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OF THE (WEAK) ENFORCEMENT
OF SALES TAX**

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ABSTRACT

The Political Economy of the (Weak) Enforcement of Sales Tax

The objective of this paper is to understand the determinants of the enforcement level of indirect taxation in a positive setting. We build a sequential game where individuals differing in their willingness to pay for a taxed good vote over the enforcement level. Firms then compete à la Cournot and choose the fraction of sales taxes to evade. We assume in most of the paper that the tax rate is set exogenously. Voters face the following trade-off: more enforcement increases tax collection but also increases the consumer price of the goods sold in an imperfectly competitive market.

We obtain that the equilibrium enforcement level is the one most-preferred by the individual with the median willingness to pay, that it is not affected by the structure of the market (number of firms) and the firms' marginal cost, and that it decreases with the resource cost of evasion and with the tax rate. We also compare the enforcement level chosen by majority voting with the utilitarian level. In the last section, we endogenize the tax rate by assuming that individuals vote simultaneously over tax rate and enforcement level. We prove the existence of a Condorcet winner and show that it entails full enforcement (i.e., no tax evasion at equilibrium). The existence of markets with less than full enforcement then depends crucially on the fact that tax rates are not tailored to each market individually.

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1 Introduction

Enforcement of indirect taxation varies widely across countries. Estimates of value added taxation (VAT) evasion for the year 2001 range from 15.7% of the estimated hypothetical VAT revenue that would be obtained with full compliance for the UK (Keen and Smith (2007)) to 29.6% for Argentina (AFIP (2008)) and 35.3% for Mexico (Hernández Trillo and Zamudio (2004)). There is also anecdotal evidence that tax enforcement varies across markets inside countries. For instance, the market place in Tepito (Mexico City, Mexico), the Salada fair (Buenos Aires, Argentina) and the truffle market in Lalbenque (Lot, France) are examples of physical markets where transactions are always made in cash. Moreover, tax audits seldom take place. Therefore, it is suspected that sellers evade most if not all of the indirect taxes that are due. Audits are much more common in other markets.

The objective of this paper is to provide a political economy analysis of why enforcement levels vary across markets, and especially of why enforcement is low in certain markets. We assume that individuals vote over the enforcement level, anticipating perfectly the impact of their choice on market equilibrium. Voters face the following trade-off: more enforcement increases tax collection (and thus the amount of a uniform lump sum transfer) but also increases the consumer price of the goods sold in an imperfectly competitive market.

To our knowledge, the political economy determination of tax enforcement when firms evade sales taxes has not been studied in the literature so far. More precisely, our paper lies at the intersection of two different strands of research. First, many articles deal with tax evasion by firms, under different market structures. Marelli (1984), Wang and Conant (1988) and Yaniv (1995) consider a risk-averse monopolist deciding simultaneously how much to produce and to evade. In particular, Yaniv (1995) presents the first rigorous analysis of the necessary conditions under which production is independent from evasion. Virmani (1989) and Cremer and Gahvari (1993) present models of risk-neutral firms that evade taxes under perfect competition. Virmani (1989) was the first to introduce concealment costs, as a way to reconcile the conventional framework with risk-neutral firms with Allingham and Sandmo (1972)'s paradigm of evasion under uncertainty. Marelli and Martina (1988), Bayer and Cowell (2006) and Goerke and Runkel (2006, 2007) analyze these same decisions in an imperfectly competitive environment. The papers closest to our are Bayer and Cowell (2006) and Goerke

and Runkel (2007) who both study two-stage Cournot models, with risk-neutral firms deciding upon output and evasion while facing the risk of being audited. All these papers either concentrate on describing the equilibrium (evasion and production) behavior of firms and on performing a comparative statics analysis, or look at the firms's decisions from a normative perspective. None of these papers adopt a positive approach to enforcement decisions.

Our paper is also related to a very recent literature on political economy models of tax enforcement. These papers study majority voting over linear income taxes when tax evasion is possible. Their main objective is to understand how the equilibrium income tax rate is affected by the presence of tax evasion. Roine (2003) and Traxler (2006) stress that the decisive agent is the one with the median taxed income, who may differ from the individual with the median before-tax income if tax evasion opportunities are not monotone in pre-tax income. Borck (2004) shows that stricter enforcement may make redistributive taxation more attractive to the decisive voter. The presence of tax evasion may have a more drastic impact on the identity of the decisive voter and on the tax rate chosen by majority voting. Roine (2006) shows that introducing tax evasion may transform the classical conflict between rich and poor (where the median income individual is decisive) into an ends-against-the-middle situation where a coalition of poor and rich agents favor higher income tax rates. Borck (2007, 2008) shows that allowing for tax evasion may result in the non existence of a majority-voting income tax rate (because individual preferences over the tax rate are neither single-peaked nor single-crossing when tax evasion is introduced).

Our paper lies at the intersection of these two strands of literature since it studies majority voting over the enforcement level when firms may evade sales taxes. We consider a continuum of individuals characterized by their willingness to pay for a good which is subject to a sales tax. Since we focus on the determination of the enforcement level, we assume for most of the paper that the tax rate is set at an exogenous level and we solve the following three-stage model. In the first stage, individuals vote over the tax enforcement level. In the second stage, the good is produced in an imperfectly competitive market, where firms compete *à la* Cournot. In the third stage, firms report to the government the amount of their sales. Firms decide either to conceal a fraction of their sales (at a resource cost) or to fully comply with the tax law, knowing that they will be audited and, if found to have evaded, penalized. In order to explain why equilibrium enforcement may be low, we bias the model in favor of full enforcement by assuming that government

audits are costless and perfect.

We solve the model backwards. In the last stage, each firm evades the same fraction of their sales. This fraction increases with the tax rate, decreases with the enforcement level but is independent of market variables such as the price of the good or the number of firms in the market. Anticipating this, in the second stage, firms compete *à la* Cournot. The equilibrium price depends positively upon the level of enforcement. Finally, we study the voting equilibrium at the first stage and we show that the equilibrium enforcement level corresponds to the one preferred by the consumer with the median willingness to pay. Performing the comparative static analysis of this level (assuming isoelastic demand functions) allows us to study its determinants and to shed some light as to why this level may differ across markets. Surprisingly, we obtain that firms' marginal cost and market structure (the number of firms) have no influence on the equilibrium enforcement level. Enforcement is also negatively correlated with the resource cost of evasion and with the value of the tax rate. We also show that the comparison between majority chosen and socially optimal enforcement levels does not depend only on the distribution of willingness to pay (i.e., it is not simply a matter of comparing the median with the average willingness to pay).

The last section of the paper studies the simultaneous determination of the tax rate and of the enforcement level. We prove the existence of a majority voting equilibrium, even though the voting space is bidimensional. We then show that the decisive voter should set the enforcement level high enough to discourage any tax evasion. In other words, the existence of low enforcement at equilibrium crucially depends on the fact that the tax rate is not chosen simultaneously by majority voting. We show how the chosen tax rate depends on the distribution of willingness to pay and on demand elasticity. We also show that, in our model with no redistribution motives (quasi-linear preferences) and no government revenue requirement, the socially optimal tax rate should be zero.

The remainder of this paper is organized as follows. Next section describes the model. Section 3 shows the equilibrium. Section 4 compares the voting equilibrium with the utilitarian optimal. Section 5 presents some comparative statics. Section 6 tackles the simultaneous determination by majority voting of the tax rate and the enforcement level. All proofs appear in the Appendix.

2 The model

2.1 Individuals

There is a continuum of individuals of measure 1. Each individual has two sources of income: an exogenous endowment¹ w and a uniform lump sum transfer G from the government. Individuals choose how much to consume of two goods, labelled 1 and 2. The price of good 1 is normalized to unity whereas the consumer price of good 2 is p . We denote by x and q the quantities bought of good 1 and 2, respectively. Individuals are characterized by θ , their willingness to pay for good 2. The parameter θ is distributed according to density $f(\theta)$ with cumulative distribution $F(\theta)$ on $[\underline{\theta}, \bar{\theta}]$. We denote the average value of θ by θ_μ and its median by θ_{med} . Utility of individual θ is

$$U_\theta = x + \theta u(q), \quad (1)$$

where the function u is strictly increasing and concave, verifying the Inada conditions.

Individual θ takes the lump sum transfer G as given and chooses the quantity q that maximizes his utility (1) subject to the individual's budget constraint

$$x \leq G + w - pq. \quad (2)$$

The first-order condition²

$$\theta u'(q) = p$$

defines $q_\theta(p)$, the demand for good 2. Aggregate demand for good 2 is given by

$$Q(p) = \int_{\underline{\theta}}^{\bar{\theta}} q_\theta(p) f(\theta) d\theta.$$

2.2 The industry

We do not explicitly formalize the market for the numeraire good. Good 2 is produced by n identical, risk neutral, firms labelled $i = 1, \dots, n$. Each firm i simultaneously decides how much to produce (q_i). Firms have constant returns to scale technologies, with the same marginal cost $c > 0$. Given

¹Our results would not be affected if endowments were heterogeneous across individuals.

²We assume that each individual's exogenous endowment is large enough so that the amount consumed of good 2 satisfies this first-order condition.

output decisions (q_1, \dots, q_n) , the price adjusts to the level that clears the market. We denote by $p(Q)$ the inverse market demand, where $Q = \sum_i q_i$ is aggregate output. The function $p(Q)$ is twice-continuously differentiable, with $p'(Q) < 0$ at all Q .

Each firm i has to pay a sales tax at a constant proportional rate $0 < \tau < 1$. We assume that the sales amount pq_i is private information, and that taxes due are computed based on the amount of sales reported by the firm. We denote by $e_i \in [0, 1]$ the fraction of sales that goes unreported by firm i . If $e_i = 0$, the firm fully complies with the tax law. But if $e_i > 0$, we say that there is tax evasion. We follow Cremer and Gahvari (1993) by assuming that, in order to be successful, concealment of the fraction e_i entails a cost of $g(e_i)$ per \$ of sale.³ We assume the following functional specification

$$g(e_i) = k \frac{e_i^2}{2},$$

where the parameter $k \geq 1$ reflects the (in)efficiency of the evasion technology.⁴

We assume that profits are distributed to unmodelled non-resident agents. We come back to this assumption in section 4.

2.3 The government

The government enforces the tax law and collects taxes and fines to finance the provision of the lump sum transfer G . The government audits each firm i with the same probability $a \in (0, 1)$. When a firm is not audited, it pays sales taxes based on the amount reported – i.e., it pays $\tau(1 - e_i)pq_i$. We assume that audits are costless and perfect, so that the government identifies the true sales amount of audited firms with certainty. Observe that this assumption biases the model in favor of full tax enforcement while our objective is to explain how equilibrium enforcement may be low. If under-reporting of sales

³Cremer and Gahvari (1993) assume that “The tax administration makes a ‘cursory examination’, at no cost, of the reported sales. The examination reveals the ‘true’ sales unless the firm spends real resources to conceal it.” Since firms caught cheating have to pay a fine above and beyond the amount of taxes due, evading without paying this cost is a dominated strategy.

⁴Our results would carry through with a generic increasing and convex concealment cost function satisfying $g(0) = 0$. The specific functional specification allows us to perform more easily comparative statics analysis.

is detected, the evading firm has to pay the tax that it legally owes, τpq_i plus an additional fine, which is a fraction $\lambda > 0$ of the amount of taxes evaded.⁵ With all revenues collected (taxes and fines), the government finances the provision of the transfer G .

2.4 Timing

We assume up to section 6 that the tax rate has been set at the exogenous rate of τ before the beginning of the game. The timing of the game we study runs sequentially as follows.

1. Individuals vote over the level of tax enforcement.
2. Firms compete à la Cournot in the market.
3. Firms decide the fraction of sales to report.
4. The government audits firms, collects taxes and fines due and finances the lump sum transfer out of the total tax and fines proceeds.

The assumption that firms decide the fraction of sales to report after having competed in the market is made for expositional convenience: all our results would carry through if the output and evasion decisions were taken simultaneously (or if evasion decisions were carried first).

In section 6, we assume that the tax rate is chosen at the same time as the enforcement level.

3 Equilibrium of the model

As is usual, we solve the game backwards. As the last stage is purely mechanical and involves no optimization, we start with the penultimate one.

3.1 Evasion

In stage 3, each firm i chooses the level of evasion e_i^* that maximizes its expected profit

$$\mathbb{E}\Pi_i = a\Pi_i^A + (1 - a)\Pi_i^{NA}, \quad (3)$$

⁵Our modelling is in line with the US legislation - see Skinner and Slemrod (1985).

where Π_i^A and Π_i^{NA} denote ex-post profits when firm i is (respectively, is not) audited. If firm i is audited, its ex-post profit is

$$\Pi_i^A = [p(Q)(1 - \tau(1 + \lambda e_i) - g(e_i)) - c]q_i$$

whereas, if it is not audited, it is given by

$$\Pi_i^{NA} = [p(Q)(1 - \tau(1 - e_i) - g(e_i)) - c]q_i.$$

Therefore, the expected profit is

$$\begin{aligned} \mathbb{E}\Pi_i &= [p(Q)(1 - \tau(1 - e_i) - a\tau e_i(1 + \lambda) - g(e_i)) - c]q_i \\ &= [p(Q)(1 - \tau(1 - e_i(1 - \xi)) - g(e_i)) - c]q_i, \end{aligned}$$

where we denote by $\xi = a(1 + \lambda)$ the expected payment rate on undeclared sales tax, expressed as a proportion of the tax rate τ . From now on, ξ will be our measure of tax enforcement.

The following first-order condition

$$\frac{\partial \mathbb{E}\Pi_i}{\partial e_i} = [\tau(1 - \xi) - g'(e_i)]p(Q)q_i = 0$$

characterizes the optimal fraction e_i^* in case of an interior solution. At an interior optimum, each firm declares a fraction of sales such that the marginal expected net benefit from evading this fraction equals the marginal cost of concealing it. Observe that e_i^* is independent of the market structure (the number of firms n), of the firms' cost c and of the variables determined in the market (price and quantities).⁶ Moreover, as all firms are audited with equal probability, they all evade the same fraction of sales taxes.⁷ Using the functional form adopted for the concealment costs function, we obtain an explicit solution for the optimal evaded share e^* . We gather the results in the following proposition.

Proposition 1 *When $\xi < 1$, each firm fails to report a fraction of sales $e^* = \tau(1 - \xi)/k$. This fraction increases with the tax rate τ and decreases with enforcement ξ and with the inefficiency of the evasion technology k . When $\xi \geq 1$, no firm evades.*

⁶Cremer and Gahvari (1993) obtain the same independence result.

⁷From now on, we save on notation by not reporting the subscript i each time a variable has the same value for all firms.

In order to evade, a firm must face an expected rate of payment on undeclared sales that is lower than the tax itself. If this is not the case, there is no evasion at equilibrium.

We now move to the second stage of the game.

3.2 Market competition

Substituting the optimal evasion share e^* into the expected profit function (3) we obtain

$$\mathbb{E}\Pi_i = [p(Q)(1 - \tau^e - g(e^*)) - c]q_i,$$

where τ^e denotes the effective tax rate paid by firms. The legal tax rate τ is modified by incorporating the evasion decision and the expected fine paid on the amount evaded:

$$\tau^e = \tau(1 - e^*(1 - \xi)).$$

Observe that

$$\tau^e + g(e^*) = \tau - \frac{\tau^2(1 - \xi)^2}{2k} < \tau \text{ when } \xi < 1,$$

so that evasion allows firms to increase the fraction of their sales that they retain. Straightforward differentiation shows that

$$\frac{\partial \tau^e}{\partial \xi} = \tau \left[\underbrace{e^*}_{\text{Enforcement effect}} - \underbrace{\frac{\partial e^*}{\partial \xi}(1 - \xi)}_{\text{Evasion effect}} \right] > 0.$$

The effective tax rate increases with enforcement through two channels: a direct “enforcement effect” (that increases the expected payment rate on undeclared sales) and an indirect “evasion effect” (that decreases the fraction of sales undeclared).

Each firm chooses q_i in order to maximize its expected profit, taking as given the decisions of all other firms $j \neq i$. The first-order condition for firm i is given by⁸

$$\frac{\partial \mathbb{E}\Pi_i}{\partial q_i} = [1 - \tau^e - g(e^*)][p(Q) + p'(Q)q_i^*] - c = 0,$$

⁸We denote equilibrium quantities with a *.

from which we see that $p(Q) + p'(Q)q_i^* > 0$ in order to obtain an interior solution. Existence and uniqueness of the Cournot equilibrium are ensured if we also assume

$$\frac{\partial^2 \mathbb{E}\Pi_i}{\partial q_i \partial q_j} = [1 - \tau^e - g(e^*)] [p'(Q) + q_i p''(Q)] < 0, \quad i \neq j,$$

i.e., the marginal revenue of firm i is decreasing in any other firm j 's production (see Vives (1999)).⁹

Using the property of symmetry,¹⁰ we sum the first-order conditions over the n firms and obtain

$$[1 - \tau^e - g(e^*)] [np(Q) + p'(Q)Q^*] = nc.$$

This equality implicitly defines the equilibrium aggregate output Q^* .¹¹ Dividing by p and rearranging, we finally obtain

$$\frac{p^* - \tilde{c}}{p^*} = \frac{1}{n\eta}, \quad (4)$$

where $p^* = p(Q^*)$, η is the (absolute value of the) elasticity of the inverse demand and

$$\tilde{c} = \frac{c}{1 - \tau^e - g(e^*)}. \quad (5)$$

Equation (4) is the usual inverse-elasticity rule, where \tilde{c} represents the effective marginal cost paid by firms. It is easy to see that $\tilde{c} > c$ when $\tau > 0$ and that \tilde{c} increases with τ and with ξ : taxation is equivalent to an increase in the marginal cost faced by the firm, but evasion allows to dampen in part this impact. As is intuitive, p^* increases with \tilde{c} . So Q^* and thus p^* are continuous functions of the enforcement parameter ξ . Straightforward use of the implicit function theorem allows us to obtain the following proposition:

⁹With this assumption, the second-order condition $\partial^2 \mathbb{E}\Pi_i / \partial q_i^2 = [1 - \tau^e - g(e^*)][2p'(Q) + p''(Q)q_i^*] \leq 0$ automatically holds.

¹⁰As firms are identical, the equilibrium production decisions q_i^* are the same. Hence, the equilibrium is symmetric, allowing us to drop the subscript i from now on.

¹¹As the market equilibrium is symmetric and both the number of firms and the prevailing price are observable, the tax authority could in theory infer sales per firm and thus whether evasion has taken place. We assume that audits are necessary to prove evasion – i.e., although sales can be inferred in equilibrium, evasion is not verifiable unless audits are commissioned.

Proposition 2 *When $\xi < 1$, the equilibrium price p^* increases with the enforcement level ξ . When $\xi \geq 1$, p^* is independent of ξ .*

As mentioned above, evasion helps firms to attenuate the impact of taxation on their marginal cost. Hence, with evasion, the consumer price is lower than the level that would have prevailed with full compliance. Therefore, the higher the enforcement, the lower the evasion and thus the higher is the equilibrium price level.

3.3 Voting over the tax enforcement level

We are now in a position to solve for the first stage decision, by majority voting, of the tax enforcement level ξ . Making use of the individual's budget constraint (2) and of the equilibrium price (4) into the utility function (1), we obtain the indirect utility¹² of individual θ

$$V_\theta = w - p^* q_\theta(p^*) + \theta u(q_\theta(p^*)) + G. \quad (6)$$

One way to prove the existence of a Condorcet winning value of ξ (i.e., a value of ξ that is preferred by a majority of citizens to any other value) consists in checking that individual preferences described by equation (6) satisfy the single-crossing property in the (ξ, G) space. Using the envelope theorem, we obtain that the marginal rate of substitution in the (ξ, G) space is given by

$$MRS(\xi, G) = -\frac{\partial V_\theta / \partial \xi}{\partial V_\theta / \partial G} = p^{*\prime}(\xi) q_\theta(p^*) > 0.$$

Intuitively, since more tax enforcement increases the consumer price of the good, agents have to be compensated with a larger transfer.

Indifference curves are single-crossing since

$$\begin{aligned} \frac{\partial MRS(\xi, G)}{\partial \theta} &= p^{*\prime}(\xi) \frac{\partial q_\theta(p^*)}{\partial \theta} > 0 \text{ if } \xi < 1, \\ &= 0 \text{ if } \xi \geq 1. \end{aligned}$$

The intuition for this result is straightforward: since higher θ individuals consume more of the good, they need more compensation following a tax

¹²Recall that firms' profits are distributed to non-residents. Alternatively, we could assume that firms' owners are resident but that they represent a set of measure zero without affecting this section's results.

enforcement-induced price increase (except if the enforcement is already so high that it has no impact on the price anymore).

We then apply the median voter theorem (see Gans and Smart (1996)) and obtain the following result.

Proposition 3 *The Condorcet winning tax enforcement level, denoted by $\hat{\xi}$, corresponds to the preferred level of enforcement of the individual with the median value of θ .*

We denote by $\xi^*(\theta)$ individual θ 's preferred value of ξ , so that $\hat{\xi} = \xi^*(\theta_{med})$. Using an envelope argument, $\xi^*(\theta)$ is given by the following first-order condition:

$$-p^{*\prime}(\xi)q_{\theta}^* + G'(\xi) = 0. \quad (7)$$

The first term describes the impact of an increase in ξ on individual θ 's expenditure, through an increase in p^* . Expenditure in good 2 increases, which is detrimental to the individual's utility. The second term shows the impact on G , and is obtained by differentiating the government's budget constraint:

$$G = \tau^e p^* Q(p^*). \quad (8)$$

Equation (8) shows that tax proceeds G can be expressed as the product of two terms: the effective tax rate τ^e and the actual value of sales, $p^*Q(p^*)$. Therefore, an increase in the enforcement level ξ has two effects on the value of G . First, the effective tax rate increases, as explained in section 3.2. Second, due to the change in p^* , the actual value of sales changes. One can surmise that the elasticity of demand will play a crucial role in determining the sign of this second impact.

Observe that condition (7) does not require G to be globally concave in ξ , but that it implies that $G'(\xi) > 0$ – i.e., we are on the upward sloping side of the “Laffer curve” $G(\xi)$. This is intuitive, since increasing tax enforcement has the drawback of increasing the consumer price p^* , and is thus only palatable to the consumer if it increases the lump sum transfer that he receives.

Assuming that the second-order condition

$$G''(\xi) - [p^{*\prime}(\xi)]^2 \frac{\partial q_{\theta}(p^*)}{\partial p^*} - p^{*\prime\prime}(\xi)q_{\theta} < 0 \quad (9)$$

holds at least locally, we obtain that

$$\text{sgn}\left(\frac{\partial \xi^*(\theta)}{\partial \theta}\right) = \text{sgn}\left(-p^{*'}(\xi) \frac{\partial q_\theta}{\partial \theta}\right) < 0, \quad (10)$$

i.e., individuals with a higher willingness to pay (who consume more of the good than individuals with a weaker taste for the good, at any given price) favor less tax enforcement as a mean to decrease the equilibrium price of the good.

4 Normative analysis of the voting equilibrium

We now compare the tax enforcement level chosen by majority voting, $\hat{\xi}$, with the level that would be socially optimal.¹³ We adopt the usual utilitarian definition of welfare, given by

$$\mathcal{W} = w + \int_{\underline{\theta}}^{\bar{\theta}} [\theta u(q_\theta(p^*)) - p^* q_\theta(p^*)] f(\theta) d\theta + G,$$

where G is given by (8). Observe that we do not include firms' profits in our definition of welfare. We do this in order not to bias the comparison with the tax enforcement level chosen by majority voting, since individuals do not take profits into account when voting. We rationalize this formulation by assuming that profits are paid to the non-resident owners of the firm. We come back to this assumption at the end of this section.

The first-order condition for the optimal tax enforcement level ξ^{opt} is

$$\frac{\partial \mathcal{W}}{\partial \xi} = -p^{*'}(\xi) Q^* + G'(\xi) = 0.$$

Observe that the comparison between this expression and the condition defining $\hat{\xi}$ (equation (7) with $\theta = \theta_{med}$) hinges on the difference between the demand of the median individual $q_{\theta_{med}}$ and the average demand Q (rather

¹³It is important to keep the timing of the game unchanged to do a proper comparison: the value of ξ is chosen at the first stage and then the game proceeds in the same sequence as previously.

than the demand of the average individual q_{θ_μ}). Measuring the derivative of welfare at $\hat{\xi}$, we obtain that

$$\frac{\partial \mathcal{W}(\hat{\xi})}{\partial \xi} = -p^{*'}(\hat{\xi})Q^* + G'(\hat{\xi})$$

so that

$$\frac{\partial \mathcal{W}(\hat{\xi})}{\partial \xi} \begin{matrix} \leq \\ \geq \end{matrix} 0 \text{ iff } Q^*(\hat{\xi}) \begin{matrix} \geq \\ \leq \end{matrix} q_{\theta_{med}}(\hat{\xi}).$$

Since individual demand is continuously increasing in θ , and since we have a unitary mass of consumers, we denote by θ_M the individual with the average demand

$$q_{\theta_M}(p(\hat{\xi})) = Q^*(p(\hat{\xi})).$$

If we assume that condition (9) is satisfied for all values of θ (including for θ_M), we obtain that the function \mathcal{W} is concave in ξ , and thus that

Proposition 4 *When the demand of the θ_{med} individual $q_{\theta_{med}}(\hat{\xi})$ is larger than the average demand $Q^*(\hat{\xi})$ (i.e., when $\theta_M < \theta_{med}$), the enforcement level chosen by majority voting, $\hat{\xi}$, is lower than the optimal enforcement level, ξ^{opt} . Otherwise, the opposite relationship between $\hat{\xi}$ and ξ^{opt} occurs.*

The intuition for this Proposition is straightforward once we recall from (10) that individuals with a larger willingness to pay (and thus a larger demand) prefer a lower enforcement level in order to bring down the price of good 2.

We now discuss how changing the assumption that profits are distributed to non-residents would affect our analysis. If profits were distributed to residents, they should be incorporated into the objective of the utilitarian planner. It is impossible to sign the derivative of total expected profits ($n\mathbb{E}\Pi_i$) with respect to the enforcement level ξ without specifying a functional form for demand functions. For example, with the isoelastic demands studied in the next sections, this derivative is always negative. This in turn means¹⁴ that the socially optimal enforcement level is lower than the value ξ^{opt} that we identify above. The comparison with the majority-chosen value of ξ then crucially depends on how profits are distributed among voters. If the set of voting citizens who own a share of the firms is of zero measure,

¹⁴Retaining of course the assumption that the planner's objective is concave.

the analysis contained in section 3.3 remains unchanged, and Proposition 4 is easily modified, with more cases in which the majority-chosen enforcement level is larger than the socially optimal one. If the set of capitalists has positive measure, we have to introduce a second dimension of heterogeneity among voters, namely the fraction of firms' profits that each one is entitled to. This would of course make the above analysis much more complex, and we leave this extension to future research.

5 Comparative static analysis

In this section, we analyze the impact of changes in parameters of the model upon the enforcement level $\hat{\xi}$. To perform this comparative static analysis, we need to impose functional forms. From now on, we restrict ourselves to isoelastic demand functions. Utilities are given by

$$u(q) = \frac{q^{1-\sigma}}{1-\sigma},$$

with individual demands of the form

$$q_\theta = \left(\frac{\theta}{p}\right)^{1/\sigma},$$

where $1/\sigma > 0$ denotes the constant (absolute value of the) demand price elasticity. Aggregate demand amounts to

$$Q(p) = \left(\frac{\theta_M}{p}\right)^{1/\sigma}, \quad (11)$$

where $\theta_M = \left(\int \theta^{1/\sigma} f(\theta) d\theta\right)^\sigma$. Observe that, due to the isoelastic specification, the identity of the individual with the average demand depends only upon the demand elasticity and the distribution of tastes, but not upon the level of enforcement. As before, $e^* = \tau(1 - \xi)/k$. Therefore, at the second stage,¹⁵ the equilibrium price p^* is given by

$$p^* = \frac{\tilde{c}}{1 - \frac{1}{n\eta}},$$

¹⁵The second-order condition for individual profit-maximization in the Cournot stage is satisfied provided $n > \sigma$. For the monopoly case, this is the well known condition that elasticity should be larger than one. As the number of firms increases, the range of admissible values of elasticities increases.

where \tilde{c} is given by (5) and is equal to

$$\tilde{c} = \frac{c}{1 - \tau + \frac{\tau^2(1-\xi)^2}{2k}}.$$

Using equation (8), the first-order condition that characterizes the (interior) solution for ξ (equation (7) with $\theta = \theta_{med}$) can be written as:

$$\tau e^* p^* Q(p^*) \left[2 + (1 - 1/\sigma) \frac{\tilde{c}}{c} \tau^e - \frac{\tilde{c}}{c} \left(\frac{\theta_{med}}{\theta_M} \right)^{1/\sigma} \right] = 0. \quad (12)$$

This expression shows the three effects of an increase in ξ on the median individual's utility, all expressed in terms of the amount of evaded taxes $\tau e^* p^* Q(p^*)$. The first two terms in brackets reflect the change in the government's expenditures G , as explained after equation (8). The positive and constant term shows how raising ξ increases the effective tax rate. The second term represents the effect through prices, since actual sales (which are the base of the tax) vary with p^* . As we surmised in the discussion of equation (8), the sign of this effect hinges on the demand elasticity: it is positive if demand elasticity is low ($1/\sigma < 1$) and negative otherwise. The last term in brackets shows how an increase in ξ affects the expenditure of the median individual through an increase in the price p^* . As we mentioned above, expenditure increases, which is detrimental to the individual's utility.

From (12), we can determine the explicit interior solution for $\hat{\xi}$ as follows:

$$\hat{\xi} = 1 - \sqrt{\frac{k \left[\left(\frac{\theta_{med}}{\theta_M} \right)^{1/\sigma} + (1 + 1/\sigma) \tau - 2 \right]}{(1/\sigma) \tau^2}}. \quad (13)$$

From this expression, we obtain the comparative statics results that we gather in the following proposition.

Proposition 5 *Assume that individual utilities are isoelastic. The majority chosen enforcement level depends neither upon the marginal cost c nor the number of firms n . When the inefficiency of the evasion technology k increases, the enforcement level chosen by majority voting decreases. For $1/\sigma \leq 1$, the majority chosen enforcement level is always decreasing in the tax rate τ .*

The intuition for this Proposition can be obtained by looking at the first-order condition (12). As the impact of increasing ξ on G through an increase in τ^e is a constant, it plays no role in the comparative static analysis of the optimal value of ξ . All comparative statics results then depend on how the other effects (on the median individual's expenditures and on the actual value of sales) are affected by the variable considered. Moreover, since the amount of sales tax evaded plays the same multiplicative role with respect to the three effects in (12), it has no influence in the comparative static analysis. This explains why the number of firms n has no influence on the optimal value of ξ , since it impacts only the amount of sales evaded. A similar result holds for the marginal cost c .

The impact of the evasion cost k is more interesting. On the one hand, increasing k increases the third term in brackets, which is related to the sensitivity of the price p^* to ξ . This increases the impact of ξ on the median individual's expenditure and calls for a smaller optimal ξ . On the other hand, increasing k raises both the effective tax rate (because it discourages evasion) and the equilibrium price p^* , resulting in an increase of (the absolute value of) the second term, which is related to the effect of ξ on actual sales volumes. The sign of this impact in turn depends on the demand elasticity (it calls for a larger value of ξ if the demand elasticity is smaller than one). Equation (13) shows that the net impact of increasing k on the optimal ξ is always negative, irrespective of the value of the demand elasticity, meaning that the first impact is larger than the second.

Increasing the tax rate τ has two impacts on the first-order condition (12). First, it increases the term related to the sensitivity of the median individual's expenditure to ξ , calling for a smaller optimal value of ξ . Second, it affects the second term, which is related to the actual values of sales. We obtain that, when measured at $\xi = \hat{\xi}$, the derivative of $\frac{\partial}{\partial \tau} \tau^e$ with respect to τ is always negative. Thus, the sign of this last impact depends upon the value of the demand elasticity. In the case of a logarithmic utility function ($\sigma = 1$), there is no impact on actual sales (since the demand elasticity is one) and increasing τ decreases the enforcement level. When demand is inelastic ($1/\sigma < 1$), the second term in brackets in (12) is negative and thus the net impact of τ on the optimal ξ is always negative. Straightforward but tedious manipulations of (13) shows that this remains the case for most (but not all) values τ and distributions of θ even when $1/\sigma \geq 1$.¹⁶

¹⁶Three inequalities must be simultaneously satisfied to have $\partial \hat{\xi} / \partial \tau > 0$: the first two

Assessing the impact of the demand elasticity and of the distribution of abilities on the majority-chosen enforcement level proves more tricky. Observe from (12) that the value of the demand elasticity $1/\sigma$ affects both the second and third terms in brackets. The sign of the impact on the second term is negative, since increasing the demand elasticity decreases the impact of ξ on G through the variation in the value of actual sales. This calls for a lower value of ξ . The sign of the impact on the third term (the price effect on the median individual's expenditures) is ambiguous. Therefore, we can not sign the impact of $1/\sigma$ over the majority-chosen enforcement level. The numerical simulations with positively skewed Beta distributions of abilities that we have run suggest that, in that case, the impact is negative: markets with a larger demand elasticity exhibit lower equilibrium enforcement levels.

The only impact of the distribution of willingness to pay on $\hat{\xi}$ is through the third term in brackets in (12), namely the price effect of ξ on the median individual's expenditures. Examination of this term shows that the distribution of θ interacts with the demand parameter $1/\sigma$. Moreover, the denominator shows that the whole distribution matters, and not only the comparison of the median with the average willingness to pay. We obtain that a larger value of the ratio of the demand of the individual with the median willingness to pay to the average demand, $(\theta_{med}/\theta_M)^{1/\sigma}$, increases the impact of ξ on the median individual's expenditures, and calls for a lower value of $\hat{\xi}$. Our numerical simulations with a family of positively skewed Beta distributions suggest that, when the skewness of the distribution increases, $(\theta_{med}/\theta_M)^{1/\sigma}$ decreases so that $\hat{\xi}$ increases. This is intuitive since an increase in the positive skewness means that a larger mass of individuals have low values of θ . Those individuals buy less of the good, while receiving the same lump sum transfer as the others. They are less sensitive to an enforcement-induced increase of the consumer price of the good and thus favor a higher value of the enforcement level.

Proposition 5 allows us to better understand why enforcement levels differ from one market to another. Consider two separate markets, with the same

are given by (15) in the proof of Proposition 5 and guarantee that $0 < \hat{\xi} < 1$, while the third one, $\alpha\tau + 2((\theta_{med}/\theta_M)^{1/\sigma} - 2) > 0$ (where $\alpha = 1 + 1/\sigma$), guarantees that the derivative of (13) with respect to τ is positive. It is far from easy to find parameters that satisfy these three equations simultaneously. For instance, with θ distributed according to a generalized Beta(2,2) distribution on $[0,2]$, $\sigma = 0.8$, $k = 1$, so that $M = 0.97$, we obtain that $\hat{\xi}(\tau) = 1$ for $\tau \leq 0.46$, $\hat{\xi}(\tau)$ decreases from 1 to 0.00095 as τ increases from 0.46 to 0.92, and then $\hat{\xi}(\tau)$ increases from 0.0095 to 0.013 as τ increases from 0.92 to 1.

tax rate set exogenously across markets, but with the enforcement level chosen by majority voting separately on each market. Proposition 5 then shows that the same equilibrium enforcement level will prevail on both markets if they differ only in the marginal cost or in the number of firms. If for some reason the concealment cost is larger in one of the markets, then the equilibrium enforcement level will be lower there. If tax rates vary exogenously across markets¹⁷ that are otherwise similar, then Proposition 5 suggests a negative correlation between tax rate and enforcement level. We stress that all these results assume that the tax rate τ is a parameter of the model –i.e., that it is set at an exogenous level before the beginning of the game. It would be of obvious interest to endogenize the choice of τ . This is what we do in the next section.

6 Simultaneous voting

We now turn to the simultaneous determination of both the tax rate τ and the enforcement level ξ by majority voting. It is well known that a Condorcet winner (i.e., a pair (τ°, ξ°) that is preferred by a majority of voters to any other pair (τ, ξ)) often fails to exist when voting simultaneously on more than one policy dimension.

In the context of our model, where voters differ according to only one dimension (the intensity of their preference for the good, measured by the parameter θ), a sufficient condition for the existence of a Condorcet winning pair (τ°, ξ°) is that utilities satisfy the Intermediate Preferences property.

Definition: Preferences are called intermediate if they can be expressed as

$$V(\theta, \tau, \xi) = J(\tau, \xi) + K(\theta)H(\tau, \xi),$$

where $K(\theta)$ is monotone in θ and where the functions $J(\tau, \xi)$ and $H(\tau, \xi)$ do not depend on θ .

When preferences are intermediate, the Condorcet winning pair (τ°, ξ°) corresponds to the preferred pair of the individual with the median willingness to pay for the good, θ_{med} (see Grandmont (1978), Persson and Tabellini (2000)).

¹⁷This is the case notably in Europe, where most countries impose different value added tax rates on different markets. This is also the case in many US states, with sales tax rates varying across products.

In order to check whether preferences are intermediate, we keep the isoelastic formulation. Starting with the indirect utility function

$$V(\theta, \tau, \xi) = w - pq_\theta(p^*) + \theta u(q_\theta(p^*)) + G,$$

where p^* and G are functions of τ and ξ and replacing $q_\theta(p^*)$ by its explicit expression, we obtain

$$\begin{aligned} V(\theta, \tau, \xi) &= w - p^* \left(\frac{\theta}{p^*} \right)^{1/\sigma} + \theta \frac{(\theta/p^*)^{(1-\sigma)/\sigma}}{1-\sigma} + G \\ &= w + p^* \frac{\sigma-1}{\sigma} \theta^{\frac{1}{\sigma}} \frac{\sigma}{1-\sigma} + G. \end{aligned}$$

Clearly, preferences are intermediate, with $K(\theta) = \theta^{\frac{1}{\sigma}} \frac{\sigma}{1-\sigma}$, $H(\tau, \xi) = p^* \frac{\sigma-1}{\sigma}$ and $J(\tau, \xi) = G$.¹⁸ We then have proved that

Proposition 6 *With isoelastic utilities, a Condorcet winning pair (τ°, ξ°) exists and corresponds to the preferred pair of the individual with $\theta = \theta_{med}$.*

We now compute the expression for (τ°, ξ°) . The first-order condition for an interior preferred value of ξ is again given by (12); while the condition for τ is

$$p^* Q(p^*) (1 - e^* \xi') \left[\frac{1 - 2e^* (1 - \xi)}{1 - e^* (1 - \xi)} + (1 - 1/\sigma) \frac{\tilde{c}}{c} \tau^e - \frac{\tilde{c}}{c} \left(\frac{\theta_{med}}{\theta_M} \right)^{1/\sigma} \right] = 0. \quad (14)$$

We show in the appendix that

Proposition 7 *With isoelastic utilities, if citizens vote simultaneously over τ and ξ , and if the resulting value of τ is non negative, then the majority chosen value of τ and ξ are given by:*

$$\begin{aligned} (i) \quad \xi^\circ &= 1 \\ (ii) \quad \tau^\circ &= \frac{1 - \left(\frac{\theta_{med}}{\theta_M} \right)^{1/\sigma}}{1/\sigma}. \end{aligned}$$

¹⁸It is straightforward that adding a constant w to the utility of each individual is immaterial when voting upon τ and ξ .

The intuition for this proposition runs as follows. The impact of an increase in τ on the median individual's utility (equation (14)) works through the same three channels as the impact of increasing ξ (equation (12)). Moreover, the comparison of (12) and (14) shows that two of these three effects are identical (up to a common multiplicative term). The only effect that differs between the two instruments is the impact on G through the effective tax rate (i.e., the first term in brackets). It is easy to see that τ has a smaller impact than ξ on this term. This is intuitive, because an increase in τ induces an increase in evasion that dampens its direct impact. Hence, if the chosen value of τ is non negative, we obtain that ξ is pushed towards its maximum level because this allows to increase the effective tax rate. In other words, it never pays to let firms evade when both the tax rate and the enforcement level can be simultaneously optimized, because one instrument dominates the other. As a consequence, the formula for τ° depends only on parameters related to the demand or to the distribution of types, and not to parameters related to evasion, such as the concealment cost k .

The majority chosen value of τ is positive provided that the demand of the individual with the median willingness is lower than the average demand –i.e., $(\theta_{med}/\theta_M)^{1/\sigma} < 1$. In the special case of logarithmic utilities (unitary demand elasticity), $\theta_M = \theta_\mu$. Hence, the majority chosen value of τ is positive in that case if $\theta_{med} < \theta_\mu$ –i.e., if the distribution of willingness to pay is positively skewed. The majority chosen value of τ decreases with the ratio $(\theta_{med}/\theta_M)^{1/\sigma}$ for the same reason that $\hat{\xi}$ decreases with this ratio - see previous section. We then obtain, for the reasons explained in the previous section, that a more positively skewed Beta distribution of θ increases τ° .

The impact of the demand elasticity is more difficult to assess, since it affects both the numerator and the denominator of τ° . For the family of positively skewed Beta distributions of θ that we have studied, increasing the demand elasticity $1/\sigma$ results in a larger value of τ° .

Observe that Proposition 7 relies on the comparison between the first-order conditions for τ° and ξ° of the individual with median willingness to pay, θ_{med} . Indeed, the identity of the individual who is decisive in the setting of τ and ξ does not matter in the proof of Proposition 7. In particular, we would obtain the same result if the individual whose utility is maximized were the one with the average demand, θ_M . Recalling from section 4 that this individual is precisely the one whose utility is maximized by a utilitarian

planner,¹⁹ we obtain the following corollary:

Corollary 1 *With isoelastic utilities, $\tau^{opt} = 0$.*

There is no revenue requirement for the government in this model, so the only reason for a social planner to introduce indirect taxation is to benefit the average consumer with the lump sum transfer.²⁰ The introduction of indirect taxation (starting from $\tau = 0$) only has second-order effects. These correspond to the first and the third term in brackets in equation (14) where θ_{med} is replaced by θ_M . These two effects have the same magnitude when $\tau = 0$: the increase in the lump sum transfer received is equal to the increase in the expenditures of the individual with the average demand.

7 Conclusion

This paper has started from the observation that the enforcement of indirect taxation varies across countries and across markets. In order to explain the determinants of tax enforcement, we build a model where individuals vote over the enforcement level anticipating how the market equilibrium is affected by their choice. As we want to explain the emergence of low equilibrium enforcement, we bias the model in favor of enforcement by assuming that tax audits are costless and perfect (i.e., that they always reveal correctly whether a firm has cheated or not, and how much). Voters differ in their willingness to pay for the taxed good and face the following trade-off: more enforcement increases tax collection but also increases the consumer price of the good sold in an imperfectly competitive market.

Our focus is the enforcement level so that we assume in most of the paper that the tax rate is set at an exogenous level. We show that there exists a Condorcet winning enforcement level –i.e., an enforcement level preferred by a majority of voters to any other enforcement level. This level corresponds to the one most-preferred by the individual with the median willingness to pay. We show that this level is affected neither by the structure of the market (the number of firms) nor by firms’ marginal cost. Interestingly, this means

¹⁹Section 4 is concerned only with the optimal enforcement level, but the same reasoning can be applied to the choice of the tax rate.

²⁰Recall that our definition of welfare does not include profits. If it were, this would be a further reason not to introduce indirect taxation, since profits are decreasing in τ with isoelastic demands.

that our analysis remains valid whether we consider a perfectly competitive market, a monopoly, or any Cournot situation in between. We show that the majority-chosen enforcement level decreases with the resource cost of tax evasion (i.e., the resources that an evading firm has to employ in order to prevent the tax administration from discovering the cheating even without audits, through a simple cursory examination), and with the value of the tax rate. We also study how the demand elasticity and the distribution of the willingness to pay affect the equilibrium enforcement level.

In the last section of the paper, we endogenize the tax rate by assuming that individuals vote simultaneously over enforcement level and tax rate. We prove the existence of a Condorcet winning pair of tax rate and enforcement level. This existence result is interesting by itself since simultaneous voting over two dimensions typically fails to generate any Condorcet winner. The reason why such a winner exists in our setting is that individuals differ on only one dimension (their willingness to pay for the taxed good) in such a way that the conflicts between them in the two-dimensional policy space can effectively be reduced to a single dimension. We obtain that, when the tax rate is optimized at the same time as the enforcement level, there is full enforcement at the equilibrium, in the sense that no firm evades any tax. In other words, it never pays to let firms evade when both the tax rate and the enforcement level can be simultaneously optimized, because one instrument dominates the other. Of course, this result of full compliance would generalize to maximal compliance if audits were costly or imperfect. For instance, if audits were costly, the equilibrium enforcement level would balance only monetary costs and benefit from audits.

Our main conclusion regarding the existence of markets with low enforcement of indirect taxation is then that this observation crucially depends on the fact that tax rates are not optimized separately on each market. Indeed, in any given country, one observes a very limited number of different tax rates. If tax rates were voted upon or optimized separately on each market, our model predicts that the equilibrium enforcement level on each market would be maximum.

8 Appendix

Proof of Proposition 2

Using the inverse demand curve, we have

$$\frac{\partial p^*}{\partial \xi} = p^{*'}(Q^*) \sum_i \frac{\partial q_i^*}{\partial \xi}$$

Recall that optimal individual production decisions are given by the following first-order condition

$$\underbrace{[1 - \tau^e - g(e^*)] [p(Q) + p'(Q) q_i^*]}_{\Phi(q_i, \xi)} = c.$$

Applying the implicit function theorem, we obtain

$$\frac{\partial q_i}{\partial \xi} = - \frac{\partial \Phi(q_i, \xi) / \partial \xi}{\partial \Phi(q_i, \xi) / \partial q_i}$$

As $\partial \Phi(q_i, \xi) / \partial q_i < 0$ by the second-order condition, we have

$$\text{sign}(\partial q_i / \partial \xi) = \text{sign}(\partial \Phi(q_i, \xi) / \partial \xi).$$

Differentiating $\Phi(q_i, \xi)$ with respect to ξ yields

$$\frac{\partial \Phi(q_i, \xi)}{\partial \xi} = -\tau e^* [p^* + p^{*'}(Q^*) q_i^*] < 0$$

because $p^* + p^{*'}(Q^*) q_i^* > 0$. Hence $\partial p^* / \partial \xi > 0$ ■

Proof of Proposition 5

Let us rewrite $\widehat{\xi}$ as

$$\widehat{\xi} = 1 - \sqrt{\frac{k \left[(\theta_{med}/\theta_M)^{1/\sigma} + \alpha\tau - 2 \right]}{(\alpha - 1) \tau^2}}.$$

where $\alpha = (1 + 1/\sigma) \geq 1$. For an interior solution, the following inequality must hold

$$\frac{(\alpha - 1) \tau^2}{k} - \alpha\tau > (\theta_{med}/\theta_M)^{1/\sigma} - 2 > -\alpha\tau. \quad (15)$$

Straightforward differentiation of $\widehat{\xi}$ with respect to τ leads to

$$\frac{\partial \widehat{\xi}}{\partial \tau} = \frac{1}{2} \sqrt{\frac{(\alpha - 1) \tau^2}{k \left[(\theta_{med}/\theta_M)^{1/\sigma} + \alpha\tau - 2 \right]}} \frac{k\tau \left[\alpha\tau + 2 \left((\theta_{med}/\theta_M)^{1/\sigma} - 2 \right) \right]}{(\alpha - 1) \tau^3}$$

So that

$$\text{sign} \left(\frac{\partial \widehat{\xi}}{\partial \tau} \right) = \text{sign} \left(\alpha \tau + 2 \left((\theta_{med}/\theta_M)^{1/\sigma} - 2 \right) \right). \quad (16)$$

Multiplying both sides of the first inequality in (15) by 2 and rearranging, one gets

$$\alpha \tau + 2 \left((\theta_{med}/\theta_M)^{1/\sigma} - 2 \right) < 2 \frac{(\alpha - 1)}{k} \tau^2 - \alpha \tau = \tau \left(2 \frac{(\alpha - 1)}{k} \tau - \alpha \right)$$

which is negative when $\sigma \geq 1$, $k \geq 1$ and $\tau < 1$. Hence, by equation (16), $\partial \widehat{\xi}/\partial \tau < 0$ ■

Proof of Proposition 7

(i) From the first-order condition for an optimal value of τ (equation (14)), we have that

$$\frac{\tilde{c}}{c} \left(\frac{\theta_{med}}{\theta_M} \right)^{1/\sigma} = \frac{1 - 2e^* \xi'}{1 - e^* \xi'} + (1 - 1/\sigma) \frac{\tilde{c}}{c} (1 - e^* \xi') \tau.$$

where $\xi' = 1 - \xi$. Substituting this expression into the first-order condition (12), we obtain

$$\frac{\partial V(\hat{\tau}(\xi), \xi)}{\partial \xi} = p^* \tau e^* Q^* \left[2 + (1 - 1/\sigma) \frac{\tilde{c}}{c} \tau^e - \frac{1 - 2e^* \xi'}{1 - e^* \xi'} - (1 - e^* \xi') \tau (1 - 1/\sigma) \frac{\tilde{c}}{c} \right].$$

Simplifying and rearranging yields

$$\frac{\partial V(\hat{\tau}(\xi), \xi)}{\partial \xi} = p^* \tau e^* Q^* \left[1 + \frac{e^* \xi'}{1 - e^* \xi'} \right] > 0.$$

So, at the optimum, $\xi^\circ = 1$ ■

(ii) When τ° is positive, $\xi^\circ = 1$ and thus $e^* = 0$. Plugging this in (14) and rearranging yields

$$p^* Q(p^*) \left[1 + (1 - 1/\sigma) \frac{\tau}{1 - \tau} - \frac{1}{1 - \tau} \left(\frac{\theta_{med}}{\theta_M} \right)^{1/\sigma} \right] = 0.$$

From this expression, we obtain the explicit solution

$$\tau^\circ = \frac{1 - \left(\frac{\theta_{med}}{\theta_M} \right)^{1/\sigma}}{1/\sigma} \quad \blacksquare$$

References

- [1] AFIP (2008) “Estimación del Incumplimiento del IVA”, mimeo, AFIP, Buenos Aires.
- [2] Allingham, M. and A. Sandmo (1972) “Income Tax Evasion: A Theoretical Analysis” *Journal of Public Economics*, 1, 323-338.
- [3] Bayer, R. and F. Cowell (2006) “Tax Compliance and Firms’ Strategic Interdependence”, DARP WP 81, London School of Economics.
- [4] Borck, R. (2004) “Stricter enforcement may increase tax evasion” *European Journal of Political Economy*, 20, 725-737.
- [5] Borck, R. (2007) “Voting, Inequality and Redistribution” *Journal of Economic Surveys*, 21, 90-109.
- [6] Borck, R. (2008) “Voting on Redistribution with Tax Evasion”, *Social Choice and Welfare*, forthcoming, DOI 10.1007/s00355-008-0334-8.
- [7] Cremer, H. and F. Gahvari (1993) “Tax evasion and the optimal commodity taxation”, *Journal of Public Economics*, 50, 261-275.
- [8] Gans, J. and M. Smart (1996) “Majority voting with single-crossing preferences”, *Journal of Public Economics*, 59, 219-237.
- [9] Goerke, L. and M. Runkel (2006) “Profit Tax Evasion under Oligopoly with Endogenous Market Structure”, *National Tax Journal* 59, 851-857.
- [10] Goerke, L. and M. Runkel (2007) “Tax Evasion and Competition”, WP 2104, CESifo.
- [11] Grandmont, J.-M. (1978) “Intermediate preferences and the majority rule”, *Econometrica*, 46, 317-330.
- [12] Hernández Trillo, F. and A. Zamudio (2004) “Evasión Fiscal en México: El caso del IVA”, mimeo, CIDE, Mexico.
- [13] Keen, M. and S. Smith (2007) “VAT Fraud and Evasion: What do we know, and What Can be Done?”, WP/07/31, IMF, Washington.
- [14] Marelli, M. (1984) “On Indirect Tax Evasion”, *Journal of Public Economics* 25, 181-196.

- [15] Marelli, M. and R. Martina (1988) “Tax Evasion and Strategic Behavior of the Firms”, *Journal of Public Economics* 37, 55-69.
- [16] Persson, T. and G. Tabellini (2000) *Political Economics. Explaining Economic Policy*. Cambridge, The MIT Press.
- [17] Roine, J. (2003) “Voting over tax schedules in the presence of tax avoidance”, *Stockholm School of Economics Working Paper Series in Economics and Finance* n° 529.
- [18] Roine, J. (2006) “The political economics of not paying taxes”, *Public Choice*, 126, 107-134.
- [19] Skinner, J. and J. Slemrod (1985) “An Economic Perspective on Tax Evasion”, *National Tax Journal*, 38, 345-353.
- [20] Traxler, C. (2006) “Voting over Taxes: The Case of Tax Evasion”, *University of Munich Discussion Papers in Economics* 2006-27.
- [21] Virmani, A (1989) “Indirect Tax Evasion and Product Efficiency”, *Journal of Public Economics* 39, 223-237.
- [22] Vives, X. (1999) *Oligopoly Pricing*. Cambridge, The MIT Press.
- [23] Wang, L. and J. Conant (1988) “Corporate Tax Evasion and Output Decisions of the Uncertain Monopolist”, *National Tax Journal* 41, 579-581.
- [24] Yaniv G. (1995) “A Note on the Tax Evading Firm”, *National Tax Journal*, 48, 113-120.