance" between every i and j as $d_{ij} = -p_{ij} > 0$. Obviously, the AB model is a special case of our setting, where all players face two choices, S and T, in absence of intrinsic preferences for two blocs[7] and a conformity component, where the dyadic influences are either $-s_i s_j d_{ij}$ if i, j belong to the same bloc, or 0, otherwise.[8]

Our model also covers the analysis of standard-setting alliances aiming to develop and sponsor technical standards (Axelrod et al. (1995)). For every firm i the set $N \backslash \{i\}$ is partitioned into two disjoint sets, C_i and D_i, i's *close* and *distant rivals*[9], respectively. The utility of a firm contemplating to join the alliance A is positively correlated with the size of A but is negatively impacted by allying with its rivals, especially the close ones: $U_i = \sum_{j \in A} s_j p_{ij}$, where p_{ij} is equal to $1 - \alpha$ if $j \in D_i$, and to $1 - \alpha - \beta$ if $j \in C_i$, where α and β are two positive parameters.[10]

4.3 Local Interaction, Statistical Physics and Ising' Model

As we indicated in subsection 3.2, the case where the realization of ϵ_i is a private information of player i, the analysis of the game requires an alternative approach. Each

[6] It is interesting to point out that AB found two stable configurations in Europe, one is the exact partition into the Axis and Allies of World War II, and another that separates the USSR, Yugoslavia and Greece from the rest of Europe.

[7] Galam (1996) removes this restriction.

[8] Note that the AB model is a special case of the multidimensional extension of the linear quadratic-model in subsection 3.1, where the set of players' pure strategies coincides with the vertices of a unit simplex.

[9] This echoes the distinction between strong and weak ties in the analysis of social networks (Granovetter (1973).

[10] Axelrod et al. (1995) test their theory by estimating the choices of nine computer companies to join one of two alliances sponsoring competing UNIX operating system standards in 1988.

player i knows the realizations of ϵ_i but has to form beliefs about the strategies of other actors. If the action can only take the values -1 and +1, we have in equilibrium

$$\text{Prob}\,(x_i = 1) = F\left(2h_i + 4E\left\{\sum_{j=1, j\neq i}^{N} J_{ij}x_j\right\}\right),$$

where F is the cumulative density function of the random variable $\epsilon_i\,(-1) - \epsilon_i\,(+1)$. There are various specifications of parameters h_i and J_{ij}. One is the *uniform global interaction model* considered by Brock and Durlauf (2001) (BD - henceforth) with $h_i = h$ and $J_{ij} = \frac{J}{2(N-1)}$, where h and J are two parameters. BD calculate the Nash equilibrium of this mean field model while assuming that each player believes that the expectation of the action of each of his opponents is identical, say m. BD derive the equilibrium condition on m, and the individual choice probability becomes $\text{Prob}\,(\eta_i = 1) = F\,(2h + 2Jm)$, where $m = \text{Prob}\,(x_i = 1) - \text{Prob}\,(x_i = -1) = 2\text{Prob}\,(x_i = 1) - 1$. BD derive the equation $m = \tanh\left(\frac{1}{2}g(h + Jm)\right)$ where $g(z) \equiv \log F(z) - \log(1 - F(z))$. Assuming, in addition, that the random terms are distributed according to the extreme value distribution with parameter β, $F(z) = 1/(1 + \exp(-\beta z))$, the equation simplifies to $m = \tanh\beta\,(h + Jm)$, which is the well-known Curie-Weiss model of magnetization in statistical physics.

Another important model of social interaction is the *uniform local interaction model* studied by Blume (1993) and Ellison (1993), where $h_i = h$ and $J_{ij} = J$ or 0, depending upon whether or not i and j are neighbors.[11] A neighborhood relation can be defined on the undirected graph with players located on the d-dimensional integer lattice Z^d, where neighbors of i are players placed at the minimal distance from i, as in the standard Ising's stochastic model.

[11] See the excellent survey of Durlauf (2004) on neighborhood effects.

5 The Multinomial Framework

In this section we consider situations where the players face more than two choices. While we focus on the discrete setting, most of the results can be extended to a continuous case. As an illustration, consider a group of co-workers who make their lunch choices among K available restaurants, varying according to the type of cuisine, food quality, price, service, etc. The restaurants offer price discounts for large groups, and the unit lunch price $l_k(s_k)$ in restaurant k is decreasing with s_k, the number of reservations it receives. We assume that the utility of player i choosing restaurant k is $U_i(\mathbf{x}) = b_i^k + \sum_{j \in S_k} p_{ij} - l_k(s_k)$, where the first term refers to the intrinsic benefit of player i from a lunch at restaurant k, the second term accounts for the social benefit derived by i from the company of her co-workers j at the lunch table, obtained through dyadic social externalities p_{ij}, whereas the last term accounts for a monetary effect of the restaurant discount.[12]

A specific case of this general framework have been examined by Florian and Galam (2000), who extended the landscape model to include a neutrality as a possible third choice. Brock and Durlauf (2002), Bayer and Timmins (2005) consider a utility specification $U_i(\mathbf{x}) = b_i^k + \alpha|S^i(\mathbf{x})| + \epsilon_i^k$, where ϵ_i^k is a random effect on player i from making choice k. When the realization of ϵ_i is the private information of player i, the equilibrium outcome is described by a probability distribution satisfying some consistency properties.

The existence of Nash equilibria in the case where $U_i(\mathbf{x}) = V_i(x_i) + H(x_i, |S(\mathbf{x}|)$, is shown in Konishi, Le Breton and Weber (1997a). In the context of jurisdiction formation, where each jurisdiction selects a local public good and finances it on its own, a Nash configuration in this setting is, in fact, a Tiebout or a sorting equilibrium (Schelling (1978)), where no resident from an existing jurisdiction desires to migrate elsewhere. An equilibrium profile is described by the partition $\pi(\mathbf{x})$ and the actions chosen by coalitions

[12] One could introduce a congestion effect that may even outweigh price advantages.

in $\pi(\mathbf{x})$. Here the social interaction game is, in fact, the *coalition formation game*, where the extent of social influence is limited to members of the same coalition. If functions V_i and H are independent of chosen actions, the payoff of a player i is fully determined by the set of players also choosing i's action, and the coalition formation game becomes *hedonic* (Banerjee, Konishi and Sömnez (2001), Bogomolnaia and Jackson (2002)). Alternatively, one can generate a hedonic game by ignoring the first and third terms in (*), and setting $W_j^i(x_i, x_j)$ equal to zero for all $x_i \neq x_j$, again restricting dyadic social influence to players choosing the same action.[13]

Finally, the multinomial setting is a natural framework to analyze strategic formation of clusters in social networks, where each player partitions the rest of the society into "friends" and "enemies". The social network is described by a graph $\mathcal{G}$ and the set of vertices N, where an (undirected) edge between i and j implies that both i and j consider the other as a friend. Let $U_i(\mathbf{x}) = \alpha \sum_{j \in S^i(\mathbf{x})} d_{ij} + H\left(|S^i(\mathbf{x})|\right)$, where α is a positive parameter and $d_{ij} = 1$ if there is an edge between i and j, and zero otherwise. In absence of intrinsic preferences over actions, the social heterogeneity is fully described by the graph $\mathcal{G}$, called a *sociogram* (Moreno (1934)). Our theorem implies the existence of a Nash equilibrium configuration, and, under certain conditions on the parameters, every Nash equilibrium splits the society into *cliques* of players, each considering the other as a friend.[14] It is also interesting to point out that by considering the complement of $\tilde{G}$ of G (i.e, players have an edge if they view each other as an enemy), we can utilize the *chromatic number* of $\tilde{G}$ (Skiena (1990)), which is the smallest number of colors needed to color the vertices of $\tilde{G}$ so that no two adjacent vertices share the same color. Indeed, since adjacent enemies have different colors, the chromatic number provides a lower bound on

[13] In this case our result has been obtained by Bogomolnaia and Jackson (2002).

[14] This notion of a *clique* (Luce and Perry (1949)) has been extensively examined in sociometry and social network studies. See, e.g., Alba (1973), Seidman and Foster (1978), Borgatti, Everett and Shirey (1990).

the number of clusters in a Nash equilibrium partition.

6 Conclusions

In this note we examine the class of social interaction games where the influence of a peer group is generated through symmetric dyadic interactions between every two members of the group. Given the scope of our note, we sketch a wide range of possible applications, whereas a detailed and more elaborated description of the environments covered by our result, is left for a longer version of the manuscript.

7 Appendix

Proof: Consider an n-dimensional strategies profile $\mathbf{x} = (x_1, ..., x_n)$ of players' strategic choices, which generates the partition $\pi(\mathbf{x}) = (S_1, \dots, S_K)$ of N, where the members of each S_k choose the same action x^k. Define the following function[15] Ψ on $\Lambda = \prod_{i=1}^n X_i$:

$$\begin{aligned}\Psi(\mathbf{x}) &= \sum_{i=1}^n V_i(x_i) + \frac{1}{2}\sum_{i=1}^n \sum_{j\in P^i} W_i^j(x_i,x_j) + \sum_{k=1}^K \sum_{r=1}^{|S_k|} H(x^k,r) \\ &= \sum_{i=1}^n V_i(x_i) + \frac{1}{2}\sum_{i=1}^n \sum_{j\in P^i} W_i^j(x_i,x_j) + \sum_{y\in B(\mathbf{x})} \sum_{r=1}^{|S(\mathbf{x},y)|} H(y,r),\end{aligned}$$

where $B(\mathbf{x})$ is the range of actions chosen at $\mathbf{x}$, i.e., $B(\mathbf{x}) = \{y \in X : \exists\, i \text{ s. t. } y = x_i\}$, and $S(\mathbf{x},y)$ is the cluster of players in $\pi(\mathbf{x})$ who choose y. Since, by A1, V_i and W_i^j are upper semi-continuous (*usc*) for all $i, j \in N$, the first two components of Ψ are *usc*. We will now show that the third term $\sum_{k=1}^K \sum_{r=1}^{|S_k|} H(x^k,r) = \sum_{y\in B(\mathbf{x})} \sum_{r=1}^{|S(\mathbf{x},y)|} H(y,r)$ is *usc* as well. Let $y \in B(x)$ and $\{\mathbf{x}^m\}$ be a sequence in Λ, converging to $\mathbf{x}$. Since for all $j \notin S(\mathbf{x},y)$, $x_j \neq y$, it follows that for m large enough, no action at $\mathbf{x}^m$ can be chosen by

[15] Note that in the AB specification in subsection 4.2, the function Φ is a weighted sum of players' payoffs, and the potential Φ coincides with the celebrated fractionalization index ELF (Atlas Narodov Mira, 1964).

both a member of $S(\mathbf{x},y)$ and a player outside of $S(\mathbf{x},y)$. Thus, again for m large enough, $S(\mathbf{x},y)$ is the union of sets in $\pi\left(\mathbf{x}^m\right)$. Let $B^m = \{z \in X : \exists \ \ i \in S(\mathbf{x},y) \quad \text{s.t.} \ \ z = x_i^m\}$. Thus, for m large enough, we obtain by A3:

$$\sum_{z\in B^m}\sum_{r=1}^{|S(\mathbf{x}^m,z)|} H(z,r) \leq \sum_{r=1}^{|S(\mathbf{x},y)|} \max_{z\in B^m} H(z,r) \equiv \sum_{r=1}^{|S(\mathbf{x},y)|} H(z_r^m,r),$$

where $H(z_r^m,r) = \max\limits_{z\in B^m} H(z,r)$ for every r. But since z_r^m converges to y for every r and H is *usc* with respect to the first argument, $\mathop{lim}\limits_{n\to\infty} H\left(z_r^m,r\right) \leq H\left(y,r\right)$. Thus,

$$\mathop{lim\ sup}_{m\to\infty} \sum_{z\in B^m}\sum_{r=1}^{|S(\mathbf{x}^m,z)|} H(z,r) \leq \sum_{r=1}^{|S(\mathbf{x},y)|} H(y,r).$$

Since X_i is compact for all $i \in N$, so is Λ, and the function Ψ being *usc*, attains its maximum $\mathbf{x}^* = (x_1^*,\ldots,x_n^*)$ over Λ. We claim that $\mathbf{x}^*$ is a Nash equilibrium. Assume, in negation, that there exists a player $i \in N$ and a strategy $x_i \in X_i$ such that $U_i(\mathbf{x}) < U_i(\widetilde{\mathbf{x}})$, where $\widetilde{\mathbf{x}} \equiv \left(x_i, x_{-i}^*\right)$:

$$V_i(x_i) + \sum_{j\in P^i} W_i^j(x_i,x_j^*) + H\left(x_i, |S^i(\widetilde{\mathbf{x}})|\right) > V_i(x_i^*) + \sum_{j\in P^i} W_i^j(x_i^*,x_j^*) + H\left(x_i, |S^i(\mathbf{x}^*)|\right).$$

Since the number of players who choose x_i at $\widetilde{\mathbf{x}}$ exceeds the number of players who chose x_i at $\mathbf{x}^*$ by one, we have:

$$\begin{aligned} & V_i(x_i) + \frac{1}{2}\sum_{j\in P^i} W_i^j(x_i,x_j^*) + \frac{1}{2}\sum_{j\in P^i} W_j^i(x_j^*,x_i) + H(x_i,|S^i(\mathbf{x}^*)|+1) \\ > \ & V_i(x_i^*) + \frac{1}{2}\sum_{j\in P^i} W_i^j(x_i^*,x_j^*) + \frac{1}{2}\sum_{j\in P^i} W_j^i(x_j^*,x_i^*) + H(x_i^*,|S^i(\mathbf{x}^*)|). \end{aligned}$$

The last inequality yields

$$\Psi\left(\widetilde{\mathbf{x}}\right) - \Psi\left(\mathbf{x}^*\right) = \left[V_i(x_i) + \frac{1}{2}\sum_{j\in P^i} W_i^j(x_i,x_j^*) + \frac{1}{2}\sum_{j\in P^i} W_j^i(x_j^*,x_i) + H(x_i,|S^i(\mathbf{x}^*|+1)\right]$$
$$-\left[V_i(x_i^*) + \frac{1}{2}\sum_{j\in P^i} W_i^j(x_i^*,x_j^*) + \frac{1}{2}\sum_{j\in P^i} W_j^i(x_j^*,x_i^*) + H(x_i^*,|S^i(\mathbf{x}^*)|)\right] > 0,$$

contradicting the fact that $\mathbf{x}^*$ is a maximum of Ψ over Λ. □

8 References

Akerlof, G. (1997) "Social distance and social decisions", *Econometrica* 65, 1005-1027.

Alba, R. (1973) "A graph-theoretic definition of a sociometric clique", *Journal of Mathematical Sociology* 3, 113-126.

Altfeld, M. and B. Bueno de Mesquita (1979) "Choosing sides in war", *International Studies Quarterly* 23, 87-112.

Atlas Narodov Mira (1964). Moscow: Miklukho-Maklai Ethnological Institute at the Department of Geodesy and Cartography of the State Geological Committee of the Soviet Union.

Axelrod, R. and D. Bennett (1993) "A landscape theory of aggregation", *British Journal of Political Science* 23, 211-233.

Axelrod, R., Mitchell, W., Thomas, R., Bennett, D. and E. Bruderer (1995) "Coalition formation in standard-setting alliances", *Management Science* 41, 1493-1508.

Ballester, C., Calvó-Armengol, A. and Y. Zenou (2006) "Who's who in networks. Wanted: the key player", *Econometrica* 74, 1403-1417.

Banerjee, S., Konishi, H. and T. Sömnez (2001) "Core in a simple coalition formation game", *Social Choice and Welfare* 18, 135-153.

Bayer, P. and C. Timmins (2005) "On the equilibrium properties of locational sorting models", *Journal of Urban Economics* 57, 462-477.

Becker, G. (1974) "A theory of social interactions", *Journal of Political Economy* 82, 1063-1093.

Blume, L. (1993) "The statistical mechanics of strategic interaction", *Games and Economic Behavior* 5, 387-424.

Blume, L. and S. Durlauf (2001) "The interactions-based approach to socioeconomic behavior", in *Social Dynamics*, S. Durlauf and P. Young, eds., MIT Press, Cambridge.

Blume, L. and S. Durlauf (2003) "Equilibrium concepts for social interaction models", *International Game Theory Review* 5, 193-209.

Bogomolnaia, A. and M. Jackson (2002) "The stability of hedonic coalition structures" *Games and Economic Behavior* 38, 201-230.

Borgatti, S., Everett, M. and P. Shirey (1990) "LS sets, lambda sets, and other cohesive subsets", *Social Networks* 12, 337-358.

Brock, W. and S. Durlauf (2001) "Discrete choice with social interactions", *Review of Economic Studies* 68, 235-260.

Brock, W. and S. Durlauf (2002) "A multinomial choice model with neighborhood effects", *American Economic Review* 92, 298-303.

Bueno de Mesquita, B. (1975) "Measuring systemic polarity", *Journal of Conflict Resolution* 19, 187-216.

Bueno de Mesquita, B. (1981) *The War Trap*, Yale University Press, New Haven.

Corbo, J., Calvó-Armengol, A. and D. Parkes (2007) "The importance of network topology in local contribution games" in *The Third International Workshop on Internet and Network Economics*, X. Tie and F. Chung Graham, eds., Springer Verlag.

Chwe, M.S. (1999) "Structure and strategy in collective action", *American Journal of Sociology* 105, 128-156.

Chwe, M.S. (2006) "Incentive compatibility implies signed covariance", UCLA, Mimeo.

Dixit, A. (2003) "Clubs with entrapment", *American Economic Review* 93, 1824-1829.

Durlauf, S. (1999) "How can statistical mechanics contribute to social science?", *Proceedings of the National Academy of Sciences* 96, 10582-10584.

Durlauf, S. (2004) "Neighborhood Effects", in *Handbook of Regional and Urban Economics*, Vol. 4, J.V. Henderson and J.F. Thisse, eds., North-Holland, Amsterdam.

Ellison, G. (1993) "Learning, local interaction and coordination", *Econometrica* 61, 1047-1072.

Fafchamps, M. and F. Gubert (2007) “The formation of risk sharing networks”, *Journal of Development Economics* 83, 326-350.

Florian, R. and S. Galam (2000) “Optimizing Conflicts in the Formation of Strategic Alliances”, *European Physical Journal* 16, 189-194.

Galam, S. (1996) “Fragmentation versus stability in bimodal coalitions”, *Physica* 230, 174-188.

Galeotti, A., Goyal, S., Jackson, M. Vega-Redondo, F. and L. Yariv (2006) “Network games”, California Institute of Technology, mimeo.

Glaeser, E., Sacerdote, B. and J. Scheinkman (1996) “Crime and social interactions”, *Quarterly Journal of Economics* 111, 507-548.

Glaeser, E., Sacerdote, B. and J. Scheinkman (2003) “The social multiplier”, *Journal of the European Economic Association* 1, 345-353.

Glaeser, E. and J. Scheikman (2003) “Non-market interventions”, in *Advances in Economics and Econometrics: Theory and Applications*, Eight World Congress, Vol. 1, M. Dewatripont, L.P. Hansen and S. Turnovsky, eds., Cambridge University Press, 339-369.

Granovetter, M. (1973) “The strength of weak ties”, *American Journal of Sociology* 78, 1360-1380.

Granovetter, M. (1978) “Threshold models of collective action”, *American Journal of Sociology* 83, 1420-1443.

Harris, J. and B. Lopez-Varcarcel (2006) “Asymmetric social interaction in economics: cigarette smoking among young people in the U.S. 1992-1993”, NBER Working Paper 10409.

Jackson, M (2008) “Average distance, diameter and clustering in social networks with homophily”, Stanford University, mimeo.

Konishi, H., Le Breton, M. and S. Weber (1997a) “Pure strategy Nash equilibria in a group formation game with positive externalities”, *Games and Economic Behavior* 21, 161-182.

Konishi, H., Le Breton, M. and S. Weber (1997b) "Equilibria in a model with partial rivalry", *Journal of Economic Theory* 72, 225-237.

Kukushkin, N. (2007) "Congestion games revisited", *International Journal of Game Theory* 36, 57-83.

Luce R. and A. Perry (1949) "A method of matrix analysis of group structure", *Psychometrika* 14, 94-116.

Manski, C. (2000) "Economic Analysis of Social Interactions", *Journal of Economic Perspectives* 14, 115-136.

Milchtaich, I. (1996) "Congestion games with player-specific payoff function", *Games and Economic Behavior* 13, 111-124.

Monderer, D. and L. Shapley (1996) "Potential games", *Games and Economic Behavior* 14, 124-143.

Moreno, J. (1934) "Who Shall Survive? A New Approach to the Problem of Human Interrelations", New York: Beacon House.

Rosenthal, R. (1973) "A class of games possessing a pure-strategy Nash equilibrium", *International Journal of Game Theory* 2, 65-67.

Schelling, T. (1969) "Models of Segregation", *American Economic Review, Papers and Proceedings* 59, 488-493.

Schelling, T. (1973) "Hockey Helmets, Concealed Weapons, and Daylight Savings: A Study of Binary Choices with Externalities", *Journal of Conflict Resolution* 17, 381-428.

Schelling, T. (1978) *Micromotives and Macrobehavior*, Norton, New-York.

Seidman, S. and B. Foster (1978) "A graph-theoretic generalization of the clique concept", *Journal of Mathematical Sociology* 6, 139-154.

Skiena, S. (1990) "Line graph" in *Implementing Discrete Mathematics: Combinatorics and Graph Theory with Mathematica*, Addison-Valley, Reading, MA.