DISCUSSION PAPER SERIES

No. 7066

PUBLIC GOODS, PARTICIPATION CONSTRAINTS, AND DEMOCRACY: A POSSIBILITY THEOREM

Hans Peter Grüner

PUBLIC POLICY

Centre for Economic Policy Research

www.cepr.org

Available online at: **www.cepr.org/pubs/dps/DP7066.asp**

ISSN 0265-8003

PUBLIC GOODS, PARTICIPATION CONSTRAINTS, AND DEMOCRACY: A POSSIBILITY THEOREM

Hans Peter Grüner, Universität Mannheim and CEPR

Discussion Paper No. 7066
December 2008

Centre for Economic Policy Research
53–56 Gt Sutton St, London EC1V 0DG, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 7066

December 2008

ABSTRACT

Public goods, participation constraints, and democracy: A possibility theorem*

It is well known that ex post efficient mechanisms for the provision of indivisible public goods are not interim individually rational. However, the corresponding literature assumes that agents who veto a mechanism can enforce a situation in which the public good is never provided. This paper instead considers majority voting with uniform cost sharing as the relevant status quo. Efficient mechanisms may then exist, which also satisfy all agents' interim participation constraints. In this case, ex post inefficient voting mechanisms can be replaced by efficient ones without reducing any individual's expected utility. Intuitively, agents with a low willingness to pay have to contribute more under majority rule than under an efficient mechanism with a balanced budget. This possibility theorem is not universal in the sense of Schweizer (Games and Economic Behavior, 2005).

JEL Classification: D02, D61, D71 and H41
Keywords: ex post efficiency, majority voting, participation constraints, possibility theorem and public goods

Hans Peter Grüner
Lehrstuhl für Wirtschaftspolitik
Universität Mannheim
68131 Mannheim
GERMANY

Email: hgruener@rumms.uni-mannheim.de

Submitted 19 November 2008

* I thank Felix Bierbrauer, Pierre Boyer, Martin Hellwig, Benny Moldovanu, Patrick Schmitz, Urs Schweizer and Michael Vogt as well as seminar participants in Bonn and Mannheim for their many helpful comments and suggestions.

1 Introduction

The present paper studies potential improvements of decision procedures for public projects. It addresses the question whether ex post inefficient voting mechanisms can be replaced by ex post efficient ones without reducing any individual's expected payoff. I show that, dependent upon the cost of the public project, there are cases in which all agents unanimously prefer an ex post efficient mechanism to simple majority voting at the interim stage.

Mechanism design theory has produced several helpful procedures for collective decision making. Two prominent examples are the second price auction and, as a generalization, the Vickrey Clarke Groves (VCG) mechanism. While the second-price auction is frequently used for the allocation of indivisible private goods, the VCG mechanism for indivisible public projects can rarely be found in practice. A standard argument why VCG mechanisms are not used for decisions about public projects is that they are too costly for some citizens.

In deed, it is a well known result that ex post efficient mechanisms for the provision of indivisible public goods are generally not interim individually rational[1]. Under a VCG mechanism, agents who do not care about the public good may have to contribute to finance its provision or, alternatively, they have to pay for the externality that they generate when the good is not provided. However, the corresponding literature assumes that agents who veto a mechanism can enforce a situation in which the public good is never provided. Today, in most economies public goods are provided and mechanisms for the provision of public goods are used.[2] Therefore, the question whether the existing mechanisms can be replaced by ex post efficient ones can not be answered on the grounds of the existing literature.

Frequently, decisions about public goods are based on some variant of a voting mechanism. Such voting mechanisms also violate some agents' participation constraints when

[1]See Güth and Hellwig (1986), Mas Colell, Whinston and Green (1995, Chapter 23), Mailath and Postlewaite (1998), and Hellwig (2003) as well as Krishna and Perry (2000) and Schweizer (2005) for simplified proofs and generalizations. A method to conveniently calculate the budget deficit under an ex-post efficient mechanism with binding interim participation constriants can be found in Krishna and Perry (2000).

[2]Note that even in absence of any "man-made" decision mechanism, all agents could choose to voluntarily contribute to the economies' public goods. However, this yields a higher interim utility to all agents than the case with no-provision. Therefore, the known impossibility theorems also apply to this case.

the cost of the public good has to be shared by all agents. Under majority voting, the procurement of a public good will either lead to an increase of the tax burden for all individuals or to a reduction of the budget for future expenditure, which can be viewed as an opportunity cost. In the present paper, I assume that this cost of the public good is shared equally among all agents. Agents decide whether they are willing to replace an existing voting procedure by an efficient mechanism after learning their own type, but before playing the voting game.

A voting mechanism asks both players for a binary signal and monotonously maps these signals into a decision. Not all voting mechanisms yield a possibility result. The possibility result holds for simple majority rule. This mechanism requires that a majority of agents favors the procurement of the public good, while, in case of a tie, the decision is made randomly. With such a mechanism in place, an alternative ex post efficient VCG mechanism may exist, which also satisfies all types' interim participation constraints. Intuitively, agents with a low willingness to pay have to contribute more under majority rule than under an efficient mechanism with a balanced budget.

My possibility theorem is not universal in the sense of Schweizer (2005). Universality would require that the participation constraints can be fulfilled independently of the distribution of types. In the present setup, it depends on the cost of the public project whether the possibility theorem holds. I also show that the type with the smallest interim gain from a new mechanism need not be located at the boundary of the type space. This is why Schweizer's (2005) method to derive (universal) possibility theorems can not be applied to the present problem.

In section 2, I begin with the analysis of the basic case of two agents with uncorrelated, uniformly distributed types. This case yields a simple analytical solution for agents' interim utility. Section 2 also has the main result - a possibility theorem for intermediate cost parameters. Section 3 presents further possibility results for normally distributed types and larger populations.

In Sections 4 and 5, I discuss the robustness of these findings in various settings. I first elaborate on the case of a large population. Possibility results for skewed distributions disappear when the population size increases. But it may also be the case that a possibility result is not robust with regards to population size when types are distributed symmetrically. I also show that the variance of the distribution of types plays a major role in shaping agents' preferences about institutions.

In section 5, I study two examples with correlated types. When types are correlated, the probability of provision is no longer monotonous in an agent's type. Skewed distributions yield impossibility results for large populations. The reason is that agents with a low willingness to pay may rely on the fact that a majority opposes the provision of the public good when the median of the distribution of types is smaller than the mean. However, when the shape of the actual distribution is not known to the agents in advance, a possibility result with positive welfare gains may obtain - even in large populations.

Related literature

The present paper contributes to the literature on the ex post efficient provision of public goods, including in particular the papers by Güth and Hellwig (1986), Mailath and Postlewaite (1992), and Hellwig (2003). Güth and Hellwig (1986) prove an impossibility result à la Myerson Satterthwaite in the public goods context. Mailaith and Postlewaite (1992) consider the case with a large number of agents when the size of the project varies with the size of the population. They find that the second best surplus of the economy goes to zero when the population size grows. Hellwig (2003) instead considers the case where the size of the project does not grow unboundedly. Positive (possibility) results can be found in papers which assume correlated types such as McAfee and Reny (1985). However, if one introduces stricter equilibrium concepts such as Bergemann and Morris's (2005) robustness criterion, the results are again negative (Bierbrauer and Hellwig, 2008).

Another way of guaranteeing participation in efficient mechanisms that has recently been studied in the literature, is the bundling of various decisions (Casella, 2005, Fang and Norman 2005, 2008, and Jackson and Sonnenschein, 2007).

The present analysis follows closely an idea that has been developed in Cramton, Gibbons, Klemperer (1993). Their paper shows in a trade context that initial conditions are key in determining whether or not a possibility or an impossibility theorem à la Myerson and Satterthwaite holds. In another related paper, Schmitz (2002) also considers the case of a public project. He shows that a possibility theorem may obtain when the status quo is a stochastic decision with an exogenously fixed provision probability. In such a case, agents' interim status quo utility is linear in an agent's type while the VCG interim utility is strictly convex. A possibility theorem holds because the VCG interim utility of the indifferent type (i.e. the type whose gross willingness to pay equals the average cost) is positive. This agent's payoff equals the average externality that he generates. As will become clear below, the status quo utility in the present paper is piecewise linear instead which makes the formal analysis more complicated.

2 The case of a uniform distribution

2.1 The setup

This section studies a case with $n = 2$ agents with uniformly distributed types. Cases with a larger number of agents will be discussed in sections 3 and 4. The notation introduced here will also be used in the rest of the paper. Consider two agents, $i = 1, 2$ who decide on the provision of a non-rival indivisible public good. The decision is denoted by $x \in \{0, 1\}$ and agents' payoff derived from the consumption of the good is

$$u_i(x, \theta_i) = \theta_i x. \tag{1}$$

The parameter θ_i is distributed uniformly on the unit interval. It is private information of agent i. An agent's total payoff v_i is additively separable in $u_i(x, \theta_i)$ and a monetary transfer t_i that is paid to the agent.

$$v_i(x, \theta_i, t) = u_i(x, \theta_i) + t_i. \tag{2}$$

A planner can procure the good at a known cost of $C = n \cdot c < 2$.

2.2 The status quo

I consider four alternative status quo situations. First, the conventional one, called no provision. In this situation the public good is never provided - independently of the realization of both agents' private information. Under the second status quo mechanism (maximum rule) both agents simultaneously and independently cast a vote $\gamma_i \in \{0, 1\}$. The good is bought if and only if at least one agent votes in favor of the provision, i.e. $x = \max\{\gamma_2, \gamma_2\}$. Moreover, in case of provision, each agent has to cover half of the cost, c. The corresponding direct mechanism would ask both agents for their type and buys the good if and only if one agent's valuation is at least c. This mechanism is incentive compatible but, in general, it is not ex post efficient.

Under the third mechanism (majority rule with tie breaking) agents also cast votes $\gamma_i \in \{0, 1\}$. The good is bought if both agents vote in favor of provision, it is not bought when both agents vote against. In case of a tie the good is provided with probability 1/2. When the good is provided, each agent has to cover half of the cost, c. Finally, I consider provision under unanimity rule: $x = \min\{\gamma_2, \gamma_2\}$.

2.3 Efficient Mechanisms

As an alternative to the status quo we consider a budget balanced Vickrey Clarke Groves (VCG) mechanism. This mechanism efficiently decides on the provision of the public good. I denote by $x = f(\theta_1, \theta_2)$ the ex post efficient social project choice and by $U_i(\theta_i)$ the interim expected utility of agent i when $f(\theta_1, \theta_2)$ gets implemented. The interim probability that the good is provided is given by

$$\pi(\theta_i) = \begin{cases} 0 & \text{if } \theta_i < 2c - 1 \\ 1 - (2c - \theta_i) & \text{if } 2c - 1 \leq \theta_i \leq 2c \\ 1 & \text{if } \theta_i > 2c \end{cases}. \tag{3}$$

Denote by $U_i(\theta_i)$ the interim expected utility of agent i when his type is θ_i. In this subsection, I calculate the value of $U_i(0)$ which balances the budget of the social planner. Two cases have to be distinguished.

2.3.1 The case where $2c \leq 1$

When $2c \leq 1$ it may be the case that a single agent has a high enough willingness to pay to make sure that the good is provided. For $\theta_i \leq 2c$ the interim expected utility (including transfers) is given by

$$\begin{aligned} U_i(\theta_i) \quad : \quad &= E_{\theta_{-i}} v_i(x(\theta), \theta_i, t(\theta)) && (4) \\ &= U_i(0) + \int_0^{\theta_i} (1 - (2c - s))\, ds && (5) \\ &= U_i(0) + (1 - 2c)\theta_i + \frac{1}{2}\theta_i^2. && (6) \end{aligned}$$

For values $\theta_i > 2c$, $U_i(\theta_i)$ increases linearly with slope 1. To summarize, interim utility is given by the following expression.

$$U_i(\theta_i) = \begin{cases} U_i(0) + (1 - 2c)\theta_i + \frac{1}{2}\theta_i^2. & \text{if } \theta_i < 2c \\ U_i(0) - \frac{1}{2}(2c)^2 + \theta_i & \text{if } \theta_i \geq 2c \end{cases}. \tag{7}$$

Under an efficient mechanism, agent i derives an interim expected utility of

$$E_{\theta_{-i}} u_i(x, \theta_i) = \begin{cases} (1 - 2c)\theta_i + \theta_i^2 & \text{if } \theta_i < 2c \\ \theta_i & \text{if } \theta_i \geq 2c \end{cases} \tag{8}$$

from the choice of the project. Taking differences, we get that he receives expected transfers of

$$E_{\theta_{-1}} t_i = \begin{cases} U_i(0) - \frac{1}{2}\theta_i^2 & \text{if } \theta_i < 2c \\ U_i(0) - 2c^2 & \text{if } \theta_i \geq 2c \end{cases}. \tag{9}$$

Total expected transfers made to one agent are

$$E_{(\theta_1,\theta_2)} t_i = U_i(0) + \int_0^{2c} -\frac{1}{2}\theta_i^2 d\theta_i + (1-2c)\left(-2c^2\right) \tag{10}$$

$$= U_i(0) + \frac{16}{6}c^3 - 2c^2. \tag{11}$$

The ex-ante probability that the good is provided is

$$p(c) = \begin{cases} 1 - \frac{(2c)^2}{2} & \text{if } 2c < 1 \\ 2(1-c)^2 & \text{if } 2c \geq 1 \end{cases}. \tag{12}$$

Hence, for $c < 1/2$ the budget is balanced in expected terms when

$$-2U_i(0) - \frac{32}{6}c^3 + 4c^2 - \left(1 - \frac{(2c)^2}{2}\right)2c = 0, \tag{13}$$

where the last term represents the expected cost of provision. This last condition yields:

$$U_i(0) = -\frac{2}{3}c^3 + 2c^2 - c. \tag{14}$$

Interim expected utility is given by (7) and (14).

2.3.2 The case where $2c > 1$

When $2c > 1$ it may be the case that a single agent's willingness to pay is so low ($\theta_i < 2c - 1$) that the public good is never provided. The interim expected utility is given by

$$U_i(\theta_i) = \begin{cases} U_i(0) & \text{if } \theta_i < 2c-1 \\ U_i(0) + \int_{2c-1}^{\theta_i} (1-(2c-s))\, ds & \text{if } \theta_i \geq 2c-1 \end{cases}. \tag{15}$$

Under an efficient mechanism, agent i derives an interim expected utility of

$$E_{\theta_{-i}} u_i(x,\theta_i) = \begin{cases} 0 & \text{if } \theta_i < 2c-1 \\ (1-2c)\theta_i + \theta_i^2 & \text{if } \theta_i \geq 2c-1 \end{cases} \tag{16}$$

from the choice of the project. Taking differences, we get that he receives expected transfers of

$$E_{\theta_{-1}} t_i = \begin{cases} U_i(0) & \text{if } \theta_i < 2c-1 \\ U_i(0) - \frac{1}{2}\theta_i^2 + \frac{1}{2}(2c-1)^2 & \text{if } \theta_i \geq 2c-1 \end{cases}. \tag{17}$$

Total expected transfers made to one agent are

$$E_{(\theta_1,\theta_2)} t_i = U_i(0) + \int_{2c-1}^{1} -\frac{1}{2}\theta_i^2 + \frac{1}{2}(2c-1)^2 \, d\theta_i \tag{18}$$

$$= U_i(0) - \frac{1}{6} + \frac{1}{6}(2c-1)^3 + (2c-1)^2(1-c). \tag{19}$$

The planner's budget constraint yields:

$$U_i(0) = \frac{2}{3}c^3 - 2c^2 + 2c - \frac{2}{3}. \tag{20}$$

One can summarize the previous results as follows.

Lemma 1 *Interim expected utility under a balanced budget VCG mechanism is given by*

$$U_i(\theta_i) = \begin{cases} -\frac{2}{3}c^3 + 2c^2 - c + (1-2c)\theta_i + \frac{1}{2}\theta_i^2 & \text{if } c \leq 1/2 \text{ and } \theta_i < 2c \\ -\frac{2}{3}c^3 + 2c^2 - c - \frac{1}{2}(2c)^2 + \theta_i & \text{if } c \leq 1/2 \text{ and } \theta_i > 2c \end{cases}. \tag{21}$$

and

$$U_i(\theta_i) = \begin{matrix} \frac{2}{3}c^3 - 2c^2 + 2c - \frac{2}{3} & \text{if } c > 1/2 \text{ and } \theta_i < 2c-1 \\ \frac{2}{3}c^3 - 2c^2 + 2c - \frac{2}{3} + (1-2c)\theta_i + \frac{1}{2}\theta_i^2 + \frac{1}{2}(2c-1)^2 & \text{if } c > 1/2 \text{ and } \theta_i > 2c-1 \end{matrix}. \tag{22}$$

Figure 1 displays the lowest interim utility $U_i(0)$ for various values of c. It shows that agents with lowest valuations are not willing to participate in an ex post efficient mechanism when the outside option is the no-provision decision. The highest loss arises for intermediate values of c because higher costs also reduce the efficient interim probability of provision.

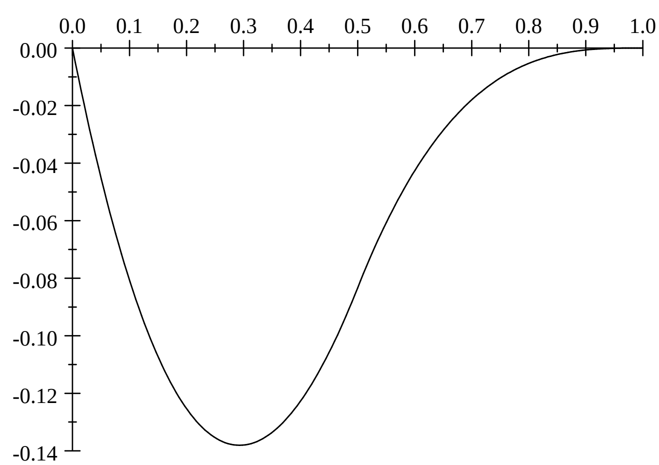

Figure 1: Minimum interim utility $U_i(0)$ as a function of the cost parameter c.

2.4 A possibility theorem

I now compare interim utility under the VCG mechanism and under the three different voting rules. Under unanimity rule, interim utility $D_i(\theta)$ is given by

$$D_i(\theta) = \begin{cases} 0 & \theta < c \\ (1-c)(\theta - c) & \theta \geq c \end{cases}. \tag{23}$$

It immediately follows that $D(0) > M(0)$ and $D(1) < M(1)$. A consequence is the following impossibility result.

Proposition 1 *When the status quo is a vote under unanimity rule, ex post efficiency, balanced budget and interim participation cannot simultaneously be satisfied.*

A similar result holds under the maximum rule. Under the maximum rule, interim utility $M(\theta_i)$ is also piecewise linear.

$$M(\theta_i) = \begin{cases} (1-c)(\theta_i - c) & \text{if } \theta < c \\ (\theta_i - c) & \text{if } \theta \geq c \end{cases}. \tag{24}$$

This status quo gives an advantage to high types who can alone enforce their desired outcome. High types prefer this status quo to an efficient mechanism.

Proposition 2 *When the status quo is a vote under the maximum rule, the highest type always opposes the ex post efficient mechanism. The lowest type favors the ex post efficient mechanism.*

PROOF See appendix.

Consider next the interim utility under majority voting with a tie breaking rule (simple majority rule). Utility is piecewise linear and given by

$$M(\theta_i) = \begin{cases} \frac{1}{2}(1-c)(\theta_i - c) & \theta < c \\ \left(1 - \frac{c}{2}\right)(\theta_i - c) & \theta \geq c \end{cases}. \tag{25}$$

The main result of this paper is that, for intermediate values of the cost parameter c, a possibility theorem holds.

Proposition 3 *For intermediate cost parameters $c \in [\underline{c}, \bar{c}]$ with $0 < \underline{c} < 1/2 < \bar{c} < 1$ (i) the efficient project choice can not be implemented with a balanced budget when all agents have to participate voluntarily at the interim stage and when the outside option is the no-provision decision (ii) the efficient project choice can be implemented with a balanced budget and under the interim participation constraints when the outside option is a vote under simple majority rule. (iii) For cost parameters c that are too high or too low, an impossibility theorem holds.*

PROOF See the appendix.

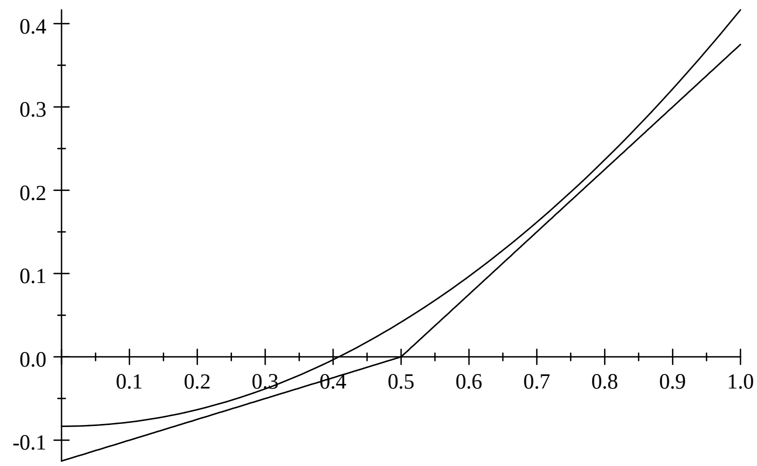

Figure 2: Agents' interim utility in a democracy (piecewise linear) and under a VCG mechanism (convex) for the case $c = 1/2$.

The previous results can be summarized as follows. When the status quo is a voting procedure, it may be possible to improve the decision process further by introducing a VCG mechanism. This yields a higher surplus and, for intermediate costs, a higher interim utility for all agents. Very strong or very weak majority requirements change this result. Under unanimity rule, everybody has to agree to the production of the public good. Therefore, the interim utility of low types is zero and the known impossibility theorem applies. When only one positive vote is required, the high types oppose the VCG mechanism.

Figure 2 compares the two interim utilities under a VCG mechanism and under majority rule for the case $c = 1/2$. The relative gain in expected welfare from a VCG mechanism is $33,\bar{3}\%$. Note that this is the gain with respect to all three voting mechanisms. However, it can only be realized when the status quo is simple majority rule. Another interesting observation in this case is that no participation constraint is binding. Accordingly, the planner could even raise extra revenues by organizing the public decision efficiently.

3 Normally distributed types

A possibility theorem also holds for another symmetric distribution, the normal distribution. I this section, I report additional results for normally distributed types. Consider

$n = m + 1$ agents whose net willingness to pay $\tilde{\theta} := \theta - c$ is normally distributed with mean zero and variance 1.[3] The cumulative density of the sum of m other agents

$$F(\theta) = \int_{-\infty}^{\theta} \frac{1}{(m \cdot 2\pi)^{\frac{1}{2}}} e^{-\frac{1}{2}\frac{x^2}{m}} dx = \frac{1}{2} \operatorname{erf}\left(\frac{1}{2m}\sqrt{2m}\theta\right) + \frac{1}{2}. \tag{26}$$

The interim probability of provision of the public good for a given valuation θ_i can then be expressed as follows.

$$\pi\left(\tilde{\theta}_i\right) = 1 - F(-\tilde{\theta}_i) = \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left(\frac{1}{2m}\sqrt{2m}\tilde{\theta}_i\right). \tag{27}$$

Agents' direct utility from the decision is

$$\pi\left(\tilde{\theta}_i\right) \cdot \tilde{\theta}_i = \left(\frac{1}{2} - \frac{1}{2} \operatorname{erf}\left(\frac{1}{2m}\sqrt{2m}\tilde{\theta}_i\right)\right) \cdot \theta. \tag{28}$$

An agent with type $\tilde{\theta}_i$ generates an expected externality of

$$x\left(\tilde{\theta}_i\right) = \int_0^{\theta_i} \left(\frac{1}{(m \cdot 2\pi)^{\frac{1}{2}}} e^{-\frac{1}{2}\frac{x^2}{m}} \cdot x\right) dx. \tag{29}$$

Figure 3 has the expected externalities for different values of m and different types θ.

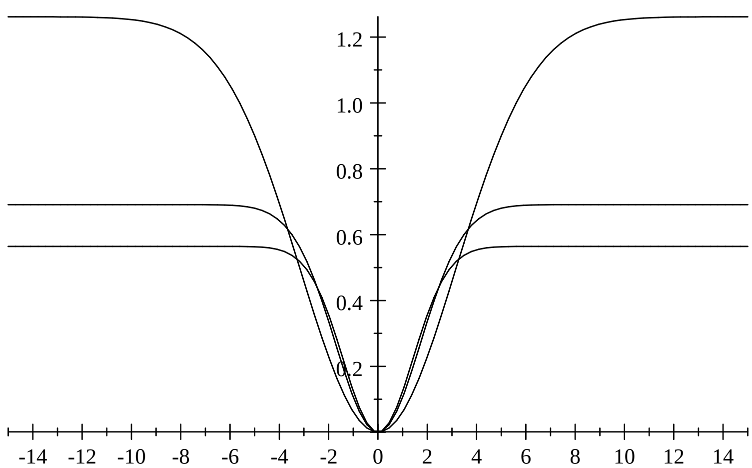

Figure 3: The expected externalities for different realizations of θ and population sizes $m = 2, 3, 4$.

[3]This implies that there is an expected benefit from consumption while some agents may actually dislike the public good.

The planner's expected revenue is

$$r := \int_{-\infty}^{\infty} \left(\frac{1}{(m \cdot 2\pi)^{\frac{1}{2}}} e^{-\frac{1}{2}\frac{x^2}{m}} \cdot x\left(\tilde{\theta}_i\right) \right) dx \tag{30}$$

This revenue can be redistributed to all agents. Hence, the interim payoff of an agent is given by $\pi(\theta) \cdot \theta - x\left(\tilde{\theta}_i\right) + r$. For odd n the interim payoff under majority vote is given by

$$M\left(\tilde{\theta}_i\right) = \begin{cases} \left(\frac{1}{2}\right)^n \sum_{i=\frac{n}{2}+1}^{n} \binom{n}{i} \tilde{\theta}_i & \tilde{\theta}_i < 0 \\ \left(\frac{1}{2}\right)^n \sum_{i=\frac{n}{2}}^{n} \binom{n}{i} \tilde{\theta}_i & \tilde{\theta}_i \geq 0 \end{cases}. \tag{31}$$

For even n it is given by

$$M\left(\tilde{\theta}_i\right) = \begin{cases} \left(\frac{1}{2}\right)^n \left(\sum_{i=\frac{n+3}{2}}^{n} \binom{n}{i} \tilde{\theta}_i + \binom{n}{\frac{n+1}{2}} \right) \tilde{\theta}_i & \tilde{\theta}_i < 0 \\ \left(\frac{1}{2}\right)^n \left(\sum_{i=\frac{n+1}{2}}^{n} \binom{n}{i} \tilde{\theta}_i + \binom{n}{\frac{n-1}{2}} \right) \tilde{\theta}_i & \tilde{\theta}_i \geq 0 \end{cases}. \tag{32}$$

Figure 4 makes clear why possibility theorems may hold in this particular setup. It shows the interim decision utility $E_{\theta_{-i}} u_i(x, \theta_i)$ as a function of an agent's types under a VCG mechanism. The utility is not monotonous because very low types may expect that their willingness to pay changes the decision in their favor. This is different under a voting mechanisms where this interim payoff is strictly monotonous and unbounded from below.

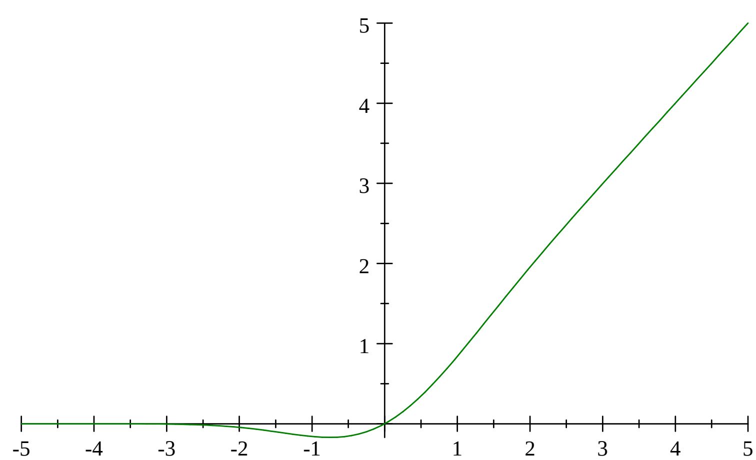

Figure 4: Decision utility. Normally distributed types, $n = 2$.

I numerically approximated the values of the per capita expected revenue r for various values of m using the lower integral sum. The values are reported in Table 1.

m	1	2	3	4	10
r	0.116 91	0.165 02	0.199 71	0.224 01	0.473 25

Table 1: Expected revenues from the expected externalities.

The following figures compare the interim utilities under majority voting and under the balanced budget VCG mechanism for different population sizes n, taking these values into account. In the cases considered, a possibility result holds.

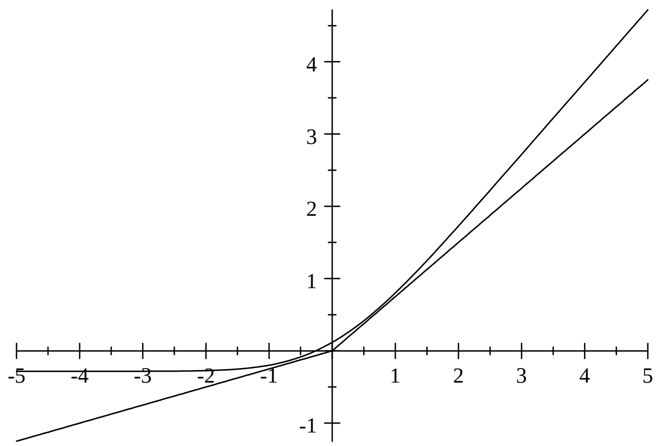

Figure 5a: Normally distributed types, $n = 2$.

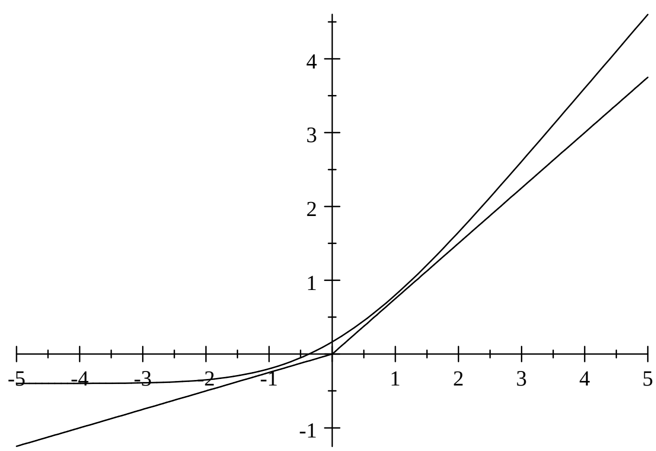

Figure 5b: Normally distributed types, $n = 3$.

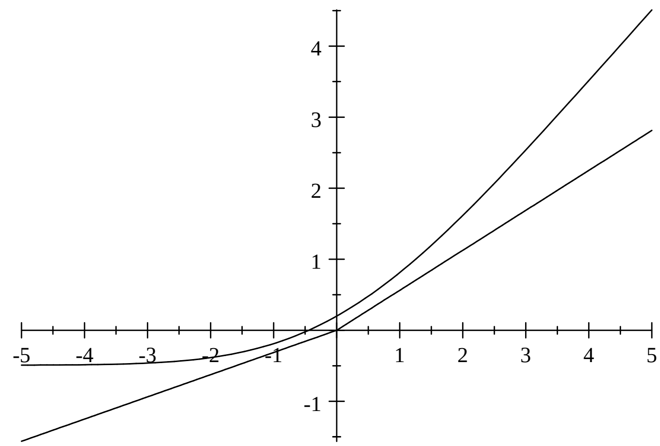

Figure 5c: Normally distributed types, $n = 4$.

4 Robustness

4.1 Increasing the number of agents

The previous analysis shows that possibility results may hold in small populations when simple majority voting is the relevant status quo. How relevant is this possibility result for larger populations? And what are the consequences when types are correlated? In this and in the following section, I discuss both questions in various settings.

It is first useful to note that agents' interim payoffs fulfill the following monotonicity property when all agents types are distributed according to the same density function $\phi(\theta_i)$.

Lemma 2 *Assume that all agents' types are distributed according to the same density function $\phi(\theta_i)$. Suppose that agent i votes in favor of the provision of a public good if and only if $\theta_i \geq 1/2$. For any distribution of types with mean $\bar{\theta}$ and median θ^M the interim probability of provision under simple majority rule converges monotonously to*

$1/2$ *for* $\bar{\theta} = \theta^M$,

0 *for* $\bar{\theta} > \theta^M$,

1 *for* $\bar{\theta} < \theta^M$,

as n goes to infinity. The interim probability of provision under a VCG mechanism equals $1/2$ for $\bar{\theta} = c$ and converges monotonously to

0 *for* $\bar{\theta} > c$,

1 *for* $\bar{\theta} < c$,

as n goes to infinity.

Proof Straightforward.

I begin with the analysis of a symmetric distribution of types $\phi(\theta_i)$ and an arbitrary finite number of agents $n > 2$. The following result describes the properties of the interim payoffs.

Lemma 3 *Consider an economy with an even number of agents. For any symmetric distribution of types $\phi(\theta_i)$ with mean $\bar{\theta} = c$, and a positive density on the support $[0, 2c]$, the difference of agents' interim payoffs with respect to simple majority rule is symmetric around $\bar{\theta}$.*

Proof The interim probability of provision for the VCG mechanism $\pi(\theta_i)$ fulfills $\pi(\theta_i) = 1 - \pi(1 - \theta_i)$. It is an easy exercise to show that the interim probability of provision for simple majority rule fulfills the same condition when $\theta_i \neq c$. For $\theta_i \neq c$, interim utility equals 0 independently of agent i's voting behavior. The lemma follows from the fact that the expected externality generated by an agent of type θ is zero for $\theta = c$ and symmetric around c. Q.E.D.

We also know from Schmitz (2002) that the interim utility of the type $\theta = c$ under the efficient mechanism exceeds zero. A consequence of this and the two Lemmata above is that, for symmetric distributions around $\bar{\theta} = c$, there is a symmetric interval around $\bar{\theta}$ on which all agents with types in this interval strictly prefer the VCG mechanism to majority voting. As n goes to infinity, all utility gains (or losses) disappear in the case of a symmetric distribution.

With a continuum of agents and when all agents' types are drawn independently from the same density distribution, the actual distribution of types coincide with the underlying probability distribution. The position of the distribution's mean and median determine the outcome under majority voting and under an efficient mechanism. With a continuum of agents there are no externalities in an equilibrium of a VCG mechanism. Therefore, the allocation of any balanced budget VCG mechanism and that of the majority voting mechanism coincide whenever the median and the mean of the distribution are on the same side of the cost parameter c. Hence, with a continuum of agents, a (trivial) possibility theorem holds if and only if the mean $\bar{\theta}$ and median θ^M of the distribution of types are on the same side of the cost parameter c. Another consequence of this is that the same asymmetric distribution of types $\phi(\theta_i)$ may yield a possibility result for small n but not for large n.

Proposition 4 *There are distributions $\phi(\theta_i)$ with $\bar{\theta} \neq \theta^M$ such that, for small enough n, a possibility theorem holds while for large enough n, an impossibility theorem holds.*

PROOF It suffices to consider one of the asymmetric distributions which yield a possibility theorem in Proposition 3. Q.E.D.

But even when the distribution of types is symmetric, a possibility result need not be robust when one increases the number of agents. Figure 5 shows the difference of the interim payoffs studied the case of a symmetric and uniform distribution of types and $n = 3$ agents. It shows that some types above and below the cost $c = 0.5$ prefer the status quo to the ex-post efficient mechanism. Figure 6 makes clear that, even though the participation constraints do not hold, the problem with participation is comparably small to the one that obtains with the conventional status quo. The highest loss is about 0.8 % of the per capita cost while it equals 100 percent of the per capita cost in the case with a no-provision status quo.

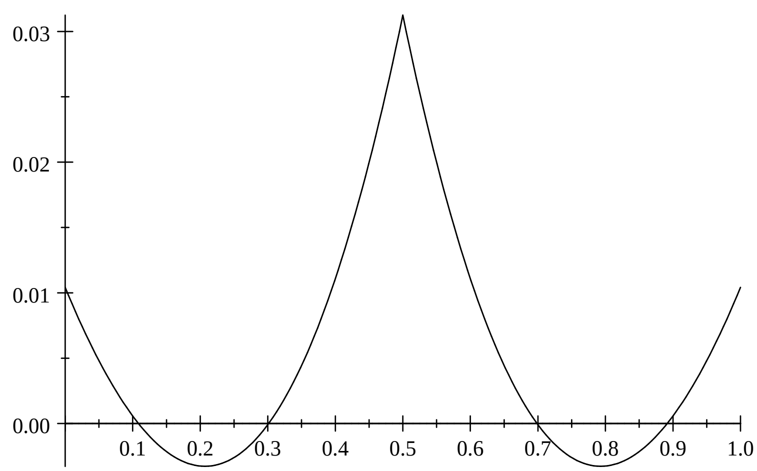

Figure 6: Difference of interim utilities, uniform distribution, $n = 3$.

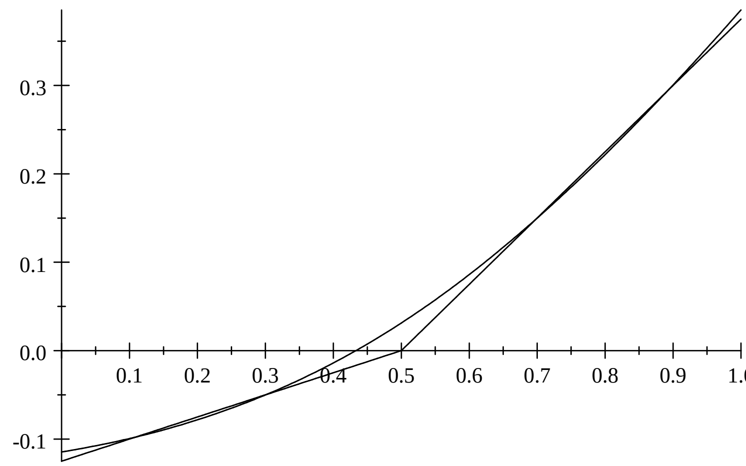

Figure 7: VCG and majority voting interim utilities, $n = 3$.

4.2 The role of the variance

With a finite number of agents, the variance of the distribution of types also plays an important role for agents' evaluation of different institutions. Consider for example the case of a symmetric distribution of types with support $[0, 2c]$. We already know that a possibility theorem holds in the case of a uniform distribution. Increasing the variance may lead to an impossibility result. To see why, consider the case with the highest possible variance. It is obtained for a binary distribution where an agent's valuation can either be 0 or $2c$. Consider an efficient expected externality mechanism which yields the provision of the public good with a 50 percent chance when the population is indifferent between provision and non-provision. In this case the expected probability of provision is the same as under majority voting. However, under a VCG mechanism, agents with high valuations have to pay for their expected externality. This externality exceeds the average externality, so agents with $\theta = 2c$ are net contributors. Therefore, for the highest possible variance, an impossibility theorem holds. With a variance going to zero instead, one can easily show that the interim utility of the VCG mechanism approaches zero below c and $\theta - c$ above c.

5 Correlated types

5.1 A possibility result for hybrid densities

When the distribution of types is common knowledge, and when the number of agents is large, one could always directly choose to implement the appropriate decision. This is why this case is of little interest. In the rest of this paper, I concentrate on the more interesting case of correlated types. When agents' types are correlated, the conventional formula (5) for agents' interim utility can no longer be applied. Agents with a low willingness to pay can conclude that it is more likely that many other agents also have a low willingness to pay. This affects their beliefs about the outcomes of the majority voting mechanism and the VCG mechanism.

I first present a general possibility result for correlated types. I then turn to one particular distribution of types for which an impossibility theorem holds. Finally, I present a case with a continuum of agents and a possibility result.

Consider the following modification of a given uncorrelated distribution of types with density $\phi\left(\tilde{\theta}_i\right)$ for which a possibility theorem holds. Call α the probability with which the joint distribution is again drawn independently according to $\phi\left(\tilde{\theta}_i\right)$. Moreover, let the types of all agents coincide with probability $1-\alpha$. In this case, assume that all types are drawn according to $\phi(\cdot)$. A possibility theorem trivially applies for values $\alpha = 0$ (perfect correlation). By assumption, a possibility theorem also holds for the case of independent types, $\alpha = 1$. In between, one has to show that the interim utility $U\left(\tilde{\theta}_i, \alpha\right)$ exceeds the interim utility under majority voting $M\left(\tilde{\theta}_i, \alpha\right)$ for all types.

It is not difficult to show that the interim utility of all agents with valuations below c is $\alpha U\left(\tilde{\theta}_i\right)$. The probability of provision equals α times the original probability with an i.i.d. distribution of types. Moreover, an externality only arises with probability α. The interim utility under majority voting $M\left(\tilde{\theta}_i, \alpha\right)$ also satisfies $M\left(\tilde{\theta}_i, \alpha\right) = \alpha M\left(\tilde{\theta}_i, 1\right)$ because, with probability $1-\alpha$ all types vote against provision of the public good. Therefore, $U\left(\tilde{\theta}_i, \alpha\right) \geq M\left(\tilde{\theta}_i, \alpha\right) \Leftrightarrow \alpha U\left(\tilde{\theta}_i, 1\right) \geq \alpha M\left(\tilde{\theta}_i, 1\right) \Leftrightarrow U\left(\tilde{\theta}_i, 1\right) \geq M\left(\tilde{\theta}_i, 1\right)$. To summarize:

Proposition 5 *Consider majority voting as the relevant status quo. Assume that the joint distribution of types is characterized as above with correlation parameter α. A possibility theorem for uncorrelated types ($\alpha = 1$) holds if and only if a possibility theorem holds for correlated types $\alpha \neq 1$.*

5.2 Shifting densities

The positive result from the previous section is not robust to changes of the underlying stochastic structure. To see why, consider the case with two agents and the following specification of the joint probability distribution of types. Both agent's types are drawn from the same uniform probability distribution $\phi(\theta_i|s)$ on with mean s and a support of size a. The parameter s in turn is drawn from another uniform distribution which also has a support of size $1-a$ and mean $\bar{\theta}=c=1$. The support of the entire type space is the interval $[0,2]$. This specification includes as special cases a setup with uncorrelated types ($a=0$) and perfectly correlated types ($a=1$). One can easily see, that a possibility theorem holds for both cases. At $a=0$, the possibility theorem from section 3 applies. At $a=1$, a possibility theorem also holds because the allocation under a VCG mechanism is the same as the one under majority voting.

I show by example that there may be intermediate values of a for which an impossibility theorem holds. Consider the case where $a=1/2$. As a consequence of Lemma 2 we may restrict our analysis of interim utilities to the lower half of the support, $\theta<1$. Consider an expected externality mechanism which redistributes expected revenues among both agents. For values $\theta<\frac{1}{2}$ an agent can be sure that the public good will not be provided. The interim expected externality that such an agent generates can be calculated as

$$\int_{\frac{1}{2}}^{\theta+\frac{1}{2}}\left(\int_{1}^{s+\frac{1}{2}}(x-1)\,dx\right)\frac{1}{\theta}ds=\frac{1}{6}\theta^{2}. \tag{33}$$

This expression takes into account, that the relevant support of s is the interval $\left[\frac{1}{2},\frac{1}{2}+\theta\right]$. Moreover, an externality arises only for types in the interval $\left[1,s+\frac{1}{2}\right]$.

Similarly, the expected externality for types $\theta\in\left[\frac{1}{2},1\right]$ is given by

$$\int_{\frac{1}{2}}^{\theta+\frac{1}{2}}\left(\int_{1}^{\min\left\{s+\frac{1}{2},2-\theta\right\}}(x-1)\,dx\right)\frac{1}{\theta}ds \tag{34}$$

$$=\frac{1}{\theta}\left(\int_{\frac{1}{2}}^{\frac{3}{2}-\theta}\left(\int_{1}^{s+\frac{1}{2}}(x-1)\,dx\right)ds+\int_{\frac{3}{2}-\theta}^{\theta+\frac{1}{2}}\left(\int_{1}^{2-\theta}(x-1)\,dx\right)ds\right) \tag{35}$$

$$=\frac{1}{\theta}\left(\frac{1}{2}(\theta-1)^{2}(2\theta-1)-\frac{1}{6}(\theta-1)^{3}\right). \tag{36}$$

Integration of (33) and (36) on the interval $[0,1]$ yields one half of the total expected externality generated by one agent.

$$2\cdot\left(\int_0^{\frac{1}{2}}\frac{1}{6}\theta^2 d\theta+\int_{\frac{1}{2}}^{1}\frac{1}{\theta}\left(\frac{1}{2}\left(\theta-1\right)^2\left(2\theta-1\right)-\frac{1}{6}\left(\theta-1\right)^3\right)d\theta\right)=\frac{1}{2}-\frac{2}{3}\ln 2. \tag{37}$$

The public good is provided with an interim probability of

$$\pi\left(\theta\right)=\begin{cases}0 & \theta<\frac{1}{2}\\ \theta-\frac{1}{2} & \frac{1}{2}\leq\theta\leq\frac{3}{2}\\ 1 & \theta>\frac{3}{2}\end{cases}. \tag{38}$$

Utility from the decision is

$$\pi\left(\theta\right)\cdot\left(\theta-1\right)=\begin{cases}0 & \theta<\frac{1}{2}\\ \left(\theta-\frac{1}{2}\right)\left(\theta-1\right) & \frac{1}{2}\leq\theta\leq\frac{3}{2}\\ 1 & \theta>\frac{3}{2}\end{cases}. \tag{39}$$

Under majority voting, the probability of provision for $\theta<1$ is instead given by

$$\frac{1}{2}\frac{\int_{\frac{1}{2}}^{\theta+\frac{1}{2}}\left(s+\frac{1}{2}-1\right)ds}{\theta}=\frac{1}{4}\theta, \tag{40}$$

and an agent's interim utility is$\frac{1}{4}\theta\left(\theta-1\right)$.

Figure 3 depicts the interim utilities for the voting mechanism and for the expected externality mechanism. The efficient project choice can not be implemented with an expected externality mechanism when all agents have to participate voluntarily at the interim stage and when the outside option is the no-provision decision. The same holds for simple majority rule. Types which are close enough to the mean of the type space conclude, that it is more likely, that the VCG mechanisms yields a positive decision about the public good than a majority vote. When they are not too close to the average cost c, the redistributed revenues of the VCG mechanism do not compensate for this. However, one can also see that the aggregate loss of the opponents to a VCG mechanism is comparably small. This indicates that the second best mechanism yields a much higher surplus when the status quo is a vote under majority rule.

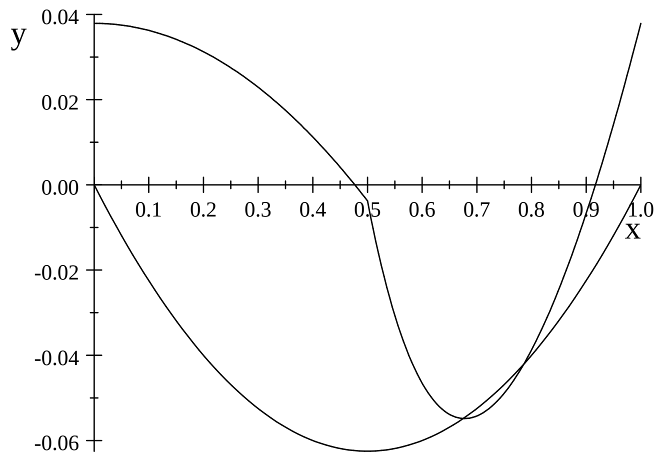

Figure 8: Interim utility in a democracy (differentiable) and with a VCG mechanism for types on the interval $[0, 1]$.

5.2.1 Large populations with correlated types

Increasing the number of agents in a setup which is parameterized as in the previous section yields a possibility theorem with no welfare gain for symmetric distributions of s and θ around c. An impossibility theorem instead obtains for all skewed distributions $\phi(\theta|s)$. When the median and mean of the conditional distribution $\phi(\theta|s)$ do not coincide, there are always some realizations of θ such that the VCG mechanism yields a strictly higher probability of provision. Therefore, an impossibility theorem holds for such skewed distributions. Types below the cost parameter c prefer the majority voting mechanism while types above the cost parameter c prefer the VCG mechanism. Therefore, there is little hope for a possibility theorem in large distributions when the shape of the conditional distribution $\phi(\theta|s)$ is known in advance.

In my last example, I consider the case where the shape of the realized distribution of types is not determined in advance. In this case, VCG mechanisms yield a strictly higher surplus than majority voting. Moreover, one can show that they may satisfy a slightly weaker concept of individual rationality. The concept that I use can be defined as follows.

Definition 1 *A mechanism is called ϵ-individually rational if all types interim expected utility is at least as high as under the outside option minus ϵ.*

I will now show that, for large enough n there always is a distribution of all agents' types such that an ex post efficient mechanism exist which is ϵ individually rational and at the same time increases aggregate surplus by a non-negligible positive amount. The trick used in this example is that the final distribution of types is also skewed, but that it is not known in advance, in which direction it will be skewed.

Proposition 6 *Consider majority voting as the relevant status quo. For large enough n there always is a continuous density with correlated types such that (i) the ex post efficient allocation can be implemented with an expected externality mechanism which is ϵ individually rational and (ii) the ex post efficient allocation generates a strictly higher surplus than majority voting. Moreover, (iii) the expected externality mechanism is not individually rational when the outside option is the no-provision decision.*

PROOF Consider a distribution which assigns almost all realizations of θ on the interval $[0, 2c]$ the same positive density which is arbitrarily close to zero. There are only four realizations of agents' types with positive density: $0, c-\epsilon, c+\epsilon, 2c$. There is a 50 percent chance, that a majority of 51 percent has a type below the value c and a 50 percent chance that a majority of 51 percent has a valuation above c. Moreover, below c there is a 50 percent chance that the remaining probability mass can be found at $\theta = 0$. Above c there also is a 50 percent chance that the remaining probability mass is allocated to $2c$.

Consider now the probability under majority voting that the mechanism chooses $x = 1$. It is 50 percent for all realizations of the valuation. Consider next the probability of provision under a VCG mechanism. When the valuation is not taken from the four discrete points above, the interim probability is exactly 50 percent. When the valuation is zero, there only is a probability of 25 percent that the good is actually provided. The same holds for a valuation which is slightly above c. The reason is that only when the low valuation is $c - \epsilon$ the good is provided when the majority is above c. This occurs with a chance of 25 percent. A similar reason can be applied to show that the remaining probabilities are at 75 percent. The rest of the proposition follows immediately. Q.E.D.

6 Conclusion

This paper studied an institutional reform. The reform, which I considered was to replace a voting mechanism for the provision of a public good by an efficient expected externality mechanism. Voting mechanisms do not always efficiently decide on the provision of public goods. The present paper shows that there are cases in which all agents unanimously accept an institutional reform which yields an ex post efficient decision. We can conclude that one may be more optimistic about efficient institutional reforms when the status quo is a democratic decision which coerces some agents to contribute to the cost of collective goods.

We have also seen that the (strong) possibility theorem of section 2 does not hold for all distributions of independent types. Moreover, there are cases where the possibility result is not robust when the size of the population or the variance of the distribution increases. Skewed distributions are more likely to produce impossibility theorems - in particular when types are correlated and when the shape of the distribution is known to all agents. Nevertheless, the overall conclusion of this paper is far more positive than the one of previous related studies. Even when a possibility theorem does not hold, the status quo will be relevant for the economy's second best welfare. According to Mailath and Postlewaite (1987), the second best outcome in a large economy in which agents could veto provision yields no positive surplus. This paper shows that this no longer needs to be the case when the status quo is a vote under majority rule.

There are also cases where the number of informed agents who oppose an ex-post efficient mechanism is relatively small. The same holds for the size of their monetary losses. This indicates that majority requirements below unanimity rule may enable democracies to switch to ex post efficient institutions.[4]

Any analysis of institutional reform under asymmetric information has to take the relevant status quo into account. The present paper applied this insight of Cramton, Gibbons, and Klemperer's paper to public economics. A useful extension will be to study and relate other institutions such as representative democracies, voluntary provision schemes, or the lobbying for collective provision in a similar analysis.

VCG mechanisms are often criticized for being too complicated and unintuitive. This may in deed be an obstacle to their implementation in practice. Another interesting option for further research is to extend the present analysis by considering more simple reform proposals.

7 Appendix: Proof of Propositions 2 and 3

Proof of Proposition 2 First consider the case where $c \leq 1/2$. Interim expected utility $U_i(\theta)$ under the VCG mechanism is increasing and convex with a slope of 1 for valuations above total cost. Low valuations prefer the efficient mechanism while high valuations don't. We have

[4]Note that democracy itself would also not be interim individually rational for all agents in the present setup.

$$\begin{aligned} U(0) &\geq M(0) && (41)\\ &\Leftrightarrow -\frac{2}{3}c^3+2c^2-c \geq -(1-c)\,c && (42)\\ &\Leftrightarrow \frac{3}{2} \geq c, && (43) \end{aligned}$$

and

$$\begin{aligned} U(1) &\geq M(1) && (44)\\ &\Leftrightarrow -\frac{2}{3}c^3+2c^2-c-2c^2+1 \geq (1-c) && (45)\\ &\Leftrightarrow -\frac{2}{3}c^3 \geq 0. && (46) \end{aligned}$$

This contradicts $c > 0$. Next consider the case where $c > 1/2$. We have

$$\begin{aligned} U(0) &\geq M(0) && (47)\\ &\Leftrightarrow \frac{2}{3}c^3-2c^2+2c-\frac{2}{3} \geq -(1-c)\,c && (48)\\ &\Leftrightarrow c \in [0.313\,86, 1] \cup \left[\frac{1}{4}\sqrt{33}+\frac{7}{4}, \infty\right). && (49) \end{aligned}$$

and

$$\begin{aligned} U(1) &\geq M(1) && (50)\\ &\Leftrightarrow \frac{2}{3}c^3-2c^2+2c-\frac{2}{3}+(1-2c)+\frac{1}{2}+\frac{1}{2}(2c-1)^2 \geq 1-c && (51)\\ &\Leftrightarrow c \in \left[-\frac{1}{2}\sqrt{3}-\frac{1}{2}, 0.366\,03\right] \cup [1,\infty). && (52) \end{aligned}$$

This contradicts $c \in [0.5, 1]$. Q.E.D.

Proof of Proposition 3

(i) is well known.

(ii) It suffices to provide an example. Consider the case of $c = 1/2$. In this case the interim utilities are $-\frac{2}{3}\frac{1}{2}^3+\frac{1}{2}\theta_i^2$. For $\theta < c$ the participation constraint becomes

$$-\frac{2}{3}\frac{1}{2}^3+\frac{1}{2}\theta_i^2 > \frac{1}{4}\left(\theta_i-\frac{1}{2}\right) \Leftrightarrow \frac{1}{2}\theta_i^2-\frac{1}{4}\theta_i > -\frac{1}{24}. \quad (53)$$

The l.h.s. has its minimum at $\theta_i = \frac{1}{4}$ with a value of $-\frac{1}{32}$. Hence, this condition holds for all θ_i. For $\theta \geq c$ the participation constraint becomes

$$-\frac{2}{3}\frac{1}{2}^3+\frac{1}{2}\theta_i^2 > \left(1-\frac{1}{4}\right)\left(\theta_i-\frac{1}{2}\right) \Leftrightarrow \frac{1}{2}\theta_i^2-\frac{3}{4}\theta_i > -\frac{7}{24} \quad (54)$$

The left hand side has its minimum at $\theta_i = \frac{3}{4}$ with a value of zero. Hence, this condition holds for all θ_i.

(iii) For cost parameters c that are too high or too low, an impossibility theorem applies. To see this, consider first the case where $c < 1/2$.We have

$$\begin{aligned} U(0) &\geq M(0) && (55)\\ &\Leftrightarrow -\frac{2}{3}c^3 + 2c^2 - c \geq -\frac{1}{2}(1-c)\,c && (56)\\ &\Leftrightarrow c \in [0.406\,93, 1.\,843\,1]\,. \end{aligned}$$

and

$$\begin{aligned} U(1) &\geq M(1) && (57)\\ &\Leftrightarrow -\frac{2}{3}c^3 + 2c^2 - c - 2c^2 + 1 > \left(1 - \frac{c}{2}\right)(1-c) && (58)\\ &\Leftrightarrow c \in (0, 0.568\,73)\,. && (59) \end{aligned}$$

Outside the intersection of both intervals, a possibility theorem does not hold.

Next consider the case where $c > 1/2$. In this case we have

$$\begin{aligned} U(1) &\geq M(1) && (60)\\ &\Leftrightarrow \frac{2}{3}c^3 - 2c^2 + 2c - \frac{2}{3} + (1-2c) + \frac{1}{2} + \frac{1}{2}(2c-1)^2 \geq \left(1 - \frac{c}{2}\right)(1-c) && (61)\\ &\Leftrightarrow c \in \left[-\frac{1}{8}\sqrt{33} - \frac{1}{8}, 0.593\,07\right] \cup [1, \infty)\,. && (62) \end{aligned}$$

This is the empty set. Q.E.D.

References

[1] D'Aspremont, Claude, Crémer, Jacques and Gérard-Varet, Louis-André (2003), "Correlation, Independence, and Bayesian Incentives" *Social Choice and Welfare*, 21, 281-310.

[2] D'Aspremont, Claude, Crémer, Jaques and Gérard-Varet, Louis-André (2004), "Balanced Bayesian mechanisms" *Journal of Economic Theory*, 115, 385-396.

[3] Bergemann, Dirk and Stephen Morris (2005), "Robust Mechanism Design", *Econometrica*, 73, 1771-1813.

[4] Bierbrauer, Felix and Sahm, Marco (2008), ”Optimal Democratic Mechanisms for Taxation and Public Good Provision” *Working Paper Series of the Max Planck Institute for Research on Collective Goods.*

[5] Bierbrauer, Felix and Martin Hellwig (2008), "Public Good Provision in a Large Economy", unpublished.

[6] Casella, Alessandra (2005), "Storable Votes", *Games and Economic Behavior*, 51, 391-419.

[7] Crémer, Jaques and McLean, Richard P., (1985), ”Optimal selling strategies under uncertainty for a discriminating monopolist when demands are interdependent” *Econometrica*, 53, 345-361.

[8] Cramton, Peter, Gibbons, Robert and Paul Klemperer (1987) "Dissolving a Partnership Efficiently" Econometrica, 55, 615-32.

[9] Güth, Werner and Hellwig, Martin F. (1986) "The Private Supply of a Public Good", *Journal of Economics.*

[10] Fang, Hanming and Peter Norman (2005) "Overcoming Participation Constraints," Cowles Foundation Discussion Papers 1511, Yale University.

[11] Fang, Hanming and Peter Norman, (2008) "Optimal Provision of Multiple Excludable Public Goods," NBER Working Papers 13797.

[12] Haimanko, Ori and Le Breton, Michel and Weber, Shlomo (2004), ”Voluntary formation of communities for the provision of public projects” *Journal of Economic Theory*, 115, 1-34.

[13] Healy, Paul J. (2005), ”Equilibrium Participation in Public Goods Allocations” *GSIA Working Papers with number 2006-E7.*

[14] Hellwig, Martin F. (2003), ”Public-Good Provision with Many Participants” *Review of Economic Studies*, 70, 589-614.

[15] Jackson, Matthew O. and Sonnenschein, Hugo F. (2007) ”Overcoming Incentive Constraints by Linking Decisions” *Econometrica*, 1, 241-257.

[16] Kosenok, Grigory and Severinov, Sergei (2008), ”Individually rational, budget-balanced mechanisms and allocation of surplus” *Journal of Economic Theory*, 140, 126-161.

[17] Krishna, Vijay and Perry, Motty (2000), "Efficient Mechanism Design " *EconWPA in its series Game Theory and Information with number 9703010.*

[18] Saijo, Tatsuyoshi (1991), "Incentive compatibility and individual rationality in public good economies" *Journal of Economic Theory*, 55, 203-212.

[19] Parreiras, Sérgio O. (2005), "Correlated information, mechanism design and informational rents" *Journal of Economic Theory*, 123, 210-217.

[20] Schmitz, Patrick W. (2002) Simple Contracts, Renegotiation under Asymmetric Information, and the Hold-up Problem", *European Economic Review*, 46, 169-188.

[21] Schweizer, Urs (2005), "Universal Possibility and Impossibility Results " *Journal of Games and Economic Behavior*, 57, 73-85.

[22] Mailaith, George J. and Andrew Postlewaite (1990) "Asymmetric Information Bargaining Problems with Many Agents," *Review of Economic Studies*, 57, 351-367.

[23] McAfee, Preston R. and Reny, Philip J. (1992), "Correlated information and mechanism design" *Econometrica*, 60, 395-421.

[24] Williams, Steven R. (1999), "A characterization of efficient, bayesian incentive compatible mechanisms" *Journal of Economic Theory*, 14, 155-180.