DISCUSSION PAPER SERIES

No. 7078

WHEN CAN CENTRAL BANKS ANCHOR EXPECTATIONS? POLICY COMMUNICATION AND CONTROLLABILITY

Andrew Hughes Hallett, Giovanni Di Bartolomeo and Nicola Acocella

INTERNATIONAL MACROECONOMICS

Centre for Economic Policy Research

www.cepr.org

Available online at: **www.cepr.org/pubs/dps/DP7078.asp**

ISSN 0265-8003

WHEN CAN CENTRAL BANKS ANCHOR EXPECTATIONS? POLICY COMMUNICATION AND CONTROLLABILITY

Andrew Hughes Hallett, George Mason University, University of St. Andrews and CEPR
Giovanni Di Bartolomeo, University of Teramo
Nicola Acocella, Università di Roma La Sapienza

Discussion Paper No. 7078
December 2008

Centre for Economic Policy Research
53–56 Gt Sutton St, London EC1V 0DG, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 7078

December 2008

ABSTRACT

When Can Central Banks Anchor Expectations?

Policy communication and controllability

Rational expectations are often used as a strong argument against policy activism, as they may undermine or neutralize the policymaker's actions. Although this sometimes happens, rational expectations do not always imply policy invariance or ineffectiveness. In fact, in certain circumstances rational expectations can enhance our power to control an economy over time. In those cases, policy announcements, properly communicated, can be used to extend the impact of conventional policy instruments. In this paper we present a general forward-looking policy framework and use it to provide a formal justification for attempting to anchor expectations, and as a possible justification for publishing interest rate forecasts or tax rate projections. This approach allows us to test when policymakers can and cannot expect to be able to manage expectations.

JEL Classification: C61, C62, E52, E61 and E62
Keywords: controllability, fiscal policy, monetary policy, policy neutrality and rational expectations

Andrew Hughes Hallett
School of Public Policy,
George Mason University
4400 University Drive, MS 3C6
Fairfax, VA 22030
USA

Email: ahughesh@gmu.edu

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=103647

Giovanni Di Bartolomeo
University of Teramo
Campus Coste S. Agostino
via del Castro Laurenziano 9
00161 Roma
ITALY

Email: gdibartolomeo@unite.it

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=163054

Nicola Acocella
Dipartimento di Economia Pubblica
Università di Roma 'La Sapienza'
Via Castro Laurenziano 9
00161 Roma
ITALY

Email: nicola.acocella@uniroma1.it

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=134542

Submitted 26 November 2008

When Can Central Banks Anchor Expectations? Policy communication and controllability*

Andrew Hughes Hallett
George Mason University, University of St Andrews and CEPR

Giovanni Di Bartolomeo
University of Teramo

Nicola Acocella
University of Rome La Sapienza

Abstract. Rational expectations are often used as a strong argument against policy activism, as they may undermine or neutralize the policymaker's actions. Although this sometimes happens, rational expectations do not always imply policy invariance or ineffectiveness. In fact, in certain circumstances rational expectations can enhance our power to control an economy over time. In those cases, policy announcements, properly communicated, can be used to extend the impact of conventional policy instruments. In this paper we present a general forward-looking policy framework and use it to provide a formal justification for attempting to anchor expectations, and as a possible justification for publishing interest rate forecasts or tax rate projections. This approach allows us to test when policymakers can and cannot expect to be able to manage expectations.

JEL Classification: C61, C62, E52, E61, E62.
Keywords: Rational expectations, controllability, policy neutrality, monetary policy, fiscal policy,

1. Introduction

Since the work of Barro (1974), Sargent and Wallace (1975) and Lucas (1976), rational expectations have been regarded as placing severe limits on what can be achieved in a world of policy conflicts; and as requiring strong policy commitments to get even that far. Time inconsistency and the Lucas critique, coupled with rational expectations, are often said to imply that such commitments cannot be considered credible and to lead inevitably to Pareto inferior outcomes.

* The authors are grateful to Reinhard Neck and Paolo Piacquadio for useful discussions and comments on an earlier draft. Nicola Acocella thanks the University of Rome "La Sapienza" for funding.

This argument however does not allow for policymakers who actively engage in managing expectations by making policy announcements, alongside policy interventions, for the express purpose of shifting the expectations path itself. If they can do that, private expectations will be exactly consistent with what the private sector/policymakers expect the outcomes to be; and no one will be required to move off their expected path (make expectation errors) for the policies to work. The literature has often used this idea informally in debates over the feasibility and desirability of trying to anchor inflation expectations for monetary policy, or in arguments over the desirability of publishing interest rate forecasts.[1] It is also an idea in the minds of the policy makers; see, for example, the European Central Bank's concern that long term policies introduced to combat the current financial crisis (greater transparency, new regulation, reduced pro-cyclicality, planned liquidity withdrawals) should have their effects now (Trichet, 2008); equally the announcement of fiscal stimulus or credit guarantee packages. But what the literature has not done is identify the conditions under which we can expect to be able to manage expectations; and also when we cannot. To do that would provide a formal justification for policies designed to anchor expectations.

This paper investigates the circumstances under which policy announcements, if properly communicated, can be used to supplement or extend the impact of conventional policy instruments. The idea is that rational expectations may, in certain cases, enhance the power to control an economy over time. Hence, contrary to conventional wisdom, rational expectations may, *but do not always* neutralize the policymaker's action.

Specifically, we consider the design of economic policy within a general rational expectations framework and show that policy invariance can only arise in specific cases. In the other cases policy announcements can be used to help steer economic behaviour, and certain targets become reachable in reduced time. The rationale for this result can be understood by using the concept of controllability, introduced in the classical theory of economic policy (Tinbergen, 1952), and its dynamic extensions. If a policymaker is able to achieve any desired vector of targets given some exogenous expectations, then he will also be to do it with endogenous expectations. In addition, he could exploit rational expectations in order to achieve his targets in a shorter time.

[1] See, among many others, Woodford (2005), Blinder *et al*. (2008) or Rudebusch and Williams (2008).

To make use of this potential benefit of rational expectations, however, another ingredient must be present. The policymakers must be able to communicate, in a clear, firm and effective way, the intent and purpose of their policies, and how exactly these policies will be carried out. This will be necessary to convince the private sector that the policy measures will in fact be undertaken when it comes to the point; and that it is reasonable to expect the intended outcomes will be achieved. Otherwise there is no reason to suppose the private sector would shift, or anchor, their expectations of future outcomes as a result of announcements of future policy actions.

Our approach differs from the recent trend in the literature on communication. In our framework, the crucial element is to reaffirm the targets and why the policies chosen can be expected to reach them, as emphasised by Eggertson and Pugsley (2006), Libich (2008), and by Woodford's (2003) observation that policy trade-offs are eased when expectations fall into line. By contrast much of the recent literature on communication has focused on the quality of forecasts, on the degree of divergence or agreement among policymakers, and on transcripts or voting records from policy committees (Ehrmann and Fratzscher 2005, 2007a; Jansen and de Haan 2006; Visser and Swank 2007).

The rest of the paper is organized as follows. Section 2 poses the communications problem by means of a simple example. Section 3 puts our example into a more general framework, deriving the reduced and final form of a general model with a single policy maker and rational expectations. In that part of the paper we deal with the conditions for static and dynamic controllability; and demonstrate that, contrary to conventional wisdom, dynamic controllability can be enhanced by rational expectations. Our purpose is to identify when that controllability is possible; *and* the conditions when it is not. Section 4 then presents two examples of how announcements of future policies may help in ensuring the static or dynamic control of the economy. Section 5 contains a specific illustration of our main point in the context of monetary policy. Section 6 concludes.

2. Controllability in a simple modern macro-model.

Let us consider a standard example of an economy with a New Keynesian structure:

(1) $$\pi_t = (1-\lambda)\beta E_t\pi_{t+1} + \lambda\pi_{t-1} + \kappa x_t + \phi f_t + v_t$$

(2) $$x_t = E_t x_t - \sigma(i_t - E_t\pi_{t+1}) + \chi f_t + \varepsilon_t$$

where ε_t and v_t are white noise, and where $x_t = y_t - \bar{y}_t$ is the output gap relative to a non-market clearing trend; or relative to the natural rate of output arising from monopolistic competition in the goods markets (Blanchard and Kiyotaki, 1987); or created by tax distortions elsewhere in the economy (Alesina and Tabellini, 1987).

If we want to analyze the role of policy announcements, and the importance of communication as a way to enhance the effectiveness of policy instruments, we must first discuss the conditions under which the policymaker can control the system given by equations (1) and (2). And to do that we need to make some generalisations.

3. A general model for analyzing controllability.

Dynamic models as (1)-(2) can be expressed in the following general form:

(3) $$y_t = Ay_{t-1} + By_{t+1/t} + Cx_t + error\ term \qquad \text{for} \quad t = 1\ldots..T$$

where T is a finite, but possibly large number; and where $y_{t+1/t} = E(y_{t+1}/\Omega_t)$ denotes the mathematical expectation of y_{t+1} conditional on Ω_t (the information set available at t). In our set up, y_t is a vector of n endogenous variables at time t; x_t is a vector of m potential policy instruments[2]; and v_t is a vector of exogenous shocks or other influences which have a known mean, but otherwise come from unspecified probability distributions. The matrices A, B and C are constant and of order n, n, and $n \times m$ respectively, and have at least some elements which are non-zero.

Model (1)-(2) can easily be written in the general form (3) by obtaining the model's reduced form

(4) $$\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} 1 & \kappa \\ 0 & 1 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} (1-\lambda)\beta & 0 \\ \sigma & 1 \end{bmatrix} E_t \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \end{bmatrix} + \begin{bmatrix} \lambda & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{pmatrix} \phi & 0 \\ \chi & -\sigma \end{pmatrix} \begin{pmatrix} f_t \\ i_t \end{pmatrix} + \begin{pmatrix} v_t \\ \varepsilon_t \end{pmatrix} \right\}.$$

We can then define the parameter matrices in (3) to be:

(5) $$A = \begin{bmatrix} \lambda & 0 \\ 0 & 0 \end{bmatrix};\ B = \begin{bmatrix} (1-\lambda)\beta + \kappa\sigma & \kappa \\ \sigma & 1 \end{bmatrix};\ C = \begin{bmatrix} \phi + \kappa\chi & -\kappa\sigma \\ \chi & -\sigma \end{bmatrix};\ \text{and} \begin{bmatrix} 1 & 0 \\ \kappa & 1 \end{bmatrix} \begin{pmatrix} v_t \\ \varepsilon_t \end{pmatrix}$$

where the last expression is the error term.

---

[2] Note the generalization of the notation for this section.

Model (3) can now be solved from the perspective of any particular period, say $t = 1$, by putting it into final form conditional on the information available in that period:[3]

$$(6)\quad \begin{pmatrix} y_{1/1} \\ : \\ : \\ : \\ y_{T/1} \end{pmatrix} = \begin{bmatrix} I & -B & 0 & . & 0 \\ -A & I & & . & . \\ 0 & & . & & 0 \\ . & . & & . & -B \\ 0 & . & 0 & -A & I \end{bmatrix}^{-1} \left\{ \begin{bmatrix} C & 0 & . & . & 0 \\ 0 & . & & & . \\ . & & . & & . \\ . & & & . & 0 \\ 0 & . & . & 0 & C \end{bmatrix} \begin{pmatrix} x_{1/1} \\ . \\ . \\ . \\ x_{T/1} \end{pmatrix} + \begin{pmatrix} v_{1/1} \\ . \\ . \\ . \\ v_{T/1} \end{pmatrix} + \begin{pmatrix} Ay_0 \\ 0 \\ . \\ . \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ . \\ . \\ 0 \\ By_{T+1/1} \end{pmatrix} \right\}$$

In the above representation, y_0 is a known initial condition for $t = 1$; and $y_{T+1/1}$ is some assumed or projected terminal condition – most likely the one that describes the economy's expected long run equilibrium state as part of Ω_t. Although (3) has been solved from the point of view of Ω_1, it must be understood that it could have been derived for each Ω_t, $t = 1....T$, in turn (where $y_{j/t} = E_t(y_j)$ if $j \geq t$, but $y_{j/t} = y_j$ if $j < t$; and similarly for x and v). However, for simplicity, we will consider the Ω_1 case only in what follows. The generalisation to any other value of t is then obvious.

Second, the equation to which (6) is the solution makes it clear that *neither* the policymakers, *nor* the private sector are required to move off their expected paths (make expectational errors) for the policies to work. In fact equations (6) and (7) below show just the opposite; that those expectations are exactly consistent with what the private sector/policymakers expect the outcomes to be. The only question is whether policies, or policy announcements, can be found that would shift the expectations path itself by the required amount. The task of this paper is to find the conditions under which that can be done. We need to determine if it is possible to shift expectations in such a way that the economy's final outcomes reach certain target values at certain points of time; and also when it is not possible.

It is easy to see that (4) always exists; that is, the matrix inverse, T_T^{-1} in (6), exists. This is demonstrated in Hughes Hallett *et al* (2008); the condition for T_T^{-1} to exist being that the matrix product AB shall not contain a unit root.[4] Given that, we rewrite (6) as:

[3] Hughes Hallett and Fisher (1988), Hughes Hallett *et al*. (1996).

[4] A weaker condition, if $T \rightarrow \infty$, would be the usual saddle point property (Hughes Hallett and Fisher, 1988). Notice that this result automatically implies that the traditional vertical Phillips curve model would not be controllable in the long run since T_T would be lower triangular with A having a unit root. It was our purpose to collect conditions when the system is not controllable, as well as when it is. The unit root condition on A (or on AB in our more general formulation) is one; the other, a failure of the rank condition in proposition1.

$$(7)\qquad \begin{pmatrix} y_{1/1} \\ . \\ . \\ . \\ y_{T/1} \end{pmatrix} = \begin{bmatrix} R_{11} & . & . & . & R_{1T} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ R_{T1} & . & . & . & R_{TT} \end{bmatrix} \begin{pmatrix} x_{1/1} \\ . \\ . \\ . \\ x_{T/1} \end{pmatrix} + \begin{pmatrix} b_{1/1} \\ . \\ . \\ . \\ b_{T/1} \end{pmatrix} \qquad \text{or} \quad y = Rx + b$$

where $R = T_T^{-1}(I \otimes C)$, $b = T_T^{-1}\{E(v/\Omega_1) + (A':0)'y_0 + (0:B')'y_{T+1/1}\}$, and "$\otimes$" denotes a Kronecker product. In this representation, each $R_{t,j} = \partial y_{t/1}/\partial x_{j/1}$ is an $n \times m$ matrix of policy multipliers for *t*, *j* = 1…*T*. But notice that *R* is not block triangular. That is, $R_{t,j} \neq 0$ even if $t < j$. Hence equation (7) implies $R_{t,j}$ is a matrix of conventional policy multipliers between $y_{t/1}$ and $x_{j/1}$ with a delay of $t - j$ periods between implementation and realization, if $t \geq j$. But $R_{t,j}$ represents a matrix of anticipatory effects, on $y_{t/1}$, of an announced or anticipated policy change $x_{j/1}$ at some point in the future if $t < j$.[5]

We now proceed in two steps:

a) We consider **static controllability** first. Static or Tinbergen controllability is normally defined (see Holly and Hughes Hallett, 1983) as the set of conditions which must hold if an arbitrary set of target values is to be reached for the endogenous variables y_t in each period – at least in expectation, given that the original model is stochastic. Define those target values to be $y_{t/1}^d$, where superscript *d* denotes a desired value from the perspective of period 1. We then define y^d to be a stacked vector of those desired values across all time periods.

Static controllability, in each period in turn, therefore requires the matrix *R* in (7) to possess an inverse:

(8) $\qquad x = R^{-1}(y^d - b)$

where *y*, *x* and *b* are all understood to be expectations conditioned on the current information set Ω_t, for each *t* = 1…*T* in turn, as noted in (7). But since $R = T_T^{-1}C_T$, where $C_T = I_T \otimes C$, we can see $R_T^{-1} = (T_T^{-1}C_T)^{-1} = C_T^{-1}T_T$ exists if and only if $C_T^{-1} = I_T \otimes C^{-1}$ exists since we already know that T_T^{-1} exists. But the instrument coefficient matrix, *C*, can only possess an inverse if *n* = *m* and C has full rank. Thus *n* = *m* is a necessary condition

[5] A conventional "backwards looking" model will have $R_{t,j} = 0$ for all $t < j$; *and* constant multipliers $R_{t,j} = R_{t,-j}$ for $t - j = 0\ldots..T{-}1$, if the model at (3) is linear. Neither of these things is true in (7).

for static controllability; and linear independence in the impacts of the instruments on the targets is sufficient. There is therefore no generalisation or change to the traditional static controllability conditions when there are rational forward looking expectations.

b) Next we consider **dynamic controllability.** An economy is said to controllable dynamically if a sequence of instrument values $x_1,.....,x_t$ can be found to reach any arbitrary value, y_t^d , for the target variables in period *t* (at least in expectation), given an arbitrary starting point y_0 (Holly and Hughes Hallett, 1983). In that case, we are no longer concerned with the period-by-period controllability of target variables between periods 1 and *t* − 1. Viewed from period 1, dynamic controllability therefore requires a sequence of intended instrument values $x_{1/1},.....,x_{T/1}$ that can guarantee $y_{t/1}^d$ is reached in period *t*. Given (7), this will be possible only if the sequence of policy multipliers and anticipatory effects, $R_{t,1}.....R_{t,T}$, is of full rank: $r(R_{t,1}....R_{t,T}) = n$, given an arbitrary initial state y_0 and a specified terminal condition $y_{T+1/1}$. This follows because $y_{t/1}^d = (R_{t,1}....R_{t/T})x + b_{t/1}$ is reachable over (1, *t*), using a Moore-Penrose generalized left inverse, if $r(R_{t,1}....R_{t,T}) = n$. But if $T \geq n$, $r(R_{t,1}...R_{t,T}) = r(R_{t,1}...R_{t,n}) = n$ which provides the result. Thus an economy represented by (3) is dynamically controllable, over (1, *t*) with $T \geq n$, if $r(R_{t,1}....R_{t,n}) = n$.

These controllability conditions contain an important generalization over the traditional case with a backward looking model. If $n > t$, which is entirely possible for small values of *t*, then dynamic controllability will be available through the reactions of $y_{t/1}$ to the implemented policy choices $x_{1/1}...x_{t/1}$; *and through the anticipatory effects of announced or anticipated policy interventions* that still lie in the future, $x_{t+1/1}....x_{n/1}$. That implies the policymaker can use policy announcements, in addition to policy interventions, to guide the course of the economy. In a conventional model that would not be possible since $R_{t,j} = 0$ for all $j > t$. In effect, the policymaker has a greater number of policy "instruments" at his disposal than in an economy without anticipations.

Thus $y_{1/1}$ itself is controllable from the first period, even if there are insufficient instruments ($m < n$), provided that $T \geq n$ and $r(R_{t,1}....R_{t,n}) = n$. The astute policymaker will quickly realise that good communication lies at the heart of the policy problem if he/she wants to reach their policy targets in the early periods or at lower cost -- a fact that has not been lost on central bank policy makers in their attempts to control or anchor the private sector's expectations of future inflation in such a way as to make their interest rate policies more effective (Woodford, 2005; Blinder *et al*. 2008; Rudebusch and Williams, 2008).

Second, dynamic controllability is possible with a much reduced instrument set compared to static controllability. There are two parts to this reduction: a) the ability to use one or more instruments repeatedly rather than a group of several instruments used once and in parallel; and b) the ability to augment or replace parts of an existing instrument set with announcements of future policy changes.

Finally it is important to note that, while the $x_{1/1} \ldots x_{t/1}$ values will be implemented decisions when it comes to the controllability of $y_{t/1}$, the $x_{t+1/1} \ldots . x_{n/1}$ values, being policy announcements, may never actually be carried out. However, because they lie in the future from the perspective of $y_{t/1}$, any subsequent time inconsistency plays no role in the controllability of $y_{t/1}$ so long as they are *genuinely* held expectations at this point.

Proposition 1: Forward looking rational expectations enhance the power to control an economy over time in that: policy announcements may be used to supplement and extend the impact of conventional policy instruments, and that controllability is now available with a reduced instrument set from $t = 1$, *if* $r(R_{11} \ldots R_{1n}) = n$.

4. An application of our general model for controllability.

We now go back to the simple macro-model introduced in section 2, and examine its controllability. We consider two cases. First we consider the simple backwards looking case by assuming $\beta = 0$. Second, we consider a more general case in which there are also rational expectations and hence forward looking variables ($\beta \neq 0$).

CASE 1: *Controllability without rational expectations.*

If $\beta = 0$ we have a recursive system in place of (1) and (2), i.e.:

(9) $$\pi_t = \lambda \pi_{t-1} + \kappa(-\sigma i_t + \chi f_t + \varepsilon_t) + \phi f_t + v_t,$$

(10) $$x_t = -\sigma i_t + \chi f_t + \varepsilon_t$$

This system has static controllability in the classic sense since the policy multipliers in

$$(11)\qquad \begin{bmatrix}\pi_t\\ x_t\end{bmatrix}=\begin{bmatrix}-\kappa\sigma & \kappa\chi+\phi\\ -\sigma & \chi\end{bmatrix}\begin{bmatrix}i_t\\ f_t\end{bmatrix}+\begin{bmatrix}v_t+\lambda\pi_{t-1}+\kappa\varepsilon_{t-1}\\ \varepsilon_t\end{bmatrix}$$

form a non-singular matrix so long as $\phi\neq 0$. Consequently any required values for π_t and x_t can be reached in expectation. However **if** only one policy instrument, i_t, is available, and no fiscal policy $(\phi=\chi=0)$, as has been the case in most of the monetary policy literature since Barro and Gordon (1983) and Rogoff (1985), **or** if fiscal policy has just one independent channel of influence $(\phi=0)$, then static controllability is lost and the desired values for π_t and x_t cannot be reached in each (and hence the current) period. The multiplier matrix in (11) is singular in either case. Nevertheless the system is still dynamically controllable over two periods. Consider the first case: $\phi=\chi=0$. Back-substituting one period in (11), we get

$$(12)\qquad \begin{bmatrix}\pi_t\\ x_t\end{bmatrix}=\begin{pmatrix}\lambda^2 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}\pi_{t-1}\\ x_{t-1}\end{pmatrix}+\begin{bmatrix}-\kappa\sigma & -\lambda\kappa\sigma\\ -\sigma & 0\end{bmatrix}\begin{bmatrix}i_t\\ i_{t-1}\end{bmatrix}+\begin{pmatrix}1 & \kappa\\ 0 & 1\end{pmatrix}\begin{pmatrix}v_t\\ \varepsilon_t\end{pmatrix}+\begin{pmatrix}\lambda & \lambda\kappa\\ 0 & 0\end{pmatrix}\begin{pmatrix}v_{t-1}\\ \varepsilon_{t-1}\end{pmatrix}$$

The policy multiplier matrix is now non-singular for *all* parameter values, and we can reach any desired values for π_t and x_t after two periods using monetary policy alone.

Essentially the same is true if fiscal policy has only one channel of influence: $\phi=0$. In this case the multiplier matrix in (11) is singular and the static controllability property is lost. But back-substituting one period implies

$$(13)\qquad \begin{bmatrix}\pi_t\\ x_t\end{bmatrix}=\begin{pmatrix}\lambda^2 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}\pi_{t-1}\\ x_{t-1}\end{pmatrix}+\begin{bmatrix}-\kappa\sigma & -\lambda\kappa\sigma\\ -\sigma & 0\end{bmatrix}\begin{bmatrix}i_t\\ i_{t-1}\end{bmatrix}+\begin{bmatrix}\kappa\chi & \lambda\kappa\chi\\ \chi & 0\end{bmatrix}\begin{bmatrix}f_t\\ f_{t-1}\end{bmatrix}+\text{ errors}$$

in which both multiplier matrices are non-singular for all nonzero parameter values. So, to reach desired values for π_t and x_t after two periods, we can either use interest rates twice; or fiscal policy twice; or first interest rates and then fiscal policy, or fiscal policy and then interest rates in an "asynchronous game". In the latter two cases, the multiplier matrices would become $\begin{bmatrix}-\kappa\sigma & -\lambda\kappa\chi\\ -\sigma & 0\end{bmatrix}\begin{bmatrix}i_t\\ f_{t-1}\end{bmatrix}$ and $\begin{bmatrix}\kappa\chi & -\lambda\kappa\sigma\\ \chi & 0\end{bmatrix}\begin{bmatrix}f_t\\ i_{t-1}\end{bmatrix}$ respectively. But it would still take two periods to reach our target values.

CASE 2: *Controllability with rational, forward-looking expectations.*

Allowing for the full effects of both policy instruments takes us back to (4); and hence to a model of the form of (3) with parameter matrices given by (5). That model has a policy multiplier matrix *C*, which is non-singular if $\phi \neq 0$. Hence we have static controllability given arbitrary initial conditions, and arbitrary values for the terminal conditions.

So far the story is the same as in the case with no rational expectations. However, things change if we have only one instrument (monetary policy) available. To see that, we write our policy problem as a two period problem with both policy instruments:

$$(14) \quad \begin{bmatrix} \pi_t \\ x_t \\ \pi_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} I & -B \\ -A & I \end{bmatrix} \left\{ \begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} f_t \\ i_t \\ E_t(f_{t+1}) \\ E_t(i_{t+1}) \end{bmatrix} + \begin{pmatrix} w_t \\ w_{t+1} \end{pmatrix} \right\} \quad \text{say,}$$

as a particular case of (6). We are interested in only the first two rows of (14) for static controllability. But we will need to use the whole system if we are to go to dynamic controllability after two periods. The policy multiplier matrix implied by (14) is then:

$$(15) \quad \begin{bmatrix} \phi + \kappa\chi & -\kappa\sigma & [(1-\Delta)\phi/\lambda + 1]\kappa\chi & -[(1-\Delta)/\lambda + 1]\kappa\sigma \\ \chi\Delta + \sigma\lambda(\phi + \kappa\chi) & -\sigma[\Delta + \kappa\lambda\sigma] & \chi[\sigma\lambda\kappa + \Delta + \sigma\phi\kappa(\lambda(1-\Delta) + \Delta)] & -\sigma[\sigma\lambda\kappa + \Delta + \sigma\kappa(\lambda(1-\Delta) + \Delta)] \\ \lambda(\phi + \kappa\chi) & -\kappa\sigma\lambda & (\phi + \lambda)\kappa\chi & -(1+\lambda)\kappa\sigma \\ 0 & 0 & \chi & -\sigma \end{bmatrix} . \Delta^{-1}$$

where $\Delta = 1 - [\lambda(1-\lambda) + \lambda\kappa\sigma] \neq 0$. The one period static controllability policy multiplier matrix (the top left 2x2 sub-matrix) is non-singular as noted above. But if there is just one policy (monetary policy) available, $\phi = \chi = 0$, then instead of (15) we get

$$(16) \quad \begin{bmatrix} 0 & -\kappa\sigma & 0 & -(1-\Delta+\lambda)\kappa\sigma/\lambda \\ 0 & -\sigma(\kappa\sigma\lambda + \Delta) & 0 & -\sigma[\sigma\kappa(1-\lambda) + 1]\Delta \\ 0 & -\kappa\sigma\lambda & 0 & -\sigma\kappa(1+\lambda) \\ 0 & 0 & 0 & -\sigma \end{bmatrix} . \Delta^{-1}.$$

Alternatively, if fiscal policy is present but has only one channel of influence $(\phi = 0)$, then we get

$$(17) \quad \begin{bmatrix} \kappa\chi & -\kappa\sigma & \kappa\chi & -(1-\Delta+\lambda)\kappa\sigma/\lambda \\ \chi(\Delta + \kappa\sigma\lambda) & -\sigma(\kappa\sigma\lambda + \Delta) & \lambda\kappa\chi & -\sigma[\sigma\kappa(1-\lambda) + 1]\Delta \\ \lambda\kappa\chi & -\kappa\sigma\lambda & \lambda\kappa\chi & -\sigma\kappa(1+\lambda) \\ 0 & 0 & \chi & -\sigma \end{bmatrix} . \Delta^{-1}.$$

Hence, for the first case, we can write the one period, static controllability problem by inserting (16) into (14) to yield

$$(18) \quad \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = -\Delta^{-1} \begin{bmatrix} \kappa\sigma & \kappa\sigma(1-\Delta+\lambda)/\lambda \\ \sigma(\kappa\sigma\lambda+\Delta) & \sigma[\sigma\kappa(1-\lambda)+1]\Delta \end{bmatrix} \begin{bmatrix} i_t \\ E_t(i_{t+1}) \end{bmatrix} + w_t$$

whose multiplier matrix is easily seen to be non-singular. In other words, we have static controllability again from the point of view of period *t*, despite having a single instrument and two targets.[6] This is achieved by using i_t *and policy announcements* of what will happen in period *t+1*. So, in this case we have static controllability even if there are insufficient policy instruments.

The same thing happens when fiscal policy is limited to one channel of influence ($\phi = 0$). If we choose to use monetary policy alone, then (16) applies again. But if we can still use fiscal policy, (18) will be replaced by inserting (17) in (14):

$$(19) \quad \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \Delta^{-1} \begin{bmatrix} \kappa\chi & \kappa\chi \\ \chi(\Delta+\kappa\sigma\lambda) & \lambda\kappa\chi \end{bmatrix} \begin{bmatrix} f_t \\ E_t(f_{t+1}) \end{bmatrix} + w_t$$

whose multiplier matrix is also non-singular. So again we have static controllability at period *t*, an option we didn't have in the non-rational expectations case. And for the same reason: credible policy announcements become a second instrument. In addition, we can easily generate examples which imply static controllability using mixed policies: for example, monetary policy and announcements about future fiscal policy; or fiscal policy plus announcements of future monetary policies.

The crucial point, however, is that case 2 requires the policymakers to exercise good communication skills in order to make their announcements credible and have some effect – that is to make the private sector shift (or anchor) their expectations of the likely outcomes to values that suit the policymakers' purpose. In other words, the private sector must be made to believe that the necessary policy changes will actually take place and achieve those outcomes. Case 1 does not require the same communication skills since there are no expectations to be anchored.

[6] Before we only had dynamic controllability over two periods in the "no rational expectations" model.

5. Communication: Should central banks try to control expectations?

One of the great debates of monetary policy is whether central banks should create or allow forecasts of future interest rates to be published. On one side, Rudebusch and Williams (2008) and Eusepi and Preston (2007) argue that this can be used to strengthen economic policy. But others have argued that to do so may imply a greater consensus or certainty about future policies than actually exists; or that it may propagate errors and make it more difficult to adjust policies again later in the face of unexpected shocks. In addition, private agents may overreact to noisy public signals, but underreact to more accurate private information (see, *inter alia*, Faust and Svensson, 2002; Amato *et al*., 2003; Walsh, 2007).

To resolve this argument, consider an economy represented by the following well-known model:

(20) $$y_t = \rho y_{t-1} + \alpha(\pi_t - E_t\pi_{t+1}) - \beta(i_t - E_t\pi_{t+1}) + \varepsilon_t$$

(21) $$i_t = c_0 + c_1(\pi_t - \pi^*) + c_2 y_t + \nu_t$$

Equation (20) is an elaboration of the standard model which has been part of the theory of monetary policy since the Barro-Gordon model was introduced in 1983. It consists of a short run Phillips curve with persistence ($\rho \neq 0$), set within a standard forward looking Lucas supply function (a long run Phillips curve) and elaborated to include the effects of real interest rate changes on output. It can therefore be interpreted as either a dynamic open economy Phillips curve; or as a forward looking new Keynesian IS curve with dynamics. In that context, y_t is the deviation of output from its natural rate ; π_t is the rate of inflation; $E_t\pi_{t+1}$ the private sector's current expectation for the rate of inflation; and π^* is the government or central bank's target inflation rate; where i_t is the nominal rate of interest, ε_t is a supply shock with mean zero and constant variance, and ν_t is a monetary policy shock with mean zero and constant variance.

The only policy instrument in this example will be i_t. Equation (21) is therefore a Taylor rule: c_0 is the constant term reflecting the equilibrium rate of interest and ν_t reflects possible control errors; π^* is the inflation target; and determinacy (the Taylor principle) suggests $c_1 > 1$.

To obtain a reduced form for (20)-(21), corresponding to (3), we renormalize (21) on π_t and set $u_t = c_1^{-1}\nu_t$. This transforms our system to:

$$(22)\qquad \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & \beta-\alpha \\ 0 & 0 \end{bmatrix} \begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} + \begin{bmatrix} -\beta \\ c_1^{-1} \end{bmatrix} i_t + \begin{bmatrix} \varepsilon_t \\ u_t \end{bmatrix}$$

which does not allow static controllability as it stands. However, we can write the two period policy problem, as we did before, using (6):

$$(23)\qquad \begin{bmatrix} y_t \\ \pi_t \\ y_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} I & B \\ -A & I \end{bmatrix}^{-1} \left\{ \begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} i_t \\ E_t i_{t+1} \end{bmatrix} + \begin{pmatrix} \varepsilon_t \\ u_t \\ \varepsilon_{t+1} \\ u_{t+1} \end{pmatrix} + \begin{pmatrix} A \begin{bmatrix} y_0 \\ \pi_0 \end{bmatrix} \\ B \begin{bmatrix} E_t y_{t+2} \\ E_t \pi_{t+2} \end{bmatrix} \end{pmatrix} \right\}$$

where *A*, *B* and *C* are the first, second and third coefficient matrices in (22). The policy multiplier matrix for this model is therefore

$$(24)\qquad \begin{bmatrix} c_1^{-1}\rho(\beta-\alpha)-\beta & c_1^{-1}(\beta-\alpha) \\ c_1^{-1} & 0 \\ c_1^{-1}\rho^2(\beta-\alpha)-\rho\beta & c_1^{-1}\rho(\beta-\alpha)-\beta \\ 0 & c_1^{-1} \end{bmatrix} \cdot \begin{bmatrix} i_t \\ E_t i_{t+1} \end{bmatrix}$$

Thus, putting (24) back into (23), the standard model tells us that (y_t, π_t) and (y_{t+1}, π_{t+1}) are both statically (immediately) controllable, using current policies and announcements or projections of future actions. That is because both the upper and lower partitions of the multiplier matrix in equation (24) are non-singular. And since (y_t, π_t) and (y_{t+1}, π_{t+1}) are current and expected future target values from the perspective of period *t*, this means that the policymakers can control not only current inflation and output (in expectation at least); *but also* inflation expectations and growth forecasts for the next period. That adds to their ability to control inflation (and output) if they can anchor those expectations.

This example is therefore entirely in line with our general theory of dynamic controllability under rational expectations, and makes the case for publishing conditional forecasts of future interest rates as a normal part of the monetary policy framework. Indeed it would become more difficult to achieve successful outcomes if inflation expectations were not tied down at the same time as inflation itself. There may be other ways of controlling the outcomes of (y_{t+1}, π_{t+1}) in period *t+1* of course; for example using current and past interest rates as implied by (24). But that is a conventional backwards looking use of dynamic controllability, not a part of the expectations story emphasized in this paper; and it may still fail to tie down/anchor expectations in the terminal period.

6. Concluding remarks.

Rational expectations have a twofold nature. Since the Lucas critique, the emphasis has been placed on the implication that the policymaker cannot fool a private sector that correctly anticipates the policies and the equilibrium. This has been done by investigating policy invariance, underlining the role of rational expectations in offsetting the policy interventions. This paper shows that, because policy invariance can only emerge when there is a conflict between the public and the private sector, this will not always be what happens.

Second, rational expectations are also a powerful mechanism, in combination with the chosen policy values, for influencing the natural dynamics of the economy. There has been much less interest in this second aspect of rational expectations, which could imply that policy announcements may be used to systematically increase the power of the policymaker's interventions. A notable exception is Woodford's *timeless perspective,* and the use of interest rate forecasts or inflation targets to anchor inflation expectations.

In proposition 1, we have obtained a rank condition to define the circumstances under which rational expectations can be used to increase a policymaker's power to control an economy over time; and how communication and policy announcements can be exploited to supplement and extend the impact of conventional policy instruments. This provides a formal justification for using policies designed to manage expectations, and also defines the circumstances in which they cannot be used.

Our approach is derived from some recent studies that apply the classical concept of controllability for the derivation of certain properties of models with strategic interactions. We have shown that rational expectations do not, *in themselves*, affect the conditions for controllability in either its static or dynamic form. Put differently, rational expectations may, in certain cases, extend the effectiveness of economic policy.

To make the point, let us assume that a policymaker *can* control an economic system under exogenous expectations; which means that, given any arbitrary values for the expectations, he can achieve any desired vector of target variables. If this is true, nothing will change if the expectations are made endogenous. In fact, for any given value of the endogenous expectations, if an equilibrium exists, the policymaker will still be able to achieve any desired target vector. If there is a policy conflict with the private sector, policy invariance need not emerge but an equilibrium can fail to exist.[7] Thus policy invariance and equilibrium non-existence derive from the conflict between policymaker and private sector, not from the assumption of rational expectations *per se*.

The examples presented in this paper have shown that the implications of our results are rather important. Any dynamic problem that implies the achievement of a given target at a certain instant of time – such as fiscal consolidation, or the achievement of a set of macroeconomic targets as a condition for creating a trade or monetary union, or reducing inflation in a specified interval – will find an important ally if expectations can be managed to help meet the policymaker's goals. But to achieve this, the conditions for dynamic controllability and good communication have to be satisfied. In such cases, the policy problem would no longer be a matter of how to find a credible commitment, but of when the targets should be announced given the lag structure of the model[8].

References

Acocella, N. and G. Di Bartolomeo (2006), "Tinbergen and Theil meet Nash: Controllability in policy games," *Economics Letters*, 90: 213-218.

Amato, J.D., S. Morris and H. S. Shin (2003), "Communication and monetary policy," *Oxford Review of Economic Policy*, 18: 495-503.

Alesina, A. and G. Tabellini (1987), "Rules and discretion with noncoordinated monetary and fiscal policies," *Economic Inquiry*, 25: 619-630.

Barro, R.J. (1974), "Are government bonds net wealth?," *Journal of Political Economy*, 82: 1095-1117.

Barro, R.J. and D. Gordon (1983), "Rules, discretion and reputation in a model of monetary policy," *Journal of Monetary Economics*, 12: 101-120.

[7] This corresponds to the case where two players simultaneously control a system, which implies the non existence of the equilibrium. See Acocella and Di Bartolomeo (2006).

[8] A point already noted in Ehrmann and Fratzscher (2007b).

Blanchard, O.J. and N. Kiyotaki (1987), "Monopolistic competition and the effects of aggregate demand," *American Economic Review*, 77: 647-666.

Blinder, A., M. Fratzscher, J. de Haan and D. J. Jansen (2008), "Central bank communication and monetary policy: A survey of theory and evidence," Working Paper No 13932, NBER, Cambridge, MA.

Eusepi, S. and B. Preston (2007), "Central bank communication and expectations stabilization," Working Paper No. 13259, NBER, Cambridge, MA.

Faust, J. and Svensson L.E.O. (2002), "The equilibrium degree of transparency and control in monetary policy," *Journal of Money, Credit and Banking* 34: 520-539.

Eggertson, G. and B. Pugsley (2006), "The mistake of 1937: A general equilibrium analysis" 4th International Research Forum on Monetary Policy, Board of Governors, Federal Reserve, Washington DC; Bank of Japan, *Monetary and Economic Studies*, 24, 151-90.

Ehrmann, M. and M. Fratzscher (2005), "How should central banks communicate?," Working Paper No. 557, European Central Bank, Frankfurt.

Ehrmann, M. and M. Fratzscher (2007a), "Communication by committee members: Different strategies, same effectiveness?," *Journal of Money Credit and Banking*, 39, 509-41.

Ehrmann, M. and M. Fratzscher (2007b), "The timing of central bank communication," *European Journal of Political Economy*, 23, 124-45.

Holly, S. and A. Hughes Hallett (1989), *Optimal control, expectations and uncertainty*, Cambridge: Cambridge University Press.

Hughes Hallett, A.J and P. G. Fisher (1988), "Efficient solution techniques for linear and non-linear rational expectations models," *Journal of Economic Dynamics and Control*, 12: 635-657.

Hughes Hallett, A.J., Y Ma and Y. P. Yin (1996), "Hybrid algorithms with automatic switching for solving nonlinear equation systems in economics," *Journal of Economic Dynamics and Control*, 20: 1051-71.

Hughes Hallett, A., G Di Bartolomeo and N Acocella (2008), "Controllability under rational expectations," Working Paper No. 42, Department of Communication, University of Teramo, Italy.

Jansen, D.J. and J. De Haan (2006), "Look who's talking: ECB communication during the first years of EMU," *International Journal of Finance and Economics*, 11, 219-28.

Libich, J. (2008) "A note on the anchoring effect of explicit inflation targets" *Macro-economic Dynamics* (forthcoming).

Lucas, R.E. (1976), "Econometric policy evaluation. A critique," *Journal of Monetary Economics*, Supplement, Carnegie-Rochester Conference Series on Public Policy, 1: 19-46.

Rogoff, K. (1985), "The optimal degree of commitment to an intermediate monetary target," *Quarterly Journal of Economics*, 100: 1169-1189.

Rudebusch, G. and J. Williams (2008), "Revealing secrets of the temple: The value of publishing central bank interest rate projections," in J. Y. Campbell (ed), *Asset Prices and Monetary Policy,* NBER-University of Chicago Press, Chicago, IL.

Sargent, T.J. and N. Wallace (1975), "Rational expectations, the optimal monetary instrument, and the optimal money supply rule," *Journal of Political Economy*, 83: 241-254.

Tinbergen, J. (1952), *On the theory of economic policy*, Amsterdam: North Holland.

Trichet, J-C. (2008), "Undervalued risk and uncertainty: some thoughts on the market turmoil," Speech to the 5th ECB Central Banking Conference, European Central Bank, Frankfurt, 13 November.

Visser, B. and O. Swank (2007), "On committees of experts," *Quarterly Journal of Economics*, 122, 337-72.

Walsh, C.E. (2007), "Optimal economic transparency," *International Journal of Central Banking*, 3: 5-36.

Woodford M. (2003), *Interest rate and prices,* Princeton University Press, Princeton.

Woodford, M. (2005), "Central bank communication and policy effectiveness," in *The Greenspan Era: Lessons for the Future,* Kansas City Fed, Kansas City, MO.