DISCUSSION PAPER SERIES

No. 7088

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INDUSTRIAL ORGANIZATION

Centre for Economic Policy Research

www.cepr.org

Available online at: www.cepr.org/pubs/dps/DP7088.asp

ISSN 0265-8003

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Discussion Paper No. 7088
December 2008

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CEPR Discussion Paper No. 7088

December 2008

ABSTRACT

Toc 'n' Roll: Bargaining, Service Quality and Specificity in the UK Railway Network*

The paper studies the regulatory design in a industry where the regulated downstream provider of services to final consumers purchases the necessary inputs from an upstream supplier. The model is closely inspired by the UK regulatory mechanism for the railway network. Its philosophy is one of vertical separation between ownership and operation of the rolling stock: the Train Operating Company (TOC) leases from a ROlling Stock COmpany (ROSCO) the trains it uses in its franchise. This, we show, increases the flexibility and competitiveness of the network. On the other hand, it also reduces the specificity of the rolling stock, thus increasing the cost of running the service, and the TOC's incentive to exert quality enhancing effort, thus reducing the utility of the final users. Our simple model shows that the UK regime of separation may indeed be preferable from the point of view of welfare.

JEL Classification: D2, L1, L5 and L92
Keywords: incomplete contracts, network regulation, railways and relation specific investment

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* We wish to thank the audience at the Italian Society in Law and Economics and in Sheffield and Newcastle.

Submitted 05 December 2008

1 Introduction

We study the interaction between two regulated firms: a downstream provider of services to final consumers and the upstream supplier of the inputs necessary for the provision of the services. While the analytical set-up is closely modelled on the scheme chosen in the UK for the regulation of the railway network[1] and therefore it can yield immediate practical interpretations for that case, it also has broader lessons for the regulatory design of the contractual relationship between upstream and downstream firms in regulated industries.

The defining regulatory feature of the UK regulatory system in place for the railway network is the separation between the Train Operating Company (TOC) and the ROlling Stock COmpany (ROSCO). In a nutshell, the national network is divided into several geographical franchises,[2] essentially separate from an operational viewpoint. In each of these areas, one TOC provides the rail services to passengers, and leases its rolling stock from one of the three ROSCOs which are allowed to operate. Each franchise is assigned to a TOC by the Department for Transport, following a competitive process which we do not consider here,[3] except to note that, to participate in the bidding, a TOC must enter a broad agreement with one of the ROSCOs, for the supply of the necessary rolling stock, if the TOC is awarded the franchise. This implies that, once a bidder is awarded the franchise for an area, it is essentially constrained to lease the rolling stock it needs from the specific ROSCO it had chosen at the earlier stage, even though the original broad agreement between them can be "finalised" (*viz*, re-negotiated). We pick up the thread from here, in a model which studies the (re-)negotiation between the two parties and the effects of the agreement they reach. In line with the UK regulatory mechanism, we rule out the possibility for the TOC to switch to a different ROSCO.

The rationale for the UK regulatory mechanism is enhancing competition. The well understood competitive effect of bidding for the award of the franchise is strengthened by the fact that participation in the bidding does not require

[1] An exhaustive and up-to-date description of the regulatory and institutional set-up of the UK railway industry is in Office of Rail Regulation (2007); see Newbery (1997) for an overview of the utilities' reform in the UK.

[2] Currently, there are 19 franchised operators, the Department for Transport is the awarding body for all of them except three. Additionally, there are 6 non-franchised operators. A full list of the train operators in the UK can be found at: www.atoc-comms.org/franchised-passenger-services.php.

[3] See ORR (2007) for details on the franchise process. For a general overview on ORR's role after the implementation of the Railways Act 2005, see Department for Transport (2007).

a TOC to incur the hefty sunk entry cost constituted by the acquisition of suitable rolling stock: this increases the number of potential bidders. Secondly, precisely because the incumbent does not own the rolling stock, it is feasible to award the franchise for a period shorter than the rolling stock's economic life,[4] ensuring that the threat to withdraw the franchise is credible and keeps the incumbent on its toes. Finally, preventing the TOCs from owning rolling stock makes it easier to transfer rolling material from one area to another, which may be required following changes in the geographical distribution of demand.

The ease with which rolling stock can be switched to a different franchise is a key determinant of the efficiency of the industry, and a crucial variable in our model. It is affected by a variety of technical factors, from the type of the train's power supply, to its clearance (which affects the compatibility between the rolling material and the rails), to operational considerations such as the maximum speed, the position of doors, the size of the trains and their configuration (a train designed with a flat rural region in mind, with few large stations, may be totally unsuited to a mountainous area with many smaller stations), to up-front costs of switching due to staff training.[5] These factors, in turn, are not technologically fixed, but are instead typically varied by the train designer, for economic or strategic reasons. We capture this through a variable, s, which is a summary measure of the level of *specificity* of the rolling stock, and we place at the centre of our model the trade-off between increasing specificity, that is designing the rolling stock in a way very suited to the geographical area to which it is destined, and increasing flexibility, opting for a design which makes it easier to operate the rolling stock in a different area.

In this paper we study the effect of the regulatory system on the choice of the level of specificity of the rolling stock, and its effects on the quality of the train service. A suitable conceptual framework to understand the interaction between TOC and ROSCO is provided by the theoretical literature on incomplete contracts.[6] The basic idea is that in many long term relationships, a party who can make a relation specific investments which reduces the costs

[4] "Engines and carriages have a working life far longer than the length of a passenger franchise contract, and are therefore not owned by the companies themselves but by private sector leasing companies.", UK Department for Transport (2004).

[5] See SRA (2004) for further details about the low degree of standardisation in the rail network in the UK.

[6] Hart and Holmstrom (1987) provided a review, and Tirole (1999) a more recent critical evaluation.

and/or increases the benefits of the other party, may refrain from doing so if it is unable to reap a share of the benefit of its investment. This happens because contracts are incomplete, in the sense that it is impossible (or prohibitively costly) to specify the obligation of each party in every conceivable eventuality in sufficient detail to allow a third party, called to enforce the contract in the event of a dispute, to determine whether a breach has occurred or not.[7]

From this perspective, the choice of the train specificity s can be viewed as a relation specific investment by the ROSCO. The cost of this investment is not due to the production process, as there is no reason to suppose that building a "flexible" train is more or less costly than building a train highly specific to the railway lines to which it is destined, and indeed we posit that the choice of s does not affect the production cost of the rolling stock. The cost of specificity is instead given by the lower net revenues which can be obtained using the rolling stock in a different region. In other words, more specificity helps produce a high quality service on the "right line", but it decreases the quality (and hence the market value) of a train's services on the "wrong line".[8] This has the subtle implication that the cost of the "investment in specificity" depends on the details of the regulatory mechanism: unlike the standard incomplete contract literature, the barriers to writing a complete contract are not due to the fact that the actions taken by the parties are not verifiable in the event of a dispute, but are determined by the UK regulatory regime, which imposes the separation between TOC and ROSCO. In this sense, the degree of separation between the TOC and the ROSCO should itself be considered a policy instrument, and the paper provides a conceptual framework to analyse its role and its effects. We therefore compare the "separation" regime chosen for the UK rail network, akin to contract incompleteness between the TOC and the ROSCO, with the "integration" regime of complete contracts, typical of most other EU countries, since they are both technologically and informationally feasible.[9] Formally, we

[7] The typical example is a clause specifying that quality must be "good" or "adequate": in the event of a dispute, even though both parties may come to the same (private) judgement as to whether quality is "adequate", an enforcer, such as a court or an arbitrator, cannot.

[8] See Ménard and Yvrande-Billon (2005) for a similar point of view: "The non-redeployability is critical here. Discrepancies between contract duration and the physical lifetime of equipment exist in many leasing industries (e.g. car and truck rental), but it is no problem as long as equipment has alternative users".

[9] The regulatory regime is however clearly under strain, to the point that the Office of Rail Regulation has recently referred to the competition authority about the prevention of competition in the leasing market of rolling stock for franchised passenger services. In August

compare two cases. In the first, the TOC and the ROSCO agree on transfer prices *and* the design of the train, that is complete contracts can be written. In the second they only agree on transfer prices, and the degree of specificity of the rolling stock is chosen by the ROSCO on its own: here the parties are not permitted to write complete long term contracts which specify in sufficient detail the characteristics of the rolling material to be supplied by the ROSCO to the TOC (or equivalently to merge).

We find that both the degree of specificity and the investment in quality increase with integration, in line with most of the literature.[10] The fact that specificity and quality increase with integration does not necessarily imply that socially a fully integrated structure should be preferable. Indeed, while our model is simple, it is not special, and it shows that there can be *over*investment in specific assets and *excessive* quality of service. Our paper therefore highlights the hitherto unnoticed trade-off between specificity and competition. This trade-off implies that a case by case analysis is in principle necessary to evaluate the best regulatory design. Indeed, in our set-up, the technological benefit of specificity is traded off the lack of flexibility and the anticompetitive effect of highly specialised rolling stock. From a social welfare point of view, there may be over-investment in specific assets, and so separation may be preferable to integration, even though, as some literature points out, the standard analytical approach would suggest that the highly specific investments typical of the relationship between the TOC and the ROSCO make integration preferable to separation.[11]

The paper is organised as follows: Section 2 introduces the model: demand, technology and the possible vertical structures in 2.1, and the bargaining mechanism in 2.2. The temporal sequence of events and decisions is summarised in Section 2.3. The policy analysis begins in Section 3 with the determination of the first best choice of specificity and effort; these are compared in Section 5 with the equilibrium values in the two regulatory regimes derived in detail in Section 4. The proofs of the more algebraic results are in the Appendix, which is preceded by a brief conclusion.

2008 the Competition Commission published its provisional findings and confirmed that some of the features of the rolling stock leasing market do indeed raise competition issues. See ORR (2007) and Competition Commission (2008).

[10] See Kain (1998), Preston (2002), Crompton and Jupe (2003), Ménard and Yvrande-Billon (2005).

[11] See, among others, Ménard and Yvrande-Billon (2005) and Preston (2002).

2 The Model

2.1 Demand, technology, and regulatory regimes

We model the interaction between three agents: a regulator, the firm franchised to supply rail transport services in a given region, shortened to TOC in what follows, and the firm who has the expertise to design and supply rolling stock, trains and locomotives, ROSCO hereafter.

The quality of the rail transport can vary, and passengers who use them pay a total amount that is exogenously given by $r > 0$: revenues are independent of quality. This may happen, for example, because the marginal consumer does not benefit from quality, while the inframarginal consumers do. It must also be the case that the price is set by the regulator and that the regulator does not use mechanisms (such as the one studied by De Fraja and Iozzi 2007) linking the allowed prices to the realised quality. This simplification helps us concentrate on the relationship between quality and specificity, leaving aside the interaction between prices and quality. Consumers' welfare is of course affected by the quality of the service;[12] this is denoted by q, and depends on the investment of the TOC, $e \in [0, 1]$ (a normalisation)[13] and on the realisation of a random variable, θ, with cumulative distribution function $\Phi(\theta)$, density $\phi(\theta) = \Phi'(\theta)$, and support also normalised to $[0, 1]$. For concreteness, in the case of the railways, we can think of quality q as given by the frequency of delayed trains, of the TOC's investment, e, as its provision and arrangements with regard to stand-by personnel and equipment, and of the random element, θ, as the external exogenous factors that affect the timely running of the train service. A high investment in stand-by provision by the TOC reduces the negative impact, in terms of inconvenience and delays, of a negative quality shock, such as a derailment.[14] For simplicity we take an additive specification:

$$q = \theta + e, \tag{1}$$

[12] Consumer surplus and total surplus are formalized in Section 3.

[13] Constraining e not to exceed 1 should be seen as capturing the idea that the cost of effort increases very rapidly when e approaches the technological maximum.

[14] θ are adverse weather shocks and freak accidents (such as the disaster caused by a SUV becoming stuck on the railway and derailing the Newcastle-London high speed train near Selby on 28/2/2001). e are the measures taken by companies to minimise the probability of accidents and reducing the disruption caused by the weather (rather than attributing delays and cancellations to leaves on the line or to the wrong "type of snow", as famously commented by a British Rail executive on 11/2/1991).

and assume $\Phi(\theta)$ to be uniform:

$$\Phi(\theta) = \theta. \tag{2}$$

Once the market structure is established, the regulator's tool-kit is the imposition of sanctions in the event of deterioration of the quality of the service. In practice, these sanctions can range from fines, up to the extreme measure of the withdrawal of the licence: this draconian punishment was imposed on Connex South Eastern, a train operator serving the South-East of England, in the second half of 2003 (National Audit Office, 2005). We capture these sanctions with the simplifying assumption that the regulator chooses a minimum quality requirement $q_m \in [0, 2]$ and the TOC's franchise is renewed if and only if its service quality is at least the minimum requirement, $q \geqslant q_m$:[15] the regulator combines price regulation with competition for the market to limit the monopoly power and promote quality.[16] Given (1) and (2), the franchise is renewed with probability

$$z = 1 - q_m + e. \tag{3}$$

By investing in quality enhancing activities e, the TOC can increase the quality of the service and hence reduce the probability that the franchise is not renewed by the regulator.

The ROSCO's investment is the degree of train specificity, denoted by s. Without loss of generality, we also normalise it to lie in $[0, 1]$.

Specificity has benefits and costs. On the one hand, it reduces the cost of providing quality, see (4) below. On the other hand, it makes it more costly to transfer rolling material to a different area: the more technically and operationally suitable a train is to network A, the less suitable it is to network B, and so we posit that the net revenue that can be obtained from using it on network B is a decreasing function of the degree of specificity to the original TOC. Formally, we denote by $P(s)$, with $P'(s) < 0$, the unit net revenues that can be obtained using a train of specificity $s \in [0, 1]$ destined to a network different from the one it was designed for. We also assume that $P(s)$ is *concave* in s: this would follow, for example, from the natural assumption

[15] An alternative, analytically identical, assumption is to posit some uncertainty on the TOC's part with regard to the regulator's preference: the regulator is satisfied and renews the franchise with a probability which is increasing in the quality level offered by the TOC.

[16] The positive effects of competition for the market on the incentives to invest have been empirically assessed by Affuso and Newbery (2002).

of convex adjustment costs. We specify $P\left(s\right)$ as:

$$P(s) = P_0 - \frac{1}{2}ks^2,$$

where $k > 0$, and $P_0 \geqslant \frac{1}{2}k$, so that the revenue from an alternative network are non-negative for every value of s. Note that there is in general no reason to presume that specificity has also a technological cost: a train with, say fixed height entry steps (specific to the design of the stations where the TOC operates) need not be more expensive to design and build than a train with variable height entry steps, which can be transferred to rail networks with a different station design. If there were an additional technological cost to specificity, it could be included as part of the function $P\left(s\right)$.

The TOC and the ROSCO are private profit maximising companies. As explained above, the TOC's revenues are exogenously given by $r > 0$. The TOC incurs two types of costs: the payments to the ROSCO for the use of its rolling material, endogenously determined and discussed in details in the next section, and the operating costs, personnel, power, and so on. The latter depend *positively* on the TOC's investment in effort for quality and *negatively* on the specificity of the rolling material supplied by the ROSCO. We assume a linear specification:

$$C(e, s) = c_0 + c_e e - c_s s, \tag{4}$$

where $c_0, c_e,\ c_s > 0$, and $c_0 > c_s$: the last ensures that operating costs are positive for every possible combination of e and s. We deliberately set the cross derivative, $\frac{\partial^2 C(\cdot)}{\partial e \partial s}$ at 0, to isolate the interaction between e and s which is caused by the institutional set-up from any technological complementarity.

While exogenously given and independent of price and quality, demand is not fixed, but may be affected by an adverse shock: for example the closing down of a local employer or the opening of a new motorway affect demand in our area, but not in the others. Without shock, total revenues are $r > 0$. The negative shock reduces demand by a proportion $u \in (0, 1)$, and happens with probability $(1 - x)$. This makes $\alpha = u\left(1 - x\right)$ the relevant measure of the TOC's *expected* loss in the demand for the final service due to the possibility of a negative demand shock. We eliminate the uninteresting possibility that the train service is shut down by assuming that the line is profitable if the demand shock occurs: $ur - c_0 > c_e$.

We assume that the areas of the network are symmetric, which imposes the analytically convenient restriction that the unit profit obtained from employing a generic train (i.e., a train with $s = 0$) in an alternative region is the same as the unit profit from using a generic train in the area considered:

$$P_0 = r - c_0 - c_e e_A,$$

where e_A is the investment in effort chosen by the passengers service operator in the alternative area. Clearly, $ur - c_0 > c_e$ implies $r - c_0 - c_e > 0$, so that the industry profit from running generic trains is always positive across the network.

The salient feature of the regulatory regime of the UK rail industry is the separation between the TOC and ROSCO, the firm who runs and the firm who owns the trains, enshrined in the rules that prevents them from entering long term agreement for the supply of rolling stock. We capture the UK regulatory regime with the assumption that s is chosen separately by the ROSCO, which does so with the aim to maximise its own profit. We refer to the UK regulatory regime as the "vertical separation" regime. If s were instead negotiated between the TOC and the ROSCO, it would be chosen to maximise their joint profit,[17] and so their choice of s would be identical to that made by an integrated company, which is the situation more common in continental Europe. We refer to this alternative regime as the "vertical integration" regime.

2.2 Bargaining

The ROSCO can be involved in three bargaining situations. To ensure compatibility across regimes, the outcome of all bargaining situations is the generalised Nash bargaining solution, with (exogenously given) bargaining power coefficient β for the ROSCO: this is the maximisation of the weighted sum of the log of the parties' surplus, with weights β for the ROSCO and $(1 - \beta)$ for its counterpart (the well known details are spelled out in Appendix 1). The first bargaining situation for the ROSCO is its negotiation with the TOC over the lease contract for the provision of the rolling stock and, in the vertical integration regime, also over the specificity of the rolling stock. We assume that the lease price negotiated by the TOC and the ROSCO is a two-part tariff (p, F), where p is the unit price of the train services actually leased, and is

[17] Whether the two companies are integrated or legally separated entities is not relevant to the choice of the train design: if the companies are legally separated they will also negotiate a side payment, which depends on the relative bargaining power of the two parties, and affects the distribution, but not the size, of the total profit.

therefore a payment conditional on the renewal of the TOC's franchise and on the realised demand, and F is a fixed fee independent of demand or quality shocks. In our simple set-up, the choice of p and F is equivalent to the choice of e, the non-contractible quality-enhancing effort by the TOC, and therefore our assumption of bargaining over a two part tariff corresponds to bargaining over e. As a consequence, for a given degree of train specificity, the effort for service quality chosen by the TOC always maximizes the joint profit generated by the TOC and ROSCO relationship irrespective of whether they are vertically integrated or separated.[18] To determine the generalised bargaining solution it is necessary to know the "disagreement payoff"of the two parties, which in the model is their outside option. The TOC's is 0: in the event of disagreement between the TOC and the ROSCO, the TOC loses the franchise, which is reassigned to an alternative TOC, say ATOC (an acronym, incidentally, reminiscent of the actual Association of TOCs). On the other hand, the ROSCO's outside option is in general strictly positive because it can lease the rolling stock to the ATOC following a broken down negotiation; it is also endogenously determined.

The second bargaining situation in which the ROSCO can be involved is the negotiation with the ATOC over the rolling stock to be used in the franchise under consideration. This can happen for two reasons: either because the TOC and the ROSCO do not reach an agreement in their negotiation or because the TOC loses its franchise due to its service quality falling short the minimum standard set by the regulator. In the first case, the ROSCO and the ATOC bargain on a lease contract for the provision of the rolling stock and, in the vertical integration regime, also on the specificity the rolling stock. In the second case, the ROSCO and the ATOC bargain only on the lease contract even in the vertical integration regime, since the train specificity has been irreversibly set in the previous agreement between the ROSCO and the TOC. In both circumstances, the bargaining between the ROSCO and the ATOC takes place after the ATOC has already set its investment in effort for quality and after the quality uncertainty has been resolved, but before the resolution of the uncertainty in demand. In other words, in both circumstances the regulator

[18] This modelling strategy is justified since information asymmetries between the TOC and the ROSCO regarding e are probably not fundamental (one presumes that this was the underlying assumption on which the separation imposed by the UK legislator is based), and the aim of the paper is to concentrate on the comparison between the regulatory regimes, not on the role of information asymmetries.

reassigns the franchise to an "average" ATOC who has already passed the quality control in another area of the network. Bargaining on a linear price is not distortionary, since the effort for quality is given, and both parties have zero outside options, as a disagreement would lead to the cancellation of the service.[19]

The ROSCO's third possible bargaining situation is caused by the adverse demand shock occurring in the area served by the TOC. The TOC needs fewer trains, and the ROSCO offers the surplus rolling stock to the operator of train service in a different area. In this case, the ROSCO and this "external" operator bargain when all the relevant variables (train specificity and the "external" operator's investment in effort, and the random shocks) are fixed and known to the two parties. As before, bargaining over a unit price p is not distortionary, and both parties have a zero outside option in this case.

To ensure symmetry across the network, all exogenously fixed parameters are the same in all areas of the network.

2.3 Timing

The timing of choices in multi-stage games affects the outcome of the interaction among players. In our set-up the timing is determined by constraints imposed by the regulatory regime and by technology. The investment required to design and build rolling stock has clearly a longer time span than the investment in quality enhancing effort, and so s must be chosen before e. Demand and quality shocks have a relatively short term nature and therefore occur after s and e have been set. The parties operate in a fixed regulatory regime, that is, they know that the regulator's rules and guidelines regarding the link between the minimum quality standard and the likelihood of sanctions being imposed will not be changed as a consequence of the parties' actions.[20]

These considerations lead to the following formal description of the timing of the game.

[19] That is, there can be at most one "negotiation breakdown". The analysis would be unaffected if we assumed instead that there is a (finite) sequence of potential ATOC's with which the ROSCO could negotiate. We would simply need to work backwards from the last bargaining process in the sequence.

[20] It is of course possible that the regulatory standard is unexpectedly tightened after the parties actions, for example as a consequence of a media campaign: conceptually, this would correspond to a negative quality shock. We restrict the range of combinations of causal effects with the assumption that the regulatory standard is not affected by the choice of the parties.

1. **Regulatory set-up.** The regulator chooses whether the TOC and the ROSCO will negotiate over the triple (s, p, F) – the vertical *integration regime* – or over (p, F) only – the *vertical separation* regime.

2. **Minimum quality.** The regulator selects q_m, the minimum quality necessary for the franchise to be continued.

3. **Train specificity.** The train specificity, s, is decided by the ROSCO in the vertical separation regime. In the vertical integration regime, specificity is negotiated by the TOC and the ROSCO at the next stage.

4. **Lease contract.** In both vertical regimes, the TOC and the ROSCO bargain over (p, F) , the terms under which the ROSCO's rolling material can be used by the TOC. In the vertical integration regime, they also bargain over the rolling stock specificity, s.

5. **Effort.** The TOC decides its effort for quality, e.

6. **Quality uncertainty resolution.** The quality shock is realised.

   (a) If the realised service quality offered by the TOC is at least q_m, the TOC retains the franchise and operates the service in the area.

   (b) If the realised service quality offered by the TOC is below q_m, then the TOC loses the franchise, which is reassigned to the ATOC. At this stage, train specificity and ATOC's effort for quality have already been determined under both vertical regimes, so that the ROSCO and the ATOC bargain only on the leasing price, p. If they reach an agreement, the ATOC operates the service in the area, otherwise the service is cancelled.

   (c) Similarly, if the TOC and the ROSCO have failed to reach an agreement in their Stage 4 negotiation, the franchise is cancelled and reassigned to the ATOC. In this case, the ROSCO and the ATOC negotiate over the leasing price, p, and, in the vertical integration regime, also over the rolling stock specificity, s. Like in stage 6(b), if they reach an agreement, then the ATOC operates the service in the area, otherwise the service is cancelled.

7. **Demand uncertainty resolution and payoffs.** The demand shock is realised.

(a) If the service is operated in the area under consideration, the realized demand is served by the TOC (or by the ATOC), which leases the rolling stock it needs from the ROSCO, pays the ROSCO according to contract (the unit price p times the amount of train services employed in the area, plus, in the case the TOC is the operator, the fixed fee F), and collects passengers revenues.

(b) If, due to adverse demand conditions, there are surplus trains in the area, the ROSCO negotiates with the franchisee of a different region.[21] If they agree, the ROSCO collects a share β of the unit profit $P(s)$ times the quantity of train services transferred to the "external" franchisee. In the case of disagreement, the unused rolling stock remains idle, and the ROSCO makes no profit from it.

3 Social welfare and first best

The aim of the paper is the comparison of the performance of different regimes against the yardstick of industry social welfare, measured by the sum of the expected consumers' and producers' surplus. In this section, we characterise the benchmark given by the first best social optimum. We consider a representative area of the network, and invoke symmetry to extend our findings to the rest of the industry.

The producers' surplus can be calculated as the profit which can be obtained from a non-specific train (a train with $s = 0$), $r - c_0 + c_e e$, augmented by the expected cost saving due to the train specificity to the area under consideration, $(1-\alpha) c_s s$, and reduced by the expected extra cost due to the train specificity when the train is used in a different area, $\alpha \frac{1}{2} k s^2$. Consumers' surplus depends only on the final service quality, q, not on the train specificity s, and can therefore be expressed as a function of e only, $S(e)$. We choose again a convenient functional form and let consumers' surplus be given by:

$$\sigma q \left(1 - \frac{b}{4} q\right).$$

[21] We assume no randomness here: the number of areas affected by the negative demand shock is known in advance, as is which franchisees will be able to lease the additional trains from the areas where the adverse demand shock has occurred: the only uncertainty is which areas will receive the adverse demand shock. The exact formalisation of this simplification would require distinguishing the analysis of TOCs who will be able to lease some of their trains from the adversely affected areas, from those who will not: this would add heavy and unrewarding notation.

$b \in (0,1]$ and $\sigma > 0$ measure the concavity of the consumers' welfare function, and the intensity of consumers' preference for quality. σ could also be interpreted as the importance of consumers' surplus relative to profit. The restriction $b \in (0,1]$ ensures that consumers' surplus increases with q in its range, the interval $[0,2]$. Recalling that $q = e + \theta$ and θ is uniformly distributed on $[0,1]$, we have:

$$S(e) = \sigma \frac{(6-b) + 3(4-b)e - 3be^2}{12}.$$

Finally, the expected social welfare, W, is given by:

$$W(e,s) = [r - c_0 - c_e e] + (1-\alpha)c_s s - \frac{\alpha k}{2}s^2 + \sigma \frac{(6-b) + 3(4-b)e - 3be^2}{12}. \quad (5)$$

The first best benchmark is the choice of investment in quality, e, and of train specificity, s, which maximizes (5). At an interior solution these are:

$$s^* = \frac{(1-\alpha)c_s}{\alpha k}, \quad (6)$$

$$e^* = \frac{4\sigma - 4c_e - \sigma b}{2\sigma b}. \quad (7)$$

The first best degree of train specificity, s^*, increases with its effectiveness in reducing the operating costs on the "right line", c_s, and the expected quantity of train services to be employed on that line, $(1-\alpha)$, and decreases with the importance of the extra costs for using the rolling stock on "wrong lines", k, and the expected quantity of train services to be moved to those lines, α. On the other hand, the optimal level of the investment in quality, e^*, increases with the intensity of consumers' preference for quality (or, in the alternative interpretation, with the importance of consumers' surplus relative to profit), σ, while it decreases with the sensitivity of the operating costs to the effort for quality, c_e, and the concavity of the consumers' welfare function, b.

Graphically, the social welfare function (5) generates in the (s,e)-plane a map of elliptic iso-welfare curves centred at the first-best point (s^*, e^*), as illustrated below in Figure 1.

4 Industry equilibrium

We now characterise the subgame perfect equilibrium of the game constructed in Section 2 for a representative franchise, working backward from the last decision stage. By symmetry, since all franchises are alike, it constitutes the industry equilibrium.

4.1 Expected profits (stage 6)

Under both vertical regimes, if the game reaches stage 6(b) or stage 6(c), that is, if the TOC's franchise is not renewed because of an adverse quality shock or because negotiations fail, then the ROSCO negotiate with ATOC a lease price p for the use of the rolling stock available. Since both parties have zero outside option, and since the price does not affect any subsequent decision, the bargained price will distribute the given joint profit, $(r - c_0 - c_e e_A + c_s s)$, according to the bargaining power coefficients. Before the resolution of demand uncertainty, the expected quantity of train services to be employed on the line equals $(1 - \alpha)$. Therefore, the ROSCO's expected profit from leasing the rolling stock to the ATOC is:

$$(1 - \alpha)\,\beta\,(r - c_0 - c_e e_A + c_s s)\,.$$

If the TOC loses the franchise because its service quality falls short the minimum standard, that is if the game reaches stages 6(b), then the train specificity is fixed in both vertical regimes, and the revenues generated by the use of the rolling stock outside the area are unrelated to the outcome of the negotiation between the ROSCO and the ATOC. In the vertical separation regime, just the same is true when the TOC loses its franchise because of a lack of agreement with the ROSCO at stage 4 (that is, if the game reaches stage 6(c)), since the rolling stock specificity is irreversibly set by the ROSCO at stage 3. In these cases, the ROSCO's expected profit from reaching an agreement with the ATOC is:

$$\pi_R^{O_S} = (1 - \alpha)\,\beta\,(r - c_0 - c_e e_A + c_s s) + \alpha\beta P(s), \tag{8}$$

where s is the rolling stock specificity set by the ROSCO at stage 3 (vertical separation) or negotiated with the TOC at stage 4 (vertical integration), and $\alpha\beta P(s)$ is the ROSCO's expected profit from leasing the expected quantity α of unused rolling stock to an external franchisee. $\pi_R^{O_S}$ in (8) is of course the ROSCO's outside option in the negotiation with TOC under vertical separation.

If, on the other hand, the TOC and the ROSCO do not reach an agreement in the vertical integration regime, in stage 6(c) the ROSCO and the ATOC bargain over both prices and the train specificity. The bargained value of s in this case maximises the expected joint profit of the ROSCO and the ATOC, $(1 - \alpha)\,(r - c_0 - c_e e_A + c_s s) + \beta\alpha P(s)$, and the ROSCO would keep a share β

of this maximized profit. Its payoff would therefore be:

$$\pi_R^{O_I} = (1-\alpha)\,\beta\,(r - c_0 - c_e e_A + c_s s) + \beta^2 \alpha P(s). \tag{9}$$

Again, (9) is the ROSCO's outside option in the initial negotiation with the TOC under the vertical integration regime.[22]

We can now write the expected profits of the TOC, Π_T, and the ROSCO, Π_R, following an agreement in their negotiation (stage 4) and after TOC's choice of the effort for quality (stage 5), but before the resolution of quality uncertainty (stage 6) and of demand uncertainty (stage 7):

$$\Pi_T = (1-\alpha)\,(1 - q_m + e)\,(r - c_0 - c_e e + c_s s - p) - F, \tag{10}$$

$$\begin{aligned}\Pi_R = {} & (1-\alpha)\,[(1 - q_m + e)p + \\ & + (q_m - e)\beta\,(r - c_0 - c_e e_A + c_s s)] + \alpha\beta P(s) + F,\end{aligned} \tag{11}$$

where $P(s) = [r - c_0 - c_e e_A] - \frac{1}{2}ks^2$.

In (10), the expected quantity of train services employed on the line, $(1-\alpha)$, and the probability that the TOC's retains the franchise, $1 - q_m + e$, weight the difference between total revenue, r, and total costs. The latter is the sum of production costs, $c_0 + c_e e - c_s s$, and the unit price paid for the lease of the rolling stock, p. In addition, the TOC pays the fixed fee F to the ROSCO. In the first term of (11), the expected quantity of train services employed on the line, $(1-\alpha)$, weights the ROSCO's expected revenues per unit of train service operated on the line: with probability $(1 - q_m + e)$, the TOC operates the service and the ROSCO collects revenue p; with probability $(q_m - e)$, on the contrary, the ATOC operates the service and the ROSCO collects revenue $\beta\,(r - c_0 - c_e e_A + c_s s)$. The second term of (11) gives the ROSCO's expected profit from leasing the expected quantity α of unused train services to an "external" franchisee, as discussed in Section 2.2, which decreases with the degree of rolling stock specificity. Finally, the ROSCO collects the fixed fee F from the TOC.

(10) and (11) give the expected joint profit generated by the TOC and

[22] As shown below, in the vertical integration equilibrium, the TOC and the ROSCO agree on the rolling stock specificity that maximizes their expected joint profit (given by equation (12) below). Therefore, the outside option $\pi_R^{O_I}$ does not affect the equilibrium values of the rolling stock specificity and the service quality: it will affect only the division of the expected joint profit between the two parties.

ROSCO relationship, $\Pi_J \equiv \Pi_T + \Pi_R$:

$$\Pi_J = (1-\alpha)\left[(1-q_m+e)(r-c_0-c_e e+c_s s)+ (q_m-e)\beta(r-c_0-c_e e_A+c_s s)\right]+\alpha\beta P(s). \tag{12}$$

4.2 Effort for service quality (stage 5)

At stage 5, the TOC takes the train specificity and the terms of the leasing contract given in (10) as fixed, and chooses e to maximise Π_T. The first order condition $\frac{\partial \Pi_T}{\partial e} = 0$ is:

$$(r-c_0-c_e e+c_s s-p)-(1-q_m+e)\,c_e = 0.$$

Hence the TOC's profit maximising effort choice is:[23]

$$e_5 = \frac{r-c_0+c_s s-p-c_e(1-q_m)}{2c_e}, \tag{13}$$

provided this is in $[0,1]$.

4.3 Lease contract (stage 4)

Inspection of equations (12) and (13) immediately reveals that, beside a pure redistributive effect, the unit price p affects the Stage 5 choice of e by the TOC, and therefore, at least in theory, it exerts a distortionary effect on the TOC and ROSCO's expected joint profit. As shown in Appendix 1, this distortionary effect disappears when the parties also negotiate over the fixed fee F. More precisely, let Π_R^O denote the ROSCO's outside option in the case of disagreement.[24] Irrespective of how the rolling stock specificity s is chosen, the two parties set their contractual terms, (p, F), in such a way that the unit-price p induces the TOC to choose the joint profit maximising level of effort e_5:

$$\frac{\partial \Pi_J}{\partial e}\frac{\partial e_5}{\partial p} = 0, \tag{14}$$

[23] The second order condition for an interior solution requires $-2c_e < 0$, which is satisfied.

[24] As shown above, the ROSCO's outside option in the negotiation with the TOC depends on the vertical regime: it is given by equation (8) under vertical separation and by equation (9) under vertical integration. This difference is, however, immaterial for our argument here. We, therefore, use the generic notation Π_R^O to indicate the ROSCO's outside option in both regimes.

and the fixed fee F distributes the surplus generated by the relationship, $\Pi_J - \Pi_R^O$, according to the bargaining power coefficients β and $1-\beta$:

$$\Pi_R = \Pi_R^O + \beta(\Pi_J - \Pi_R^O), \tag{15}$$

$$\Pi_T = (1-\beta)(\Pi_J - \Pi_R^O). \tag{16}$$

From (13), $\frac{\partial e_5}{\partial p} = -(2c_e)^{-1} < 0$, and, from the first order condition of the TOC's optimisation problem at stage 5, $\frac{\partial \Pi_T}{\partial e} = 0$. Hence, condition (14) reduces to $\frac{\partial \Pi_R}{\partial e} = 0$, where Π_R is given in (11). This leads to:

$$p_4 = \beta\left(r - c_0 - c_e e_A + c_s s\right). \tag{17}$$

From equations (13) and (17) we can now derive the equilibrium level of the effort for quality, as a function of the rolling stock specificity:[25]

$$e_4 = \frac{(1-\beta)\left(r - c_0 + c_s s\right) + \beta c_e e_A - c_e\left(1 - q_m\right)}{2c_e}. \tag{18}$$

In the symmetric equilibrium of the industry, the rolling stock specificity and the effort for quality are the same in all areas of the network, so that $e_4 = e_A$. Then, equation (18) provides the following expression for the equilibrium level of the effort for quality, $\widehat{e}$ (common to all TOCs in the industry), as a function of the equilibrium level of the rolling stock specificity, $\widehat{s}$ (also common to all franchises):

$$\widehat{e} = \frac{(1-\beta)\left(r - c_0 + c_s \widehat{s}\right) - c_e\left(1 - q_m\right)}{c_e(2-\beta)}. \tag{19}$$

4.4 Choice of specificity (stage 3)

In the vertical integration regime, the rolling stock specificity is part of the negotiation between the TOC and the ROSCO. In equilibrium, the TOC and the ROSCO agree on the level of specificity which maximises their joint profit Π_J, given in (12). (Details are spelled out in Appendix 1). The first order condition $\frac{\partial \Pi_J}{\partial s} = 0$ yields:[26]

$$\beta + (1-\beta)(1 - q_m + e) = \beta \frac{\alpha k}{(1-\alpha)\, c_s} s. \tag{20}$$

[25] In alternative, solving in e the condition $\frac{\partial \Pi_J}{\partial e} = 0$, and using the solution in system with (13), would lead the same expressions for e_4 and p_4 as in equations (18) and (17), respectively.

[26] The second order condition for an interior solution requires: $\beta \frac{\alpha k}{(1-\alpha)c_s} - \frac{1}{2}\frac{c_s}{c_e}(1-\beta)^2 > 0$.

On the other hand, in the vertical separation regime, the rolling stock specificity, s, is unilaterally set by the ROSCO, to maximize its expected profit which would follow an agreement with the TOC:

$$\max_s \; \Pi_R = \Pi_R^{O_S} + \beta(\Pi_J - \Pi_R^{O_S}), \tag{21}$$

where Π_J and $\Pi_R^{O_S}$ are given by equations (12) and (8), respectively. (We refer again to Appendix 1 for details). Notice that the ROSCO's outside option, $\Pi_R^{O_S}$, itself depends on the rolling stock specificity.[27] Using (12) and (8), the first order condition $\frac{\partial \Pi_R}{\partial s} = 0$ yields:

$$1 + (1-\beta)(1 - q_m + e) = \frac{\alpha k}{(1-\alpha)\, c_s} s. \tag{22}$$

The price for the simplification obtained thanks to the linearity assumptions that we made is the need to consider many corner cases at stage 3, which makes the presentation of the results slightly cumbersome: in practice, of course, costs and benefits never jump suddenly, as we have assumed, but rises progressively more rapidly.

To compact notation, in what follows we denote:

$$K \equiv \frac{\alpha k}{(1-\alpha)\, c_s}, \qquad R \equiv \frac{r - c_0}{c_e} > 1, \qquad C \equiv \frac{c_s}{c_e}, \qquad z \equiv 1 - q_m + e, \tag{23}$$

and:

$$\delta = \begin{cases} \beta & \text{for the integration regime} \\ 1 & \text{for the separation regime} \end{cases}.$$

Note that z is the probability that TOC's franchise is continued, given in (3), while δ is a parameter characterising the vertical regime. The following values of the variables characterise a "fully interior equilibrium" − an equilibrium where s, e and z all lie in the interior of their respective range − and are obtained by solving the first order conditions (20) and (22) to determine the equilibrium values of s, and then substituting in (19) and (3) to obtain e and z:

$$\hat{s} = \frac{(1-\beta)^2 \left[R + (1 - q_m)\right] + \delta\,(2-\beta)}{\delta\,(2-\beta)\, K - C\,(1-\beta)^2}, \tag{24}$$

[27] Since $\Pi_R^{O_S}$ does not depend on e and $\frac{\partial \Pi_J}{\partial e} = 0$, any indirect effect of s on Π_R via e vanishes in the first order condition $\frac{\partial \Pi_R}{\partial s} = 0$. The second order condition for an internal equilibrium requires: $\frac{\alpha k}{(1-\alpha) c_s} - \frac{1}{2}\frac{c_s}{c_e}(1-\beta)^2 > 0$.

$$\hat{e} = \frac{\delta(1-\beta)(RK+C) - (1-q_m)[\delta K - C(1-\beta)^2]}{\delta(2-\beta)K - C(1-\beta)^2}, \quad (25)$$

$$\hat{z} = \frac{\delta(1-\beta)[K(1-q_m) + RK + C]}{\delta(2-\beta)K - C(1-\beta)^2}. \quad (26)$$

Given the linearity of the cost functions, the equilibrium is not necessarily characterised by the first order conditions, but it may be given by values of the choice variables at the boundary of the choice set. Lemma 1 addresses this point. Let:

$$\Gamma = \delta(1-\beta)[(R-1)K + C] - [\delta K - C(1-\beta)^2]. \quad (27)$$

Lemma 1 *There exist threshold values, $q_1^z, q_1^e, q_0^e \in (0,2)$, such that:*

i) If $\Gamma \geqslant 0$:

$q_m \in [0,1]$	*implies*	$e = q_m$	$s = \frac{1+\delta-\beta}{\delta K}$	*and* $z = 1$,
$q_m \in [1,2]$	*implies*	$e = 1$	$s = \frac{\delta+(1-\beta)(2-q_m)}{\delta K}$	*and* $z = 2 - q_m$.

ii) If $0 \geqslant \Gamma \geqslant -(1-\beta)\delta K$:

$q_m \in [0, q_1^z]$	*implies*	$e = q_m$	$s = \frac{1+\delta-\beta}{\delta K}$	*and* $z = 1$,
$q_m \in [q_1^z, q_1^e]$	*implies*	$e = \hat{e}$	$s = \hat{s}$	*and* $z = \hat{z}$,
$q_m \in [q_1^e, 2]$	*implies*	$e = 1$	$s = \frac{\delta+(1-\beta)(2-q_m)}{\delta K}$	*and* $z = 2 - q_m$.

iii) If $-(1-\beta)\delta K \geqslant \Gamma$:

$q_m \in [0, q_0^e]$	*implies*	$e = 0$	$s = \frac{\delta+(1-\beta)(1-q_m)}{\delta K}$	*and* $z = 1 - q_m$,
$q_m \in [q_0^e, q_1^e]$	*implies*	$e = \hat{e}$	$s = \hat{s}$	*and* $z = \hat{z}$,
$q_m \in [q_1^e, 2]$	*implies*	$e = 1$	$s = \frac{\delta+(1-\beta)(2-q_m)}{\delta K}$	*and* $z = 2 - q_m$.

Moreover, the threshold values are given by:

$$q_1^z = 1 + \frac{\delta(1-\beta)[(R-1)K + C] - [\delta K - C(1-\beta)^2]}{\delta(1-\beta)K}, \quad (28)$$

$$q_1^e = 2 - \frac{\delta(1-\beta)[(R-1)K + C]}{\delta K - C(1-\beta)^2}, \quad (29)$$

$$q_0^e = 1 - \frac{\delta(1-\beta)(RK+C)}{\delta K - C(1-\beta)^2}. \quad (30)$$

The proof is in Appendix 2. In words, according to (26), if $q_m \leq q_1^z$ the equilibrium probability that TOC's franchise is renewed is 1.[28] Similarly, according to (25), if $q_m \leq q_0^e$ the equilibrium level of the effort for quality becomes zero, while, if $q_m \geq q_1^e$, it is 1.

[28] Similarly, imposing $\hat{z} \geq 0$ in equation (26), we find $q_m \leq 1 + R + \frac{C}{K}$. However, since $1 + R + \frac{C}{K} > 2$, this threshold value never constrains the industry equilibrium.

5 Optimal regulation

When the regulator cannot directly choose the first best values of the rolling stock specificity and the effort for service quality, she will try to influence them indirectly by setting the minimum quality standard, q_m. The regulator takes as given the vertical regime (separation or integration) and chooses the quality standard that maximises the social welfare function (5) under the constraint that the values of the rolling stock specificity and the effort for quality be the equilibrium values for that regime.

To begin, we show that, in both vertical regimes, the regulator faces a trade-off between specificity and effort for quality in the determination of the minimum quality standard q_m.

Definition 1 *The* regulation possibility locus *is the locus of points in the* (s, e)*-plane representing the combinations of effort for quality and rolling stock specificity achievable through the regulation of the minimum quality standard.*

Recall that by choosing q_m the regulator affects the interaction between TOC and ROSCO, and therefore it indirectly affects their choice of e and s. However, only combinations (s, e) on the regulation possibility locus can be induced by the choice of q_m. These are described in next result.

Proposition 1 *In both vertical regimes, the regulation possibility locus is non-increasing, and it is strictly decreasing when the equilibrium values of* s, e *and* z *all lie in the interior of their respective ranges.*

Proof. For all cases where the equilibrium is not "fully interior", inspection of Lemma 1 suffices to prove that, as q_m increases, either the effort for quality increases while the rolling stock specificity remains constant, or the rolling stock specificity decreases while the effort for quality remains constant. Consider now fully interior equilibria. Solving (24) for q_m and substituting in (25), yields:

$$\widehat{e} = R + \frac{\delta}{(1-\beta)^2} - \frac{1}{(1-\beta)^2}\left[\delta K - C(1-\beta)^2\right]\widehat{s}, \tag{31}$$

that is, the relation between $\widehat{e}$ and $\widehat{s}$ is linear. Next, according to Lemma 1, a fully interior equilibrium requires $\Gamma < 0$, which clearly implies $\delta K - C(1-\beta)^2 > 0$. Therefore, (31) is a downward sloping line in the $(s, e) -$ plane. Finally, it is clear from (24) and (25) that an increase in q_m corresponds to a movement along this line where $\widehat{s}$ falls and $\widehat{e}$ rises. ■

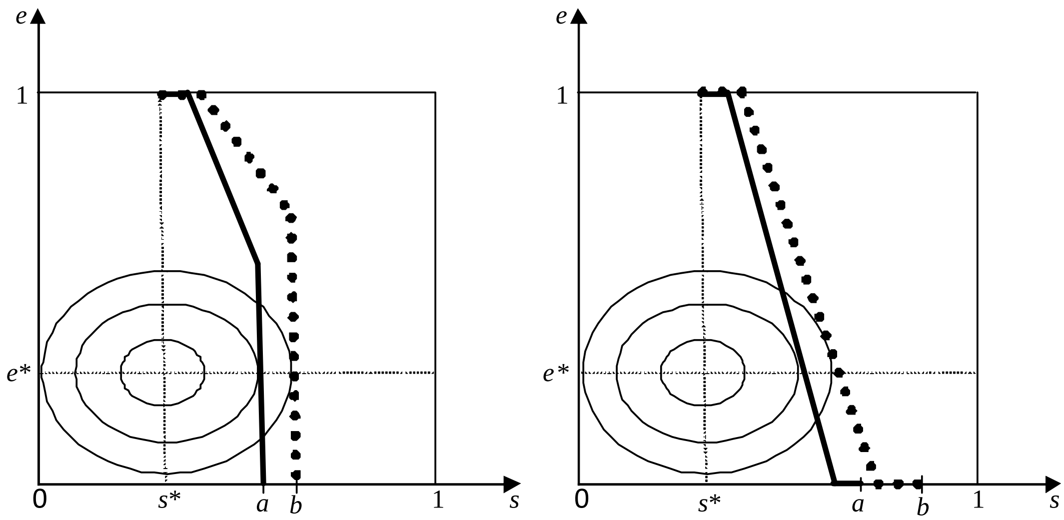

Figure 1: The iso-welfare loci and the regulation possibility loci.

Figure 1 illustrates two possible shapes of the regulation possibility loci in the separation regime (the solid line) and in the integration regime (the dotted line). The LHS diagram illustrates the second part of Lemma 1. For low values of q_m, the probability that TOC renews its franchise is 1, and the loci are vertical: as q_m increases, e increases but s remains constant. For intermediate values of q_m, the equilibrium is fully interior, and the loci take the shape of a downward sloping line. For high values of q_m, the effort for quality is at its maximum possible level, and the rolling stock specificity and the probability that TOC renews its franchise both decrease with q_m. The loci become horizontal on the $e = 1$ line. The RHS diagram corresponds to the third part of Lemma 1. When q_m is low, the effort for quality takes its minimum possible value, $e = 0$, with z and s still in the interior of their respective ranges, and the loci exhibit a horizontal portion on the e-axis. For intermediate values of q_m, the equilibrium is "fully interior" and the two loci are downward sloping. For high values of q_m, the effort for quality is fixed at its maximum possible level: this is the horizontal portion of the two loci at $e = 1$. The picture illustrating the first part of Lemma 1 is different from the diagram on the LHS only in that the downward sloping portion of the curve is perfectly vertical.

The intuition behind Proposition 1 is the following. As the regulator raises the quality standard, the probability that TOC's franchise is renewed decreases, even though the TOC partly offset the higher quality threshold with its effort for quality. This reduces the private expected return of specificity in both regimes. This is because whoever chooses the level of s (the ROSCO on its own or the TOC and the ROSCO in concert) has a lower marginal return from specificity if the ATOC rather than the TOC operates the service. Under vertical integration, the reason is that the cost saving due to more specificity is fully internalized in the TOC and ROSCO joint profit when the TOC operates the service, while a share $(1-\beta)$ of it will accrue to the ATOC if the TOC's franchise is not renewed. Under vertical separation, the ROSCO has a positive outside option in the negotiation with the TOC which increases with the rolling stock specificity. Therefore, an increase in specificity raises the ROSCO's share of the joint profit realised with the TOC.

Our next result shows that, irrespective of the vertical regime, the industry equilibrium always exhibits over-investment in rolling stock specificity unless the regulator sets q_m at its maximum value, $q_m = 2$, in which case that TOC's franchise is renewed with probability 0.

Proposition 2 *Let* $0 < \frac{c_s(1-\alpha)}{k\alpha} < 1$ *and* $\beta < 1$. *Then, in both vertical regimes,* $q_m < 2$ *implies* $\widehat{s} > s^*$.

Proof. Using the notation introduced in (23), we first rewrite the first order conditions (20) and (22) as:

$$\delta + (1-\beta)\widehat{z} = \delta \frac{\alpha k}{(1-\alpha)\, c_s} \widehat{s},$$

where $\widehat{z}$ is the equilibrium probability that TOC's franchise is continued. Since $\frac{\alpha k}{(1-\alpha)c_s} = \frac{1}{s^*}$ (see equation (6)), we have:

$$\widehat{s} = s^* \left[1 + \frac{(1-\beta)}{\delta} \widehat{z} \right],$$

which assures that, in both regimes, there is over-investment in specificity unless $\beta = 1$ or $\widehat{z} = 0$. Finally, by Lemma 1, $\widehat{z} = 0$ if and only if $q_m = 2$. ■

The conditions in the statement ensure that the first-best socially optimal level of rolling stock specificity is interior, $0 < s^* < 1$, and the ROSCO does not have full bargaining power.

Proposition 2 is illustrated in Figure 1. All possible cases formalised in Lemma 1 generate second-best loci with the qualitative characteristic depicted in the figure: irrespective of the vertical regime, to achieve any value of $e < 1$, the regulator must accept over-investment in specificity. Only by setting

$q_m = 2$ the regulator can eliminate the over-investment in s, and implement the first-best socially optimal level of specificity, with the resulting value of e at its maximum possible level.

To understand the over-investment in specificity shown by Proposition 2, recall that specificity offers the social benefit of lowering the cost of operating the rolling stock on the network it was designed for, at the social cost of additional operating costs if the train is moved to a different network. The private choice of s would replicate the first-best socially optimal choice only when the social cost and the social benefit of specificity are internalised in the profit function of the private decision maker exactly in the same proportion. This is clearly the case when the ROSCO has full bargaining power, in which case the social cost and benefit of specificity are fully internalised in the ROSCO's profit in both vertical regimes. If the ROSCO does not have full bargaining power (the condition $\beta < 1$ in Proposition 2), only a share β of the social cost of specificity is internalised in the profit function of the private decision maker for the choice of s, the remaining $1 - \beta$ share is borne by an ATOC operating the service in a different network. On the benefit side, if the TOC's franchise is not renewed ($z = 0$), then the TOC and the ROSCO (in the integration regime) or the ROSCO alone (in the separation regime) collect the fraction β of the profit generated by the ATOC on the "right" network, and thus the share β of the social benefit of specificity: the social cost and benefit of specificity are internalised in the same proportion by the private decision maker, and the private choice of s is socially optimal. If, on the contrary, $z > 0$, then, irrespective of the vertical regime, the private decision maker of s internalises (in expectation) a share of the social benefit of specificity larger than β:[29] the private decision maker of s internalises a higher proportion of the social benefit than of the social cost of specificity, which leads to over-investment in specificity in both regimes.

We are now in the position to analyze the regulator's optimal choice of the vertical regime.

Proposition 3 *Assume that the first-best social optimum is interior in both s and e. Then the vertical separation regime is socially preferable to the vertical integration regime.*

[29] As argued in the discussion of Proposition 1, the private marginal return of specificity is always higher when the TOC rather than the ATOC operates the service on the "right" network, and it equals the fraction β of social marginal benefit when the ATOC operates the service.

The "interior" condition implies that the centre point of the ellipses drawn in the picture is not on the sides of the $[0,1] \times [0,1]$ square in the (s,e)-plane. The regulator selects the minimum quality standard to obtain the combination of e and s given by the point of tangency of the separation locus and the highest possible iso-welfare curve. As the Figure illustrates, the reason why the vertical separation regime (solid line) is preferable is that it limits the over-investment in specificity. While the intuition is easily illustrated, the formal proof is more complex, and relegated to Appendix 2. In essence, it requires showing that each of the possible pairs (one for the integration and one for the separation regime) of regulation possibility loci has the property that the locus for the separation regime always lies on the left of the locus for the integration regime for every $0 \leq e < 1$. In view of Proposition 2, this implies that the separation regime entails a lower over-investment in specificity for any value of the effort for quality (except the maximum) the regulator might want to implement by setting the minimum quality standard. Notice also that, irrespective of the shape of the regulation possibility loci, the regulator's preferred value of e is 1 only when the first best is also 1. In this case, the optimal regulation policy achieves the first-best levels of both s and e by setting $q_m = 2$ in both regimes.

The intuition of Proposition 3 is straightforward given our discussion of Propositions 1 and 2. The over-investment in specificity is always stronger under vertical integration: if the TOC operates the service on the "right line", the social benefit of specificity is fully appropriated by the ROSCO and the TOC in the vertical integration regime, whilst only a fraction of it is appropriated by the ROSCO in the vertical separation regime. On the other hand, as we have already explained, in both regimes whoever sets s internalises the fraction β of the social benefit of specificity when the ATOC operate the service on the "right line", and the fraction β of the social cost of specificity in all circumstances.

In the second-best solution, there is always over-investment in specificity (unless the regulator can achieve the first-best), while the level of the effort for quality can be greater or equal to the first-best value. This is shown in Figure 1. If the tangency point between the separation locus and the lowest possible iso-welfare curve is on a downward sloping (respectively, vertical) portion of the locus – as on the RHS (respectively, LHS) –, then the second best is a fully interior equilibrium and there is (respectively, there is no) over-investment also in effort for quality. Notice also that the regulation of the quality standard, linked to the threat of franchise termination, is an effective way to fine tune the effort for quality, which can take any value in $[0, 1]$, and the regulator adjusts the investment in effort for quality with the aim to reduce the over-investment in specificity.[30]

6 Concluding remarks.

This paper examines the effects of imposing separation on the vertically related suppliers of the outputs necessary in a regulated industry. The stylised model is inspired by the structure of the UK railway industry, where TOCs and ROSCOs, the suppliers of train services to passengers, and of rolling stock are obliged to maintain a substantial degree of separation from each other. This is unlike most of the rest of the world, where instead the suppliers of train services also own the trains used to supply those services. It can shed light on several other regulated industries, where it is technologically feasible to separate the vertical stages of production, in some cases (such as British Gas) imposed as a remedy to non-competitive practices.

Our paper confirms the view that the UK system provides a weaker incentive to specific train design and effort for quality than a more vertically integrated system. However, unlike the existing literature (see footnote 10 above), we show that the benefit of a competitive and flexible structure, both ex-ante and ex-post, may well outweigh the negative effects of lower specificity and the lower quality effort that occur in the separation regime, as also suggested by Affuso and Newbery (2002).

[30] We have so far assumed that the equilibrium level of specificity is interior: $\widehat{s} \in (0, 1)$. This is not restrictive. We have already shown that $\widehat{s} \geq s^*$, and therefore the second best optimum, $\widehat{s}$, can be 0 only when the first best, s^*, is also 0, that is when specificity has no social value. This is not realistic for the railway industry. $\widehat{s} < 1$ is not restrictive either, provided the the first best level, s^*, is also lower than 1. Allowing for $\widehat{s} = 1$ simply means that our second best loci now can reach the $s = 1$-line, and coincide with it in a region. However, for all other portions of the loci, our comparative results would still apply. This means that the integration locus will never lie below the separation locus, and the second best solution under integration never dominates the second best solution under separation. More precisely, the two regimes woud offer the same social value: (i) for $e^* = 1$ (as shown in our second-best analysis), since the over-investment in specificity can be completely eliminated without any cost in term of over-investment in effort; (ii) for sufficiently low values of e^*, since, for low values of e, both regimes would offer the same level of specificity, i.e. the upper bound $s = 1$. In all other cases (i.e., for intermediate values of e^*) vertical separation still dominates vertical integration for the same reasons illustrated in this Section. Finally, the case where $s^* = 1$ is a limit case where the two regimes are always equivalent. This shows that the regime of integration is weakly dominated by the regime of vertical separation.

In the competitive environment which characterises the UK railways, specificity is set partly with the aim of reducing the probability that the franchise is lost by the Train Operating Company: a high specificity makes it easier (i.e., cheaper) for the TOC to meet the quality standard set by the regulator. To the extent that the ROlling Stock COmpany can extract some of the surplus from the TOC being abler to comply with the quality standard, the ROSCO can gain by increasing the rolling stock specificity even though it will mean lower revenues in the event that trains need to be switched to a different area. Now there is a difference between the two vertical regimes in this respect. In the integration regime, the ROSCO can extract more of the TOC's surplus, because it can do so in the direct negotiation. In the separation regime, it must instead do it indirectly, by adjusting s itself, which affects ROSCO's outside option in the negotiation with TOC over the lease contract. The surplus it can extract is less, and hence the downside of specificity (the lower revenues in the case specific trains need to be switched to a different line) keeps its level down in the separation regime.

In other words, train specificity is used by the TOC (possibly in concert with the ROSCO) to blunt the regulatory threat of the withdrawal of the franchise (by making it cheaper for the TOC to meet the service quality standard). This over-investment in specificity (which is greater in the integration regime, because the TOC and the ROSCO are better able to agree on this strategic use of s) disappears when it is not possible to positively affect the probability that the TOC's franchise is continued, that is when, although the equilibrium probability that the franchise is renewed is 0, the effort for quality is already at its maximum value. On the other hand, the equilibrium probability that the TOC's franchise is continued can be 1 only if the TOC adjusts the effort for quality one-to-one to any increase in the quality standard. The cost savings from more specificity play a crucial role in incentivising the TOC to follow this strategy. This explain why, in both regimes, the over-investment in specificity is maximum when $z = 1$.

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Appendix 1.

In this appendix, we first show that the generalized Nash bargaining between the ROSCO and the TOC over the two part pricing schedule (p, F) leads to the solution adopted in the text: p induces the joint profit maximising level of effort for quality and F splits the resulting surplus, $\Pi_J - \Pi_R^O$, according to the bargaining power coefficients β and $(1-\beta)$. Afterward, we show that the rolling stock specificity bargained, along with the lease contract, by the TOC and ROSCO in the vertical integration regime maximizes their joint profit.

In both vertical regime, the TOC and the ROSCO set p and F to solve the maximization problem:

$$\underset{p,F}{Max}\left(\Pi_R - \Pi_R^O\right)^{\beta}\Pi_T^{1-\beta}, \tag{A1.1}$$

where Π_R and Π_T are given by equations (11) and (10), respectively, and Π_R^O is the ROSCO's outside option in the negotiation with TOC. For the moment, we do not need to distinguish between the values of the ROSCO's outside option in the two vertical regimes, but only to note that the ROSCO's outside option is independent of the pricing schedule under negotiation with the TOC. The first order conditions for problem (A1.1) yield:

$$\beta\left(\Pi_R - \Pi_R^O\right)^{\beta-1}\Pi_T^{1-\beta}\frac{\partial\Pi_R}{\partial F} + (1-\beta)\left(\Pi_R - \Pi_R^O\right)^{\beta}\Pi_T^{-\beta}\frac{\partial\Pi_T}{\partial F} = 0$$
$$\beta\left(\Pi_R - \Pi_R^O\right)^{\beta-1}\Pi_T^{1-\beta}\frac{\partial\Pi_R}{\partial p} + (1-\beta)\left(\Pi_R - \Pi_R^O\right)^{\beta}\Pi_T^{-\beta}\frac{\partial\Pi_T}{\partial p} = 0,$$

or

$$\beta\Pi_T\frac{\partial\Pi_R}{\partial F} + (1-\beta)\left(\Pi_R - \Pi_R^O\right)\frac{\partial\Pi_T}{dF} = 0 \tag{A1.2}$$

$$\beta\Pi_T\frac{\partial\Pi_R}{\partial p} + (1-\beta)\left(\Pi_R - \Pi_R^O\right)\frac{\partial\Pi_T}{\partial p} = 0. \tag{A1.3}$$

Now, from equations (11) and (10), we calculate $\frac{d\Pi_R}{dF} = -\frac{d\Pi_T}{dF} = 1$, which means that F has a pure distributive role with no effect on the joint profit. Then, condition (A1.2) reduces to

$$\beta\Pi_T = (1-\beta)\left(\Pi_R - \Pi_R^O\right), \tag{A1.4}$$

which, with the definition of the joint profit $\Pi_J = \Pi_T + \Pi_R$, leads the distribution of the surplus stated in equation (15):

$$\Pi_R = \Pi_R^O + \beta\left(\Pi_J - \Pi_R^O\right)$$
$$\Pi_T = (1-\beta)\left(\Pi_J - \Pi_R^O\right).$$

Next, (A1.4) and (A1.3) imply:

$$\frac{\partial\Pi_R}{\partial p} + \frac{\partial\Pi_T}{\partial p} = 0,$$

that is,

$$\frac{\partial \Pi_J}{\partial p} = 0. \tag{A1.5}$$

Obviously, the pure distributive effect of the unit-price p on the TOC's and the ROSCO's profits cancels out in the joint profit. Therefore, Π_J depends on p only through the effort for quality chosen by TOC at the subsequent stage 5, and (A1.5) is equivalent to:

$$\frac{\partial \Pi_J}{\partial e}\frac{\partial e_5}{\partial p} = 0,$$

that is, condition (14) in the text.

Consider now the vertical integration regime. The ROSCO and TOC bargain over the pricing schedule and the rolling stock specificity simultaneously. Their maximisation problem becomes:

$$\underset{p,F,s}{Max}\left(\Pi_R - \Pi_R^{O_I}\right)^{\beta}\Pi_T^{1-\beta}, \tag{A1.6}$$

where the ROSCO's outside option, $\Pi_R^{O_I}$, is given by equation (9). Notice that, beside being independent of the pricing schedule, $\Pi_R^{O_I}$ is also independent of the rolling stock specificity under negotiation between the ROSCO and the TOC, since, in the case of disagreement, the rolling stock specificity will remain indeterminate and will enter the successive negotiation between the ROSCO and the ATOC. In addition to conditions (A1.2) and (A1.3), the optimisation problem (A1.6) requires the first order condition in s:

$$\beta\Pi_T\frac{\partial \Pi_R}{\partial s} + (1-\beta)\left(\Pi_R - \Pi_R^O\right)\frac{\partial \Pi_T}{\partial s} = 0, \tag{A1.7}$$

which, using (A1.4), becomes:

$$\frac{\partial \Pi_R}{\partial s} + \frac{\partial \Pi_T}{\partial s} = 0.$$

In other words, the TOC and the ROSCO will agree on the specificity level that maximizes their joint profit.

Appendix 2.

Proof of Lemma 1. Part (*i*). We first prove that $\Gamma \geqslant 0$, where Γ is given in (27), precludes fully interior equilibria for any $q_m \in [0,2]$. To this end, it is convenient to reformulate equation (25) as:

$$\widehat{e} = 1 + \frac{\delta(1-\beta)((R-1)K+C) - (\delta K - C(1-\beta)^2) - (1-q_m)(\delta K - C(1-\beta)^2)}{\delta(2-\beta)K - C(1-\beta)^2}. \tag{A2.1}$$

Recall also that:

$$\delta(1-\beta)[(R-1)K+C] > 0 \ \text{ (since } R>1 \text{ and } C>0\text{)}, \tag{A2.2}$$

$$\beta(2-\beta)K - C(1-\beta)^2 > 0 \ \text{ (for } \widehat{s} \text{ to be positive and finite)}. \tag{A2.3}$$

If $\Gamma \geqslant 0$, $\widehat{e} < 1$ would require $(\delta K - C(1-\beta)^2) > 0$ and $q_m < 1$, as it is apparent from (A2.1). On the other hand, (28) would imply $q_1^z > 1$. Since $\widehat{z}$ is decreasing in q_m (see (26)), $\widehat{z}$ is corner at 1 for $q_m \leq q_1^z$. With $q_1^z > 1$, $\widehat{z}$ will be corner at 1 for any $q_m \leq 1$. Then, over the $[0, 2]$ range of q_m, the the equilibrium is either corner in e (for $q_m \in (1, 2]$), or corner in z (for $0 \leq q_m \in [0, 1)$), or corner in both e and z (for $q_m = 1$), and we never find a fully interior equilibrium.

To characterise the equilibrium, we first re-write the first order conditions in s of the two regimes (equations (20) and (22)) in the compact form:

$$\delta + (1-\beta)z = \delta K s. \tag{A2.4}$$

For $q_m \in [0, 1]$, $z = 1$, so that $z = 1 - q_m + e$ implies $e = q_m$. The equilibrium level of s must solve condition (A2.4) with $z = 1$, yielding: $s = \frac{1+\delta-\beta}{\delta K}$.

For $q_m \in [1, 2]$, $e = 1$ so that $z = 1 - q_m + e$ implies $z = 2 - q_m$. The equilibrium level of s must solve condition (A2.4) with $z = 2 - q_m$, yielding: $s = \frac{\delta+(1-\beta)(2-q_m)}{\delta K}$.

Parts *(ii)* and *(iii)*. We first prove that, if $\Gamma < 0$, then it always exists an interval in the $[0, 2]$ range of q_m where the industry equilibrium is fully interior.

Notice that, $\Gamma < 0$ and (A2.2) imply $\delta K - C(1-\beta)^2 > 0$. Then, from (A2.1), $\widehat{e}$ is increasing in q_m.

Next, $\Gamma < 0$ implies $1 < q_1^e < 2$ (from (29)) and $0 < q_0^e < 1$ (from (30)). This ensures that $\widehat{e}$ takes interior values in a connected interval of q_m around 1. On the other hand, $\Gamma < 0$ implies $q_1^z < 1$ (from (28)). Since $\widehat{z}$ is decreasing in q_m and cannot be zero for $0 < \widehat{e} < 1$ (as explained in footnote 28, p.19), $\widehat{z}$ also takes interior values in a (connected) interval of q_m around 1. There must therefore exist a connected interval of q_m around 1 where both $\widehat{z}$ and $\widehat{e}$ are interior. Since $\widehat{s}$ is interior by assumption, the equilibrium is fully interior in such an interval.

The upper extreme of interval of q_m where the equilibrium is fully interior is always q_1^e. Therefore, for $q_m \in [q_1^e, 2], e = 1$, and the equilibrium is characterised as in the analogous case of part (*i*): $e = 1$, $z = 2 - q_m$, and $s = \frac{\delta+(1-\beta)(2-q_m)}{\delta K}$ (third line of both part (*ii*) and part (*iii*)).

The lower extreme of the fully interior equilibrium interval is clearly given by the maximum between q_1^z and q_0^e. Using equations (28) and (30), we find that $q_1^z > q_0^e$ is equivalent to $\Gamma > -(1-\beta)\delta K$.

In **part *(ii)*,** $\Gamma > -(1-\beta)\delta K$, so that the fully interior equilibrium arises for $q_m \in [q_1^z, q_1^e]$ (second line of part (*ii*)).

For $q_m \in [0, q_1^z]$, $z = 1$, and the equilibrium is characterised as in the analogous case of part (*i*): $z = 1$, $e = q_m$, and $s = \frac{1+\delta-\beta}{\delta K}$ (first line of part (*ii*))

In **part *(iii)***, $\Gamma < -(1-\beta)\delta K$, so that the fully interior equilibrium arises for $q_m \in [q_0^e, q_1^e]$ (second line of part (*iii*)).

For $q_m \in [0, q_0^e]$, $e = 0$, so that $z = 1 - q_m + e$ implies $z = 1 - q_m$. The equilibrium level of s must solve condition (A2.4) with $z = 1 - q_m$, yielding: $s = \frac{\delta+(1-\beta)(1-q_m)}{\delta K}$ (first line of part (*iii*)). ■

Proof of Proposition 3. Building upon proposition 1 (over-investment in specificity in both regimes), it is sufficient to show that in any parameter region of the model, by moving q_m, the regulator can implement a lower (or, at least, not higher) level of specificity in the separation rather than in the integration regime for any level of effort for quality she would like to achieve.

According to Lemma 1, we can have three alternative shapes for the second best $(s, e) - locus$ of each regime (made explicit in our discussion of proposition 1). Hence, abstracting from the consistency of the parameter conditions, we can in principle pair the integration and the separation loci in nine different ways. We proceed by distinguishing two classes of cases. In the first class, the separation locus always has a downward sloping portion (corresponding to fully interior equilibria), as in the two diagrams of Figure 1, while the integration locus can either have a downward sloping portion (like in the two diagrams of Figure 1) or be inverted-L shaped (i.e., the shape arising from the first part of Lemma 1). This class comprises 6 of the 9 cases we counted above.

We show that, in all cases of this class, the separation locus always lies on the left of the integration locus (except for $e = 1$, where the two loci always overlap). Assume first that also the integration locus has a downward sloping portion. Then, using equation (31) with $\delta = 1$ for the separation and $\delta = \beta$ for the integration regime, it is easy to check that the linear downward sloping portion is flatter (recall that $\beta < 1$) and intersects the $e = 1 - line$ for a higher value of s, under integration. Then, irrespective of the shape of the remaining portions of the two loci (i.e., a vertical segment, as in the right diagram of Figure 1, or an horizontal segment lying on the $e = 0 - line$, as in the left diagram of Figure 1), the separation locus is entirely on the left of the integration locus if

$$\frac{1}{\beta K} > \frac{2-\beta}{K},$$

where the left (right) hand side of the inequality is the measure of the segment $\overline{0b}$ (resp., $\overline{0a}$) in the diagrams of Figure 1 (by Lemma 1). This inequality is indeed always satisfied for $\beta < 1$.

We are left only with two cases in the first class: the integration locus is inverted L-shaped, while the separation locus can either be as in the left or as in the right diagram of Figure 1. If so, the inequality above is clearly sufficient for the separation locus always being on the left of the integration locus.

The second class consists of the three cases where the separation locus is always inverted-L shaped, and the integration locus has, alternatively, an inverted L-shape, the shape shown in the left or in the right diagram of Figure 1. If both loci are inverted L-shaped, the inequality above is again sufficient to prove the statement. The remaining two cases could, in principle, invalidate the statement. In these cases, the separation locus is inverted L-shaped, that is, the level of s is fixed at $\frac{2-\beta}{K}$ for any $e < 1$. The integration locus, on the contrary, always exhibits a downward sloping portion (corresponding to fully interior equilibria). We prove, however, that the parameter condition for a fully interior equilibrium in the integration regime is inconsistent with the parameter condition required for the separation locus to be inverted L-shaped, so that these cases are impossible. From parts (i) and (ii) of Lemma 1, a fully interior equilibrium in the integration regime requires:

$$\beta (1-\beta) [(R-1)K + C] - [\beta K - C (1-\beta)^2] < 0.$$

On the other hand, the separation locus is inverted L-shaped only if the condition of part (i) of Lemma 1 holds when $\delta = 1$, that is:

$$(1-\beta) [(R-1)K + C] - [K - C (1-\beta)^2] \geq 0.$$

Hence, it should be:

$$(1-\beta) [(R-1)K + C] - K + \frac{C (1-\beta)^2}{\beta} < 0$$
$$(1-\beta) [(R-1)K + C] - K + C (1-\beta)^2 > 0,$$

which is impossible since $\frac{C(1-\beta)^2}{\beta} > C (1-\beta)^2$ as $\beta < 1$. ■