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ABSTRACT

Monetary Policy Regimes and the Term Structure of Interest Rates*

This paper proposes to investigate whether US monetary policy changed over time by evaluating evidence from the entire yield curve. A regime-switching no-arbitrage term structure model relies on inflation, output and the short interest rate as factors. In a departure from the finance literature, the model is complemented with identifying assumptions that allow the private sector (inflation and output dynamics) to be separated from monetary policy (short interest rate). The model posits regime changes in the volatility of exogenous output and inflation shocks, in the monetary policy rule, and in the volatility of monetary shocks. The monetary policy regimes cannot be identified correctly if the yield curve is ignored during estimation. Counterfactual analysis uses the disentangled regimes in policy and shocks to understand their importance for the great moderation. The low-volatility regime of exogenous shocks during the last two decades plays an important role, while monetary policy contributes by trading off asymmetric responses of output and inflation under different regimes.

JEL Classification: C52 and E43

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1 Introduction

That monetary policy matters for the real economy is widely accepted in modern macroeconomics (e.g., Woodford, 2003). Moreover, many researchers believe that monetary policy has improved over time. In particular, the great moderation – a post-1982 decline in the volatility of output and inflation – is an outcome of changing monetary policy. However, this point is actively debated in the literature. Earlier studies (e.g., Clarida, Gali, and Gertler, 2000) assume a break point and find different reaction to expected inflation in the interest rate rules estimated before and after the break. More recent work (e.g., Sims and Zha, 2006) explicitly models regime changes in monetary policy and in the volatility of exogenous shocks and finds that the regimes most likely affected the economy via the changing shocks to the private sector.

This paper contributes to the debate by arguing that monetary policy regimes may not be well-identified if one uses information from the short interest rate only. We propose to incorporate the information from the cross-section of yields, which by its nature is forward-looking and, therefore, reflects market-based expectations of the future monetary policy. We do so by proposing a novel no-arbitrage term structure model, which allows for regimes shifts in the monetary policy and in the shocks to the private sector. We show via Monte Carlo analysis that using the yields of several maturities is instrumental for identifying the monetary policy regimes. We find that the US monetary policy can be characterized as switching between active and passive regimes, judging by the differential response of the interest rate to expected inflation.

The finance literature has produced a number of important contributions on the yield curve modelling with regime shifts.¹ However, all of the existing finance models belong to the class of reduced-form models. This means that one cannot isolate the regime switches in the structural shocks to the economy from the regime switches in the monetary policy. For this reason, we complement the traditional setup with identifying assumptions.

Our identifying assumptions are imposed in the spirit of the structural VAR literature. That is, we explicitly posit a monetary policy reaction function and the dynamics of the macro economy.

¹Latent factor regime-switching no-arbitrage models of term structure are represented by the works of Bansal and Zhou (2002), Bansal, Tauchen, and Zhou (2003) and Dai, Singleton, and Yang (2007), among others. Ang, Bekaert, and Wei (2008) and Evans (2003) develop regime-switching models to study the term structure of real interest rates and inflation risk premia by combining the latent and macroeconomic factors.

However, our specification is silent about investors' preferences and how they are connected to the model's parameters. The advantage of this approach is that the model is less restrictive than explicit structural models and thus allows for a greater degree of realism.

We dispense with the traditional latent factors encountered in finance models and rely only on three observable variables: inflation, output, and the short interest rate. The economy's (inflation and output) behavior is driven by past, current, and expected future values of inflation and output. We assume that the short interest rate is the monetary policy instrument. Similar to forward-looking Taylor rules, the monetary policy responds to expected future inflation and current output. We also allow for some degree of policy inertia by positing a response to the past interest rate.

We allow for three regime variables in our model. The first shifts the volatilities of exogenous inflation and output shocks. The second switches the parameters in the systematic monetary policy reaction function modeled as a forward-looking interest rate rule. The third affects the volatility of the monetary policy shock.

We are first to provide a numerical rational expectations solution of the model with regime shifts.² The solution helps relating the cross-section of yields to the current values of the three observed variables using the tools of the regime-switching affine models. Because of our identifying assumptions, we are not free to choose convenient parametrization of the dynamics of the state vector that allows closed-form bond prices to be obtained.³ We propose a new approximate solution, which we show to be more precise than the log-linearization typically used in the literature.⁴

Our estimation results indicate the presence of at least two regimes for the volatilities of inflation and output shocks, two for the systematic monetary policy, and two for the volatility of the monetary shock. Because any combination of these regimes can be realized at a particular time, there are a total of eight possible regimes in the economy. In the high volatility regime for inflation and output shocks, both shocks are more volatile than in the low-volatility regime, which is mainly associated with oil shocks and recessions. The two monetary policy regimes are clearly

²This solution is consistent with a view that if policy regimes changed in the past then rational economic agents would form their expectations about the future shifts in policy and act accordingly.

³See, e.g., Ang, Bekaert, and Wei (2008) and Dai, Singleton, and Yang (2007) for examples of parameterizations that lead to analytical expressions for bond prices.

⁴See, e.g., Bansal and Zhou (2002) for an example of an approximate solution based on log-linearization.

distinguished by how the the Federal Reserve (the Fed hereafter) reacts to expected inflation. In the “active” policy regime, the Fed reacts aggressively to expected inflation in order to stabilize the price level. This regime occurred in the seventies and throughout Volcker’s disinflation period. It has also prevailed since 2002. In the other, “passive”, regime the reaction to expected inflation is far less strong. The “passive” regime appeared in the seventies and prevailed during the monetary experiment and the internet bubble of 1995-2001. The high and low regimes for the volatility of monetary shock are interpreted as “discretion” and “commitment” respectively. These two regimes interchanged sporadically in the sample.

When confronted with a dataset that does not include long-term bonds, our model yields similar results for volatility regimes but very different estimates of monetary policy regimes. Intuitively, the yield curve contains information about expected future interest rates, which, in particular, reflect the the probabilities that a particular regime will be present. A simulation study suggests that using the yield curve reduces the bias of the estimated monetary policy regime by a factor of 20. These findings indicate the importance of using the whole term structure for identifying policy regimes.

Because our model specification allows monetary and private sector regimes to be disentangled, we can evaluate the effects of these different regimes on the economy and the yield curve. We do so by simulating counterfactual economies, i.e., economies that are driven by the shocks realized in our sample, but in which only one of the eight possible regimes prevails throughout the full sample. Therefore, we can contrast active and passive monetary policies by holding other regimes constant. For example, we can assume low volatility of output and inflation shocks, the commitment regime of monetary policy shocks, and then allow either passive or active policy to prevail throughout the full sample. Comparing the resulting output, inflation, and yields allows us to characterize the effect of a specific monetary policy.

Using the counterfactual analysis as a basis, we report that a nearly permanent transition from high to low volatility of exogenous shocks to output and inflation made a large contribution to the great moderation. However, this is an incomplete explanation of the real economy’s improvement over the last two decades.

We show via impulse responses that output and inflation react asymmetrically in the two monetary policy regimes, depending on the type of exogenous shock (output or inflation). In

the case of output shock, inflation declines much faster in the active regime, while output reacts in a similar fashion across all the regimes. In the case of inflation shock, there is a monetary policy trade-off as inflation declines fast in the active regime, while output does not decline in the passive regime. Building on this observation, we show, using the mean and volatility of macro variables and yields as basis, that the realized shocks were such that the Fed had to face the noted trade-off between active and passive policies, because there is no single regime that uniformly dominated others in our sample. Because the inflation realized during the post-1982 sample is, on average, lower and less volatile than inflation in any of the individual regimes, the regime-switching monetary policy contributed to the great moderation in addition to the “lucky” low-volatility exogenous shocks.

A recent work by Ang, Boivin, and Dong (2007) is interested, similarly to us, in evaluating the impact of changing monetary policy on the entire term structure. They address this question in the reduced-form no-arbitrage model with drifting coefficients in the interest rate rule (see, for example, Boivin, 2005 or Cogley, 2005). However, Ang, Boivin, and Dong do not impose structural identifying assumptions in their model and, therefore, cannot separate the changes in the private sector from changes in the policy.

The rest of the paper is organized as follows. Section 2 develops the model, including the state dynamics, bond pricing issues, and the strategy for estimation. Section 3 presents the findings. Section 4 concludes. All technical details are presented in the appendices.

2 The model

2.1 State dynamics

We assume that the joint behavior of the Treasury bond yields and macroeconomic fundamentals is captured by the state vector $x_t = (g_t, \pi_t, r_t)'$, where g_t is the detrended real output, π_t is inflation, and r_t is the nominal short rate process. The model parameters switch according to a Markov chain process $S_t = (s_t^e, s_t^m, s_t^d)$ that consists of three regime variables. Regime variable s_t^e affects the volatilities of exogenous shocks. The s_t^m and s_t^d determine the systematic and discretionary parts of the monetary policy, respectively. These two regimes are controlled by the Fed.

The state variables satisfy the following forward-looking regime switching model:

$$g_t = m_g + (1 - \mu_g)g_{t-1} + \mu_g E_t g_{t+1} - \phi(r_t - E_t \pi_{t+1}) + \sigma_g(s_t^e)\varepsilon_t^g \quad (2.1)$$

$$\pi_t = m_\pi + (1 - \mu_\pi)\pi_{t-1} + \mu_\pi E_t \pi_{t+1} + \delta g_t + \sigma_\pi(s_t^e)\varepsilon_t^\pi \quad (2.2)$$

$$r_t = m_r(s_t^m) + (1 - \rho(s_t^m))[\alpha(s_t^m)E_t \pi_{t+1} + \beta(s_t^m)g_t] + \rho(s_t^m)r_{t-1} + \sigma_r(s_t^d)\varepsilon_t^r \quad (2.3)$$

This model represents an empirical specification of the economy dynamics rather than a micro-based general equilibrium.⁵

Equations (2.1)-(2.2) represent the outcome of the behavior in the private sector. In this paper, we assume that the private sector parameters are not regime-dependent. However, because the private sector expectations about future state variables are conditional upon the realization of the current regime, the reduced-form representation of dynamics of the private sector will be regime-dependent. Thus, such a specification is consistent with empirical findings in the literature on macro-finance term structure (see, for example, Ang, Bekaert, and Wei, 2008).

Equation (2.1) postulates that the current output depends on lagged and expected output and on the real interest rate. Higher expected output leads to higher consumption today that raises the current aggregate demand. When the real rate, $r_t - E_t \pi_{t+1}$, is high people save more and consume less, which drives the output down. Equation (2.2) specifies that the current inflation depends on lagged and expected inflation as well as the current output. Higher expected inflation will lead to higher prices today in the presence of price stickiness. Higher output affects the aggregate price level and therefore raises current inflation.

In this paper, we refer to shocks $\sigma_g(s_t^e)\varepsilon_t^g$ and $\sigma_\pi(s_t^e)\varepsilon_t^\pi$ as output and inflation shocks, respectively. Stochastic innovations ε_t^g and ε_t^π are assumed to be i.i.d. standard normal variables. In principle, the inflation shock could be conditionally correlated with the output shock. However, we do not allow for such correlation for identification reasons.⁶ Thus, ε_t^g and ε_t^π are mutually

⁵The model nests the New IS-LM model (see King, 2000 for a review). While there exist microfounded, or “New Keynesian,” versions of the New IS-LM model, the specification is generally ad hoc. While our specification is related to the literature on “New Keynesian” macro term structure (see, e.g., Hordahl, Tristani, and Vestin, 2006 or Rudebusch and Wu, 2003 for single-regime term structure applications), we do not attempt to interpret the model equations in terms of micro-based parameters but rather take them as a purely empirical specification to be confronted with the data.

⁶We tried to estimate a more general specification allowing for the conditional correlation of the shocks. However,

independent.

Assuming that the short interest rate is the monetary policy instrument, equation (2.3) represents a monetary policy rule.⁷ We borrow the specification of the monetary policy reaction function (2.3) from a single-regime baseline model of Clarida, Gali, and Gertler (2000). It has the form of a forward-looking Taylor rule that allows for some degree of monetary policy inertia captured by parameter ρ . The first three terms in (2.3) are a systematic part of the monetary policy, which switches with regime s_t^m . The monetary policy shock, interpreted as a random outcome of the policy decision making process, $\sigma_r(s_t^d)\varepsilon_t^r$, is driven by its own regime s_t^d , which affects the volatility of the policy shock. When the volatility $\sigma_r(s_t^d)$ is high, the Fed is more willing to deviate from the systematic rule. The stochastic innovation ε_t^r is modelled as an i.i.d. standard normal variable. Given that the exogenous policy shock is independent of the rest of the economy, ε_t^r is not correlated with either ε_t^g or ε_t^π .

As emphasized by Sims and Zha (2006) and Cogley and Sargent (2005), it is essential to account for the stochastic volatility of exogenous shocks when trying to identify shifts in monetary policy, because a specification of constant volatility may easily lead to spurious instabilities in the drift term. Following Sims and Zha (2006) and Ang, Bekaert, and Wei (2008), we allow the drift and volatility terms to be driven by their own regime variables. For reasons of tractability, the regime variables s_t^e , s_t^m and s_t^d are assumed to be mutually independent.

2.2 Rational expectations solution

We assume that the private sector and the Fed have the same information when forming their expectations about future values of the state variables. In addition, both the Fed and private sector know the current realization of regimes.⁸ Finally, to achieve the tractability of solutions, empirically it is hard to disentangle the effect of this correlation from the unconditional correlation due to changes in regimes. This prompted us to restrict the conditional correlation of shocks to zero.

⁷One might argue that during the monetary experiment of 1979-1982 the Fed explicitly targeted the unborrowed reserves and that, therefore, the funds rate was not an acceptable indicator of the monetary policy stance in that period. However, many authors provide evidence that the funds rate remained a valid, albeit imperfect, measure of the monetary policy (for example, Cook, 1989 and Romer and Romer, 2004).

⁸These assumptions are clearly not innocuous. The importance of imperfect knowledge and learning has been emphasized by Sargent (1999), Schorfheide (2005) and Primiceri (2004) among many others.

the probabilities of regime switches are assumed to be constant and, therefore, independent of the state variable x_t .

Under the above assumptions, it can be shown that the rational expectations solution of (2.1)-(2.3) where agents form the expectations of future state variables taking into account the future shifts in regimes is given by the following regime-switching vector autoregressive process:

$$x_t = \mu(S_t) + \Phi(S_t)x_{t-1} + \Sigma(S_t)\varepsilon_t \tag{2.4}$$

where $\varepsilon_t = (\varepsilon_t^g, \varepsilon_t^\pi, \varepsilon_t^r)'$ and reduced form parameters μ , Φ and Σ are non-linear functions of the parameters of the original model (2.1-2.3). Appendix A derives the sufficient existence conditions for the above problem and suggests a numerical method for finding the parameters for the reduced form.

2.3 Structural vs reduced-form representations

Because our model can be cast as a regime-switching VAR with non-linear parameter constraints, one might think that a better empirical approach would be to estimate a more flexible unrestricted VAR in order to avoid the potential misspecification of the model (2.1)-(2.3). However, if the economic agents' behavior is forward-looking, as in the model (2.1)-(2.3), the reduced form parameters will change whenever any one of the regime variables switches. For example, it is not possible to attribute a shift in the conditional volatility of the reduced-form state dynamics to either shifts in monetary policy or switches in volatilities of the exogenous shocks without making assumptions about identification.

By distinguishing private sector, monetary policy, and volatilities of exogenous shocks explicitly in (2.1)-(2.3), we effectively impose the requisite identification assumptions on the reduced-form representation (2.4). This structural identification is what distinguishes our approach from the existing term structure models. For the first time, we can explicitly separate the effects of changes in the monetary policy from the effects of changes in shocks on the yield curve.

Ideally, we would have liked to have derived explicitly the dynamic equations that describe the economy from fundamental microeconomic principles in a general equilibrium framework.⁹ However, for the set of the state variables used in this paper, the existing general equilibrium

⁹Woodford (2003) is a comprehensive exposition of this subject. Bekaert, Cho, and Moreno (2005) propose a

studies were not able to produce market prices of risk flexible enough to fit the yield curve accurately. Moreover, to the best of our knowledge, no one has studied general equilibrium in a regime-switching framework. We think that our approach is sufficient for distinguishing between the private sector and monetary policy blocks of equations and that it is an empirically-relevant and internally consistent alternative to a general equilibrium model.

2.4 Market prices of risk

In order to be able to value bonds, we complete the model with a stochastic discount factor $M_{t,t+1}$:

$$\log M_{t,t+1} = -r_t - \frac{1}{2}\Lambda'_{t,t+1}\Lambda_{t,t+1} - \Lambda'_{t,t+1}\varepsilon_{t+1} \quad (2.5)$$

The market price of risk $\Lambda_{t,t+1}$ is specified as the product of the volatility of the state variable x_t and time-varying vector $\Pi(x_t)$:

$$\Lambda_{t,t+1} = \Sigma'(S_{t+1})\Pi(x_t) \quad (2.6)$$

This specification recognizes the fact that investors require greater compensation for holding bonds in a more volatile economic environment. As we discuss in appendix B, time-varying parameters $\Pi(x_t)$ could be related, in a reduced-form sense, to the attitude towards risk on the part of the investors. Therefore, with some abuse of language, we will refer to $\Pi(x_t)$ as “preferences” or “risk aversion.”¹⁰

Following Duffee (2002) and Dai, Singleton, and Yang (2007), we assume an essentially affine structure of “preferences” $\Pi(x_t)$

$$\Pi(x_t) = \Pi_0 + \Pi_x x_t \quad (2.7)$$

single-regime “New Keynesian” general equilibrium model that is based on a number of observable and unobservable state variables in order to study the term structure of interest rates.

¹⁰Dai, Singleton, and Yang (2007) note that $\Lambda_{t,t+1}$ cannot be straightforwardly interpreted as market prices of risk because $\Lambda_{t,t+1}$ are not in the investors’ information set at time t . As equation (2.4) and our discussion of equation (2.5) in appendix B show, our specific timing is a rational-expectations outcome of our structural formulation. We attach a label “market prices of risk” to $\Lambda_{t,t+1}$, but do not attempt to interpret these parameters directly.

The “preferences” do not depend explicitly on the regimes, which is consistent with our assumption that the dynamics of the private sector are stable.¹¹ Indeed, in a general equilibrium model, the parameters in private sector equations (2.1), (2.2) would be functions of the agents’ preferences. For the same reason, the changes in the regimes themselves are not priced in this paper.¹²

This specification of the stochastic discount factor is what is commonly referred to as “no-arbitrage restrictions” in the finance literature. For our purposes, the main attraction of these restrictions is the ability to translate, in an internally consistent manner, expectations of monetary policy implied by our model (2.1)-(2.3) into market expectations reflected in the yield curve. As we argue in the sequel, bringing the information from the yield curve is crucial for understanding the monetary policy.

2.5 Bond valuation

Having defined $M_{t,t+1}$ we also introduce a n -period stochastic discount factor as

$$M_{t,t+n} = \prod_{i=1}^n M_{t+i-1,t+i}. \quad (2.8)$$

Then a price of a zero-coupon bond with maturity n can be found as

$$B_t^n(x_t, S_t) = E[M_{t,t+n}|x_t, S_t]. \quad (2.9)$$

The bond prices do not have a closed-form solution. We deviate from the standard strategy of computing an analytical approximate solution that is based on the log-linearization. Appendix C describes our approximate pricing method and provides evidence of its accuracy.

¹¹This assumption by no means implies that the agents’ preferences cannot vary over time. In fact, it is exactly the time-varying effect that is captured by the essentially-affine specification in (2.7).

¹²Dai, Singleton, and Yang (2007) argue, in a reduced-form model, that allowing the risk of changing regimes to be priced may substantially increase the model’s ability to fit the data. In their setup, investors require a premium for possible changes of future regimes even if there is no uncertainty about the state variables. This assumption means that the agents’ preferences are explicit functions of the regime variables. Ang, Boivin, and Dong (2007) make similar points in the context of a reduced-form drifting coefficients model. Such an extension is not possible with our structural identification assumptions about the constant parameters in the private sector equations.

2.6 Empirical approach

We use quarterly series of macro and bond data from 1970:Q1 to 2004:Q4. The annual log difference of an implicit price deflator is used as a proxy for inflation. We use a linearly detrended log of real per capita GDP as a proxy for detrended real output. Both the price deflator and GDP series were obtained from the FRED database. Our yields data are an unsmoothed Fama-Bliss series.¹³ Four different maturities are used: three months and two, five, and 10 years.

To study the information content of long-term yields, we also consider a smaller dataset that does not include long-term yields. This dataset consists of inflation, detrended output, and a three-month yield only. We refer to a model estimated on this set as a “short-rate model”, or *SRM*, while the same model estimated on the set that includes yields is referred to as a “term-structure model,” or *TSM*.

The macro variables are assumed to be observed without errors. One might argue against this assumption, given that inflation and GDP are not perfectly observed. However, our model is not grounded in equilibrium foundation and does not require observations of the true inflation and output. We can interpret our model as a description of the specific macro variables that we use.

The theoretical yield of the three-month yield computed from (2.9) coincides with the short rate because of the quarterly frequency of the data. Therefore, it is practical to assume that this yield is observed without any measurement error, so that the state vector x_t is observed exactly. It is assumed that the other yields are measured with errors. We choose the simplest correlation structure of the yield measurement errors: all errors are independent and identically normally distributed with a standard deviation of σ_y . The rationale for this specification is to put as much pressure on the model as possible to fit the yield curve.

Our estimation method is maximum likelihood. For any given set of “structural” parameters from model (2.1)-(2.3), we compute reduced form parameters μ , Φ and Σ as in (2.4). The latter parameters and “risk aversion” coefficients Π_0 and Π_x are then used to compute the bond prices and construct the likelihood function. The details of the procedure are provided in Appendix D.

Consistent with previous studies on regime-switching term structure, we found that the means of state variables are identified poorly. Following Ang, Bekaert, and Wei (2008) and Dai, Singleton,

¹³We are grateful to Robert Bliss for providing us with the data.

and Yang (2007), we pinned down these parameters to match the long-run means of the state variables in the data. Appendix E shows how to compute the long-run means of the state variables in our model.

Because regime-switching models are known to have many local optima, it is necessary to search for the global optimum carefully. We used the following optimization approach. First, we selected starting values by generating 1,000,000 Sobol points from a range of reasonable parameter values.¹⁴ Second, we chose 10,000 points that led to the largest likelihood values. Finally, starting from each of these points, we ran local optimization and selected the best result.

Because both macroeconomic and yield data are highly persistent, the asymptotic inference theory is likely to be unreliable. Therefore, following Conley, Hansen, and Liu (1997), we implemented a parametric bootstrap approach. We simulated 1000 data paths from the estimated model and reestimated the model along each path. This approach gave the finite sample distribution of the parameters that was used to construct the confidence bounds.

3 Results

Appendix F provides goodness-of-fit analysis of the *TSM*. The overall conclusion is that one can trust the implications of the model, because it is sufficiently realistic. We analyze the model properties in Section 3.1. Then, in Section 3.2 we investigate the issue of whether the yield curve contains information that is useful for identifying monetary policy regimes. Finally, in Section 3.3, we present the counterfactual policy analysis.

3.1 The model properties

3.1.1 Estimates of the state dynamics

In contrast to the reduced-form macro-finance models, we can meaningfully interpret the estimated parameters because of our structural assumptions. The parameter estimates, which correspond

¹⁴The Sobol sequence is a deterministic sequence that appears to be random. The use of the Sobol sequence instead of random numbers avoids the unpleasant clustering property of the conventional Monte-Carlo approach. See Glasserman (2003) for the exact definition of the Sobol sequence.

to the dynamics of the state variables, are displayed in Table 1. The parameter values are scaled so that they correspond to annual changes in macro variables and yields, measured in percentage points. For example, volatilities σ_i , $i = g, \pi, r$, are reported as $\sigma_i/\sqrt{\Delta}$ where $\Delta = 1/4$. In this section, we focus on the *TSM* version.

The constant parameter estimates in the private sector equations are generally consistent with single-regime New-Keynesian term-structure models (e.g., Bekaert, Cho, and Moreno, 2005, Hor-dahl, Tristani, and Vestin, 2006, Rudebusch and Wu, 2003). The degrees of forward-lookingness of inflation and output, μ_π and μ_g , respectively, are both approximately equal to 0.5. This value is similar to Bekaert, Cho, and Moreno (2005), but is higher than that found by other authors. The sensitivity of inflation to the output δ is small but significant, which is consistent with Rudebusch and Wu (2003). The sensitivity of output to real rate, ϕ , is small but significant, as is the case in much of the macroeconomic literature. Parameters m_g and m_π are insignificantly different from zero. Our model (2.1) - (2.3) implies that $m_\pi = -\delta E(g)$ and $m_g = \phi(E(r) - E(\pi))$. Therefore, the estimate of m_π is consistent with our use of detrended real output. The estimate of m_g is consistent with a small value of ϕ . In addition, the values of m_g and ϕ imply that the long-run real rate is equal to 2% (m_g/ϕ), which is consistent with the classic Taylor rule and is right in the middle of the various estimates reported in the macro literature.

We see that in one state of the regime variable s_t^e , the volatilities of exogenous shocks $\sigma_g(1) = 1.10$ and $\sigma_\pi(1) = 0.56$, which correspond to the output and inflation equations, respectively, are greater than the ones in the other regime, where $\sigma_g(2) = 0.63$ and $\sigma_\pi(2) = 0.23$. Therefore, we refer to the first state as a “high-volatility” regime.

The volatility $\sigma_r(1) = 2.73$ of the exogenous monetary policy shock in state 1 of s_t^d is greater than $\sigma_r(2) = 1.31$ in state 2. Thus, in the first state, which we call a “discretionary” regime, the Fed significantly deviates from the systematic interest rate rule. In the second state, the monetary authorities tend to follow the systematic rule more closely. We refer to this regime as “commitment.”

The regimes of the systematic monetary policy have very different properties. In one regime, the response to inflation $\alpha(1) = 3.22$ is greater than unity. In a single regime framework, this condition is known to guarantee the uniqueness of the rational expectation solution of model (2.1)-(2.3) and to rule out non-fundamental equilibria. We call this state an “active” monetary

policy regime, because of the strong reaction on the part of the Fed to one-quarter expected inflation. The other regime has a coefficient that is less than unity, $\alpha(2) = 0.38$. In a single-regime setup, this condition may lead the economy to be prone to non-fundamental sunspot fluctuations. Intuitively, if parameter α is less than 1, the Fed reduces its real rate long-term target when expected inflation rises. A lower real interest rate will accelerate economic growth, which will result in higher inflation. Thus, expectations about high inflation are self-fulfilling. We refer to this state as a “passive” monetary policy regime.¹⁵

Clarida, Gali, and Gertler (2000) estimate a constant coefficient model similar to the one specified in this paper for pre- and post-1979 subsamples. For the pre-1979 (pre-Volcker) subsample, they found the response to inflation to be less than unity, while for post-1979 (Volcker and Greenspan) subsample the response turned out to be greater than unity. Clarida, Gali, and Gertler argue that this finding may indicate the effect of non-fundamental sunspot shocks, which could explain the high volatility of the macroeconomic environment in the pre-Volcker era. However, this argument does not take into account the fact that economic agents may have expected the change from a passive regime to an active one. If this were the case, rational agents would have formed their expectations about inflation accordingly, thereby eliminating sunspot dynamic paths. Consistent with this view, our model has a unique rational expectations solution, despite the occasional switch of the monetary policy into the passive regime.

The values of parameters $\beta(1) = 2.12$ and $\beta(2) = 1.33$ suggest that the Fed reacted more aggressively to the real output in the active regime than in the passive one. We can also see that in the active regime, the degree of monetary policy inertia captured by parameter $\rho(1) = 0.97$ was higher than $\rho(2) = 0.79$ in the passive regime.

The discussion of the “preference” parameters is provided in Appendix G.

¹⁵We checked for indeterminacy of the rational expectations solutions conditional on both regimes using the formal method of Blanchard and Kahn (1980). Following Blanchard and Kahn, we constructed a matrix that defines the joint dynamics of predetermined and non-predetermined variables and computed its eigenvalues. For the set of parameters that corresponds to the “active” regime, the number of the eigenvalues outside the unit circle is equal to the number of non-predetermined variables. This property guarantees the existence and uniqueness of the solution. For the “passive” regime, the number of eigenvalues outside the unit circle turned out to be less than the number of non-predetermined variables, which points to the indeterminacy of the solution.

3.1.2 Regime probabilities

The estimated transition probabilities are displayed in Table 2. The monetary policy regimes, s_t^m , are the most persistent ones. For example, the probability that the active regime will continue is 99%. The monetary shocks regimes, s_t^d , are the least persistent ones: the probability of staying in either discretion or commitment regime is 82%. We can view the realizations of the regimes in our sample by computing the smoothed regime probabilities. Figure 1 displays the smoothed probabilities of the active policy, high-volatility, and discretionary regimes.

The active policy regime appeared in the 1970s and during the Volcker disinflation period after the end of the monetary policy experiment. As to the Greenspan tenure as the Fed chairman, the active regime prevailed from 1991 to 1995 and has been in place since 2002 to the end of the sample period. The period from 1973 to 1975, the monetary experiment of 1979-1982, the period from 1988 to 1991, and the internet bubble 1995-2001 period, are characterized by the passive regime.

The discretionary regime appeared sporadically throughout the sample. These regimes are well-identified because despite the frequent switching, there is little uncertainty whether a regime is discretionary: the smoothed regime probabilities are always clearly 0 or 1. Further, Figure 1 shows the Romer dates by vertical dashed lines. Those dates correspond to tightening monetary shocks obtained from the reading of the FOMC Minutes.¹⁶ In the context of our model, the Romer dates are interpreted as substantial deviations from the interest rate rule and, therefore, should induce the volatility of the monetary policy shock to rise. Indeed, Figure 1 shows that all Romer dates correspond to the discretionary regime. Our model allows many more discretionary regimes to be uncovered.

We can see that the volatility of exogenous shocks was high throughout the 1970s (during which major oil shocks occurred), during the monetary policy experiment, in 1998 (Russian default and the LTCM collapse), in 2000-2001 (the burst of the Nasdaq bubble and the terrorist attack of September 11th), and in the second half of 2004 (oil price shock). Note that all the recessions except the one of 1991 were associated with high-volatility regimes.

¹⁶See Romer and Romer (1989) for the exact definition of Romer dates.

3.2 Is the term structure important for identifying monetary policy regimes?

3.2.1 The short-rate model

We can see from Figure 1 that *TSM* delivers well-identified monetary policy regimes: at any point of time the smoothed probability of the regime is close to either 0 or 1. Does the term structure help in identifying the regimes? Intuitively, it does, because the long-term yields contain information about expected monetary policy and, in particular, about the probabilities of future monetary regimes.

To address this question more formally, we estimate *SRM*, that is, model (2.1)-(2.3), with the same specification of regime variables as in *TSM* but with the smaller dataset of inflation, output, and the short rate only. Because yields are not used, the “preference” parameters are not identified and, therefore, cannot be estimated.

Table 1 reports the parameter estimates for this model. As in *TSM*, there are two regimes of the systematic monetary policy. Using the monetary policy response to the one-quarter expected inflation as a basis, we can classify the regimes as active and passive. There are still two volatility regimes for exogenous shocks (high and low) and two regimes for the volatility of policy shocks (discretion and commitment).

Figure 2 shows the smoothed probabilities of the regimes for the short-rate model. We see that the high-volatility regime is very similar to the one obtained from the term structure model: the volatility was high in the 1970s and low afterwards. However, the monetary policy regimes are drastically different. The transition probability matrices in Table 2 and Figures 1 and 2 indicate that the regimes of the systematic monetary policy that are obtained from the *SRM* are much less persistent than those that are obtained from the *TSM*. In addition, in contrast to the term-structure model, where the systematic policy regime is well defined, the probability of the active regime from *SRM* hovers around 0.5 in many subperiods, for example, from 1987 to 1995. This property is in contrast to the discretionary regime in *TSM* as the latter has clearly delineated probabilities of either 0 or 1 despite the frequent switching of regimes.

Because of this indeterminacy of the active regime, it is easy to see how a more parsimonious model, one that does not allow for monetary policy changes, could be preferred to *SRM*. This

observation is consistent with the conclusions reached by Sims and Zha (2006) in a regime-switching model estimated without the yield curve. They find that a model with a best fit allows for shifts in disturbances only.

3.2.2 Informational advantage of using the yield curve

The previous subsection demonstrated that using the yield curve for estimation leads to very different conclusions about the historical monetary policy regimes. However, because the true monetary policy regime is unknown, we cannot evaluate the informativeness of the yield curve in quantitative terms. In order to address this issue, we performed the following simulation exercise. We simulated 1000 paths of data from the estimated term-structure model *TSM*. Then, we estimated both *TSM* and *SRM* along each path. For every simulated data sample, we computed the bias

$$\text{MSE}[\text{model}] = \left(\frac{1}{T} \sum_{t=1}^T (Q(t) - \hat{Q}(t))^2 \right)^{1/2} \quad (3.1)$$

of the smoothed probability $\hat{Q}(t)$ of the regime from the known simulated regime probability $Q(t)$ (equal to either 0 or 1 at each point of time) for both models and constructed the ratio of these biases:

$$R = \frac{\text{MSE}[SRM]}{\text{MSE}[TSM]}. \quad (3.2)$$

This procedure yields a finite sample distribution of the ratio R . If the yield curve does not contain information about the regimes additional to that provided by the state variables, the ratio R must be equal to 1.

Table 3 reports 95% confidence bounds of R for all the three regime variables of our model. We can see that these bounds lie above 1. The bias of the systematic monetary policy regime obtained from the *TSM* is, on average, 20 times smaller than that given by the *SRM*. We conclude that the yield curve and no-arbitrage restrictions are instrumental in identifying the monetary policy regimes.

3.3 Counterfactual policy analysis

Because our structural identification approach allows us to estimate parameters corresponding to all eight regimes, we can perform a model-based analysis of the effects of monetary policy regimes vis-a-vis exogenous volatility regimes. For a given combination of regimes, e.g., active monetary policy with discretionary shocks during high-volatility macro shocks, we can compute impulse responses.¹⁷ By comparing such impulse responses across the regime triples, that is, by performing counterfactual policy analysis, we can establish which aspects of the state dynamics are important for changes in macro variables or yields. Alternatively, we can perform a time-series counterfactual analysis. In this case, we would retain the shocks ϵ_t that were realized in the economy and change one of the regimes, say active monetary policy, to an alternative, for example, passive monetary policy. Such a hypothetical exercise allows to assess the importance of the exercised policy conditional on the specific historical experiences.

3.3.1 Counterfactual impulse responses

Figure 3 shows the impulse-response functions of the state variables in response to one standard deviation shocks in state variables themselves for *TSM*.¹⁸ The rightmost column of the panels shows the outcomes of the monetary policy shocks. These outcomes depend on the volatility of the policy shock and the stance of the systematic monetary policy, which determines the transmission of the shocks. However, they are independent of the volatility of exogenous shocks.

The observed change in the monetary transmission mechanism is mainly due to a change in the monetary policy. Indeed, there is almost no difference between the discretion and commitment regimes when the policy is passive (thin lines). When the policy is active (thick lines) the response is not identical, but qualitatively similar across the policy shock regimes. Output initially declines, but then increases in the long run. Inflation declines.¹⁹ Consistent with these changes in the

¹⁷Such regime conditioning for impulse response is consistent with the generalized impulse response approach of Koop, Pesaran, and Potter (1996).

¹⁸Another popular alternative is to consider a 1% shock. However, in this case we cannot distinguish between different volatility regimes and, therefore, we focus on one standard deviation shocks.

¹⁹This behavior is consistent with the private sector dynamics (2.1) and (2.2): a strong reaction of the interest rate to expected inflation in the active regime ($\alpha > 1$) leads to a higher real rate, which leads to decline in output and inflation.

private sector, the short rate initially increases, but then gradually declines in the active regime. In fact, the magnitudes of these changes can be attributed almost entirely to the active state of the monetary policy, because the response in the passive state is close to zero (except for the brief spike in the short rate). The effect of a policy shock on the macroeconomy is stronger under discretion than under commitment, conditional on the systematic policy regime. This is an expected result, given that the discretionary regime is characterized by the higher volatility of the policy shock.

The two left columns of the panels in Figure 3 depict the effects of the exogenous shocks. The volatility of the monetary policy shock does not affect the impulse responses in these plots. In the passive regime, the Fed reacts to an output shock strongly, by raising the short rate. However, this effect is transitory. In contrast, in the active regime, the rise in interest rates is more prolonged. As a result, the total effect on inflation and output is stronger in the active regime because both revert back to their intimal state eventually. Similarly, when facing a shock in the inflation equation in the active regime, the Fed raises the short rate in a more aggressive and protracted way compared to the reaction in the passive regime. In the high-volatility regime, the Fed always reacts more strongly to both types of shock than in the low-volatility regime.

We gauge the behavior of the yield curve in the different regimes by computing the impulse responses of the slope. The slope is defined as the 10-year yield minus the three-month yield. As before, we condition the responses on a given combination of regime states. Accordingly, the bond prices are computed conditional on a path of fixed regime states s . Under this assumption, standard affine pricing is applied so that the yield of a bond with maturity τ is a linear function of the state vector x_t :

$$y(x_t, \tau, s) = a(\tau, s) + b(\tau, s)'x_t. \tag{3.3}$$

Figure 4 shows the impulse-response functions. Overall, the slope responses suggest that in the active regime, the monetary policy steers the economy in a more stable way. Specifically, in comparison with the passive policy, the long-term yield rises more strongly than the short rate in response to all types of shock in the active regime. As a result of a monetary shock in the active regime, the term structure shifts in a parallel way so that the slope is essentially unaffected. In contrast, in the passive regime, a monetary policy shock leads the short end of the term structure to rise more strongly than the long end. Hence, the slope becomes negative.

As we saw in Figure 3, the short rate increases faster in the passive than in the active regime in response to the output shock. This leads to a sharp decline of the slope in the passive regime. The short-rate adjustment is more gradual in active regime, which results in a moderately positive impact on the slope. These observations, when combined with the fact that inflation declines in the active regime (Figure 3), suggest that that active monetary policy handles overheating economy more effectively than the passive one. Inflation shock leads to a decline in the slope in the passive regime, which suggests expectations of recession. In the active regime, the slope increases. This is consistent with a more aggressive and prolonged increase in the short rate and a faster decline in inflation, as shown in Figure 3.

3.3.2 Counterfactual economies

Impulse responses reveal how the macro variables and yields respond to exogenous shocks across the different regimes. However, they do not provide a full picture, because analysts are typically interested in understanding how the different regimes interact with each other, given the realized sequence of shocks. Simulating counterfactual economies allows us to achieve precisely this goal.

In particular, counterfactual economies allow us to address the debate regarding the sources of the great moderation, which was a decline in the volatility of economic activity after 1982. Many analysts argue that beginning with the Volcker chairmanship of the Fed, the monetary policy has improved and has led to lower inflation and lower volatility of output. However, Stock and Watson (2003) attribute a large part of the reduced volatility to the reduced volatility of the exogenous macroeconomic shocks.

Most authors evaluate the impact of monetary policy by splitting the sample at 1982, which marks the end of monetary experiment. Evaluating averages and the standard deviations of inflation and output before and after this date has led many researchers to announce the arrival of the great moderation. Indeed, as Table 4 documents, the volatility of both inflation and output falls after 1982, average output increases, and average inflation declines. Can one attribute this moderation to the monetary policy?

To address this question, we fix all the shocks as they were realized in our dataset. Similar to our estimation results, we assume that the high volatility of private sector shocks prevailed

pre-1982, and then switched to low volatility afterwards. In addition, we commit ourselves to one of the four possible policy-shock combinations: active policy and discretion (AD), active policy and commitment (AC), passive policy and discretion (PD), and passive policy and commitment (PC).

Table 4 provides the evidence from counterfactual economies. The results imply that if any of the four monetary regimes was fixed throughout the entire sample, one would observe the same signs of moderation as in the actual data. Therefore, a large driver of the moderation is the change in the nature of the private sector shocks. This conclusion is consistent with Stock and Watson (2003).

Does this finding imply that monetary policy was irrelevant? To address this question we revisit the impulse responses and conduct additional counterfactual exercises. The analysis reveals a nuanced picture that suggests that monetary policy plays an important role, in addition to the “lucky” realizations of the exogenous shocks.

First, as we noted above, impulse responses reveal that monetary policy plays an important role, because the differences between responses are due mainly to the type of policy regime (active or passive). Second, the impulse responses in Figure 3 highlight the asymmetries in the responses of output and inflation in the two monetary policy regimes, depending on the type of shock. In the case of inflation shock, there is a trade off between the macro variables behavior depending on the monetary policy regime. This is because output declines (an undesirable outcome) and inflation declines fast (a desirable outcome) in the active regime, while there are minimal changes in output (a desirable outcome) and inflation declines slowly (an undesirable outcome) in the passive regime. In the case of output shock, there is no trade off, because there are almost no differences in the output behavior across the regimes, while inflation declines much faster in the active regime. Therefore, depending on the type of shock that is realized, either active policy will dominate other strategies or there will be a non-trivial trade off.

To investigate these effects in greater detail, we examine the counterfactual analysis of the time series once again. This time, in addition to fixing all the shocks as they were realized in our dataset, we also fix the realized volatility regimes of the exogenous shocks (high or low). As before, we commit ourselves to one of the four possible policy-shock combinations: AD, AC, PD, or PC. By fixing one of the policies throughout the sample, but allowing for the realized exogenous

shocks, we can evaluate whether the Fed had to trade the active regime off against the passive one and whether it was handled efficiently.

Figure 5 displays the time series of the simulated counterfactual economies. We observe that in the passive regimes (PD or PC), there is almost no difference between the realized and counterfactual outputs, while counterfactual inflation is higher overall than the realized one. In the active regimes, the counterfactual inflation is lower until 1996. The counterfactual output is slightly higher than the realized one after 1991 in the AC economy, while it is sporadically greater or smaller than the realized one in the AD economy. These plots imply that shock realizations were such that the Fed had to face the trade off discussed above.

To gauge how effective the Fed was in addressing the trade off, Table 4 compares the average inflation and output and the respective standard deviations from the full sample to those obtained from the counterfactual economies. The aforementioned trade off in managing output and inflation is clear, because the average output is higher in the passive economies while the average inflation is lower in the active economies. Moreover, in the active economies, there is an additional trade off between the discretionary and commitment regimes, because the discretionary regime leads to lower inflation and higher output at the cost of both variables being more volatile than in the commitment regime.

Evaluating the impact of the various policies on yields informs us about further differences between the regimes. Table 4 reports the averages and standard deviations of the short and long yields and Figure 6 complements it with the time-series plots. The AD regime, which seemed to be attractive, given the average inflation and output, appears to be not likely in the role of the single prevailing regime because it generates highly volatile yields. In the PD regime, the averages and volatilities are very similar to those in the data. The PC economy can generate even less volatile yields. Figure 3 shows that discretionary shock helps to cut inflation faster, but it appears that, because of the high inflation and yield volatility in the discretionary regime, the Fed does not continue to use such a policy for long.

In the context of the post-1982 moderation, the realized inflation is, on average, lower and less volatile than inflation in any of the individual regimes. The volatility of realized output is similar to the single-regime counterfactual economies. However, the average realized output is lower than it would have been under the active monetary policy and the realized interest rates

are more volatile than the ones in the passive regime. On balance, the Fed appears to have been relatively successful in considering the described trade offs.

4 Conclusion

We have presented a no-arbitrage term-structure model that is complemented with structural identification assumptions in the spirit of the structural VAR literature. The model incorporates the regime-switching dynamics of monetary policy and the changing volatilities of exogenous shocks that drive the economy.

We demonstrated the presence of at least two regimes of systematic policy: active and passive. The active regime is characterized by a stronger response to the expected one-quarter inflation. This regime prevailed in the 1970s, throughout the Volcker's disinflation, in 1991-1995, and in 1995-2004. We also found the presence of two volatility regimes of exogenous shocks driving the economy: high- and low-volatility regimes. The high-volatility regime is mostly associated with oil shocks and recessions. Finally, we detected two regimes of the volatility of the monetary policy shock, interpreted as discretion and commitment. These two regimes were interchanged sporadically throughout the sample period.

The yield curve and no-arbitrage restrictions are instrumental in identifying the monetary policy regimes. We found drastically different policy regimes when the yields were not involved. The simulation study that we performed demonstrates that the yield curve reduces the bias of the estimated monetary policy regimes by an order of magnitude. No-arbitrage restrictions help in distinguishing market-based expectations of monetary policy from the objective expectations.

Our identifying assumptions helped us to separate monetary policy and private sector regimes; hence, we were able to address the issue of what their relative roles were in the great moderation. We simulated counterfactual economies that maintain realized shocks, but allowed only one set of regimes to prevail throughout the full sample. Low volatility of the private-sector shocks during the post-1982 period makes a clear contribution to moderation. We identified important trade offs between active and passive monetary policy for managing inflation and output growth, and trade offs between discretionary and commitment policy shocks for containing the volatility of the state variables and cutting the inflation rate. Overall, our results suggest that monetary policy was

important for the moderation, in addition to the fortunate realizations of the exogenous shocks.

References

- Ang, Andrew, Geert Bekaert, and Min Wei, 2008, The term structure of real rates and expected inflation, *Journal of Finance*, forthcoming.
- Ang, Andrew, Jean Boivin, and Sen Dong, 2007, Monetary policy shifts and the term structure, Working paper, Columbia Business School.
- Bansal, Ravi, George Tauchen, and Hao Zhou, 2003, Regime-shifts, risk premiums in the term structure, and the business cycles, working paper.
- Bansal, Ravi, and Hao Zhou, 2002, Term structure of interest rates with regime shifts, *Journal of Finance* 57, 1997–2043.
- Bekaert, Geert, Seonghoon Cho, and Antonio Moreno, 2005, New keynesian macroeconomics and the term structure, NBER working paper No. 11340.
- Blanchard, Oliver, and Charles M. Kahn, 1980, The solution of linear difference models under rational expectations, *Econometrica* 48, 1305–1312.
- Boivin, Jean, 2005, Has u.s. monetary policy changed? evidence from drifting coefficients and real-time data, NBER working paper No. 11314.
- Cho, Seonghoon, and Antonio Moreno, 2003, A structural estimation and interpretation of the new keynesian macro models, working paper, Columbia University.
- Clarida, Richard, Jordi Gali, and Mark Gertler, 2000, Monetary policy rules and macroeconomic stability: Evidence and some theory, *Quarterly Journal of Economics* 115, 147–180.
- Cogley, Timothy, 2005, Changing beliefs and the term structure of interest rates: Cross-equation restrictions with drifting parameters, *Review of Economic Dynamics* 8, 420–451.
- , and Thomas Sargent, 2005, Drifts and volatilities: Monetary policies and outcomes in the post war U.S., *Review of Economic Dynamics* 8, 262–302.
- Conley, Timothy, Lars P. Hansen, and W.F. Liu, 1997, Bootstrapping the long run, *Macroeconomic Dynamics* 1, 279–311.

- Cook, Timothy, 1989, Determinants of the federal funds rate: 1979-1982, *Federal Reserve Bank of Richmond Economic Review* 75, 3–19.
- Dai, Qiang, Kenneth Singleton, and Wei Yang, 2004, Predictability of bond risk premia and affine term structure models, Working Paper, Stanford University.
- , 2007, Regime shifts in a dynamic term structure model of U.S. Treasury bond yields, *Review of Financial Studies*.
- Duffee, Gregory, 2002, Term premia and interest rate forecasts in affine models, *Journal of Finance* 57, 405–443.
- Evans, M.D.D., 2003, Real risk, inflation risk, and the term structure, *The Economic Journal* 113, 345–389.
- Farmer, Roger, Daniel Waggoner, and Tao Zha, 2006, Minimal state variable solutions to Markov-switching rational expectations models, Working paper, Federal Reserve Bank of Atlanta.
- Glasserman, Paul, 2003, *Monte-Carlo Methods in Financial Engineering*. (Springer).
- Hordahl, Peter, Oreste Tristani, and David Vestin, 2006, A joint econometric model of macroeconomic and term structure dynamics, *Journal of Econometrics* 131, 405 – 444.
- King, Robert G., 2000, The new IS-LM model: Language, logic, and limits, *Federal Reserve Bank of Richmond Economic Quarterly* 86, 45–103.
- Koop, G., M.H. Pesaran, and S.M. Potter, 1996, Impulse response analysis in nonlinear multivariate models, *Journal of Econometrics* 74, 119147.
- McCallum, Bennett, 1983, On non-uniqueness in rational expectations models: An attempt at perspective, *Journal of Monetary Economics* 11, 139–168.
- Primiceri, G., 2004, Why inflation rose and fell: Policymakers’ beliefs and postwar stabilization policy, unpublished manuscript, Northwestern University.
- Romer, Christina D., and David H. Romer, 1989, Does monetary policy matter? a new test in the spirit of friedman and schwartz., *NBER Macroeconomic Annual* 4, 121–170.

- , 2004, A new measure of monetary shocks: Derivation and implications, *American Economic Review*.
- Rudebusch, Glenn, and Tao Wu, 2003, A macro-finance model of the term structure, monetary policy, and the economy, working paper, Federal Reserve Bank of San Francisco.
- Sargent, Thomas J., 1999, *The Conquest of American Inflation* (Princeton Univ. Press: Princeton).
- Schorfheide, Frank, 2005, Learning and monetary policy shifts, *Review of Economic Dynamics* 8, 392–419.
- Sims, Christopher A., and Tao Zha, 2006, Were there regime switches in U.S. monetary policy, *American Economic Review* 96, 54–81.
- Stock, James, and Mark Watson, 2003, Has the business cycle changed? Evidence and explanations, in *Monetary Policy and Uncertainty* (Federal Reserve Bank of Kansas City).
- Woodford, Michael, 2003, *Interest and Prices: Foundations of a Theory of Monetary Policy* (Princeton Univ. Press: Princeton).

A Rational Expectations Solution

This appendix suggests a method to find the rational expectations solution of problem (2.1)-(2.3).

First, the problem can be rewritten in the following compact matrix form

$$B_0(s_t^m)x_t = m_0(s_t^m) + B_{-1}(s_t^m)x_{t-1} + B_1(s_t^m)E_t[x_{t+1}] + \Gamma(s_t^e, s_t^m, s_t^d)\varepsilon_t \quad (\text{A.1})$$

where

$$B_0 = \begin{bmatrix} 1 & 0 & \phi \\ -\delta & 1 & 0 \\ -(1 - \rho(s_t^m))\beta(s_t^m) & 0 & 1 \end{bmatrix}, \quad B_{-1} = \begin{bmatrix} 1 - \mu_g & 0 & 0 \\ 0 & 1 - \mu_\pi & 0 \\ 0 & 0 & \rho(s_t^m) \end{bmatrix},$$

$$B_1 = \begin{bmatrix} \mu_g & \phi & 0 \\ 0 & \mu_\pi & 0 \\ 0 & (1 - \rho(s_t^m))\alpha(s_t^m) & 0 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \sigma_g(s_t^e) & 0 & 0 \\ 0 & \sigma_\pi(s_t^e) & 0 \\ 0 & 0 & \sigma_r(s_t^d) \end{bmatrix},$$

$$m_0 = (m_g, m_\pi, m_r)' \text{ and } \varepsilon_t = (\varepsilon_t^g, \varepsilon_t^\pi, \varepsilon_t^r)'$$

Since matrix $B_0(s_t^m)$ is generally non-singular (A.1) can be recasted in a more convenient form

$$x_t = m_1(S_t) + F_1(S_t)x_{t-1} + E_t[A_1(S_t)x_{t+1}] + \Sigma_1(S_t)\varepsilon_t \quad (\text{A.2})$$

with $m_1(S_t) = B_0^{-1}(s_t^m)m_0(s_t^m)$, $F_1(S_t) = B_0^{-1}(s_t^m)B_{-1}(s_t^m)$, $A_1(S_t) = B_0^{-1}(s_t^m)B_1(s_t^m)$, $\Sigma_1(S_t) = B_0^{-1}(s_t^m)\Gamma(s_t^e, s_t^m, s_t^d)$ and where $S_t = (s_t^e, s_t^m, s_t^d)$ denotes the compound regime variable.

We first demonstrate that the rational expectations solution of (A.2) has a form of the regime switching vector autoregressive process (2.4). Substituting (2.4) in (A.2) and keeping in mind that S_t is independent of the state variable x_t we can see that (2.4) is the solution of (A.2) if there exist reduced form parameters μ , Φ and Σ satisfying the following set of equations:

$$\begin{aligned} [I - A_1(S_t)E_t\Phi(S_{t+1})]\Phi(S_t) &= F_1(S_t) \\ [I - A_1(S_t)E_t\Phi(S_{t+1})]\mu(S_t) &= m_1(S_t) + E_t\mu(S_{t+1}) \\ \Sigma(S_t) &= [I - A_1(S_t)E_t\Phi(S_{t+1})]^{-1}\Sigma_1(S_t), \end{aligned}$$

which can be rewritten in terms of transition regime probabilities π_{ij} as

$$[I - A_1(i)\sum_j \pi_{ij}\Phi(j)]\Phi(i) = F_1(i) \quad (\text{A.3})$$

$$[I - A_1(i) \sum_j \pi_{ij} \Phi(j)] \mu(i) = m_1(i) + \sum_j \pi_{ij} \mu(j) \quad (\text{A.4})$$

$$\Sigma(i) = [I - A_1(i) \sum_j \pi_{ij} \Phi(j)]^{-1} \Sigma_1(i). \quad (\text{A.5})$$

To find the solution directly from the above equations one would have to solve the system of coupled quadratic matrix equations (A.3) for $\Phi(i)$ and then use $\Phi(i)$ to find $\mu(i)$ and $\Sigma(i)$ from (A.4) and (A.5). However, because the direct solution of (A.3) is complicated we do not follow this approach. Instead, we generalize a method proposed in Cho and Moreno (2003) to the case of regime switching dynamics. The idea of the solution method is to substitute the left hand side of (A.2) into the expectation in the right hand side recursively. At step n of this recursive procedure we obtain

$$x_t = m_n(S_t) + F_n(S_t)x_{t-1} + E_t[A_n(S_{t+1}, \dots, S_{t+n-1})x_{t+n}] + \Sigma_n(S_t)\varepsilon_t \quad (\text{A.6})$$

By substituting (A.6) into the right hand side of (A.2) we obtain the following recursive formulas for coefficients m_n , F_n , A_n and Σ_n :

$$\begin{aligned} C_n(S_t) &\equiv I - A_1(S_t)E_t F_n(S_{t+1}) \\ F_{n+1}(S_t) &= C_n^{-1}(S_t)F_1(S_t) \\ m_{n+1}(S_t) &= C_n^{-1}(S_t) [m(S_t) + A_1(S_t)E_t m_n(S_{t+1})] \\ A_{n+1}(S_t, \dots, S_{t+n}) &= C_n^{-1}A_1(S_t)A_n(S_t, \dots, S_{t+n-1}) \\ \Sigma_{n+1}(S_t) &= C_n^{-1}(S_t)\Sigma_1(S_t) \end{aligned}$$

Suppose the above iteration procedure converges, i.e. there exist $\Phi(S_t)$, $\mu(S_t)$ and $\Sigma(S_t)$ such that

1. For $S_t = 1, \dots, N$ $\lim_{n \rightarrow \infty} F_n(S_t) = \Phi(S_t)$
2. For $S_t = 1, \dots, N$ $\lim_{n \rightarrow \infty} \mu_n(S_t) = \mu(S_t)$
3. For $S_t = 1, \dots, N$ $\lim_{n \rightarrow \infty} \Sigma_n(S_t) = \Sigma(S_t)$

Also, assume that the following condition is satisfied

$$\lim_{n \rightarrow \infty} E_t[A_n(S_{t+1}, \dots, S_{t+n-1})x_{t+n}] = 0. \quad (\text{A.7})$$

Then (2.4) is obviously the solution of the (A.2) and, therefore, of the original model (2.1-2.3). Cho and Moreno (2003) refer to condition (A.7) as the no-bubble condition.²⁰ They report that, in the case of a unique stationary solution, the above iterative method delivers the same solution as obtained with the QZ decomposition method.

In case the rational expectations solution is not unique the iterative procedure presented above yields the so-called minimum state variable solution proposed in McCallum (1983). Indeed, when matrices $F_1(i)$ tend to zero so that (A.2) becomes purely forward-looking matrices $\Phi(i)$ tend to zero as well. Hence, the solution becomes a random walk driven only by the current shocks ε_t . As discussed in McCallum (1983) this property characterizes the minimal state variable solution.²¹

B Risk premia

This appendix provides motivation for our formulation of risk premia $\Lambda_{t,t+1}$ in (2.6). From the prospective of structural modeling, (2.6) can be seen as a local approximation of a general nonlinear form of the stochastic discount factor $\log M_{t,t+1} = g(x_{t+1}, x_t)$ that does not explicitly depend on regime S_t :

$$\begin{aligned}
\log M_{t,t+1} &\approx g(x_t, x_t) + \frac{\partial g}{\partial x}(x_{t+1} - x_t) \\
&= g(x_t, x_t) + \frac{\partial g}{\partial x}\mu(S_{t+1})\frac{\partial g}{\partial x}x_t(\Phi(S_{t+1}) - I) + \frac{\partial g}{\partial x}\Sigma(S_{t+1})\varepsilon_{t+1} \\
&= g(x_t, x_t) + \frac{\partial g}{\partial x}\mu(S_{t+1}) + \frac{\partial g}{\partial x}x_t(\Phi(S_{t+1}) - I) + \frac{1}{2}\left(\frac{\partial g}{\partial x}\Sigma(S_{t+1})\right)'\frac{\partial g}{\partial x}\Sigma(S_{t+1}) \\
&\quad - \frac{1}{2}\left(\frac{\partial g}{\partial x}\Sigma(S_{t+1})\right)'\frac{\partial g}{\partial x}\Sigma(S_{t+1}) + \frac{\partial g}{\partial x}\Sigma(S_{t+1})\varepsilon_{t+1}
\end{aligned} \tag{B.1}$$

It must be the case that $E_t[M_{t,t+1}] = \exp(-r_t)$. Using log-linearization, this property implies that

$$-r_t \approx g(x_t, x_t) + \sum_j \pi_{S_t, j} \left[\frac{\partial g}{\partial x}\mu(j) + \frac{\partial g}{\partial x}(\Phi(j) - I)x_t + \frac{1}{2}\left(\frac{\partial g}{\partial x}\Sigma(j)\right)'\frac{\partial g}{\partial x}\Sigma(j) \right] \tag{B.2}$$

²⁰To make sure that condition (A.7) holds we checked it by simulation for the set of estimated parameters. We found that the condition is satisfied.

²¹Farmer, Waggoner, and Zha (2006) identify a class of minimum state variable solutions that are also solutions to the regime-switching rational expectations model and provide a set of necessary and sufficient conditions for the solution to be unique. While the class of identified solutions is large, it is not exhaustive.

Based on these expressions, we conclude that the continuous-time formulation of (B.1) would imply the reduced-form formulation of the stochastic discount factor in the expressions (2.5)-(2.6), where $\Pi(x_t) = \frac{\partial g}{\partial x}(x_t, x_t)$. In discrete time, the drift of the log stochastic discount factor, that is, the next-to-last line in (B.1), must be close to $-r_t$, as its expectation is equal to $-r_t$.

C Bond Pricing

In order to facilitate the bond pricing we introduce risk-neutral notations. Following Dai, Singleton, and Yang (2007), we introduce an equivalent risk neutral measure \mathbb{Q} such that the Radon-Nikodym derivative is given by

$$\left(\frac{d\mathbb{Q}}{d\mathbb{P}}\right)_{t,t+1} = \exp\left(-\frac{1}{2}\Lambda'_{t,t+1}\Lambda_{t,t+1} - \Lambda'_{t,t+1}\varepsilon_{t+1}\right) \quad (\text{C.1})$$

where \mathbb{P} denotes the physical measure. It can be shown that a random variable $\varepsilon_{t+1}^{\mathbb{Q}} = \varepsilon_{t+1} - \Lambda_{t,t+1}$ has a standard Gaussian distribution under \mathbb{Q} .²² Thus, the risk neutral dynamics of the state variable x_t has the following form

$$x_t = \mu^{\mathbb{Q}}(S_t) + \Phi^{\mathbb{Q}}(S_t)x_{t-1} + \Sigma(S_t)\varepsilon_t^{\mathbb{Q}} \quad (\text{C.2})$$

where the risk neutral parameters are

$$\mu^{\mathbb{Q}}(S_t) = \mu(S_t) - \Sigma(S_t)\Sigma'(S_t)\Pi_0 \quad (\text{C.3})$$

$$\Phi^{\mathbb{Q}}(S_t) = \Phi(S_t) - \Sigma(S_t)\Sigma'(S_t)\Pi_x \quad (\text{C.4})$$

Standard arguments then imply that the price of a zero-coupon bond with maturity n is given by

$$B_t^n(x_t, S_t) = E^{\mathbb{Q}}[e^{-\sum_{i=0}^{n-1} r_{t+i}} | x_t, S_t] \quad (\text{C.5})$$

with $r_t = \delta'x_t$ and $\delta = (0, 0, 1)'$.

We can reinterpret the price (C.5) as being equal to $B_t^n(x, i) = E^{\mathbb{Q}}[\exp(-\xi_{t,n}) | x_t = x, S_t = i]$ where x and i are time- t values of the state and regime variables respectively and $\xi_{t,n} = \delta'(x_t +$

²²See, for example, Dai, Singleton, and Yang (2007) for details. Dai, Singleton, and Yang use a different timing for regime switches but the change of measure can be done in the same way for the timing used in our paper.

$\dots + x_{t+n-1}$). The yield of the bond can be represented in term of cumulants of random variable $\xi_{t,n}$.

$$Y_t^n(x, i) = -\frac{1}{n} \log \left(E^{\mathbb{Q}}[\exp(-\xi_{t,n}) | x, i] \right) = -\frac{1}{n} \sum_{k=1}^{\infty} \frac{(-1)^k}{k!} \mu_n^{(k)}(x, i) \quad (\text{C.6})$$

Cumulants $\mu_n^{(k)}(x, i)$ are related to the moments $m_n^{(k)}(x, i) = E^{\mathbb{Q}}[\xi_{t,n}^k | x_t = x, S_t = i]$ of $\xi_{t,n}$ in the following recursive way:

$$m_n^{(k)}(x, i) = m_n^{(k)}(x, i) - \sum_{j=1}^{k-1} \binom{k-1}{j-1} \mu_n^{(j)}(x, i) m_n^{(k-j)}(x, i) \quad (\text{C.7})$$

Note that although the price of a bond $B_t^n(x, i)$ does not have a closed form solution the moments of $\xi_{t,n}$ do. It can be shown that the n -th moment $m_n^{(k)}(x, i) = E^{\mathbb{Q}}[\xi_{t,n}^k | x_t = x, S_t = i]$ can be represented as

$$m_n^{(k)}(x, i) = A_n^0(i) + \sum_{i_1} A_n^{i_1}(i) x^{(i_1)} + \sum_{i_1, i_2} A_n^{i_1, i_2}(i) x^{(i_1)} x^{(i_2)} + \dots + \sum_{i_1, i_2, \dots, i_k} A_n^{i_1, i_2, \dots, i_k}(i) x^{(i_1)} x^{(i_2)} \dots x^{(i_k)} \quad (\text{C.8})$$

By substituting (C.8) into the definition of moment $m_{n+1}^{(k)}(x, i)$ for maturity $n+1$ one can find the recursive formulas for coefficients $A_n^{i_1, i_2, \dots, i_k}$. The idea of our approximation method is to compute a few first moments to approximate bond prices. In what follows we demonstrate that taking just two first cumulants in (C.6) produces accurate enough prices.

The first two moments $m_n^{(k)}(x, i)$ can be represented using the following matrix notations:

$$\begin{aligned} m_n^{(1)}(x, i) &= A_n(i) + B_n'(i)x \\ m_n^{(2)}(x, i) &= C_n(i) + D_n'(i)x + x'F_n(i)x \end{aligned}$$

where coefficients $A_n(i)$, $B_n(i)$, $C_n(i)$, $D_n(i)$ and $F_n(i)$ satisfy the following recursive equations:

$$\begin{aligned} A_{n+1}(i) &= \sum_j \pi_{ij} [A_n(j) + \mu^{\mathbb{Q}}(j)' B_n(j)] \\ B_{n+1}(i) &= \delta + \sum_j \pi_{ij} \Phi^{\mathbb{Q}}(j)' B_n(j) \\ C_{n+1}(i) &= \sum_j \pi_{ij} [C_n(j) + \mu^{\mathbb{Q}}(j)' D_n(j) + \mu^{\mathbb{Q}}(j)' F_n(j) \mu^{\mathbb{Q}}(j) + \text{tr}(F_n(j) \Sigma(j) \Sigma(j)')] \\ D_{n+1}(i) &= 2A_{n+1}(i)\delta + \sum_j \pi_{ij} \Phi^{\mathbb{Q}}(j)' [D_n(j) + 2F_n'(j) \mu^{\mathbb{Q}}(j)] \\ F_{n+1}(i) &= -\delta\delta' + \delta B_{n+1}'(i) + \delta' B_{n+1}(i) + \sum_j \pi_{ij} \Phi^{\mathbb{Q}}(j)' F_n(j) \Phi^{\mathbb{Q}}(j) \end{aligned}$$

with initial conditions $A_1(i) = 0$, $B_1(i) = \delta$, $C_1(i) = 0$, $D_1(i) = 0$ and $F_1(i) = \delta\delta'$.

We performed a simulation study to check how accurate our quadratic approximation is. To find a bond price by simulation we note that the bond price has a close form solution conditional on a regime path. Indeed, the price of a bond (C.5) can be represented as

$$B_t^n(x, i) = E^{\mathbb{Q}} [E^{\mathbb{Q}}[\exp(-\xi_{t,n})|x_t = x, \mathcal{S}_t^{t+n-1}]|S_t = i] \quad (\text{C.9})$$

where the outer expectation is taken with respect to regime paths $\mathcal{S}_t^{t+n-1} = \{S_t, \dots, S_{t+n-1}\}$. The inner expectation in (C.9) can be represented as

$$\mathcal{B}_t^n(x, \mathcal{S}_t^{t+n-1}) \equiv E^{\mathbb{Q}}[\exp(-\xi_t, n)|x_t = x, \mathcal{S}_t^{t+n-1}] = \exp(-A_n(\mathcal{S}_t^{t+n-1}) - B_n(\mathcal{S}_t^{t+n-1})'x) \quad (\text{C.10})$$

where coefficients $A_n(\mathcal{S}_t^{t+n-1})$ and $B_n(\mathcal{S}_t^{t+n-1})$ satisfy the following recursive equations:

$$\begin{aligned} A_n(\mathcal{S}_t^{t+n-1}) &= A_{n-1}(\mathcal{S}_{t+1}^{t+n-1}) + \mu^{\mathbb{Q}}(S_{t+1})B_{n-1}(\mathcal{S}_{t+1}^{t+n-1})' \\ &\quad - \frac{1}{2}B_{n-1}(\mathcal{S}_{t+1}^{t+n-1})\Sigma(S_{t+1})\Sigma(S_{t+1})'B_{n-1}(\mathcal{S}_{t+1}^{t+n-1})' \\ B_n(\mathcal{S}_t^{t+n-1}) &= \delta + \Phi^{\mathbb{Q}}(S_{t+1})'B_{n-1}(\mathcal{S}_{t+1}^{t+n-1}) \end{aligned}$$

with boundary conditions $A_1(\mathcal{S}_{t+n-1}^{t+n-1}) = 0$ and $B_1(\mathcal{S}_{t+n-1}^{t+n-1}) = \delta$. Based on the above formulas the simulation strategy is clear: we simulate regime paths $\mathcal{S}_t^{t+n-1} = \{S_t, \dots, S_{t+n-1}\}$, compute the bond prices (C.10) conditional on the paths, and take their average over a number of simulations. This ‘‘conditional Monte-Carlo’’ strategy improves the convergence properties drastically compared with the direct sampling of the state variables.

Equipped with the benchmark simulation pricing method we compared the accuracy of the quadratic approximation suggested above. The design of the experiment was as follows. We constructed a range of parameter values and state variables within three standard deviations from the estimated parameters and unconditional means of state variables. We generated 5,000 Sobol points inside this range. For each generated set of parameters we computed bond prices based on simulation, quadratic approximation, and a log-linear approximation proposed in Bansal and Zhou (2002). Then, we calculated mean, median and maximal absolute deviations of yields obtained with both approximate methods from the exact (simulated) yields. To make sure that the simulated prices are found accurately enough we repeated the exercise several times increasing the number of simulated paths. We found that 100,000 simulations are sufficient: the results did

not change when the number of simulations was increased further. Table 5 reports the results. The maximum error produced by our method is 2.8 basis points which is less than the accuracy of the Fama-Bliss zero yield bootstrapping procedure and by far less than the pricing errors generated by our model (see table 6).

D Likelihood Function

Let $\Omega_t = \{w_1, \dots, w_t\}$ be the econometrician dataset at time t , $t = 1, \dots, T$, where T is the sample size. The econometrician observes the state variable x_t that consists of detrended real GDP, inflation, and the short rate (equal to the three month bill yield) as well as the set of yields $Y_t = (Y_t^1, \dots, Y_t^n)$ with maturities longer than 3 months: τ_1, \dots, τ_n . Thus, $w_t = (x_t, Y_t)$. The state vector x_t satisfies a regime-switching VAR (2.4) with parameters μ , Φ and Σ being the non-linear functions of structural parameters from model (2.1)-(2.3). Yields Y_t are observed with errors and have the following form

$$Y_t^{(i)} = f(x_t, S_t, \tau_i) + \epsilon_t^i, \quad (\text{D.1})$$

where $f(x_t, S_t, \tau_i)$ are the yields generated by the model. Measurement errors are assumed to be mutually independent i.i.d. normally distributed random shocks with standard errors σ_y .

Let $Q_t^i = \Pr(S_t = i | \Omega_t)$ be the filtered probability of regime i . Following Ang, Bekaert, and Wei (2008) and Dai, Singleton, and Yang (2007) we use a recursive algorithm in order to compute the likelihood function. At time $t = 1$ we initialize Q_t^i at the stationary probabilities π_i^* of the Markov chain S_t . At time $t + 1$ we compute the following conditional density

$$p(w_{t+1}, S_{t+1} = j | \Omega_t) = \sum_i \pi_{ij} Q_t^i p(x_{t+1} | x_t, S_t = i, S_{t+1} = j) p(Y_{t+1} | x_{t+1}, S_t = i, S_{t+1} = j), \quad (\text{D.2})$$

where $\pi_{i,j}$ are the transition probabilities of S_t and the conditional densities of x_{t+1} and Y_{t+1} are equal to

$$\begin{aligned} p(x_{t+1} | x_t, S_t = i, S_{t+1} = j) &= \frac{1}{(2\pi)^{3/2} |\Sigma_j \Sigma_j'|^{1/2}} \\ &\times \exp \left[-\frac{1}{2} (x_{t+1} - \mu_j - \Phi_j x_t)' (\Sigma_j \Sigma_j')^{-1} (x_{t+1} - \mu_j - \Phi_j x_t) \right] \\ p(Y_{t+1} | x_{t+1}, S_t = i, S_{t+1} = j) &= \frac{1}{(2\pi \sigma_y^2)^{n/2}} \exp \left[-\frac{1}{2\sigma_y^2} \sum_{k=1}^n (Y_{t+1}^{(k)} - f(x_{t+1}, j, \tau_k))^2 \right] \end{aligned}$$

The filtered probabilities of regimes are then updated using the Bayes rule:

$$Q_{t+1}^j = \Pr(S_{t+1} = j | \Omega_{t+1}) = \frac{p(w_{t+1}, S_{t+1} = j | \Omega_t)}{p(w_{t+1} | \Omega_t)} \quad (\text{D.3})$$

The density of w_{t+1} conditional on the information set at time t is given by

$$p(w_{t+1} | \Omega_t) = \sum_j p(w_{t+1}, S_{t+1} = j | \Omega_t). \quad (\text{D.4})$$

Finally, the log-likelihood function is equal to

$$\mathcal{L} = \frac{1}{T-1} \sum_{t=1}^{T-1} \log p(w_{t+1} | \Omega_t). \quad (\text{D.5})$$

E Long-Run Means of The State Variables

This appendix demonstrates how to compute the unconditional mean of the state variable x_t based on the reduced-form representation (2.4) of the model dynamics. We essentially follow Ang, Bekaert, and Wei (2008). First, let us compute conditional means $m_i = E[x_t | S_t = i]$, $i = 1, \dots, N$. From (2.4) it follows that

$$E[x_{t+1} | S_{t+1} = i] = \mu_i + \Phi_i E[x_t | S_{t+1} = i]. \quad (\text{E.1})$$

The conditional expectation in the right hand side can be rewritten as

$$E[x_t | S_{t+1} = i] = \sum_j \Pr(S_t = j | S_{t+1} = i) E[x_t | S_t = j]. \quad (\text{E.2})$$

Probabilities $b_{ji} = \Pr(S_t = j | S_{t+1} = i)$ define the “backward” transition matrix. These probabilities are related to the “forward” transition probabilities $\pi_{ij} = \Pr(S_{t+1} = i | S_t = j)$:

$$b_{ji} = \pi_{ij} \frac{\pi_j^*}{\pi_i^*}, \quad (\text{E.3})$$

where π_i^* are the stationary probabilities of π_{ij} . Combining (E.1) and (E.2) we obtain the following equations for conditional means m_i :

$$m_i = \mu_i + \Phi_i \sum_j b_{ji} m_j \quad (\text{E.4})$$

By introducing matrix notations $\bar{m} = [m'_1, \dots, m'_N]'$, $\bar{\mu} = [\mu'_1, \dots, \mu'_N]'$ and

$$b\Phi = \begin{bmatrix} b_{11}\Phi_1 & \cdots & b_{N1}\Phi_1 \\ & \cdots & \\ b_{1N}\Phi_N & \cdots & b_{NN}\Phi_N \end{bmatrix}$$

we can represent the solution to (E.4) as $\bar{m} = (I - b\Phi)^{-1}\bar{\mu}$. Finally, the unconditional mean of x_t is given by $Ex_t = \sum_i \pi_i^* m_i$.

F Goodness of Fit

To evaluate the model's ability to fit the data we first review the pricing errors of the yields of different maturities. Table 6 indicates that the absolute pricing errors do not exceed 30 basis points. This is a reasonable accuracy given the magnitude of the noise in the approximated zero yields (e.g., Dai, Singleton, and Yang, 2004). Table 6 also reports the correlation of model-implied slope and curvature with their data counterparts.²³ The correlations are 97.5% and 64.1% respectively. Thus, the model captures the slope almost perfectly and does a decent job in fitting the curvature. This performance is on par with single regime term structure models that use several latent variables as well as observed macro variables.

We also evaluate the model's ability to fit various unconditional moments, such as the mean, standard deviation, skewness, kurtosis, and autocorrelations of the state variables, yields, slope, and curvature. We use the parametric bootstrap strategy to compute the finite sample confidence bounds of the model-implied moments.²⁴ Table 7 reports the results. The model is able to match all aspects of the data except for the kurtosis of the curvature. Note, that although the state dynamics (2.4) involves only one quarter lag of the state variables the one year autocorrelations are perfectly matched by our model.

We conclude that, overall, model *TSM* fits the data well.

²³In this paper the slope of the yield curve is defined as $y(40) - y(1)$, i.e. the ten year yield minus the three month yield. The curvature is defined as $y(40) + y(1) - 2y(8)$, where $y(8)$ is the two year yield.

²⁴We do not take into account parameter uncertainty. With parameter uncertainty the confidence bounds would be even wider.

G Risk Premia Estimates

Because our model specification is not derived from the micro fundamentals, our results are silent about “deep” parameters that connect investor’s preferences and state variables dynamics. In particular, we cannot provide economic interpretation to our “preferences” parameters. Therefore, we simply display them in Table 8 for completeness. We note that half of the parameters are insignificant. The essential risk premia specification is warranted, nonetheless, as both the vector Π_0 and the matrix Π_x have statistically significant parameters.

Table 1 : Parameter estimates. Dynamics of the state variables

The table reports parameter estimates for two versions of our model

$$\begin{aligned}
 g_t &= m_g + (1 - \mu_g)g_{t-1} + \mu_g E_t g_{t+1} - \phi(r_t - E_t \pi_{t+1}) + \sigma_g(s_t^e)\varepsilon_t^g \\
 \pi_t &= m_\pi + (1 - \mu_\pi)\pi_{t-1} + \mu_\pi E_t \pi_{t+1} + \delta g_t + \sigma_\pi(s_t^e)\varepsilon_t^\pi \\
 r_t &= m_r(s_t^m) + (1 - \rho(s_t^m))[\alpha(s_t^m)E_t \pi_{t+1} + \beta(s_t^m)g_t] + \rho(s_t^m)r_{t-1} + \sigma_r(s_t^d)\varepsilon_t^r.
 \end{aligned}$$

The *TSM* version is a no-arbitrage term structure model with independent regimes of volatilities of exogenous shocks, the systematic monetary policy, and volatility of the monetary policy shock. This model is estimated with output, inflation and a cross-section of yields. The *SRM* version is identical to *TSM* but is estimated without long-term yields using the short interest rate (three month yield) only. The bootstrapped 95% confidence intervals are presented in parentheses.

Private Sector								
	$m_g \times 10^3$	m_π	μ_g	μ_π	$\phi \times 10^4$	δ		
<i>TSM</i>	1.77	0.00	0.53	0.48	8.49	0.01		
	(-1.10, 4.00)	(-0.01,0.01)	(0.52,0.57)	(0.42,0.51)	(0.97,19.00)	(0.00,0.03)		
<i>SRM</i>	0.00	0.00	0.54	0.50	0.00	0.01		
	(-0.01,0.01)	(-0.01,0.01)	(0.52,0.57)	(0.50,0.51)	(0.00, 0.00)	(0.00,0.01)		
Monetary Policy								
	$m_r(1) \times 10^2$	$m_r(2) \times 10^2$	$\rho(1)$	$\rho(2)$	$\alpha(1)$	$\alpha(2)$	$\beta(1)$	$\beta(2)$
<i>TSM</i>	-0.17	0.81	0.97	0.79	3.22	0.38	2.12	1.33
	(-0.33,0.07)	(0.13,4.20)	(0.95,0.99)	(0.07,0.86)	(1.14,6.60)	(0.01,1.10)	(0.58,4.60)	(0.06,1.90)
<i>SRM</i>	-0.16	1.65	0.97	0.53	12.01	0.30	5.20	0.80
	(-1.20,0.77)	(1.10,1.90)	(0.90,0.99)	(0.47,0.57)	(1.30,45.00)	(0.14,0.42)	(2.20,32.00)	(0.41,1.10)
Volatilities								
	$\sigma_g(1)$	$\sigma_g(2)$	$\sigma_\pi(1)$	$\sigma_\pi(2)$	$\sigma_r(1)$	$\sigma_r(2)$		
<i>TSM</i>	1.10	0.63	0.56	0.23	2.73	1.31		
	(0.88,1.30)	(0.54,0.74)	(0.46,0.67)	(0.19,0.26)	(2.30,3.20)	(1.10,1.50)		
<i>SRM</i>	1.20	0.50	0.52	0.23	2.09	0.40		
	(1.00,1.30)	(0.32,0.60)	(0.31,0.62)	(0.09,0.29)	(1.40,2.70)	(0.25,0.47)		

Table 2 : Parameter estimates. Transition probabilities.

The table reports the estimates of the transition probabilities for two models. *TSM* is a no-arbitrage term structure model with independent regimes of volatilities of exogenous shocks, the systematic monetary policy, and volatility of the monetary policy shock. This model is estimated with output, inflation and a cross-section of yields. *SRM* is identical to *TSM* but estimated without long-term yields using the short interest rate (three month yield) only. The bootstrapped 95% confidence intervals are presented in parentheses.

	<i>TSM</i>		<i>SRM</i>	
Systematic monetary policy regime variable s_t^m				
	active	passive	active	passive
active	98.59 (97.60,99.97)	1.41 (0.033,2.44)	67.39 (54.20,79.80)	32.61 (20.10,45.60)
passive	8.55 (3.38,17.60)	91.45 (82.30,96.60)	73.53 (59.70,92.50)	26.47 (6.62,40.20)
Volatility of exogenous shocks regime variable s_t^e				
	high vol	low vol	high vol	low vol
high vol	89.46 (77.00,94.80)	10.54 (5.19,23.00)	99.02 (77.00,99.98)	0.98 (0.02,20.70)
low vol	5.53 (2.09,9.40)	94.47 (90.50,97.90)	0.74 (0.01,14.30)	99.26 (84.80,99.99)
Volatility of monetary policy shock regime variable s_t^d				
	discretion	commitment	discretion	commitment
discretion	82.03 (73.10,88.60)	17.97 (11.30,26.70)	90.02 (66.90,96.90)	9.98 (3.04,32.30)
commitment	18.33 (11.30,24.70)	81.67 (75.10,88.60)	7.14 (2.28,21.30)	92.86 (78.60,97.70)

Table 3 : How Important Is the Information Contained in Yields? Simulation Study.

This table presents the results of a simulation exercise designed to evaluate the informativeness of the yield curve about monetary policy regimes. We simulated 1000 paths from the estimated term structure model TSM . For every path model TSM and short rate model SRM (without long-term yields) were estimated. We then computed the ratio $\frac{SRM \text{ bias}}{TSM \text{ bias}}$ of the biases of the smoothed regimes from the exact ones for the two models. The table below presents 95% confidence bounds and the mean of this ratio.

Regime Variable	Confidence Bounds	Mean
Monetary Policy s_t^m	(2.94, 82.1)	19.3
Volatilities of Exogenous Shocks s_t^e	(1.07, 13.8)	4.02
Volatility of Monetary Policy Shock s_t^d	(1.41, 3.18)	2.12

Table 4 : Sample statistics in the data and counterfactual economies

This table provides summary statistics of output g , inflation π , short yield $y(1)$, and long yield $y(40)$ for counterfactual economies. The last panel considers the full sample, assumes that exogenous shocks are exactly the same as they were realized in the sample and assumes that only one of the monetary policy regimes AD (active policy - discretionary shocks), AC (active policy - commitment shocks), PD (passive policy - discretionary shocks), or PC (passive policy - commitment shocks) prevailed throughout the full sample. The first panel considers the pre-1982 sample, and, in addition, assumes that the high volatility regime has prevailed for the output and inflation shocks. The second panel considers the post-1982 sample and assumes that the low volatility regime has prevailed for the output and inflation shocks.

				Average				Standard deviation			
				g	π	$y(1)$	$y(40)$	g	π	$y(1)$	$y(40)$
Subsample 1 (1970-1982)				-0.08	13.46	15.48	16.93	4.34	3.56	6.52	4.75
Model	s_t^m	s_t^d	s_t^e								
	A	D	H	-1.01	12.99	16.91	20.50	5.30	3.98	12.17	9.40
	A	C	H	-1.04	11.83	16.56	18.18	4.97	3.39	6.84	5.30
	P	D	H	-0.24	14.52	13.67	14.39	4.64	3.82	7.41	3.86
	P	C	H	-0.23	14.33	13.37	13.11	4.61	3.81	5.10	2.81
Subsample 2 (1983-2004)				0.05	5.03	10.48	14.21	2.22	1.66	4.76	4.33
Model	s_t^m	s_t^d	s_t^e								
	A	D	L	0.26	5.31	9.22	15.64	2.16	2.31	8.17	6.17
	A	C	L	0.20	5.51	9.78	14.54	2.19	2.04	5.50	4.33
	P	D	L	0.07	6.36	10.24	14.14	2.20	2.07	4.47	2.50
	P	C	L	0.06	6.39	10.48	13.21	2.22	2.06	3.05	2.07
Full sample (1970-2004)				0.00	8.16	12.33	15.22	3.16	4.80	5.97	4.66
Model	s_t^m	s_t^d									
	A	D		0.08	6.45	11.37	16.06	3.49	6.45	11.97	9.24
	A	C		0.05	6.85	11.60	14.84	3.31	4.84	7.68	5.94
	P	D		0.12	10.46	11.82	14.63	3.11	4.65	6.20	3.69
	P	C		0.12	10.55	11.86	13.61	3.12	4.54	4.42	2.99

Table 5 : Accuracy of the Approximate Pricing Method.

This table presents the results of a simulation exercise designed to evaluate the accuracy of the bond pricing method proposed in this paper. We generated 5,000 Sobol points of parameters and state variables inside a range of three standard deviations around estimated parameter values and long-run means of the state variables. For each generated point we computed an exact yield based on simulations as well as the yields obtained with the quadratic approximation method used in this paper and log-linear method proposed in Bansal and Zhou (2002). The table reports the mean, median and maximal absolute deviations of the approximate yields from the exact one.

	mean, bp	median, bp	maximum, bp
8 quarters maturity			
Log-Linear Approximation	0.04	0.01	0.53
Quadratic Approximation	0.13e-4	3.78e-4	0.02
20 quarters maturity			
Log-Linear Approximation	0.19	0.04	2.91
Quadratic Approximation	0.01	0.17e-4	0.19
40 quarters maturity			
Log-Linear Approximation	0.49	0.05	16.20
Quadratic Approximation	0.09	0.32e-4	2.77

Table 6 : Pricing Errors.

The table reports the pricing errors measured as mean absolute errors (MAE) for yields of maturities two, five and ten years for model *TSM*. We also report the correlations of slope and curvature implied by the model with their data counterparts.

Yield maturity, qtrs	MAE, bp
1	0.00*
8	24.03
20	29.14
40	23.21
	Correlations, %
slope	97.46
curvature	64.07

* one quarter yield is assumed to be observed exactly.

Table 7 : Unconditional Moments Tests.

This table reports various unconditional moments of the observables computed from the dataset (quarterly observations from 1970 to 2004) and implied by estimated model *TSM*. Bootstrapped 95% confidence bounds are shown in parentheses.

Means, %							
	output	inflation	y(1)	y(8)	y(40)	slope	curvature
data	0.00	4.08	6.17	6.89	7.61	1.45	0.00
model	0.01	4.14	6.20	7.31	8.33	2.13	-0.09
	(-0.5,0.5)	(0.5,7.8)	(2.8,9.6)	(3.8,11.0)	(5.2,11.0)	(0.6,3.7)	(-0.7,0.5)
Standard Deviation, %							
	output	inflation	y(1)	y(8)	y(40)	slope	curvature
data	1.58	2.40	2.98	2.83	2.33	1.43	0.86
model	1.37	2.66	4.28	4.27	3.79	1.79	1.04
	(1.0,1.8)	(1.2,5.1)	(2.3,7.2)	(2.3,7.0)	(2.1,6.1)	(1.1,2.9)	(0.8,1.4)
Skewness							
	output	inflation	y(1)	y(8)	y(40)	slope	curvature
data	-0.32	0.91	0.81	0.59	0.86	-0.67	0.10
model	-0.01	0.02	0.01	-0.00	0.08	-0.07	-0.09
	(-0.7,0.7)	(-1.0,1.0)	(-0.9,1.0)	(-0.9,0.9)	(-0.8,0.9)	(-1.2,0.9)	(-0.6,0.4)
Kurtosis							
	output	inflation	y(1)	y(8)	y(40)	slope	curvature
data	3.66	2.76	4.06	3.60	3.56	3.51	4.15
model	2.82	2.32	2.59	2.52	2.53	3.08	2.84
	(2.1,4.0)	(1.5,4.0)	(1.7,4.3)	(1.7,4.1)	(1.7,4.0)	(2.0,5.4)	(2.3,3.7)
One Quarter Autocorrelation							
	output	inflation	y(1)	y(8)	y(40)	slope	curvature
data	0.87	0.99	0.92	0.94	0.96	0.77	0.68
model	0.82	0.98	0.96	0.95	0.94	0.81	0.46
	(0.7,0.9)	(0.9,1.0)	(0.9,1.00)	(0.9,1.00)	(0.9,1.0)	(0.6,0.9)	(0.2,0.7)
Four Quarter Autocorrelation							
	output	inflation	y(1)	y(8)	y(40)	slope	curvature
data	0.28	0.86	0.78	0.81	0.83	0.44	0.41
model	0.43	0.91	0.81	0.81	0.79	0.54	0.33
	(0.2,0.7)	(0.8,1.0)	(0.6,0.9)	(0.6,0.9)	(0.6,0.9)	(0.2,0.8)	(0.1,0.6)

Table 8 : Parameter estimates. Risk premia

The table reports the risk premia estimates for *TSM*. *SRM* uses short rate only for estimation and, therefore, risk premia cannot be estimated for this version of our model. The bootstrapped 95% confidence intervals are presented in parentheses.

	Π_0	g	Π_x	r
			π	
g	4.22 (1.60,9.80)	1.29 (0.10,2.50)	-0.17 (-1.90,0.42)	-0.53 (-1.10,-0.18)
π	-4.42 (-17.00,1.70)	-2.09 (-4.80,0.32)	1.43 (0.10,4.90)	0.80 (0.03,1.90)
r	-0.36 (-0.60,-0.17)	0.03 (-0.02,0.08)	0.00 (-0.03,0.04)	-0.01 (-0.03,0.01)

Figure 1. Smoothed Probabilities of Regimes for Model *TSM*.

This figure shows the smoothed probabilities of regime variables for term structure model *TSM*, which is model (2.1)-(2.3) estimated with output, inflation and the cross-section of yields. The model has three mutually independent regime variables that switch the volatilities of exogenous shocks, systematic monetary policy, and the volatility of the monetary policy shock. The graph shows the smoothed probability of an active monetary policy regime in which the Fed aggressively stabilizes the price level, the probability of a “high” volatility regime (with high conditional variances of exogenous shocks), and the probability of a “discretionary” monetary policy regime corresponding to a high volatility of the policy shock. Shaded regions indicate NBER recessions. Dashed vertical lines show the Romer dates.

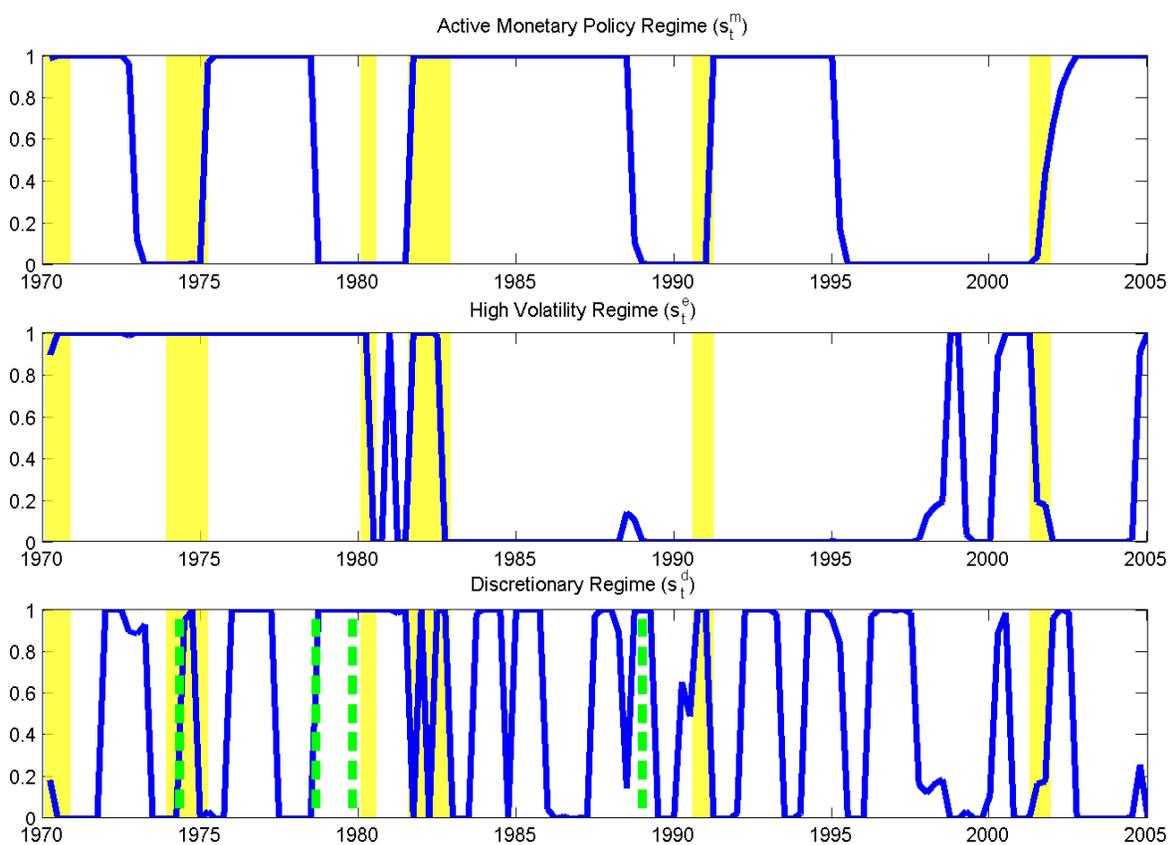


Figure 2. Smoothed Probabilities of Regimes for Time Series Model *SRM*.

This figure shows the smoothed probabilities of regime variables for the short rate model *SRM*, which is model (2.1)-(2.3) estimated with output, inflation and the short interest rate. The model has three mutually independent regime variables that switch the volatilities of exogenous shocks, systematic monetary policy, and the volatility of the monetary policy shock. The graph shows the smoothed probability of an active monetary policy regime in which the Fed aggressively stabilizes the price level, the probability of a “high” volatility regime (with high conditional variances of exogenous shocks), and the probability of a “discretionary” monetary policy regime corresponding to a high volatility of the policy shock. Shaded regions indicate NBER recessions.

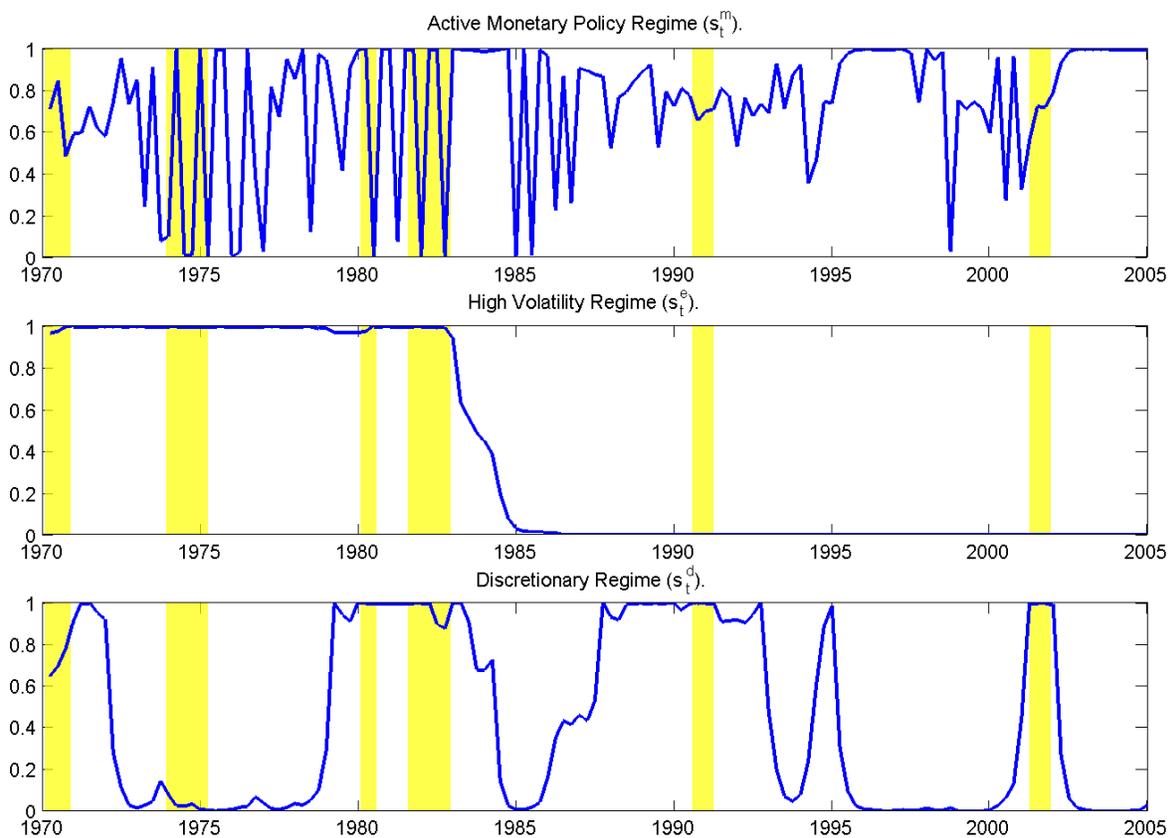


Figure 3. Impulse Response Functions of The State Variables.

This figure presents the impulse responses of the state variables to one standard deviation shocks in state variables themselves. The responses are conditional on regimes as if the economy would stay in a given combination of regimes forever. The rightmost column shows the responses from the monetary policy shock. These responses are independent of the volatilities of exogenous shocks. The two left columns show the responses from the exogenous shocks. These responses are independent of the volatilities of the policy shock (discretionary vs commitment). The notation in the legend corresponds to the combination of two out of the three regimes: A and P correspond to active and passive regimes, correspondingly; H and L correspond to high and low volatility regimes, correspondingly; D and C correspond to discretion and commitment regimes, correspondingly.

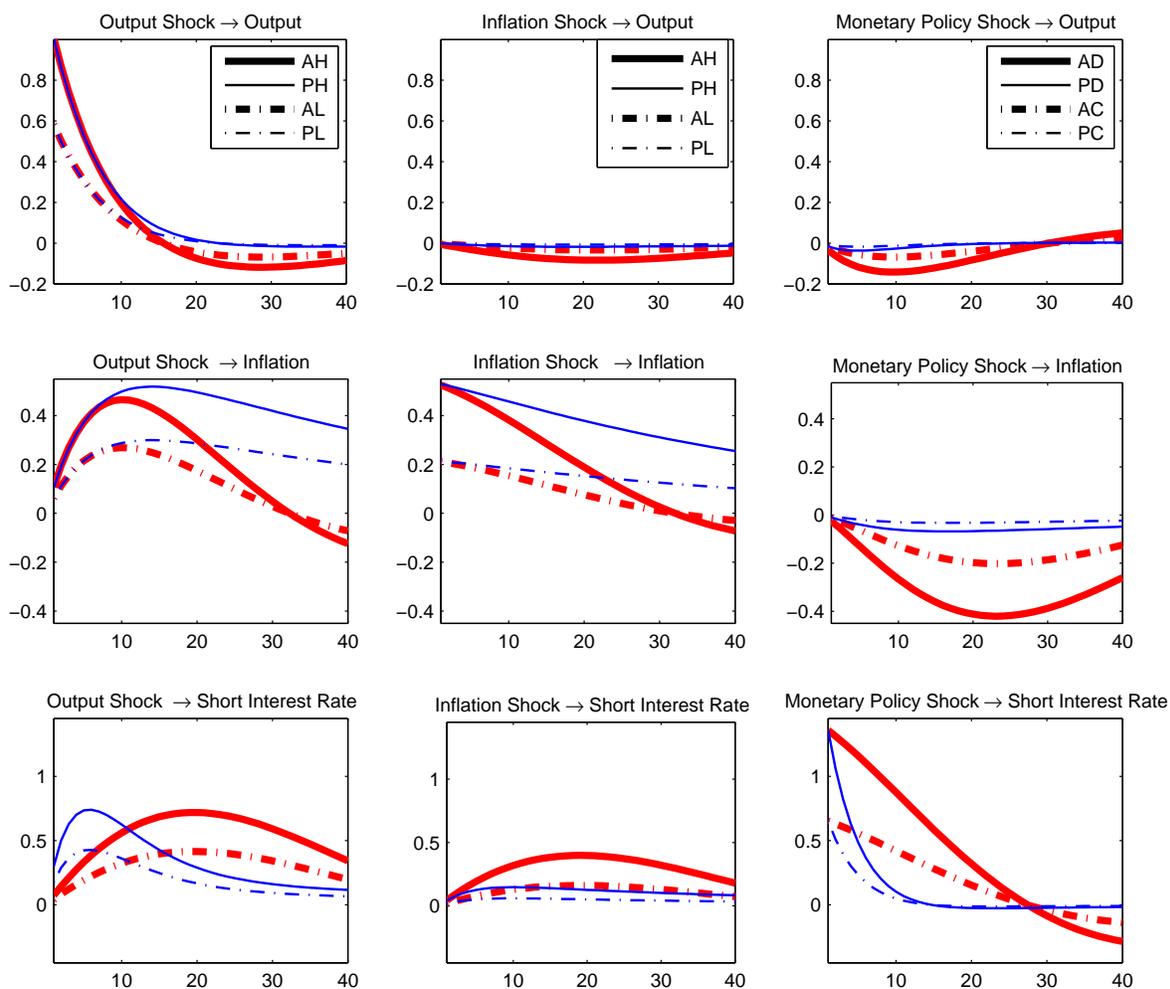


Figure 4. Impulse Response Functions of the Slope.

This figure shows the impulse responses of the slope of the term structure to one standard deviation shocks in state variables. The responses are conditional on regimes as if the economy would stay in a given combination of regimes forever. The first and second row of panels correspond to the active and passive monetary policy regimes, respectively. The notation in the legend corresponds to the combination of the other two regimes: H and L correspond to high and low volatility regimes, correspondingly, and D and C correspond to discretion and commitment regimes, correspondingly.

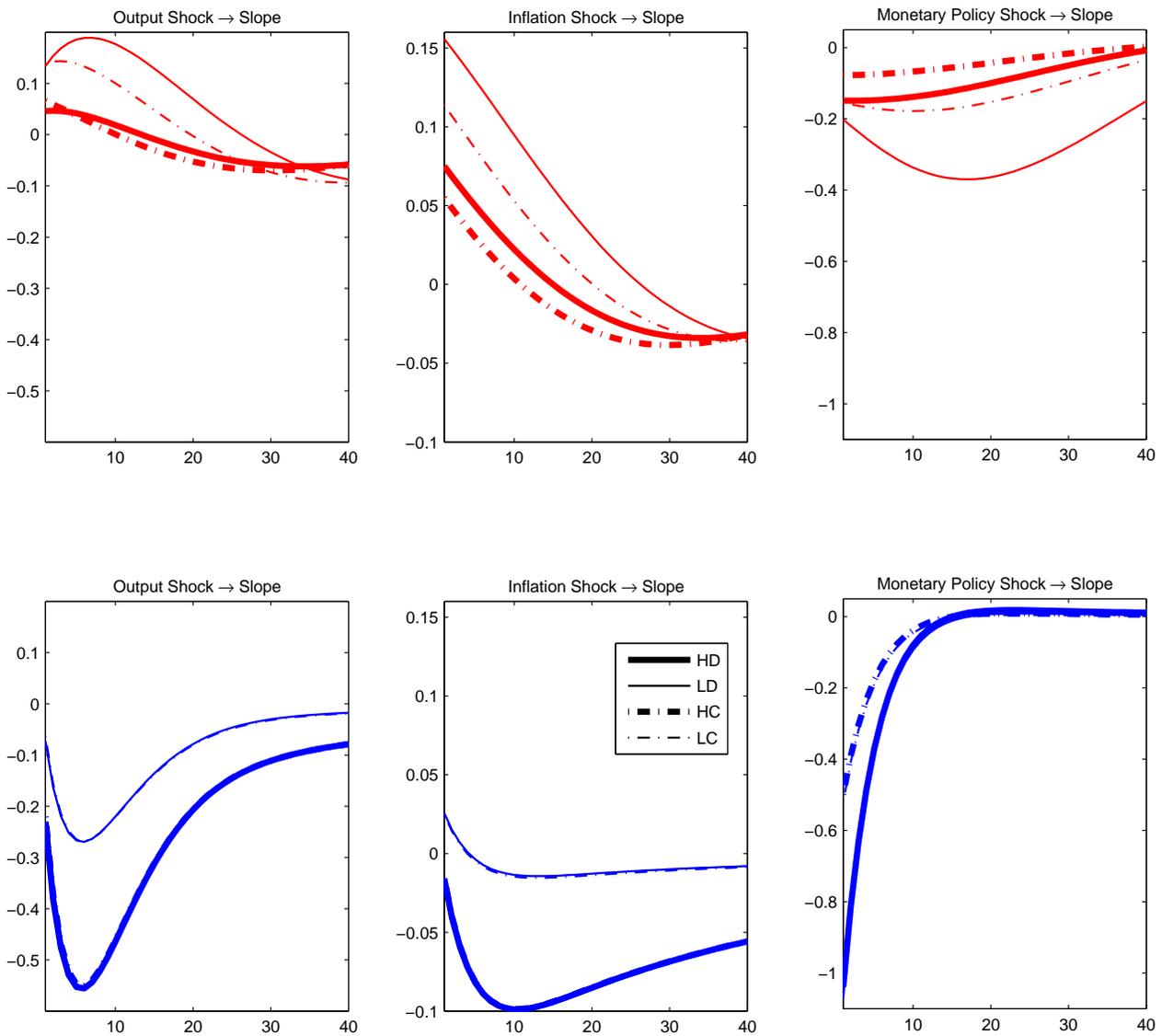


Figure 5. Counterfactual Inflation and Output

This figure displays the time series of inflation and output under the assumption that one of the monetary policy regimes AD (active policy - discretionary shocks), AC (active policy - commitment shocks), PD (passive policy - discretionary shocks), or PC (passive policy - commitment shocks) prevailed throughout the full sample. Thin lines represent realized output and inflation and thick lines represent counterfactuals.

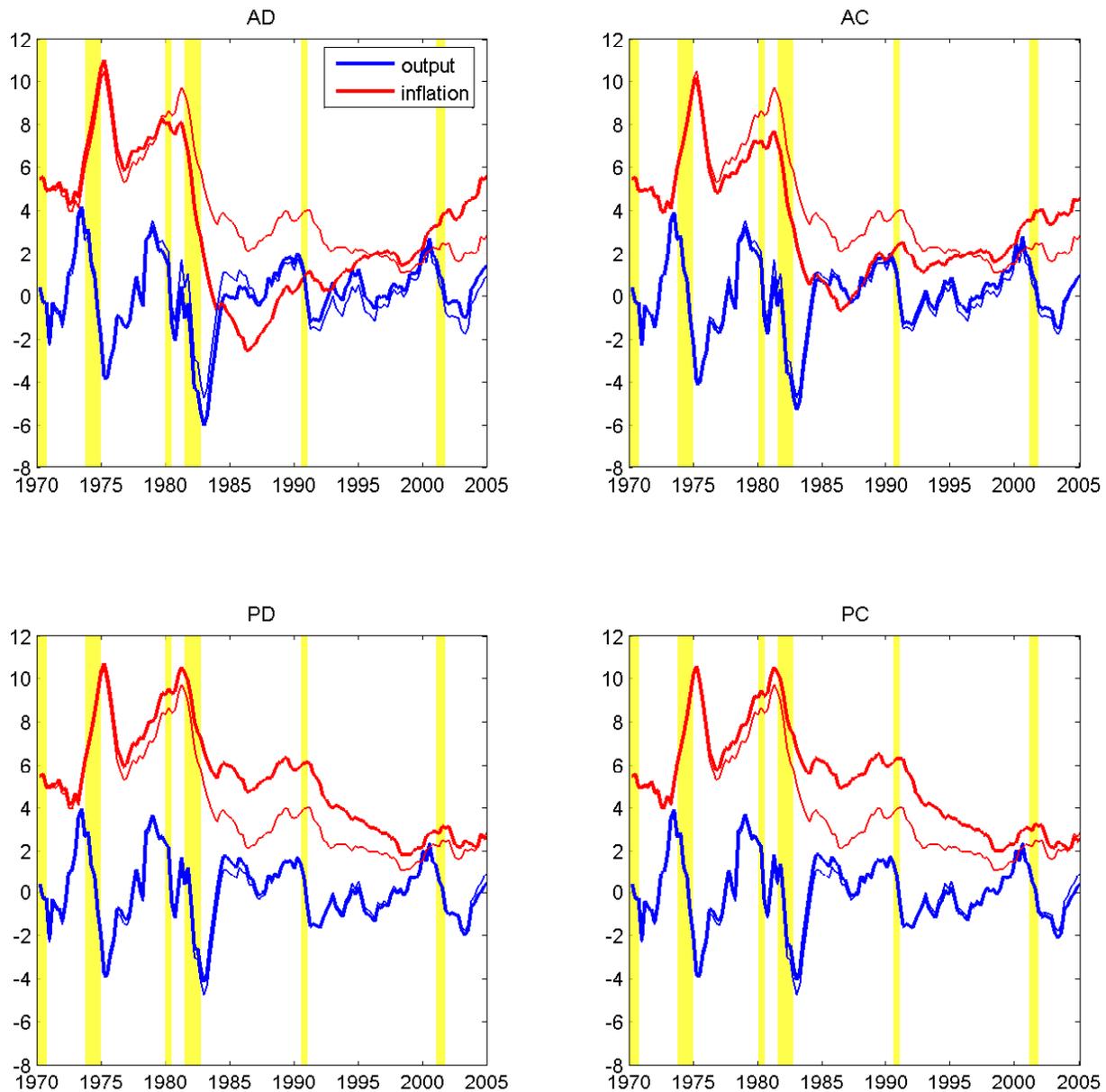


Figure 6. Counterfactual yields

This figure displays the time series of short ($y(1)$) and long ($y(40)$) yields under the assumption that one of the monetary policy regimes AD (active policy - discretionary shocks), AC (active policy - commitment shocks), PD (passive policy - discretionary shocks), or PC (passive policy - commitment shocks) prevailed throughout the full sample. Thin lines represent realized yields and thick lines represent counterfactuals.

