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ABSTRACT

Waiting-time targets in healthcare markets: How long are we waiting?

Waiting-time targets are frequently used by policy makers in the healthcare sector to monitor provider's performance. Such targets are based on the distribution of the patients on the list. We compare and link such distribution with the distribution of waiting time of the patients treated, as opposed to on the list, which is arguably a better measure of welfare or total disutility from waiting (although it can only be calculated retrospectively). We show that the latter can be estimated from the former, and viceversa. We also show that, depending the hazard function, one distribution may be more or less favourable than the other. However, empirically we find that the proportion of patients waiting on the list more than x months is a downward estimate of the proportion of patients treated waiting more than x months, therefore biasing downwards the total disutility from waiting.

JEL Classification: I11 and I18
Keywords: duration, targets and waiting times

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1 Introduction

Waiting times are a major health policy concern in many OECD countries. Mean waiting times for non-emergency care are above three months in several countries and maximum waiting times can stretch into years (Siciliani and Hurst, 2004). Increasingly, information on waiting times is made available to patients, who would like to take informed decisions when choosing where to seek treatment, and researchers, who would like test the effect of policy interventions on waiting times. Information on waiting times is also used by policy makers as a target or as a performance indicator at provider's level (hospitals, general practitioners). Typically providers with longer waiting times are penalised or monitored more strictly. Many policies have recently been tailored at reducing waiting times (Siciliani and Hurst, 2005). Waiting times can be recorded at various levels and in many different ways. They are measured either by specialty (like general surgery, ophtalmology, and orthopaedics) or by procedure (for example cataract surgery, hip replacement, and varicose veins).

There are two commonly used measures of waiting times. The first measure is the waiting time of patients *treated* in a given year. This takes all of the patients treated throughout the year, and measures the difference between the time the patient was added to the waiting list and the time the patient receives the treatment (the completed waiting time).[1] The second common measure is the waiting time of the patients *on the list* at a census date (usually the end of the month or quarter): it is a cross-sectional measure which takes the list of patients at a point in time (census date) and measures the difference between that time and when the patient was added to the waiting list (for most patients this is an incomplete waiting duration since they will still be waiting after the census date). These two measures are fundamentally different: this study is devoted at clarifying exactly what these two different measures capture and investigates the link between the *distribution of waiting times* under the two measures. If we look at the two measurements

[1] The exact definition would be "the waiting time between the time the patient is added to the waiting list and the time the patient is admitted to the hospital for treatment". For expositional simplicity we refer to the "waiting time of the patients treated". The difference between the two definitions is the length of stay.

in demographic terms, the first is a measure of the completed lifetimes (age at death) of all persons who died in a particular year; the second is the age at the census date. Each of the two waiting-time measures has its own advantages and drawbacks, depending on our purpose.

The waiting time of the patients *on the list* is often used as a performance indicator at provider's level for monitoring purposes (Propper et al, 2008). In the past it was measured quarterly, and more recently monthly. This measure has the valuable property that it is available with a very short time lag. It gives an up-to-date description, a snapshot, of the waiting times of patients currently on the list. Not surprisingly, this feature means the measure is used for monitoring purposes (both internally and externally). Many targets are based on the proportion of patients waiting more than x months (where x can be six, nine, twelve or eighteen months). The higher is this proportion, the worse is the performance, and the higher is the likelihood that the provider will be strictly monitored. Hospital managers might loose their job if their performance is too poor. The disadvantage of the list measure is that it uses mainly incomplete waiting times.

From a patient's and policy maker's perspective, what matters is the *completed waiting time,* since this captures the overall effect on the welfare of the patient. Therefore the waiting time of patients *treated* seems to be a preferable measure: it captures the expected waiting time (and indeed the distribution of waiting times) faced by the representative patient at the beginning of the wait. However, the disadvantage of the waiting time of patients treated is that it can only be calculated retrospectively, i.e. only once the patients have finished waiting. There is often a substantial publication lag before the waiting time of the patients treated is made available: usually it is published one year later. If used for monitoring purposes, it gives a description only of the *past performance* of the providers, whilst policy makers are understandably more interested in *current performance.*

In this study we first investigate the theoretical link between the distribution of the waiting time of patients *on the list* with the distribution of the waiting time of patients *treated.* We show that in steady-state there is a one-to-one mapping of the two distributions, so that given one of the distributions the other one can be derived. This has the important implication that policymakers can use the up-to-date waiting time of the patients on the list not only for monitoring purposes (or performance assessment), but also to infer or predict the waiting time of patients treated long before it is available.

Moreover, we compare common statistics like the average and median waiting times, and the proportion of patients waiting over x months under the two distributions, which, as mentioned, are commonly used to set waiting-time targets. Intuitively, we might expect the average waiting time of the patients treated to be higher than the average waiting time of the patients on the list, as the first refers to the complete waiting while the second one to the *incomplete* waiting. However, this is not necessarily the case. Since the waiting time of the patients on the list tends to oversample long-waiting patients, in general the average wait of the patients on the list may be higher or lower than the average wait of the patients treated.

We then apply the theory using list data from the English National Health Service. Using data at hospital speciality level, we find that the estimated average waiting time of the patients treated is higher than the average waiting time of the patients on the list across all the specialties considered. Moreover, the proportion of patients treated waiting more than x months is also higher than the corresponding proportion of patients waiting on the list. Therefore, waiting-time targets based on the proportion of patients on the list is below our estimate of the time waited by the patients treated. We also compare our estimates of the waiting time of patients treated with the actual one, using data from the Hospital Episodes Statistics (HES). We find that our estimated mean is indeed a reasonable predictor of the HES data, and in 4 out of 6 specialities it gets to within 5%. This means, for some specialities at least, that we can use the waiting list data and create as a reliable and accurate predictor of the patients treated data before it becomes available.

The existing literature on waiting times is extensive (for a review see Cullis, Jones and Propper, 2000). The theoretical literature has focused on the role of waiting times as a rationing mechanism to help bringing the demand for and supply of health care in equilibrium.[2] The empirical literature has focused on investigating the responsiveness of demand for and supply of health services to waiting times (Lindsay and Feigenbaum, 1984; Martin and Smith, 1999; 2003; Gravelle, Smith and Xavier, 2003; Martin, Rice, Jacobs and Smith, 2007), and on assessing the effectiveness of different policy interventions, like the introduction of GP fundholding (Propper, Croxson and Shearer, 2002; Gravelle, Dusheiko and Hutton, 2002; Dusheiko, Gravelle and Jacobs, 2004); targets (Propper, Sutton, Whitnall and Windmeijer, 2008; Dimakou, Parkin, Devlin and Appleby, 2008); provider's choice (Dawson et al, 2007; Siciliani and Martin, 2007; Propper, Burgess and Gossage, 2008); and willingness to pay for wait reductions (Propper, 1990, 1995; Johannesson, Johansson, and Söderqvist, 1998; Bishai and Lang, 2000).

Informal discussions on the link between the distribution of the waiting time of the patients treated and on the list are provided by Don, Lee and Goldacre (1987), Hurst and Siciliani (2003), and Sanmartin (2001). However no formal model is provided. Dimakou, Parkin, Devlin and Appleby (2008) use data from the Hospital Episode Statistics (HES) to estimate hazard and survival curves using the Kaplan-Meier estimator. They show that the hazard rate (the probability of being admitted for treatment) is higher when the waiting-time target approaches while reduces if the patient's wait is already above the target. Differently from our work, they do not focus on the link between the two waiting-time distributions. There is an analogy between the duration of unemployment spells and waiting times, and indeed statistical models of transition (see Lancaster, 1992). There is also a literature on duration of price spells in dynamic pricing models (see Dixon, 2006, and Dixon and Kara, 2007).

The study is organised as follows. Sections 2 and 3 describe the distribution of the patients treated and of the patients on the list as a function of the probability of waiting in each period (the hazard rate). Section 4 shows how the distribution of the patients

[2] Lindsay and Feigenbaum, 1984; Iversen, 1993; 1997; Goddard, Malek and Tavakoli, 1995; Martin and Smith, 1999; Van Ackere and Smith, 1999; Olivella, 2002; Smith and Van Ackere, 2002; Gravelle, Smith and Xavier, 2003; Xavier, 2003; Hoel and Sæther, 2003; Barros and Olivella, 2005; González, 2005; Marchand and Schroyen, 2005; Siciliani, 2005, 2008; Gravelle and Siciliani, 2008a and 2008b; 2009; Brekke, Siciliani and Straume, 2008.

treated can be derived as a function of the patients on the list, and viceversa. Section 5 uses data from the English National Health Service on the distribution of the patients on the list for different specialties and for two financial years 2004-5 and 2005-2006, to derive the distribution of the patients treated. Section 6 concludes.

2 The waiting time of patients treated

Following Carlson and Horrigan (1983) and Dixon (2006), define N as the number of patients entering the waiting list in any given period t. It is easiest to think of a continuum of patients[3] and normalise $N = 1$. In period 1, a proportion p_1 continues waiting the following period while a proportion $(1 - p_1)$ receives treatment. Of those who continue to wait in the second period, a proportion p_2 keeps waiting, while a proportion $(1 - p_2)$ receives treatment, and so on. More generally, define p_i as the proportion of individuals on the list for i periods who keep waiting one more period (the survival rate), and $(1 - p_i)$ as the proportion of patients on the list for i periods who receive treatment in period i (the hazard rate), where $i = 1, ..., I$ (I being the longest time a patient can wait).

Define Ω_i as the proportion of patients waiting for at least i periods from the time they enter the waiting list to the time they receive treatment:

$$\Omega_i = \prod_{j=1}^{i-1} p_j \tag{1}$$

By convention $\Omega_1 = 1$: a patient cannot wait less than 1 period. That is because we count the period in which the patient is treated as part of the total waiting time.[4] The proportion of patients waiting for two periods is given by $\Omega_2 = p_1$: of those who waited for one period (i.e. everyone), a proportion $(1 - p_1)$ receives treatment in period 1, leaving the

[3]Since we will be applying this to English data, the numbers are very large and the continuum is appropriate. However, at the individual hospital level we would want to use precise integer numbers and focus more on "small sample" properties.

[4]This is really a trivial issue as a result of using discrete time. A waiting time of 1 means that the waiting ended within the first period. Thus, if we are using weekly data, we do not know which day of the week the patient was admitted for treatment, and all admissions that week are treated as ending waiting times within that week.

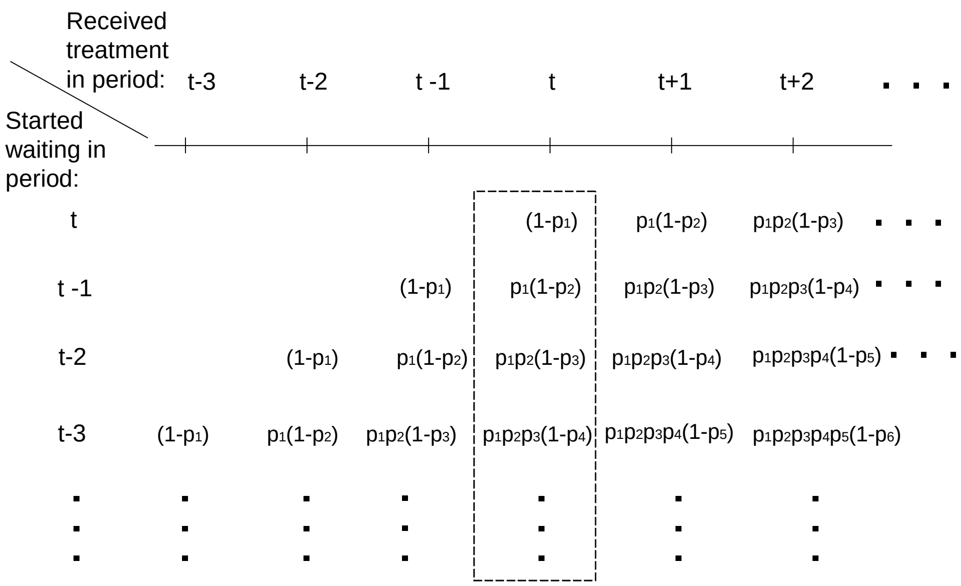
remaining p_1 waiting for a second period. Similarly, the proportion of patients waiting for three periods is equal to $\Omega_3 = p_1 p_2$: of those who waited for two periods (p_1), a proportion $(1 - p_2)$ receives treatment, leaving the remaining proportion $p_1 p_2$ waiting for one more period, and so on.

Define $f^{Tr}(i)$ as the density function of the patients treated in any given period t who have been waiting for i periods (where the superscript Tr stands for *treated*):

$$
\begin{aligned}
f^{Tr}(1) &= (1 - p_1), \\
f^{Tr}(2) &= p_1(1 - p_2), \\
f^{Tr}(3) &= p_1 p_2 (1 - p_3), \\
f^{Tr}(4) &= p_1 p_2 p_3 (1 - p_4), \ ...
\end{aligned}
\tag{2}
$$

Thus among the cross-section of patients who have been treated now (i.e. in period t) there are $(1 - p_1)$ patients who have entered the waiting list in the same period t and are treated straightaway; there are $p_1(1 - p_2)$ who have started waiting in period $(t - 1)$ but are treated in the following period t; there are $p_1 p_2 (1 - p_3)$ who have started waiting in period $(t - 2)$ but are treated after two periods (with probability p_1 they were not treated in the first period, and with probability p_2 they were not treated the second period; with probability $(1 - p_3)$ they were treated in period 3); there are $p_1 p_2 p_3 (1 - p_4)$ who have started waiting in period $(t - 3)$ but are treated after three periods. Figure 1 illustrates.

Figure 1. Proportion of patients receiving treatment in period t



In general terms, we can write more compactly the density function of the patients treated as:

$$f^{Tr}(i) = (1 - p_i)\,\Omega_i \quad \text{with} \quad i = 1, ..., I. \tag{3}$$

Therefore, the proportion of patients treated in any given period t who have waited i periods is given by the probability of being treated in period i, $(1 - p_i)$, times the probability of waiting i periods, Ω_i.

The average waiting time of the patients *treated*, w^{Tr}, in any single period, is then given by:

$$\begin{aligned} w^{Tr} &= \sum_{i=1}^{I} i \times f^{Tr}(i) \qquad (4)\\ &= 1 \times (1 - p_1) + 2 \times p_1\,(1 - p_2) + 3 \times p_1 p_2\,(1 - p_3) + 4 \times p_1 p_2 p_3 (1 - p_4)...\\ &= 1 + p_1 + p_1 p_2 + p_1 p_2 p_3 + ...\\ &= \Omega_1 + \Omega_2 + ... + \Omega_I\\ &= \sum_{i=1}^{I} \Omega_i \end{aligned}$$

The cumulative density function $F^{Tr}(i)$, ie the proportion of patients treated who have waited less than i periods is given by:

$$F^{Tr}(i) = \sum_{j=1}^{i-1} f^{Tr}(j) = \sum_{j=1}^{i-1} (1 - p_j)\, \Omega_j, \tag{5}$$

which, after cancelling out, reduces to $F^{Tr}(i) = 1 - \Omega_i$.

A commonly-used measure for policy purposes is the proportion of patients who waited more than (or at least) i periods, which is given by

$$1 - F^{Tr}(i) = \Omega_i = \prod_{j=1}^{i-1} p_j. \tag{6}$$

(see (1) above). Therefore p_1 wait more than one period, $p_1 p_2$ wait more than two periods, $p_1 p_2 p_3$ wait more than three periods, and so on.

Finally, notice that if a researcher or a policy maker can observe the distribution of the patients treated f^{Tr}, then she can recover the proportion of the patients who are treated in each period $(1 - p_i)$, and the proportion of patients who keep waiting (p_i), as a function of f^{Tr}: simply inverting (2) we get

$$\begin{aligned} p_1 &= 1 - f^{Tr}(1) \\ p_2 &= \frac{1 - f^{Tr}(1) - f^{Tr}(2)}{1 - f^{Tr}(1)} \\ p_i &= \frac{1 - \sum_{j=1}^{i} f^{Tr}(j)}{1 - \sum_{j=1}^{i-1} f^{Tr}(j)} \\ p_I &= 0. \end{aligned} \tag{7}$$

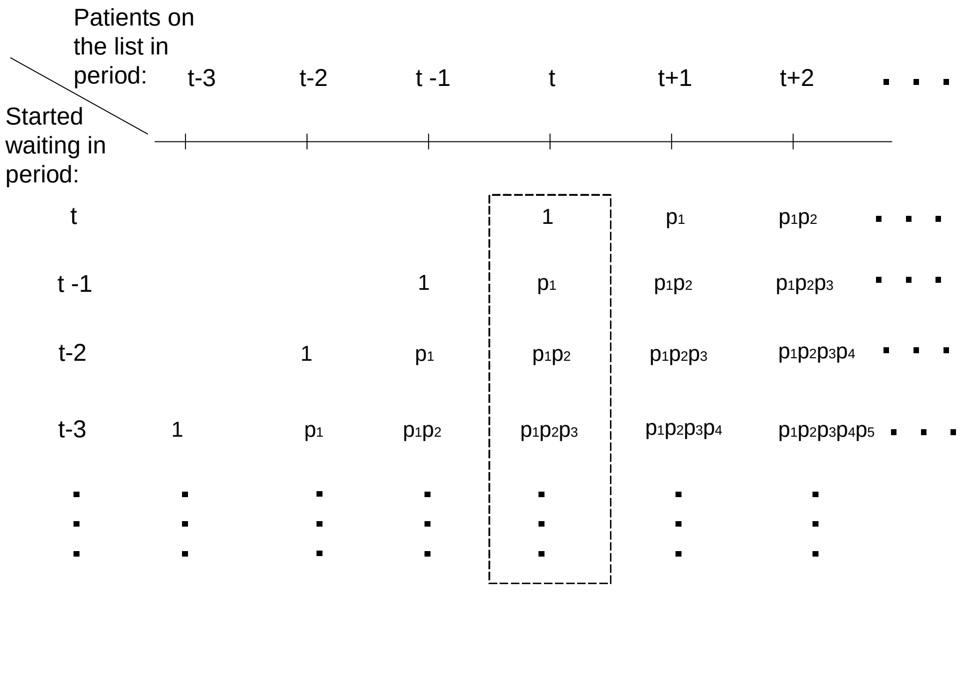
3 The waiting time of patients on the list

A second common measure used by researchers and policy makers is the waiting time of the patients *on the list*. This is a cross-sectional measure: it refers to the whole cross section of patients that are on the list at a point in time. It includes all the patients who entered the list at earlier periods and have been waiting (i.e. they have 'survived') and not been treated. The current cohort who just entered the list at time t is normalised to size 1. There are p_1 patients left of the previous cohort who entered the list at $t-1$, and $p_1 p_2$ patients left of the cohort who entered the list at $t-2$ and so on. The total number of patients on the list, defined as L, is thus:

$$L = (1 + p_1 + p_1 p_2 + p_1 p_2 p_3 + ...) = \sum_{i=1}^{I} \Omega_i = w^{Tr} \tag{8}$$

Figure 2 illustrates. Notice that since the number of patients treated in each period is equal to one (since $\sum_{i=1}^{I} f^{Tr}(i) = 1$), then Equation (8) suggests that in the steady state the average waiting time of patients treated w^{Tr} is equal to the number of patients on the list L divided by the number of patients treated in each period (equal to one), as intuitively expected.

Figure 2. Patients on the list in period t



The density function of the waiting time of the patients on the list $f^L(i)$ at any given time t gives the proportion of the patients on the list who have been waiting for i periods:

$$f^L(1) = \frac{1}{L},\ \ f^L(2) = \frac{p_1}{L},\ f^L(3) = \frac{p_1 p_2}{L},\ f^L(4) = \frac{p_1 p_2 p_3}{L},\ ... \tag{9}$$

Therefore, a proportion equal to $1/L$ have been waiting on the list for one period, a proportion equal to p_1/L has been waiting on the list for two periods, and so on. From (9), it is obvious that the density function $f^L\left(i\right)$ is monotonic and decreasing over waiting time. This implies that the proportion of patients who have been on the list for $i+1$ periods has to be (weakly) lower than the proportion of patients who have been on the list for only i periods. Intuitively, since every period which passes a proportion of patients is treated, the remaining number of patients left on the list has to be lower.

A more compact and general formulation of the density function of the patients on the list is given by:

$$f^L\left(i\right) = \frac{\Omega_i}{\sum_{i=1}^{I} \Omega_i} \text{ with } i = 1, ..., I. \tag{10}$$

The *average* waiting time of the patients on the list, defined with w^L, is then given by:

$$w^L = \frac{1 + 2 \times p_1 + 3 \times p_1 p_2 + 4 \times p_1 p_2 p_3 + ...}{1 + p_1 + p_1 p_2 + p_1 p_2 p_3 + ...}$$

which can be written more compactly as:

$$w^L = \frac{\Sigma_{i=1}^{I} i \times \Omega(i)}{L} = \frac{\Sigma_{i=1}^{I} i \times \Omega(i)}{\Sigma_{i=1}^{I} \Omega(i)} \tag{11}$$

The cumulative density function $F^L(i)$, i.e. the proportion of patients waiting *on the list* (strictly) less than i periods is given by $F^L(i) = \frac{\Sigma_{j=1}^{i-1} \Omega(j)}{L}$ for $i > 1$, with $F^L\left(1\right) = 0$, since a patient cannot be treated in less than 1 period (the waiting period is inclusive of the period in which treatment commences), while the proportion of the patients on the list waiting (weakly) more than i periods is equal to:

$$1 - F^L(i) = 1 - \frac{\Sigma_{j=1}^{i-1}\Omega(j)}{L} \text{ for } i > 1, \text{ with } 1 - F^L(1) = 1 \tag{12}$$

Therefore there is a proportion of patients $1 - F^L(2) = 1 - \frac{p_1}{L}$ on the list who have been waiting for more than one period; there is a proportion of patients $1 - F^L(3) = 1 - \frac{p_1 + p_1 p_2}{L}$ on the list who have been waiting for more than two periods; $1 - F^L(4) = 1 - \frac{p_1 + p_1 p_2 + p_1 p_2 p_3}{L}$ who have been waiting more than three periods and so on.

Notice that if a researcher or a policy maker can observe the distribution of waiting times elapsed for patients on the list $f^L(i)$, then she can recover the proportion of the patients who get treated in each period i, i.e. $(1 - p_i)$, and the proportion of patients that keep waiting in each period i, i.e. p_i:

$$(1 - p_i) = \frac{f^L(i) - f^L(i+1)}{f^L(i)}; \; p_i = \frac{f^L(i+1)}{f^L(i)} : i = 1...I \tag{13}$$

where $f^L(I+1) = 0$. This is a steady-state identity: if we are not in steady-state, then we cannot move from one distribution to the other in this way.

4 The comparison of waiting time measures

The two distributions of waiting time described in sections 2 and 3 are both cross-sectional in nature. It turns out that there is a steady-state identity between the two distributions, which means that if we know either distribution (the density across the patients treated or the patients on the list) we can recover the other (see Dixon, 2006). The following two propositions describe such identity. Proposition 1 determines the distribution of the patients treated as a function of the distribution of the patients on the list.

Proposition 1 *Suppose that we observe $f^L(i)$, i.e. the distribution of the waiting time of the patients on the list. Then, the distribution of the waiting time of the patients treated* *is given by:*

$$f^{Tr}(i) = \frac{f^L(i) - f^L(i+1)}{f^L(1)} = [f^L(i) - f^L(i+1)]L. \tag{14}$$

Proof. Comparing the two distributions, it is immediate that $f^{Tr}(i) = (1-p_i)w^{Tr}f^L(i)$. Now, recall that $w^{Tr} = L = \frac{1}{f^L(1)}$ and note that $p_i = \frac{f^L(i+1)}{f^L(i)}$. By substitution, $f^{Tr}(i) = \left(1 - \frac{f^L(i+1)}{f^L(i)}\right)\frac{f^L(i)}{f^L(1)}$ and the result is obtained. ■

Proposition 1 suggests that the proportion of the patients who are treated in any given period after having waited for i periods is equal to the difference between the proportion of the patients on the list who have waited for i periods and $(i+1)$ periods divided by the proportion of the patients on the list in the first period. Equivalently, and perhaps more intuitively, it is equal to the additional number of patients that have been taken out of the list, and therefore treated, between period i and period $i+1$.

Proposition 2 determines the distribution of the patients on the list as a function of the distribution of the patients treated.

Proposition 2 *Suppose that we observe $f^{Tr}(i)$, i.e. the distribution of the waiting time of the patients treated. Then, the distribution of the waiting time of the patients on the list is given by:*

$$f^L(1) = \frac{1}{w^{Tr}} \text{ and } f^L(i) = \frac{1 - \Sigma_{j=1}^{i-1} f^{Tr}(j)}{w^{Tr}} \text{ for } i > 1. \tag{15}$$

See appendix for proof. The intuition is that there is a flow of new entrants onto the list equal to $1/w^{Tr}$ each period. Hence at time t the people on the list waiting for one period are those arriving at t. The people who are waiting for two periods at time t are those who arrived in the previous period $(t-1)$, less the proportion $f^{Tr}(1)$ who were treated in $(t-1)$. The people who are waiting for 3 periods at t are those who arrived in period $t-2$, less the proportion $f^{Tr}(1)$ treated in period $(t-2)$ and the proportion $f^{Tr}(2)$ treated in $(t-1)$. The waiting list at a point in time just represents the people who arrived in the past and have yet to be treated.

To compare the average waiting times across the two distributions, recall that:

$$w^{Tr} = \sum_{i=1}^{I} \Omega(i); \;\; w^{L} = \frac{\Sigma_{i=1}^{I} i \times \Omega(i)}{\Sigma_{i=1}^{I} \Omega(i)} \tag{16a}$$

It is apparent that in general the average waiting times for the two distributions will be different. To make this point more clearly, we can write the average waiting time of the patients treated as a function of the average waiting time of the patients on the list:[5]

$$w^{Tr} = \frac{w^{L} - \Sigma_{i=1}^{I} i f^{L}(i+1)}{f^{L}(1)} \tag{17}$$

Since the denominator is less than one, while the second term in the numerator is negative, the average waiting time of the patients treated can in general be higher, lower or equal to the average waiting time of the patients on the list. Why does this result arise? On the one hand, the full length of waiting of any patients measured under the "waiting time of the patients treated" always exceeds the partial length of any patient measured under the "waiting time of the patients on the list" (also known as *interruption bias*). On the other hand, it is patients with longer than average full length of waiting who are more likely to be in progress when the "waiting time of the patients on the list" is measured (also known as *length bias*).

The concept of "the proportion of patients waiting more than i periods" also differs across the two measure. The proportion of patients *on the list* who waited more than (or equal to) i periods as a function of the proportion of patients treated who waited more than (or equal to) i periods is given by:

$$1 - F^{L}(i) = 1 - \frac{\Sigma_{j=1}^{i} \Omega(j)}{L} = 1 - \frac{\Sigma_{j=1}^{i} \left(1 - F^{Tr}(j)\right)}{w^{Tr}} \tag{18}$$

It is again apparent that in general the two measures will be different, although it is difficult to predict in which direction with further assumptions. The next section provides

[5] $w^{Tr} = \Sigma_{i=1}^{I} i f^{Tr}(i) = \Sigma_{i=1}^{I} i \frac{f^{L}(i) - f^{L}(i+1)}{f^{L}(1)} = \frac{\Sigma_{i=1}^{I} i f^{L}(i) - \Sigma_{i=1}^{I} i f^{L}(i+1)}{f^{L}(1)} = \frac{w^{L} - \Sigma_{i=1}^{I} i f^{L}(i+1)}{f^{L}(1)}$, where $f^{L}(I+1) = 0$.

some examples.

4.1 Some examples

Example 1: Constant hazard rate (constant probability of being treated)

Suppose that the proportion of patients that keep waiting from one period to the following (the hazard rate) is constant, i.e. $p_i = p$ for any period i, and that the maximum number of periods that patients can wait goes to infinity, $I \to \infty$ (the patient is never treated). It is straightforward to show that under this assumption the distribution of the waiting time of the patients on the list and of the patients treated coincide, i.e. $f^{Tr}(i) = f^L(i)$.

By comparison with equation (2) notice that $f^{Tr}(i) = (1-p)p^{i-1}$ for $i = 1, ..., \infty$. Moreover, notice that the number of patients on the list is equal to $L = 1+p+p^2+p^3+... = \Sigma_{i=1}^{\infty} p^{i-1} = 1/(1-p)$. It follows then that:

$$f^L(1) = \frac{1}{L} = (1-p), \;\; f^L(2) = \frac{p}{L} = p(1-p), \; f^L(3) = \frac{p^2}{L} = p^2(1-p), \; ... \qquad (19)$$

or in general $f^L(i) = (1-p)p^{i-1}$. We conclude therefore that the two distributions coincide, i.e. $f^{Tr}(i) = f^L(i)$, from which it follows that the proportion of waiting longer than i periods for the two distributions also coincide, i.e. $1 - F^{Tr}(i) = 1 - F^L(i)$, as well as the average waiting time, i.e. $w^{Tr} = w^L$.

More precisely, the average waiting time of patients treated is equal to $w^{Tr} = 1 + p + p^2 + p^3 + ... = \Sigma_{i=1}^{\infty} p^{i-1} = 1/(1-p)$. The average waiting time of the patients on the list is equal to $w^L = \frac{1+2p+3p^2+4p^3+...}{1+p+p^2+p^3+...} = \frac{\Sigma_{i=1}^{\infty} i p^{i-1}}{\Sigma_{i=1}^{\infty} p^{i-1}} = \frac{1/(1-p)^2}{1/(1-p)} = \frac{1}{1-p}$. The interruption bias is completely offset by the length bias. Note finally that a higher per-period probability of being treated (i.e. when $(1-p)$ is higher) implies that the average waiting time is lower, as intuitively expected.

In summary, when the hazard rate is constant, the two distributions are exactly equivalent, so we have:

$$\begin{aligned} w^{Tr} &= w^L = \frac{1}{1-p} \\ f^{Tr}(i) &= f^L(i) = (1-p)p^{i-1} \\ 1 - F^{Tr}(i) &= 1 - F^L(i). \end{aligned} \tag{20}$$

Example 2: Monotonic hazard rate (probability of being treated increases or decreases with time waited)

Suppose now that the hazard rate, rather than being constant as for example 1, varies over time, but, crucially, is weakly monotonically increasing over time. More formally assume that $p_{i+1} \geq p_i$, i.e. the probability of waiting (weakly) increases with the time waited (or, equivalently, the probability of being treated decreases with the time waited) and strictly increases for at least one period i, then:

$$w^L > w^{Tr}.$$

The average waiting time of the patients on the list is higher than the average waiting time of the patients treated. In this case the interruption bias is more than offset by the length bias (Carlson and Horrigan, 1983). Similarly, it can be shown that if $p_{i+1} \leq p_i$, i.e. the probability of waiting weakly decreases over time, and strictly decreases for at least one period, then:

$$w^L < w^{Tr}.$$

The average waiting time of the patients on the list is now *lower* than the average waiting time of the patients treated. In this case the interruption bias dominates over the length bias.

Example 3: Suppose that all patients wait the same time to be treated: let us say 4 periods.

In this case we have:

$$\begin{array}{lll} p_1 = 1 & f^{Tr}(1) = 0 & f^{L}(1) = \frac{1}{4} \\ p_2 = 1 & f^{Tr}(2) = 0 & f^{L}(2) = \frac{1}{4} \\ p_3 = 1 & f^{Tr}(3) = 0 & f^{L}(3) = \frac{1}{4} \\ p_4 = 0 & f^{Tr}(4) = 1 & f^{L}(4) = \frac{1}{4} \end{array}$$

and therefore

$$w^{Tr} = 4 > w^{L} = \frac{5}{2} = 2.5.$$

The waiting time of the patients treated is higher than the waiting time of the patients on the list. The result holds more generally. Suppose that all the patients wait I periods, then:

$$\begin{array}{lll} p_1 = 1 & f^{Tr}(1) = 0 & f^{L}(1) = \frac{1}{I} \\ p_2 = 1 & f^{Tr}(2) = 0 & f^{L}(2) = \frac{1}{I} \\ ... & ... & ... \\ p_I = 0 & f^{Tr}(I) = 1 & f^{L}(I) = \frac{1}{I} \end{array}$$

Then the average waiting time of the patients treated is $w^{Tr} = I$, and the average of the patients on the list is $w^{L} = \frac{\Sigma_{i=1}^{I} i}{I} = \frac{1+I}{2}$. Therefore, it follows that:

$$w^{Tr} = I > w^{L} = \frac{1+I}{2} \text{ for } I > 1.$$

The average waiting time of patients treated is generally higher unless everyone waits for one period only (i.e. $I = 1$), in which case the two measures coincide.

Example 4: The distribution of the waiting time of the patients treated is uniform: $f^{Tr}(i) = \frac{1}{4}, i = 1...4$.

$$\begin{array}{lll} p_1 = \frac{3}{4} & f^{Tr}(1) = \frac{1}{4} & f^{L}(1) = 0.4 \\ p_2 = \frac{2}{3} & f^{Tr}(2) = \frac{1}{4} & f^{L}(2) = 0.3 \\ p_3 = \frac{1}{2} & f^{Tr}(3) = \frac{1}{4} & f^{L}(3) = 0.2 \\ p_4 = 0 & f^{Tr}(4) = \frac{1}{4} & f^{L}(4) = 0.1 \end{array}$$

The average waiting time is in this case:

$$w^{Tr} = \frac{5}{2} = 2.5 > w^L = 2$$

and the average waiting time of the patients treated is higher than the average waiting time of the patients on the list. The result holds more generally for any I.

$$\begin{array}{rcc} p_1 = & f^{Tr}(1) = \frac{1}{I} & f^L(1) = \frac{2}{I+1} \\ p_2 = & f^{Tr}(2) = \frac{1}{I} & f^L(2) = \frac{2}{I+1}(1 - \frac{1}{I}) \\ ... & ... & ... \\ p_i = & f^{Tr}(i) = \frac{1}{I} & f^L(i) = \frac{2}{I+1}(1 - \frac{i-1}{I}) \\ ... & ... & ... \\ p_I = & f^{Tr}(I) = \frac{1}{I} & f^L(I) = \frac{2}{I+1}(1 - \frac{I-1}{I}) \end{array}$$

The average waiting time of the patients treated is $w^{Tr} = \frac{\Sigma_{i=1}^{I} i}{I} = \frac{1+I}{2}$ and the average waiting time of the patients on the list is $w^L = \frac{2}{I+1}\Sigma_{i=1}^{I} i(1 - \frac{i-1}{I}) = \frac{2+I}{3}$. Therefore, it follows that:

$$w^{Tr} = \frac{1+I}{2} > w^L = \frac{2+I}{3} \text{ if } I > 1.$$

The average waiting time of patients treated is generally higher unless everyone waits for one period only (i.e. $I = 1$), in which case the two measures coincide.

Example 5: 90% of the patients treated wait for 1 period, 10% for 4 periods.

$$\begin{array}{ccc} p_1 = 0.1 & f^{Tr}(1) = 0.9 & f^L(1) = 10/13 = 0.77 \\ p_2 = 1 & f^{Tr}(2) = 0 & f^L(2) = 1/13 = 0.08 \\ p_3 = 1 & f^{Tr}(3) = 0 & f^L(3) = 1/13 = 0.08 \\ p_4 = 0 & f^{Tr}(4) = 0.1 & f^L(4) = 1/13 = 0.08 \end{array}$$

$$w^{Tr} = 1.3 < w^L = 1.46$$

The distribution f^L is given in exact fractions and to 2 decimal points. The intuition here is that if we look at the list at a point in time, it consists of the 90% of patients who arrive at time t and will be treated during t. However, there are also the 10% who have just arrived at t and will have to wait 4 periods. Together, these add up 77% of those on the list. The remaining 23% on the list consist of those who arrived in the previous 3 periods and are awaiting their treatment in their fourth period of waiting. Unlike examples 3 and 4, we have that the average waiting time on the patients on list is longer than the average waiting time of the patients treated.

In fact, for the case of $f^{Tr}(2) = f^{Tr}(3) = 0$, we have $w^{Tr} = w^L = 2$ when $f^{Tr}(1) = 2/3$; $w^{Tr} > w^L$ when $f^{Tr}(1) < 2/3$; $w^{Tr} < w^L$ when $f^{Tr}(1) > 2/3$. Hence when there are two groups, with one waiting 4 periods and the other just one period, if the proportion who get treated in the first periods exceeds 2/3, then the average time on the list will be bigger than the average of those treated.

Example 6 80% of the patients treated wait 1 period; 10% wait for 2 periods and 10% for 4 periods. $w^{Tr} < w^L$.

$$\begin{array}{lll} p_1 = 0.2 & f^{Tr}(1) = 0.8 & f^L(1) = 0.71 \\ p_2 = 0.5 & f^{Tr}(2) = 0.1 & f^L(2) = 0.14 \\ p_3 = 1 & f^{Tr}(3) = 0 & f^L(3) = 0.07 \\ p_4 = 0 & f^{Tr}(4) = 0.1 & f^L(4) = 0.07 \end{array}$$

$$w^{Tr} = 1.4 < w^L = 1.5$$

Clearly, these examples show that it is possible for average waiting time across all the list to be greater than or less than the average waiting time of those on the list. The exact relationship depends on the empirical distribution of waiting times.

5 Empirical evidence

The Department of Health regularly publishes information on waiting times statistics. Figures on the waiting time of the patients on the list have been collected for at least the last ten years. Over time, the format has changed. Initially the data were available quarterly and at specialty level. More recently, the data are available monthly, but the speciality breakdown is not available anymore.

In this section, we use quarterly data for England on the distribution of the waiting time of the patients on the list. We mainly focus on financial years 2004-2005 and 2005-2006 as the data are available at specialty level. For each specialty, we have information on the number of patients that have been waiting on the list for less then 1 month, between 2 and 3 months, between 3 and 4 months, ..., and more than 12 months. We truncate the distribution at 9 months, since the proportion of patients waiting longer than 9 months is virtually zero (therefore $i = 1, ..., 8$, and the last category is patients waiting 9 months or more). We focus on six major specialties where waiting times and waiting lists are large numerically: these are General Surgery, Urology, 'Trauma and Orthopaedics', 'Ear, Nose and Throat', Ophtalmology, and Gynaecology (these six specialities cover more than 75% of the total waiting list). This data is freely available from the Department of Health,[6] and originates from the KH07 quarterly returns which all NHS Trusts in England are required to submit. Quarter 1 refers to patients on the list on the 1^{st} April, while quarters 2-4 refer respectively to patients on the list on 1^{st} July, 1^{st} October, and 1^{st} January of each year.

The specialty breakdown is not available for more recent years (from 2006-2007), which provide only aggregate figures across all specialties. For years before 2004-2005, the specialty breakdown is available. However, the data is presented in broader time categories: patients waiting less than 3 months, between 3 and 5 months, between 6 and 8 months, and so forth (one period being three rather than one month). We therefore prefer to focus on 2004-2005 and 2005-2006 with the monthly breakdown by speciality.

The raw data of the number of patients on the list for specialties with longest waiting times are provided in Table A1 in the Appendix. 'General Surgery' and 'Trauma and Orthopaedics' are the specialties with longest waiting lists. Taking 'General Surgery' as an example, there are 148,853 patients on the list on 1 April of 2004, of which 44,120 have been waiting less than one month, 29,250 have been waiting between one and two months, 21,965 have been waiting between two and three months, and so on.

[6] www.performance.doh.gov.uk/waitingtimes/index.htm.

From the theory developed in section 3, we know that in steady-state, the proportion of patients waiting i periods is weakly monotonically decreasing in i. This is confirmed by the data in Table A1, with very few exceptions. The first anomaly is the "Christmas effect": recall that quarter 3 refers to all the patients on the list on 1^{st} January. As hospitals see less patients in December, there are less decisions to admit, and therefore the number who have been waiting under 1 month at the end of December will be low in this particular quarter. The Christmas effect shows up in the raw data in quarter 3 of both years in three specialties ('Trauma and Orthopaedics', 'Ear, Nose and Throat', and 'Ophtalmology'). Take for example quarter 3 of Ophtalmology in year 2004-05. The number of patients waiting less than one month is 24,741, which is lower than the number of patients waiting between one and two months which is 25,217. Whilst the Christmas effect does not cause a non-monotonicity in other specialities, it will almost certainly be present but simply not strong enough to make the proportions of patients waiting non-monotonic. To counter the Christmas effect, we replace the number of patients waiting less than one month with the number of patients waiting between one and two months, which implies that all the patients on the list wait at least one month (the probability of being admitted in the first month is zero). The numbers are reported in Table A2 in bold. Finally, notice that this correction is marginal for 'Ear, Nose and Throat' and Ophtalmology (less than 0.1% and 1% of patients on the list respectively), and more substantial for 'Trauma and Orthopaedics only (a little less than 5% of patients on the list).

The second anomaly occurs towards the end of the period when there are very few patients left on the list. Due to the small numbers, the sequence is "noisy". Whilst the quantitative importance of such small numbers of patients is negligible, we tidied up the data to make it monotonic. Take 'general surgery' quarter 4 in year 2005-06. The number of patients waiting between 6 and 7 months is 16, it is 19 between 7 and 8 months, and it is 36 between 8 and 9 months. These data would imply negative hazard rate. To avoid this, we take the number of the individuals waiting between 6-9 months (73) and divide by three (the number of periods). This procedure is used for quarters 3 and 4 in year

2005-06 for all specialties. Table A2 reports the 'adjusted' data. The adjusted numbers are in bold. It is important to emphasize that the proportion of patients waiting between 6-9 months is always tiny, below 0.1%, and therefore these adjustment have virtually no impact on the results.

5.1 The Results

Tables 1 to 6 describe one of the main results of our analysis. They provide for each specialty and each of the four quarters the distribution of waiting times for the patients on the list (first column, $1 - F^L(i)$) and, using Proposition 1, *estimates* of the distribution of the patients treated (second column, or $1 - F^T(i)$). Notice that the first column, which gives the *proportion* of patients on the list waiting more than i months, is simply obtained by dividing the number of patients on the list waiting more than i months, by the total number of patients on the list. To calculate the numbers in the second column ($1 - F^T(i)$), we first compute the density function of the patients on the list $f^L(i)$ by dividing the number of patients on the list waiting i months by the total number of patients on the list. For example, for General Surgery in quarter 1 of year 2004-05, there were 14.8% of patients that waited between two and three months (21,965/148,853 = 0.148). Second, we use Proposition 1 to transform the distribution of the patients *on the list* ($f^L(i)$ with $i = 1, ..., I$) into the distribution of of the patients *treated* ($f^T(i)$ with $i = 1, ..., I$). Third, we compute the cumulative density function of the patients treated waiting more than i months, which provides the second column ($1 - F^T(i)$).

For each quarter and speciality, we also provide the average waiting time of the patients on the list and of the patients treated. In this calculation, we assume that all the patients waiting for less than a month waited for 0.5 months, all patients waiting between 1 and 2 months waited for 1.5 months, all patients waiting between 2 and 3 months waited for

2.5 months, and so forth.[7]

Tables 1-6 here

Two main results emerge from Tables 1-6. First, *the estimated average waiting time of the patients treated is higher than the actual average waiting time of the patients on the list.* This holds across all specialties and quarters (the only exception being Urology in quarter 2 of year 2004-05, where the difference is virtually zero). For example, in quarter 1 of 2004-05 for General Surgery the average waiting time of the patients on the list was 2.60 months, while the average waiting time of patients treated was 2.87 months. This indicates that the "interruption bias" outweighs the "length bias". Across all quarters the difference between the two averages for General Surgery is between 0.22-0.84 months. For Trauma and Orthopaedics the difference is more substantial, between 1.10-2.34 months. For the remaining specialties, the difference is in the range 0.53-1.11 months for Ear, Nose and Throat, 0.33-0.72 months for Ophtalmology, 0.14-074 months for Gynaecology, between -0.01 and 0.57 months for Urology. Table 7 presents the average across each year for each specialty.

The second main result is that the proportion of patients *treated* that have been waiting longer than 5, 6, 7, 8 or 9 months is higher than the proportion of patient *on the list* that have been waiting more than 5, 6, 7, 8 or 9 months in the vast majority of the quarters and specialities considered. For example, in quarter 1 of year 2004-05 we have that 8.2% of the patients on the list waited more than six months in General Surgery, while 13% of the patients treated waited more than six months. Of course, these are just two different distributions which look at the same phenomenon in a different way. However, we might well think that what the government is really interested in is the average waiting time of patients treated. In that case the distribution of the patients on the list is generally more favourable than the distribution of the patients treated. In some cases the distribution

[7] We could alternatively assume either that: i) all patients waiting less than one month waits zero, all patients waiting between 1 and 2 months wait 1 month, ..., and so on; or that ii) all patients waiting less than one month waits one month, all patients waiting between 1 and 2 months wait 2 months, ..., and so on. If we chose the first approach, both averages (of patients on the list and treated) would reduce by 0.5 months. If we chose the second approach, the average would increase by 0.5 months. However, and crucially, the difference between the two averages would not be affected.

of the patients on the list first-order stochastically dominates the distribution of patients treated. This is for example the case for all specialties in quarters 3 and 4 of year 2004-05. Hence measures of the proportion of *patients on the list* that have been waiting more than six or nine months are a downward estimate of the proportion of *patients treated* that have been waiting more six or nine months. *The size of this bias is not negligible.*

Table 7 here

In Tables 1-6 the average waiting time of the patients treated has been estimated. In Table 7 we compare our *estimated* average waiting time of patients treated with the *actual* waiting time of patients treated. The latter is available from the Hospital Episodes Statistics (HES), which is published annually. It should be noted that this is an entirely different data set to the waiting list statistics. HES gives aggregate figures on average waiting times of patients treated broken down by specialty which are freely available from the Department of Health.[8] The data are normally available with a lag of six months from the end of the financial year and include all the patients that were admitted for treatment during the financial year (therefore the data are recorded annually rather than quarterly, in contrast to the patients on the list). Only the average (and the median) waiting time of the patients treated is reported in the public domain (i.e. there is no information on the proportion of patients treated waiting more than i months). The last column of Table 7 compares our estimated measure of the average waiting time of the patients treated (column B)[9] with the actual average waiting time reported by HES (column C) for each specialty and year. For General Surgery and Urology, the difference is never higher than 5.2%. The difference is higher for Ophtalmology, where the actual waiting times for patients treated was even longer than our estimate. For example, in 2004-5, the average waiting time on the Opthalmology list was a mere 1.9 months; our estimate of the average waiting time of patients treated was 2.4 months; the HES data shows that the

[8] www.hesonline.nhs.uk/Ease/servlet/ContentServer?siteID=1937&categoryID=207.
[9] This is calculated as the average across the four quarters.

actual waiting time of patients treated averaged 2.9 months, 50% longer than the waiting time on the list. The main exception is Gynaecology, where the average waiting time of patients treated from HES data is lower than our estimate. If we aggregate over all six specialities, the average waiting time on the list was 3.14 months in 2004-05, and 2.67 months in 2005-06.

We would conclude that for some specialities, our estimate is a good one: for others it is less accurate. In all cases, it is a good "ball park" estimate, getting to within 21% of the actual value. In General Surgery, ENT and Urology our estimate was very accurate (within 5%). Recall, the two sources of data are completely different. To be able to get good ball park estimates for all specialities, and be very accurate for others is quite a surprising result.

Figures 1-6 here

Finally, Figures 1-6 plot for each specialty the hazard rates, i.e. the probability of being admitted in each period. To interpret these figures it is useful to remind that maximum waiting-times targets were set at six months during years 2004-05 and 2005-06.[10] We report the results only for six quarters, the four quarters of 2004-05 and the first two of 2005-06. We do not use the last two quarters for 2005-06 as the proportion of patients waiting more than six months is virtually zero across all quarters and specialities.

Note that the hazard rate is always one in the last period (eventually everyone gets treated). The hazard rate is generally constant or mildly decreasing during the first five months. Interestingly, the hazard rate has a 'peak' at 6 months. It increases between 5-6 months and it decreases between 6-7 months. Providers seem to increase effort as target approaches, and decrease effort if the patients are already passed the target. This echoes the effect found by Dimakou et al (2008) using the HES microdata of how the hazard rate responded to targets. However, our non-parametric estimates of the hazard rate are derived from cross-sectional aggregate waiting list data. It is reassuring that the hazard rates derived from the two different waiting-time sources suggest a similar behaviour of

[10] Maximum-waiting time targets was 9 months in 2003-04, 12 months in 2002-03, 15 months 2001-02, 18 months in 2000-01 and before.

the providers.

Table 8 uses data from year 2007-08: this measures waiting times in weeks, but does not provide a breakdown by speciality.[11] Table 8 aggregates the data across the four quarters of 2007-08. Weekly data are much more prone to "Christmas" like effects: any public holiday, industrial action or bad weather can affect admissions for a particular week, leading to a non-monotonicity in the list data. Furthermore, there is a "bureaucratic effect": it takes a few days or in some cases more than a week for a patient to appear on the list, at which point they appear on the list with a waiting time of over a week. This means that in nearly all quarters, the number of people on the list waiting for one week is less than those waiting for two or three weeks. This bureaucratic lag means that it is impossible to estimate the hazard for the first 2-3 weeks accurately from the list data. What we do in this case is set the hazard to zero for the first two weeks: to do this we set the number of patients on the waiting list equal to the number waiting for three weeks. As in the monthly data, there are very few patients left on the list after 28 weeks, so for the last 3 weeks, we take the average of the patients waiting between 28-30 weeks. The main results are qualitatively similar to those obtained above. The estimated proportion of patients *treated* waiting more than i weeks is higher than the proportion of patients *on the list* waiting more than i weeks. Moreover, the estimated average waiting time of the patients treated is 9.321 weeks which is higher than the average waiting time of the patients on the list (6.681 weeks). The average waiting time from HES data across all specialities was 8.35 weeks, which is 10.4% lower than our estimate, but still above the waiting time on the list.[12]

Finally, Figure 7 plots the hazard rate (probability of being treated) in the different weeks. We can see that the hazard jumps around a bit, with a peak at 20 weeks (by which time 98.7% of patients have been treated), and remains high for 23-26 weeks (the small

[11] We do not use data from year 2006-07 because data are available at specialty level but patients fall within the following catergories: between 1-13 weeks, 13-26 weeks, more than 26 weeks, which is very aggregate. We do not use 2008-09 because only quarter 1 is available.

[12] This comparison should be treated with some caution: the two datasets are quite different and we do not know in detail exactly how the "all specialities" category in the HES data compares with the list data.

numbers problem mean that at this stage the hazard estimates are less reliable).

Table 8 here

6 Conclusions

Waiting-time targets are used by policy makers to monitor performance. Such targets are based on the distribution of the patients on the list. We have compared and linked the distribution of waiting times of the patients on the list with the distribution of waiting time of the patients treated under the assumption of steady-state. We showed that if you know one you can retrieve the other. We also showed how the hazard rates, which give the probability of being treated in each period, can be derived from either one of the two distributions of waiting times. Depending on the hazard rate, the mean wait on the list could be larger or smaller than the mean wait of patients treated, depending on the relative importance of the interruption bias and the length bias.

We argued that the waiting time of patients treated is a better measure of welfare (or total disutility from waiting), as it refers to the full duration of waiting that patients experience. However, in practice it can only be calculated retrospectively (i.e. with a delay), and it is therefore of limited usefulness for monitoring current performance.

Our main contribution has been to show that under the assumption of steady state, the distribution of the patients treated can be estimated from the distribution of the patients on the list. This means that the instant snapshot taken of the waiting lists data can be used to estimate the waiting time of patients treated, which would otherwise only be available with the subsequent analysis of the HES data set many months later. We have found that for six specialities which cover at least 75% of the patients on the waiting list, the estimate of the mean waiting time of patients is accurate to within 20% of the actual ex post mean wait from the HES data; in four out of six specialities it was accurate to within 5%.

We have also shown that theoretically, depending on the hazard function, one distribution may be more or less favourable than the other. However, empirically we find that the proportion of patients waiting on the *list* more than x months is a downward estimate of the proportion of patients *treated* waiting more than x months, therefore biasing downwards the total disutility from waiting.

In subsequent work, our model might be used to develop comparative figures of waiting times at international level, where normally only one of the two distributions is available for each country. This would facilitate international comparative work on health policies and outcomes.

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8 Appendix. Proof of Proposition 2

Recall that:

$$f^L(1) = \frac{1}{w^{Tr}},\ f^L(2) = \frac{p_1}{w^{Tr}},\ f^L(3) = \frac{p_1 p_2}{w^{Tr}},\ f^L(4) = \frac{p_1 p_2 p_3}{w^{Tr}},\ ... \tag{21}$$

$$f^{Tr}(1) = (1-p_1),\ \ f^{Tr}(2) = p_1(1-p_2),\ \ f^{Tr}(3) = p_1 p_2 (1-p_3),\ \ f^{Tr}(4) = p_1 p_2 p_3 (1-p_4),\ ... \tag{22}$$

From the second we derive:

$$p_1 = 1-f^{Tr}(1),\ \ p_2 = \frac{1 - f^{Tr}(1) - f^{Tr}(2)}{1 - f^{Tr}(1)},\ \ p_3 = \frac{1 - f^{Tr}(1) - f^{Tr}(2) - f^{Tr}(3)}{1 - f^{Tr}(1) - f^{Tr}(2)},\ ... \tag{23}$$

Then,

$$f^L(1) = \frac{1}{w^{Tr}}, \tag{24}$$

$$f^L(2) = \frac{1 - f^{Tr}(1)}{w^{Tr}}, \tag{25}$$

$$f^L(3) = \frac{1 - f^{Tr}(1) - f^{Tr}(2)}{w^{Tr}} \tag{26}$$

$$f^L(4) = \frac{1 - f^{Tr}(1) - f^{Tr}(2) - f^{Tr}(3)}{w^{Tr}},\ ... \tag{27}$$

$$... \tag{28}$$

$$f^L(i) = \frac{1 - f^{Tr}(1) - f^{Tr}(2) - f^{Tr}(3) - ... - f^{Tr}(i-1)}{w^{Tr}},\ ... \tag{29}$$

which can be re-written more compactly as

$$f^L(1) = \frac{1}{w^{Tr}} \text{ and } f^L(i) = \frac{1 - \Sigma_{j=1}^{i-1} f^{Tr}(j)}{w^{Tr}} \text{ for } i > 1. \tag{30}$$

Table 1. Speciality: General Surgery

Proportion waiting more than i months	Quarter 1, 2004-2005		Quarter 2, 2004-2005		Quarter 3, 2004-2005		Quarter 4, 2004-2005	
	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$
1 month	70.4%	66.3%	69.0%	63.2%	72.8%	88.8%	71.0%	76.5%
2 months	50.7%	49.8%	49.4%	50.3%	48.7%	58.5%	48.9%	55.9%
3 months	36.0%	41.6%	33.7%	37.9%	32.8%	44.1%	32.7%	40.2%
4 months	23.6%	29.6%	22.0%	26.1%	20.8%	29.3%	21.1%	34.9%
5 months	14.8%	22.6%	13.9%	18.7%	12.9%	21.6%	11.0%	21.3%
6 months	8.2%	13.0%	8.1%	14.6%	7.0%	14.5%	4.8%	9.0%
7 months	4.3%	9.1%	3.5%	8.0%	3.0%	8.1%	2.2%	5.1%
8 months	1.6%	5.4%	1.1%	3.4%	0.8%	3.1%	0.7%	2.5%
9 months	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
average	2.595	2.874	2.506	2.723	2.489	3.180	2.424	2.954
	Quarter 1, 2005-2006		Quarter 2, 2005-2006		Quarter 3, 2005-2006		Quarter 4, 2005-2006	
1 month	68.1%	66.4%	66.9%	64.9%	70.2%	92.7%	66.8%	69.6%
2 months	47.0%	50.7%	45.4%	47.3%	42.6%	61.3%	43.6%	53.2%
3 months	30.8%	35.3%	29.7%	37.3%	24.3%	44.6%	26.0%	36.9%
4 months	19.5%	27.2%	17.4%	25.1%	11.0%	26.0%	13.7%	30.1%
5 months	10.8%	17.0%	9.1%	15.0%	3.3%	10.8%	3.7%	11.0%
6 months	5.4%	9.1%	4.1%	6.6%	0.1%	0.1%	0.1%	0.0%
7 months	2.5%	5.8%	1.9%	3.8%	0.0%	0.0%	0.0%	0.0%
8 months	0.7%	2.2%	0.6%	1.9%	0.0%	0.0%	0.0%	0.0%
9 months	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
average	2.349	2.637	2.251	2.519	2.016	2.856	2.037	2.508

Table 2. Speciality: Urology

Proportion waiting more than i months	Quarter 1, 2004-2005		Quarter 2, 2004-2005		Quarter 3, 2004-2005		Quarter 4, 2004-2005	
	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$	Patients on the list $(1-F^L(i))$	Patients on the list $(1-F^T(i))$	Patients treated $(1-F^L(i))$	Patients on the list $(1-F^T(i))$
1 month	66.2%	62.2%	62.9%	55.4%	67.8%	79.3%	65.9%	69.9%
2 months	45.2%	44.6%	42.3%	41.3%	42.4%	48.0%	42.1%	44.5%
3 months	30.2%	33.9%	26.9%	26.0%	26.9%	32.1%	27.0%	28.4%
4 months	18.8%	22.2%	17.3%	17.0%	16.6%	20.0%	17.3%	24.1%
5 months	11.3%	15.3%	11.0%	12.8%	10.2%	15.5%	9.1%	15.0%
6 months	6.1%	8.2%	6.2%	9.5%	5.2%	8.6%	4.0%	6.1%
7 months	3.3%	6.4%	2.7%	5.1%	2.4%	5.3%	2.0%	3.8%
8 months	1.2%	3.5%	0.8%	2.2%	0.7%	2.2%	0.7%	2.0%
9 months	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
average	2.323	2.462	2.201	2.193	2.222	2.610	2.182	2.437
	Quarter 1, 2005-2006		Quarter 2, 2005-2006		Quarter 3, 2005-2006		Quarter 4, 2005-2006	
1 month	63.7%	63.0%	61.7%	58.5%	65.9%	82.4%	60.6%	61.3%
2 months	40.9%	42.4%	39.3%	39.2%	37.7%	50.3%	36.5%	39.3%
3 months	25.5%	27.2%	24.3%	26.5%	20.5%	33.3%	21.0%	24.7%
4 months	15.6%	18.6%	14.2%	18.1%	9.2%	18.7%	11.2%	20.7%
5 months	8.9%	12.4%	7.2%	10.3%	2.8%	8.0%	3.1%	7.8%
6 months	4.4%	6.5%	3.2%	5.0%	0.1%	0.0%	0.0%	0.0%
7 months	2.0%	4.0%	1.3%	2.3%	0.0%	0.0%	0.0%	0.0%
8 months	0.5%	1.5%	0.4%	1.1%	0.0%	0.0%	0.0%	0.0%
9 months	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
average	2.115	2.255	2.017	2.111	1.862	2.430	1.825	2.039

Table 3. Speciality: Trauma and orthopaedics

Proportion waiting more than i months	Quarter 1, 2004-2005		Quarter 2, 2004-2005		Quarter 3, 2004-2005		Quarter 4, 2004-2005	
	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$	Patients on the list $(1-F^L(i))$	Patients on the list $(1-F^T(i))$	Patients treated $(1-F^L(i))$	Patients on the list $(1-F^T(i))$
1 month	82.0%	95.9%	80.8%	87.2%	80.3%	100.0%	80.3%	100.0%
2 months	64.6%	82.6%	64.1%	82.0%	60.7%	78.8%	60.4%	88.4%
3 months	49.7%	79.4%	48.4%	72.5%	45.2%	69.9%	43.0%	69.2%
4 months	35.4%	62.4%	34.5%	58.3%	31.4%	51.1%	29.4%	66.7%
5 months	24.1%	52.1%	23.4%	44.1%	21.4%	42.8%	16.3%	44.9%
6 months	14.7%	36.0%	14.9%	39.6%	12.9%	33.4%	7.4%	19.5%
7 months	8.2%	29.9%	7.3%	25.4%	6.4%	22.3%	3.6%	11.5%
8 months	2.9%	15.8%	2.5%	12.8%	2.0%	10.1%	1.3%	6.6%
9 months	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
average	3.317	5.041	3.260	4.718	3.102	4.583	2.916	4.581
	Quarter 1, 2005-2006		Quarter 2, 2005-2006		Quarter 3, 2005-2006		Quarter 4, 2005-2006	
1 month	78.2%	90.9%	76.8%	88.6%	75.3%	100.0%	76.2%	97.2%
2 months	58.4%	80.9%	56.2%	74.5%	50.7%	79.2%	53.0%	80.8%
3 months	40.8%	63.3%	38.9%	63.2%	31.1%	64.4%	33.7%	61.1%
4 months	27.0%	51.4%	24.3%	46.4%	15.3%	42.0%	19.1%	56.6%
5 months	15.8%	35.7%	13.5%	29.6%	4.9%	18.6%	5.6%	22.3%
6 months	8.0%	18.8%	6.6%	14.8%	0.3%	0.4%	0.3%	0.6%
7 months	3.9%	12.9%	3.2%	9.2%	0.2%	0.4%	0.2%	0.3%
8 months	1.1%	5.2%	1.1%	4.6%	0.1%	0.4%	0.1%	0.3%
9 months	0.000	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
average	2.833	4.090	2.706	3.809	2.279	3.554	2.381	3.693

Table 4. Speciality: Ear, Nose and Throat

Proportion waiting more than i months	Quarter 1, 2004-2005		Quarter 2, 2004-2005		Quarter 3, 2004-2005		Quarter 4, 2004-2005	
	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$	Patients on the list $(1-F^L(i))$	Patients on the list $(1-F^T(i))$	Patients treated $(1-F^L(i))$	Patients on the list $(1-F^T(i))$
1 month	75.2%	78.1%	74.9%	75.0%	77.2%	100.0%	73.6%	85.9%
2 months	55.9%	63.4%	56.2%	63.8%	54.4%	71.0%	51.0%	61.0%
3 months	40.1%	55.0%	40.2%	53.3%	38.2%	54.9%	34.9%	43.6%
4 months	26.5%	38.6%	26.8%	38.0%	25.7%	39.7%	23.4%	39.2%
5 months	16.9%	29.9%	17.3%	28.8%	16.6%	31.3%	13.1%	26.8%
6 months	9.5%	17.9%	10.1%	23.1%	9.5%	22.9%	6.0%	12.1%
7 months	5.1%	14.0%	4.3%	12.4%	4.3%	13.4%	2.8%	6.7%
8 months	1.6%	6.5%	1.2%	4.8%	1.2%	5.3%	1.0%	3.9%
9 months	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
average	2.809	3.534	2.810	3.492	2.770	3.883	2.558	3.293
	Quarter 1, 2005-2006		Quarter 2, 2005-2006		Quarter 3, 2005-2006		Quarter 4, 2005-2006	
1 month	71.3%	76.0%	72.1%	78.7%	72.9%	100.0%	69.6%	79.9%
2 months	49.4%	59.7%	50.1%	59.9%	45.9%	69.0%	45.3%	59.4%
3 months	32.3%	40.9%	33.3%	48.3%	27.2%	52.2%	27.3%	42.6%
4 months	20.5%	31.1%	19.8%	33.0%	13.1%	33.6%	14.3%	34.2%
5 months	11.6%	19.6%	10.6%	20.1%	4.0%	14.7%	3.9%	12.7%
6 months	5.9%	11.0%	5.0%	9.9%	0.1%	0.1%	0.1%	0.1%
7 months	2.8%	7.0%	2.2%	5.7%	0.1%	0.1%	0.0%	0.1%
8 months	0.8%	2.7%	0.6%	2.2%	0.0%	0.1%	0.0%	0.0%
9 months	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
average	2.445	2.981	2.436	3.079	2.134	3.197	2.105	2.790

Table 5. Speciality: Ophthalmology

Proportion waiting more than i months	Quarter 1, 2004-2005		Quarter 2, 2004-2005		Quarter 3, 2004-2005		Quarter 4, 2004-2005	
	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$	Patients on the list $(1-F^L(i))$	Patients on the list $(1-F^T(i))$	Patients treated $(1-F^L(i))$	Patients on the list $(1-F^T(i))$
1 month	70.3%	80.4%	63.6%	73.2%	64.4%	100.0%	60.2%	81.2%
2 months	46.3%	61.5%	36.9%	50.8%	28.8%	48.1%	27.9%	43.5%
3 months	28.0%	44.8%	18.4%	24.3%	11.7%	14.8%	10.6%	11.2%
4 months	14.7%	26.4%	9.5%	12.0%	6.5%	8.0%	6.2%	8.4%
5 months	6.8%	14.0%	5.1%	7.5%	3.6%	5.4%	2.8%	4.4%
6 months	2.7%	4.7%	2.4%	4.1%	1.7%	2.9%	1.1%	1.6%
7 months	1.3%	3.2%	0.9%	1.8%	0.7%	1.3%	0.4%	0.6%
8 months	0.3%	1.1%	0.3%	0.7%	0.2%	0.7%	0.2%	0.5%
9 months	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
average	2.205	2.862	1.869	2.244	1.677	2.311	1.595	2.013
	Quarter 1, 2005-2006		Quarter 2, 2005-2006		Quarter 3, 2005-2006		Quarter 4, 2005-2006	
1 month	58.3%	72.1%	59.0%	74.3%	64.3%	100.0%	58.7%	74.2%
2 months	28.2%	42.1%	28.6%	42.0%	28.6%	51.2%	28.0%	40.6%
3 months	10.6%	11.9%	11.4%	14.0%	10.3%	17.1%	11.2%	14.1%
4 months	5.6%	7.1%	5.7%	7.4%	4.2%	8.4%	5.4%	9.9%
5 months	2.7%	3.9%	2.6%	4.0%	1.1%	3.1%	1.3%	3.1%
6 months	1.1%	1.5%	1.0%	1.4%	0.0%	0.0%	0.0%	0.0%
7 months	0.4%	0.8%	0.4%	0.6%	0.0%	0.0%	0.0%	0.0%
8 months	0.1%	0.2%	0.1%	0.3%	0.0%	0.0%	0.0%	0.0%
9 months	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
average	1.569	1.896	1.588	1.941	1.584	2.299	1.546	1.919

Table 6. Speciality: Gynaecology

Proportion waiting more than i months	Quarter 1, 2004-2005		Quarter 2, 2004-2005		Quarter 3, 2004-2005		Quarter 4, 2004-2005	
	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$	Patients on the list $(1-F^L(i))$	Patients treated $(1-F^T(i))$
1 month	67.5%	66.3%	65.6%	60.2%	70.1%	87.9%	68.2%	73.4%
2 months	45.9%	47.4%	44.9%	46.4%	43.8%	53.2%	44.8%	50.7%
3 months	30.4%	36.6%	28.9%	32.6%	27.9%	38.4%	28.7%	34.0%
4 months	18.5%	23.2%	17.7%	21.0%	16.4%	22.0%	17.9%	28.3%
5 months	11.0%	16.4%	10.5%	13.6%	9.9%	16.0%	8.8%	15.9%
6 months	5.7%	9.0%	5.8%	9.8%	5.1%	9.9%	3.8%	6.8%
7 months	2.7%	6.0%	2.5%	5.2%	2.1%	5.3%	1.6%	3.4%
8 months	0.8%	2.4%	0.7%	2.0%	0.6%	1.9%	0.5%	1.7%
9 months	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
average	2.325	2.573	2.266	2.408	2.260	2.846	2.243	2.643
	Quarter 1, 2005-2006		Quarter 2, 2005-2006		Quarter 3, 2005-2006		Quarter 4, 2005-2006	
1 month	65.2%	65.3%	64.0%	64.6%	68.5%	91.4%	64.0%	67.3%
2 months	42.4%	45.3%	40.7%	43.5%	39.6%	55.8%	39.8%	47.9%
3 months	26.6%	30.0%	25.0%	31.2%	22.0%	38.8%	22.6%	31.3%
4 months	16.2%	20.7%	13.8%	20.0%	9.8%	22.7%	11.4%	23.9%
5 months	9.0%	13.0%	6.6%	10.8%	2.6%	8.2%	2.8%	7.6%
6 months	4.4%	6.7%	2.7%	4.3%	0.0%	0.1%	0.0%	0.0%
7 months	2.1%	4.4%	1.2%	2.4%	0.0%	0.0%	0.0%	0.0%
8 months	0.6%	1.6%	0.3%	0.9%	0.0%	0.0%	0.0%	0.0%
9 months	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
average	2.164	2.370	2.044	2.276	1.927	2.671	1.907	2.280

Table 7. Comparison of average waiting						
		Patients on the list	Patients treated (estimated)	Difference	Patients treated (HES)	Difference
		A	B	(B-A)/A	C	(C-B)/B
General surgery	2004-2005	2.504	2.932	17.1%	2.800	-4.53%
	2005-2006	2.166	2.636	21.4%	2.633	0.11%
Urology	2004-2005	2.232	2.426	8.7%	2.300	-5.17%
	2005-2006	1.957	2.209	12.9%	2.200	-0.40%
Trauma and orthopaedics	2004-2005	3.180	4.731	48.8%	4.867	2.88%
	2005-2006	2.580	3.787	46.8%	4.367	15.33%
Ear, nose and throat	2004-2005	2.741	3.551	29.5%	3.500	-1.42%
	2005-2006	2.286	3.012	31.7%	3.200	6.25%
Ophthalmology	2004-2005	1.867	2.358	26.3%	2.867	21.61%
	2005-2006	1.572	2.014	28.1%	2.400	19.18%
Gynaecology	2004-2005	2.273	2.618	15.2%	2.233	-14.69%
	2005-2006	2.013	2.399	19.2%	2.133	-11.10%

Table 8. Waiting times with weekly data (year 2007-08)				
Weeks Waited	Patients on the list	Proportion of patients on the list waiting i weeks	Proportion of patients on the list waiting more than i weeks	Proportion of patients treated waiting more than i weeks
1	67168	10.73%	89.3%	100.0%
2	67168	10.73%	78.5%	100.0%
3	67168	10.73%	67.8%	92.8%
4	62317	9.96%	57.9%	74.2%
5	49837	7.96%	49.9%	71.5%
6	48028	7.67%	42.2%	62.8%
7	42194	6.74%	35.5%	52.9%
8	35532	5.68%	29.8%	49.4%
9	33171	5.30%	24.5%	41.8%
10	28108	4.49%	20.0%	36.3%
11	24403	3.90%	16.1%	27.7%
12	18612	2.97%	13.1%	23.5%
13	15769	2.52%	10.6%	19.0%
14	12729	2.03%	8.6%	17.9%
15	12015	1.92%	6.7%	15.4%
16	10329	1.65%	5.0%	12.4%
17	8340	1.33%	3.7%	9.4%
18	6307	1.01%	2.7%	7.2%
19	4861	0.78%	1.9%	5.6%
20	3758	0.60%	1.3%	3.0%
21	2045	0.33%	1.0%	2.7%
22	1815	0.29%	0.7%	2.1%
23	1438	0.23%	0.5%	1.8%
24	1181	0.19%	0.3%	1.2%
25	832	0.13%	0.1%	0.8%
26	546	0.09%	0.0%	0.1%
27	81	0.01%	0.0%	0.1%
28	76	0.01%	0.0%	0.1%
29	76	0.01%	0.0%	0.1%
30	76	0.01%	0.0%	0.1%

Figure 1. General surgery

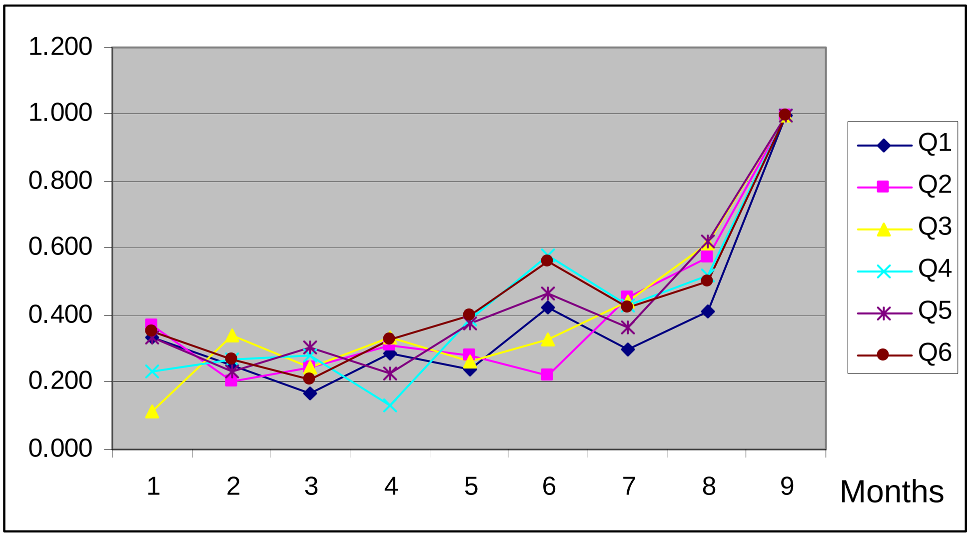


Figure 2. Urology

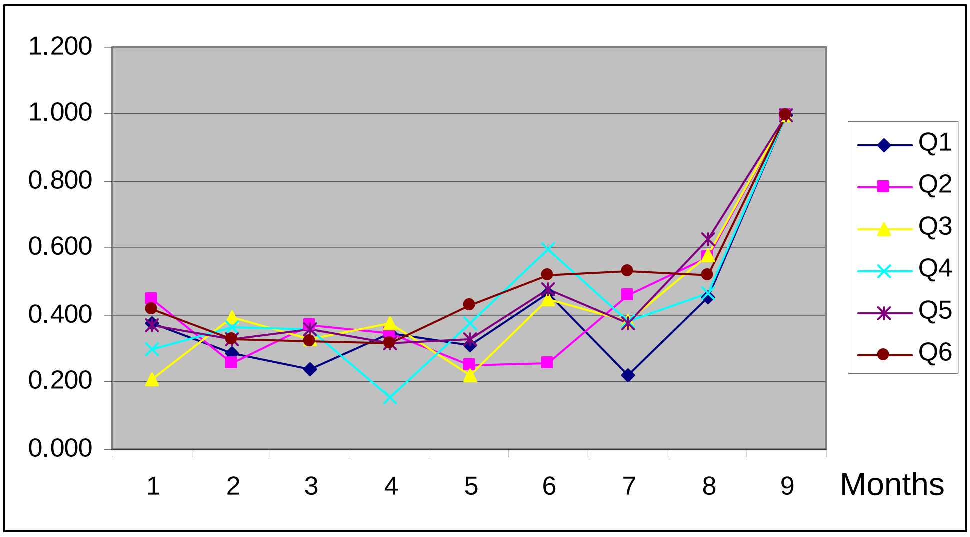


Figure 3. Trauma and Orthopaedics

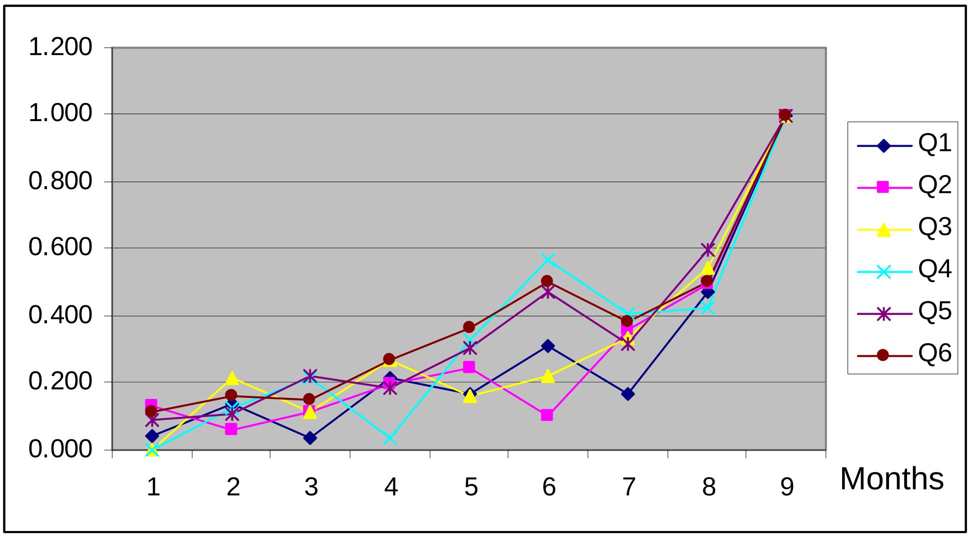


Figure 4. Ear, Nose and Throat

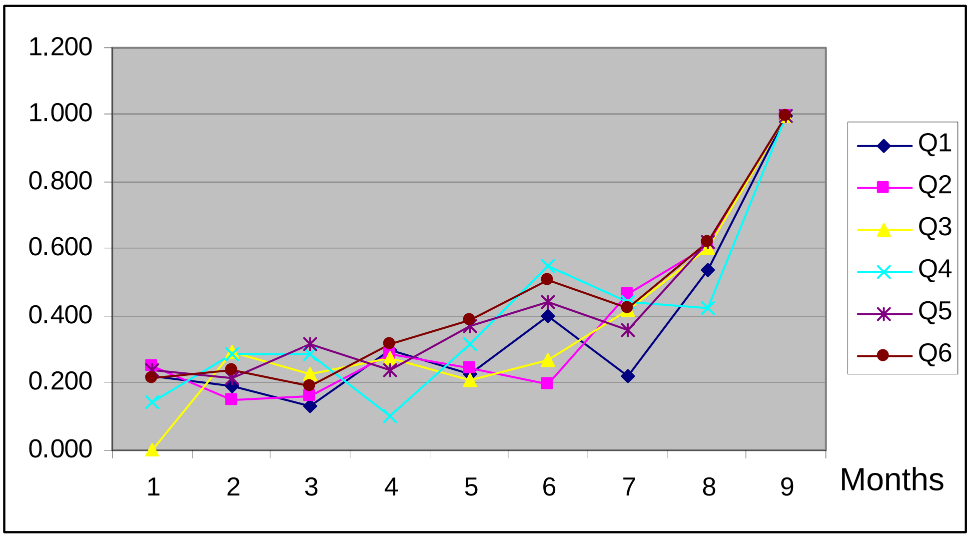


Figure 5. Ophthalmology

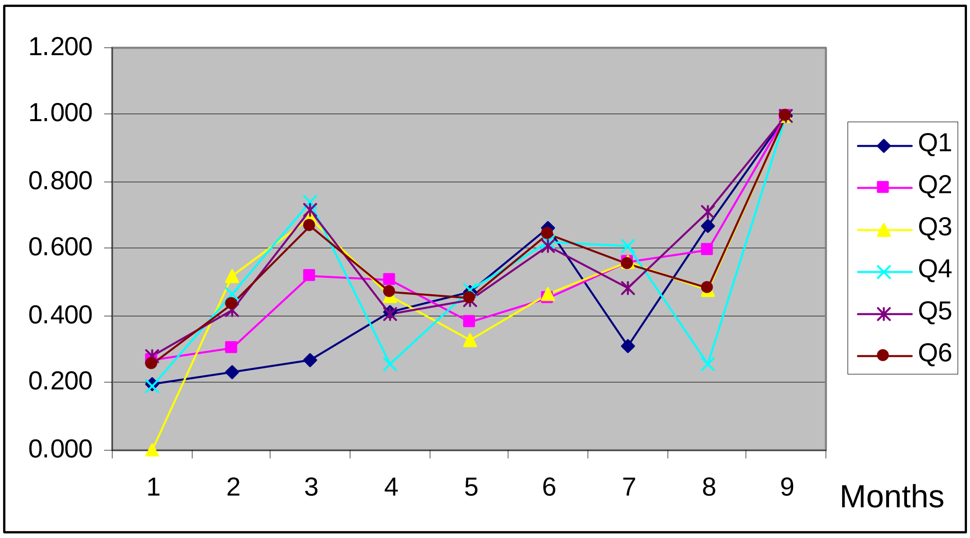


Figure 6. Gynaecology

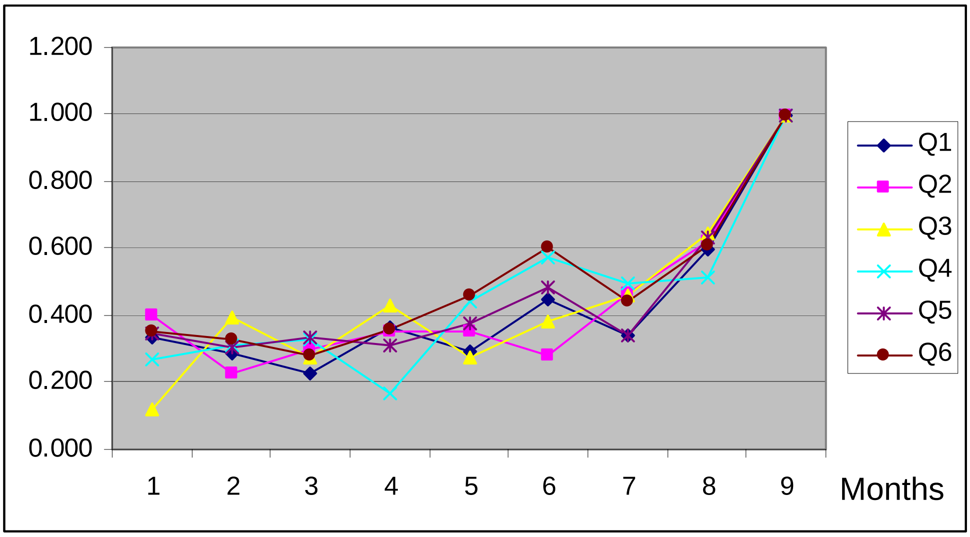
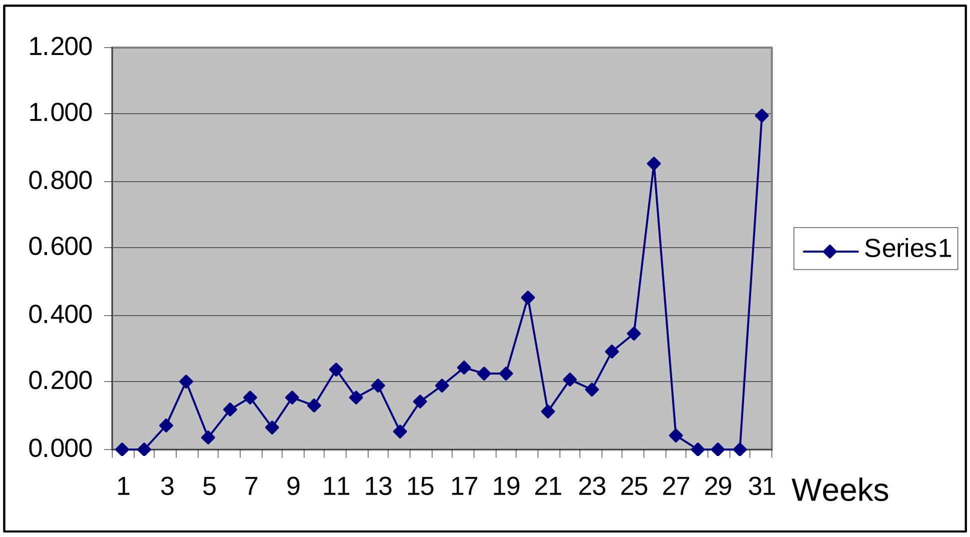


Figure 7.


1.200
1.000
0.800
0.600
0.400
0.200
0.000
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31
Weeks
Series1

Appendix

Table A1. Patients waiting for admission by month waiting (England)

Specialty and year	Quarter	Total Number of patients waiting	Less than 1 Month	1 to <2 months	2 to <3 months	3 to <4 months	4 to <5 months	5 to <6 months	6 to <7 months	7 to <8 months	>8 months
General Surgery 20004/05	1	148853	44120	29250	21965	18358	13064	9960	5749	4026	2361
	2	145189	45042	28476	22673	17093	11753	8435	6585	3591	1541
	3	149160	40537	35982	23695	17883	11887	8769	5864	3281	1262
	4	144853	41937	32085	23458	16868	14639	8929	3767	2139	1031
General Surgery 20005/06	1	140653	44842	29757	22747	15836	12215	7625	4069	2588	974
	2	137536	45551	29558	21530	16978	11446	6849	3016	1741	867
	3	136090	40554	37582	24846	18087	10554	4379	28	8	52
	4	132944	44191	30748	23501	16285	13299	4849	16	19	36
Urology 20004/05	1	63429	21415	13324	9542	7251	4751	3287	1752	1364	743
	2	62212	23102	12800	9550	6011	3927	2949	2189	1181	503
	3	63306	20354	16137	9767	6542	4067	3158	1747	1081	453
	4	63170	21512	15033	9565	6115	5177	3221	1307	809	431
Urology 20005/06	1	63482	23039	14504	9766	6256	4279	2864	1493	932	349
	2	60493	23165	13545	9074	6147	4201	2397	1158	544	262
	3	59511	20314	16745	10227	6764	3798	1632	9	5	17
	4	60094	23671	14505	9312	5847	4904	1836	5	2	12
Trauma and Orthopaedics 20004/05	1	236630	42703	40952	35269	33908	26655	22246	15385	12768	6744
	2	233533	44754	39009	36685	32441	26087	19718	17744	11349	5746
	3	233592	38653	47744	37609	33363	24384	20437	15936	10633	4833
	4	215507	42415	42919	37507	29340	28298	19055	8271	4890	2812
Trauma and Orthopaedics 20005/06	1	211976	46178	41981	37378	29214	23747	16465	8686	5940	2387
	2	206353	47887	42440	35670	30267	22214	14186	7109	4398	2182
	3	200122	40858	52149	41304	33563	21911	9707	153	120	357
	4	199884	47670	46338	38541	29109	27003	10607	291	122	203
Ear, Nose and Throat 20004/05	1	83066	20590	16076	13051	11319	7956	6165	3688	2883	1338
	2	76917	19270	14452	12286	10270	7326	5544	4458	2382	929
	3	74160	16904	16927	11994	9287	6714	5297	3881	2261	895
	4	72304	19064	16371	11635	8311	7479	5105	2314	1285	740
Ear, Nose and Throat 20005/06	1	72985	20968	15942	12523	8581	6530	4108	2300	1477	556
	2	69202	19336	15226	11581	9335	6389	3892	1920	1105	418
	3	65223	17581	17667	12186	9216	5930	2589	13	11	30
	4	68382	20784	16607	12342	8859	7110	2642	13	6	19
Ophthalmology 20004/05	1	93072	27685	22252	17033	12416	7314	3875	1302	897	298
	2	79005	28796	21071	14635	7002	3469	2148	1168	511	205
	3	70400	24741	25217	12130	3728	2015	1357	727	318	167
	4	69088	27491	22315	11945	3083	2296	1199	452	176	131
Ophthalmology 20005/06	1	70856	29573	21328	12457	3506	2090	1150	452	233	67
	2	70897	29047	21575	12203	4057	2145	1168	418	187	97
	3	70345	25060	25169	12887	4301	2125	792	3	1	7
	4	69493	28724	21322	11660	4039	2851	878	7	3	9
Gynaecology 20004/05	1	73682	23975	15904	11372	8766	5562	3928	2165	1431	579
	2	72441	24913	15010	11565	8115	5237	3388	2432	1293	488
	3	73705	22030	19371	11713	8469	4853	3519	2170	1168	412
	4	72644	23114	16973	11728	7861	6551	3674	1570	789	384
Gynaecology 20005/06	1	70959	24728	16137	11198	7421	5118	3203	1662	1092	400
	2	68508	24677	15948	10729	7690	4929	2657	1057	591	230
	3	67341	21239	19408	11850	8243	4829	1740	14	2	16
	4	67957	24441	16439	11700	7655	5830	1861	8	7	16

Table A2. Patients waiting for admission by month waiting (England)											
Specialty and year	Quarter	Total Number of patients waiting	Less than 1 Month	1 to <2 months	2 to <3 months	3 to <4 months	4 to <5 months	5 to <6 months	6 to <7 months	7 to <8 months	>8 months
General Surgery 20004/05	1	148853	44120	29250	21965	18358	13064	9960	5749	4026	2361
	2	145189	45042	28476	22673	17093	11753	8435	6585	3591	1541
	3	149160	40537	35982	23695	17883	11887	8769	5864	3281	1262
	4	144853	41937	32085	23458	16868	14639	8929	3767	2139	1031
General Surgery 20005/06	1	140653	44842	29757	22747	15836	12215	7625	4069	2588	974
	2	137536	45551	29558	21530	16978	11446	6849	3016	1741	867
	3	136090	40554	37582	24846	18087	10554	4379	**29**	**29**	**29**
	4	132944	44191	30748	23501	16285	13299	4849	**24**	**24**	**24**
Urology 20004/05	1	63429	21415	13324	9542	7251	4751	3287	1752	1364	743
	2	62212	23102	12800	9550	6011	3927	2949	2189	1181	503
	3	63306	20354	16137	9767	6542	4067	3158	1747	1081	453
	4	63170	21512	15033	9565	6115	5177	3221	1307	809	431
Urology 20005/06	1	63482	23039	14504	9766	6256	4279	2864	1493	932	349
	2	60493	23165	13545	9074	6147	4201	2397	1158	544	262
	3	59511	20314	16745	10227	6764	3798	1632	**10**	**10**	**10**
	4	60094	23671	14505	9312	5847	4904	1836	**6**	**6**	**6**
Trauma and Orthopaedics 20004/05	1	236630	42703	40952	35269	33908	26655	22246	15385	12768	6744
	2	233533	44754	39009	36685	32441	26087	19718	17744	11349	5746
	3	**242683**	**47744**	47744	37609	33363	24384	20437	15936	10633	4833
	4	**216011**	**42919**	42919	37507	29340	28298	19055	8271	4890	2812
Trauma and Orthopaedics 20005/06	1	211976	46178	41981	37378	29214	23747	16465	8686	5940	2387
	2	206353	47887	42440	35670	30267	22214	14186	7109	4398	2182
	3	**211413**	**52149**	52149	41304	33563	21911	9707	**210**	**210**	**210**
	4	199884	47670	46338	38541	29109	27003	10607	291	**163**	**163**
Ear, Nose and Throat 20004/05	1	83066	20590	16076	13051	11319	7956	6165	3688	2883	1338
	2	76917	19270	14452	12286	10270	7326	5544	4458	2382	929
	3	**74183**	**16927**	16927	11994	9287	6714	5297	3881	2261	895
	4	72304	19064	16371	11635	8311	7479	5105	2314	1285	740
Ear, Nose and Throat 20005/06	1	72985	20968	15942	12523	8581	6530	4108	2300	1477	556
	2	69202	19336	15226	11581	9335	6389	3892	1920	1105	418
	3	**65309**	**17667**	17667	12186	9216	5930	2589	**18**	**18**	**18**
	4	68382	20784	16607	12342	8859	7110	2642	**13**	**13**	**13**
Ophthalmology 20004/05	1	93072	27685	22252	17033	12416	7314	3875	1302	897	298
	2	79005	28796	21071	14635	7002	3469	2148	1168	511	205
	3	**70876**	**25217**	25217	12130	3728	2015	1357	727	318	167
	4	69088	27491	22315	11945	3083	2296	1199	452	176	131
Ophthalmology 20005/06	1	70856	29573	21328	12457	3506	2090	1150	452	233	67
	2	70897	29047	21575	12203	4057	2145	1168	418	187	97
	3	**70454**	**25169**	25169	12887	4301	2125	792	**4**	**4**	**4**
	4	69493	28724	21322	11660	4039	2851	878	**6**	**6**	**6**
Gynaecology 20004/05	1	73682	23975	15904	11372	8766	5562	3928	2165	1431	579
	2	72441	24913	15010	11565	8115	5237	3388	2432	1293	488
	3	73705	22030	19371	11713	8469	4853	3519	2170	1168	412
	4	72644	23114	16973	11728	7861	6551	3674	1570	789	384
Gynaecology 20005/06	1	70959	24728	16137	11198	7421	5118	3203	1662	1092	400
	2	68508	24677	15948	10729	7690	4929	2657	1057	591	230
	3	67341	21239	19408	11850	8243	4829	1740	14	**9**	**9**
	4	67957	24441	16439	11700	7655	5830	1861	**10**	**10**	**10**