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FUNCTIONAL DIFFERENTIATION

Susanna Sällström

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Susanna Sällström, University of Cambridge and CEPR

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Centre for Economic Policy Research
53–56 Gt Sutton St, London EC1V 0DG, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Functional Differentiation*

Models of product differentiation typically assume a demand for variety. This paper derives the demand for variety in a model where a representative consumer chooses how many specialised varieties to purchase for the pursuit of different activities. In contrast with previous models this generates a demand for variety that is price and income elastic. In applications to monopoly and duopoly I find that whilst a duopoly will choose efficient characteristics it will offer too many specialised varieties, whereas a monopoly will either offer excessively specialised varieties or too few specialised varieties on the assumption of no fixed costs of variety.

JEL Classification: D31, D42, D43 and L11

Keywords: general purpose goods, product differentiation, specialised goods and variety

Susanna Sällström
Department of Economics and Politics
University of Cambridge
Austin Robinson Building
Sidgwick Avenue
Cambridge
CB3 9DD
Email: se.sallstrom@econ.cam.ac.uk

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FUNCTIONAL DIFFERENTIATION

SUSANNA SÄLLSTRÖM MATTHEWS

1. INTRODUCTION

Bils and Klenow (2001a) found, using US Bureau of Labor Statistics item substitution rates for 160 product categories over the period 1980-1996, that ‘new varieties do increase spending on a category, as well as drive out or replace incumbent varieties.’ Whilst an increase in spending is consistent with a ‘love of variety’ the pattern that has resulted of an increase in variety rather than quality is not. Furthermore since most models assume that consumers buy one variety only or that a representative consumer buys everything,¹ they do not capture the evidence in Jackson (1984) and Gronau (1986,1997) that consumers with higher incomes demand more variety. To understand these empirical regularities we therefore need micro foundations for the demand for variety.

The demand for variety is an important area that has received little attention hitherto, since ‘one has to define the nature of the product, something most economists tend to shy away from’ in the words of Gronau and Hamermesh (2008). Economists have instead focused their attention to firms’ supply of variety, taking the demand for variety as given. As a result standard models of product differentiation cannot explain why attributes of *new* varieties are such that consumers wish to buy more varieties than they did before, and stop buying *old* varieties with different attributes even if those were of higher material quality.

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¹Deneckere and Rotschild (1992) and Anderson et al. (1985) could therefore establish results that all models for demand for variety could be generated from a model with heterogeneous consumers who buy one variety only.

For example there is no analytical difference between new and old spokes in the Spokes model of Chen and Riordan (2007). There is no difference between new and old varieties in the classical models of Dixit and Stiglitz (1977), Perloff and Salop (1985), and Spence (1976) either, or on Salop's (1979) circle where firms allocate symmetrically. Hence, these models cannot explain why new varieties drive out old varieties, since they assume consumers love variety *per se*. The literature on quality up-grades² cannot explain either why many up-grades take the form of added functions and thus result in variety rather than quality growth, as has been shown by Bils and Klenow (2001a).

This paper provides micro foundations for preferences for variety that build on recent empirical evidence in Gronau and Hamermesh (2008) that demand for variety comes from the fact that consumers engage in different activities.³ These preferences are then used to derive the demand for variety in a model where a representative consumer chooses *how many* specialised varieties he should purchase for the pursuit of different activities. For example how many different pairs of shoes should a consumer optimally buy if he wishes to pursue different physical activities like running or hiking.

In this paper, I find that the resulting demand has properties that are consistent with the evidence, i.e. consumers demand more varieties the higher their real income, and consumers prefer more specialised varieties the more varieties they buy in total. Since these preferences give rise to demand for goods that are differentiated to better match specialised functions, this form of product differentiation will be called, *functional differentiation*.

Whether there will be too much or too little variety has typically been studied in models that assume that consumers will buy all varieties that are supplied, and have exogenously given reservation values for each variety (for example Chen and Riordan

²See e.g. Johnson and Myatt (2003) with references, Gordon and Griliches (1997), Grossman and Helpman (1991) and Aghion and Howitt (1992).

³They found that the reason why consumers with higher incomes demand more variety is because they engage in more activities. Note that this would not be consistent with e.g. Dixit and Stiglitz (1977) since lower income consumers would simply buy a smaller amount of each variety rather than a smaller number of varieties in a model based on the assumption that everybody loves variety.

2007, Deneckere and Rotschild (1992), Perloff and Salop 1985). These models have therefore assumed a fixed cost for each variety to limit the total number of varieties. The evidence, as noted above, is however inconsistent with the demand generated from these assumptions.

It is therefore of interest to determine what the effect from market power (monopoly) and strategic interaction (duopoly) will be on product characteristics and the number of varieties when reservation values for varieties are endogenous⁴ and the demand for variety is elastic.

In this paper, I find that a duopoly will choose efficient product attributes but that it will offer too many specialised varieties. A monopoly, on the other hand, will offer excessively specialised varieties of too low quality. Since the demand for variety is elastic these results can be derived without assuming that there is a fixed cost for each variety.⁵

The model in this paper has similarities with the household production model of Becker (1966) as further developed in Gronau and Hamermesh (2008). Thus I model varieties as goods that are used to produce services in different states, where the consumer derives utility from time and the services produced. In their recent study, Gronau and Hamermesh (2008) provided a mechanism that could explain why individuals with higher incomes would engage in more activities. This paper analyses a different problem which is how many different specialised varieties should be bought taking the activities and the time spent in each activity as given.⁶ I furthermore analyse the implications for the equilibrium selection of specialised varieties.

⁴The value of a variety in the model in this paper is endogenously determined by what else the consumer buys.

⁵Since all varieties that are supplied will be demanded in models that assume some form of 'love of variety', it has been necessary to assume a fixed cost of each variety to analyse whether there will be too many or too few.

⁶This has the important implication that the consumer will derive utility from the joint attributes of the varieties they buy as in Lancaster (1966), rather than from the goods *per se*. It implies that the utility from a good is determined endogenously rather than exogenously given as in Deneckere and Rotschild's (1992) general framework. It is a crucial property of the model that drives most of the results in this paper. It is not shared with any of the other models of demand for variety

That consumers engage in different activities is modeled as having multiple addresses on the unit interval and thus resembles Hotelling's (1929) model. However, this paper shows that a different mechanism generates higher prices in equilibrium the more differentiated the goods. This is because the Hotelling model represents a situation where firms are competing for demand from a heterogeneous population of consumers, where the demand for variety is independent of prices. In contrast, in the present paper the demand for variety depends on individual prices as well as the characteristics of individual varieties. What is constraining firms in this case is how to induce consumers to buy *more* varieties rather than inducing them to switch supplier. This difference will be shown to have important implications for the nature of the distortions that arise in a monopoly and duopoly respectively.

The multi-purpose nature of consumption when consumers pursue different activities has important analytical implications for the modeling of product quality. For example if consumption is single-purpose making cycling shoes less wind resistant or more durable can be represented by a one dimensional variable, since they both have the effect of increasing the consumer's utility when cycling. However, in a multi-purpose consumption framework they will be analytically distinct, since a more specialised function that will improve the performance under specific circumstances such as cycling will make the product perform less well under other circumstances such as walking or running. As long as consumption remains multi-purpose, e.g. if the shoes are also used to walk in, there will be a tradeoff between making goods fit for one particular purpose and still usable for other purposes. Improving one particular function of a good comes at a price of making the good less useful for other purposes.

This is an important aspect of product differentiation that has not been modeled before.⁷ Goods are not just produced in new locations, but they can be made more or less specialised to perform a particular function. As in the above example, cycling and is therefore part of the explanation for why this model gives different predictions from previous localised and non-localised models for differentiated products.

⁷Von Ungern Sternberg (1988) noted that different locations could be interpreted as different needs. However he did not model that an increase in specialisation increases utility of a good that is used for its intended use, only that it would lower its value for alternative uses.

shoes can be more or less specialised. By endogenising not only location and the number of varieties but also the product attributes, the theory for functional differentiation provides an explanation for why the optimal attributes of any location will change when consumers buy more varieties in total and therefore why new varieties will drive out or replace old varieties.⁸

The intuition underlying why optimal attributes will change with an increase in the number of varieties is that they depend on usage rather than location, and the usage will change when the consumer owns a larger number of functionally differentiated goods. For example if a child has only one toy, it should optimally be very durable and general-purpose so that it can play multiple roles in the child's imagination and last for many years. If a child, on the other hand, has many different toys, their optimal characteristics should be more specialised to better match specific needs, and since they will be used less frequently they should also optimally be of lower quality. The point is that one single plastic toy is not preferable to a high quality toy, but that several specialised plastic toys may be preferable to one high quality general purpose toy.⁹

The outline of the paper is as follows. Section 2 provides micro foundations for preferences for variety and derives the demand for variety. Section 3 characterises optimal product attributes and derives a condition for when it is optimal to functionally differentiate a good. Section 4 analyses the implications from elastic demand for variety on product selection in a monopoly. Section 5 does the same for a price setting duopoly. Section 6 summarises the main results. Section 7 concludes the paper with a discussion. All proofs can be found in an appendix.

⁸This paper thus shows that people are induced to buy more toys or pairs of shoes not just because they have become more specialised, but also because they have optimally become less durable.

⁹The toy industry is a good example for these phenomena. It is an industry with a few dominant firms and high turnover of small firms (see Pesendorfer (2005)). There is no stability in product varieties and the industry has experienced periods of dramatic variety growth and doubling of sales (see Welsh (1992) for the case of soft toys).

2. A MODEL FOR FUNCTIONAL DIFFERENTIATION

This section provides micro foundations for preferences for variety. These preferences are then used to derive the demand for variety in a model where a representative consumer chooses a subset of varieties, rather than selecting one ‘best buy’, from all available varieties in the same product category. Since all varieties are valuable but also substitutable this model generates a demand that, unlike existing models of variety,¹⁰ is consistent with the evidence that consumers demand more variety the higher their real income (Jackson (1984) and Gronau (1986,1997)). The section concludes by outlining the supply side.

2.1. Micro foundations for preferences for variety. Suppose that there are n varieties of a good in the same product category indexed by the subscript $i = 1, 2, \dots, n$. These varieties have three observable product attributes $a_i = (x_i, t_i, r_i)$ that are analytically distinct. First, the primary function $x_i \in [0, 1]$, where $x_i < x_{i+1}$. Second, a continuous variable $t_i > 0$, which represents how specialised a good is to perform its primary function. Third, a continuous variable $r_i > 0$, which is the quality. The set of the attributes of all available varieties is denoted $A = \{a_1, \dots, a_n\}$. The price of a variety is denoted $p(a_i)$.

A representative consumer can use any subset $B \subset A$ of these varieties for the production of services in heterogeneous states $s \in S = [0, 1]$, where the fraction of time spent in each state is distributed according to $F(s)$ with density $f(s)$.¹¹ The total time is normalised to one. The services produced in each state $q = h(s - x_i, r_i, t_i)$ are a function of the state and the product attributes of the variety used. This production function $h(\cdot)$ has the following properties:

- (1) $h(\cdot) \geq 0$,
- (2) $h_r > 0$ and $h_{rr} \leq 0$,
- (3) $h_{|s-x|} < 0$,

¹⁰This is because models such as ‘love of variety’ and generalisations of Hotelling including Chen and Riordan (2007) assume a demand for variety. All varieties supplied will therefore be in demand in those models.

¹¹It is assumed that the needs are given and are not influenced by the attributes of the tools.

(4) $h_t(0, r, t) > 0$ and $h_{tt} \leq 0$. If $|s - x| > 0$ then $\exists \tilde{t}(|s - x|)$ such that $h_t > 0$ for $t < \tilde{t}$, $h_t = 0$ for $t = \tilde{t}$ and $h_t < 0$ for $t > \tilde{t}$, where $\tilde{t}'(\cdot) < 0$.¹²

(5) $h_{rt} \geq 0$.

It is not possible to produce negative services.¹³ Services h are furthermore increasing in the quality r regardless of the state s , decreasing in the distance $|s - x|$ between the state and the good's primary function and increasing in the degree of specialisation t for a perfect match. If it is not a perfect match the optimal degree of specialisation \tilde{t} will be decreasing with the distance. The difference between r and t is that the former will shift h whereas the latter will change its shape. Thus whilst a general purpose variety with a low t would produce a similar level of services in all states, a specialised variety with a high t would do better in some states and worse in other states *ceteris paribus*.¹⁴ The degree of specialisation and quality are weakly complementary in the production of services.¹⁵

If a consumer has several varieties to choose from the consumer will choose for each state s the variety that can deliver the highest q . The highest services that can be produced in a state s using varieties from B thus solves,

$$(2.1) \quad q(s, B) = \max\{h(s - x_i, r_i, t_i) \mid a_i \in B\}.$$

Thus $q(s, B)$ is the upper envelope of all the h functions for each variety $a_i \in B$. It therefore represents the efficient production plans. The solution to this problem gives us the optimal *usage* $S_i = [s_{i-1}, s_i]$ of each variety a_i . Let j and k be the closest substitutes to a_i in B where $x_k < x_i < x_j$. The marginal states s_{i-1} and s_i can then

¹²Any technology of the following form $h(s - x, r, t) = \max\{v(r, t) - g(t, |s - x|), 0\}$ will satisfy these assumptions.

¹³If a consumer could only produce negative services in a state the consumer would prefer not to produce any services at all. hence in those cases h would be zero.

¹⁴Specialised and general purpose varieties have been modeled in different ways in the literature. For example Bar-Isaak (2007) models general purpose as interval on the Hotelling line, and specialised as the quality at the end points of the lines, whereas von Ungern Sternberg (1988) lets the cost of transport measure how general purpose a good is.

¹⁵For example for some goods like hiking boots quality and degree of specialisation may be complementary.

be found by solving,

$$(2.2) \quad h(s_i - x_i, t_i, r_i) = h(s_i - x_j, t_j, r_j)$$

$$(2.3) \quad h(s_{i-1} - x_i, t_i, r_i) = h(s_i - x_k, t_k, r_k).$$

Hence the services produced in a marginal state will be the same using either variety. The optimal usage plays an essential role in explaining why the demand for variety is elastic. It captures the intuition that the larger the number of varieties a consumer has, the less use will he make of each of them.

The consumer's preferences over different subsets B , $U(B) = \theta \bar{q}(B)$, depend on the consumer's marginal valuation of services $\theta > 0$ and a weighted average of the services produced in different states,

$$(2.4) \quad \bar{q}(B) = \int_0^1 q(s, B) f(s) ds.$$

Thus the services delivered in states that are more frequent are assigned a greater weight.¹⁶ How much a consumer values a variety a_i therefore depends on the marginal product of this variety in the bundle it will be part of. The marginal product is

$$(2.5) \quad R_{a_i}(B) = \bar{q}(B) - \bar{q}(B \setminus a_i) = \int_{s_{i-1}}^{s_i} [h(s - x_i, r_i, t_i) - q(s, B \setminus a_i)] f(s) ds.$$

It is increasing in the marginal increment in services $h(s - x, r_i, t_i) - q(s, B \setminus a_i)$, the optimal usage S_i and the frequency $f(s)$.

If the consumer buys more than one variety, the consumer will therefore only include varieties that can perform better in a subset of states than other varieties, i.e. goods that are *specialised* to perform a particular function better than other goods. Since this form of product differentiation is neither vertical nor horizontal we shall define it as follows:¹⁷

¹⁶Note the difference with a standard production function that defines the maximum output from a set of inputs for the production of one good. Here we are producing an infinite number of services using a fixed number of inputs. Hence, the main difference is that an input that has been used to produce services in one state is not 'consumed' but can also be used to produce services in another state, for example the same pair of shoes can be used in different activities.

¹⁷The model can also be extended to analyse vertical differentiation, by introducing heterogeneity with respect to θ and varieties that are strictly preferred to another cheaper variety. It could also

Definition 2.1. Two varieties $\{a_i, a_j\} \subset A$ are *functionally differentiated* if their usage S_i and S_j are both non-empty sets.

The marginal product can furthermore provide an explanation for why the optimal attributes of individual varieties will change when a consumer purchases more varieties in total. For example if a consumer can only afford to buy one car the consumer will prefer more general purpose attributes, whereas if the consumer can afford to buy several different cars he would prefer more specialised cars.¹⁸

Thus whilst the marginal product of a general purpose variety will be higher than the marginal product of a specialised variety if the consumer buys only one variety, the marginal product of a specialised variety will be higher than the marginal product of a general purpose variety if the consumer buys two. This provides one possible explanation for why a larger number of new varieties have driven out old varieties.

The fact that an increase in total number of varieties reduces the marginal product of other varieties a consumer buys has one important implication, which ensures uniqueness, namely that

Lemma 2.2. *The marginal product of each variety $a_i \in A$, is lowest in a basket containing all varieties, $R_{a_i}(A) \leq R_{a_i}(B), \forall B \subset A$.*

This follows since the usage will be the lowest and the marginal increment the lowest when the consumer buys all varieties. Thus whilst the consumer can increase \bar{q} by buying more functionally differentiated varieties, the more varieties he buys the less will he be willing to pay for each of them. This is why the demand for variety will depend on how much the consumer can pay for services θ .

These preferences will be used to derive the demand in a model where a representative consumer chooses how many varieties to buy.

2.2. Demand for variety. The demand for variety is the solution to a discrete choice problem where a representative consumer chooses a subset of all available varieties

be extended to allow for horizontal differentiation by introducing heterogeneity with respect to the distribution over states.

¹⁸See the empirical evidence in Pakes (2003) and Petrin (2002) on attributes of new automobiles.

$B \subset A$ with the objective to maximise the net surplus,

$$(2.6) \quad \theta \bar{q}(B) - P(B),$$

where $P(B) = \sum_{a_i \in B} p(a_i)$ denotes the price of the basket containing a subset B of all available varieties.¹⁹

The optimal subset of varieties B^* has to satisfy the following set of inequalities,

$$(2.7) \quad \theta \bar{q}(B^*) - P(B^*) \geq \theta \bar{q}(B) - P(B) \quad \forall B \subset A.$$

These conditions can alternatively be written

$$(2.8) \quad \theta [\bar{q}(B^*) - \bar{q}(B)] \geq P(B^*) - P(B).$$

Hence, for all subsets of varieties that are part of the optimal basket it has to be true that the marginal value of that subset is higher than the price. Similarly for all subsets of varieties that are not part of the optimal bundle it has to be the case that their marginal value is less than the price. Since each variety will be worth less the larger the total number of varieties, the consumer will buy more varieties in total the higher is θ and the lower the prices.

In equilibrium demand for variety must equal supply. Hence, the case that is relevant for the derivation of the equilibrium selection of varieties A^* is when the consumer optimally demands all varieties.

Proposition 2.3. *The consumer's maximisation problem has a unique solution which is to buy all varieties $B^* = A$ if $p(a_i) \leq \theta R_{a_i}(A)$ for all $a_i \in A$.*

For prices such that it is optimal to buy all available varieties, all other subsets would make the consumer less satisfied. Thus for those prices we get a unique prediction for demand. However, if $p(a_i) > \theta R_{a_i}(A)$ the solution may be non-unique. In this case we shall assume that the demand is split equally among baskets that give the same net surplus. The demand for an individual variety i , y_i thus depends on whether $a_i \in B^*$ and whether B^* is unique.

¹⁹Hence we shall consider the case of linear pricing only. However, the model could easily be extended to bundling by allowing for non-linear pricing.

Proposition 2.4. *Demand y_i is elastic for $p(a_i) > \theta R_{a_i}(A)$ and inelastic for $p(a_i) \leq \theta R_{a_i}(A)$.*

When goods are functionally differentiated demand is only sensitive to price and income if the optimal bundle does not contain all varieties. Hence, when consumers buy all available varieties the firm cannot increase the demand for his variety by lowering the price further. The demand therefore becomes inelastic when the price is equal to the marginal value of that variety in a basket containing all varieties. This demand differs from the previous literature in that it captures the insight that a lower price does not only increase the demand for one particular variety, but that it also increases the number of varieties a consumer will buy in total.

For the rest of the paper I shall confine my attention to two cases. The first is, the demand when there is just one general purpose variety $A = \{a_G\}$. In this case demand will be given by,

$$(2.9) \quad y_G = \begin{cases} 1 & \text{if } p(a_G) \leq \theta \bar{q}(a_G), \\ 0 & \text{otherwise.} \end{cases}$$

The second is the case in which there are two functionally differentiated varieties, $A = \{a_1, a_2\}$. The demand for variety $i = 1, 2$ is then given by,

$$(2.10) \quad y_i = \begin{cases} 1 & \text{if } p(a_i) \leq \theta R_{a_i}(A), \text{ or } \theta R_{a_i}(A) < p(a_i) < \min\{\theta \bar{q}(a_i), \Delta_{ij} + p(a_j)\} \\ \frac{1}{2} & \text{if } \theta R_{a_i}(A) < p(a_i) = \Delta_{ij} + p(a_j) < \theta \bar{q}(a_i) \\ 0 & \text{otherwise} \end{cases}$$

where $\Delta_{ij} = \theta [\bar{q}(a_i) - \bar{q}(a_j)]$ is the difference in utility that can be produced in all states using a_i and a_j respectively.

These two cases are sufficient to analyse whether there are any distortions in the decision whether or not to offer two specialised varieties in place of a general purpose variety.

2.3. Supply side. One important implication of the fact that demand for variety is downward sloping in price is that the number of varieties will be limited in equilibrium even if there are no fixed costs of product variety. Hence I shall assume that there is no

fixed cost of variety. Instead there is a constant per unit cost of a good that depends on quality and degree of specialisation $c(r, t)$, where $c_r > 0, c_{rr} \geq 0, c_t \geq 0, c_{tt} \geq 0, c_{rt} \geq 0$.

To determine the effect of market power and strategic behaviour on product selection when the demand for variety is elastic I shall derive the equilibrium selection of varieties A^* for three cases. First, I derive the social optimum where the price of each variety is equal to marginal cost, and varieties have attributes that maximise social surplus. Next, I derive the monopoly supply of functionally differentiated goods, where the monopolist chooses A and $p(a_i)$. Finally, I consider product selection in a price setting duopoly.

3. SOCIAL OPTIMUM

In order to determine whether market power or strategic behaviour give rise to any distortions in product selection in the case of specialised goods, and the nature of such distortions when they arise, this section presents the conditions that determine socially optimal attributes and whether it is optimal from a social point of view to functionally differentiate a good.

The optimal product attributes maximise

$$(3.1) \quad \max_{a_i \in A} U(A) - C(A).$$

where $C(A) = \sum_{a_i \in A} c(a_i)$.

Optimal attributes of each variety i solves the following first order conditions:²⁰

$$(3.2) \quad \frac{\partial U(A)}{\partial x_i} = \int_{s_{i-1}}^{s_i} \theta \frac{\partial h(s - x_i, t_i, r_i)}{\partial x_i} f(s) ds = 0,$$

$$(3.3) \quad \frac{\partial U(A)}{\partial t_i} - c_{t_i} = \int_{s_{i-1}}^{s_i} \theta \frac{\partial h(s - x_i, t_i, r_i)}{\partial t_i} f(s) ds - c_{t_i}(r_i, t_i) = 0$$

$$(3.4) \quad \frac{\partial U(A)}{\partial r_i} - c_{r_i} = \int_{s_{i-1}}^{s_i} \theta \frac{\partial h(s - x_i, t_i, r_i)}{\partial r_i} f(s) ds - c_{r_i}(r_i, t_i) = 0.$$

²⁰Note that we only need to consider the direct effect, since the indirect effect on s_{i-1} and s_i are zero by the envelope theorem.

Hence the location x_i is chosen such that it maximises the weighted average of services for its optimal usage S_i . Degree of specialisation t_i is determined by its impact on the average performance for its optimal usage. Note that there exists a t that maximises the weighted utility for any usage which is decreasing in the number of different uses.

Remark 3.1. Functionally differentiated goods will be more specialised to perform their primary function the more specialised their usage if $c_t = 0$.

However if $c_t > 0$ t will be optimally chosen where the marginal utility from t is positive, hence it will be lower than the t that would maximise performance for those states. There are two effects from a good being used less. For example there is, on the one hand, an incentive to make a bridal dress very specialised because it is single purpose. On the other hand, the fact that it will only be used once will still limit the resources that are allocated to making it more specialised for this purpose and this purpose only.

Quality r_i is chosen such that the marginal value of an increase in quality given the goods optimal usage is equal to the marginal cost.

Remark 3.2. The quality of a specialised variety will be lower than a general purpose variety unless quality is complementary to function $h_{rt} > 0$.

The less the good will be used the lower the optimal quality. This is a mechanism that can explain why general quality attributes, such as durability of shoes may be optimally reduced when consumers buy more shoes in total. However, for some specialised shoes such as hiking boots where quality is complementary with function quality will not be reduced. Hence the model predicts that whether or not a more specialised variety that is used less frequently will optimally be of higher or lower quality depends on whether quality is complementary to function or not.

Socially optimal attributes are denoted $a_i^* = (x_i^*, t_i^*, r_i^*)$.

To functionally differentiate a good can be described without loss of generality²¹ as replacing a ‘general’ purpose variety G that is used in all states S with two functionally differentiated varieties $i = 1, 2$ that are used in two mutually exclusive subsets S_i , where $S_1 \cap S_2 = \emptyset$. Thus $A_1^* = a_G^*$ and $A_2^* = \{a_1^*, a_2^*\}$. Furthermore let $\Delta_c = c(a_1^*) + c(a_2^*) - c(a_G^*)$.

It is optimal to replace a general purpose variety with optimal characteristics with two functionally differentiated varieties if,

$$(3.5) \quad \theta [\bar{q}(\{a_1^*, a_2^*\}) - \bar{q}(a_G^*)] > \Delta_c.$$

Thus if it is costly to functionally differentiate, i.e. $\Delta_c > 0$, it will only be socially optimal to functionally differentiate if θ is sufficiently high to motivate the marginal increase in services produced across states \bar{q} .

Thus the evidence in Bilal and Klenow (2001a) whereby a larger number of new varieties have replaced old ones can be explained by optimal changes in product attributes when goods become functionally differentiated. The question then is what the incentives are for firms to functionally differentiate goods and whether there are any distortions. On the one hand, it is possible to increase utility in a product category by differentiating goods functionally. On the other hand, the more functionally differentiated a good is the less surplus can be extracted. The first provides an incentive for functional differentiation. The second provides an incentive for not differentiating. In what follows I shall demonstrate how these two incentives distort choices in a monopoly and a price setting duopoly respectively.

4. MONOPOLY

This section shows that the standard textbook results of product selection in a monopoly (Tirole 1988) rely on an assumption of inelastic demand for variety. With an endogenous demand for variety a monopolist will no longer choose efficient locations, nor will it choose an efficient quality when the the marginal valuation of quality is

²¹As long as consumption remains multi-purpose functional differentiation of any good could be described as replacing a variety that is being used in ‘all’ states with varieties that are only used in a subset of states.

the same for the average and the marginal consumer.²² On the contrary when the consumer can choose how many varieties to purchase the monopolist will have an incentive to distort all product characteristics to maximise what he can charge for all varieties subject to the constraint that the monopolist needs to induce the consumer to buy all varieties. Thus this section presents a mechanism that generates distortions in a monopoly when a consumer can choose how many varieties to buy. This mechanism has not been present in earlier models since they treat the number of varieties a consumer can buy as exogenous.²³

Since distortions in product characteristics happen when functionally differentiated goods are sold separately the results in this section shows why there is a strong incentive to bundle functionally differentiated goods. Hence, it provides a rationale for the evidence in Bils and Klenow (2001a) for why ‘quality improvements’ frequently takes the form of added functions, and therefore result in variety rather than quality growth.

The monopolist chooses product characteristics that maximises the profit. If the monopoly only sells one variety the maximisation problem becomes

$$(4.1) \quad \max_{x,t,r} [p_G - c(q, t)] y_G$$

where $y_G = 1$ if and only if $p_G \leq \theta \bar{q}(a_G)$. The first order conditions to this problem are the same as those that maximise social welfare (3.2). Hence there are no distortions for a general purpose variety.

However, when the monopolist offers two specialised varieties it distorts all characteristics. In this case the profit is given by,

$$(4.2) \quad \max_{x_i, t_i, r_i} [p_1 - c(r_1, t_1)] y_1 + [p_2 - c(r_2, t_2)] y_2$$

²²It is well established that distortions from this marginal condition will arise when consumers are heterogeneous both in terms of variety (White 1977) and quality (Mussa and Rosen (1978) and Donnenfeldt and White (1990)). This paper however shows that when goods are functionally differentiated distortions from the marginal condition will also arise when consumers are homogeneous.

²³One honorable exception is Alger (1999) who established a general result for quantity discounts by allowing for multiple purchases in a model of quantity discrimination.

subject to the demand (2.10). This demand states that $y_1 = y_2 = 1$ if and only if,

$$(4.3) \quad p_1 \leq \theta R_{a_1}(\{a_1, a_2\}),$$

and

$$(4.4) \quad p_2 \leq \theta R_{a_2}(\{a_1, a_2\}).$$

The monopolist will therefore maximise the difference between $\theta R_{a_i} - c(a_i)$ for each variety. Let the solution to this problem be denoted $a_i^M = (x_i^M, t_i^M, r_i^M)$.

Proposition 4.1. *The monopoly will choose excessively extreme locations: $x_1^M < x_1^*$, $x_2^M > x_2^*$, too low a quality $r_i^M < r_i^*$, and too high a degree of specialisation $t_i^M > t_i^*$.*

The intuition for this result is that the monopolist will internalise the externality on the price for the other variety when choosing product attributes. The monopoly can charge more for each variety the less valuable they are if used as general purpose. This can be achieved by choosing more extreme locations, making them more specialised and reducing the quality. As a result the monopoly will not maximise the gross surplus from functionally differentiated goods.

Thus if a monopoly functionally differentiates the pattern for changes in product characteristics will be similar to the social optimum. However they will be accentuated. Hence a reduction in quality when goods become functionally differentiated may either be due to market power or separability of r and t in the production of services. Similarly an increase in the degree of specialisation may be due to a low marginal cost of t , or be a distortion as a result of market power.

However, consumers may nonetheless be better off from functional differentiation in a monopoly.

Proposition 4.2. *Consumers will never be worse off but may be better off from functional differentiation in a monopoly.*

This is because consumers have always the option to only buy one variety. Thus as long as the monopoly does not distort varieties so much that they are completely useless for other purposes, the consumer will be left with a positive surplus if the

monopoly differentiates. However, the fact that a monopolist can extract less surplus if it differentiates implies that it may choose not to offer functionally differentiated goods even if it would be socially optimal to do so.

Proposition 4.3. *A monopoly offers too few varieties unless it can bundle.*

If a monopoly can bundle functionally differentiated goods there will be no distortions in either attributes nor in the decision whether or not to differentiate. This is because the monopolist's objective function then coincides with the social welfare.

5. BERTRAND DUOPOLY

This section shows that endogenous demand for variety also generates novel and interesting results in a Bertrand duopoly. With the standard assumption that product differentiation only affects the cross price elasticity of demand, equilibrium prices will be higher if the firms differentiate due to the effect it has on the slope and intercept of the reaction functions.²⁴ Firms will therefore generally have an incentive to differentiate if they compete in price. Hence, in these models the incentive to differentiate does not depend on the parameter values of the problem. These models can therefore not explain why firms would choose to differentiate more over time.

With endogenous demand for variety, on the other hand, firms may or may not choose to functionally differentiate depending on parameter values. Hence, elastic demand for variety will also, in the Bertrand model, explain why there will be more differentiation when the parameter values such as income change.²⁵

The reason for this is that, while functional differentiation will not change the slope of the reaction function, it may change the intercept depending on the parameter values. As a result, firms may be able to charge more than the marginal cost in equilibrium. However if they charge too high prices, the best response function of each firm will still be as aggressive as in regular Bertrand Competition. Hence, firms will not wish to functionally differentiate goods in general. It is only if consumers value

²⁴See e.g. Shy (1996).

²⁵If the income elasticity of demand with respect to function is more than one, $\epsilon_t > 1$, the individual will be prepared to pay more for improved services, i.e. θ increase.

function enough to support both varieties in equilibrium that they will differentiate. This can be shown to have important implications for the nature of the distortions that do arise in a price setting duopoly.

Consider a two stage version of a Bertrand game. In the first stage, two firms $i = 1, 2$ choose product attributes a_i simultaneously and independently. In the second stage they can observe all attributes prior to choosing prices p_i simultaneously and independently. The payoff to each firm i is $\pi_i = [p_i - c(a_i)]y_i$ where the demand is given in (2.10). This game will be solved backwards.

In the final stage there are three possibilities. First, if firms have chosen the same attributes we get regular Bertrand competition and thus prices $p_i = c(a_i)$. When firms have chosen different attributes of their product, the best response correspondence of firm i , $B_i(p_j)$ depends on whether the marginal value of a specialised variety $\theta R_{a_i}(\{a_1, a_2\})$ covers its unit cost of production $c(a_i)$.

If functional differentiation is not viable, that is $\theta R_{a_i}(\{a_1, a_2\}) < c(a_i)$, the best response correspondence is,

$$(5.1) \quad B_i(p_j) = \begin{cases} c(a_i) & \text{if } p_j \leq c(a_i) - \Delta_{ij} \\ \Delta_{ij} + p_j - \varepsilon & \text{if } p_j > c(a_i) - \Delta_{ij}. \end{cases}$$

The firms will in this case compete for demand until the price reaches their marginal cost. Thus in this case the problem is similar to the standard Bertrand case, where firms will offer a better deal and get all the demand as long as this results in a positive profit, i.e., $\Delta_{ij} + p_j - \varepsilon - c(a_{ij}) > 0$.

Whereas if functional differentiation is viable, that is $\theta R_{a_i}(\{a_1, a_2\}) \geq c(a_i)$, the best response correspondence is

$$(5.2) \quad B_i(p_j) = \begin{cases} \theta R_{a_i}(\{a_1, a_2\}) & \text{if } p_j \leq \theta R_{a_j}(\{a_1, a_2\}) \\ \Delta_{ij} + p_j - \varepsilon & \text{if } p_j > \theta R_{a_j}(\{a_1, a_2\}). \end{cases}$$

In this case firms have no incentive to choose $p_i < \theta R_{a_i}(\{a_1, a_2\})$ since the demand is then inelastic. However, if the other firm charges a price in excess of the reservation price for their variety $p_j > \theta R_{a_j}(\{a_1, a_2\})$, firm i could increase its profit by increasing its price and still sell one variety. Firm j would in this case sell nothing, since the

consumer would only buy one variety. Thus the only equilibrium happens when both firms charge $p_i = \theta R_{a_i}(\{a_1, a_2\})$ for $i = 1, 2$.

Remark 5.1. When functional differentiation is viable $\theta R_{a_i}(a_1, a_2) \geq c(a_1)$, Bertrand competition drives prices down to the value of their marginal product.

Thus, when deciding on product attributes the firms know, depending on parameter values, that they will either end up competing for demand if functional differentiation is not viable, or choosing a price that is sufficiently low to induce the consumer to buy both varieties if it is viable.

There are two possible sub-game perfect equilibria of the two stage Bertrand game that are stated in the next proposition.

Proposition 5.2. *If $\theta R_{a_i^*}(\{a_1^*, a_2^*\}) < c(a_i^*)$ then the two stage Bertrand game has a sub-game perfect equilibrium in which both firms choose a_G^* and $p = c(a_G^*)$ and gets demand $y_i^* = \frac{1}{2}$. Whereas, if $\theta R_{a_i^*}(\{a_1^*, a_2^*\}) \geq c(a_i^*)$ firms will functionally differentiate their goods and choose attributes a_i^* and price $p_i = \theta R_{a_i^*}(\{a_1^*, a_2^*\})$ and get demand $y_i^* = 1$ each.*

The intuition for why attributes will be efficient runs as follows. If functional differentiation is not viable, both firms will choose to produce a general purpose variety with efficient product attributes. This is because optimal attributes enable a firm to either offer a better deal or to match any strategy of the competitor. In this case the representative consumer will only buy one unit in total.

If functional differentiation is viable, the firms will differentiate and sell products with different attributes that are efficient. This is because the reservation price $\theta R_{a_i}(\{a_1, a_2\})$ taking the attributes of the other variety as given will be maximised for a_i^* . If functional differentiation is viable firms will make non-negative profits in equilibrium and sell one unit each. Hence, in this case the representative consumer will buy two units.

The reason for efficient product characteristics is a Prisoner's Dilemma-style mechanism. The firms would collectively be better off maximising the sum of the marginal

products, however they do each individually have an incentive to deviate and offer efficient characteristics since this maximises their individual profit.

Hence there is no incentive to distort product attributes. However, it can be shown that the incentive to differentiate is too strong.

Proposition 5.3. *Firms in a Bertrand duopoly will offer too many varieties.*

The mechanism that leads to excessive differentiation in a duopoly is the positive externality on price from the other firm choosing to offer a functionally differentiated variety. This implies that the firm can charge more than the social marginal value of functional differentiation for the goods. From this follows that:

Corollary 5.4. *Consumers may be worse off as a result of functional differentiation in a Bertrand Duopoly.*

Since consumers pay the marginal cost for a general purpose variety, and more than the marginal cost if they are functionally differentiated in a Bertrand duopoly, the effect on consumers' surplus may be negative.

6. SUMMARY OF MAIN RESULTS

This paper has shown that the empirical patterns for demand for variety can be explained by a model where a consumer chooses how many specialised varieties to purchase for the pursuit of different activities. For example, it explains why those who can pay more for services will demand more specialised varieties. It also explains why attributes of new varieties are such that consumers wish to buy a larger number of varieties in total and cease to be interested in the old varieties. It furthermore explains why 'quality improvements' often take the form of added functions rather than a genuine increase in quality. Finally, it explains why the quality of some products may be reduced when they become functionally differentiated. The model can also explain cross-country differences in product selection, since it would, for example, predict that there will be more varieties catering for cold conditions in a country with extensive periods of cold weather.

The analysis of a monopoly and a duopoly furthermore provides insights into the nature of the distortions in product selection which occur when demand for variety is elastic.

The table below summarises the distortions that arise in a monopoly and Bertrand duopoly respectively from functional differentiation of goods, and the implications for consumers' surplus.

	Monopoly	Bertrand Duopoly
Attributes	Distorts all	No distortion
Varieties	Too few	Too many
Consumers' Surplus	Weakly improved	Reduced

There are two types of distortions that may arise. First, there is an incentive to distort attributes. Second, there is an incentive to distort number of varieties. The most interesting observation to be derived from these distortions is their effect on efficiency versus consumer's surplus. In the monopoly there are more distortions, however consumers may nonetheless be better off from functional differentiation since the monopoly can extract less consumers' surplus if it differentiates. In the price setting duopoly on the other hand, attributes are optimal. However consumers may nevertheless be worse off for two reasons. First, the firms have an incentive to differentiate even if it is optimal not to do so. Second, functional differentiation enables firms to charge more than the marginal cost if it is viable in equilibrium.

Thus the direction of change in welfare and consumers' surplus are the opposite in a monopoly and duopoly respectively.

7. DISCUSSION

Product differentiation has typically been analysed in models that rely on the assumption that a consumer buys at most one unit of one variety.²⁶ This paper has

²⁶Deneckere and Rotschild (1992) showed that several standard models of product differentiation can be generated from the set of possible preference patterns when the consumer buys at most one

provided micro foundations for preferences for variety that allows us to derive the demand for variety in a model where a consumer can choose any subset of varieties from all available varieties. The resulting patterns for demand differ from other models in a way that is consistent with the empirical evidence, thus supporting the plausibility of the set up.

This paper has thus shown that the general patterns for product variety can be explained as functional differentiation of goods. The paper has confined attention to functional differentiation of goods with homogeneous consumers. However the set up renders itself suitable for various extensions. For example it could easily be extended to consider consumers that are heterogeneous with respect to how much they are willing to pay for services (i.e. vertical differentiation) or to consumers who differ with respect to how frequently they need to produce services in different states (i.e. horizontal differentiation). These are two lines of research which could provide new potentially useful insights about product selection.

One important prediction of the model is that it will not give rise to a ‘pure love of variety’, which is the standard assumption in the classical models of product differentiation due to Spence (1976), Dixit and Stiglitz (1977), Hotelling (1929), and Salop (1979) as well as more recent models such as Chen and Riordan (2007). Instead, this paper has shown that the optimal attributes of a product depend on the total number of varieties an individual buys. New varieties will therefore replace or drive out old varieties not because they are individually better, but because they are collectively better.

In applications of the model to monopoly and a price setting duopoly this paper also finds novel and interesting implications from elastic demand for variety. This is because the mechanism that gives rise to distortions in the model for functional differentiation differs from that described in the literature.

In the Hotelling model there are two strategic effects that give a higher price when firms are located further afield and transport costs are higher, whereas with elastic

unit of one variety only. These preferences are described by exogenously given reservation values for different brands.

demand for variety the higher prices are due to a positive externality that allows the other firm to charge more for their variety.

Since this externality is not internalised in a duopoly, product characteristics will be efficient, whereas in the Hotelling model they will be distorted.²⁷ The distortions in product characteristics therefore arise in a monopoly since the externality is then internalised, whereas in the Hotelling model they will be efficient since there are no strategic effects on price in the monopoly.²⁸

Furthermore, the positive externality on the other firm's price from offering a more specialised variety implies that the incentive to offer a new specialised variety will be higher if a firm expects the other firm to offer a specialised variety as well. The price that can be charged for a new variety will therefore also be higher than the marginal social value of having the good functionally differentiated. As a result there will be too much functional differentiation in a duopoly. A monopoly, on the other hand, will differentiate too little, since functional differentiation, even with distorted characteristics allows the monopoly to extract less surplus. These results can be contrasted with the study of Deneckere and Rothschild (1992), who showed that too much variety arises in Salop (1985) because it is a less competitive environment, and too little variety happens in Dixit and Stiglitz (1977) because it is a more competitive environment.

Hence the prediction of the present model is that companies who specialise in one particular variety, such as hiking boots, are more likely to choose optimal product characteristics than a company that does everything. Alternatively a specialised toy manufacturer is less likely to distort characteristics, whereas indeed a monopoly like

²⁷This result can also be contrasted with Von Ungern Sternberg (1988) who found that there was an incentive to make products too general purpose when firms compete for customers. Note that general purpose has two meanings. For example a good that is a bundle of several specialised varieties is multi-purpose, which is different from regular shoes with no particular features. Thus what Von Ungern Sternberg modeled was in the terminology of this paper more like a bundle of functionally differentiated goods. Which is also consistent with his assumption that it is costly to make a good more multi-purpose.

²⁸Some of the externalities would also be internalised in a multi-product duopoly.

Lego do have an incentive to offer excessively specialised products, such as sets to build just one specific character.

More generally the model predicts that the extent of variety growth should be related to the multi-purpose nature of consumption, and to whether or not new functions can be bundled. This is consistent with empirical data on new buys and on buying more than one item in Blis and Klenow (2001b). The product category with the lowest number of varieties were sewing machines where most new functions can easily be bundled. In this case only 4.4 percent of the 1.8 percent households who made a purchase bought more than one. This can be compared with cars, where only a limited degree of bundling is possible, as exemplified in the combi. In this case 20.7 percent of households bought a new car, which a higher proportion of 13.5 bought more than one. In contrast, for shoes, where no bundling is possible, the largest number of varieties were purchased. Shoes are indeed the product category with the highest percentage of new buys. For example for women's shoes, some 63.3 percent of all households purchased new shoes, and 62.8 percent of those bought several pairs over a twelve month period, in a survey of 65,189 US households over the period 1980-1996.

The distortions that arise in a monopoly can be alleviated if a company can bundle. Hence, my analysis also provides an argument in favour of bundling. When there are high barriers to entry due to, for example, research and development costs, there will be less extensive distortions in product selection over time if firms can bundle. Hence, by bundling their software Microsoft will have less distorted incentives to optimise the characteristics of each individual software product. This is an argument which complements the literature on bundling following Adams and Yellen (1976). This literature has examined various ways in which bundling enables a firm to extract more surplus for given product characteristics through leverage or price discrimination, or

through strategic effects on price or quantity.²⁹ Thus the results in this paper imply that a monopoly who could bundle would choose characteristics and the number of varieties that maximise welfare, an implication that differs from previous papers.³⁰

The paper also makes a conceptual contribution to the literature on product innovation, by adding a third class of successful innovations which encourages consumers to buy more goods rather than switching suppliers.³¹

Functional differentiation differs from all other models of product differentiation in one important respect: it cannot be generated with a characteristics model and heterogeneous consumers.³² On the contrary, this paper shows that it does matter whether the demand for different varieties comes from the same consumer who can choose any subset, or whether the demand arises from the preferred locations of heterogeneous consumers. Hence it illustrates that previous results on the equivalence between demand from a representative consumer and heterogeneous consumers relies upon the assumption that consumers cannot choose how many varieties they purchase.

The model for functional differentiation thus shows that a reason for why individuals have demanded more variety instead of quality with higher incomes is because

²⁹Carbajo, de Meza and Seidmann (1990) look at the strategic effects on price and quantity, and find that the welfare effects will depend on the nature of product market competition. Whinston (1990) explored a justification for the leverage theory through its strategic implications. See also McAfee et al (1989) on multi-product monopoly and bundling.

³⁰On the contrary, when bundling is used to deter entry, as in Choi (1996) and Choi and Stefandis (2001) who showed that the leverage theory could be understood through its impact on innovation in the case of complementary goods, there will be a negative effect on consumers' surplus as well as welfare.

³¹The nature of successful product innovations is an issue that brings industrial economists and business strategists together (see Caves (1984)). Porter (1980) points out two ways for an innovation to be successful, either by improving quality (see e.g. Fudenberg et.al.(1983)) or to better match the taste of a market segment. In both cases some consumers will switch from one supplier to the new one.

³²In an important paper Anderson et.al. (1989) showed that all existing models used in empirical and analytical work could be generated with a characteristics model and heterogeneous consumers, implying that the source to the demand for heterogeneity would not matter. Deneckere and Rotschild (1992) derived a similar result.

of the multi-purpose nature of consumption that generates a need for functional differentiation of goods. When individuals can afford to spend more on each product category they will therefore prefer function over higher quality, for example, most individuals prefer to own several specialised shoes to one seriously expensive shoe precisely because of the multi-purpose nature of walking. This paper thus finds that the empirical pattern is consistent with function being more income elastic than quality. Hence, this paper finds that the general principle in Rosen (2002) that ‘rising incomes increase the demand for quality’ should be modified to ‘rising incomes increase the demand for function!’

APPENDIX A. PROOFS

Proof of Lemma 2.2 The marginal product of variety $a_i \in A$ if the consumer buys all varieties is given by,

$$(A.1) \quad R_{a_i}(A) = \int_{s_{i-1}}^{s_i} [h(s - x_i, r_i, t_i) - q(s, A \setminus a_i)] f(s) ds,$$

where $q(s, A) = \max\{h(s - x_j, r_j, t_j) \mid a_j \in A \setminus a_i\}$. From this follows that $q(s, B) \leq q(s, A)$. Hence, the marginal increment in services can never be smaller in a basket that offers less choice. The optimal usage $S_i^* = [s_{i-1}^*, s_i^*]$ is furthermore smallest in A . To see this note that it is determined by the conditions,

$$(A.2) \quad h(s_i^* - x_i, t_i, r_i) = h(s_i^* - x_{i+1}, t_{i+1}, r_{i+1})$$

$$(A.3) \quad h(s_{i-1}^* - x_i, t_i, r_i) = h(s_{i-1}^* - x_{i-1}, t_{i-1}, r_{i-1}).$$

If the consumer instead could choose from a set that did not include a_{i+1} , the right hand side of (A.2) would be less than the left hand side for s_i^* . Since $h|_{s-x} < 0$ s_i would have to be increased to restore equality. A similar argument could be made for s_{i-1} . Hence, the consumer will never use the variety in a smaller subset than he does when he has the largest choice of varieties. Q.E.D.

Proof of Proposition 2.3 This solution is unique if all other constraints in (2.7) are strict inequalities. Hence, if

$$(A.4) \quad \theta [\bar{q}(A) - \bar{q}(A \setminus a_i)] = p(a_i)$$

then

$$(A.5) \quad \theta [\bar{q}(A) - \bar{q}(A \setminus \{a_i, a_j\})] > p(a_i) + p(a_j)$$

should also be true. We can verify that this is indeed true by substitution of (A.4) in (A.5) and rearranging which gives

$$(A.6) \quad \underbrace{\theta [\bar{q}(A \setminus a_j) - \bar{q}(A \setminus \{a_i, a_j\})]}_{R_{a_i}(A \setminus a_j)} > \underbrace{\theta [\bar{q}(A) - \bar{q}(A \setminus a_i)]}_{R_{a_i}(A)}.$$

This is true by Lemma 2.2. Q.E.D.

Proof of Proposition 2.4 If $p(a_i) > \theta R_{a_i}(A)$, the variety may or may not be in positive demand depending on prices of other varieties. Hence, the demand can be increased if the price p is reduced.

To show that it is inelastic for $p(a_i) \leq \theta R_{a_i}(A)$ we will use proof by contradiction. Suppose that $a_i \notin B^*$, then f.o.c. implies that $p(a_i) > \theta R_{a_i}(B^*)$, which is a contradiction since $R_{a_i}(A) \leq R_{a_i}(B^*)$. Q.E.D.

Proof of Proposition 4.1: The direction of distortions in product attributes can be found by evaluating first order conditions at a^* and determine whether the monopoly profit is increasing or decreasing at the social optimum.

Substitution of the reservation prices in the monopolist's profit gives,

$$(A.7) \quad R_{a_1}(\{a_1, a_2\}) + R_{a_2}(\{a_1, a_2\}) - c(t_1, r_1) - c(t_2, r_2)$$

First note that there will be a direct effect on $h(\cdot)$ and an indirect effect on s_1 from changing product attributes. However, the indirect effect will be zero since

$$(A.8) \quad \frac{\partial \pi^M}{\partial s_1} = (h(s_1 - x_1, t_1, r_1) - h(s_1 - x_2, t_2, r_2)) f(s_1) - (h(s_1 - x_2, t_2, r_2) - h(s_1 - x_1, t_1, r_1)) f(s_1) = 0$$

which follows from first order conditions for s_1 .

Thus we only need to include the direct effect in the first order conditions for the problem.

First consider the conditions for the optimal location of the varieties which is

$$(A.9) \quad \frac{\partial R_{a_1}(\{a_1, a_2\})}{\partial x_1} + \frac{\partial R_{a_2}(\{a_1, a_2\})}{\partial x_1} = 0$$

$$(A.10) \quad \frac{\partial R_{a_1}(\{a_1, a_2\})}{\partial x_2} + \frac{\partial R_{a_2}(\{a_1, a_2\})}{\partial x_2} = 0$$

This condition is different from social optimum since the monopoly will not only choose x to maximise the surplus the consumer derives from the product, but will also take into

account its impact on the price it can charge for the other variety. Hence the monopolist will not choose the location that maximises the utility from using the good.

For x_1^* the first term in the first order condition will be zero by definition, whereas the second term is negative. Hence the function is decreasing for x_1^* . Strict concavity of the maximisation problem thus implies that $x_1^M < x_1^*$. Similarly for x_2^* the second term in the first order condition will be zero by definition, whereas the first term is increasing. Hence the function is increasing for x_2^* . Thus $x_2^M > x_2^*$.

The first order conditions with respect to quality are,

$$(A.11) \quad \frac{\partial R_{a_1}(\{a_1, a_2\})}{\partial r_1} + \frac{\partial R_{a_2}(\{a_1, a_2\})}{\partial r_1} - c_{r_1} = 0$$

$$(A.12) \quad \frac{\partial R_{a_1}(\{a_1, a_2\})}{\partial r_2} + \frac{\partial R_{a_2}(\{a_1, a_2\})}{\partial r_2} - c_{r_2} = 0$$

Since quality increases utility in all states, higher quality does not only increase the value of a good when it is used for its intended use, but is also increases its value in alternative uses. The monopoly will therefore distort quality as well and not choose quality where the consumers marginal valuation of quality is equal to its marginal cost.

For r_1^* the first order condition becomes

$$(A.13) \quad \frac{\partial R_{a_2}(\{a_1, a_2\})}{\partial r_1} = - \int_{s_1}^1 h_{r_1}(s - x_1, t_1, r_1) f(s) ds < 0$$

which is negative, since $h_r > 0$. Hence the function is decreasing for r_1^* . The same argument be used to show that the function is decreasing for r_2^* .

The first order conditions with respect to the degree of specialisation are

$$(A.14) \quad \frac{\partial R_{a_1}(\{a_1, a_2\})}{\partial t_1} + \frac{\partial R_{a_2}(\{a_1, a_2\})}{\partial t_1} - c_{t_1} = 0$$

$$(A.15) \quad \frac{\partial R_{a_1}(\{a_1, a_2\})}{\partial t_2} + \frac{\partial R_{a_2}(\{a_1, a_2\})}{\partial t_2} - c_{t_2} = 0$$

As with the other varieties the monopoly will not only take into account its effect on surplus in use but also how valuable it would be for its alternative use. In this case a higher degree of specialisation will reduce the value for the alternative use, thus giving the monopolist an incentive to invest too much in function.

For t_1^* the first order condition becomes

$$(A.16) \quad \frac{\partial R_{a_2}(\{a_1, a_2\})}{\partial r_1} = - \int_{s_1}^1 h_{t_1}(s - x_1, t_1, r_1) f(s) ds > 0$$

which is positive since $h_{t_1} < 0$ a more specialised primary function reduces the utility for alternative uses. Hence the function is increasing for t_1^* . A similar argument can be used to show that the first order condition is increasing for t_2^* . Q.E.D.

Proof of Proposition 4.2: The consumer gets no surplus if the monopoly sells a general purpose variety. Since $h \geq 0$, attributes can never be so distorted that the monopolist can charge more for a variety than it is worth to a consumer. Since it may not be optimal to distort characteristics so much that they have no use in other activities it is possible that they remain to some extent substitutable, i.e., the a $h(\cdot) > 0$ for alternative uses. In this case the consumer will be left with a positive surplus given by

$$(A.17) \quad \theta [\bar{q}(\{a_1, a_2\}) - R_{a_1}(\{a_1, a_2\}) - R_{a_2}(\{a_1, a_2\})] = \int_0^{s_1} h(a_2)f(s)ds + \int_{s_1}^1 h(a_1)f(s)ds > 0.$$

Hence, consumers can extract more surplus as a result of differentiation. Q.E.D.

Proof of Proposition 4.3: A monopoly will choose to functionally differentiate a good if

$$(A.18) \quad p_1^M - c(r_1^M, t_1^M) + p_2^M - c(r_2^M, t_2^M) \geq p_G - c(r_G^M, t_G^M)$$

The left hand side is less than the gross surplus since the consumer will be left with some positive surplus. The gross surplus is furthermore less than the surplus when characteristics are optimally chosen by definition, hence

$$(A.19) \quad p_1^M - c(r_1^M, t_1^M) < \int_0^{s_1} h(a_1^M)f(s)ds - c(q_1^M, t_1^M) < \int_0^{s_1} h(a_1^*)f(s)ds - c(q_1^*, t_1^*).$$

This in turn implies that a monopoly may choose not to differentiate when it would have been optimal to do so. Q.E.D.

Proof of Proposition 5.2: If $\theta R_{a_i^*}(\{a_1^*, a_2^*\}) < c(a_i^*)$ then a_i^* is a weakly dominant strategy. If $a_j \neq a_G^*$ then from the best response correspondence (5.1) we can infer that the equilibrium occurs for firm i charging a price $p_i = \Delta_{ij} - \Delta c > 0$ and selling one unit whereas firm j sells nothing. Hence the firm will make a profit $\pi_i = \Delta_{ij} - \Delta c > 0$. Whereas if the other firm chooses a_G^* they will split the demand and make zero profit.

If $\theta R_{a_i^*}(\{a_1^*, a_2^*\}) \geq c(a_i^*)$ then a_i^* is a dominant strategy. The firm's equilibrium payoff is in this case $\pi_i = \theta R_{a_i^*}(\{a_1^*, a_2^*\}) - c(a_i^*)$. Note that $\theta R_{a_i}(\{a_i, a_j\}) - c(a_i)$ is maximised for a_i^* regardless of a_j since the services that the alternative variety would have produced in those states is constant and therefore has no effect on the optimum. Hence a_i^* is best

response to all a_j . Thus the unique subgame perfect Nash equilibrium in this case is a_i^* and $p = \theta R_{a_1^*}(\{a_1^*, a_2^*\})$. Q.E.D.

Proof of Proposition 5.3: It is socially optimal to differentiate if

$$(A.20) \quad \theta [\bar{q}(\{a_1^*, a_2^*\}) - \bar{q}(a_G^*)] \geq c(a_1^*) + c(a_2^*) - c(a_g^*).$$

Whereas it is viable to differentiate if

$$(A.21) \quad \theta [\bar{q}(\{a_1^*, a_2^*\}) - \bar{q}(a_1^*)] \geq c(a_2^*).$$

Now suppose that it is just optimal to differentiate. Then we can substitute (A.21) in (A.20) and rearrange which gives

$$(A.22) \quad \theta \bar{q}(a_1^*) - c(a_1^*) > \theta \bar{q}(a_G^*) - c(a_G^*)$$

which is a contradiction since a_G^* maximises the surplus when only one variety is being used in all states. Hence it is possible that firms will choose to differentiate even if it is not socially optimal. Q.E.D.

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FACULTY OF ECONOMICS

E-mail address: SE.Sallstrom@econ.cam.ac.uk