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ABSTRACT

Tax Contracts and Government Formation*

We introduce tax contracts and examine how they affect government formation and welfare of voters in a democracy with proportional elections. A tax contract specifies a range of tax rates a party is committed to if in government. We develop a new model of party competition in which parties choose tax rates, public-good provision, and perks, and we show that the introduction of tax contracts has two effects: a perks effect and a policy-shift effect. The former plays a central role in societies with a low degree of political polarization, where it tends to reduce politicians' perks. If a society is highly polarized, tax contracts can yield more moderate political outcomes. However, there are also circumstances in which tax contracts induce more extreme policies.

JEL Classification: D72, D82 and H55
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1 Introduction

Should politicians or parties be allowed to sign political contracts that make their campaign promises binding? In this paper, we study how political contracts affect government formation and the welfare of voters in democracies with proportional elections. A political contract is a written promise made by a politician or a party, coupled with sanctions and bonuses depending on whether the promises are kept when they are in office. For instance, the office-holder may lose his right to stand for reelection if he fails to fulfill the promises in the political contract, or he may receive extra rewards if he honors the contract.

Campaign promises, however, can only qualify as political contracts if a public certification body approves them. This, in turn, requires that hard and non-manipulable information will be available when the contract is up for review. It is clear that only a subset of political decisions are certifiable and are thus susceptible of inclusion in political contracts. In particular, financial promises are easier to certify than promises regarding the level of public goods. For instance, the promise not to raise taxes is certifiable, whereas the promise to increase the strength and the morale of the military is not. Similar examples are reforms of the judical system or investments in the health-care system. While the financial scheme is certifiable, the output is not.

In this paper, we examine this partial certification problem in a setting where political parties can make binding promises regarding the financing of public goods in the form of tax contracts, but no binding promises regarding the amount of public goods they will provide once in office. A tax contract specifies a range of tax rates a party is committed to if it is part of the government.

To study the impact of tax contracts, we develop a new type of model of party competition in a democracy with proportional elections, where two large and one smaller conventional parties compete for office. Parties have policy preferences about the level of public-good provision and benefit from perks when in office. A government raises taxes for both purposes. A fourth and extremist party may enter parliament and may force the large conventional parties to enter into a grand coalition.

We first examine the model where no tax contracts are allowed. We show that if the extremist party does not enter, a small coalition of conventional parties will form. The chosen policy is a convex combination of the coalition partners' political orientation and will hence either be to the left or to the right of the center. If the extreme party does enter, a grand coalition is formed that implements more moderate policies. In both cases, tax rates are higher than desired by voters, since parties use some of the tax revenues to fund their perks.

Second, we examine how this democracy performs when parties can sign political contracts regarding taxes during the campaign, i.e. parties can commit to the set of tax rates they are allowed to choose from if they are part of the government. The parties, however, cannot commit to a particular amount of public-good provision or to the level of perks, since such promises are not certifiable.[1] Tax contracts have the following effects: (a) a policy-shift effect, as tax contracts may allow a party to influence the policy of the coalition government in favor of its own ideal point, and (b) a perks effect that can lead to lower tax rates, since the parties in the coalition government perceive the perks of their coalition partner as a negative externality to public-good provision.

We find that tax contracts may involve substantial welfare gains in highly polarized societies when the policy-shift effect tends toward the center, as this keeps policies moderate. However, under certain circumstances the policy shift with tax contracts will be towards the left or right. Then tax contracts yield more extreme outcomes and may not be welfare-improving. The perks effect plays an important role in societies with low a degree of polarization. However, it will depend on the concrete parameter values whether this leads to considerable welfare improvements. Our numerical examples suggest rather moderate relative welfare differences from the perks effect. The relative welfare impacts of the policy-shift effect seem to be substantially higher.

There is a famous example from the recent past where tax contracts would probably have made a difference. In the general election in Germany in 2005, the large parties CDU/CSU and SPD made campaign announcements regarding the value-added tax rate. The CDU/CSU announced a two-percentage-point increase, while the SPD had pledged not to increase the tax rate at all. The election outcome forced both parties into a grand coalition, since the extreme party "Die Linke" entered parliament and no party wanted to form a coalition with them. The grand coalition increased the value-added tax by three percentage points. So both large parties reneged on their campaign promises. Had tax contracts been feasible, tax rates would either have been lower after the government was formed or parties would have made different binding

[1] There are instances where the amount of public goods is certifiable, e.g. if the public good is the building of a bridge. However, even in these cases the specific construction is not certifiable.

promises regarding the tax rate.

The paper is organized as follows: In section 2 we relate our analysis to the literature. The model is developed in section 3. In section 4 the formation of coalition governments without contracts is examined. We introduce tax contracts into the legislative game in section 5 and characterize the equilibria with tax contracts. In section 6 the perks effect and the policy-shift effect are identified. Thereafter, in section 7, we compare the different regimes with respect to a utilitarian welfare function. We illustrate our results with some numerical examples in section 8. In appendix B we also discuss the welfare consequences involved if parties were additionally permitted to make their contracted tax sets conditional on the event of the extreme party entering parliament. Section 9 concludes.

2 Relation to the Literature

Our paper is related to four strands of the literature. First, some recent literature suggests that political contracts might be a viable supplement to democracies (e.g. Gersbach (2005)). This literature assumes that the output of policy actions is certifiable or can be made certifiable. In this paper, we examine the democratic provision of public goods when financing is certifiable while output is not. We introduce tax contracts constraining the set of tax rates a party can choose from if it is part of the government.

Second, in order to account for uncertain coalition formation, we introduce a new type of model of party competition where three conventional parties and an extremist party compete for office. As the policy that separates the conventional parties from the extremist party is indivisible, only conventional parties will form a coalition. The formation process builds on the theory of governmental coalition formation. We refer to Bandyopadhyay and Chatterjee (2006) and Austen-Smith and Banks (2005) for surveys. Our approach in specifying coalition formation involving a formateur follows in particular Roemer (2001).

Third, we suggest a new type of tax competition that may deliver welfare improvements. The standard theme since Tiebout (1956) has been that local or national jurisdictions compete on taxes and public inputs for mobile factors. In particular, a large body of literature on international tax competition suggests that mobility of capital restricts national tax autonomy and puts pressure on governments to reduce taxes on mobile factors and to improve the efficiency of public-good provision.[2] We develop a new form of tax competition by parties that has the potential to increase the efficiency of the combination of tax and public spending. Moreover, there is an extensive range of literature on how democratic institutions affect rent extraction by political agents. A general theme in this literature is that, in a representative democracy, office-holders can extract rents and that referendums will reduce the representative's opportunities for rent extraction (see e.g. Feld et al. (2008)). We suggest a new type of institution that can help to limit rent extraction by agents in a representative democracy.

Fourth, we focus on proportional electoral systems. In these widespread systems, a multitude of parties are usually present in parliament and governments are formed by coalitions. A common theme in the literature on proportional election systems is how such coalitions affect spending and public goods allocation in comparison to majoritarian systems (Persson and Tabellini (2003), Milesi-Ferretti et al. (2002)). In this paper, we examine how tax contracts affect policies and perks in proportional electoral systems.

3 The Model

3.1 Voters and policies

We consider a society consisting of a set of voters, $\mathcal{I}$, who differ in their income levels y_i, $i \in \mathcal{I}$. Income is distributed over the interval $[y_{min}, y_{max}]$ according to density $f(y)$. Aggregate income Y is given by

$$Y = |\mathcal{I}| \int_{y_{min}}^{y_{max}} y f(y) dy = |\mathcal{I}| \bar{y},$$

where $\bar{y}$ denotes the average income and $|\mathcal{I}|$ is the measure of set $\mathcal{I}$. The government is organized as a parliamentary democracy where a majority coalition in the legislature elects the ministers of the executive branch. The total number of seats in the parliament is given by S^p and there are S (S even) seats in government (e.g. S ministers). The government decides on

- the level of a public good $g \in \{0, \infty)$ and the perks for the parties in power;

[2] See Wilson (1999) for a survey and Benassy-Quer et al. (2007) for a recent assessment of whether a "race to the bottom" regarding corporate tax rates is taking place in Europe.

both are financed by an income tax with associated tax distortion λ; the costs of providing the amount g of the public good is gq, where $q > 0$ is the unit cost.

- a binary policy $d \in \{0, \bar{d}\}$, where $d = 0$ represents the status quo and $d = \bar{d}$ represents an indivisible policy change, such as waging a war or stopping immigration.

The utility of a voter with income y_i is

$$U_i = U(y_i, \delta_i) = A \ln g - t y_i - \delta_i d,$$

where $\delta_i \in \{-1, 1\}$. The variable t denotes the tax rate on income.

The desired policy of a voter characterized by $(y_i, \delta_i = 1)$ is the solution of the following problem:

$$\max_{t,g,d \geq 0} U_i$$
$$\text{s.t. } tY = (1 + \lambda)(gq).$$

It is obvious that the optimal choice is $d_i^* = 0$. Substituting t in U_i, the first-order condition with respect to g is

$$\frac{A}{g} - y_i \frac{(1 + \lambda)q}{Y} = 0,$$

which yields

$$t_i^* = \frac{A}{y_i}, \quad (1)$$
$$g_i^* = \frac{YA}{y_i(1 + \lambda)q}, \quad (2)$$
$$d_i^* = 0. \quad (3)$$

The solution for a voter characterized by $(y_i, \delta_i = -1)$ is identical, except that $d_i^* = \bar{d}$.

3.2 Parties

There are three conventional parties denoted by $j, k = \{L, M, R\}$ with ideal platforms $(y_L, 1), (y_M, 1), (y_R, 1)$. We assume that $y_L < y_M < y_R$. M is the middle party, while L (R) is left (right) of center. Parties are interested in the policies the government enacts and in the perks from holding office. Both these motives for politicians are well-documented in the literature. In particular, the utility of a party is assumed to be given by[3]

$$V_k = U(y_k, 1) + 2\theta s_k \sqrt{b}, \tag{4}$$

where θ is a weighting factor and b are the perks in office per member of party k if party k is part of the government. The variable s_k denotes the number of party k' s governmental seats. If party k were the sole party in power, i.e. if $s_k = S$, the party's policy would result from the following utility-maximization problem:

$$\max_{t,g,b,d} V_k$$
$$\text{s.t. } tY = (1+\lambda)(gq + s_k b)$$

Accordingly, party k will choose

$$t_k^* = \frac{A}{y_k} + \frac{\theta^2 Y s_k}{y_k^2(1+\lambda)}, \tag{5}$$

$$g_k^* = \frac{YA}{y_k(1+\lambda)q}, \tag{6}$$

$$b_k^* = \frac{\theta^2 Y^2}{y_k^2(1+\lambda)^2}, \tag{7}$$

$$d_k^* = 0. \tag{8}$$

The parties' optimal policy with regard to the public good corresponds to the preferred policy of the respective platform voter. The tax rate, however, is higher than voter y_k's preferred tax rate, since the party finances perks for its members in government.

Finally, there is a protest or extreme party E with platform $(y_E, -1)$ and utility

$$V_E = U(y_E, -1) + 2\theta s_E \sqrt{b}.$$

The extreme party would like to implement $\bar{d}$. If the extreme party could form a government, it would choose $\bar{d}$ and $t_E^*(y_E), g_E^*(y_E), b_E^*(y_E)$. The latter correspond to equations (5)-(7) evaluated at income level y_E. We assume that policy d is important to voters in the sense that a voter with $\delta_i = -1$ is better off with the optimal choice of party E than with any choice of the conventional parties, independently of the income level. In turn, voters with $\delta_i = 1$ are better off with any of the optimal choices of the conventional parties than with the optimal choice of E.

[3] The rationale for this utility function is as follows: Suppose party k consists of S^k members with political preferences $U(y_k, 1)$ and s_k members additionally enjoy utility $2\tilde{\theta} s_k \sqrt{b}$ from perks when the party is part of the government. Then aggregate utility for party k is given by $S^k U(y_k, 1) + 2\tilde{\theta} s_k \sqrt{b}$. Dividing by $S^k$and defining $\theta = \tilde{\theta}/S^k$ yields V_k as in equation (4).

3.3 The legislative game

We allow for parties to commit to tax contracts, i.e. a party can voluntarily restrict its tax policy to an arbitrary range of tax rates. We assume that there is an independent institution that enforces the contracts. If the party violates the contract, the institution imposes penalties that are sufficiently large to ensure that violation never occurs.[4] Penalties may take the form of reduction or elimination of party financing, or even exclusion from government. We consider the following game structure:

Stage 1: Contract choice

Stage 2: Election

Stage 3: Government formation

With respect to the contract choice stage, our analysis proceeds in two steps. First, as a benchmark, we examine the standard case where parties cannot sign contracts, i.e. in which stage 1 is omitted. Second, we allow parties to sign tax contracts. At the election stage, we assume that voters vote sincerely, i.e. they vote for the party whose policy would yield the highest utility if it were the sole party in power. That is, a voter supports the party whose representative voter is closest to him.[5] Finally, at the third stage a government forms. It is assumed that only conventional parties can form a government, i.e. party E is excluded from the government formation process. This is justified by the indivisibility of the policy option d and the role of ideology as a constraint on bargaining by parties, which is a prominent theme and important argument in political economy (see Mueller (2003), Benabou (2008)). The defining characteristic of party E is the policy $d = \bar{d}$. Hence, for ideological reasons a compromise in this political dimension is not possible for the extreme party.

We turn next to the formalization of government. There are several ways of modelling coalition formation in multi-party democracies with proportional election schemes. Our procedure is based on the following premises: First, the party with the highest number of seats is called upon to form a coalition. This is usually done by the head of the

[4] In the event of catastrophe or crisis, contracts may be renegotiated or cancelled, if all parties in parliament vote for such a proposal.

[5] For a recent empirical justification of the sincere voting assumption in elections, see Degan and Merlo (2007).

state.[6] Second, the coalition-forming party will have an advantage in coalition negotiations.[7] We capture this advantage by allowing the coalition-forming party to approach a potential coalition partner and to propose a specific tax rate. Third, actual negotiations on how to spend government revenues are based on the amount of seats a party has gained in the election.[8] Fourth, bargaining should be efficient for the coalition partners. As we have two independent policy dimensions, we allow a party to reject a tax offer and to opt for comprehensive bargaining where t, g, and perks are determined simultaneously. This ensures that bargaining outcomes are efficient.[9] Taking all four premises together yields the following government formation process:

The party that has received the highest vote share is recognized as the first formateur where ties are broken by fair randomization. Then the government forms according to the following process:

Stage 3.1 The formateur chooses a potential coalition partner and suggests a tax rate t_F.

Stage 3.2a) If the coalition partner accepts t_F:
Bargaining: Parties maximize

$$\sigma_k V_k + \sigma_j V_j$$

over $(g, b_j, b_k, d = 0)$ subject to the budget constraint

$$t_F Y = (1 + \lambda)(gq + s_j b_j + s_k b_k),$$

where $\sigma_k = \frac{s_k}{s_k + s_j}$ represents party k's share of seats within the coalition.

Stage 3.2b) If the coalition partner does not accept t_F:
Bargaining: Parties maximize

$$\sigma_k V_k + \sigma_j V_j$$

[6]See Mueller and Strom (2000) for recent examples in western Europe.

[7]There is ample evidence that being the first-mover in coalition negotiations tends to be advantageous (see e.g. Ansolabehere et al. (2005))

[8]This is a standard feature of coalition bargaining processes (see Bandyopadhyay and Chatterjee (2006), Austen-Smith and Banks (2005) and Roemer (2001))

[9]As we have two independent policy dimensions, e.g. t and g, where perks are residually determined by the budget constraint, efficiency would not otherwise be guaranteed.

over $(t, g, b_j, b_k, d = 0)$ subject to the budget constraint

$$tY = (1 + \lambda)(gq + s_j b_j + s_k b_k).$$

Stage 3.3 Vote of confidence. The proposed government coalition is elected if it receives a majority of votes in parliament.

If the vote of confidence fails, the party with the second-highest vote share becomes the new formateur and the process described above repeats itself. If every party has been the formateur once and all votes of confidence have failed, a "caretaker government" assumes power. The caretaker government consists of S bureaucrats and implements policy vector $(t_{ct}, g_{ct}, b_{ct}, d_{ct}) = (\frac{A}{y_{ct}} + \frac{\theta^2 YS}{y_{ct}^2(1+\lambda)}, \frac{YA}{y_{ct}(1+\lambda)q}, \frac{\theta^2 Y^2}{y_{ct}^2(1+\lambda)^2}, 0)$. In other words, the caretaker government acts like a single-party government of bureaucrats with the political orientation y_{ct}. We assume that the parties allocate the seats in government in proportion to their seats in parliament. That is, if parties k and j with parliamentary seats s_k^p and s_j^p form a government, the seats are allocated as follows:[10]

$$s_k = \frac{s_k^p}{s_k^p + s_j^p} S \ ,$$
$$s_j = \frac{s_j^p}{s_k^p + s_j^p} S \ .$$

The rationale for the specification of stage 3.2 in the government formation process is that the two potential coalition partners bargain over policy and that in this process their bargaining power corresponds to their share of seats in government. We allow the formateur to propose restriction of bargaining to a certain tax rate, in order to account for exit options on the part of its potential coalition partner.

3.4 Elections

The voters' preferences are distributed in such a way that there are two possible election outcomes. With probability p_e the extreme party exceeds the vote-share threshold required to enter parliament[11], whereas the middle party M does not. The resulting distribution of seats in parliament will be $(s_R^p, s_L^p, 0, s_E^p)$, where $s_j^p > 0,\ j \in \{R, L, E\}$. With complementary probability $1 - p_e$, party M will obtain positive legislative weight but the extreme party will not. The parliament then consists of $(s_R^p, s_L^p, s_M^p, 0)$.[12]

[10] For convenience, we allow s_k and s_j to be real numbers.

[11] Many parliaments stipulate a vote-share threshold a party must achieve in order to enter parliament.

[12] Examples of such voter distributions are available upon request.

We assume that $s_M^p < s_j^p$, $j \in \{R, L\}$ and – to simplify the exposition – that $s_R^p = s_L^p$ as well as $s_E^p = s_M^p$. The latter ensures that the number of parliamentary seats for the large parties are the same in both situations, i.e. regardless of whether the extreme party enters parliament or not. Whenever the extreme party obtains seats in parliament, a majority of the conventional parties can only be achieved by a grand coalition.

4 Government Formation without Tax Contracts

We start with the benchmark case where no tax contracts are allowed and the first stage of the legislative game is absent. Recall that voters vote sincerely and choose the party whose representative voter's ideal point is closest to theirs. We start with the outcome produced when comprehensive bargaining takes place in 3.2b.

Lemma 1

Suppose party k is the formateur and offers a coalition to party j. Then the bargaining outcome in stage 3.2b is

$$t_{kj}^b = \frac{A}{y_{jk}} + \frac{\theta^2 Y}{(1+\lambda)y_{jk}^2}(s_j\sigma_j^2 + s_k\sigma_k^2), \tag{9}$$

$$g_{kj}^b = \frac{YA}{q(1+\lambda)y_{jk}}, \tag{10}$$

$$b_{k,kj}^b = \frac{\sigma_k^2\theta^2 Y^2}{(1+\lambda)^2 y_{jk}^2}, \tag{11}$$

$$b_{j,kj}^b = \frac{\sigma_j^2\theta^2 Y^2}{(1+\lambda)^2 y_{jk}^2}, \tag{12}$$

where $y_{jk} = \frac{s_j y_j + s_k y_k}{s_j + s_k}$.

The proof follows from the specification of the objective function of the coalition and the budget constraint expressed in stage 3.2b of the government-formation process. For later use, we note that the bargaining problem in stage 3.2b of the legislative game can also be solved in two steps. In a first step, parties choose g, b_j, b_k for a given budget tY. That is, they determine the optimal allocation of resources to the public good and the perks for a given tax rate t. The solution of this optimization problem is unique and independent of the coalition partners' platforms y_k and y_j. Comparing (10) and (11), we observe that the relation between the amount of the public good provided and the parties' perks per person is independent of the available budget and given by

$$g = \frac{A\sqrt{b_k}}{\sigma_k \theta q} = \frac{A\sqrt{b_j}}{\sigma_j \theta q}. \tag{13}$$

Inserting this equation into the budget constraint, we obtain

$$tY = (1+\lambda)(qg + \left(\frac{\theta q}{A}\right)^2 (s_j \sigma_j^2 + s_k \sigma_k^2) g^2). \tag{14}$$

This defines a bijection between taxes and public-good provision, $t_{kj} : \mathcal{R}_+ \rightarrow \mathcal{R}_+$, where the indices k and j indicate the coalition partners.[13] Use $g_{kj}(t)$ to denote the inverse function, that is, the amount of the public good for a given tax rate. Using (13), we can now define the functions

$$b_{k,kj}(t) = \left(\frac{\theta q \sigma_k}{A}\right)^2 (g_{kj}(t))^2, \tag{15}$$

$$b_{j,kj}(t) = \left(\frac{\theta q \sigma_j}{A}\right)^2 (g_{kj}(t))^2. \tag{16}$$

Hence the policy vector in stage 3.2b of a coalition of conventional parties $(t, g, b_k, b_j, 0)$ is fully characterized by the tax rate t and by the relation between the coalition parties' governmental seats. The coalition maximizes the objective function with respect to t using the solution of the first problem, $g_{kj}(t), b_{j,kj}(t), b_{k,kj}(t)$.[14]

We next characterize the outcome in stage 3.2a.

Lemma 2

(i) Suppose party k has offered a coalition and t_F to party j. If party j accepts, the bargaining outcome is $(t_F, g_{kj}(t_F), b_{j,kj}(t_F), b_{k,kj}(t_F))$.

(ii) With a given budget $t_F Y$, a grand coalition provides a higher amount of the public good than a small coalition, and a small coalition provides higher levels of the public good than a single-party government: $g_{RL}(t_F) > g_{kM}(t_F) > g_k^*(t_F)$; $k \in \{R, L\}$ and $g_k^*(t_F)$ denote public-good provision by a single-party government for given t_F.

The proof of lemma 2(ii) can be found in the appendix. In accordance with lemma 2(i), we can abbreviate a policy outcome in stages 3.2a and 3.2b to (t, kj) and denote the

[13] I.e. the mapping between t and g differs with the distribution of governmental seats, σ_k and σ_j.

[14] With the previous notation this problem writes as
$\max_t \ \sigma_k V_k(t, kj) + \sigma_j V_j(t, kj) = Aln[g_{kj}(t)] - t(\sigma_k y_k + \sigma_j y_j) + 2\theta(s_k \sigma_k \sqrt{b_{k,kj}(t)} + s_j \sigma_j \sqrt{b_{j,kj}(t)})$.

value of a policy for a party k by $V_k(t, kj)$. If the identity of the parties is irrelevant, we may also denote the policy as (t, sc) or (t, gc) for a small and a grand coalition respectively and will do so in the following when convenient. Further, let $V_k(ct)$ be k's value of a caretaker government.

We next determine the tax rate t_F. In general, an agent's problem in finding the most preferred tax rate of a coalition of parties k and j can be witten as

$$\max_t V_h(t, kj) = Aln[g_{kj}(t)] - ty_h + 2\theta s_h \sqrt{b_{h,kj}(t)},$$

where $s_h > 0$ if $h \in \{k, j\}$ and is zero otherwise, i.e. if h were a voter or a party that is not part of the government.

Before we discuss equilibrium government formation without tax contracts, we can state the following lemma:

Lemma 3

There always exists a value of t such that $V_k(t, kj) > V_k(ct)$ and $V_j(t, kj) > V_j(ct)$, $k, j \in \{R, M, L\}$.

The proof of lemma 3 can be found in the appendix. Intuitively, in a coalition government the parties enjoy perks and a higher level of public-good provision than under a caretaker government. This is a direct consequence of lemma 2(ii).

The equilibrium policy outcomes without tax contracts depend on the coalition that assumes power. This in turn depends on the coalitions that are feasible. Two situations may arise that differ according to whether the extreme party is represented in parliament (referred to as event E) or not (denoted by ¬E).

Suppose ¬E occurred. Then either L or R will be the first formateur. The formateur will choose party M as a partner because the large parties' relative weight in the bargaining process is higher and the political positions between either of the large parties and the middle party are closer. Accordingly, a large party can achieve a higher utility with M than with the other large party.

In equilibrium, the first formateur $F \in \{R, L\}$ will suggest the tax rate

$$\begin{aligned} t^{nc}_{FM} &= \text{argmax}_t V_F(t, FM) \\ & \quad s.t.\ V_M(t, FM) = \max\{V_M(t^b_{FM}, FM), V_M(t^b_{jM}, jM), V_M(ct)\}, \end{aligned}$$

where $j \in \{R, L\}, j \neq F$ and the superscript "nc" indicates the situation without tax contracts. At the second stage, M accepts the formateur's proposal and bargaining starts over g, b_j, b_k, then followed by a successful vote of confidence. Accordingly, in situation ¬E the policy outcome will be (t^{nc}_{LM}, LM) or (t^{nc}_{RM}, RM) with a probability of $\frac{1}{2}$ each.

In accordance with our earlier assumption only a grand coalition will be feasible in the situation where the extreme party is part of the parliament. In this case, the first formateur offers the other large party a tax rate

$$t^{nc}_{RL} = \begin{cases} \text{argmax}_t V_j(t, RL) \\ s.t. V_F(t, RL) \geq V_F(ct), & \text{if } \exists k \in \{L, R\} : V_k(ct) > V_k(t^b_{RL}, RL), \\ \text{argmax}_t V_F(t, RL) \\ s.t. V_j(t, RL) \geq V_j(t^b_{RL}, RL), & \text{else,} \end{cases}$$

which will be accepted by the potential coalition partner.[15] Consequently, the policy (t^{nc}_{LR}, LR) will be realized.

We summarize the preceding observations in the following proposition:

Proposition 1
If the extreme party E does not possess legislative weight, the policy outcome will be either (t^{nc}_{LM}, LM) or (t^{nc}_{RM}, RM), each with a probablity of $\frac{1}{2}$. If the extreme party is part of the parliament, a grand coalition with a policy vector (t^{nc}_{LR}, LR) will materialize.

5 Government Formation with Tax Contracts

5.1 General considerations

We will now analyse the game where the parties can voluntarily sign tax contracts. By signing contracts, each party k restricts its tax policy to an interval $\tau_k = [\underline{t}_k, \bar{t}_k]$ to which the party is committed if it is part of the government. A tax contract $\tau_k = [0, 1]$ is equivalent to signing no contract. We assume the following:

Assumption 1
For each party k, $t^*_k \in \tau_k$.

[15] Note that whenever one party favors a caretaker government over the comprehensive bargaining outcome, the last formateur can implement his most preferred tax rate, subject to the condition that the coalition partner will not be worse off than with a caretaker government. In anticipation, the first formateur will already offer this tax rate at the first stage.

This assumption implies that the parties' ideal policy if they were the sole party in power is feasible under the tax contract. The assumption can be justified either by the role of the party basis (the party rank and file would not accept an exclusion of the party's ideal point), or by ideological reasons as discussed earlier.[16] As a consequence of assumption 1 and sincere voting, tax contracts do not affect vote shares and the number of seats in parliament. This keeps the focus of the analysis on government formation.

5.2 Contract choice

We assume that, at the first stage of the legislative game, the contracts are chosen sequentially:

Stage 1.1: With probability 0.5, party $k \in \{R, L\}$ signs a contract containing a choice τ_k.

Stage 1.2: Party $j \in \{R, L\}$, $j \neq k$ signs τ_j.

Stage 1.3: Party M chooses τ_M.

The fact that the contract announcement game starts with the large parties first simplifies our analysis.[17] We neglect the contract choice of party E, as it does not affect the game played by the other parties.

The sequential structure of the contract choice game can be justified as follows: First, a simultaneous game seems rather unrealistic, as tax contracts have to be adopted at party conventions. Even if tax contracts had to be signed simultaneously, there will be earlier discussions within parties about which tax contract to sign and how to adjust to the proposals of other parties. Second, as the equilibria in the sequential game are also equilibria in a simultaneaous game (though by no means the only ones), the outcomes of the sequential game could be interpreted as an equilibrium refinement to the simultaneous game.

[16] A further reason outside the model could be that otherwise the extreme party would become too strong because voters at the margins become dissatisfied with the conventional parties.

[17] It would not affect our main results if all three parties had the same chance of being the first to sign a contract.

5.3 Equilibria with tax contracts

We will now proceed to solve the legislative game by backward induction. Consider the last stage of the legislative game. The only difference in the outcomes with tax contracts is that they satisfy the additional constraint that the tax rate must be within the intersection of the contracted tax sets. In other words, with tax contracts the comprehensive bargaining outcome of parties k and j in stage 3.2b is the solution to the following problem:

$$\begin{aligned} &\max_{t,g,b_j,b_k} \sigma_k V_k + \sigma_j V_j \\ &\text{s.t. } tY = (1+\lambda)(gq + s_j b_j + s_k b_k), \\ &\quad t \in \tau_k \cap \tau_j. \end{aligned} \tag{17}$$

For stage 3.2a, when a tax rate $t_F \in \tau_k \cap \tau_j$ has been offered by the formateur, parties k and j maximize with respect to g, b_j, b_k. As the vote shares in the election are not influenced by the parties' contract choices, we can directly move to the first stage of the legislative game where the parties decide on their tax contracts. We assume the following tie-breaking rule:

Assumption 2
If a party is indifferent between two contracts τ_k^1 and τ_k^2, it will choose the contract with the smaller range. E.g. if $|\tau_k^1| < |\tau_k^2|$, then τ_k^1 is chosen. If $|\tau_k^1| = |\tau_k^2|$, each contract will be selected with a probability of 0.5.

We next introduce some notation and three conditions that will be crucial for the type of equilibria that can occur. The definitions and conditions are given formally in appendix A.3.

In the following, the index $k \in \{L, R\}$ stands for the party that will sign its tax contract first and $j \in \{L, R\}$, $j \neq k$ for the party moving second. We use $t^*_{h,co}$, $h \in \{R, M, L\}$, $co \in \{sc, gc\}$ to denote the most preferred tax-rate of party h, given that it is part of a (small or large) coalition. That is, $t^*_{h,co}$ is the tax rate that maximizes the utility of h, under the condition that the coalition, co, maximizes (17) given this tax rate. For example, $t^*_{M,sc}$ stands for the most preferred tax rate of the middle party when it is part of a small coalition. The most interesting cases for our analysis are societies that exhibit a degree of political polarization captured by the following assumption:

Assumption 3
The difference between the incomes of the representative voters of the parties is such that the parties' most preferred tax rates – given that they are part of a coalition government – satisfy $t^*_{R,sc} \leq t^*_{M,sc} \leq t^*_{L,sc}$ *and* $t^*_{R,gc} \leq t^*_{L,gc}$.[18]

Further, it can be shown that $t^*_{M,sc} < t^*_M$, as will become clear in section 6. We introduce particular tax rates. We use $t^*_{M,(sc)}$ to denote the bargaining outcome constrained by $\tau_M = [t^*_{M,sc}, t^*_M]$ between M and a formateur who is either L or R. Note that $t^*_{M,(sc)}$ may differ from $t^*_{M,sc}$ when L is the formateur and forms a coalition with M.

With regard to party k's decision problem at the first stage of the contract-choice game, let t_{kj} be the most preferred tax rate of party k in a grand coalition with j, given that j will include this tax rate in its tax contract. We use the superscript $-$, (t^-_{kj}), to indicate whether t_{kj} is in the interval $[t^*_k, t^*_{M,sc})$, i.e. t_{kj} is closer to k' s ideal policy t^*_k than $t^*_{M,sc}$, and $+$, (t^+_{kj}), whenever it is in $[t^*_j, t^*_{M,sc})$, i.e. t_{kj} is farther from k' s ideal policy than $t^*_{M,sc}$.

We use t'_j to denote the most preferred tax rate of party j within a given tax set τ_k, such that k enjoys at least as much utility as from a caretaker government. Further, for a given tax contract of k, t_{jM} is the most preferable tax rate for party j in a small coalition to ensure that M is still willing to coalesce with j rather than with k.

With this notation we can distinguish the following conditions: To sharpen intuition, some conditions are illustrated for the case where $k = R$ and $j = L$.

Condition C1: For all $t \in [t^*_k, t^*_{M,sc}]$, $V_j(t, RL) < V_j(ct)$.

This condition states that party j will prefer a caretaker government to a grand coalition with a policy characterized by a tax rate that is farther from t^*_j than $t^*_{M,sc}$.

Condition C2: If k chooses $[t^*_k, t^-_k]$, where $t^-_k \in [t^*_k, t^*_{M,sc})$, party j will choose $[t^*_j, t^-_k]$ instead of $[t^*_j, t_{jM}]$.

This means that party j will value certain participation in a small coalition with $t^*_{M,(sc)}$ in event $\neg$E and a grand coalition with t^-_k in E more than certain participation in a small coalition with t_{jM} in $\neg$E but a caretaker government in E.

[18] As will become clear later, in the case of symmetric vote shares for the large parties, the latter condition follows directly from $y_R \geq y_L$.

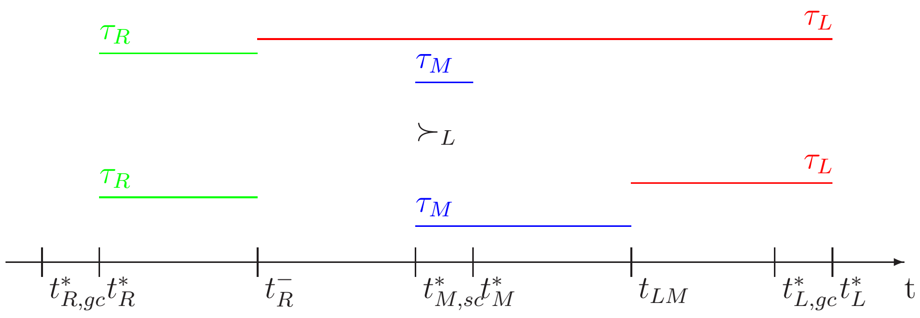

Figure 1: Illustration of Condition C2

The label $\succ_L$ means that the set of tax contracts at the top is preferred by party L to the set of tax contracts below $\succ_L$.

Condition C3: Party k prefers $[t^*_k, t^*_{M,sc}]$ to $[t^*_k, t^-_{kj}]$.

This means that k's expected utility from potential participation in a small coalition with $t^*_{M,sc}$ in ¬E and a grand coalition with $t^*_{M,sc}$ in E[19] is higher than the utility from a small coalition of j and M with $t^*_{M,(sc)}$ in event ¬E and a grand coalition with t^-_{kj} in E.

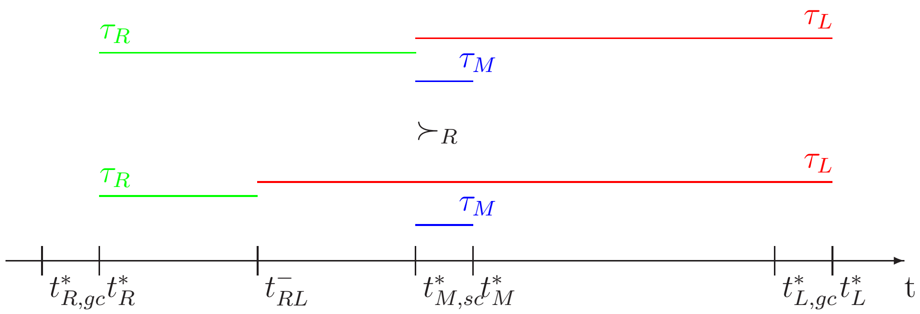

Figure 2: Illustration of Condition C3

The label $\succ_R$ means that the set of tax contracts at the top is preferred by party R to the set of tax contracts below $\succ_R$.

We begin our characterization of the equilibrium tax-contract choices with the following lemma:

[19] To be more precise, there will be a grand coalition with t'_j where $t'_j = t^*_{M,sc}$ if $V_k(t^*_{M,sc}, RL) \geq V_k(ct)$ and $t'_j \neq t^*_{M,sc}$ otherwise.

Lemma 4

If the extreme party does not enter parliament (event ¬E) and $\tau_R \cap \tau_L \neq \emptyset$*, the equilibrium outcome will be a small coalition.*

The proof can be found in the appendix. The lemma expresses that even if a grand coalition is possible in event ¬E, the formateur will prefer a small coalition. With this lemma and the above conditions, we can now characterize the equilibria where the label "u" denotes equilibrium choices.

Proposition 2

There are three types of subgame-perfect Nash equilibria:

1.

$$\begin{aligned} \tau_k^u &= [t_k^*, t_{kj}^+], \\ \tau_j^u &= [t_j^*, t_{M,sc}^*], \\ \tau_M^u &= [t_M^*, t_{M,sc}^*], \end{aligned}$$

if and only if (C1).

2.

$$\begin{aligned} \tau_k^u &= [t_k^*, t_{M,sc}^*], \\ \tau_j^u &= [t_j^*, t_L'], \\ \tau_M^u &= [t_M^*, t_{M,sc}^*], \end{aligned}$$

if and only if ¬(C1) and (C3) or ¬(C1) and ¬(C2).

3.

$$\begin{aligned} \tau_k^u &= [t_k^*, t_{kj}^-], \\ \tau_j^u &= [t_j^*, t_{kj}^-], \\ \tau_M^u &= [t_M^*, t_{M,(sc)}^*], \end{aligned}$$

if and only if ¬(C1), (C2), and ¬(C3).

The proof is given in the appendix. The intuition of the equilibria can be summarized as follows:

If the value of a caretaker government to party j is sufficiently high, j would prefer a caretaker government in the event E to a grand coalition with a tax rate farther away from its ideal point than $t^*_{M,sc}$. Hence, party k includes $t_{kj} > t^*_{M,sc}$ into its tax set, as it will be better off with a grand coalition government with t_{kj} than with a caretaker government according to lemma 3. Party j, however, still includes $t^*_{M,sc}$ in τ_j in order to be able to form a small coalition with M in the event ¬E[20] and equilibrium 1 realizes. The equilibrium choices are illustrated in figure 3.

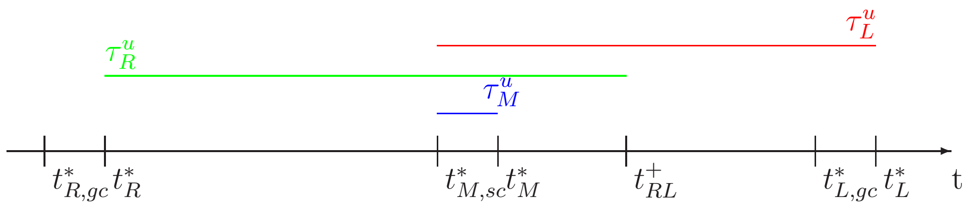

Figure 3: Illustration of Equilibrium 1

If a caretaker government possesses a small enough value for j, then either equilibrium 2 or 3 will be realized. Which of the two finally occurs depends on the tradeoff reflected in C3.

If C3 holds, a grand coalition with $t^*_{M,sc}$ in E and potential participation in a small coalition with $t^*_{M,sc}$ in ¬E creates higher value for party k than a more favorable policy in a grand coalition in E but no possibility to form a small coalition in ¬E. This yields equilibrium 2. In the opposite case (i.e. C3 does not hold), equilibrium 3 will be realized. Note that (C2) must also hold to ensure the existence of equilibrium 3. For an illustration of Equililibrium 3, see the situation depicted in figure 2 below the preference sign $\succ_R$.

Given the tax-set $\tau_k = [t^*_k, t^*_{M,sc}]$, party k will induce a caretaker government by denying the vote of confidence if $\tau_j = [t^*_j, t^*_{M,sc}]$ and $V_k(t^*_{M,sc}, RL) < V_k(ct)$. Hence, in equilibrium 2, we have $t'_j \neq t^*_{M,sc}$ if $V_k(t^*_{M,sc}, RL) < V_k(ct)$ and $t'_j = t^*_{M,sc}$ otherwise.[21] The tax-set choices in equilibrium 2 with $t'_j = t^*_{M,sc}$ are illustrated in figure 2 above the

[20]Note that j always has the possibility of inducing a caretaker government by rejecting a coalition agreement in stage 3 when the vote of confidence takes place. Hence k cannot gain by reducing its contract to $[t^*_k, t^*_{M,sc}]$.

[21]Of course we must also have ¬ (C1) for equilibrium 2 to come about.

preference sign. The figure below depicts the case $t'_j \neq t^*_{M,sc}$.

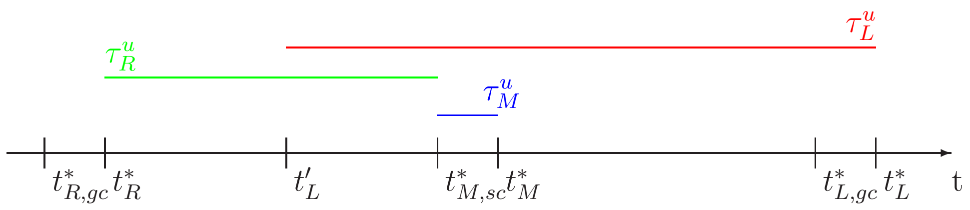

Figure 4: Illustration of Equilibrium 2

From the proposition we can immediately obtain

Corollary 1

In an equilibrium with tax contracts, a caretaker government will never assume power.

The set of equilibria can be further reduced by the following assumption.

Assumption 4

For any tax-rate $t \in [t^*_R, t^*_L]$, the large parties will value a grand coalition more highly than a caretaker government.

We obtain

Corollary 2

Suppose assumption 4 holds. Then,

(i) only equilibrium 2, where $t'_j = t^*_{M,sc}$, and equilibrium 3 can occur.

(ii) for each of the large parties, there exists a critical value $\hat{p}_{e,k}$, such that if k is the first to choose a tax contract, equilibrium 2 will be realized for $p_e < \hat{p}_{e,k}$ and equilibrium 3 for $p_e \geq \hat{p}_{e,k}$.

(iii) $\hat{p}_{e,k}$ is decreasing in the distance between y_k and y_M for given y_M and y_j.

The reasoning in corollary 2 is as follows: As any coalition is better than a caretaker government, condition C1 is violated, which rules out equilibrium 1. Further, assumption 4 implies that $V_k(t^*_{M,sc}, RL) > V_k(ct)$ and hence $t'_j = t^*_{M,sc}$.

As equilibrium 2 will come about if C3 holds, $\hat{p}_{e,k}$ is defined as the value of p_e for which condition ¬(35) in appendix A.3 holds with equality. This is equivalent to the condition that C3 holds for $p_e < \hat{p}_{e,k}$ but not for $p_e \geq \hat{p}_{e,k}$.

Finally, if the distances between the platforms of the large parties and party M increase, a grand coalition with a policy characterized by $t^*_{M,sc}$ becomes less attractive for the large parties relative to their own best choice t^-_{kj}. Hence it becomes more attractive to forgo participation in a small coalition with $t^*_{M,sc}$ and to opt for a more favorable policy in a grand coalition. Hence the critical value $\hat{p}_{e,k}$ declines.[22]

Corollary 2 also implies that from an ex ante perspective, that is, before the first mover in the contract-choice game is known, the probability that equilibrium 2 will be realized is 1 if $p_e < \min_{k \in \{R,L\}} \hat{p}_{e,k}$, 0.5 if $\hat{p}_{e,h} > p_e > \hat{p}_{e,n}$, $h, n \in \{R, L\}$, $h \neq n$, and 0 if $p_e > \max_{k \in \{R,L\}} \hat{p}_{e,k}$.

We summarize the policy outcomes in the next proposition:

Proposition 3
The equilibria at the contract-choice stage lead to the following policy outcomes of the legislative game:

Equilibrium 1:	¬ *E:*	$(t^*_{M,sc}, sc)$
	E:	(t^+_{kj}, gc)
Equilibrium 2:	¬ *E:*	$(t^*_{M,sc}, sc)$
	E:	(t'_L, gc)
Equilibrium 3:	¬ *E:*	$(t^*_{M,(sc)}, sc)$
	E:	(t^-_{kj}, gc)

6 The Effects of Tax Contracts on Policy Outcomes

6.1 The set-up

In this section, we identify two key effects of tax contracts on equilibrium policy outcomes. This can be done most easily in situations where assumption 4 and the following assumption hold:

[22]A formal proof follows directly from differentiating C3 as given in appendix A.3 with respect to y_k. Details are available on request.

Assumption 5

The conventional parties' platforms $(y_j, 1)$ are distributed such that the positions y_R and y_L are symmetric around y_M, i.e. M's utility level in a coalition with R is the same as in a coalition with L.[23]

The two assumptions imply that, according to proposition 2 and corollary 1, the equilibrium policy outcomes with tax contracts will be either k's most preferred implementable tax rate in the event of a grand coalition or M's most preferred implementable tax rate in the event of a small coalition.

The addition 'implementable' refers to the fact that a party's most preferred tax rate may not be the coalition's policy, as due to assumption 1 the party is not entirely free with respect to the choice of its contracted set of tax rates. The constraint imposed by assumption 1 is called the implementation constraint. As will become obvious in the following paragraphs, the most preferred tax rates of a party k, $k \in \{R, L, M\}$ if it is part of a coalition is always lower than if it were the sole party in power, i.e. $t^*_{k,co} < t^*_k$. This implies that there is one situation in which the implementation constraint has bite, which is if R is the first to choose its tax contract and equilibrium 3 comes about, i.e. $\hat{p}_{e,R} < p_e$. In this case, R cannot choose $t^*_{R,gc}$ in event E, while in $\neg$E a small coalition of L and M will implement $t^*_{M,(sc)}$, which differs from $t^*_{M,sc}$. To identify the effects of tax contracts on policy outcomes we abstract from this implementability restriction and will discuss what would change with its inclusion at the end of this section. At this point, we can forestall that the qualitative results remain unaffected.

We can then use $(t^*_{h,\hat{co}}, co)$ to describe the equilibrium outcomes with tax contracts, where $h \in \{R, L, M\}$ and $\hat{co}, co$ are the labels for the type of coalition, i.e. $\hat{co}, co \in \{sc, gc\}$. In order to compare the policy outcomes of the regimes with and without tax contracts, it will be useful to introduce the following problem:[24]

$$\max_{t,g,b_k,b_j} \quad Alng - t(\alpha y_k + (1-\alpha)y_j) + 2\theta(\alpha s_k\sqrt{b_k} + (1-\alpha)s_j\sqrt{b_j}) \tag{18}$$

$$\text{s.t. } tY = (1+\lambda)(gq + s_k b_k + s_j b_j), \tag{19}$$

$$g = \frac{A\sqrt{b_k}}{\theta q \sigma_k}, \tag{20}$$

$$g = \frac{A\sqrt{b_j}}{\theta q \sigma_j}. \tag{21}$$

[23] More formally, this assumption can be written as: The parties' platforms $(y_j, 1)$, $j \in \{L, R\}$ satisfy $V_M(t^b_{RM}, RM) = V_M(t^b_{LM}, LM)$.

[24] The problem could also be written as $\max_t \alpha V_k(t, kj) + (1-\alpha)V_j(t, kj)$.

In this formulation, $\alpha \in [0,1]$ represents the weight given to party k's preferences. For $\alpha = \sigma_k$ the maximization problem is identical to the bargaining problem without tax contracts. For $\alpha = 1$, it reflects k's problem of choosing its most preferred tax rate, given that it is part of a coalition government with j.

In the following, it will be convenient to express the income levels as the relative distance from the average voter's income. For example, party L's representative voter would be characterized by $l_L = \frac{y_L}{\bar{y}}$, where $\bar{y}$ denotes the income of the average voter. Similarly, we define $l_{kj} = \sigma_k l_k + \sigma_j l_j$ and thus have $y_{kj} = (\sigma_k l_k + \sigma_j l_j)\bar{y} = l_{kj}\bar{y}$. We refer to l_k as the political orientation of party k.

With this definition, the solution to the maximization problem can be characterized by the following equation system:

$$\frac{\bar{y}}{Y} = G_{kj}(g;\alpha), \tag{22}$$

$$b_k = \left(\frac{\theta q \sigma_k g}{A}\right)^2, \tag{23}$$

$$b_j = \left(\frac{\theta q \sigma_j g}{A}\right)^2, \tag{24}$$

$$tY = (1+\lambda)(gq + s_k b_k + s_j b_j), \tag{25}$$

where

$$G_{kj}(g;\alpha) = \frac{1}{\alpha l_k + (1-\alpha) l_j} \frac{A^2 + 2\theta^2 gq[(1-\alpha)s_j\sigma_j + \alpha s_k \sigma_k]}{A(1+\lambda)gq} \frac{0.5A^2}{0.5A^2 + \theta^2(s_j\sigma_j^2 + s_k\sigma_k^2)gq}.$$

Recall that $\sigma_k = \frac{s_k}{S}$. We note that the maximization problem is strictly concave and hence possesses a unique solution. This can be verified as follows: Equations (23) and (24) can be interpreted as bijections $b_{k,kj}(g)$ and $b_{j,kj}(g)$ in the positive quadrant. Inserting (23) and (24) into (25) we obtain $t_{kj}(g)$ as already defined in section 4. Hence, as $\frac{\partial G(g;\alpha)}{\partial g} < 0$[25], equation (22) yields a unique optimal amount of the public good, implying that the problem solution is a unique policy vector.[26] The term $G_{kj}(g;\alpha)$ captures the way in which public-good provision is related to the weights placed on the parties' preferences. For later use we also note that the amount of perks per

[25]The verification of this claim is available on request.

[26]Another way to show uniqueness is to examine the second derivative. We use the constraints to write the objective function with only one control, g: $A ln g - \left[(1+\lambda)(gq + \left(\frac{\theta q}{A}\right)^2 (s_k\sigma_k^2 + s_j\sigma_j^2)g^2\right] \frac{\alpha y_k + (1-\alpha)y_j}{Y} + 2\theta^2 \frac{q}{A}(\alpha s_k \sigma_k + (1-\alpha)s_j\sigma_j)g$. The second derivative is $-\frac{A}{g^2} - 2\left(\frac{\theta q}{A}\right)^2 (1+\lambda)\frac{\alpha y_k + (1-\alpha)y_j}{Y}(s_k\sigma_k^2 + s_j\sigma_j^2)$, which is negative for any $g > 0$.

unit of public-good provision is lower in a grand coalition than in a small coalition, $t_{gc}(g) < t_{sc}(g)$. This follows from lemma 2(ii). Intuitively, the lemma's statement that, for a given tax rate, public-good provision is higher in a grand coalition than in a small coalition implies that the amount of perks per unit of public-good provision is higher in a small coalition than in a grand coalition. Consequently, to provide a certain amount of the public good, a small coalition would levy higher taxes than a grand coalition.

6.2 Policy shift and perks effect

The difference in tax rates implemented by a coalition government formed (a) without and (b) with tax contracts can be attributed to two sources. One is the coalition's political orientation. This may differ because tax contracts can give a single party considerable power to influence bargaining in its favor. This we call the policy-shift effect. The other effect subsumes all the differences in tax rates with and without tax contracts that result from the parties' having preferences for perks. That is, it would vanish if θ, the parties' weight on perks in their utility functions, were zero. Accordingly, this effect is referred to as the perks effect. It can be broken down into three sub-effects discussed later.

We will now identify the policy-shift effect and the perks effect using the first-order conditions of problem (18). We start with the situation where p_e is sufficiently large for equilibrium 3 to be realized. In this situation, the perks effect only comprises two sub-effects. Then we introduce the third sub-effect that additionally arises in equilibrium 2. Thereafter we will give a formal definition of the policy-shift effect and the perks effect and discuss their direction.

Suppose p_e is high enough for equilibrium 3 to be realized. The difference between the policy outcomes without and with tax contracts can be identified by the differences between $G_{kj}(g;\sigma_k)$ and $G_{kj}(g;1)$. In particular,

$$G_{kj}(g;\sigma_k) = \frac{1}{l_{kj}} \frac{A}{(1+\lambda)qg} \stackrel{!}{=} \frac{\bar{y}}{Y}, \tag{26}$$

$$G_{kj}(g;1) = \frac{1}{l_k} \frac{A}{(1+\lambda)qg} \underbrace{\frac{0.5A^2 + s_k\sigma_k\theta^2 gq}{0.5A^2 + s_k\sigma_k^2\theta^2 gq} \left(1 - \frac{s_j\sigma_j^2\theta^2 gq}{0.5A^2 + s_j\sigma_j^2\theta^2 gq + s_k\sigma_k^2\theta^2 gq}\right)}_{P(g,\sigma_k)} \stackrel{!}{=} \frac{\bar{y}}{Y}. \tag{27}$$

From equations (26) and (27) we can identify two reasons why policy outcomes with tax contracts may differ from those without. First, without contracts the political orientation of a coalition government is a convex combination between the two coalition partners, e.g. l_{kj}. With tax contracts, it is that of a single party, e.g. l_k. We refer to this difference as the **policy-shift effect** of tax contracts. The second difference arises from the term $P(g, \sigma_k)$ as defined in equation (27) and is called the **perks effect**. Using (23), the first quotient of $P(g, \sigma_k)$ can be rewritten as

$$\frac{A + 2s_k\theta\sqrt{b_k}}{A + 2s_k\sigma_k\theta\sqrt{b_k}}. \tag{28}$$

This term reflects the relation between the weight that party k attaches to perks in its utility function and the weight given to its perks in coalition bargaining. Hence, this term is always greater than 1. That is, ceteris paribus party k would like to implement a higher tax rate than the coalition government without tax contracts in order to finance more perks. This is the first sub-effect of the perks effect. The second sub-effect is reflected in the second multiplier of $P(g, \sigma_k)$, which is always smaller than 1. It reflects a negative externality, as with the budget for a coalition government, party k has to finance the coalition partner's perks although these are not part of its utility. The first force becomes dominant if σ_k is high, as the negative externality then becomes small. The opposite occurs when σ_k is small. This is expressed in the following lemma which shows that forces balance at $\sigma_k = 0.5$.

Lemma 5
$P(g, \sigma_k) < 1$, *if* $\sigma_k \in (0, 0.5)$
$P(g, \sigma_k) = 1$, *if* $\sigma_k = 0.5$
$P(g, \sigma_k) > 1$, *if* $\sigma_k \in (0.5, 1)$

The proof is given in the appendix. Lemma 5 implies that if we compare a small coalition formed under tax contracts with one formed without tax contracts and if we suppose that both had the same political orientation (in our case l_M), then the amount of public-good provision is lower with tax contracts than without, as $\sigma_M < 0.5$. The reason is that the (negative) externality dominates the (positive) under-representation of perks in $P(g, \sigma_k)$.

We now consider the situation where p_e is sufficiently small for equilibrium 2 to be realized. Here we observe the third sub-effect on perks. It results from the different amounts of perks associated with public-good provision in small and grand coalitions. To precisely identify this sub-effect, let us denote the bargaining solution of (26) as a function of the coalition's political orientation: $g^b(l)$, where $g^b(l_{kj}) = g^b_{kj}$. Further, the solution to (27) is referred to as $g_{k,kj}$. In equilibrium 2, a grand coalition implements a tax rate $t^*_{M,sc}$ that can be written as $t_{sc}(g_{M,sc})$. Although a grand coalition without tax contracts and political orientation l_M would implement $g^b(l_M) > g_{M,sc}$, this does not necessarily involve $t_{gc}(g^b(l_M)) > t_{sc}(g_{M,sc})$. As indicated previously, the reason is that, in a grand coalition, the relationship of perks per unit of public-good provision is lower than in a small coalition. Accordingly, this sub-effect is only present in equilibrium 2 when a grand coalition is formed.

Next we give a formal definition of the effects. For this purpose, let us write $t_{kj}(l)$ for $t_{kj}(g^b(l))$.

Definition 1
Let $t^*_{h,co}$, $h \in \{R, M, L\}$ be the resulting tax rate of a coalition of parties k and j under tax contracts.

Policy-shift effect:	$\Delta t_{ps} = t_{kj}(l_h) - t^{nc}_{kj}$
Perks effect:	$\Delta t_{pr} = t^*_{h,co} - t_{kj}(l_h)$

Note that $k, j, h \in \{R, M, L\}$ and $k \neq j$. To illustrate the effects, consider the case of a grand coalition in equilibrium 2. The tax rate in the equilibrium with tax contracts would be $t^*_{M,sc}$. Consequently, $h = M$. A grand coalition with political orientation l_M would like to implement $t_{RL}(l_M)$. As in a grand coalition $P(g, \sigma_k) = 1$ (see lemma 5), the first two sub-effects of the perks effect cancel each other out. The remaining (third) sub-effect is captured by $t^*_{M,sc} - t_{RL}(l_M)$. Clearly, as with a given coalition kj the tax rate maps one-to-one into the corresponding public-good provision and the corresponding amount of perks via $g_{kj}(t), b_{j,kj}(t), b_{k,kj}(t)$, the effects can also be formulated in the latter dimensions.

Intuitively, the policy-shift effect represents the difference between the tax rate without tax contracts and the comprehensive bargaining outcome of that coalition if it had a political orientation l_h. As, due to assumption 4, the policy of a coalition formed without tax contracts is the comprehensive bargaining outcome, the policy-shift effect would vanish for $l_h = l_{kj}$. The perks effect captures the difference between the actual policy of a government formed under tax contracts and the comprehensive bargaining outcome of this coalition if it had the political orientation of party h. This difference comprises the three sub-effects described above and would vanish if θ were zero.

Concerning the signs of the effects, we know from section 4 that the bargaining solution of a coalition with a political orientation geared more towards the rich would provide fewer public goods and vice versa. With tax contracts, a single party has more power, but it still is the case that $\frac{\partial g_{k,kj}}{\partial l_k} < 0$. Hence, whether the policy-shift effect is positive or negative depends on the direction of the shift, i.e. towards which party's preferences the policy shifts. According to our previous observations, the perks effect can also go both ways.

Proposition 4
The perks effect is negative in event $\neg E$. In E, it is zero if equilibrium 3 is realized and ambiguous in its sign if equilibrium 2 occurs.

A detailed look at the cases can be summarized as follows: Independent of p_e, in small coalitions a policy shift towards M's preferences occurs and – as $\sigma_M < 0.5$ – there is a negative perks effect. There are differences between equilibria 2 and 3 with respect to the situation where a grand coalition comes about. In equilibrium 2, the policy shift is still towards the center. However, the perks effect can be positive due to a different relationship between public-good provision and perks in small and grand coalitions. If p_e is sufficiently high for equilibrium 3 to occur, a grand coalition implies only a policy shift to either the left or right and no perks effect due to symmetric seat shares.

From an ex-ante perspective, we can say that the perks effect tends to be negative. The reason is that the perks effect is at least weakly negative in all situations except one. Only in a grand coalition in equilibrium 2 will the perks effect not necessarily be positive, and the probability of this situation arising (p_e), must be low, because otherwise equilibrium 3 would come about, and the perks effect will be weakly negative again. Hence in the expectation operator the weight attached to a possibly positive perks effect is low.

6.3 Discussion

The policy-shift effect and the perks effect have been identified in a particular setting. In this section we discuss whether the definition is robust for more general settings and how a relaxation of assumptions 5 and 4 influences the sizes of the effects.

First, we address the question whether definition 1 also captures the effects if we relax assumption 5. For example, we assume that k's platform is farther away from M's than that of j so that M's utility in a small coalition with j is higher than with k. We answer this question in the affirmative. In event ¬E, even the sizes of the effects should remain the same. To see this, suppose the extreme party did not enter parliament and party k became the first formateur. In the regime without tax contracts, which tax rate would k offer to M? Due to the parties' single peaked preferences on tax rates if a small coalition is realized, there will be a unique tax rate that maximizes k's utility under the restriction that M enjoys at least the utility level of a small coalition with j. This tax rate must be identical with the bargaining outcome if k's platform were symmetric with j's around M. Hence the size of the policy-shift effect would not change in this situation.

Second, we ask whether the implementation restriction has any influence on the previously identified effects. A comparison of the policy of a single-party government of party k with the most preferred policy of k in a coalition government makes it clear that $t^*_{k,gc} < t^*_k$.[27] This means that the implementation restriction only has influence in equilibrium 3 if party R is the first to choose its tax contract. Then R would not be able to implement $t^*_{R,gc}$ in a grand coalition but only t^*_R, and a small coalition would implement $t^*_{M,(sc)}$ instead of $t^*_{M,sc}$. Using definition 1, the implementation restriction would only affect the perks effect but not the policy-shift effect. The direction is clear: the magnitude of a negative perks effect will become smaller (or the effect may even turn positive), and a positive perks effect will increase. Hence we conclude that the sum of the policy-shift effect and the perks effect is smaller in a situation where the implementation restriction has bite.

Third, suppose that assumption 4 does not hold, i.e. that for some tax rates at least one of the large parties prefers a caretaker government to a grand coalition. This implies that equilibrium 1 and equilibrium 2 with $t'_j \neq t^*_{M,sc}$ could occur and that $t^-_{kj} \neq t^*_{k,gc}$. The major difference to the setting with assumption 4 is that policy shifts towards the left and right in equilibrium 3 are no longer possible. The simple reason is that such a policy shift must give both coalition partners a higher utility than a caretaker government, otherwise the vote of confidence would fail.[28] This implies that

[27]Comparing the first-order condition of a single-party government (6) with (27) reveals that $g_k \geq g_{k,co}$. From a comparison of the budget constraints we obtain

$$(1+\lambda)(gq + s_k(\theta q \sigma_k g/A)^2 + s_j(\theta q \sigma_j g/A)^2)/Y = t_{kj}(g) < t_k(g) = (1+\lambda)(gq + s_k(\theta q g/A)^2)/Y$$

for a given value of g. These two arguments taken together yield $t^*_{k,co} < t^*_k$.

[28]In fact, a comparison between the equilibrium policies in a grand coalition formed with and without tax contracts in the situation where the caretaker government is more favorable for one of the large parties and $\exists k \in \{R, L\}$ with $V_k(t^b_{RL}, RL) < V_k(ct)$ reveals that $t_{kj} = t^{nc}_{RL}$ if j is the first

the policies of a grand coalition government formed with or without tax contracts show little if any differences. However, in event ¬E, the policy-shift effect and the perks effect will remain unchanged.

7 Welfare Comparison

In this section we examine the consequences of tax contracts for the aggregate welfare of voters measured by a utilitarian welfare function:

$$W = |\mathcal{I}| \int_{y_{min}}^{y_{max}} (Alng - ty) f(y) dy.$$

For a given policy vector $(t, g, b, 0)$, this transforms to

$$W = A \ln g \, |\mathcal{I}| - tY \;\Leftrightarrow\; W/|\mathcal{I}| = Alng - t\bar{y},$$

where $\bar{y}$ is the average income in the economy. Hence, we can take the utility of the average-income voter as a measure of overall welfare.[29] His most preferred policy if a coalition of parties k and j comes about is therefore characterized by

$$\frac{\bar{y}}{Y} = \frac{A}{(1+\lambda)qg} \underbrace{\frac{0.5A^2}{0.5A^2 + s_k\sigma_k^2\theta^2 gq}\left(1 - \frac{s_j\sigma_j^2\theta^2 gq}{0.5A^2 + s_j\sigma_j^2\theta^2 gq + s_k\sigma_k^2\theta^2 gq}\right)}_{P_v(g,\sigma_k)}, \tag{29}$$

and (23), (24), and (25). Note that in the first-order condition of the voter's problem $P_v(g, \sigma_k) < 1$, as perks are weighted too heavily in governmental bargaining over and against his preference for zero perks. Hence the average-income voter would like a lower level of public-good provision than a government with the same political orientation (l=1). The immediate upshot of this is

Lemma 6
Only a coalition with a political orientation $l > 1$ can achieve the welfare-optimal level of the public good.

We now inquire into the political constellations (l_R, l_M, l_L) under which government formation with tax contracts yields higher welfare than government formation without tax contracts. Let us again consider situations where assumptions 4 and 5 hold. For

formateur without tax contracts and $t'_j = t^{nc}_{RL}$ if k is the first formateur without tax contracts.

[29] The problem of a voter with $y_i = \bar{y}$ is $\max_{t,g,b_j,b_k} A \ln g - t\bar{y}$ s.t. (23), (24), (25).

the comparison we have to distinguish whether an equilibrium of type 2 or 3 comes about in the regime with tax contracts.

If p_e is not too high and thus equilibrium 2 occurs with tax contracts, these yield higher welfare if and only if

$$\begin{aligned} & p_e U_{\bar{y}}(t^*_{M,sc}, gc) + (1-p_e) U_{\bar{y}}(t^*_{M,sc}, sc) \\ & \quad > p_e U_{\bar{y}}(t^{nc}_{RL}, gc) + \frac{1-p_e}{2}[U_{\bar{y}}(t^{nc}_{RM}, sc) + U_{\bar{y}}(t^{nc}_{LM}, sc)], \end{aligned} \tag{30}$$

where $U_{\bar{y}}(t, co)$ denotes the utility of the average-income voter derived from the policy (t, co). If equilibrium 3 occurs, tax contracts yield higher welfare if and only if

$$\begin{aligned} & \frac{p_e}{2}[U_{\bar{y}}(t^*_{L,gc}, gc) + U_{\bar{y}}(t^*_{R}, gc)] + \frac{1-p_e}{2}[U_{\bar{y}}(t^*_{M,sc}, RM) + U_{\bar{y}}(t^*_{M,(sc)}, LM)] \\ & \quad > p_e U_{\bar{y}}(t^{nc}_{RL}, gc) + \frac{1-p_e}{2}[U_{\bar{y}}(t^{nc}_{RM}, sc) + U_{\bar{y}}(t^{nc}_{LM}, sc)] \end{aligned} \tag{31}$$

holds.[30]

To identify the political constellations (l_R, l_M, l_L) under which these inequalities hold, we define the political distance of the large parties as

$$\Delta_l := l_R - l_L.$$

Δ_l measures political polarization.[31] We obtain

Proposition 5

(i) If $p_e < \min_{k\in\{R,L\}} \hat{p}_{e,k}$, tax contracts will improve welfare for sufficiently large Δ_l.

(ii) If p_e is sufficiently large, tax contracts will lower welfare for large values of Δ_l.

The proof is provided in appendix A.7.

Intuitively, if p_e is small enough for equilibrium 2 to occur with tax contracts (case (i)), such contracts will guarantee a policy characterized by $t^*_{M,sc}$ independently of whether the extreme party enters the legislature. Without contracts, the policies are relatively moderate in a grand coalition, but more to the left or right in small coalitions, depending on how strong the political polarization of the large parties is. Hence for large political differences Δ_l, the average voter's utility from the small coalitions' policies without tax contracts is smaller than with tax contracts.

[30]These conditions also apply without the symmetry assumption for the large parties' platforms. Note that, in equation 31, we have t^*_R instead of $t^*_{R,gc}$ due to the implementation restriction.

[31]Note that for a given position of the middle party, l_M, the positions of R and L are fully specified by Δ_l due to assumption 5.

The result is the opposite in case (ii), where p_e is large enough for equilibrium 3 to occur. Then, in highly polarized societies, we would have widely diverging policies with tax contracts in the case of event E. Although the small coalitions' policies will also become more extreme without contracts, they are more moderate as party M still has a certain say in coalition bargaining. Hence, when p_e is sufficiently large, tax contracts will lower social welfare.

If we relax the symmetry assumption with respect to the parties' platforms, the lowest degree of political polarization that still satisfies assumption 3 would imply $t^*_{M,sc} \approx t^*_{R,sc}$ and $l_L \approx l_M$. We denote this degree of polarization by Δ_l^{min}. In this case, we have

Proposition 6

If $p_e < \min_{k\in\{R,L\}} \hat{p}_{e,k}$, $l_M \leq 1$ and Δ_l close to Δ_l^{min}, tax contracts are welfare-improving for $t^{nc}_{RL} > t^*_{M,sc}$ and otherwise yield (weakly) lower welfare than a regime without tax contracts.

The proof can be found in the appendix.

Intuitively, if political polarization is very low, the perks effect plays a central role. If $p_e < \min_{k\in\{R,L\}} \hat{p}_{e,k}$ equilibrium 2 will occur if tax contracts are allowed. As Δ_l^{min} implies that $g^*_{R,sc} \approx g^*_{M,sc}$, it follows that $g^*_{M,sc} = g^{nc}_{RM} = g^{nc}_{LM}$ implying a tax rate of $t^*_{M,sc}$. Since $l_M \leq 1$, the average voter's preferred level of public-good provision is lower than that desired by the middle party in a small coalition: $g_{\bar{y}} < g^*_{M,sc}$ or $t_{\bar{y}} < t^*_{M,sc}$.[32] However, only if $t^{nc}_{RL} > t^*_{M,sc}$ will the perks effect be sufficiently negative for tax contracts to be welfare-improving. The opposite occurs for $t^{nc}_{RL} < t^*_{M,sc}$. Note that $l_M \leq 1$ is the plausible case, as median income is usually below average income and the middle party's political position is likely to be close to that of the median voter.

8 Numerical Examples

In this section we illustrate our welfare analysis with some numerical examples in order to assess the magnitudes of the effects. We consider situations in which assumptions 4 and 5 hold and use a 'baseline' society, which we examine for different degrees of political polarization and different probabilities of the extreme party entering parliament.

[32] Although it can be shown that for Δ_l close enough to Δ_l^{min}, $g^{nc}_{RL} > g^*_{M,sc}$, this does not necessarily imply $t^{nc}_{RL} > t^*_{M,sc}$ due to the different relation between perks per unit of public-good provision in grand and small coalitions.

The baseline society is characterized by the following parameter values: $A = 10$, $q = 1$, $\lambda = 0.05$, $\theta = 0.005$, $|\mathcal{I}| = 8.\bar{3}$, $Y = 2500$, $l_M = 1$. The relation $Y = \bar{y} * |\mathcal{I}|$ reflects the GDP and GDP per capita of Germany, the other values are chosen ad hoc.

We let $l_M = 1$ and start with an initial l_R value of 1.2. The corresponding political orientation of the left party, l_L, is implied by the symmetry assumption 5, which then yields the initial level of polarization Δ_l. Each iteration increases l_R by 0.25 and the corresponding position of party L is adjusted so that symmetry holds. We consider 50 iterations leading to a maximal degree of polarization of 13.26. We indicate the results for cases where the probability of the extreme party entering parliament is low (8%), medium (33%) and high (75%). As the expected welfare under tax contracts depends on whether equilibrium 2 or 3 will come about, we define the binary variables D_{2R} and D_{2L}, which assume the value 1 if equilibrium 2 occurs when R (L) is the first party to choose its tax contracts, and zero otherwise. The expected welfare of a regime with tax contracts can then be written as

$$\begin{aligned} EW_{tc} \quad := \quad & \frac{p_e}{2}[(D_{2R} + D_{2L})U_{\bar{y}}(t^*_{M,sc}, gc) + (1 - D_{2R})U_{\bar{y}}(t^*_R, gc) + (1 - D_{2L})U_{\bar{y}}(t^*_{L,gc}, gc)] \\ & + \frac{1 - p_e}{2}[(1 + D_{2R})U_{\bar{y}}(t^*_{M,sc}, sc) + (1 - D_{2R})U_{\bar{y}}(t^*_{M,(sc)}, sc)]. \end{aligned}$$

Recall that expected welfare without tax contracts (EW_{nc}) is given by the right-hand sides of (30) and (31). We start with the case where the probability of the extreme party entering parliament is low.

8.1 p_e low

For this case, our analytical results predict that equilibrium 2 will occur, inducing welfare improvements from tax contracts for high degrees of polarization.

As figure 5 illustrates, with tax contracts, both a small coalition and a grand coalition would implement $t^*_{M,sc}$, which is equal to $t^*_{M,sc} = 3.14\%$. Accordingly, the expected welfare level is independent of the degree of political polarization. By contrast, welfare is decreasing without tax contracts since the equilibrium policies are convex combinations of the coalition partners' preferred policies and the distance between them increases with increasing political polarization. In our example, a coalition of R and M would choose a tax rate of 2.6% for the lowest level of polarization and 0.3% for the highest polarization level of 13.26 if no tax contracts can be written. A coalition between L and M starts out at a tax rate of 3.73% for the lowest polarization level and ends up

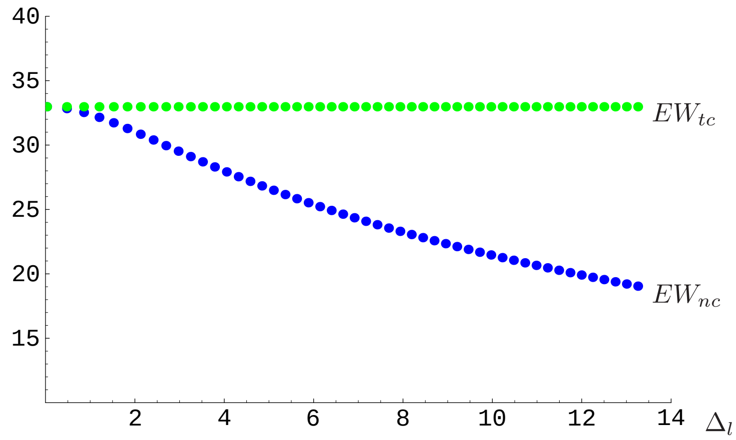

Figure 5: Expected welfare with and without tax contracts when p_e is low

at 11.9% if the degree of polarization is maximal. Accordingly, expected welfare gains of a regime with tax contracts increases with Δ_l as depicted in the upper left part of figure 6.

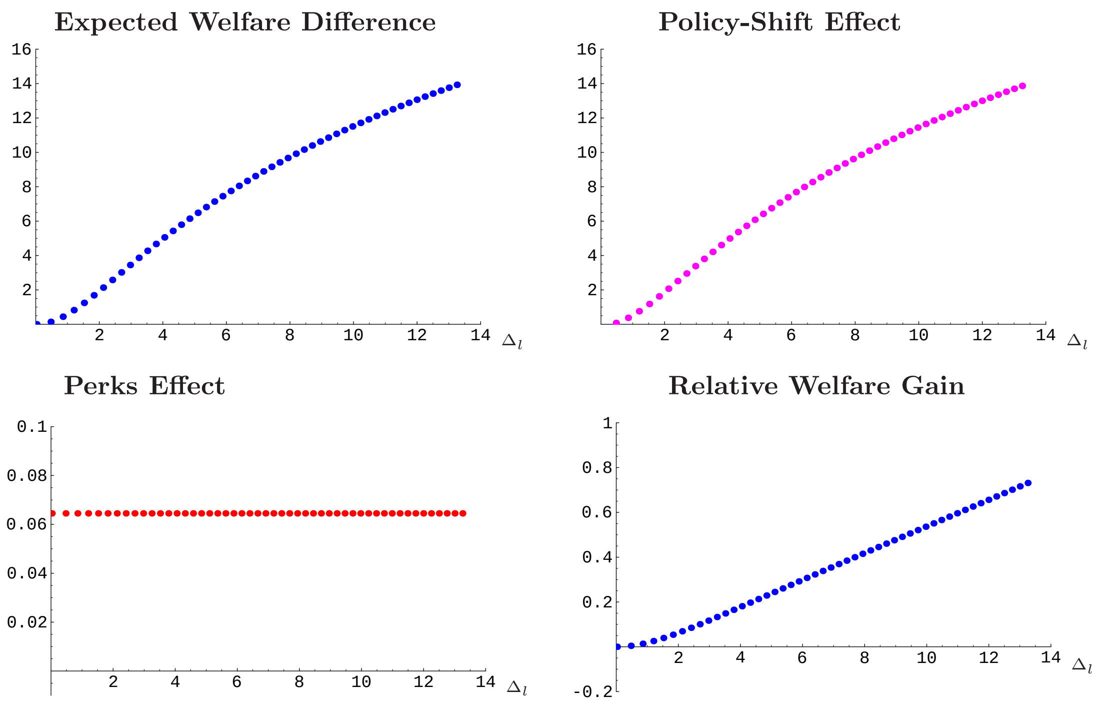

Figure 6: Expected welfare differences between regimes with and without tax contracts when p_e is low

The graph in the upper right part of figure 6 displays the welfare difference associated with the policy-shift effect, while the graph on the lower left displays the expected welfare difference resulting from the perks effect. We observe that the welfare impact of the perks effect is quite low. Only in societies with very small polarization does it match the welfare impact of the policy-shift effect. Finally, the expected welfare gain relative to expected welfare without tax contracts is depicted in the lower right part of figure 6. As our analytical results predict, the relative welfare gains increase with the degree of political polarization. In this particular specification, there are substantial welfare gains of up to 70% if polarization is large when the tax contracts are introduced. The intuition is that the policy-shift towards the middle party will keep policies moderate. By contrast, the absence of tax contracts would make the political orientation of the small coalition deviate widely from the average income when E does not enter.

8.2 p_e very high

We now consider the case $p_e = 75\%$.

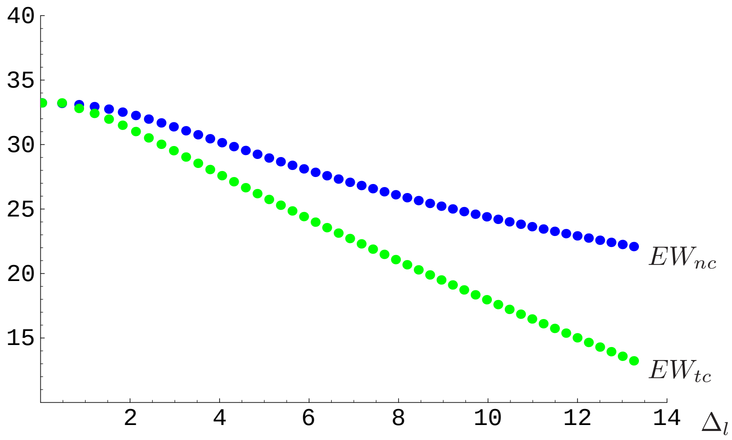

Figure 7: Expected welfare with and without tax contracts when p_e is high

In this case, equilibrium 3 occurs, implying that policies are only moderate in small coalitions but strongly tend to the left or right in grand coalitions. As indicated in figure 7, these policy shifts towards L and R, rather than towards the middle, yield increasing welfare losses with higher degrees of political polarization when tax contracts are introduced.

8.3 p_e medium-sized

Finally we consider an intermediate value of $p_e = 1/3$.

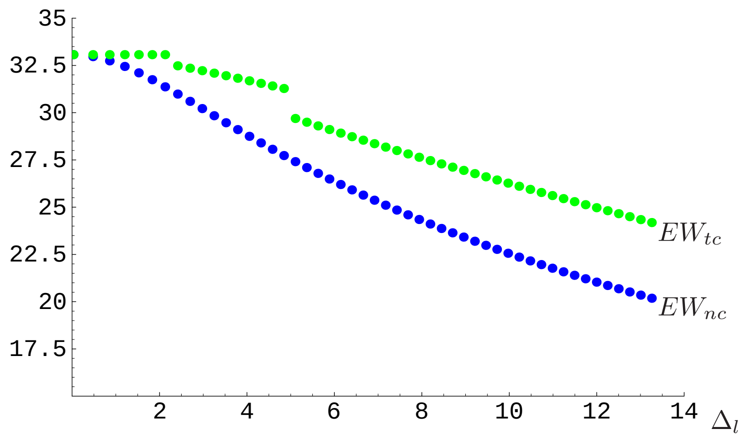

Figure 8: Expected welfare with and without tax contracts when p_e is medium-sized

For low degrees of polarization, the parties play equilibrium 2. As stated in corollary 2, the critical value $\hat{p}_{e,k}$ will fall below $p_e = 1/3$, if polarization increases. In the example, this occurs first for $\hat{p}_{e,L}$. If $\hat{p}_{e,L} < p_e < \hat{p}_{e,R}$, which occurs for degrees of polarization above 2, equilibrium 3 occurs if party L chooses its tax contract first. Otherwise equilibrium 2 obtains. With higher levels of polarization, both $\hat{p}_{e,L}$ and $\hat{p}_{e,R}$ are smaller than $p_e = 1/3$ and equilibrium 3 will be played in all cases. The jumps in expected welfare under tax contracts reflect the switch from one equilibrium to another.

Figure 9 displays the absolute and relative welfare gains obtained by allowing parties to sign tax contracts. At low levels of polarization, the welfare increase with tax contracts is again caused by a moderation of policies due to the policy shift towards the middle in equilibrium 2. At a polarization level of about 2, there is a slight drop in the expected welfare gains because equilibrium 3 occurs when L is the first to sign its tax contract. Finally, for polarization levels greater than 5, only equilibrium 3 is played. Though this causes another drop in expected welfare gains, the expected welfare difference between the regime with and without tax contracts is increasing, as indicated by the upper left graph in figure 9. The reason is that in equilibrium 3, there is a shift towards the middle in small coalitions and this has a positive impact on welfare. The magnitude rises for increasing levels of political polarization, as the weight of event ¬E, $(1 - p_e) = 0.667$, is higher than that of event E. The increase in expected welfare resulting from small

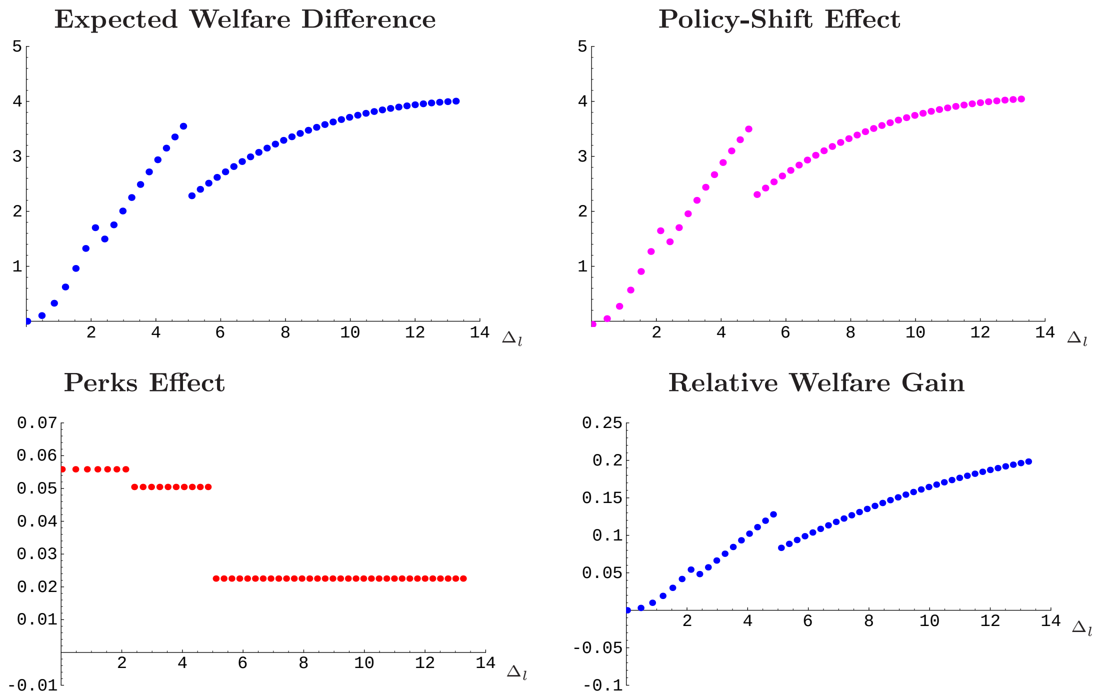

Figure 9: Expected welfare differences between regimes with and without tax contracts when p_e is medium-sized

coalitions formed under tax contracts outweighs the decreases from the policy shifts towards the left or right in grand coalitions under tax contracts.[33]

9 Conclusions

In this paper we have developed a new type of party competition model to examine the role of tax contracts for government formation in a parliamentary democracy. The model involves three conventional parties and a fourth extremist party. If the extremist party enters parliament, it will force the two large conventional parties to form a grand coalition.

We show that allowing parties to sign tax contracts has two effects: a policy-shift effect and a perks effect. The first captures the incentive for a party to influence the policy of the coalition government in favor of its own political preferences by committing to a range of tax rates. The latter effect mainly captures two opposing incentives for a party.

[33]However, for even larger degrees of political polarization than depicted above, the expected welfare difference will decline and eventually turn negative.

On the one hand, it would like to commit to higher tax rates to increase its perks, which from the party's perspective have insufficient weight in coalition bargaining. On the other hand, the coalition partner's perks act as a negative externality to the provision of public goods and hence as an incentive to lower taxes.

From a utilitarian welfare perspective, we find that tax contracts may yield substantial welfare gains in a highly polarized society where the political positions of the right and left party differ widely but the probability of the extremist party entering parliament is not high. In this case, tax contracts are a means of generating moderate political outcomes. However, there may be welfare losses if the probability of the extremist party obtaining seats in parliament is very high, because then the policy shift in grand coalitions is to the left or to the right, rather than towards the middle.

In societies exhibiting a low degree of political polarization, the perks effect may play an important role. Our numerical examples suggest that the relative welfare gains from the perks effect are quite moderate, whereas substantial welfare changes are effected by the policy shifts.

Our model can be extended in several directions. To keep our focus on the effects of tax contracts on government formation, we have assumed that the electorate will vote sincerely. It would be interesting, but not trivial, to extend our model to strategic voting. One might also consider the possibility of tax contracts being conditional on whether E or ¬E occurs. What would happen in this case is explored in appendix B, where we show that such conditional contracts may further improve welfare if political polarization is relatively small.

Appendix

A Conditions and Proofs

A.1 Proof of Lemma 2(ii)

A coalition of parties k and j optimally allocates a given budget $t_F Y$ between public-good provision and perks according to (13). Inserting this relation between public-good provision and perks into the budget constraint and using $s_j = S - s_k$,[34] we obtain

$$t_F Y = (1+\lambda)\left(qg + \left(\frac{\theta q}{AS}\right)^2 (s_k^3 + (S-s_k)^3)g^2\right). \qquad (32)$$

The right-hand side is increasing in g. The term $s_k^3 + (S - s_k)^3$ is strictly convex in s_k and reaches its unique minimum within $[0, S]$ at $s_k = 0.5S$. From the implicit function theorem we hence obtain that public-good provision by coalition governments with a more unequal distribution of seats provides fewer public goods. Accordingly, we have $g_{RL}(t_F) > g_{kM}(t_F)$, $k \in \{R, L\}$. As the optimal solution of a single-party government is reflected in the case $s_k = S$ (i.e., the most unequal distribution of seats), we obtain $g_{kM}(t_F) > g_k^*(t_F)$. □

A.2 Proof of Lemma 3

There are two reasons why potential coalition partners can find a policy vector that yields more utility to both than a caretaker government. Suppose t_{ct} is fixed.

1 A coalition with budget $t_{ct}Y$ would enjoy a higher level of public-good provision than under a caretaker government. As the caretaker government acts as a single-party government of bureaucrats, this follows from lemma 2(ii).

2 Both coalition partners would enjoy a positive amount of perks, whereas in a caretaker government they would receive no perks.

Hence there exists a non-empty set of tax rates that would leave both coalition partners better off than under a caretaker government. □

[34] Note that $s_j = S - s_k$ implies $\sigma_j^2 = (S - s_k)^2/S^2$.

A.3 Formal Definition of Conditions C2 and C3

$$\begin{aligned} t^*_{M,sc} \;&:=\; \arg\max_t V_M \\ &\quad \text{s.t. } \; tY \geq (1+\lambda)(g(t)q + (s_M b_M(t) + s_k b_k(t)), \end{aligned}$$

where $k \in \{R, L\}$.

$$t^*_{M,(sc)} := \begin{cases} t^{bc}_{FM} & \text{, if either } \; \forall t \in \tau_R \cap \tau_L : t > t^*_{M,sc} \text{ and } t^*_M < t^*_{M,sc} \\ & \quad \text{or } \; \forall t \in \tau_R \cap \tau_L : t < t^*_{M,sc} \text{ and } t^*_M > t^*_{M,sc} \\ t^*_{M,sc} & \text{, else.} \end{cases} \tag{33}$$

where

$$t^{bc}_{FM} = \arg \max_{t \in \tau_M \cap \tau_F} \sigma_M V_M(t, FM) + \sigma_F V_F(t, FM),$$

and F is the formateur with $\tau_M \cap \tau_F \neq \emptyset$.

For a given set $\tau_k = [t^*_k, t^-_k]$ with $t^-_k \in [t^*_k, t^*_{M,sc})$:[35]

$$\begin{aligned} t_{jM} &:= \arg\max_t V_j(t, jM) \\ \text{s.t.} V_M(t, jM) &> V_M(t^-_k, kM), \\ V_M(t, jM) &> V_M(ct). \end{aligned}$$

Further, we have

$$\begin{aligned} t'_j \;&:=\; \arg\max_{t \in \tau_k} V_j(t, RL) \\ &\quad \text{s.t. } \; V_k(t, RL) > V_k(ct). \end{aligned}$$

$$\begin{aligned} t^+_{kj} \;&:=\; \arg \max_{t \in [t^*_j, t^*_{M,sc})} V_k(t, RL) \\ &\quad \text{s.t. } \; V_j(t, RL) > V_j(ct). \end{aligned}$$

$$\begin{aligned} t^-_{kj} \;&:=\; \arg \max_{t \in [t^*_k, t^*_{M,sc})} V_k(t, RL) \\ &\quad \text{s.t. } \; \frac{1-p_e}{p_e} < \frac{V_j(t, RL) - V_j(ct)}{V_j(t_{jM}, jM) - V_j(t^*_{M,(sc)}, jM)}. \end{aligned}$$

[35]We are aware that due to the strict inequality constraint and the continuity of $V_M(\cdot, co)$ the maximization problem that defines t_{jM} has no solution. In this paper we understand all inequality constraints in the way that there is an arbitrarily small "grid size" ε.

Note that the last constraint uses equation (34) expressing that j will find a grand coalition with t and a small coalition with $t_{M,(sc)}$ better than a caretaker government in event E and a small coalition with t_{jM} in event ¬E. Conditions C2 and C3 can be formalized as follows:

Condition C2: For a given $t_k^- \in [t_k^*, t_{M,sc}^*]$,

$$\frac{1-p_e}{p_e} < \frac{V_j(t_k^-, RL) - V_j(ct)}{V_j(t_{jM}, jM) - V_j(t_{M,(sc)}^*, jM)}. \tag{34}$$

Condition C3:

$$\frac{1-p_e}{p_e} > \frac{V_k(t_{kj}^-, RL) - V_k(t_L', RL)}{\frac{1}{2}(V_k(t_{M,sc}^*, kM) - V_k(t_{M,sc}^*, jM)) + V_k(t_{M,sc}^*, jM) - V_k(t_{M,(sc)}^*, jM)}. \tag{35}$$

A.4 Proof of Lemma 4

Suppose that event ¬E comes about and party $k \in \{L, R\}$ has been chosen as the first formateur in the government-formation stage of the legislative game. Due to $\tau_R \cap \tau_L \neq \emptyset$ and assumption 1, a grand coalition and a small coalition will be possible. Without loss of generality, assume that k can choose between suggesting a grand coalition or a small coalition and there is one common tax-rate t that can be implemented either with the small party M or the other large party j. Party k will choose M as a coalition partner for two reasons:

1) Party k will occupy more seats in government than in a grand coalition.

2) Bargaining over expenditures (given revenues tY)

$$\sigma_k V_k + \sigma_j V_j$$

over $(g, b_j, b_k, d = 0)$ subject to the budget constraint

$$t_F Y = (1 + \lambda)(gq + s_j b_j + s_k b_k)$$

involves higher weight for k's preferences.

As for a given tax rate a small coalition will yield higher utility for party k than a grand coalition, the only reason for k to suggest a grand coalition would be if the tax rate it can implement in a grand coalition, $\hat{t}_{RL}$, provides a sufficiently greater utility level than the tax rate ($\hat{t}_{kM}$) it can implement in a small coalition, i.e. $V_k(\hat{t}_{RL}, gc) > V_k(\hat{t}_{kM}, sc)$.

This, however, implies that $V_j(\hat{t}_{RL}, gc) < V_j(t_{jM}, sc)$. Consequently, j would not support the vote of confidence and offer a small coalition to M when it is the formateur itself. □

A.5 Proof of Proposition 2

The proof is established by backward induction.

Stage 1.3

Consider party M at the third stage. As τ_R and τ_L are given, M cannot influence the policy in a grand coalition. Hence party M contracts to the set of tax rates that yield the highest utility if ¬ E. M may face two situations: Either $\tau_R \cap \tau_L = \emptyset$ or $\tau_R \cap \tau_L \neq \emptyset$.

If $\tau_R \cap \tau_L \neq \emptyset$, party M knows that any possible set τ_M will intersect with at least one of the large parties' tax-sets, so a small coalition will be feasible. According to lemma 4, a small coalition will result in the case of ¬E and M can contract to its preferred tax rate $t^*_{M,sc}$. Note that, due to assumption 1, it may be the case that in a coalition with party k, if $\{t^*_M, t^*_{M,sc}\} \in \tau_k$, the coalition outcome will not be $t^*_{M,sc}$ but a different tax rate t^{bc}_{FM}.[36] In these situations, M will be indifferent between the sets $[t^*_M, t^*_{M,sc}]$ and $[t^*_M, t^{bc}_{FM}]$ and due to assumption 2 will sign $\tau_M = [t^*_M, t^{bc}_{FM}]$. For $\tau_R \cap \tau_L \neq \emptyset$, we can generally write

$$(t^*_{M,sc} > t^*_M \ \wedge \ \forall t \in \tau_R \cap \tau_L : t > t^*_{M,sc}) \ \vee \ (t^*_{M,sc} < t^*_M \ \wedge \ \forall t \in \tau_R \cap \tau_L : t < t^*_{M,sc}) \\ \rightarrow \ \tau_M = [t^*_M, t^{bc}_{FM}]$$

In all other cases, we have $\tau_M = [t^*_M, t^*_{M,sc}]$.

If $\tau_R \cap \tau_L = \emptyset$, M solves

$$\max_{t, j \in \{L,R\}} V_M(t, Mj) \quad \text{s.t. } t \in \tau_j.$$

Denote the solution by t_{Mj}, where j indicates the preferred coalition partner. If $V_M(t_{Mj}, Mj) > V_M(ct)$, then $\tau_M = [t^*_M, t_{Mj}]$, otherwise, $\tau_M = \{t^*_M\}$.

[36] t^{bc}_{FM} is defined in appendix A.3.

Note that we have neglected the case $[t^*_M, t_{Mk}] \subset \tau_k$ for $k \in \{L, R\}$, as it will never be chosen in equilibrium.

Stage 1.2

Suppose without loss of generality that $k = R$.[37] There are three possible cases for the upper bound of R's tax contract, $\bar{t}_R$: $\bar{t}_R > t^*_{M,sc}$, $\bar{t}_R = t^*_{M,sc}$, or $\bar{t}_R < t^*_{M,sc}$. Anticipating the reaction of M, party L takes the following choices:

1) $\tau_R = [t^*_R, t^*_{M,sc}]$

   In this case, the candidates for best tax sets of L are $\hat{\tau}_L = [t^*_L, t'_L]$ and $\bar{\tau}_L = \{t^*_L\}$, where

$$t'_L \;:=\; \arg\max_{t\in\tau_R} V_L(t, RL) \quad \text{s.t. } V_R(t, RL) \geq V_R(ct).$$

   With $\hat{\tau}_L$, L may be part of a small coalition with $t^*_{M,sc}$ if ¬E and will join a grand coalition with t'_L if E. If $V_L(t^*_{M,sc}, RL) < V_L(ct)$, L would let the vote of confidence fail and hence force a caretaker government into power. Choosing $\bar{\tau}_L$ yields a small coalition of R and M with $t^*_{M,(sc)}$ in event ¬E and a care-taker government in event E. Hence $\bar{\tau}_L$ is dominated by $\hat{\tau}_L$.

2) $\tau_R = [t^*_R, t^+_R]$, where $t^+_R > t^*_{M,sc}$.

   In order to keep the exposition as simple as possible, assume that[38]

$$t^+_R \;:=\; \arg\max_{t\in[t^*_L, t^*_{M,sc}]} V_R(t, RL) \quad \text{s.t. } V_L(t, RL) \geq V_L(ct).$$

   Candidates for best tax contracts are $\hat{\tau}_L = [t^*_L, t^*_{M,sc}]$, $\tau'_L = [t^*_L, t^+_R]$ and $\bar{\tau}_L = \{t^*_L\}$.

[37] Note that, due to the implementation restriction, it may make a difference whether R or L is the first to choose its tax contract. With respect to the generality of the proof, this is only a problem if the direction of the implementation restriction is known, i.e. whether t^*_k is greater or smaller than $t^*_{k,co}$. Hence, although we know that $t^*_{k,co} < t^*_k$, for this proof we neglect this knowledge in order to establish generality in the sense that we do not know whether the first mover R suffers from the implementation restriction. (The transfer to the general case is that we do not know which party will be the first mover at the contract-choice stage and hence we do not know whether the implementation restriction applies to this party.)

[38] This is the only case that occurs in equilibrium. For clarity of exposition, we here omit the out-of-equilibrium responses by L to other choices made by party R.

The coalition outcomes for the different choices are as follows:

set	¬E	E
$\hat{\tau}_L$	$V_L(t^*_{M,sc}, LM)$ or $V_L(t^*_{M,sc}, RM)$	$V_L(t^+_R, RL)$
τ'_L	$V_L(t^*_{M,(sc)}, RM)$	$V_L(t^+_R, RL)$
$\bar{\tau}_L$	$V_L(t^*_{M,(sc)}, RM)$	$V_L(ct)$

The table reveals that L will choose $\hat{\tau}_L$ since it dominates the other choices.

3) $\tau_R = [t^*_R, t^-_R]$, where $t^-_R < t^*_{M,sc}$.

In this case there are three candidates for best responses of L: $\tau'_L = [t^*_L, t'_L]$, $\tau''_L = [t^*_L, t_{LM}]$ and $\bar{\tau}_L = \{t^*_L\}$, where t_{LM} is defined by

$$\begin{aligned} t_{LM} &:= \arg\max_t V_L(t, LM) \\ \text{s.t. } & V_M(t, LM) > V_M(t^-_R, RM), \\ & V_M(t, LM) > V_M(ct). \end{aligned}$$

The outcomes with the different sets are:

set	¬E	E
τ'_L	$V_L(t^*_{M,(sc)}, LM)$	$V_L(t'_L, RL)$, if $V_L(ct) \leq V_L(t'_L, RL)$ $V_L(ct)$, else
τ''_L	$V_L(t_{LM}, LM)$	$V_L(ct)$
$\bar{\tau}_L$	$V_L(t^-_R, RM)$ or $V_L(ct)$	$V_L(ct)$

It is clear that $V_L(t_{LM}, LM) > V_L(t^-_R, RM)$ and due to lemma 3, $V_L(t_{LM}, LM) > V_L(ct)$. Hence $\bar{\tau}_L$ is strictly dominated by τ''_L. Consequently, L will compare the sets τ'_L and τ''_L and decide for τ'_L if

$$\frac{1-p_e}{p_e} < \frac{V_L(t'_L, RL) - V_L(ct)}{V_L(t_{LM}, LM) - V_L(t^*_{M,(sc)}, LM)}. \tag{36}$$

Stage 1.1

At the first stage, R decides on τ_R, anticipating the reactions of L and M. The candidates for optimal tax sets are $\hat{\tau}_R = [t^*_R, t^*_{M,sc}]$, $\tau'_R = [t^*_R, t_{RL}]$ and $\bar{\tau}_R = [t^*_R, \tilde{t}_R]$, where

$$t_{RL} := \begin{cases} t^+_{RL}, & \text{if } \{t^-_R \in [t^*_R, t^*_{M,sc}] : (36)\} = \emptyset, \\ t^-_{RL}, & \text{if } \{t^-_R \in [t^*_R, t^*_{M,sc}] : (36)\} \neq \emptyset, \end{cases}$$

and

$$t^+_{RL} := \arg\max_{t>t^*_{M,sc}} V_R(t, RL)$$
$$\text{s.t. for}\, V_L(t, RL) \geq V_L(ct).$$

$$t^-_{RL} := \arg\max_{t<t^*_{M,sc}} V_R(t, RL)$$
$$\text{s.t. for}\, t = t'_L,\ \ (36).$$

$$\tilde{t}_R := \arg\max_{t<t^*_{M,sc}} V_R(t_{LM}, LM)$$
$$\text{s.t.}\, t = t^-_R,\ \ \neg(36).$$

Consider the case where $t_{RL} > t^*_{M,sc}$, i.e. $t_{RL} = t^+_{RL}$. From the definition of t^+_{RL}, this implies

$$\forall t \in [t^*_R, t^*_{M,sc}],\ V_L(t, RL) < V_L(ct). \tag{37}$$

Using (37), we obtain the outcomes associated with the three tax sets:

set	¬E	E
$\hat{\tau}_R$	$V_R(t^*_{M,sc}, RM)$ or $V_R(t^*_{M,sc}, LM)$	$V_R(ct)$
τ'_R	$V_R(t^*_{M,sc}, RM)$ or $V_R(t^*_{M,sc}, LM)$	$V_R(t^+_{RL}, RL)$
$\bar{\tau}_R$	$V_R(t_{LM}, LM)$	$V_R(ct)$

As $V_R(t^*_{M,sc}, LM) > V_R(t_{LM}, LM)$, $\bar{\tau}_R$ is strictly dominated by $\hat{\tau}_R$. Further, R will prefer τ'_R to $\hat{\tau}_R$ if

$$V_R(t^+_{RL}, RL) > V_R(ct)$$

This inequality must hold due to lemma 3. Hence, if $t_{RL} > t^*_{M,sc}$, R will choose τ'_R.

If $t_{RL} < t^*_{M,sc}$, i.e. $t_{RL} = t^-_{RL}$. Then, the three tax sets yield

set	¬E	E
$\hat{\tau}_R$	$V_R(t^*_{M,sc}, RM)$ or $V_R(t^*_{M,sc}, LM)$	$V_R(t'_L, RL)$
τ'_R	$V_R(t^*_{M,(sc)}, LM)$	$V_R(t^-_{RL}, RL)$
$\bar{\tau}_R$	$V_R(t_{LM}, LM)$	$V_R(ct)$

As $V_R(t'_L, RL) \geq V_R(ct)$, $\hat{\tau}_R$ dominates $\bar{\tau}_R$. A comparison of $\hat{\tau}_R$ with τ'_R leads to R adopting $\hat{\tau}_R$ over τ'_R if

$$\frac{1-p_e}{p_e} > \frac{V_R(t^-_{RL}, RL) - V_R(t'_L, RL)}{\frac{1}{2}(V_R(t^*_{M,sc}, RM) - V_R(t^*_{M,sc}, LM)) + V_R(t^*_{M,sc}, LM) - V_R(t^*_{M,(sc)}, LM)}. \tag{38}$$

To sum up, we obtain the following types of equilibria:

1.

$$\begin{aligned} \tau^u_R &= [t^*_R, t^+_{RL}], \\ \tau^u_L &= [t^*_L, t^*_{M,sc}], \\ \tau^u_M &= [t^*_M, t^*_{M,sc}]. \end{aligned}$$

if (37).

2.

$$\begin{aligned} \tau^u_R &= [t^*_R, t^*_{M,sc}], \\ \tau^u_L &= [t^*_L, t'_L], \\ \tau^u_M &= [t^*_M, t^*_{M,sc}]. \end{aligned}$$

if ¬(37) and (38) or ¬(37) and ¬(36).

3.

$$\begin{aligned} \tau^u_R &= [t^*_R, t^-_{RL}], \\ \tau^u_L &= [t^*_L, t^-_{RL}], \\ \tau^u_M &= [t^*_M, t^*_{M,(sc)}]. \end{aligned}$$

if ¬(37), (36) and ¬(38).

Note that the equilibrium conditions correspond to the formal conditions given in appendix A.3. □

A.6 Proof of Lemma 5

Here we show that for all $g > 0$, $P(g, \sigma_k) < 1$ if $\sigma_k < 0.5$, $P(g, 0.5) = 1$ and $P(g, \sigma_k) > 1$ if $\sigma_k > 0.5$.

Recall that

$$P(g, \sigma_k) = \frac{0.5A^2 + s_k \sigma_k \theta^2 gq}{0.5A^2 + s_j \sigma_j^2 \theta^2 gq + s_k \sigma_k^2 \theta^2 gq}.$$

Suppose $\sigma_k < 0.5$. Then

$$s_k < \frac{S}{2}. \tag{39}$$

Using $s_j = S - s_k$, we obtain

$$s_k \sigma_k < s_j \sigma_j. \tag{40}$$

This implies that

$$0.5A^2 + s_k \sigma_k \theta^2 gq < 0.5A^2 + s_j \sigma_j^2 \theta^2 gq + s_k \sigma_k^2 \theta^2 gq$$

and consequently $P(g, \sigma_k) < 1$. The results for $\sigma_k = 0.5$ and $\sigma_k > 0.5$ can be derived in the same way. □

A.7 Proof of Proposition 5

Consider case (i), where p_e is small enough for equilibrium 2 to be realized in the tax contracts regime.

Using (30), we can define the welfare difference as follows:

$$\begin{aligned} \Delta W^{unoc2} &= p_e U_{\bar{y}}(t^*_{M,sc}, gc) + (1 - p_e) U_{\bar{y}}(t^*_{M,sc}, sc) \\ &\quad - \left[p_e U_{\bar{y}}(t^{nc}_{RL}, gc) + \frac{(1 - p_e)}{2} (U_{\bar{y}}(t^{nc}_{RM}, sc) + U_{\bar{y}}(t^{nc}_{LM}, sc)) \right]. \end{aligned}$$

An increase in polarization affects the welfare difference according to

$$\begin{aligned} \frac{\partial \Delta W^{unoc2}}{\partial \Delta_l} = \frac{d\Delta W^{unoc2}}{dl_R} &= -p_e \left(\frac{\partial U_{\bar{y}}(t^{nc}_{RL}, gc)}{\partial l_R} + \frac{\partial U_{\bar{y}}(t^{nc}_{RL}, gc)}{\partial l_L} \frac{dl_L}{dl_R} \right) \\ &\quad - \frac{(1 - p_e)}{2} \left(\frac{\partial U_{\bar{y}}(t^{nc}_{RM}, sc)}{\partial l_R} + \frac{U_{\bar{y}}(t^{nc}_{LM}, sc)}{\partial l_L} \frac{dl_L}{dl_R} \right). \end{aligned}$$

As due to assumption 5 $\frac{dl_L}{dl_R}$ must be smaller than 0, the welfare difference between a tax contracts regime and one without tax contracts will increase if

$$\frac{\partial U_{\bar{y}}(t_{RL}^{nc}, gc)}{\partial l_R} + \frac{\partial U_{\bar{y}}(t_{RL}^{nc}, gc)}{\partial l_L}\frac{dl_L}{dl_R} < 0 \ ,$$
$$\frac{\partial U_{\bar{y}}(t_{RM}^{nc}, sc)}{\partial l_R} < 0 \ , \ \frac{U_{\bar{y}}(t_{LM}^{nc}, sc)}{\partial l_L} > 0 \ .$$

With

$$\frac{\partial U_{\bar{y}}(t_{RM}^{nc}, sc)}{\partial l_R} = \left[\frac{A}{g_{RM}^{nc}} - \frac{\partial t(g_{RM}^{nc})}{\partial g_{RM}^{nc}}\right]\frac{\partial g_{RM}^{nc}}{\partial l_R},$$

the condition

$$\frac{\partial U_{\bar{y}}(t_{RM}^{nc}, sc)}{\partial l_R} < 0$$

implies

$$A > \frac{\partial t(g_{RM}^{nc})}{\partial g_{RM}^{nc}} g_{RM}^{nc}. \tag{41}$$

The right-hand side is a strictly increasing quadratic function in g_{RM}^{nc}. As g_{RM}^{nc} is declining with l_R, condition (41) will be satisfied for sufficiently large values of l_R.

Using the same steps as above, the condition

$$\frac{\partial U_{\bar{y}}(t_{LM}^{nc}, sc)}{\partial l_L} > 0$$

can be written as

$$A < \frac{\partial t(g_{LM}^{nc})}{\partial g_{LM}^{nc}} g_{LM}^{nc}. \tag{42}$$

Since g_{LM}^{nc} is decreasing with l_L, it is guaranteed that the condition will be satisfied for small enough l_L.

Finally, we determine how the policy under a grand coalition changes with increasing political polarization. We can write

$$\frac{dU_{\bar{y}}(t_{RL}^{nc}, gc)}{dl_R} = \left[\frac{A}{g_{RL}^{nc}} - \frac{\partial t(g_{RL}^{nc})}{\partial g_{RL}^{nc}}\right]\frac{\partial g_{RL}^{nc}}{\partial l_R}\left(1 + \frac{dl_L}{dl_R}\right).$$

In our setting, we must have $\left|\frac{dl_L}{dl_R}\right| < 1$, which implies that l_{RL} increases with Δ_l and that g_{RL}^{nc} consequently decreases. For

$$\frac{dU_{\bar{y}}(t_{RL}^{nc}, gc)}{dl_R} < 0$$

to hold, we require

$$A > \frac{\partial t(g_{RL}^{nc})}{\partial g_{RL}^{nc}} g_{RL}^{nc}. \tag{43}$$

As g^{nc}_{RL} decreases in Δ_l, the condition (43) will hold for a sufficiently large degree of political polarization. Hence, tax contracts are socially preferable for sufficiently large political polarization Δ_l.

The comparison is similar but more involved in case (ii), where p_e is large enough for equilibrium 3 to occur in the regime with tax contracts. The argument can be most easily demonstrated for high values of p_e.

Suppose that p_e is arbitrarily close to 1 and thus equilibrium 3 occurs. As the probability of ¬E is negligible, we can concentrate on what happens for event E. With high polarization, the policy outcome of a grand coalition under tax contracts is much more extreme than without tax contracts. Hence for sufficiently high p_e and sufficiently high polarization tax contracts are less socially desirable than a regime without tax contracts.[39]

A.8 Proof of Proposition 6

The condition $p_e < \min_{k\in\{R,L\}} \hat{p}_{e,k}$ guarantees that equilibrium 2 will occur under a regime with tax contracts, thus producing a policy characterized by $t^*_{M,sc}$ in both a small and a grand coalition. Δ_l being close enough to Δ_l^{min} implies that $t^*_{R,sc} \approx t^*_{M,sc}$ and consequently that $t^*_{M,sc} \approx t^{nc}_{RM} \approx t^{nc}_{LM}$. Hence the policies with and without tax contracts coincide in event $\neg E$. Policy differences occur for event E. In E, the grand coalition implements $t^*_{M,sc}$ with tax contracts, whereas in the absence of tax contracts will produce the bargaining outcome t^{nc}_{RL}.

Since $l_M \leq 1$, the average voter's preferred tax rate is lower than that of the middle party's platform voter: $t_{\bar{y}} < t_{y_M}$. From equation (29) we deduce that $t_{y_M} < t^*_{M,sc}$. Whether government-formation with tax contracts will yield a higher level of welfare than without tax contracts depends on whether $t^*_{M,sc} < t^{nc}_{RL}$.[40]

[39] A formal analysis of the argument is available on request.

[40] Note that although it can be shown that $g^*_{M,sc} < g^{nc}_{RL}$ for $\Delta_l \approx \Delta_l^{min}$, this does not imply $t^*_{M,sc} < t^{nc}_{RL}$. The reason is that the amount of perks associated with the provision of one unit of the public good by a grand coalition is lower than it would be with a small coalition, as stated in lemma 2(ii).

B Extension – Conditional Tax Contracts

In this appendix, parties are allowed to condition their tax contracts on event E. We denote the contracted sets of tax-rates by the index e (event E occurs) and ne (event ¬E occurs), respectively. We will first derive the equilibria and then discuss their welfare implications.

B.1 Equilibria with Conditional Tax Contracts

To derive the equilibria we can draw upon proposition 2. We first consider the equilibrium where $p_e \approx 0$, which produces the contract choice if E does not occur. As corollaries 1 and 2 suggest, this is the type 2 equilibrium. Setting $p_e = 1$ produces the contract choice conditional on E. This would correspond to equilibria of type 1 or type 3, depending on the value of the caretaker government. Hence we obtain

Proposition 7
With conditional tax contracts, the subgame-perfect Nash equilibria are characterised by

$$\begin{aligned}\tau_k^{ne} &= [t_k^*, t_{M,sc}^*], \tau_k^e = [t_k^*, t_{kj}^e],\\ \tau_j^{ne} &= [t_j^*, t_{M,sc}^*], \tau_j^e = [t_j^*, t_{kj}^e],\\ \tau_M^{ne} &= [t_M^*, t_{M,sc}^*], \tau_M^e = \{t_M^*\},\end{aligned}$$

where $k, j \in \{L, R\}$ and k is the party that chooses its tax-set in the first stage. t_{kj}^e is defined as

$$\begin{aligned}t_{kj}^e \quad := \quad & \arg\max_{t \in [t_k^*, t_j^*]} V_k(t, RL)\\ & s.t. \;\; V_j(t, RL) \geq V_j(ct).\end{aligned}$$

A more formal proof can be found in appendix B.3. The outcomes of the legislative game are summarized in the next proposition.

Proposition 8
The legislative game with conditional tax contracts yields the following policy outcomes:

¬ *E:* $(t_{M,sc}^*, sc)$
E: (t_{kj}^e, RL).

B.2 Welfare Consequences of Conditional Tax Contracts

We next compare conditional tax contracts with unconditional tax contracts in terms of expected welfare and then discuss when they are more favorable than government formation without tax contracts. Again, we restrict our analysis to situtions where assumption 4 holds.

In order to compare the two kinds of tax contract, we have to distinguish whether equilibrium 2 or 3 occurs in the case with unconditional tax contracts. In equilibrium 2, both the small coalition and the grand coalition will agree on $t^*_{M,sc}$, whereas with conditional tax contracts this tax-rate will only be implemented by a small coalition. If equilibrium 3 occurs with unconditional tax contracts, then – abstracting from the implementation restriction of party M in a small coalition with L – the equilibrium outcomes under both types of tax contracts are the same.[41]

Consequently, our comparison focuses on situations where equilibrium 2 is realized with unconditional tax contracts. In this case, the two types of tax contract yield the same outcome in event ¬E. In E, however, there will be a grand coalition characterized by $t^*_{M,sc}$ in the unconditional tax-contract equilibrium and a probability of $\frac{1}{2}$ for a grand coalition either with $t^*_{L,gc}$ or with t^*_R in the case of conditional tax contracts.[42] Hence unconditional tax contracts would be more favorable from a welfare perspective if

$$U_{\bar{y}}(t^*_{M,sc}, gc) > \frac{1}{2}U_{\bar{y}}(t^*_{L,gc}, gc) + \frac{1}{2}U_{\bar{y}}(t^*_R, gc), \tag{44}$$

We thus obtain

Proposition 9

For small degrees of polarization, conditional tax contracts may be socially preferable to unconditional tax contracts. If political polarization is sufficiently large, unconditional tax contracts will yield higher welfare than conditional tax contracts.

The proof is given in the appendix.

Intuitively, if equilibrium 2 comes about, unconditional tax contracts will induce moderate policies even if the degree of political polarization increases. This is different

[41]Note that the implementation restriction would make conditional tax contracts socially preferable if p_e were large enough for equilibrium 3 to occur and if $l_M \leq 1$. In the case of l_M being sufficiently larger than 1, the result would be the opposite.

[42]Note that due the implementation restriction a grand coalition with R the first to sign its tax contract yields an outcome t^*_R instead of $t^*_{R,gc}$.

in the case of conditional contracts, since the large parties will take the opportunity to implement their preferred policies when the extreme party enters. Hence unconditional tax contracts are socially preferable over conditional tax contracts if polarization is sufficiently large.

With proposition 9 and welfare comparisons of policies by coalition governments formed (a) without tax contracts and (b) under unconditional tax contracts, we can now conclude that a situation where conditional tax contracts may be favorable is one in which both the probability of the extreme party entering parliament and political polarization are low. The reason is as follows: Depending on the position of the middle party, it might be the case that equilibrium 3 is welfare-better than equilibrium 2 and also welfare-better than the outcome without tax contracts. However, with p_e sufficiently low in a regime with unconditional tax contracts, equilibrium 2 would come about.

B.3 Proof of Proposition 7

As in the proof of proposition 2, we use backward induction. The difference now is that the parties choose two tax contracts conditional on events E and $\neg E$.

Stage 1.3

In event E, party M will not be part of the parliament. Hence it is indifferent with respect to the set of contracted tax rates and chooses $\tau_M^e = \{t_M^*\}$ according to assumptions 1 and 2.

In event ¬E, the situation is identical to that in the unconditional case and we can hence adopt both the same line of argument and the resulting optimal sets by just adding the index ne, i.e. $\tau_M^{ne} = [t_M^*, t_{M,sc}^*]$.

Stage 1.2

For given sets $\tau_k^e = [t_k^*, t_k^e]$, $\tau_k^{ne} = [t_k^*, t_k^{ne}]$, party j chooses the following conditional tax sets:

– Event ¬E:

if $t_k^{ne} \in [t_k^*, t_{M,sc}^*)$, j signs the set $\tau_j^{ne} = [t_j^*, t_{jM}]$, where

$$\begin{aligned} t_{jM} &:= \arg\max_t V_j(t, jM) \\ \text{s.t. } V_M(t, jM) &> V_M(t_k^{ne}, kM), \\ V_M(t, jM) &> V_M(ct). \end{aligned}$$

if $t_k^{ne} = t_{M,sc}^*$, j chooses the set $\tau_j^{ne} = [t_j^*, t_{M,sc}^*]$ and

if $t_k^{ne} \in [t_j^*, t_{M,sc}^*)$, it is also optimal to contract for the set $\tau_j^{ne} = [t_j^*, t_{M,sc}^*]$.

– Event E: Party j has to decide whether to choose set $\hat{\tau}_j^e = [t_j^*, t_k^e]$ or $\bar{\tau}_j^e = \{t_j^*\}$. It will select $\hat{\tau}_j^e$ if and only if

$$V_j(t_k^e, RL) > V_j(ct).$$

Stage 1.1

Knowing how the other parties will react, party k will choose the set $\tau_k^{ne} = [t_k^*, t_{M,sc}^*]$ since otherwise it will not with certainty be part of the government in event ¬E. Conditional on E, k will choose the set $\tau_k^e = [t_k^*, t_{kj}^e]$, where

$$\begin{aligned} t_{kj}^e &:= \arg\max_t V_k(t, RL) \\ \text{s.t. } V_j(t, RL) &\geq V_j(ct). \end{aligned}$$

That is, k chooses a tax contract for event E that will have elements in common with the tax contract chosen by j for this event. This is due to lemma 3

□

B.4 Proof of Proposition 9

We define the welfare gain from unconditional tax contracts as opposed to conditional tax contracts by

$$\Delta W^{uc} = p_e U_{\bar{y}}(t_{M,sc}^*, RL) - \frac{p_e}{2}[U_{\bar{y}}(t_{R,gc}^*, RL) + U_{\bar{y}}(t_{L,gc}^*, RL)]. \tag{45}$$

ΔW^{uc} changes in Δ_l according to

$$\frac{d\Delta W^{uc}}{dl_R} = -\frac{p_e}{2}\left[\left(\frac{A}{g_{L,gc}^*} - \bar{y}\frac{\partial t_{L,gc}(g_{L,gc}^*)}{\partial g_{L,gc}^*}\right)\frac{\partial g_{L,gc}^*}{\partial l_L}\frac{dl_L}{dl_R} + \left(\frac{A}{g_{R,gc}^*} - \bar{y}\frac{\partial t_{R,gc}(g_{R,gc}^*)}{\partial g_{R,gc}^*}\right)\frac{\partial g_{R,gc}^*}{\partial l_R}\right]. \tag{46}$$

Although conditional tax contracts might become more favorable from a welfare perspective for small values of Δ_l, this cannot be the case for major political polarization. The simplest way to show this is to consider separately the impact of a change in political polarization to the right and to the left. That is, we examine how the welfare difference changes if, starting from a small degree of polarization, we keep l_M and one of the large parties' position fixed and vary the position of the other large party.

If l_L is constant, we can derive from (46) that $\frac{\partial \Delta W^{uc}}{\partial l_R}$ is positive iff

$$\bar{y}\frac{\partial t(g^*_{R,gc})}{\partial g^*_{R,gc}}g^*_{R,gc} < A. \tag{47}$$

As $\frac{\partial g^*_{R,gc}}{\partial l_R} < 0$ and $g^*_{R,gc}$ converges to zero for $l_R \to \infty$, this condition must be satisfied for all values higher than a certain threshold of l_R.

If l_R is constant and we vary l_L, $\frac{\partial \Delta W^{uc}}{\partial l_L}$ is positive iff

$$\bar{y}\frac{\partial t(g^*_{L,gc})}{\partial g^*_{L,gc}}g^*_{L,gc} > A. \tag{48}$$

If l_L increases, the corresponding optimal amount of public-good provision $g^*_{L,gc}$ rises as well. Hence there exists a threshold for l_L such that for all l_L smaller than this threshold ΔW^{uc} will increase in l_L.

Since with widening political polarization, Δ_l, the policy changes to the left and to the right separately yield lower welfare, taking them together must also result in welfare losses. Hence for a sufficiently high degree of polarization, unconditional tax contracts are socially preferable to conditional tax contracts. □

C List of Tax Rates

Symbol	Meaning
t_k^*	ideal tax policy of party k if it is the sole party in power
t_{kj}^{nc}	tax rate agreed upon by party k and party j when bargaining without tax contracts
$\underline{t}_k$	lower bound of contracted tax-set of party k
$\bar{t}_k$	upper bound of contracted tax-set of party k
t_{ct}	tax policy in the caretaker government
$t_{M,sc}^*$	most preferred tax-rate of the center party M, if it is part of a small coalition
$t_{M,(sc)}^*$	bargaining outcome in a small coalition formed with tax contracts
$t_{k,gc}^*$	most preferred tax-rate of party k if it is part of a grand coalition
t_{kj}	most preferred tax-rate of (first-mover) party k in a grand coalition with j, given that j will include this tax-rate in its contracted set at the second stage
t_{kj}^-	indicates that t_{kj} is in the interval $[t_k^*, t_{M,sc}^*)$. That is, t_{kj} is closer to k's ideal policy t_k^* than $t_{M,sc}^*$
t_{kj}^+	indicates that t_{kj} is in the interval $[t_j^*, t_{M,sc}^*)$. That is, t_{kj} is farther from k's ideal policy t_k^* than $t_{M,sc}^*$
t_{kj}^b	comprehensive bargaining solution of parties k and j without tax contracts
t_{kj}^{bc}	comprehensive bargaining solution of parties k and j constrained by the set $\tau_k \cap \tau_j$
t_j'	most preferred tax-rate of (second-mover) party j within (first-mover) party k's tax set τ_k, such that party k's utility is weakly higher than in a caretaker government
t_{Mj}	center party's most preferred tax rate within the tax-set τ_j of its preferred coalition partner j
t_{jM}	most preferable tax rate of (second-mover) party j in a small coalition, such that M is willing to have a coalition with j rather than with (first-mover) k
$\widetilde{t}_k$	boundary of k's tax-set implying the most preferable t_{jM} from k's viewpoint provided that $\tau_j \cap \tau_k = \emptyset$
t_{kj}^e	t_{kj} conditioned on the event that party E enters parliament

References

Stephen Ansolabehere, Jr. Snyder, James M., Aaron B. Strauss, and Michael M. Ting. Voting weights and formateur advantages in the formation of coalition governments. *American Journal of Political Science*, 49(3):550–563, 2005.

David Austen-Smith and Jeffrey S. Banks. *Positive Political Theory II: Strategy and Structure*. University of Michigan Press, 2005.

Siddhartha Bandyopadhyay and Kalyan Chatterjee. Coalition Theory and its Applications: A Survey. *Economic Journal*, 116(509):136–155, 2006.

Roland Benabou. Ideology. *Journal of the European Economic Association*, 6:321–352, 2008.

Agnes Benassy-Quer, Nicolas Gobalraja, and Alain Trannoy. Tax and Public Input Competition. *Economic Policy*, 22(50):385–429, 2007.

Arianna Degan and Antonio Merlo. Do Voters Vote Sincerely? *mimeo*, 2007.

Lars P. Feld, Christoph A. Schaltegger, and Jan Schnellenbach. On Government Centralization and Fiscal Referendums: A Theoretical Model and Evidence from Switzerland. *European Economic Review (forthcoming)*, 2008.

Hans Gersbach. *Designing Democracy: Ideas for Better Rules*. Springer, Heidelberg, 2005.

Gian Maria Milesi-Ferretti, Roberto Perotti, and Massimo Rostagno. Electoral systems and public spending. *The Quarterly Journal of Economics*, 117(2):609–657, 2002.

Dennis C. Mueller. *Public Choice III*. Cambridge University Press, Cambridge, UK, 2003.

Wolfgang C. Mueller and Kaare Strom, editors. *Coalition Governments in Western Europe*. Oxford University Press, Oxford, UK, 2000.

Torsten Persson and Guido E. Tabellini. *The Economic Effects of Constitutions*. MIT Press, Cambridge, MA, 2003.

John E. Roemer. *Political Competition: Theory and Applications*. Harvard University Press, Cambridge, MA, 2001.

Charles M. Tiebout. A Pure Theory of Local Expenditures. *The Journal of Political Economy*, 64(5):416–424, 1956.

John D. Wilson. Theories of Tax Competition. *National Tax Journal*, 52:269–304, 1999.