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ABSTRACT

On the International Dimension of Fiscal Policy

This paper analyses the international dimension of fiscal policy using a small open economy framework in which the government finances its spending by levying distortionary taxation and issuing non-state-contingent debt. The main finding of the paper is that, once the open economy aspect of the policy problem is considered, it is not optimal to smooth taxes following idiosyncratic shocks. Even when prices are flexible and inflation can costlessly act as a shock absorber to restore fiscal equilibrium, the presence of a terms of trade externality lead to movements in the tax rate. Also in contrast with the closed economy, the introduction of sticky prices, can reduce the optimal volatility of taxes.

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1 Introduction¹

This paper aims at analyzing the international dimension of fiscal policy. The neoclassical literature on optimal fiscal policy has focused mainly on closed economy models suggesting that, when taxes are distortionary, welfare would be maximized if taxes are smoothed over time and across states of nature (see Barro, 1979 and Lucas and Stokey, 1983). In these models, if possible, taxes would be essentially invariant (see Lucas and Stokey, 1983 and Chari, Christiano and Kehoe, 1991) or would follow a random walk (see Barro, 1979, Aiyagari et al. 2002).

Our analysis shows that in a small open economy optimal policy departs from tax smoothing. As emphasised in Corsetti and Pesenti (2001) and Obstfeld and Rogoff (1998), policymakers in open economies are influenced by a “terms of trade externality”. In an open economy that is a monopolist producer of its own goods, a real exchange rate appreciation can lead to higher welfare by allowing domestic agents to consume more for lower levels of domestic production. Thus, policymakers have an incentive to use fiscal policy to exploit this externality. As a result, distortionary taxes vary over time and across states of nature under the optimal plan.

The afore-mentioned incentive is similar to the one discussed in the recent international monetary policy literature. Considering a model in which taxes are lump sum, Tille (2002), Benigno and Benigno (2003), Sutherland (2005), De Paoli (2008), among others, find that optimal policy deviates from price stability in order to allow policymakers to strategically manage the terms of trade. That is, in the monetary stabilization case, inflation varies over time and across states of nature. Our analysis also explores the implications of the terms of trade externality for the behavior of inflation when both fiscal and monetary stabilization issues are present.

Our work is also related to recent contributions that analyse monetary and fiscal policy interactions in open economies. Beetsma and Jensen (2005) study these interactions in a two-country monetary union model in which public spending delivers utility to the consumer but taxes are lump sum. Similar assumptions are considered in Gali and Monacelli (2008), who examine a continuum of small economies in a currency union setting. Lombardo and Sutherland (2004) investigate the costs and benefits from fiscal cooperation in a two-period version similar in the structure as in Beetsma and Jensen (2004). Ferrero (2008) presents a currency union model in which lump sum taxes are not available to fiscal authorities and derives the optimal plan under commitment for the fiscal and monetary stabilization problem. In another interesting work revisiting the issues in the optimal currency area literature, Adao, Correia and Teles (2006) examine the implications of the choice of exchange rate regimes for fiscal policy. In contrast to these studies, our work concentrates on the analysis of a small open economy, in which strategic interactions between countries can be dismissed.

Our small open economy is characterized by a set of standard assumptions. Firms are monopolistic producers of differentiated goods and the economy is perfectly integrated with the rest of the world. Indeed there are no trade frictions (i.e. the law of one price holds) and capital markets are perfect (i.e. asset markets are complete). Following the recent contributions by Benigno and Woodford (2003), Schmitt-Grohe and Uribe (2004) and Siu (2004) we allow for distortionary income taxation and restrict

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government debt to one period nominal or real riskless bonds.²

In order to highlight the fiscal dimension of the optimal policy problem, our analysis focuses initially on the case of flexible prices. In this setting, policymakers in the small open economy face different policy incentives. On the one hand, there is an incentive to avoid distortions on households' consumption-leisure choice caused by fluctuations in income taxes. And on the other hand, there is an incentive to exploit the "terms of trade externality" by varying income taxes. This is because movements in the level of distortionary taxation may induce a real exchange rate appreciation, which, as mentioned above, may lower domestic disutility of production without a corresponding decline in the utility of consumption.³ Thus, while in a closed economy tax smoothing is optimal; in an open economy varying taxes may improve welfare.

Once we allow for sticky prices, inflation generates inefficiency in the allocation of resources. In this case, both inflation and taxes create distortions in agents' consumption-leisure decisions. But, for this exact reason, inflation, like taxes, can also be used to exploit the terms of trade externality. So the introduction of price rigidities could reduce the variability of taxes under the optimal plan.

Our quantitative results show that, compared with taxes, the optimal volatility of inflation is significantly lower, suggesting that the cost of inflation overshadows the inefficiency caused by taxation. This result is an outcome of the trade-off between price stability and tax smoothing that arises from the presence of the terms of trade externality. Thus, it is not an outcome of a trade-off (emphasized by the Benigno and Woodford (2003), Schmitt-Grohe and Uribe (2004) and Siu (2004)) between price stability and tax smoothing that arises when inflation can be used as a fiscal stabilizer, i.e., when the government issues nominal bonds. Our quantitative results hold independently of the denomination of government bonds.

In terms of the methodology, we follow a linear-quadratic approach (as proposed by Benigno and Woodford (2003) and Sutherland (2002)) and characterize a utility-based loss function for a small open economy. Our work encompasses as special cases the closed economy framework (Benigno and Woodford, 2003) and the small open economy case in which there are endogenous lump sum taxes (De Paoli, 2008). Under flexible prices the loss function can be written as a quadratic expression of output and real exchange rate gaps. With price rigidities, the variability of domestic producer inflation also affects welfare.

The linear quadratic approach also allows us to derive simple policy rules that describe the optimal state-contingent responses to shocks. We do so by deriving targeting rules *a la* Svensson (2003). In particular, the optimal plan is composed by two rules: one that specifies targeting a linear combination of domestic producer inflation, domestic output growth and changes in the real exchange rate and another that seeks to stabilize expected producer inflation to zero.

The remainder of the paper is structured as follows. Section 2 describes the structure of the model; section 3 present the model in log-linear approximation; Section 4 discusses the policy problem while the analysis of the optimal policy plan is conducted in Section 5. Section 6 concludes.

²Our approach, however, does not consider state-contingent debt and alternative forms of distortionary taxes, as in Correia et al. (2003). These authors assess how this rich set of policy instruments could lead to equivalence between the flexible-price and the sticky-price equilibrium.

³The validity of this claim depends on the specified values for the structural parameter (in particular the elasticity of intertemporal and intratemporal substitution).

2 Model

2.1 Household behavior

We consider a two-country framework, a small open economy and the rest of the world. Each country is populated by agents who consume a basket of goods consisting of home and foreign produced goods. The model follows closely the one proposed by De Paoli (2008) and Gali and Monacelli (2005) for the case in which there is no fiscal policy stabilization problem. We consider a very simple small open economy model in which markets are complete and producer currency pricing holds.

There is a measure n of agents in our small open economy, that have the following utility function:

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} [U(C_s^j) - V(y_s(h), \varepsilon_{Y,s})] \quad (1)$$

Households obtain utility from consumption $U(C)$ and contribute to the production of a differentiated good $y(h)$ attaining disutility $V(y(h), \varepsilon_Y)$. Productivity shocks are denoted by $\varepsilon_{Y,s}$. We abstract from any monetary frictions by considering a cashless economy as in Woodford (2003, chapter 2). The consumption index C is a Dixit-Stiglitz aggregator of home and foreign goods as

$$C = \left[v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

where $\theta > 0$ is the intratemporal elasticity of substitution and C_H and C_F are the two consumption sub-indexes that refer, respectively, to the consumption of home-produced and foreign-produced goods. v is a function of the relative size of the small economy with respect to the rest of the world, n , and of the degree of openness, λ : $(1-v) = (1-n)\lambda$.

Similar preferences are specified for the rest of the world:

$$C = \left[v^*{}^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v^*)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (2)$$

with $v^* = n\lambda$.

Note that the specification of v and v^* gives rise to home bias in consumption, as in Sutherland (2002). The size of the bias decreases with degree of openness λ . Moreover, the proportion of foreign-produced goods in Home consumption preferences is proportional to Foreign size and the proportion of home-produced goods in Foreign consumption preferences is proportional to Home size.

We have

$$C_H = \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, C_F = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}} \quad (3)$$

where $\sigma > 1$ is the elasticity of substitution for goods produced within a country. The consumption-based index for the small open economy that corresponds to the above specifications of preferences is given by:

$$P = \left[v P_H^{1-\theta} + (1-v) (P_F)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \theta > 0 \quad (4)$$

where P_H is the price sub-index for home-produced goods expressed in the domestic currency and P_F is the price sub-index for foreign produced goods expressed in the domestic currency.

$$P_H = \left[\left(\frac{1}{n} \right) \int_0^n p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, P_F = \left[\left(\frac{1}{1-n} \right) \int_n^1 p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}} \quad (5)$$

The law of one price holds: $p(h) = Sp^*(h)$ and $p(f) = Sp^*(f)$, where S is the nominal exchange rate (the price of foreign currency in terms of domestic currency). We define the real exchange rate as $RS = \frac{SP^*}{P}$.

We assume, as in Chari et al. (2002), that markets are complete domestically and internationally. As a result, marginal utilities of income are equalized across countries at all times and states of nature:

$$\frac{U_C(C_{t+1}^*)}{U_C(C_t^*)} \frac{P_t^*}{P_{t+1}^*} = \frac{U_C(C_{t+1})}{U_C(C_t)} \frac{S_{t+1}P_t}{S_tP_{t+1}} \quad (6)$$

Given our preference specification, the total demands of the generic good h , produced in country H, and of the good f , produced in country F, are respectively:

$$y^d(h) = \left[\frac{p(h)}{P_H} \right]^{-\sigma} \left\{ \left[\frac{P_H}{P} \right]^{-\theta} \left[vC + \frac{v^*(1-n)}{n} \left(\frac{1}{RS} \right)^{-\theta} C^* \right] + G \right\}$$

$$y^d(f) = \left[\frac{p(f)}{P_F} \right]^{-\sigma} \left\{ \left[\frac{P_F}{P} \right]^{-\theta} \left[\frac{(1-v)n}{1-n} C + (1-v^*) \left(\frac{1}{RS} \right)^{-\theta} C^* \right] + G^* \right\}$$

where G and G^* are country-specific government purchase shocks.

To characterize our small open economy we use the definition of v and v^* and take the limit for $n \rightarrow 0$, so that

$$y^d(h) = \left[\frac{p(h)}{P_H} \right]^{-\sigma} \left\{ \left[\frac{P_H}{P} \right]^{-\theta} \left[(1-\lambda)C + \lambda \left(\frac{1}{RS} \right)^{-\theta} C^* \right] + G \right\} \quad (7)$$

$$y^d(f) = \left[\frac{p(f)}{P_F} \right]^{-\sigma} \{C^* + G^*\}$$

2.2 Price setting mechanism

Prices follows a partial adjustment rule a la Calvo in which in each period a fraction $\alpha \in [0, 1)$ of randomly chosen firms is not allowed to change the nominal price of the good it produces. The remaining fraction of firms $(1-\alpha)$ choose prices optimally by maximizing the expected discounted value of profits⁴. Therefore, the optimal choice of producers that can set their price $\tilde{p}_t(j)$ at time T is:

$$E_t \left\{ \sum (\alpha\beta)^{T-t} U_c(C_T) \left(\frac{\tilde{p}_t(j)}{P_{H,t}} \right)^{-\sigma} Y_{H,T} \left[\frac{\tilde{p}_t(j)}{P_{H,T}} \frac{P_{H,T}}{P_T} - \frac{\sigma mu_T}{(1-\tau_T)(\sigma-1)} \frac{V_y(\tilde{y}_{t,T}(j), \varepsilon_{Y,T})}{U_c(C_T)} \right] \right\} = 0 \quad (8)$$

Monopolistic competition in production leads to a wedge between marginal utility of consumption and marginal disutility of production, represented by $\frac{\sigma mu_t}{(1-\tau_t)(\sigma-1)}$. Movements in the tax rate τ_t affect this wedge and generate distortions in agents choice between consumption and labor. However, differently from to the case studied in De Paoli (2008), changes in the tax rate are no longer exogenous. We allow for exogenous fluctuations in this wedge by assuming a time varying mark-up shock mu_t ⁵. Given this price setting specification a la Calvo, the price index evolves according to the following law of motion:

⁴All households within a country that can modify their price at a certain time face the same discounted value of the streams of current and future marginal costs under the assumption that the new price is maintained. Thus they will set the same price.

⁵This mark up shock is introduced in order to allow for the evaluation of pure cost push shocks. It can be interpreted as a shock to the level of monopolistic power of firms. Alternatively, it may be thought as a shock to wage mark up in an environment where the labour market is also characterized by imperfect competition and differentiated labour input.

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha)(\tilde{p}_t(h))^{1-\sigma} \quad (9)$$

2.3 Government budget constraint

In the present framework we consider two alternative specifications for government debt. In particular, we consider the case in which the government issues bonds denominated in domestic currency and the case in which public debt is denoted in real terms (or, in other words, debt is indexed to consumer price inflation). The structure of the debt denomination is exogenously given. Moreover, we abstract from the existence of seigniorage revenues.

In the first case we focus on the situation in which the government issues one period nominal risk free bonds expressed in local currency units, collects taxes and faces exogenous expenditure streams. Government debt D_t^n , expressed in nominal terms, follows the law of motion:

$$D_t^n = D_{t-1}^n(1 + i_{t-1}) - P_{H,t}s_t$$

where s_t is the real primary budget surplus:

$$s_t \equiv \tau_t Y_t - G_t - Tr_t$$

and G_t and Tr_t are exogenously given government purchases and (lump-sum) government transfers, and τ_t denotes income tax rate. We define

$$d_t^n \equiv \frac{D_t^n(1 + i_t)}{P_t},$$

in order to rewrite the government budget constraint as

$$d_t^n = d_{t-1}^n \frac{(1 + i_t)}{\Pi_t} + \frac{P_{H,t}}{P_t} s_t (1 + i_t) \quad (10)$$

Besides the case of nominal bonds, we consider the case in which the government issues a riskless real one-period bonds (D_t^r), or, equivalently, the case in which government bonds are indexed to consumer price inflation. Under this specification, the government budget constraint can be written as:

$$d_t^r = d_{t-1}^r (1 + i_t) + \frac{P_{H,t}}{P_t} s_t (1 + i_t^r) \quad (11)$$

where

$$d_t^r \equiv D_t^r (1 + i_t^r)$$

The implication for fiscal and monetary policy for the different denomination of debt are explored later.

Note that we can obtain expressions analogous to (8), (9), (10) and (11) for the foreign economy.

3 A log-linear representation of the model

We approximate the model around a steady state in which the exogenous variables $\varepsilon_{y,t}$, G_t and mu_t take constant values $\bar{\varepsilon}_y$, $\bar{G} > 0$ and $\bar{mu} \geq 1$. We further focus on a steady-state in which $\Pi_{H,t} \equiv$

$P_{H,t}/P_{H,t-1} = 1$. In this steady-state $\overline{RS} = 1$, $\bar{C} = \bar{C}^*$, $\bar{Y} = \bar{Y}^*$ and $\bar{U}_C(\bar{C}, 0) = \bar{\mu}\bar{V}_y(\bar{Y}, 0)$ ⁶. Log deviations from the steady state are denoted with a hat.

The small open economy system of equilibrium conditions derived from log linearizing equations (8), (7), (6), (10), (11) and (4) is given by the following set of equations (see Table 1):

Table 1: System of log-linear equilibrium conditions	
Phillips Curve	$\hat{\pi}_t^H = k \left(\rho \hat{C}_t + \eta \hat{Y}_t - \hat{p}_H + \widehat{mu}_t + \omega \hat{\tau}_t - \eta \widehat{\varepsilon}_{Y,t} \right) + \beta E_t \hat{\pi}_{t+1}^H$
Demand	$\hat{Y}_t = -\theta \hat{p}_H + (1 - \lambda) \hat{C}_t + \lambda \hat{C}^* + \theta \lambda \widehat{RS}_t + \hat{g}_t$
Risk Sharing Condition	$\hat{C}_t = \hat{C}_t^* + \frac{1}{\rho} \widehat{RS}_t$
Government Budget Constraint	$\tilde{d}_t = d_{ss}(1 - \beta)(-\rho \hat{C}_t - \frac{\lambda}{1 - \lambda} RS_t) + \bar{\tau}(\hat{\tau}_t + \hat{Y}_t) - \hat{g}_t + \beta E_t \tilde{d}_{t+1}$
Price Index	$(1 - \lambda) \hat{p}_H + \lambda \widehat{RS} = 0$
*where $\tilde{d}_t = \hat{d}_{t-1} - d_{ss}(a \Delta RS_t + b \hat{\pi}_t^H) - \rho d_{ss} \hat{C}_t$	

The first equation is derived from the price setting condition (8) and represents the Phillips Curve for our small open economy⁷. By inspection of this equation we can see that a policy of pure domestic price stabilization, i.e a policy that sets $\hat{\pi}_t^H = 0$ in every state, and for every period leads to the same equilibrium allocation that would arise in the case of perfectly flexible prices, i.e. when $\alpha = 0$ and therefore $k \rightarrow \infty$. The demand equation is derived from log-linearizing condition (7). The government budget constraint is represented in a compact form in order to allow for different type of bonds. We can set $a = \lambda/(1 - \lambda)$ and $b = 1$ to retrieve the nominal bonds case and $a = b = 0$ for the case of real bonds. The last equation is derived from the price index (4) and describes the relationship between domestic relative prices and the real exchange rate.

Note that in the case of zero steady state government debt, the denomination of government debt is irrelevant for the dynamics of the small open economy. In this case, the government budget constraint becomes:

⁶This specification implies a specific level of initial distribution of wealth across countries. Appendix A contains the full characterization of the steady state.

⁷We denote: $\hat{p}_{H,t} = \ln(P_{H,t}/P_t)$, $\hat{\pi}_{H,t} = \ln(P_{H,t}/P_{H,t-1})$, $\hat{g}_t = \frac{G_t - \bar{G}}{\bar{Y}}$ and $\hat{d}_t = \frac{d_t - \bar{d}}{\bar{Y}}$. Moreover, as shown in the appendix, ρ represents the coefficient of relative risk aversion and η the inverse of the elasticity of goods production. Also we define $k = \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha(1 + \sigma\eta)}$, $\omega = \frac{\bar{\tau}}{1 - \bar{\tau}}$ and $d_{ss} = \frac{\bar{d}}{\bar{Y}}$. Appendix B has a detailed derivation of the approximations.

$$\widehat{d}_{t-1} = \bar{\tau}(\widehat{\tau}_t + \widehat{Y}_t) - \widehat{g}_t + \beta\widehat{d}_t \quad (12)$$

The system of structural equilibrium conditions is closed by specifying the monetary and fiscal policy rules. Given the domestic exogenous variables $\widehat{\varepsilon}_{y,t}, \widehat{g}_t, \widehat{m}u_t$ and the external shock \widehat{C}_t^* , we can determine the dynamics of $\widehat{Y}_t, \widehat{R}S_t, \widehat{C}_t, \widehat{\pi}_t^H, \widehat{d}_t$ and $\widehat{p}_{H,t}$ ⁸.

Foreign dynamics are governed by the foreign Phillips curve, demand condition and government budget constraint:

Table 2: Foreign system of log-linear equilibrium conditions

Phillips Curve
$\widehat{\pi}_t^* = k \left(\rho\widehat{C}_t^* + \eta\widehat{Y}_t^* + \omega\widehat{\tau}_t^* + \widehat{m}u_t^* - \eta\widehat{\varepsilon}_{Y,t}^* \right) + \beta E_t \widehat{\pi}_{t+1}^*$
Demand
$\widehat{Y}_t^* = \widehat{C}_t^* + \widehat{g}_t^*$
Government Budget Constraint
$\widetilde{d}_t^* = -\rho d_{ss}(1 - \beta)\widehat{C}_t^* + \bar{\tau}^*(\widehat{\tau}_t^* + \widehat{Y}_t^*) - \widehat{g}_t^* + \beta E_t \widetilde{d}_{t+1}^*$
where $\widetilde{d}_t^ = \widehat{d}_{t-1}^* - d_{ss}^* a \widehat{\pi}_t^* - \rho d_{ss}^* \widehat{C}_t^*$

The specification of the foreign policy rules complete the system of equilibrium conditions which determine the evolution of $\widehat{Y}_t^*, \widehat{C}_t^*, \widehat{d}_t^*, \widehat{\tau}_t^*$ and $\widehat{\pi}_t^*$. We should note that the dynamics of the rest of the world is not affected by *Home* variables. Therefore, policymakers in the small open economy can treat \widehat{C}_t^* as an exogenous shock. Moreover, the policy choice of the rest of the world and the denomination of foreign public debt do not influence how \widehat{C}_t^* affects the small open economy.

4 Welfare measure

In a microfounded model a natural measure of welfare is the expected utility of agents belonging to the economy. In our small open economy this can be written as:

$$W = E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} [U(C_t) - V(y_t(h), \varepsilon_{Yt}) dh] \right\}. \quad (13)$$

We assume that the policy authority's goal is to maximize domestic welfare. We obtain the objective function from a second-order approximation of the utility function, or, equivalently, a second-order Taylor expansion of Equation (13). As shown in the Appendix, the Taylor expansion implies

$$W_{t_0} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\begin{array}{c} \widehat{C}_t - \frac{1}{\mu} \widehat{Y}_t + \frac{1}{2}(1 - \rho) \widehat{C}_t^2 \\ -\frac{1}{2} \frac{(\eta+1)}{\mu} (\widehat{Y}_t - \frac{\eta}{(\eta+1)} \widehat{\varepsilon}_{Y,t})^2 - \frac{1}{2} \frac{\sigma}{k\mu} (\pi_t^H)^2 \end{array} \right] + t.i.p + O(\|\xi\|^3) \quad (14)$$

⁸In order to retrieve the value of the nominal exchange rate and interest rate we can use households' intertemporal choice (i.e. the Euler equation) and the definition of the real exchange rate.

where the term *t.i.p* stands for terms independent of policy, that is, constants or function of exogenous shocks that are not affected by the policy choice. The term $O(\|\xi\|^3)$ refers to terms of order strictly higher than two. And the parameter μ denotes the steady state degree of monopolistic distortion, i.e. $\mu = \frac{\sigma \bar{m} \bar{u}}{(1-\bar{\tau})(\sigma-1)}$.

Following the method of Benigno and Woodford (2003) and Sutherland (2002), we use a second order approximation to some of the structural equilibrium conditions in order to eliminate the linear terms that appear in Equation (14). It follows that the loss function for our small open economy can be expressed as a quadratic function of \widehat{Y}_t , \widehat{RS}_t , and $\widehat{\pi}_t^H$:

$$L_{t_0}^i = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} \Phi_Y \widehat{y}_t^2 + \frac{1}{2} \Phi_{RS} \widehat{r}s_t^2 + \frac{1}{2} \Phi_\pi (\widehat{\pi}_t^H)^2 \right] + t.i.p, \quad (15)$$

where the $\widehat{r}s_t = (\widehat{RS}_t - \widehat{RS}_t^T)$, $\widehat{y}_t = (\widehat{Y}_t - \widehat{Y}_t^T)$ are the welfare-based gaps, and \widehat{Y}_t^T and \widehat{RS}_t^T are the target output and real exchange rate that are functions of exogenous shocks while the weights Φ_Y , Φ_{RS} and Φ_π depend on structural parameters of the model. These are defined in appendix B.

The welfare function (15), indicates that policymakers should seek to minimize the discounted value of a weighted sum of squared deviations of inflation from zero and squared fluctuations in the output gap and real exchange rate gap. As in De Paoli (2008), the open economy dimension of the model implies the existence of a real exchange rate gap in the policy objective function.

The current model features three economic frictions that are common to the closed economy counterpart (which can be replicated in our framework by setting $\lambda = 0$): (a) monopolistic competition which generates an inefficient level of production; (b) staggered prices, which create dispersion of output across the differentiated goods; and (c) distortionary income taxes that give rise to a wedge in agents' consumption-labour decisions. Therefore, parameters such as the degree of monopolistic competition μ , the degree of price stickiness α , and the steady state level of government taxes $\bar{\tau}$ are important determinants of the weights Φ_π and Φ_Y in the loss function.

In an open economy, however, another policy incentive arises. As described in Corsetti and Pesenti (2001) "*In an open economy there exists an economic distortion that is directly associated with openness, namely, a country's power to affect its terms of trade by influencing the supply of labour product. [...] the improved terms of trade allow domestic agent to finance higher consumption for any given level of labour effort.*" The linear terms in equation (14) capture analytically this policy incentive: the welfare expression in a small open economy is affected by the unconditional means of consumption and output, and these terms depend directly by the real exchange rate. In particular when we set $\mu = (1 - \lambda)^{-1}$, the term $E[\widehat{C}_t - \frac{\widehat{Y}_t}{\mu}]$ can be rewritten as a function of $E[(1 - \rho\theta)\widehat{RS}_t]$.⁹ That is, the unconditional E mean of the real exchange rate has a direct impact on welfare. This term represent the "terms of trade externality" as in highlighted in Obstfeld and Rogoff (1998).¹⁰ Note that in Gali and Monacelli (2005), where the policy problem in an open economy is isomorphic to a closed economy, the linear term is cancelled by

⁹ As shown in De Paoli (2004), the assumption of $\mu = (1 - \lambda)^{-1}$ guarantees that the steady-state is efficient when $\rho\theta = 1$. Moreover, when $\mu = (1 - \lambda)^{-1}$, using the first order approximation of the demand equation and the risk sharing condition, this term $E[\widehat{C}_t - \frac{\widehat{Y}_t}{\mu}]$ can be expressed as a function of $E[(1 - \rho\theta)\widehat{RS}_t]$. Thus, to a first order approximation, the linear terms in equation (14) can be expressed by $E[(1 - \rho\theta)\widehat{RS}_t]$.

¹⁰ For alternative works that similarly identify the terms of trade rate externality, see Corsetti and Pesenti, 2001, Benigno and Benigno, 2003, Arseneau, 2004, among others).

imposing $\theta = \rho = 1$ so that the external distortion peculiar to the open economy disappears.

When domestic and foreign goods are substitutes in the utility (i.e. when $\rho\theta > 1$), an appreciated real exchange rate, on average, can improve welfare by decreasing the disutility of producing at home without an equivalent fall in the utility of consumption. This is because the appreciation induces lower levels of domestic production and lower consumption of domestic goods, while leading to larger consumption of foreign goods. That is to say, the small open economy, as a monopolist over the goods it produces, can gain from reducing the supply and increasing the price of its goods. When goods are complements in the utility (i.e. when $\rho\theta < 1$), however, a fall in consumption of domestic goods is accompanied by a fall in foreign goods consumption. An appreciation of the real exchange rate is unable to divert consumption towards foreign goods. In this case, a higher unconditional mean of real exchange rate (i.e. a depreciation of the real exchange rate) increases welfare, by creating a rise in consumption larger than the rise in output. Finally, when the marginal utility of consuming one good does not depend on the consumption of the other good (i.e. when $\rho\theta = 1$), and in this case where $\mu = (1 - \lambda)^{-1}$, welfare does not depend on the level of the real exchange rate (i.e. $\Phi_{RS} = 0$).

But what are the implications of this externality for optimal policy? In our framework, monetary and fiscal policy can influence the overall volatility of macro variables in order to minimise the loss function (15). As suggested above, when $\rho\theta > 1$, the terms of trade externality implies that a more appreciated exchange rate can improve welfare. Using a second order approximation to the equilibrium conditions, we can show that the mean of the real exchange rate depends positively on its volatility. Therefore, as long as $\rho\theta > 1$, inducing a less volatile exchange rate can produce a more appreciated exchange rate on average.

To illustrate this result we focus initially on the flexible price specification of the model, i.e. $\alpha = 0$ (the value of the other parameters are specified in Table 3). In general, by using a the second order approximation of the equilibrium conditions, it is possible to express the unconditional mean of the real exchange rate as a function of second moments of the different macro variable. Under flexible prices, the unconditional mean of the real exchange rate can actually be expressed solely as a function of the real exchange rate volatility. In particular figures (1) and (2) show the relationship between the mean and variance of the real exchange rate for different values of θ and λ . The figures show that the effect of the real exchange rate volatility on its mean is increasing on the degree of openness (λ) and the elasticity of substitution between home and foreign-produced goods (θ). More importantly, regardless of the value of λ and θ , the higher the level of the exchange rate volatility, the more appreciated is the real exchange rate on average. However, the welfare effect of this volatility depend critically on parameter values. In particular it depends on whether $\rho\theta$ is bigger or smaller than unity. Figure (3) plots the linear term in equation (14) as a function of the real exchange rate volatility. This is done for different values of θ , holding $\rho = 1$. Since the unconditional mean of the real exchange rate is increasing in the volatility of the real exchange rate, higher volatility of the real exchange rate improves welfare when a more depreciated exchange rate is welfare improving (i.e. when $\rho\theta < 1$). On the other hand, when a more appreciated exchange rate improves welfare (i.e. when $\rho\theta > 1$), lower volatility of the real exchange rate is welfare improving.

[INSERT HERE FIGURES 1,2, 3]

5 Optimal Policy

In this section we analyze optimal policy under alternative specifications of the model. We start by characterizing the flexible price case and then turn to sticky prices. Throughout the analysis we explore the differences between the open economy and the closed economy cases. We also explore the implications of having real versus nominal government debt. In what follows our analysis is restricted to the case in which monopolistic competition inefficiencies in steady-state output are set to $\mu = (1 - \lambda)^{-1}$ (as in Gali and Monacelli (2005)) in order to facilitate the analytical illustration of the results.¹¹ We will concentrate on the policy implication of *distortionary taxation*, *sticky prices* and *the terms of trade externality*.

Once we explore the properties of the optimal allocation we calibrate our model as in Table 3. For the calibration of the shocks, we follow Gali and Monacelli (2005) who fit AR(1) processes to (log) labor productivity in Canada (their proxy for domestic productivity) using quarterly, HP-filtered data over the sample period 1963:1–2002:4 with the following estimates $\widehat{\varepsilon}_{Y,t} = 0.66(0.06)\widehat{\varepsilon}_{Y,t-1} + a_t$, $\sigma_{a_t} = 0.0071$. In order to compute the volatilities of the variable of interests, we generate simulated time series of length T for the variables of interest and compute the standard deviation. We repeat this procedure J times and then compute the average of the moments. We set $T = 400$ quarters and $J = 500$. We compute the moments based on these Monte Carlo simulations because, under certain specifications, our model is non stationary.

5.1 The case of Flexible Prices

When prices are flexible (that is, $\alpha = 0$), the loss function derived in the previous section simplifies to:

$$\min U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} \Phi_Y \widehat{y}_t^2 + \frac{1}{2} \Phi_{RS} \widehat{r} s_t^2 \right] + t.i.p. \quad (16)$$

Alternatively, using the relationship between distortionary taxes and output dictated by the Phillips curve, it is possible to rewrite the objective function (16) as

$$\min U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} \Phi_\tau (\widehat{\tau}_t - \widehat{\tau}_t^T)^2 + \frac{1}{2} \Phi_{RS} \widehat{r} s_t^2 \right] + t.i.p + O(\|\xi\|^3),$$

where $\Phi_\tau = -\frac{\omega}{\eta + \rho(1+l)^{-1}} \Phi_Y$ and $\widehat{\tau}_t^T$ is the desired level for the level of distortionary taxation, which also depend on exogenous shocks. Under this specification, domestic producer inflation is not costly (the assumption that $\alpha = 0$ implies that $\Phi_\pi = 0$), but policymakers' incentives are affected by tax distortions and the terms of trade externality.

The constraints of the policy problem are given by the equilibrium conditions presented in Table 1, noting that under flexible prices $k^{-1} = 0$. We define φ_t as the Lagrange multiplier associated with the government budget constraint, representing the marginal value, measured in utility terms, of one unit of the government revenue in any given period. As shown in the Appendix, we impose further constraints, associated with initial conditions $\widehat{\pi}_{t_0}$ and \widehat{RS}_{t_0} , to ensure that the first order conditions to

¹¹This parameterization guarantees that the steady state is efficient when the elasticities of intratemporal and intertemporal substitution are unitary, or when the economy is closed.

the problem are time invariant.¹² Thus, the first order conditions of the policy problem can be written as follows:

$$\Phi_{RS}\widehat{r}_t + \frac{(1+l)}{\rho(1-\lambda)}\Phi_Y\widehat{y}_t = -m_0\varphi_t + (a+1)d_{ss}(\varphi_t - \varphi_{t-1}), \quad (17)$$

$$-bd_{ss}(\varphi_t - \varphi_{t-1}) = 0, \quad (18)$$

and

$$E_t\varphi_{t+1} = \varphi_t, \quad (19)$$

where m_0 is a constant defined in the Appendix. The above equations represent the general optimal plan under flexible prices.¹³ In what follows we illustrate both graphically and analytically the properties of the optimal plan for the various cases.

The case of nominal government debt

When bonds are denominated in nominal terms, inflation affects the real value of government debt and therefore has direct fiscal consequences. Under this specification (i.e. when $d_{ss} \neq 0$ and $b \neq 0$), the optimal plan can be summarized by

$$\Phi_{RS}\widehat{r}_t + \frac{(1+l)}{\rho(1-\lambda)}\Phi_Y\widehat{y}_t = 0, \quad (20)$$

or, equivalently,

$$\Phi_{RS}\widehat{r}_t + \frac{(1+l)}{\rho(1-\lambda)}\Phi_\tau(\widehat{\tau}_t - \widehat{\tau}_t^{T'}) = 0. \quad (21)$$

The above equation can be interpreted as the small open economy targeting rule *à la* Gianonni and Woodford (2003) and Svensson (2003). This rule prescribes stabilizing a linear combination of the output gap and the real exchange rate gap. Policymakers should allow some fluctuations in output, or, alternatively, some fluctuations in the tax rate around the desired target. These fluctuations are a consequence of the terms of trade externality described in the previous section. Indeed, if we were in a closed economy framework ($\lambda = 0$) equation (20) becomes:

$$\widehat{y}_t = (\widehat{\tau}_t - \widehat{\tau}_t^{T'}) = 0. \quad (22)$$

The output gap is fully stabilized (as in Benigno and Woodford (2003)) and the first best can be achieved. It is easy to show that this result holds also for the small open economy under the further restriction $\theta\rho = 1$, for which $\Phi_{RS} = 0$ so that the loss function is not affected by real exchange rate fluctuations. Indeed this isomorphism among the closed and open economy optimal policy problems is

¹²This method follows Woodford's (1999) timeless perspective approach and ensures that the policy problem does not constitute a time inconsistent problem. The policymaker's stated future course of action is such that the same policy process tomorrow will result in actions consistent with the announced plan. McCallum 2005 refers to this approach as "strategic coherence" and argues that the timeless perspective does not coincide with the concept of time-consistency described by Kydland and Prescott (1977). "Time-inconsistency" has been understood in the prior literature to mean that it will not be optimal at a date later than time t to implement the policy which was determined as optimal at t . As stated in Taylor 2005 (mimeo, Bank of England) the "timeless perspective" concept can be thought of a way of choosing a policy which is constrained-optimal. The constraints capture the idea that in an ongoing policy regime, policymakers should not take advantage of the fact that expectations are predetermined.

¹³In the exercises to follow, the second order conditions are verified numerically.

Table 3: Parameter Values used in the Quantitative Analysis

Parameter	Value	Notes:
β	0.99	Specifying a quarterly model
η	0.47	Following Rotemberg and Woodford (1997)
θ	2.3	Following the range as in Obstfeld and Rogoff (1998) (unless specified otherwise)
λ	0.2	This implies a 20% import share of the GDP
ρ	1	Specifying a Log utility function
α	0.66	Characterizing an average length of price contract of 3 quarters
σ	10	Following Benigno and Woodford (2003)
d_{ss}	2.4	Steady state debt to GDP of 60% (unless specified otherwise)
$sdv(\varepsilon_y)$	0.71%	Consistent with Galí and Monacelli (2005) and Kehoe and Perri (2002)
τ	0.2	Steady state taxes of 20% of GDP

identical to the case analyzed by Galí and Monacelli (2005) and De Paoli (2008) for the monetary policy stabilization policy in a framework in which taxes are lump sum. When $\theta = \rho = 1$ the substitution and income effects associated with terms of trade movements cancel out and there is no terms of trade externality. Under these restrictions, there is only one policy incentive: to smooth taxes across states and times in order to minimize distortions in agents' labour-leisure decisions.

Since inflation is not costly, optimal policy can induce unexpected variations in domestic prices in order to restore fiscal equilibrium. The resulting allocation is the same as the one that would prevail if state-contingent debt were available. This finding is consistent with the ones of Bohn (1990), Chari, Christiano and Kehoe (1991) and Benigno and Woodford (2003).

On the other hand when $\theta\rho \neq 1$, it is possible to improve welfare by combined changes in the tax rate gap and the real exchange rate gap. Figure (4) displays the relationship between the sample standard deviation of the tax instrument and the intratemporal elasticity of substitution, θ . The figure shows that, unless $\theta = \rho = 1$, taxes vary over time. This is because, outside this knife-edge case, there is trade off between stabilizing taxes, and minimizing the fiscal distortion, and exploiting the terms of trade externality.

[INSERT FIGURE 4 HERE]

In terms of the dynamic properties of the variables of interests, as shown in the appendix, if the government only issues nominal bonds and prices are flexible, φ_t is time-invariant and all economic variables follow a stationary process. But these assumptions also imply that the evolution of expected producer inflation and nominal debt are indeterminate; we can only determine the evolution of real debt. This can be verified by inspection of the government budget constraint.

The case in which the government solvency condition is independent of inflation

When the government only issues real debt (i.e. $b = 0$) or the steady state debt is zero (i.e. $d_{ss} = 0$), the path of inflation does not affect the government budget constraint and it is not possible to use inflation as a shock-absorber. In fact, under flexible prices, inflation does not affect the entire system of equilibrium condition (specified in Table 1). That is to say, producer price inflation is *indeterminate*.

Combining equations (17), (18) and (19) we obtain the following expression:

$$\Phi_{RS} E_t \Delta \widehat{r}_{s_{t+1}} + \frac{(1+l)}{\rho(1-\lambda)} \Phi_Y E_t \Delta \widehat{y}_{t+1} = 0, \quad (23)$$

and in the case of a closed economy ($\lambda = 0$), we have:

$$E_t \Delta \widehat{y}_{t+1} = E_t \Delta (\widehat{\tau}_{t+1} - \widehat{\tau}_{t+1}^T) = 0. \quad (24)$$

In a closed economy, as emphasized in the analysis shown in Barro (1979) and Aiyagari et al. (2002), if a shock creates fiscal imbalances and inflation cannot act as a shock absorber by affecting the value of real debt, taxes have to do the adjustment. But in order to minimize distortions in the consumption/leisure trade-off, the expected tax gap should be kept constant. For example following a one period productivity shock, the tax rate would follow a random walk. Taxes vary across states but they remain constant after the shock hits the economy.

In an open economy, it is no longer optimal to fully stabilize the expected tax gap. As illustrated in Equation (23), policymakers should stabilize expected movements in the output and real exchange rate gap. As in the case of nominal bonds, there is a trade-off between reducing fiscal distortions and exploiting the terms of trade externality. In terms of dynamic properties of the variables of interest now, differently from the case in which inflation affects the government budget constraint, government debt follows a unit root process (see Appendix). Moreover, taxes, output and the real exchange rate face permanent changes following a temporary shock to fiscal conditions.

5.2 The case of sticky prices

We now turn to the optimal policy problem in the case of sticky prices (i.e. $\alpha > 0$) and characterize the general optimal fiscal and monetary plan. The policy problem consists of choosing the path of $\{\widehat{\pi}_t^H, \widehat{Y}_t, \widehat{RS}_t, \widehat{d}_t, \widehat{\tau}_t\}$ as to minimize (15), subject to: the equilibrium conditions specified in table 1; the initial condition for \widehat{d}_{t_0-1} ; and the constraints on $\widehat{\pi}_{t_0}$ and \widehat{RS}_{t_0} that ensure a time-invariant optimal policy problem (see Appendix for details).

As before, we express the optimal state-contingent response to shocks in the form of targeting rules. In particular the optimal plan can be written as follows

$$\left[\frac{(1+l)\Phi_y}{(1-\lambda)\rho} \right] \Delta \widehat{y}_t + \Phi_{RS} \Delta \widehat{r}_{s_t} + \left[\frac{k\Phi_\pi}{(1-\bar{\tau}) + bd_{ss}k} \right] (\gamma \widehat{\pi}_t^H + d_{ss}(a+1)\widehat{\pi}_{t-1}^H) = 0 \quad (25)$$

$$E_t \widehat{\pi}_{t+1}^H = 0, \quad (26)$$

where $\gamma = \left[d_{ss} \left(\frac{(1-\beta)}{(1-\lambda)} - (1+a) \right) + \left(\frac{(1+l)(\eta(1-\bar{\tau})-\bar{\tau})}{\rho(1-\lambda)} + \frac{1-\bar{\tau}}{(1-\lambda)} \right) \right]$.¹⁵

We first note that the variables of interest in this targeting rule are: current and past domestic producer inflation, the rate of change in the real exchange rate gap, and the rate of change of the output gap. Also, equation (26) states that expected producer inflation is set to zero under the optimal plan.

¹⁴Differently from the case presented before this equation does not characterise a targeting rule, because, if specified by itself, the equilibrium dynamic is not determined under this rule. Equation (23) is simply an equilibrium condition implied by the optimal plan, which is illustrated by equations (17) to (19).

¹⁵See appendix for full derivation of the optimal policy problem.

As shown in the Appendix, equations (25) and (26) together with the constraints specified in Table 1 imply that the dynamics of output, the real exchange rate and taxes follows a non stationary process. This result contrasts with the case of flexible prices, in which, when bonds are nominal, all the variables follow a stationary process, since inflation can work as a shock absorber following exogenous shocks.

Closed versus Open Economy

When the economy is closed, the loss function is only affected by inflation and output variability. Although there are two policy incentives - reducing the inefficiencies created by distortionary taxation and minimizing the price distortions - and two policy instruments - an active fiscal and monetary policies - the first best cannot be achieved. If we further assume that debt is zero in steady state, i.e. $d_{ss} = 0$, the optimal plan implies

$$\omega E_t \Delta(\hat{\tau}_{t+1} - \hat{\tau}_{t+1}^{T'}) + k^{-1} \pi_t^H = 0. \quad (27)$$

In general, it's not possible to keep simultaneously inflation and taxes constant across states and over time.¹⁶ In our small open economy the optimal policy, given by equations (25) and (26), implies

$$\Phi_{RS} E_t \Delta \hat{r}_{s_{t+1}} + \frac{(1+l)}{\rho(1-\lambda)} \Phi_Y E_t \Delta \hat{y}_{t+1} = 0, \quad (28)$$

and

$$E_t \hat{\pi}_{t+1} = 0 \quad (29)$$

Equation (28) is identical to equation (23) obtained under flexible prices. Therefore, when there is no nominal burden from existing debt (i.e. $d_{ss} = 0$), the optimal policy under both flexible and sticky prices prescribes the stabilization of expected growth rate of the output gap and expected change in the real exchange rate gap. A difference among these two cases arises because while under flexible prices only taxes affect the labor leisure trade-off, with sticky prices, both domestic producer inflation and taxes do so. Therefore, under sticky prices, movements in both taxes and inflation can lead to changes in the real exchange rate. Policymakers have, thus, a "choice" between allowing some fluctuations in inflation or fluctuations in taxes in order to exploit the exchange rate externality. While optimal inflation volatility is decreasing in the degree of nominal rigidity, for our parametrization, the volatility of taxes initially tend to decrease and then increases with α (see figure 5). When price rigidities are introduced, inflation can be used to affect the real exchange rate and this fact reduces the required movement in taxes. But as the degree of nominal rigidities increases, so does the distortions generated by them. So, for significantly high levels of α , inflation is practically constant and only taxes are used to exploit the terms of trade externality (figure 5).

[INSERT FIGURE 5 HERE]

From a quantitative point of view, however, our framework suggests that the cost of inflation will overshadow the inefficiency caused by varying distortionary taxation and, therefore, changes in domestic

¹⁶ Nor is it possible, as in the flexible price case, to move tax rates permanently (and smooth them in subsequent periods). By inspection of the Phillips curve we note that, when prices are sticky, a permanent change in taxes would imply a non stationary process for inflation (and an explosive path for the domestic price level).

producer inflation are quantitatively small. Note that this result holds even in a model with real bonds and is a consequence of the conflict between price stability and the incentive to strategically affect the real exchange rate. This is different, however, from the trade-off (emphasized by the Benigno and Woodford (2003), Schmitt-Grohe and Uribe (2004) and Siu (2004)) between price stability and the use of inflation as insurance that arise only in models in which the government issues nominal bonds.

6 Conclusion

This paper presents an integrated analysis of fiscal and monetary policy in a small open economy. The literature on optimal policy in open economies has extensively analysed the monetary stabilization problem when inflation is costly and taxation is non-distortionary. In the present work, we start our analysis by characterizing the opposite scenario. That is, we study the optimal policy problem in an environment in which prices are perfectly flexible (and therefore inflation is costless) and production taxation affects households labour-leisure trade off (i.e. taxes are distortionary). We lay out this specification in order to highlight the international dimension of fiscal policy. Our results show that, whereas it is optimal to perfectly smooth taxes in a closed economy, the optimal tax rate varies over time in an open economy. Managing the level of proportional taxes may improve welfare by affecting the real exchange rate and the overall level of consumption utility and production disutility.

When prices are sticky, movements in the inflation rate affects individuals' production and consumption choices. Thus, once nominal rigidities are incorporated into the analysis, both inflation and taxes can be used strategically to affect the terms of trade and the overall level of utility. As a result, the introduction of price rigidity can decrease the required variability of tax rates. But the presence of nominal rigidities also reduces the policy incentive to use inflation to affect the level of real government debt.

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A Appendix: Steady state equations

In this appendix we derive the steady state conditions and define some parameters that depend on these conditions. All variables in steady state are denoted with a bar.

From the demand equation at Home, we have:

$$\begin{aligned} y^d(h) &= \left[\frac{p(h)}{P_H} \right]^{-\sigma} \left[\frac{P_H}{P} \right]^{-\theta} \left[vC + \frac{v^*(1-n)}{n} \left(\frac{1}{RS} \right)^{-\theta} C^* \right] \\ y^d(f) &= \left[\frac{p(f)}{P_F} \right]^{-\sigma} \left[\frac{P_F}{P} \right]^{-\theta} \left[\frac{(1-v)n}{1-n} C + (1-v^*) \left(\frac{1}{RS} \right)^{-\theta} C^* \right] \end{aligned} \quad (\text{A.1})$$

Normalizing $\bar{P}_H = \bar{P}_F$, we have:

$$\bar{Y} = v\bar{C} + \frac{v^*(1-n)}{n}\bar{C}^* + \bar{G} \quad (\text{A.2})$$

$$\bar{Y}^* = \frac{(1-v)n}{1-n}\bar{C} + (1-v^*)\bar{C}^* \quad (\text{A.3})$$

If we specify the proportion of foreign-produced goods in home consumption as $1-v = (1-n)\lambda$ and the proportion of home-produced goods in foreign consumption is $v^* = n\lambda$, and take the limiting case where $n = 0$, we have:

$$\bar{Y} = (1-\lambda)\bar{C} + \lambda\bar{C}^* + \bar{G} \quad (\text{A.4})$$

And from the Foreign demand:

$$\bar{Y}^* = \bar{C}^* \quad (\text{A.5})$$

For further reference, we define the following steady state dependent constants:

$$\begin{aligned} d_g &= \frac{\bar{G}}{\bar{Y}} \\ d_b &= (1-\lambda)\frac{\bar{C}}{\bar{Y}} \end{aligned}$$

Moreover, using equation (A.4), we can notice that:

$$\frac{\lambda\bar{C}^*}{\bar{Y}} = 1 - d_b - d_g$$

From the government budget constraint we have:

$$\delta\bar{D} = \bar{P}\bar{s} \quad (\text{A.6})$$

where $\delta = \beta^{-1}(1-\beta)$ and the steady state fiscal surplus defined as:

$$\bar{s} = \bar{\tau}\bar{Y} - \bar{G} - \bar{T}r \quad (\text{A.7})$$

Thus, the steady state constants defined in the text are:

$$s = \bar{s}^{-1}\bar{Y}$$

$$s_\tau = \bar{s}^{-1} \bar{\tau} \bar{Y}$$

The Symmetric Steady State:

From the complete asset market assumption we have:

$$RS_t = \kappa_0 \left(\frac{C_t}{C_t^*} \right)^\rho \quad (\text{A.8})$$

where

$$\kappa_0 = RS_0 \left(\frac{C_0}{C_0^*} \right)^\rho \quad (\text{A.9})$$

Assuming an initial level of wealth such that $\kappa_0 = 1$, the steady state version of (A.8) implies $\bar{C} = \bar{C}^*$. If, moreover we assume $\bar{G} = 0$ we have:

$$d_g = 0$$

$$d_b = (1 - \lambda)$$

And in the case of $\bar{Tr} = 0$, we have:

$$s = 1/\bar{\tau}$$

$$s_\tau = 1$$

Finally, applying our normalization to the price setting equations we have:

$$U_C(\bar{C}) = \mu V_y (\lambda \bar{C}^* + (1 - \lambda) \bar{C}) \quad (\text{A.10})$$

$$U_C(\bar{C}^*) = \mu^* V_y (\bar{C}^*). \quad (\text{A.11})$$

where

$$\mu = \frac{\sigma \bar{m} \bar{u}}{(1 - \bar{\tau})(\sigma - 1)}$$

$$(1 - \phi) = \frac{1}{\mu}$$

$$\phi = 1 - \frac{(1 - \bar{\tau})(\sigma - 1)}{\sigma \bar{m} \bar{u}}$$

$$0 \leq \phi < 1; \mu > 1$$

B Appendix: Welfare derivation

In this appendix, we derive the 1st and 2nd order approximation of the equilibrium conditions of the model. Moreover, we show the second order approximation of the utility function in order to address welfare analysis. To simplify and clarify the algebra, we use the following isoelastic functional forms:

$$U(C_t) = \frac{C_t^{1-\rho}}{1-\rho}$$

$$V(y_t(h), \varepsilon_{Y,T}) = \frac{\varepsilon_{y,t}^{-\eta} y_t(h)^{\eta+1}}{\eta+1}$$

B.1 Demand

As shown in the text, home demand equation is:

$$Y_{H,t} = \left[\frac{P_{H,t}}{P_t} \right]^{-\theta} \left[(1-\lambda)C_t + \lambda \left(\frac{1}{RS_t} \right)^{-\theta} C_t^* \right] + g_t \quad (\text{B.12})$$

Therefore, the first order approximation to the above equation is:

$$\hat{Y}_H = -\theta(1-d_g)\hat{p}_H + d_b\hat{C} + (1-d_b-d_g)\hat{C}^* + \theta(1-d_b-d_g)\hat{RS} + \hat{g} \quad (\text{B.13})$$

In the symmetric steady state, where $d_b = 1-\lambda$ and $d_g = 0$, equation (B.13) becomes:

$$\hat{Y}_H = -\theta\hat{p}_H + (1-\lambda)\hat{C} + \lambda\hat{C}^* + \theta\lambda\hat{RS} + \hat{g} \quad (\text{B.14})$$

And the second order approximation to the demand function is:

$$\sum \beta^t \left[d'_y y_t + \frac{1}{2} y'_t D_y y_t + y'_t D_e e_t \right] + t.i.p + O(\|\xi\|^3) = 0$$

where

$$y_t = \begin{bmatrix} \hat{Y}_t & \hat{C}_t & \hat{p}_{Ht} & \hat{\tau}_t & \hat{RS}_t \end{bmatrix}$$

$$e_t = \begin{bmatrix} \hat{\varepsilon}_{yt} & \hat{m}u_t & \hat{g}_t & \hat{C}_t^* \end{bmatrix}$$

$$d'_y = \begin{bmatrix} -1 & d_b & -\theta(1-d_g) & 0 & \theta(1-d_b-d_g) \end{bmatrix}$$

$$D'_y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & (1-d_b)d_b & -\theta d_b d_g & 0 & -\theta(1-d_b-d_g)d_b \\ 0 & -\theta d_b d_g & \theta^2(1-d_g)d_g & 0 & -\theta^2 d_g(1-d_b-d_g) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\theta(1-d_b-d_g)d_b & -\theta^2 d_g(1-d_b-d_g) & 0 & \theta^2(1-d_b-d_g)(d_b+d_g) \end{bmatrix}$$

$$D'_e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -d_b & -(1-d_b-d_g)d_b \\ 0 & 0 & 0 & \theta(1-d_g) & -\theta d_g(1-d_b-d_g) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\theta(1-d_b-d_g) & \theta(1-d_b-d_g)(d_b+d_g) \end{bmatrix}$$

Moreover, in the symmetric equilibrium:

$$d'_y = \begin{bmatrix} -1 & 1-\lambda & -\theta & 0 & \theta\lambda \end{bmatrix}$$

$$D'_y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda(1-\lambda) & 0 & 0 & -\theta\lambda(1-\lambda) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\theta\lambda(1-\lambda) & 0 & 0 & \theta^2\lambda(1-\lambda) \end{bmatrix}$$

$$D'_e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(1-\lambda) & -\lambda(1-\lambda) \\ 0 & 0 & 0 & \theta & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\theta\lambda & \theta\lambda(1-\lambda) \end{bmatrix}$$

B.2 Risk Sharing Equation

In a perfectly integrated capital market, the value of the intertemporal marginal rate of substitution is equated across borders:

$$\frac{U_C(C_{t+1}^*)}{U_C(C_t^*)} \frac{P_t^*}{P_{t+1}^*} = \frac{U_C(C_{t+1})}{U_C(C_t)} \frac{S_{t+1}P_t}{S_tP_{t+1}} \quad (\text{B.15})$$

Assuming the symmetric steady state equilibrium, the log linear approximation to the above condition is:

$$\hat{C}_t^* = \hat{C}_t + \frac{1}{\rho} \widehat{RS}_t \quad (\text{B.16})$$

Given our utility function specification, equation (B.15) gives rise to a exact log linear expression and therefore the first and second order approximation are identical. In matrix notation, we have:

$$\sum E_t \beta^t \left[c'_y y_t + \frac{1}{2} y'_t C_y y_t + y'_t C_e e_t \right] + t.i.p + O(\|\xi\|^3) = 0$$

$$c'_y = \left[0 \quad -1 \quad 0 \quad 0 \quad \frac{1}{\rho} \right]$$

$$c'_e = \left[0 \quad 0 \quad 0 \quad 1 \right]$$

$$C'_y = 0$$

$$C'_e = 0$$

B.3 The Real Exchange Rate

Given our preference specification for the small open economy, and knowing that in the rest of the world $P_F = SP^*$, we can write the price level in the following form:

$$\left(\frac{P}{P_H} \right)^{1-\theta} = (1-\lambda) + \lambda \left(RS \frac{P}{P_H} \right)^{1-\theta} \quad (\text{B.17})$$

Therefore, the first order approximation to the above expression is:

$$\hat{p}_H = -\frac{\lambda \widehat{RS}}{1-\lambda} \quad (\text{B.18})$$

Moreover, the second order approximation to equation (B.17) is:

$$\sum E_t \beta^t \left[f'_y y_t + \frac{1}{2} y'_t F_y y_t + y'_t F_e e_t \right] + t.i.p + O(\|\xi\|^3) = 0$$

$$f'_y = \left[0 \quad 0 \quad -(1-\lambda) \quad 0 \quad -\lambda \right]$$

$$F'_y = \lambda(\theta - 1) \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & (1 - \lambda/(1 - \lambda)) \end{bmatrix}$$

B.4 Price Setting

The first and second-order approximation to the price setting equation follow Benigno and Benigno (2003). The introduction of the tax component is done in the same manner as in Benigno and Woodford (2003). The optimal price setting condition of sellers that can reset their prices is:

$$E_t \left\{ \sum (\alpha\beta)^{T-t} U_c(C_T) \left(\frac{\tilde{p}_t(h)}{P_{H,t}} \right)^{-\sigma} Y_{H,T} \left[\frac{\tilde{p}_t(h) P_{H,T}}{P_{H,T} P_T} - \frac{\sigma mc_t V_y(\tilde{y}_{t,T}(h), \varepsilon_{Y,t})}{(1-\sigma)(1-\tau_t) U_c(C_T)} \right] \right\} = 0 \quad (\text{B.19})$$

where

$$\tilde{y}_t(h) = \left(\frac{\tilde{p}_t(h)}{P_{H,t}} \right)^{-\sigma} Y_{H,t} \quad (\text{B.20})$$

and mu_t is a markup shock, and income taxes are represented by τ_t . Therefore the evolution of the domestic price level is:

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha) (\tilde{p}_t(h))^{1-\sigma} \quad (\text{B.21})$$

We can write the second order approximation for equation (B.19) as follows:

$$V_o = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t z_t + \frac{1}{2} z_t X_t + \frac{1}{2} \frac{\sigma(1+\eta)}{k} (\pi_t^H)^2 \right\} + t.i.p + O(\|\xi\|^3) \quad (\text{B.22})$$

where:

$$z_t = \eta \hat{Y}_t + \rho \hat{C}_t - \hat{p}_H + \widehat{m}u_t - \hat{q}_t - \eta \hat{\varepsilon}_{Y,t}$$

$$X_t = (2+\eta) \hat{Y}_t - \rho \hat{C}_t + \hat{p}_H + \widehat{m}u_t + \hat{q}_t - \eta \hat{\varepsilon}_{Y,t}$$

We define $q_t = 1 - \tau_t$ and, therefore:

$$\hat{q}_t = -\omega \hat{\tau}_t - \frac{1}{2} \frac{\omega}{1-\bar{\tau}} \hat{\tau}_t^2$$

where $\omega = \frac{\bar{\tau}}{1-\bar{\tau}}$.

The first order approximation to the price setting equation can be written in the following way:

$$\hat{\pi}_t^H = k \left(\rho \hat{C}_t + \eta \hat{Y}_t - \hat{p}_H + \widehat{m}u_t + \omega \hat{\tau}_t - \eta \hat{\varepsilon}_{Y,t} \right) + \beta E_t \hat{\pi}_{t+1}^H \quad (\text{B.23})$$

where $k = (1-\alpha\beta)(1-\alpha)/\alpha(1+\sigma\eta)$.

And the second order approximation to the price setting can be written as follows:

$$Q_{to} = \phi \sum E_t \beta^t \left[a'_y y_t + \frac{1}{2} y'_t A_y y_t + y'_t A_e e_t \right] + t.i.p + O(\|\xi\|^3) \quad (\text{B.24})$$

with

$$a'_y = [\eta \quad \rho \quad -1 \quad \omega \quad 0]$$

$$A'_y = \begin{bmatrix} \eta(2+\eta) & \rho & -1 & \omega & 0 \\ \rho & -\rho^2 & \rho & -\rho\omega & 0 \\ -1 & \rho & -1 & \omega & 0 \\ \omega & -\rho\omega & \omega & \omega & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A'_e = \begin{bmatrix} -\eta(1+\eta) & 1+\eta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

B.5 Government Budget Constraint

We assume at first that all public debt consists of riskless nominal one-period bonds. The law of motion of government debt is:

$$D_t = D_{t-1}(1 + i_{t-1}) - P_{H,t}s_t,$$

where

$$s_t \equiv \tau_t Y_t - G_t - Tr_t.$$

Defining

$$d'_t \equiv \frac{D_t(1 + i_t)}{P_t},$$

we can rewrite the government budget constraint as

$$d'_t = d'_{t-1} \frac{(1 + i_t)}{\Pi_t} + \frac{P_{H,t}}{P_t} s_t (1 + i_t).$$

In log-linear terms the government budget constraint is given by

$$\beta \hat{d}'_t = \hat{d}'_{t-1} + \frac{\beta s^{-1}}{1 - \beta} i_t - s^{-1} \hat{p}_{H,t} - s_\tau s^{-1} (\hat{Y}_t + \hat{\tau}_t) + \hat{g}_t + \hat{T}r - \frac{s^{-1}}{1 - \beta} (\hat{\pi}_t^H + \lambda/(1 - \lambda) \Delta R S_t).$$

In order to derive a second order approximation to the intertemporal government solvency condition we define:

$$NW_t = \frac{d'_{t-1}}{\Pi_t} U_C(C_T, \xi_{C,T}), \quad (\text{B.25})$$

and, therefore:

$$NW_t = E_t \sum_{T=t}^{\infty} U_C(C_T, \xi_{C,T}) s_t p_{H,t} \quad (\text{B.26})$$

The second order approximation to condition (B.26) is

$$\begin{aligned} U_C(C_T, \xi_{C,T}) s_t p_{H,t} &= U_{c\bar{s}} \left\{ \begin{array}{l} 1 - \rho \hat{C} + \hat{p}_H + [s_\tau(\hat{\tau} + \hat{Y}) + s_\tau(\hat{\tau} + \hat{Y})^2 - s\hat{g}] \\ + \frac{1}{2} \rho^2 \hat{C}^2 + [s_\tau(\hat{\tau} + \hat{Y}) + s_\tau(\hat{\tau} + \hat{Y})^2 - s\hat{g}] (-\rho \hat{C} + \hat{p}_H) \end{array} \right\} \\ &U_{c\bar{s}} \left\{ \begin{array}{l} 1 + s_\tau \hat{Y} - \rho \hat{C} + \hat{p}_H + s_\tau \hat{\tau} + \frac{1}{2} s_\tau \hat{Y}^2 - \rho s_\tau \hat{Y} \hat{C} + s_\tau \hat{Y} \hat{p}_H \\ + s_\tau \hat{\tau} \hat{Y} + \frac{1}{2} \rho^2 \hat{C}^2 + \frac{1}{2} \hat{p}_H^2 + s_\tau \hat{\tau}^2 \\ - \rho s_\tau \hat{\tau} \hat{C} + s_\tau \hat{\tau} \hat{p}_H + \rho s\hat{g} \hat{C} - s\hat{p}_H \hat{g} \end{array} \right\} \\ &+ t.i.p + O(\|\xi\|^3) \end{aligned}$$

Therefore, defining $\widetilde{NW}_t = \frac{NW_t - \overline{NW}}{\overline{NW}}$, we have:

$$\widetilde{NW}_t = (1 - \beta) \left[b'_y y_t + \frac{1}{2} y'_t B_y y_t + y'_t B_e e_t \right] + \beta E_t \widetilde{NW}_{t+1} + t.i.p + O(\|\xi\|^3)$$

$$b'_y = [s_\tau \quad -\rho \quad 1 \quad s_\tau \quad 0]$$

$$B'_y = \begin{bmatrix} s_\tau & -\rho s_\tau & s_\tau & s_\tau & 0 \\ -\rho s_\tau & \rho^2 & 0 & -\rho s_\tau & 0 \\ s_\tau & 0 & 1 & s_\tau & 0 \\ s_\tau & -\rho s_\tau & s_\tau & s_\tau & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B'_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \rho s & 0 \\ 0 & 0 & -s & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Note that

$$\pi_t = \widehat{\pi}_t^H + \frac{\lambda}{1-\lambda} \Delta R S_t,$$

therefore, the first order approximation to condition (B.25) is

$$\widetilde{NW}_t = -\rho \widehat{C}_t + \widehat{d}'_{t-1} - \widehat{\pi}_t = -\rho \widehat{C}_t + \widehat{d}'_{t-1} - \left(\frac{\lambda}{1-\lambda} \Delta R S_t + \widehat{\pi}_t^H \right)$$

Hence, the first order approximation of the intertemporal budget constraint can be written as:

$$\begin{aligned} -\rho \widehat{C}_t + \widehat{d}'_{t-1} - \left(\frac{\lambda}{1-\lambda} \Delta R S_t + \widehat{\pi}_t^H \right) &= (1-\beta)(-\rho \widehat{C}_t + \widehat{p}_{H,t} + s_\tau(\widehat{\tau}_t + \widehat{Y}_t) - s\widehat{g}_t) \\ &\quad + \beta E_t \left[-\rho \widehat{C}_{t+1} + \widehat{d}'_t - \left(\frac{\lambda}{1-\lambda} \Delta R S_{t+1}^* + \widehat{\pi}_{t+1}^H \right) \right] \end{aligned}$$

Throughout the text we use an alternative representation of the budget constraint in order to allow for a zero steady state government debt. The above equation is rescaled, using $\widehat{d}'_t = d_{ss} \widehat{d}'_t$ (note that $s_\tau = \frac{\bar{\tau}}{d_{ss}(1-\beta)}$).

$$\begin{aligned} -\rho d_{ss} \widehat{C}_t + \widehat{d}'_{t-1} - \left(\frac{\lambda}{1-\lambda} d_{ss} \Delta R S_t + d_{ss} \widehat{\pi}_t^H \right) &= (1-\beta) d_{ss} (-\rho \widehat{C}_t + \widehat{p}_{H,t}) + \bar{\tau}(\widehat{\tau}_t + \widehat{Y}_t) - \widehat{g}_t \\ &\quad + \beta E_t d_{ss} \left[-\rho \widehat{C}_{t+1} + \widehat{d}'_t - \left(\frac{\lambda}{1-\lambda} \Delta R S_{t+1}^* + \widehat{\pi}_{t+1}^H \right) \right] \end{aligned}$$

An analogous derivation can be conducted in the case of **Real Bonds**. In order to derive the second order approximation to the government budget constraint we use a the recursive formulation, in which:

$$RW_t = d'_{t-1} U_C(C_t) \tag{B.27}$$

and

$$RW_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} U_C(C_T) s_t p_{H,t} \tag{B.28}$$

Defining $\widetilde{RW}_t = \frac{RW_t - \overline{RW}}{\overline{RW}}$, we have:

$$\widetilde{RW}_t = (1-\beta) \left[r b'_y y_t + \frac{1}{2} y'_t R B_y y_t + y'_t R B_e e_t \right] + \beta E_t \widetilde{RW}_{t+1} + t.i.p + O(\|\xi\|^3) \tag{B.29}$$

$$r b'_y = [s_\tau \quad -\rho \quad 1 \quad s_\tau \quad 0]$$

$$R B'_y = \begin{bmatrix} s_\tau & -\rho s_\tau & s_\tau & s_\tau & 0 \\ -\rho s_\tau & \rho^2 & 0 & -\rho s_\tau & 0 \\ s_\tau & 0 & 1 & s_\tau & 0 \\ s_\tau & -\rho s_\tau & s_\tau & s_\tau & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$RB'_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \rho s & 0 \\ 0 & 0 & -s & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

And the first order approximation of the intertemporal budget constraint is:

$$\begin{aligned} -\rho\widehat{C}_t + \widehat{d}'_{t-1} &= (1 - \beta)(-\rho\widehat{C}_t + \widehat{p}_{H,t} + s_\tau(\widehat{\tau}_t + \widehat{Y}_t) - s\widehat{g}_t) \\ &+ \beta E_t \left[-\rho\widehat{C}_{t+1} + \widehat{d}'_t \right] \end{aligned} \quad (\text{B.30})$$

We should note that welfare function just depend on rb'_y, RB'_y and RB'_e which are equal to b'_y, B'_y and B'_e . Therefore the loss function formulation is independent of the denomination of government debt. However, the first order approximation to the government budget constraint changes with the bond denomination. Hence, the constraint of the policy problem varies according to the type of bond being issued by the government.

Morover, we can write the budget constraint as follows:

$$\begin{aligned} -\rho\widehat{C}_t + \widehat{d}'_{t-1} - (a\Delta RS_t + b\widehat{\pi}_t^H) &= (1 - \beta)\left(-\rho\widehat{C}_t - \frac{\lambda}{1 - \lambda}RS_t + s_\tau(\widehat{\tau}_t + \widehat{Y}_t) - s\widehat{g}_t\right) \\ &+ \beta E_t \left[-\rho\widehat{C}_{t+1} + \widehat{d}'_t - (a\Delta RS_{t+1} + b\widehat{\pi}_{t+1}^H) \right] \end{aligned} \quad (\text{B.31})$$

where: $a = \lambda/(1 - \lambda)$ and $b = 1$ in the case of nominal bonds; and $a = b = 0$ in the case of real bonds (in this case $\widehat{d}'_t = \widehat{d}_t$). Alternatively, rescaling the above equation using $\widehat{d}'_t = d_{ss}\widehat{d}_t$ we obtain the government budget constraint as specified in the text (see table 1).

B.6 Welfare

Following Benigno and Benigno (2003), the second order approximation to the utility function can be written as:

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} [U(C_s^j) - V(y_s^j, \xi_{Y,s})] \quad (\text{B.32})$$

$$W_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[w'_y y_t - \frac{1}{2} y'_t W_y y_t - y'_t W_e e_t - \frac{1}{2} w'_\pi \pi_t^2 \right] + t.i.p + O(\|\xi\|^3) \quad (\text{B.33})$$

$$w'_\pi = \frac{\sigma}{\mu k}$$

$$w'_y = [-1/\mu \quad 1 \quad 0 \quad 0 \quad 0]$$

$$W'_y = \begin{bmatrix} \frac{(1+\eta)}{\mu} & 0 & 0 & 0 & 0 \\ 0 & -(1-\rho) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$W'_e = \begin{bmatrix} -\frac{\eta}{\mu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Using the second order approximation of the equilibrium condition, we can eliminate the term $w'_y y_t$. Do so, we will derive the vector Lx , such that:

$$\begin{bmatrix} a_y & d_y & f_y & c_y & b_y \end{bmatrix} Lx = w_y$$

Giving the values of a_y , b_y , f_y , c_y , and d_y defined in this appendix, we have:

$$\begin{aligned} Lx_1 &= \frac{((-1 + \phi)(1 - \theta\rho)\lambda^2 + (-(-1 + \phi)2\theta\rho - 1 + 2\phi)\lambda - \phi)(-1 + \tau)}{\Omega} \\ Lx_2 &= \frac{\Sigma\rho(-1 + \phi) + (\Psi(-1 + \lambda))}{\Omega} \\ Lx_3 &= \frac{(-1 + \phi)\Sigma(1 - \theta\rho)\lambda - \theta(-1 + \phi)\Sigma\rho + \Psi\theta + \phi\Sigma}{\Omega} \\ Lx_5 &= \frac{(-1 + \phi)(1 - \theta\rho)\lambda^2 + (-(-1 + \phi)2\theta\rho - 1 + 2\phi)\lambda - \phi}{\Omega} \end{aligned}$$

where:

$\Psi = ((\eta + 1)\tau - \eta)$, $\Sigma = (-1 + \tau - d_{ss} + d_{ss}\beta)$, $\Omega = -\Psi l - \Sigma\rho - \Psi$, $l = (\rho\theta - 1)\lambda(2 - \lambda)$ and $1 - \phi = 1/\mu$.

Note: these parameters were derived under the special case where there is a symmetric steady state ($G = 0$).

And the loss function L_{t_0} can be written as follows

$$L_{t_0} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} y'_t L_y y_t + y'_t L_e e_t + \frac{1}{2} l_\pi \pi_t^2 \right] + t.i.p + O(\|\xi\|^3) \quad (\text{B.34})$$

where:

$$L_y = W_y + Lx_1 A_y + Lx_2 D_y + Lx_3 F_y + Lx_5 B_y$$

$$L_e = W_e + Lx_1 A_e + Lx_2 D_e$$

$$L_\pi = w_\pi + Lx_1 a_\pi$$

Note that Lx_4 is irrelevant since $C_y = 0$

To write the model just in terms of the output, real exchange rate, taxes and inflation, we define the matrixes N and N_e mapping all endogenous variables into $[Y_t, T_t]$ and the errors in the following way:

$$y'_t = N [Y_t, RS_t, \tau_t] + N_e e_t \quad (\text{B.35})$$

$$N = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -\frac{l+\lambda}{\rho(1-\lambda)} & 0 \\ 0 & -\frac{\lambda}{(1-\lambda)} & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$N_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Equation (B.34) can therefore be expressed as:

$$L_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} \left[\widehat{Y}_t, \widehat{RS}_t, \widehat{\tau}_t \right]' L'_y \left[\widehat{Y}_t, \widehat{RS}_t, \widehat{\tau}_t \right] + \left[\widehat{Y}_t, \widehat{RS}_t, \widehat{\tau}_t \right]' L'_e e_t + \frac{1}{2} l_\pi \pi_t^2 \right] \quad (\text{B.36})$$

$$+ t.i.p + O(\|\xi\|^3) \quad (\text{B.37})$$

where:

$$L'_y = N' L_y N$$

$$L'_e = N' L_y N_e + N' L_e$$

The last step is to eliminate the cross variables terms $\widehat{Y}_t \widehat{RS}_t$. For that we use the following identity (derived from combining the demand function with the risk sharing condition):

$$2\widehat{Y}_t \widehat{RS}_t = \frac{\rho(1-\lambda)}{(1+l)} \widehat{Y}_t^2 + \frac{(1+l)}{\rho(1-\lambda)} \widehat{RS}_t^2 + t.i.p + O(\|\xi\|^3) \quad (\text{B.38})$$

and, therefore:

$$\left[\widehat{Y}_t, \widehat{RS}_t, \widehat{\tau}_t \right]' L'_y \left[\widehat{Y}_t, \widehat{RS}_t, \widehat{\tau}_t \right] = \left[\widehat{Y}_t, \widehat{RS}_t, \widehat{\tau}_t \right]' \begin{bmatrix} l_{yy} & l_{yt} & 0 \\ l_{yr} & l_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \left[\widehat{Y}_t, \widehat{RS}_t, \widehat{\tau}_t \right] \quad (\text{B.39})$$

$$= (l_{yy} + \frac{\rho(1-\lambda)}{(1+l)} l_{yt}) \widehat{Y}_t^2 + (l_{tt} + \frac{(1+l)}{\rho(1-\lambda)} l_{yr}) \widehat{RS}_t^2 + t.i.p + O(\|\xi\|^3) \quad (\text{B.40})$$

Substituting (B.39) into (B.36), we have:

$$L_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} (l_{yy} + \frac{\rho(1-\lambda)}{(1+l)} l_{yt}) \widehat{Y}_t^2 + (l_{tt} + \frac{(1+l)(1-\lambda)}{\rho} l_{yr}) \widehat{RS}_t^2 + \left[\widehat{Y}_t, \widehat{RS}_t \right]' L'_e e_t + \frac{1}{2} l_\pi \pi_t^2 \right] + t.i.p + O(\|\xi\|^3) \quad (\text{B.41})$$

Finally, we rewrite the previous equation as deviations from the target variables:

$$L_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} \Phi_Y (\widehat{Y}_t - \widehat{Y}_t^T)^2 + \frac{1}{2} \Phi_{RS} (\widehat{RS}_t - \widehat{RS}_t^T)^2 + \frac{1}{2} \Phi_\pi (\widehat{\pi}_t^H)^2 \right] \quad (\text{B.42})$$

$$+ t.i.p + O(\|\xi\|^3) \quad (\text{B.43})$$

where:

$$\begin{aligned} \Phi_Y &= Lx5 \left\{ -2\rho\tau + \rho^2 d_{ss} (1-\beta) + \tau + \frac{[\rho\tau - \rho^2 d_{ss} (1-\beta)] (l+\lambda) - \tau\lambda\rho}{1+l} \right\} \\ &+ Lx2 \left\{ (1-\lambda)\lambda + \frac{(-\lambda)(l+\lambda)(1-\lambda) - \theta\lambda(1-\lambda)^2\rho}{1+l} \right\} \\ &+ Lx1 \left\{ 2\rho - \rho^2 + (2+\eta)\eta + \frac{-(\rho - \rho^2)(l+\lambda) - (-1+\rho)\lambda\rho}{1+l} \right\} \\ &+ (\eta+1)(1-\phi) - 1 - \frac{(-1+\rho)(l+\lambda)}{1+l} + \rho^2 \end{aligned}$$

$$\begin{aligned}
\Phi_{RS} = & Lx5 \left\{ \frac{d_{ss}(1-\beta) \left[(l+\lambda)^2 + \lambda^2 - (1+l)(l+\lambda) \right] + \frac{1}{\rho}(1+l)l\tau}{(1-\lambda)^2} \right\} \\
& + Lx3 \left\{ \frac{\lambda(\theta-1)}{1-\lambda} \right\} \\
& + Lx2 \left\{ \frac{(l+\lambda)\lambda \left(\theta - \frac{1}{\rho} \right) - \lambda\theta(1-\lambda)}{\rho} + \theta^2\lambda(1-\lambda) \right\} \\
& + Lx1 \left\{ \frac{l(l+2\lambda) + \frac{1+l}{\rho}l(\rho-1)}{(1-\lambda)^2} \right\} \\
& - \frac{(l+\lambda)(-1+\rho)}{(1-\lambda)\rho^2}
\end{aligned}$$

$$\Phi_{\pi} = \frac{\sigma(1-\phi)}{k} + (1+\eta)\frac{\sigma}{k}Lx1$$

and

$$\widehat{Y}_t^T = q_y^e e_t$$

$$\widehat{RS}_t^T = q_{rs}^e e_t$$

with

$$q_y^e = \frac{1}{\Phi_{RS}} \left[\begin{array}{cccc} \frac{n}{\mu} + Lx1(1+\eta)\eta & -Lx1(1+\eta) & -q_{yg}^e & Lx2(1-\lambda)\lambda \end{array} \right]$$

$$\begin{aligned}
q_{yg}^e = & Lx5 \{ \rho(1+\tau) - \rho^2 d_{ss}(1-\beta) \} \\
& + Lx1 \{ \rho(\rho-1) \} \\
& + Lx2 \{ \lambda^2 - 1 \} \\
& + 1 - \rho
\end{aligned}$$

$$q_t^e = \frac{1}{\Phi_{RS}} \left[\begin{array}{cccc} 0 & 0 & -q_{rg}^e & -Lx2 \left\{ \frac{(l+\lambda)\lambda}{\rho} + \theta\lambda(1-\lambda) \right\} \end{array} \right]$$

$$\begin{aligned}
q_{rg}^e = & Lx5 \left\{ \frac{(l+\lambda)\rho d_{ss}(1-\beta) - l}{1-\lambda} \right\} \\
& + Lx2 \left\{ \frac{(l+\lambda)(1+\lambda)}{\rho} - \frac{\lambda\theta}{1-\lambda} - \theta\lambda^2 \right\} \\
& + Lx1 \left\{ \frac{-l\rho}{1-\lambda} \right\} \\
& + \frac{(l+\lambda)(-1+\rho)}{\rho(1-\lambda)}
\end{aligned}$$

B.6.1 Special Case

Assumptions*: $\rho = \theta = 1$ and $\phi = \lambda$

$$\frac{\Phi_Y}{1-\lambda} = (\eta+1)$$

$$\frac{\Phi_{RS}}{1-\lambda} = 0$$

$$\frac{\Phi_{\pi}}{1-\lambda} = \frac{\sigma}{k}$$

$$\widehat{Y}_t^T = q_y^e e_t$$

where:

$$q_y^e = \frac{1}{1+\eta} \begin{bmatrix} \eta & 0 & -1 & 0 \end{bmatrix}$$

and

$$\widehat{RS}_t^T = 0$$

***Note:** In the text we use the above specification (as in Galí and Monacelli, 2005). However, by inspection of the weights presented in this Appendix, we can verify that the necessary conditions for a zero weight of the exchange rate in the loss function are $\rho\theta = 1$ and $d_{ss}(\phi - \lambda) = 0$.

C Appendix: Optimal Fiscal Policy under Flexible Prices

The optimal policy can be represented by the following Lagrangian: The optimal policy can be represented by the following Lagrangian:

$$\mathcal{L} = E_{t_0} \sum \beta^{t-t_0} \left[\begin{aligned} & \frac{1}{2} \Phi_Y (\widehat{Y}_t - \widehat{Y}_t^T)^2 + \frac{1}{2} \Phi_{RS} (\widehat{RS}_t - \widehat{RS}_t^T)^2 + \varphi_{1,t} \left(\eta \widehat{Y}_t + (1-\lambda)^{-1} \widehat{RS}_t - \omega \widehat{\tau}_t \right) + \\ & + \varphi_{2,t} \left(\begin{aligned} & -d_{ss} \widehat{RS}_t + \widehat{d}_{t-1} - d_{ss} (a \Delta \widehat{RS}_t + b \widehat{\pi}_t^H) + d_{ss} (1-\beta) \left(\frac{1}{1-\lambda} \widehat{RS}_t \right) \\ & - \bar{\tau} (\widehat{\tau}_t + \widehat{Y}_t) + \beta E_t \left[d_{ss} \widehat{RS}_{t+1} - \widehat{d}_t + d_{ss} (a \Delta \widehat{RS}_{t+1} + b \widehat{\pi}_{t+1}^H) \right] \end{aligned} \right) \\ & + \varphi_{3,t} \left(\widehat{Y}_t - \frac{(1+l)}{\rho(1-\lambda)} \widehat{RS}_t \right) \\ & + b d_{ss} \varphi_{2,t_0-1} \widehat{\pi}_{t_0}^H + d_{ss} (a+1) \varphi_{2,t_0-1} \widehat{RS}_{t_0}^H \end{aligned} \right] \\ + t.i.p + O(\|\xi\|^3). \end{aligned}$$

The last line of the Lagrangian contains the constraints for the optimal policy that ensure that the problem is time invariant. And these are the first order conditions:

$$-b d_{ss} (\varphi_{2,t} - \varphi_{2,t-1}) = 0, \quad (C.44)$$

$$\Phi_y (\widehat{Y}_t - \widehat{Y}_t^T) + \eta \varphi_{1,t} - \bar{\tau} \varphi_{2,t} + \varphi_{3,t} = 0, \quad (C.45)$$

$$\Phi_y (\widehat{RS}_t - \widehat{RS}_t^T) + \frac{1}{(1-\lambda)} \varphi_{1,t} - d_{ss} (a+1) (\varphi_{2,t} - \varphi_{2,t-1}) \quad (C.46)$$

$$+ d_{ss} \frac{(1-\beta)}{(1-\lambda)} \varphi_{2,t} + \beta a d_{ss} (E_t \varphi_{2,t+1} - \varphi_{2,t}) - \frac{(1+l)}{\rho(1-\lambda)} \varphi_{3,t} = 0,$$

$$\omega \varphi_{1,t} - \bar{\tau} \varphi_{2,t} = 0, \quad (C.47)$$

and

$$-\varphi_{2,t} + E_t \varphi_{2,t+1} = 0. \quad (C.48)$$

In addition, the first order condition at time t_0 implies $b d_{ss} (\varphi_{2,t_0} - \varphi_{2,t_0-1}) = 0$. Substituting $\widehat{C}_t = \widehat{C}_t^* + \frac{1}{\rho} \widehat{RS}_t$ into the government budget constraint we have

$$\begin{aligned} -\rho d_{ss} \widehat{C}_t^* - d_{ss} \widehat{RS}_t + \widehat{d}_{t-1} - d_{ss} (a \Delta \widehat{RS}_t + b \widehat{\pi}_t^H) &= d_{ss} (1-\beta) \left(-\rho \widehat{C}_t^* - \frac{1}{1-\lambda} \widehat{RS}_t \right) + \\ + \tau (\widehat{\tau}_t + \widehat{Y}_t) - \widehat{g}_t + \beta E_t \left[-\rho d_{ss} \widehat{C}_{t+1}^* - d_{ss} \widehat{RS}_{t+1} + \widehat{d}_t - d_{ss} (a \Delta \widehat{RS}_{t+1} + b \widehat{\pi}_{t+1}^H) \right] \end{aligned} \quad (C.49)$$

Furthermore, under the assumption that $\alpha = 0$ the Phillips curve implies

$$-\omega^{-1} \left(\frac{\eta(1+l) + \rho}{(1+l)} \right) (\hat{Y}_t - \hat{Y}_t^T) = (\hat{\tau}_t - \hat{\tau}_t^{T'}) \quad (\text{C.50})$$

By integrating equation (C.49) forward we can rewrite the intertemporal budget constraint of the government as

$$\begin{aligned} \hat{d}_{t-1}^l - d_{ss} b \hat{\pi}_t^H &= \hat{f}_t + d_{ss} a \Delta \frac{\rho(1-\lambda)}{(1+l)} (\hat{Y}_t - \hat{Y}_t^T) + \\ &\frac{d_{ss} \rho(1-\lambda)}{1+l} (\hat{Y}_t - \hat{Y}_t^T) + \frac{(1-\beta)}{(1+l)} d_{ss} E_t \sum_{s=0}^{\infty} \beta^{t+s} \left[m (\hat{Y}_t - \hat{Y}_t^T) \right], \end{aligned} \quad (\text{C.51})$$

where

$$m \equiv \left(\frac{\bar{\tau}(1-\omega^{-1}\eta)(1+l)}{(1-\beta)} - d_{ss} \rho(1+\omega^{-1}) \right)$$

and

$$\begin{aligned} \hat{f}_t &\equiv d_{ss} a \Delta \frac{\rho(1-\lambda)}{(1+l)} \left[-\hat{C}_t^* - \hat{g}_t + \hat{Y}_t^T \right] + \frac{\rho(l+\lambda)}{1+l} d_{ss} \hat{C}_t^* - \frac{\rho(1-\lambda)}{1+l} d_{ss} g_t + \frac{\rho(1-\lambda)}{1+l} d_{ss} Y_t^T + \\ &+ (1-\beta) d_{ss} E_t \sum_{s=0}^{\infty} \beta^{t+s} \left[\left(\frac{\bar{\tau}}{(1-\beta) d_{ss}} - \frac{\rho}{1+l} \right) \hat{Y}_t^T + \frac{\bar{\tau}}{(1-\beta) d_{ss}} \hat{\tau}_t^{T'} + \frac{\rho - \bar{s}(1+l)}{1+l} g_t - \frac{\rho l}{1+l} C_t^* \right] \end{aligned}$$

The combination of the first order condition implies:

$$\begin{aligned} \frac{\Phi_{RS} \rho(1-\lambda)}{(1+l)} (\hat{Y}_t - \hat{Y}_t^T - \delta_t) + \frac{(1+l)}{\rho(1-\lambda)} \Phi_Y (\hat{Y}_t - \hat{Y}_t^T) \\ + m_0 \varphi_{2,t} - d_{ss} (1+a) (\varphi_{2,t} - \varphi_{2,t-1}) = 0, \end{aligned} \quad (\text{C.52})$$

where $\delta_t \equiv \hat{C}_t^* + \hat{g}_t - \hat{Y}_t^T + \frac{(1+l)}{\rho(1-\lambda)} \widehat{RS}_t^T$ and $m_0 = \frac{1}{1-\lambda} \left(\bar{s} + \frac{\eta(1+l)\bar{\tau}}{\omega\rho} + \frac{\bar{\tau}}{\omega} - \frac{\bar{\tau}(1-l)}{\rho} \right)$. Alternatively we can write:

$$(\hat{Y}_t - \hat{Y}_t^T) = m_1 \delta_t - m_2 \varphi_{2,t} + (1+a) d_{ss} m_3 (\varphi_{2,t} - \varphi_{2,t-1}) \quad (\text{C.53})$$

where

$$\begin{aligned} m_1 &= \left(\frac{(1+l)^2 \Phi_Y + \Phi_{RS} \rho^2 (1-\lambda)^2}{\Phi_{RS} \rho^2 (1-\lambda)^2} \right)^{-1} \\ m_2 &= \left(\frac{(1+l)^2 \Phi_Y + \Phi_{RS} \rho^2 (1-\lambda)^2}{\rho(1+l)} \right)^{-1} m_0 \\ m_3 &= \left(\frac{(1+l)^2 \Phi_Y + \Phi_{RS} \rho^2 (1-\lambda)^2}{\rho(1-\lambda)(1+l)} \right)^{-1} \end{aligned}$$

Moreover, substitution equation (C.53) into equation (C.51) we have:

$$\varphi_{2,t} = \frac{-(1+l)}{(n_1+n_2)} \hat{d}_{t-1} + \frac{1}{(n_1+n_2)} f'_t + \frac{n_1}{(n_1+n_2)} \varphi_{2,t-1} \quad (\text{C.54})$$

and using (C.48) we have

$$\varphi_{2,t} = \frac{-(1+l)}{n_2} \hat{d}_t + \frac{1}{n_2} E_t f'_{t+1} \quad (\text{C.55})$$

where:

$$n_1 = -((1 - \beta)m - d_{ss}\rho(1 - \lambda))m_3d_{ss}$$

$$n_2 = (d_{ss}\rho(1 - \lambda) + m)m_2$$

Therefore the evolution of the lagrange multiplier and government debt can be written as

$$\varphi_{2,t} = \frac{1}{(n_1 + n_2)}(f'_t - E_{t-1}f'_t) + \varphi_{2,t-1} \quad (\text{C.56})$$

$$\widehat{d}_t = \widehat{d}_{t-1} - \frac{n_2}{(1+l)} \frac{1}{(n_1 + n_2)}(f'_t - E_{t-1}f'_t) + \frac{1}{(1+l)}E_t f'_{t+1} \quad (\text{C.57})$$

Case (i) : $d_{ss} = 0$

In this case, equation (C.54) implies:

$$\varphi_{2,t} = \frac{-(1+l)}{n_2}\widehat{d}_{t-1} + \frac{1}{n_2}f'_t \quad (\text{C.58})$$

and

$$\widehat{d}_t = \widehat{d}_{t-1} + \frac{1}{(1+l)}E_t \Delta f'_{t+1} \quad (\text{C.59})$$

Case (ii) : $d_{ss} \neq 0$ and of Nominal Bonds

In this case, we define $a = \frac{\lambda}{(1-\lambda)}$ and $b = -1$. The first order condition at time t_0 combined with (C.44) implies that $\varphi_{2,t}$ is constant over time. In this case, the first order conditions (C.44) to (C.48) can be expressed as the following targeting rule:

$$\Phi_{RS}(\widehat{RS}_t - \widehat{RS}_t^T) + \frac{(1+l)}{\rho(1-\lambda)}\Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T) = 0.$$

Moreover, in the special case where $\rho = \theta = 1$ and $\mu = 1/(1 - \lambda)$,

$$\widehat{Y}_t = \widehat{Y}_t^T. \quad (\text{C.60})$$

Equation (C.51) implies

$$\widehat{d}_{t-1} - d_{ss}b\widehat{\pi}_t^H = \widehat{f}_t; \quad (\text{C.61})$$

that is, if the underlying structural disturbances composing \widehat{Y}_t^T are stationary, output and the real exchange rate will also be stationary. Finally, Equation (C.61) determines the evolution of real debt.

D Appendix: Optimal Fiscal and Monetary Policy when Prices are Sticky:

In this case the Lagrangian is:

$$\mathcal{L} = E_{t_0} \sum \beta^{t-t_0} \left[\begin{aligned} & \frac{1}{2}\Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T)^2 + \frac{1}{2}\Phi_{RS}(\widehat{RS}_t - \widehat{RS}_t^T)^2 + \frac{1}{2}\Phi_\pi \left(\widehat{\pi}_t^H \right)^2 \\ & + \varphi_{1,t} \left(-k^{-1}\pi_t^H + \eta\widehat{Y}_t + (1-\lambda)^{-1}\widehat{RS}_t - \omega\widehat{\tau}_t + \beta E_t \pi_t^H \right) + \\ & + \varphi_{2,t} \left(\begin{aligned} & -d_{ss}\widehat{RS}_t + \widehat{d}_{t-1} - d_{ss}(a\Delta RS_t + b\widehat{\pi}_t^H) + d_{ss}(1-\beta)\left(\frac{1}{1-\lambda}\widehat{RS}_t\right) \\ & -\bar{\tau}(\widehat{\tau}_t + \widehat{Y}_t) + \beta E_t \left[d_{ss}\widehat{RS}_{t+1} - \widehat{d}_t + d_{ss}(a\Delta RS_{t+1} + b\widehat{\pi}_{t+1}^H) \right] \end{aligned} \right) \\ & + \varphi_{3,t} \left(\widehat{Y}_t - \frac{(1+l)}{\rho(1-\lambda)}\widehat{RS}_t \right) \\ & + \varphi_{1,t_0-1}k^{-1}\widehat{\pi}_{t_0}^H + bd_{ss}\varphi_{2,t_0-1}\widehat{\pi}_{t_0}^H + d_{ss}(a+1)\varphi_{2,t_0-1}\widehat{RS}_{t_0}^H \end{aligned} \right] \\ + t.i.p + O(\|\xi\|^3).$$

As before, the last line in the Lagrangian contain constraints in the initial conditions that ensure a time-invariant policy problem. The first order conditions are:

$$\Phi_\pi \widehat{\pi}_t^H - (\varphi_{1,t} - \varphi_{1,t-1})k^{-1} - bd_{ss}(\varphi_{2,t} - \varphi_{2,t-1}) = 0, \quad (\text{D.62})$$

$$\Phi_y(\widehat{Y}_t - \widehat{Y}_t^T) + \eta\varphi_{1,t} - \bar{\tau}\varphi_{2,t} + \varphi_{3,t} = 0, \quad (\text{D.63})$$

$$\begin{aligned} & \Phi_y(\widehat{RS}_t - \widehat{RS}_t^T) + \frac{1}{(1-\lambda)}\varphi_{1,t} - d_{ss}(a+1)(\varphi_{2,t} - \varphi_{2,t-1}) \\ & + d_{ss}\frac{(1-\beta)}{(1-\lambda)}\varphi_{2,t} + \beta ad_{ss}(E_t\varphi_{2,t+1} - \varphi_{2,t}) - \frac{(1+l)}{\rho(1-\lambda)}\varphi_{3,t} = 0, \end{aligned} \quad (\text{D.64})$$

$$-\omega\varphi_{1,t} = \bar{\tau}\varphi_{2,t}, \quad (\text{D.65})$$

and

$$-\varphi_{2,t} + E_t\varphi_{2,t+1} = 0. \quad (\text{D.66})$$

These equations imply that:

$$E_t\widehat{\pi}_{t+1}^H = 0 \quad (\text{D.67})$$

And the first order conditions can be combined and written as:

$$\left[\frac{(1+l)\Phi_y}{(1-\lambda)\rho} \right] \Delta\widehat{y}_t + \Phi_{RS}\Delta\widehat{r}_t + \left[\frac{k\Phi_\pi}{(1-\bar{\tau}) + bd_{ss}k} \right] (\gamma\widehat{\pi}_t^H + d_{ss}(a+1)\widehat{\pi}_{t-1}^H) = 0 \quad (\text{D.68})$$

where $\widehat{r}_t = (\widehat{RS}_t - \widehat{RS}_t^T)$, $\widehat{y}_t = (\widehat{Y}_t - \widehat{Y}_t^T)$, and $\gamma = \left[d_{ss} \left(\frac{(1-\beta)}{(1-\lambda)} - (1+a) \right) + \left(\frac{(1+l)(\eta(1-\bar{\tau}) - \bar{\tau})}{\rho(1-\lambda)} + \frac{1-\bar{\tau}}{(1-\lambda)} \right) \right]$

Moreover, combining the first order condition with the government budget constraint and the Phillips Curve leads to the following expressions:

$$\varphi_{2,t} = \frac{f'_t - E_{t-1}f'_t}{(n'_1 + n'_2)} + \varphi_{2,t-1} \quad (\text{D.69})$$

$$\widehat{d}_t = \frac{E f'_{t+1}}{1+l} - \frac{n'_2\varphi_{2,t}}{1+l} + \frac{n_3 d_{ss}}{(1+l)}(\varphi_{2,t} - \varphi_{2,t-1}) \quad (\text{D.70})$$

$$\Phi_{RS}\widehat{r}_t + \frac{(1+l)}{\rho(1-\lambda)}\Phi_Y\widehat{y}_t = -m_0\varphi_{2,t} + (a+1)d_{ss}(\varphi_{2,t} - \varphi_{2,t-1}), \quad (\text{D.71})$$

where:

$$n'_1 = -((1-\beta)m + d_{ss}\rho(1-\lambda))(a+1)m_3d_{ss} + d_{ss}m_4 + m_5)$$

$$n'_2 = (d_{ss}\rho(1-\lambda) + m)m_2$$

$$n_3 = -a\rho(1-\lambda)(a+1)d_{ss}m_3$$

$$m_4 = \left[a\rho(1-\lambda)(-m_2 + (a+1)d_{ss}m_3) + b\frac{(1+l)}{\Phi_\pi} \left(\frac{\bar{\tau}}{k\omega} + bd_{ss} \right) \right]$$

$$m_5 \equiv \frac{\bar{\tau}(1+l)}{(1-\beta)} \frac{\omega^{-1}}{k} \frac{1}{\Phi_\pi} \left(\frac{\bar{\tau}}{k\omega} + bd_{ss} \right)$$

Figure D.1: Figure 1

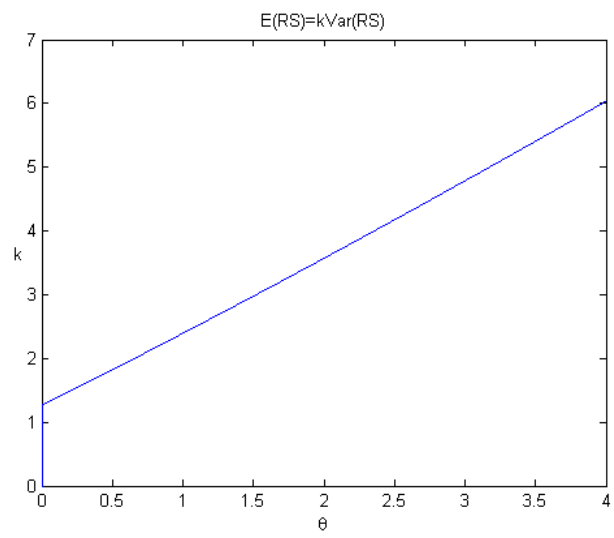
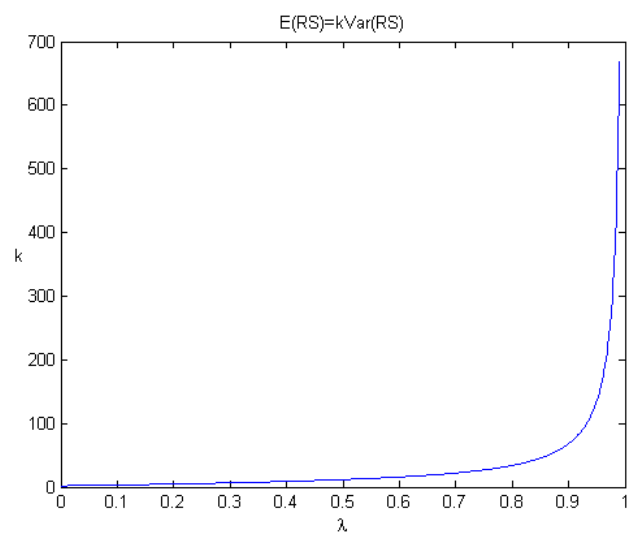


Figure D.2: Figure 2



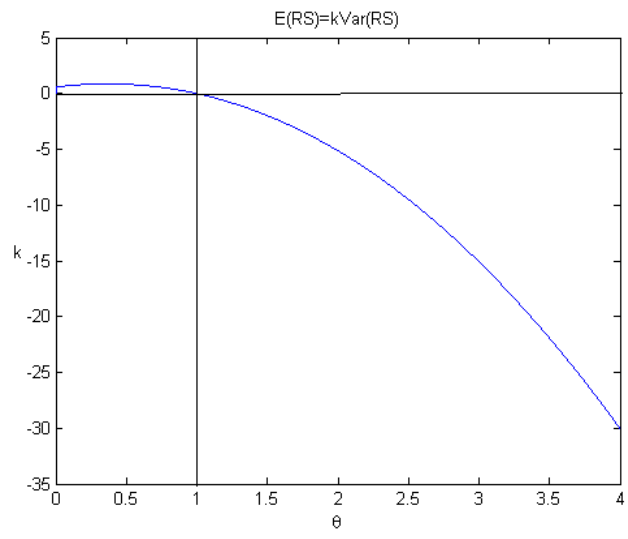


Figure D.3: Figure 3

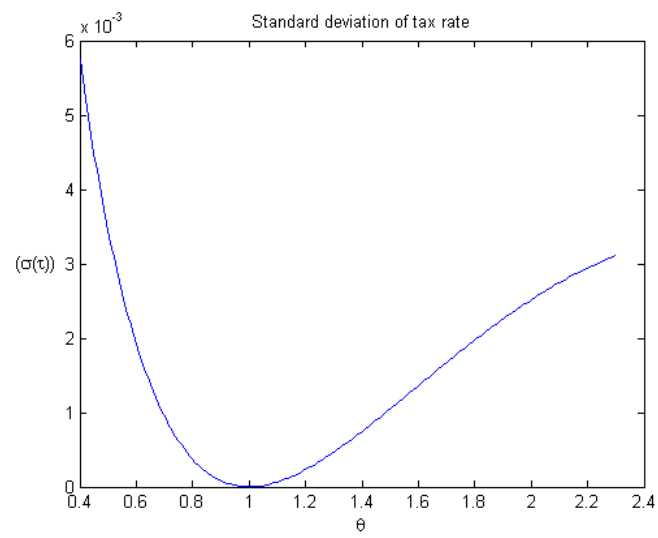


Figure D.4: Figure 4

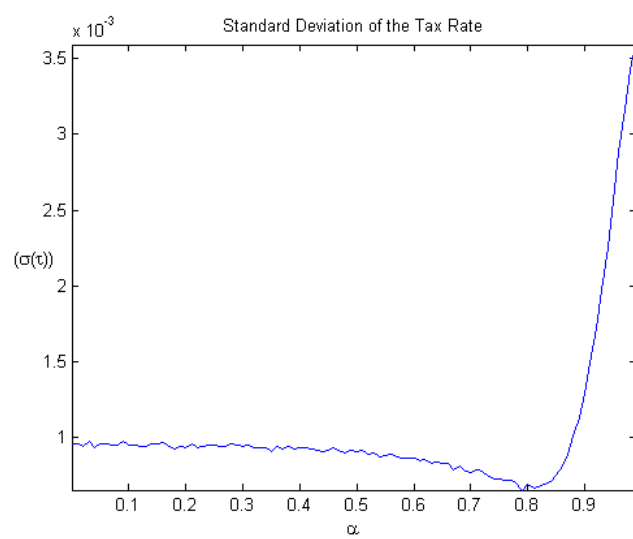


Figure D.5: Figure 5