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ABSTRACT

Repeated electoral competition over non-linear income tax schedules*

We consider a repeated electoral competition game between two parties, each representing a constituent with a given income level. Parties are unable to commit to any policy before the election; they choose a non-linear income tax schedule once elected. In each period, citizens cast a vote either for the incumbent or for the challenger. We first show that there exist (pure strategy) subgame perfect equilibria where both parties choose the most-preferred tax schedule of their constituent, subject to the constraint that they are reelected. We characterize a specific class of these BPR (Best Policy with Reelection) equilibria in which one of the parties plays its constituent's unconstrained optimal tax function. Equilibrium tax schedules are always piecewise linear. Depending on the income levels of the two parties' constituents, we obtain either classical left-vs-right equilibria (where poorer people vote for one party and richer people for the other one) or ends-against-the-middle equilibria (where both poor and rich people vote for one party while the middle class vote for the other party). In both types of equilibria both parties propose the same tax schedule to a subset of the population.

JEL Classification: D72 and H24
Keywords: ends-against-the-middle, no commitment, piecewise linear income tax and postelection politics

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1 Introduction

The determination of income taxes through a democratic process has been extensively studied over the last three decades. In their seminal contributions Romer (1975) and Roberts (1977) studied the determination of a linear income tax schedule under majority voting. This problem being unidimensional, a Condorcet winner (majority voting equilibrium) exists under quite weak assumptions. It is given by the most-preferred tax of the individuals with median productivity. This implies that the electoral competition game between two parties who are able to commit at the campaign stage has a unique equilibrium in pure strategies, which is given by the Condorcet winner.

The restriction to a linear tax scheme is purely artificial. It is made for simplicity but cannot be justified by informational or practical considerations. In reality income tax schedules are typically non-linear (most often piecewise linear), and marginal tax rates vary with income. A satisfactory treatment of the democratic choice of income taxes should thus allow for non-linear taxes. However, with a non-linear tax, the policy space becomes multi-dimensional, leading to voting cycles and the non-existence of a pure strategy Nash equilibrium of the electoral competition game (Plott (1967)).

Confronted with this difficulty, two different approaches have been adopted. The first one consists in keeping the framework of the Downsian electoral competition but to study solution concepts different from the pure-strategy Nash equilibrium. Recall that key features of Downsian models is that they concentrate on one-shot games and that parties are able to commit to their announced policy. The alternative solution concepts that have been considered include the uncovered set, obtained by eliminating weakly dominated strategies (De Donder and Hindriks (2003)) or the bipartisan set, which is the support of mixed strategies played in equilibrium (De Donder and Hindriks (2003), Carbonell-Nicolau and Ok (2007)). Roemer (2001) does depart from the strict Downsian framework but keeps the assumption that parties can commit to any policy at the electoral stage. This area of research has been dubbed “preelection politics” by Persson and Tabellini (2000).

The second direction of research, referred to as “postelection politics”, assumes that electoral promises are not binding or too vague to matter. Consequently, parties are effectively unable to commit to any policy before the election. In particular, some authors have studied one-shot games in which candidates (who have preferences over the different policies) if elected implement their own most-preferred policy. The policy space is thus reduced to the set of individually optimal policies so that we essentially return to a single dimensional problem. Roëll (1996) and Bohn and Stuart (2005) show that a Condorcet winning non-linear taxation schedule exists under rather mild conditions.

We follow this second strand of research and we take the analysis a step forward by considering an *infinitely repeated electoral competition game with no policy commitment*, building on the work by Duggan (2000) and Banks and Duggan (2008). In our model, two parties with known preferences compete repeatedly. There is an election in each period and the party receiving the most votes is elected for that period (majority rule). Parties are not able to commit to the policy they would implement if elected. The voters then cast their votes on the basis of the past performances of the parties while in power and/or on their expectations about the future choices of these parties. The elected party chooses a tax schedule for the current period and this process is repeated indefinitely.

To select among the potentially many equilibria of this game, we assume that the voters act as though pivotal in each election: they vote for the candidates offering them the highest continuation value (*prospective voting*). In addition, we assume that if a party deviates in a given period from its equilibrium policy, it plays this deviation in all future periods. Under these two assumptions, we show that, when the parties are sufficiently patient, the incumbent party in any given period adopts the policy that maximizes the utility of its constituent under the constraint that it is reelected (i.e., that it receives at least 50% of the votes). In these equilibria, labeled *BPR* (Best Policy with Reelection), the initial incumbent remains in power forever.

We apply this equilibrium concept to a model where citizens differ only according to their exogenous income level. The income distribution is positively skewed, with the median income below the mean. The set of admissible tax schedules consists of all functions that *(i)* are continuous, *(ii)* entail marginal tax rates that are nonnegative and do not exceed one[1] and *(iii)* satisfy the government budget constraint. The identity of the parties constituents are exogenously given. One of the parties, A, represents a constituent with income below the median level, while party B stands for a constituent with income above the median level.

We focus on BPR equilibria in which one of the two parties proposes the most-preferred tax schedule of the constituent it represents (absent any reelection consideration). We distinguish between four types of equilibria and show that the specific type that emerges depends on the income levels of the constituents of the two parties. The partition of the space of constituents' income we provide shows that depending on the income levels of the constituents it is party A or party B who plays its unconstrained optimal strategy. In each case we obtain either classical left-vs-right equilibria (where poorer people vote for one party and richer people for the other one) or ends-against-the-middle equilibria (where both poor and rich people vote for one party while the middle class vote for the other party).

All BPR equilibria, whatever the identity of the two constituents, exhibit piecewise linear tax functions proposed by both parties. Furthermore, in all equilibria, both parties propose the same tax policy to a subset of the electorate. We discuss these important features of BPR equilibria in the concluding section.

2 The model

2.1 The economic model

The economic modelling is borrowed from Roemer (2000). There is a continuum of citizens/voters identified by their exogenous income y, which is distributed according to a continuous cumulative distribution function F and a density function f on the support $[y_-, y_+]$. The associated Lebesgue measure is denoted by H. We assume that the distribution function is positively skewed, with the median income y_m strictly lower than the average income $\overline{y}$.

Let $T(y)$ denote a generic tax function. We restrict the set of admissible tax func-

[1] where derivatives exist: we do not impose differentiability.

tions, denoted by Ω, to functions that are *(i)* continuous, *(ii)* satisfy[2]

$$0 \leq T'(y) \leq 1 \tag{1}$$

where differentiable, and *(iii)* are purely redistributive (there is no revenue requirement) so that[3]

$$\int_{y_-}^{y_+} T(y)f(y)dy = 0. \tag{2}$$

We do not explicitly impose the limited liability condition $T(y) \leq y$ since this constraint is not binding at any of the equilibria we consider. Consumption of an individual with pre-tax income y is denoted by $c(y) = y - T(y)$.[4] Utility is increasing in consumption and denoted by the function $u(c)$ with $u' > 0$ and $u'' \leq 0$. Let $v(y,T) = u(y-T)$ denote the utility level of an individual with pre-tax income y who pays a tax of T.

2.2 The political competition game

There are two political parties denoted by A and B. Following Duggan (2000) or Banks and Duggan (2008), we study a *repeated elections model with infinite horizon.* The players in this game are the two parties and the voters. At the beginning of each period t, an election takes place in which voters decide whether to reelect the incumbent party or to appoint the challenger. The party obtaining the most votes is elected (majority rule) and gets to choose a tax schedule T_t for the current period. When the two parties receive the same proportion of votes, the incumbent party is reelected for sure.[5] In the

[2]This condition implies that both tax liabilities and after-tax income are non-decreasing functions of pre-tax income. Such a condition is usually derived instead of assumed in the optimal tax literature with endogenous income. Alternatively, we could replace the upperbound on the marginal tax rate by $\alpha < 1$ as in Roemer (2008). One interpretation of α is that both parties agree not to enact policies with too large marginal tax rates (to avoid the distortionary impact of unmodelled labor-leisure substitution). With this interpretation, marginal tax rates below α create few if any labor supply supply distortions. Introducing such an α would not change substantially our results.

[3]Adding an exogenous revenue requirement R would not change our results.

[4]Given the one-to-one relationship between tax functions T and after-tax consumption functions c, the game we describe is equivalent to one where parties announce consumption functions rather than tax functions. In the rest of the paper, we sometimes adopt the shortcut "a party proposes a consumption function" rather than writing that "a party proposes a tax function that results in the following consumption function".

[5]This assumption is crucial to our results. If the incumbent were reelected with a probability lower than 1, equilibria would cease to exist. The incumbent would indeed have an incentive to deviate from the candidate equilibrium policy, in order to get strictly more votes than the challenger.

following period $t+1$, the same sequence of events is repeated. There is no last period to this game which is repeated infinitely.

An important feature of this model is that elections are not preceded by a campaign stage with policy announcements. Parties are not able to commit before the election to the policy they would implement if elected. This is in contrast with most election models in the literature.

We now describe the payoffs and strategies of the two types of actors (namely parties and voters), starting with the two parties. Party i ($i = A, B$) represents constituents with income y_i, with $y_A < y_m < y_B$. Its payoff from the sequence $\{T_t(.)\}$ of outcomes is given by

$$(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}\left[v(y_i, T_t(y_i)) + \beta\omega_{it}\right]. \tag{3}$$

The parties' payoffs come from two sources. First, parties care about the tax function enacted because it determines the utility of its constituents. Parties also care about winning the elections per se, enjoying an additional payoff $\beta > 0$ (spoils of office, ego rents, etc.) when they win the elections at time t. Accordingly, ω_{it} denotes the indicator function taking the value 1 if party i is elected in period t (and 0 otherwise). The discount factor is denoted by $\delta < 1$. A (pure) strategy of party i specifies the policy $T_t(.)$ chosen once elected in period t.

A type y voter's payoff from the sequence $\{T_t(.)\}$ of policy outcomes is the discounted sum of per-period utility levels:

$$(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}v(y, T_t(y)),$$

where $\delta < 1$ is the same time discount factor as for parties.[6]

A (pure) strategy of voter j specifies the action chosen in period t, $a_{j,t} \in \{I, C\}$, where $a_{j,t} = I$ (resp. C) means that this individual votes for the incumbent (resp. challenger). We assume that voters act as though pivotal in the election.[7] They thus

[6] The assumption of identical discount factors is made purely for convenience. Considering different discount factors for the parties and the citizens would not affect the results.

[7] This condition can be seen as the analogous of the elimination of weakly dominated strategies, a criterion widely used in static games to get rid of "unreasonable" equilibria. This also corresponds to stage-undominated strategies in dynamic models with a finite number of voters (see Baron and Kalai (1993)).

vote for the incumbent if and only if the utility from re-electing him is at least as high as the utility of electing the challenger.[8] This property is called *prospective voting*.

We are now in a position to characterize a special class of equilibria of this game.

2.3 Best Policy with Reelection equilibria

In these equilibria, called *BPR* (Best Policy with reelection) equilibria, a party always plays at equilibrium the same policy once elected. Moreover, if a party deviates in a given period, it is supposed to play this deviation policy in all subsequent periods.

We now define the optimal tax schedules of a given party i. We distinguish between the unconstrained optimum and the constrained optimum. Let $T_i^{opt}(.)$ denote the *unconstrained* optimal tax schedule of party i absent electoral considerations. It is obtained by maximizing the utility of the party's constituent (with income y_i) over the entire set of admissible strategies:

$$T_i^{opt}(.) = \arg\max_{T \in \Omega} v(y_i, T(y_i)), i = A, B. \tag{4}$$

The *constrained* optimum $T_i^{BPR}(., T_j)$, on the other hand, is the tax function that maximizes party i's constituent's utility under the constraint that at least 50% of the voters prefer this tax function to T_j:

$$\begin{aligned} T_i^{BPR}(., T_j) = &\arg\max_{T \in \Omega} v(y_i, T(y_i)), \quad i, j = A, B, \quad i \neq j \\ \text{s. t.} \quad &H(y : v(y, T(y)) \geq v(y, T_j(y))) \geq 1/2. \end{aligned}$$

The following proposition, established in the Appendix, states that when the discount factor is sufficiently large, there exists a (pure strategy) subgame perfect equilibrium of the electoral competition game where both parties offer their preferred policy *under the constraint of reelection.*

Proposition 1 *Assume that there exist two tax schedules $T_A^*(.)$ and $T_B^*(.)$ such that $T_A^*(.) = T_A^{BPR}(., T_B^*)$ and $T_B^*(.) = T_B^{BPR}(., T_A^*)$. Then there exists a subgame perfect equilibrium, called BPR equilibrium, with the parties playing $T_i^*(.)$ when in office if and*

[8] We assume that, when indifferent, they vote for the incumbent.

only if

$$\delta \geq \max \left\{ \frac{v(y_A, T_A^{opt}(y_A)) - v(y_A, T_A^*(y_A))}{v(y_A, T_A^{opt}(y_A)) - v(y_A, T_B^*(y_A)) + \beta}, \frac{v(y_B, T_B^{opt}(y_B)) - v(y_B, T_B^*(y_B))}{v(y_B, T_B^{opt}(y_B)) - v(y_B, T_A^*(y_B)) + \beta} \right\}. \tag{5}$$

Intuitively this result can be understood as follows. By definition, any incumbent proposing its *BPR* policy is reelected at each period and obtains a payoff which consists of the utility associated to this policy plus the spoils of office term (defined by expression (3)). Its best deviation is to play its unconstrained most-preferred policy, which yields a larger one-shot payoff but is followed by a stream of lower payoffs (as the opposing party wins for sure at each and every election after the deviation is played for the first time).[9] It is then intuitive that if a party is patient enough (i.e., δ is sufficiently large), the one-shot gain it can achieve by deviating is outweighed by the associated long-term losses. Consequently, it will stick to its *BPR* policy forever.

In the remainder of the paper, we characterize a specific and particularly interesting class of *BPR* equilibria, namely those in which one of the parties is *not* effectively constrained by electoral considerations. In other words, this party offers its constituents' unconstrained optimal policy. We distinguish between four types of equilibria which we first characterize (Sections 4 and 5) before establishing their existence (Section 6).

3 Individually optimal tax schedules

Before studying equilibria of the electoral competition game, it is necessary to describe the optimal tax schedules of individual voters. Ideally, individuals with income y_i would like to completely expropriate people with income below and above y_i, setting their net income to 0. However, they face the constraints that the tax function must be continuous and that marginal tax rates (where they exist) have to be between 0 and 1. Under these restrictions, it is plain that they should set a marginal tax rate of 0 to all individuals with income below y_i and of 1 to individuals with higher incomes. The optimal tax

[9]Recall that the strategy supporting this equilibrium is such that the deviating party is supposed to play this deviation in all subsequent elections.

schedule of an individual with income y_i is thus given by

$$T_i^{opt}(y) = \max[y - y_i, 0] - I_i^{opt}, \ (y, y_i) \in [y_-, y_+]^2 \tag{6}$$

where $I_i^{opt} > 0$ is the transfer (negative income tax) received by the individual with the lowest income, y_-. Put differently it is the difference between this individual's after-tax (net) and before-tax (gross) income. Graphically, I_i^{opt} measures the intercept of the net income schedule in the (gross, net) income space $[y_-, y_+] \times [y_-, y_+]$. It is obtained from the government budget constraint (hereafter GBC) which, using (2) and (6) can be written as

$$\int_{y_-}^{y_i} (y + I_i^{opt}) f(y) dy + \int_{y_i}^{y_+} (y_i + I_i^{opt}) f(y) dy = \int_{y_-}^{y_+} y f(y) dy \equiv \overline{y}.$$

Solving this expression yields

$$I_i^{opt} = \int_{y_i}^{y_+} (y - y_i) f(y) dy.$$

Consequently, the difference between net and gross income of the poorest individual in party i's $(i = A, B)$ unconstrained optimal tax schedule is given by

$$I_A^{opt} = \int_{y_A}^{y_+} (y - y_A) dF, \tag{7}$$

$$I_B^{opt} = \int_{y_B}^{y_+} (y - y_B) dF. \tag{8}$$

For some of our arguments it is convenient to define $c_i^{opt}(.)$, the optimal consumption function of individuals with income y_i:

$$c_i^{opt}(y) = I_i^{opt} + \min[y, y_i], \ (y, y_i) \in [y_-, y_+]^2.$$

It is optimal in the sense that it is induced by the individual's optimal tax schedule. The unconstrained optimal consumption schedules of parties A and B are represented on Figure 1.

The remainder of the paper concentrates on situations where one party proposes its *unconstrained* most-preferred policy. To be more precise we consider *BPR* equilibria which are such that the reelection constraint is not effectively binding for one of the parties. These equilibria have a number of interesting features. We characterize them and show that they effectively exist which in itself is a very interesting (and rather surprising) property.

In section 4 party B (representing the richer constituent) offers its unconstrained most-preferred policy. We first identify a simple tax proposal by the other party that gathers one half of the votes, with poorer people voting for one party and richer people supporting the other one. We then study the best deviation by party A, and show that it consists in proposing a policy that obtains the support of both low and high income individuals, while individuals with intermediate income support party B. Section 5 performs the same exercise when party A proposes its unconstrained most-preferred policy. Section 6 studies the conditions (on the identity of the parties' constituents y_A and y_B) under which these pairs of policies constitute a *BPR* equilibrium of the electoral game.

Observe that the Best Policy with Reelection $T_i^{BPR}(., T_j)$ and the unconstrained optimal policy $T_i^{opt}(.)$ do not depend on the shape of the utility functions v and u. The analysis contained in the rest of the paper is thus valid for any increasing utility function that satisfies condition (5).

4 Party B proposes its unconstrained most-preferred policy

Assume for the time being that B plays its unconstrained optimal strategy T_B^{opt}. With this hypothesis, party B's behavior is well defined and to characterize the equilibrium we only have to determine party A's strategy. In other words, we will have to determine party A's *BPR* reply to T_B^{opt}. Interestingly it will turn out that this strategy can obey two quite distinct patterns. Under pattern I, voters will be divided simply along the income line, with the "poor" supporting party A while the "rich" vote for B. The other pattern, II, is of the "end-against-the-middle type" with the poor and the rich supporting party A while the middle class supports B.

Turning to the formal construction of the equilibria, let $c_i^j(x, y_B, y_A)$ denote the consumption level of an individual with income x, induced by the tax schedule played in a type j equilibrium by party i, when party B represents an individual with income y_B and party A an individual with income y_A. When no ambiguity can arise, we simply use the notation $c_i^j(.)$ (and $T_i^j(.)$ for the corresponding tax function). The after-tax schedule corresponding to T_B^{opt} is denoted by c_B^{opt} on Figure 2.

Let us consider I_B^{opt} and recall that this variable represents the transfer received by the poorest individual, that is $I_B^{opt} = c_B^{opt}(y_-) - y_- = -T_B^{opt}(y_-)$. Further recall that the level of I_B^{opt} is given by (8).

We now identify a simple tax function that allows party A to be reelected with the support of all individuals with an income below the median y_m. Party A's corresponding after-tax schedule, denoted by c_A^{IB}, is constructed as follows (see Figure 2). All individuals with an income lower than y_A face a zero marginal tax rate. Individuals with income between y_A and y_m face the highest marginal tax rate compatible with $c_A^{IB}(y) \geq c_B^{opt}(y)$. As can be seen from Figure 2, we denote by y_A' the pre-tax income threshold such that individuals with $y_A \leq y < y_A'$ face a marginal tax of one while individuals with $y_A' < y < y_m$ face a zero marginal tax rate. Individuals whose income is between y_A' and y_m, which we call the middle class, are indifferent between $T_A^{IB}(.)$ and $T_B^{opt}(.)$ because they obtain the same after-tax income with both tax schedules. Above y_m, marginal tax rates under party A's policy are all equal to one. Finally, observe that if A is the incumbent, he receives exactly one half of the votes while, if B is the incumbent, he receives strictly more than one half.

A limit case of such a schedule occurs when $y_A' = y_m$, in which case the tax schedule described above corresponds to T_A^{opt}. It is easy to see from Figure 2 that a necessary and sufficient condition for a schedule like c_A^{IB} to satisfy the GBC is that c_A^{opt} intersects c_B^{opt} to the left of y_m. If this were not the case, c_A^{IB} would be everywhere below c_A^{opt} which by definition satisfies the GBC. Consequently, the total after-tax income under c_A^{IB} would be strictly less than the average income. We will come back to this very important point in section 6, but for the moment we assume that the following assumption holds, so that the schedule c_A^{IB} exists:

Assumption 1: $c_A^{opt}(y_m) < c_B^{opt}(y_m)$.

We now show that the function c_A^{IB} is uniquely defined given I_B^{opt}. The function c_A^{IB}

is defined by the parameters I_2^{IB} and y'_A. These two variables are the solution to the following two equations

$$\int_{y_-}^{y_A} (y + I_B^{opt} + I_2^{IB})dF + \int_{y_A}^{y'_A} (y'_A + I_B^{opt})dF$$
$$+ \int_{y'_A}^{y_m} (y + I_B^{opt})dF + \int_{y_m}^{y_+} (y_m + I_B^{opt})dF = \overline{y}$$

and

$$y'_A = y_A + I_2^{IB}.$$

The first constraint is the GBC which gives I_2^{IB} as a function of I_B^{opt} and y'_A. The second constraint is obtained by construction ($y_A + I_B^{opt} + I_2^{IB} = y'_A + I_B^{opt}$) and gives y'_A as a function of I_2^{IB}. Solving them simultaneously, we obtain

$$I_B^{opt} + \int_{y_-}^{y_A} (y + I_2^{IB})dF + \int_{y_A}^{y_A + I_2^{IB}} (y_A + I_2^{IB})dF + \int_{y_A + I_2^{IB}}^{y_m} y dF + \int_{y_m}^{y_+} y_m dF - \overline{y} = 0,$$

which gives I_2^{IB} as a function of I_B^{opt} and thus indirectly y'_A also as a function of I_B^{opt}.

Let us now consider potential deviations from these schedules. Obviously, B would not want deviate, as it is always reelected with its optimal policy. As to party A, observe first that, for a deviation to be profitable, it must satisfy simultaneously two requirements: it must give a larger post-tax income to individual A (compared with $c_A^{IB}(y_A)$), and it must garner exactly 50% of the vote when party A is an incumbent faced with policy c_B^{opt}. If party A were to obtain strictly more than one half of the votes, it could reallocate after-tax consumption in such a way as to increase $c(y_A)$ while at the same time retaining a majority.

We now show that a "marginal" deviation, close to c_A^{IB}, cannot satisfy both of these requirements. If A wants to increase the after-tax consumption level of individuals with income y_A, it necessarily has to give a little less income to people with income above y'_A (in order to satisfy the GBC - see Figure 2). But then, the proportion of people (weakly) preferring this tax schedule to c_B^{opt} is strictly less than 50%.

It follows that the deviations potentially attractive to A are of the ends-against-the-middle type, that is such that low and high income individuals vote for A while middle-class individuals strictly prefer B's policy. We illustrate such a deviation, called c_A^{IIB}, on

Figure 2. Starting at a pre-tax income level $y_1 < y_m$, party A imposes a marginal tax rate of one. The additional tax proceeds from these individuals (compared to c_A^{IB}) are redistributed towards low income people (in order to increase $c(y_A)$) and toward high income people (in order to garner exactly 50% of the votes in the case of incumbency). From Figure 2, it is clear that party A has two degrees of freedom when devising such a deviation: it has to set the income level $y_1 < y_m$ above which the marginal tax rate is one, and the income level y_A'' above which the marginal tax rate is one and the after-tax income is the same under both parties' proposed schedules. The best such deviation c_A^{IIB} is formally defined in Appendix B. From Figure 2, we see that some individuals have the same after-tax income with c_A^{IIB} and with c_B^{opt}: middle-class individuals with $\hat{y}_A \leq y \leq y_1$ and richer individuals with an income at least equal to y_A''.

We have thus shown that, when party B plays c_B^{opt}, c_A^{IIB} is the best deviation available to party A when it plays c_A^{IB}. Assume now that party A plays c_A^{IIB} while party B plays c_B^{opt}. Obviously, party B has no incentive to deviate since (by assumption) it is reelected with its unconstrained most-preferred policy. As for party A, c_A^{IIB} is by construction the best feasible winning policy of an ends-against-the-middle type. Any fruitful deviation is such that party A attracts the votes of the bottom half of the population. Observe that *(i)* by construction of c_A^{IIB}, the individual with income y_A obtains a higher after-tax income with c_A^{opt} than with c_A^{IIB} while *(ii)* c_A^{opt} satisfies by definition the GBC. It follows that party A will deviate towards c_A^{opt} unless this policy gathers less than 50 percent of the votes when faced with c_B^{opt}– i.e., unless Assumption 1 holds. If this assumption holds, we then have that c_A^{IB} is the best deviation from c_A^{IIB} available to party A when party B plays c_B^{opt}.

To sum up, we have two candidates for party A's best (*BPR*) strategies and the induced consumption profiles are c_A^{IB} and c_A^{IIB} respectively. To determine which of these two candidates effectively represents the party's best policy we simply have to compare respective consumption (net income) levels they imply for its constituent (an individual with income y_A). We have that c_A^{IB} is better when $c_A^{IB}(y_A) > c_A^{IIB}(y_A)$, while c_A^{IIB} is better if the opposite relationship occurs. We summarize the results obtained in this section in the following Proposition.

Proposition 2 *Under Assumption 1, if party B proposes its unconstrained most-preferred schedule c_B^{opt}, then party A's best response is to offer either c_A^{IB} or c_A^{IIB}. In either case, (i) a subset of the population obtains the same after-tax income under the tax schedules proposed by both parties, and (ii) party B has no incentive to deviate from c_B^{opt}.*

5 Party A proposes its unconstrained most-preferred policy

In this section, we assume that A plays its unconstrained optimal strategy T_A^{opt}. The corresponding after-tax schedule c_A^{opt} is depicted on Figure 3. We first identify a simple tax function that allows party B to receive the votes of all individuals with an income above the median y_m. Party B's schedule is denoted by c_B^{IA} and is constructed as follows (see Figure 3). All individuals with pre-tax income above y_B face a marginal tax rate of one. We denote by y'_B the threshold income level such that all individuals with income in between y_m and y'_B face a marginal tax rate of one, while individuals with income in-between y'_B and y_B face a zero marginal tax rate. Note that middle-class individuals with $y_m \leq y \leq y'_B$ receive the same after-tax income with the tax functions proposed by both parties. Lower-than-median income individuals are then faced with a zero marginal tax rate. Finally, observe that, if B is the incumbent, it receives exactly one half of the votes while, if A is the incumbent, it receives strictly more than one half.

¿From Figure 3, we have

$$I_1^{IA} + I_2^{IA} = I_A^{opt} = \int_{y_A}^{y_+} (y - y_A)dF.$$

Observing that $y_m + I_1^{IA} = y_A + I_1^{IA} + I_2^{IA}$, we get

$$I_1^{IA} = y_A - y_m + \int_{y_A}^{y_+} (y - y_A)dF. \tag{9}$$

A limit case of such a schedule occurs when $y'_B = y_m$, in which case the considered tax schedule corresponds to c_B^{opt}. It is easy to see from Figure 3 that a necessary and sufficient condition for a schedule like c_B^{IA} to satisfy the GBC is that c_A^{opt} intersects c_B^{opt} to the right of y_m. If it were not the case, c_B^{IA} would be everywhere below c_B^{opt} which by definition satisfies the GBC, so that the total after-tax income under c_B^{IA} would integrate to strictly less than the average income. We will come back to this point in section 6, but for the moment we assume that the following assumption holds, so that the schedule c_B^{IA} exists:

Assumption 2: $c_A^{opt}(y_m) > c_B^{opt}(y_m)$.

We now show that c_B^{IA} is uniquely defined given c_A^{opt}. The function c_B^{IA} is defined by the parameters I_0^{IA} and y'_B (see Figure 3). These two variables are the solution to the following two equations

$$\int_{y_-}^{y_m} (y + I_1^{IA})dF + \int_{y_m}^{y'_B} (y_m + I_1^{IA})dF + \int_{y'_B}^{y_B} (y + I_0^{IA})dF + \int_{y_B}^{y_+} (y_B + I_0^{IA})dF = \overline{y} \tag{10}$$

and

$$y'_B + I_0^{IA} = y_m + I_1^{IA}. \tag{11}$$

The first constraint is the GBC. The second constraint is obtained by construction.

Solving (11) for y'_B and substituting in (10) yields

$$\int_{y_-}^{y_m} (y + I_1^{IA})dF + \int_{y_m}^{y_m + I_1^{IA} - I_0^{IA}} (y_m + I_1^{IA})dF + \int_{y_m + I_1^{IA} - I_0^{IA}}^{y_B} (y + I_0^{IA})dF + \int_{y_B}^{y_+} (y_B + I_0^{IA})dF - \overline{y} = 0. \tag{12}$$

Let us now turn to the potential deviations for party B (keeping in mind that party A would not want to deviate as it is always reelected with its optimal policy). These deviations are obtained along the same lines as those considered in the previous section. In any fruitful deviation, party B receives the support of low and high income people whereas middle-class individuals strictly prefer to vote for A. For a given c_A^{opt}, this function, denoted by c_B^{IIA} is depicted on Figure 3: it is the best possible tax schedule yielding 50% of the votes to B, when this party's voters consist of a coalition of the poor and the rich. A formal definition of c_B^{IIA} is provided in Appendix C. ¿From Figure 3, we see that some individuals have the same after-tax income with c_B^{IIA} and with c_A^{opt}: poor individuals with a pre-tax income at most equal to y''_B and middle-class individuals with $y_2 \leq y \leq \hat{y}_B$.

We have thus shown that, when party A plays c_A^{opt}, c_B^{IIA} is the best deviation available to party B when it plays c_B^{IA}. Assume now that party B plays c_B^{IIA} while party A plays c_A^{opt}. Obviously, party A has no incentive to deviate since (by assumption) it is reelected with its unconstrained most-preferred policy. As to party B, c_B^{IIA} is by construction the best feasible winning policy of an ends-against-the-middle type. Any fruitful deviation is such that party B attracts the votes of the bottom half of the population. Observe that *(i)* by construction of c_B^{IIA}, the individual with income y_B obtains a higher after-tax income with c_B^{opt} than with c_B^{IIA}, while *(ii)* c_B^{opt} satisfies by definition the GBC. It follows that party B will deviate towards c_B^{opt} unless this policy gathers less than 50 percent of the votes when faced with c_A^{opt}– i.e., unless Assumption 2 holds. If this assumption holds, we then have that c_B^{IA} is the best deviation from c_B^{IIA} available to party B when party A plays c_A^{opt}. Obviously, what will determine whether c_B^{IA} or c_B^{IIA} is best for party B is the after-tax income each scheme gives to an individual with income y_B: c_B^{IA} is better when $c_B^{IA}(y_B) > c_B^{IIA}(y_B)$, while c_B^{IIA} is better if the opposite relationship occurs. We summarize the results from this section in the following proposition.

Proposition 3 *Under Assumption 2, if party A proposes its unconstrained most-preferred schedule* c_A^{opt}*, then party B's best response is to offer either* c_B^{IA} *or* c_B^{IIA}*. In either case, (i) a subset of the population obtains the same after-tax income under the tax schedules proposed by both parties, and (ii) party A has no incentive to deviate from* c_A^{opt}*.*

So far we have characterized policy pairs that potentially constitute *BPR* equilibria of the electoral game. In each case we have posited that one of the parties (B in Section 4 and A in Section 5) was playing its unconstrained optimal policy. We have constructed the optimal *BPR* of the other party according to two possible patterns which would arise according to Assumption 1 and 2. These assumptions have implied a comparison of endogenous variables. The next section studies the circumstances under which the policy pairs described here and in the previous section effectively constitute a *BPR*. For this we provide conditions on the income levels y_A and y_B of the parties' respective constituents which are the fundamental exogenous variables underlying our analysis.

To make the argument crisper some extra terminology is useful. An outcome will be referred to as "Type I" equilibrium when parties receive the support of either the poorer or the richer part of the electorate, while "Type II" equilibria are of the ends-against-the-middle variety (one party receives the support of the poorer and richer voters while voters in the middle of the income distribution support the other party). We also index the equilibrium type by the identity of the party proposing its unconstrained optimal policy, to obtain the following definitions:

Definition 1

In a type I_A equilibrium, party A proposes c_A^{opt} while party B proposes c_B^{IA}.
In a type II_A equilibrium, party A proposes c_A^{opt} while party B proposes c_B^{IIA}.
In a type I_B equilibrium, party B proposes c_B^{opt} while party A proposes c_A^{IB}.
In a type II_B equilibrium, party B proposes c_B^{opt} while party A proposes c_A^{IIB}.

6 Equilibrium partition

We are now in a position to state our main result.

Proposition 4 *The space (y_B, y_A) can be partitioned into four regions, each corresponding to a different equilibrium. There exist three decreasing functions of y_B, denoted by Φ_A, Φ and Φ_B, that are such that*

if $y_- \leq y_A \leq \Phi_B(y_B)$, the equilibrium is of type II_B;
if $\Phi_B(y_B) \leq y_A \leq \Phi(y_B)$, the equilibrium is of type I_B;
if $\Phi(y_B) \leq y_A \leq \Phi_A(y_B)$, the equilibrium is of type I_A;
if $\Phi_A(y_B) \leq y_A$, the equilibrium is of type II_A.

The three functions and the partition of equilibria they delineate are represented on Figure 4. The formal proof of Proposition 4 is provided in Appendix D. Here we restrict ourselves to presenting the underlying intuition.

If the median income individuals are indifferent between both parties' unconstrained most-preferred policies (T_A^{opt} and T_B^{opt}), then the unique equilibrium is such that each party proposes its unconstrained most-preferred policy. This is because under this policy any incumbent party will be reelected with one half of the votes. The locus of pairs (y_B, y_A) for which this is true is represented by the function $y_A = \Phi(y_B)$. To the left of this schedule (in the (y_B, y_A) space - see Figure 4), Assumption 1 holds so that by Proposition 3 and Definition 1, the equilibrium of the game is of Type I_B or II_B; it is party B who proposes its unconstrained optimal policy. Similarly, to the right of Φ, Assumption 2 holds and, by Proposition 2 and Definition 1, the equilibrium of the game is of Type I_A or II_A, and it is party A who proposes its unconstrained optimal policy.

Intuitively, we obtain that party A proposes its unconstrained optimal policy (i.e., Assumption 2 holds) if y_B is large enough (i.e., very different from y_m) for a given y_A, or if y_A is large enough (i.e., close enough to y_m) for a given y_B. The proof of Proposition 4 also shows that the function Φ is decreasing in y_B.[10] This means that, the lower y_A is (i.e., the more different it is from y_m), the larger (i.e., the more different from y_m) y_B has to be in order for party A to propose its unconstrained optimal policy at equilibrium. Furthermore, we obtain that, if y_B is large enough (close enough to the maximum income y_+), then whatever the value of $y_A \leq y_m$, the equilibrium is of type I_A or II_A.[11]

We now turn to the policy offered by the party not playing its unconstrained most-preferred policy at the equilibrium. We obtain from Proposition 4 and Figure 4 that policies that divide the electorate between poorer and richer voters (Type I) are played if the parties' constituents are close to the locus $y_A = \Phi(y_B)$. The intuition for this result comes from the observation that policies c_B^{IA} and c_A^{IB} respectively tend toward c_B^{opt} and c_A^{opt} when y_A and y_B become arbitrary close to Φ. This is not the case for policies c_B^{IIA} and c_A^{IIB}. For any point such that $y_A = \Phi(y_B)$, the only equilibrium consists in

[10] To understand why Φ decreases with y_B, observe first that the space of most-preferred after-tax income schedules is unidimensional and can be indexed by the pre-tax income of the individual (since there is a unique kink in the most-preferred after-tax income function, with zero marginal tax rate below and unitary marginal tax rate above — see Figure 1). We show in the Appendix that individual preferences are single-peaked in the space of most-preferred policies. That is, the utility that an individual with gross income y achieves with the most-preferred tax schedule of an individual with gross income x increases with x when $x < y$ and decreases with x when $x > y$. This in turn implies that, as y_B increases, the value of y_A for which the median income individual is indifferent between the most-preferred policies of y_A and y_B decreases.

[11] This is due to the fact that the median income individual strictly prefers the unconstrained optimal tax schedule of party A (whatever the value of $y_A < y_m$) to the one of party B when y_B is arbitrarily close to y_+. This preference in turn comes from the observation that, under any individual y's most-preferred policy, all individuals with an income above y obtain at least the average income $\bar{y}$. On the other hand, individual y_+'s most preferred policy is the *laissez-faire*, so that with a positively skewed income distribution the median income prefers the policy advocated by party A to the *laissez-faire*.

both parties offering their unconstrained optimal policy. As we move slightly away from Φ by, say increasing y_A, the schedule c_B^{IA} remains very close to c_B^{opt} (while ensuring that party B is reelected if it is the incumbent), and thus gives more utility to party B's constituents than the schedule c_B^{IIA}.

The fact that both Φ_A and Φ_B decrease with y_B is intuitive once we look at Figure 4. We see that the locus Φ_B gives, for any value of y_A, the minimum value of y_B that is compatible with a Type I equilibrium. Similarly, Φ_A gives, for values of y_A large enough, the maximum value of y_B compatible with a Type I equilibrium. In other words, Φ_B and Φ_A determine the boundaries of an interval of values of y_B for which we have a Type I equilibrium. The fact that both functions Φ_B and Φ_A are decreasing in y_B means that this interval moves to the right (its two extreme values increase) as y_A decreases. This is intuitive since the value of y_B for which both parties play their unconstrained most-preferred policy also increases when y_A decreases.

Finally, observe that there is some asymmetry in the model because of the assumption that the median income is lower than the mean. This asymmetry benefits party A (whose constituent has a lower-than-median income) in the following sense: there exist equilibria where party A proposes its unconstrained optimal policy even when y_A is extreme (i.e., very low – provided of course that y_B is also extreme, i.e., large enough) while there is no equilibrium where party B proposes its unconstrained optimal policy when y_B is extreme (i.e., large enough, whatever the value of y_A).

7 Conclusion

In this paper, we develop a repeated electoral competition game between two parties, each maximizing the utility of a different citizen. Citizens differ only according to their exogenous income level. Parties are unable to commit to any policy before the election; they choose a non-linear income tax schedule once elected. In each period, citizens cast a vote either for the incumbent or for the challenger.

We first show that there exist (pure strategy) subgame perfect equilibria where both parties choose the most-preferred tax schedule of their constituent subject to the constraint that they are reelected. We characterize a class of these (Best Policy with

Reelection, or *BPR*) equilibria for which one of the two parties plays the unconstrained optimal tax schedule of its constituents. We obtain that tax schedules in such *BPR* equilibria are always piecewise linear. We distinguish between four types of equilibria and show that the specific type that emerges depends on the income levels of the constituents of the two parties. The partition of the (y_A, y_B) space we provide shows that depending on the income levels of the constituents it is party A or party B who plays the unconstrained optimal strategy. In each case we obtain either classical left-vs-right equilibria (where poorer people vote for one party and richer people for the other one) or ends-against-the-middle equilibria (where both poor and rich people vote for one party while the middle class vote for the other party). All types of equilibria are such that both parties propose the same tax schedule for a subset of the population.

How do these results relate to the income tax schedules observed in reality? Piecewise linearity is a very common feature in practice. It is worth emphasizing that we do not assume that tax functions have this property, but rather that we start from a much larger set of admissible non-linear functions and that we obtain piecewise linearity as a property of all tax functions proposed in BPR equilibria. In all equilibria we consider, one party proposes a tax function with only two brackets while the other party proposes a tax function with more than two brackets. One way to generate more brackets at equilibrium would be to assume that parties represent more than one constituent (i.e. income level). We leave such an extension for future research.

We also obtain that, in all equilibria studied, one party proposes a tax function (with minimum marginal tax rate up to a threshold income level and the maximum marginal rate above) that exhibits both marginal and average progressivity. The tax function proposed by the other party may not be progressive (except for very specific combinations of y_A and y_B). Marginal progressivity is a characteristic of many real world tax schedules and is notoriously difficult to obtain as a solution in an optimal taxation framework. Average progressivity, on the other hand, is an even more common characteristic of real world tax schedules but our setting offers just one of the many possible explanations (which include essentially the entire optimal income tax literature).

Finally, in all equilibria studied both parties propose the same tax schedule to a subset of the population. This characteristic of equilibrium tax schedules is also obtained by Roemer (2008) who studies the same economic environment and the same set of admissible tax functions but who develops another electoral competition game where parties can commit to electoral promises. Roemer (2008) also uses data on income taxation in the US from 1960 to 2004 to check whether both parties propose the same tax schedule for a subset of the population. Since parties do not compete by announcing precise tax schedules,[12] Roemer (2008) looks at major tax reforms, attributing the reform to the party of the President at the time of reform. The reform then becomes the status quo until a new reform is enacted. He can then attribute both the status quo legislation and the reform to political parties. He obtains that most reforms do not change the effective tax-and-transfer rates faced by a substantial subset of the population, which then translates into both parties proposing the same tax rate for this subset. His empirical evidence then supports this characteristic of our equilibria.

This paper falls short on several accounts. First, we restrict the set of equilibria to *BPR* equilibria where one party proposes the most-preferred policy of its constituent, absent any electoral consideration. It would be worthwhile to look at non-*BPR* equilibria in this setting.

In our model, there is no political alternation since the incumbent party stays in power forever in *BPR* equilibria. Political alternation would require the introduction of shocks and/or uncertainty into the electoral process.

Finally, incentive effects on the labor supply decision are absent from our model. Introducing a preference for leisure and thus a distortionary impact of income taxation would constitute an interesting extension. This would certainly yield less extreme results which would have a more realistic flavor. In particular marginal tax rates of 1 would no longer emerge and be replaced by the tax revenue maximizing rates (maximum of Laffer curve).

[12]This observation is much more in line with the postelection politics approach developed here than with the preelection politics model developed in Roemer (2008).

APPENDIX

A Proof of proposition 1

Assume that party A proposes $T_A^*(.)$ if elected while party B proposes $T_B^*(.)$ if elected. Moreover, if a party deviates in a given period, it is assumed to play this deviation in all subsequent periods.

Note that, because the equilibrium choice of a party in a given period does not depend on the identity of elected politicians in preceding periods, the continuation value of the voters in any period $t+1$ does not depend on their decision in period t. Under prospective voting, this implies that a voter with income y will prefer to vote for the incumbent i rather than the challenger j if and only if $v(y, T_{i,t}(y)) \geq v(y, T_{j,t}(y))$.

We now derive the necessary and sufficient conditions under which such strategies constitute an equilibrium. We start with party A.

1) Suppose that A is in power in period $t = 1$. If A plays the equilibrium policy, T_A^*, it is, by definition, reelected since this policy garners at least 50% of the votes against T_B^*. According to its strategy, A continues to play T_A^* in the next period and is then reelected. Making the same reasoning in all future periods, we can conclude that A is reelected forever when playing the equilibrium strategy and obtains the payoff:

$$(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}\left[v(y_A, T_A^*(y_A)) + \beta\right] = v(y_A, T_A^*(y_A)) + \beta. \tag{13}$$

Consider now the possibility of a deviation by A. If $T_A^* = T_A^{opt}$, no deviation can improve its payoff. Let us then assume $T_A^* \neq T_A^{opt}$. Obviously, A should not deviate toward another policy that leads to reelection since T_A^* is by definition the best policy guaranteeing reelection. However, A may want to deviate to a policy that does not ensure reelection. The best possible deviation in this class is A's optimal tax schedule, T_A^{opt}. According to A's strategy, if A deviates to T_A^{opt} today, it should continue to play this policy in all future periods in case it is elected. As T_A^{opt} does not lead to reelection, it follows that A will never be reelected if it deviates to T_A^{opt} today. Its payoff following the deviation is thus given by

$$(1-\delta)\left(v(y_A, T_A^{opt}(y_A)) + \beta\right) + (1-\delta)\sum_{t=2}^{\infty}\delta^{t-1}v(y_A, T_B^*(y_A))$$
$$= (1-\delta)\left(v(y_A, T_A^{opt}(y_A)) + \beta\right) + \delta v(y_A, T_B^*(y_A)). \tag{14}$$

Comparing the payoffs associated with the equilibrium strategy, (13), with that achieved in case of a deviation (14), shows that A should play the equilibrium policy if and only if

$$\delta \geq \frac{v(y_A, T_A^{opt}(y_A)) - v(y_A, T_A^*(y_A))}{v(y_A, T_A^{opt}(y_A)) - v(y_A, T_B^*(y_A)) + \beta},$$

which is true under condition (5).

2) Repeating the same analysis for party B (assuming that it is in power in period $t = 1$), we obtain that B should play its equilibrium policy if and only if

$$\delta \geq \frac{v(y_B, T_B^{opt}(y_B)) - v(y_B, T_B^*(y_B))}{v(y_B, T_B^{opt}(y_B)) - v(y_B, T_A^*(y_B)) + \beta}.$$

Therefore under (5), both parties should play the equilibrium policy.

B Formal definition of c_A^{IIB}

Formally, the best deviation c_A^{IIB} solves

$$\max_{I_2^{IIB}, I_3, y_1, \hat{y}_A, \tilde{y}_A, y_A''} \quad I_2^{IIB}$$

s.t.

$$\int_{y_-}^{y_A}(y + I_B^{opt} + I_2^{IIB})dF + \int_{y_A}^{\hat{y}_A}(y_A + I_B^{opt} + I_2^{IIB})dF + \int_{\hat{y}_A}^{y_1}(y + I_B^{opt})dF$$
$$+ \int_{y_1}^{\tilde{y}_A}(y_1 + I_B^{opt})dF + \int_{\tilde{y}_A}^{y_A''}(y + I_3)dF + \int_{y_A''}^{y_+}(y_A'' + I_3)dF = \overline{y},$$
$$\int_{y_-}^{y_1}dF + \int_{y_A''}^{y_+}dF = \frac{1}{2},$$
$$\hat{y}_A = y_A + I_2^{IIB},$$
$$\tilde{y}_A + I_3 = y_1 + I_B^{opt},$$
$$y_A'' + I_3 = y_B + I_B^{opt},$$
$$y_1 \geq \hat{y}_A.$$

The first constraint is the GBC. The second one means that c_A^{IIB} must yield exactly half of the votes. The last four conditions are obtained by construction.

After some manipulations, we obtain the following program:

$$\max_{I_2^{IIB}, I_3, y_1} \quad I_2^{IIB}$$

s.t.

$$\int_{y_-}^{y_A} (y + I_B^{opt} + I_2^{IIB}) dF + \int_{y_A}^{y_A + I_2^{IIB}} (y_A + I_B^{opt} + I_2^{IIB}) dF$$
$$+ \int_{y_A + I_2^{IIB}}^{y_1} (y + I_B^{opt}) dF + \int_{y_1}^{y_1 + I_B^{opt} - I_3} (y_1 + I_B^{opt}) dF$$
$$+ \int_{y_1 + I_B^{opt} - I_3}^{y_B + I_B^{opt} - I_3} (y + I_3) dF + \int_{y_B + I_B^{opt} - I_3}^{y_+} (y_B + I_B^{opt}) dF = \overline{y},$$
$$\int_{y_-}^{y_1} dF + \int_{y_B + I_B^{opt} - I_3}^{y_+} dF = \frac{1}{2},$$
$$y_1 \geq \hat{y}_A.$$

C Formal definition of c_B^{IIA}

The function c_B^{IIA} then solves the following program

$$\max_{I_0^{IIA}, I_4, y_B'', \tilde{y}_B, y_2, \hat{y}_B} \quad I_0^{IIA}$$

s.t.

$$\int_{y_-}^{y_B''} (y + I_A^{opt}) dF + \int_{y_B''}^{\tilde{y}_B} (y_B'' + I_A^{opt}) dF + \int_{\tilde{y}_B}^{y_2} (y + I_4) dF$$
$$+ \int_{y_2}^{\hat{y}_B} (y_2 + I_4) dF + \int_{\hat{y}_B}^{y_B} (y + I_0^{IIA}) dF + \int_{y_B}^{y_+} (y_B + I_0^{IIA}) dF = \overline{y},$$
$$\int_{y_-}^{y_B''} dF + \int_{y_2}^{y_+} dF = \frac{1}{2},$$
$$\hat{y}_B + I_0^{IIA} = y_A + I_A^{opt},$$
$$y_B'' + I_A^{opt} = \tilde{y}_B + I_4,$$
$$y_2 + I_4 = y_A + I_A^{opt},$$
$$y_2 \leq \hat{y}_B.$$

This program simplifies to

$$\max_{I_0^{IIA}, I_4, y_B''} \quad I_0^{IIA}$$

s.t.

$$\int_{y_-}^{y_B''} (y + I_A^{opt})dF + \int_{y_B''}^{y_B''+I_A^{opt}-I_4} (y_B'' + I_A^{opt})dF$$
$$+ \int_{y_B''+I_A^{opt}-I_4}^{y_A+I_A^{opt}-I_4} (y + I_4)dF + \int_{y_A+I_A^{opt}-I_4}^{y_A+I_A^{opt}-I_0^{IIA}} (y_A + I_A^{opt})dF$$
$$+ \int_{y_A+I_A^{opt}-I_0^{IIA}}^{y_B} (y + I_0^{IIA})dF + \int_{y_B}^{y_+} (y_B + I_0^{IIA})dF - \overline{y} = 0,$$
$$\int_{y_-}^{y_B''} dF + \int_{y_A+I_A^{opt}-I_4}^{y_+} dF = \frac{1}{2},$$
$$y_2 \leq \hat{y}_B.$$

D Proof of proposition 4

We first prove the properties of the function Φ before turning to those of Φ_A. The characteristics of Φ_B are obtained in a similar way. Proposition 4 follows from the definitions and properties of these three curves.

Step 1: The function $\Phi(y_B)$

The function Φ is defined in the following way: it gives the (unique) value of y_A that is such that the median income individual is indifferent between the unconstrained optimal tax schedule of party A and the unconstrained optimal tax schedule of party B. We first prove that, in the space (y_B, y_A), this curve goes through the point (y_m, y_m), is decreasing and crosses the horizontal axis at (α, y_-) with $y_m < \alpha < y_+$.

The after-tax income of the median voter under A's optimal tax is

$$y_A + \int_{y_A}^{y_+} (y - y_A)dF$$

whereas it is

$$y_m + \int_{y_B}^{y_+} (y - y_B)dF$$

under B's optimal tax, where we have used (7) and (8). The condition $y_A = \Phi(y_B)$ can thus be written

$$y_A + \int_{y_A}^{y_+} (y - y_A) dF = y_m + \int_{y_B}^{y_+} (y - y_B) dF. \tag{15}$$

a) When $y_A = y_m$ and $y_B = y_m$, it is satisfied with equality. Therefore the curve Φ goes through the point (y_m, y_m).

b) We then determine how the LHS of (15) varies with y_A:

$$\frac{\partial}{\partial y_A} \left(y_A + \int_{y_A}^{y_+} (y - y_A) dF \right) = F(y_A) \geq 0.$$

The derivative of the RHS of (15) with respect to y_B is:

$$\frac{\partial}{\partial y_B} \left(y_m + \int_{y_B}^{y_+} (y - y_B) dF \right) = F(y_B) - 1 \leq 0.$$

These two properties imply that the function $\Phi(y_B)$ is continuous and decreasing.

c) We now argue that

$$y_A + \int_{y_A}^{y_+} (y - y_A) dF > y_m + \int_{y_B}^{y_+} (y - y_B) dF$$

for all $y_A \leq y_m$ when y_B is close to y_+.This comes from the observation that

$$y_A + \int_{y_A}^{y_+} (y - y_A) dF = \int_{y_-}^{y_A} y_A dF + \int_{y_A}^{y_+} y dF \geq \overline{y} > y_m = \lim_{y_B \to y_+} y_m + \int_{y_B}^{y_+} (y - y_B) dF.$$

This inequality, together with the continuity of Φ, implies that the curve Φ crosses the horizontal axis at the point (α, y_-) where $y_m < \alpha < y_+$.

The paragraph after the statement of the proposition in section 4 explains why we have type I_A or II_A equilibria when $y_m \geq y_A > \Phi(y_B)$, and type I_B or II_B equilibria when $y_- < y_A < \Phi(y_B)$.

Step 2: The function $\Phi_A(y_B)$

We now construct the curve Φ_A and show that it is decreasing and goes through the points (y_m, y_m) and (y_+, β), where $y_- < \beta < y_m$. Along this curve (i.e., when $y_A = \Phi_A(y_B)$), party B is indifferent between playing the strategy c_B^{IA} or c_B^{IIA} when party A proposes its optimal tax schedule. As the utility level attained by party B is continuous in y_B under both policies c_B^{IA} and c_B^{IIA}, the function Φ_A is also continuous in y_B.

Take any point on Φ except (y_m, y_m) and consider an increase in y_B: $y_B \to y_B + \delta$. We argue that for δ arbitrarily small, B prefers to play c_B^{IA} to c_B^{IIA} when A proposes its ideal policy. The argument is that when δ gets small, c_B^{IA} gets arbitrarily close to B's ideal policy c_B^{opt} (recall that along Φ the median voter is indifferent between the ideal policies of the two parties) while c_B^{IIA} does not. Then it is easy to see that, for small δ enough, B strictly prefers c_B^{IA} to c_B^{IIA}.

Next consider the point (y_m, y_m). We argue that at any point to the right of (y_m, y_m), B prefers c_B^{IIA} to c_B^{IA}. This follows from the fact that c_A^{IA} is the ideal policy of the median voter, c_m^{opt}, when $y_A = y_m$. Consequently, c_B^{IA} must also be equal to c_m^{opt} for B to be reelected by the median voter. It then immediate follows that c_B^{IIA} is preferred by B to c_B^{IA}, as it gives less income to y_m individuals and more to y_B individuals.

These two facts imply that Φ_A is to the right of Φ and crosses the axis $y_A = y_m$ at (y_m, y_m) only. Therefore Φ_A is decreasing over some subset of its support.

We now argue that Φ_A cannot increase, by showing that at any point to the right of Φ_A, B prefers c_B^{IIA} to c_B^{IA}.

Consider a point (y_B, y_A) on Φ_A (corresponding tax schedules are represented on figure 5) and increase slightly $y_B : y_B \to y_B + \delta$. The initial point being on Φ_A, we have $c_B^{IA}(y_B, y_B, y_A) = c_B^{IIA}(y_B, y_B, y_A)$, namely B obtains the same after-tax income by playing c_B^{IA} or c_B^{IIA}. We then construct $c_B^{IA}(y, y_B + \delta, y_A)$. It is represented on Figure 5. It is identical to $c_B^{IA}(y, y_B, y_A)$ for values of $y \leq y'_B$; for $y > y'_B$, it corresponds to the dashed green curve. Then consider the strategy for $y_B + \delta$ that corresponds to the dashed green curve for y larger than y'_B and to $c_B^{IIA}(y, y_B, y_A)$ for lower values of y. This tax function satisfies by construction the GBC and yields 50% of the votes to B. Moreover, it provides exactly the same net income as $c_B^{IA}(y, y_B + \delta, y_A)$ to people with gross income $y_B + \delta$. Therefore there exists a tax schedule in the family c_B^{IIA} that is at least as good as c_B^{IA} for individuals with income $y_B + \delta$. This means that c_B^{IIA} is weakly preferred to c_B^{IA} and thus that the equilibrium cannot be IA to the right of Φ_A (either the curve Φ_A is flat or the equilibrium is IIA).

Finally we prove that Φ_A cannot cross the horizontal axis $y_A = y_-$, that is $\Phi_A(y_+) > y_-$ for any $y_B \leq y_+$. To do so, it is sufficient to show that B strictly prefers c_B^{IA} to c_B^{IIA}

when $y_A \rightarrow y_-$. When $y_A = y_-$, A's optimal tax policy consists in giving the same net income, $\overline{y}$, to everyone (the function c_A^{IA} is flat). This has a direct implication for strategy c_B^{IIA}: when y_A gets arbitrarily close to y_-, B has no other choice than proposing a tax schedule c_B^{IIA} arbitrarily close to c_A^{IA}. It is then easy to see from Figure 3 why party B prefers the schedule c_B^{IA}: this schedule gives less than $\bar{y}$ to individual with income $y < y_m$ in order to increase after-tax consumption of individuals $y'_B < y \leq y_+$. We then have that $c_B^{IA}(y_B, y_B, y_-) > c_B^{IIA}(y_B, y_B, y_-)$.

Step 3: The function $\Phi_B(y_B)$

The technique used for constructing Φ_B is similar to the one used for Φ_A.

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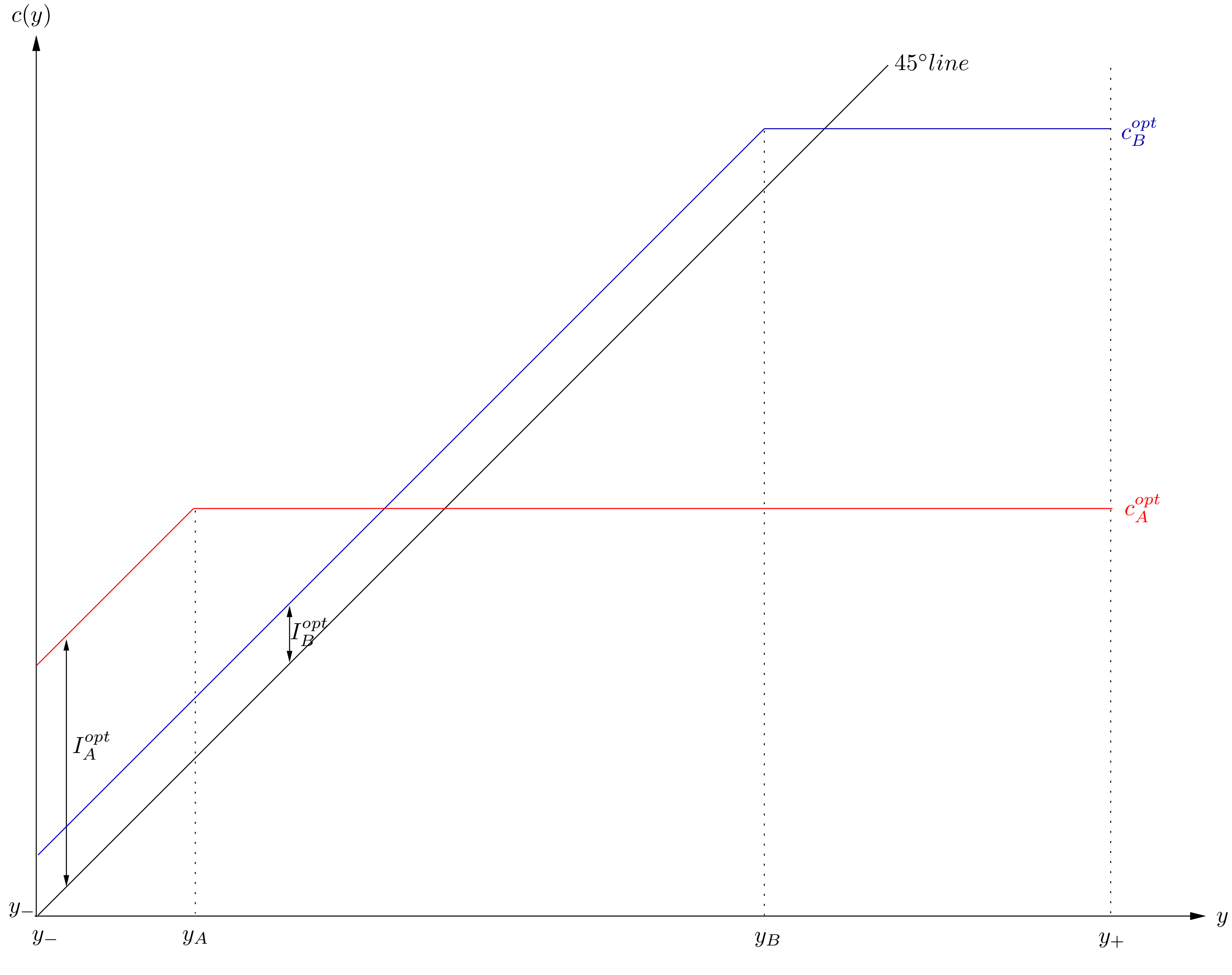

Figure 1: Optimal tax schedules of the two parties

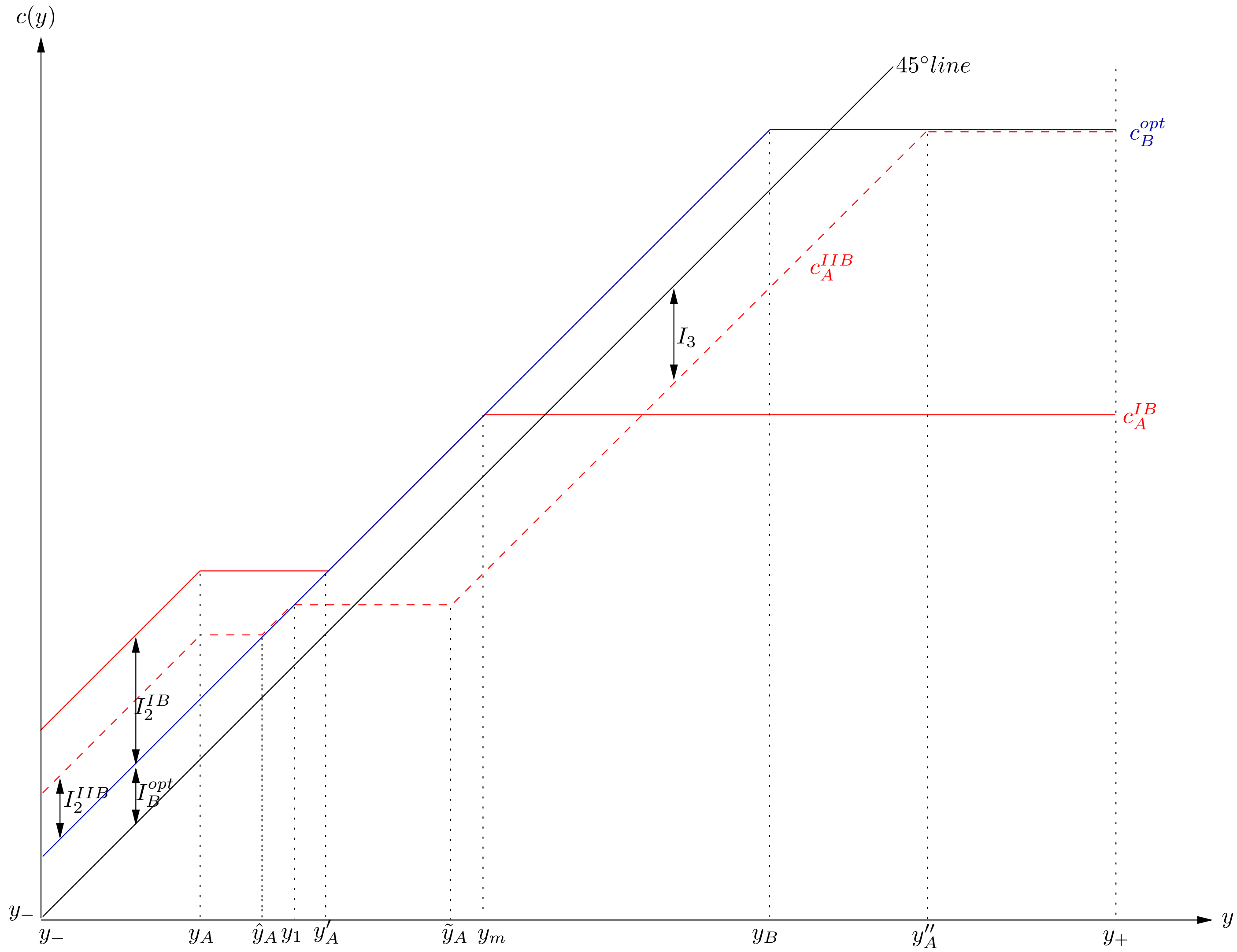

Figure 2: Party B proposes its unconstrained optimal policy

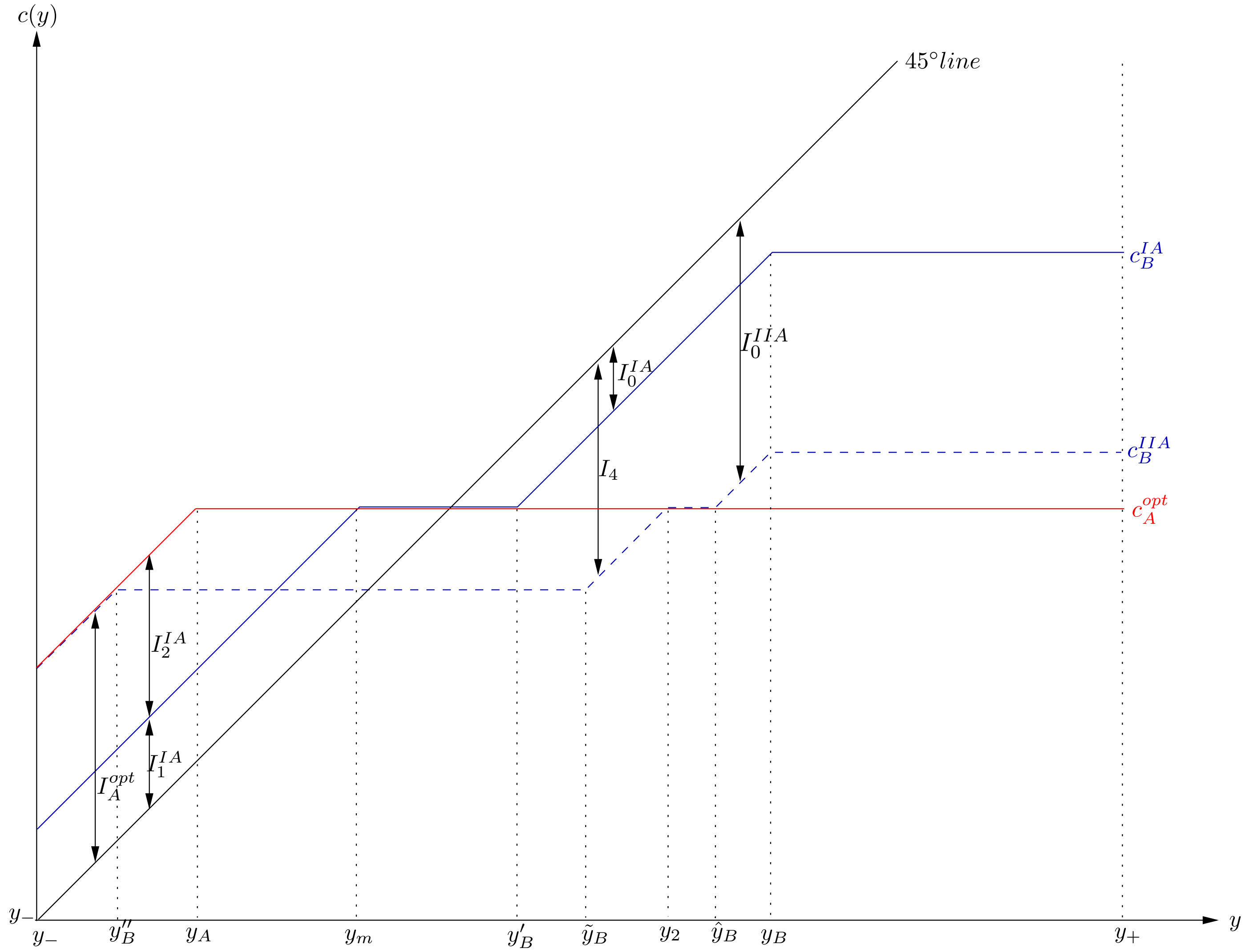

Figure 3: Party A proposes its unconstrained optimal policy

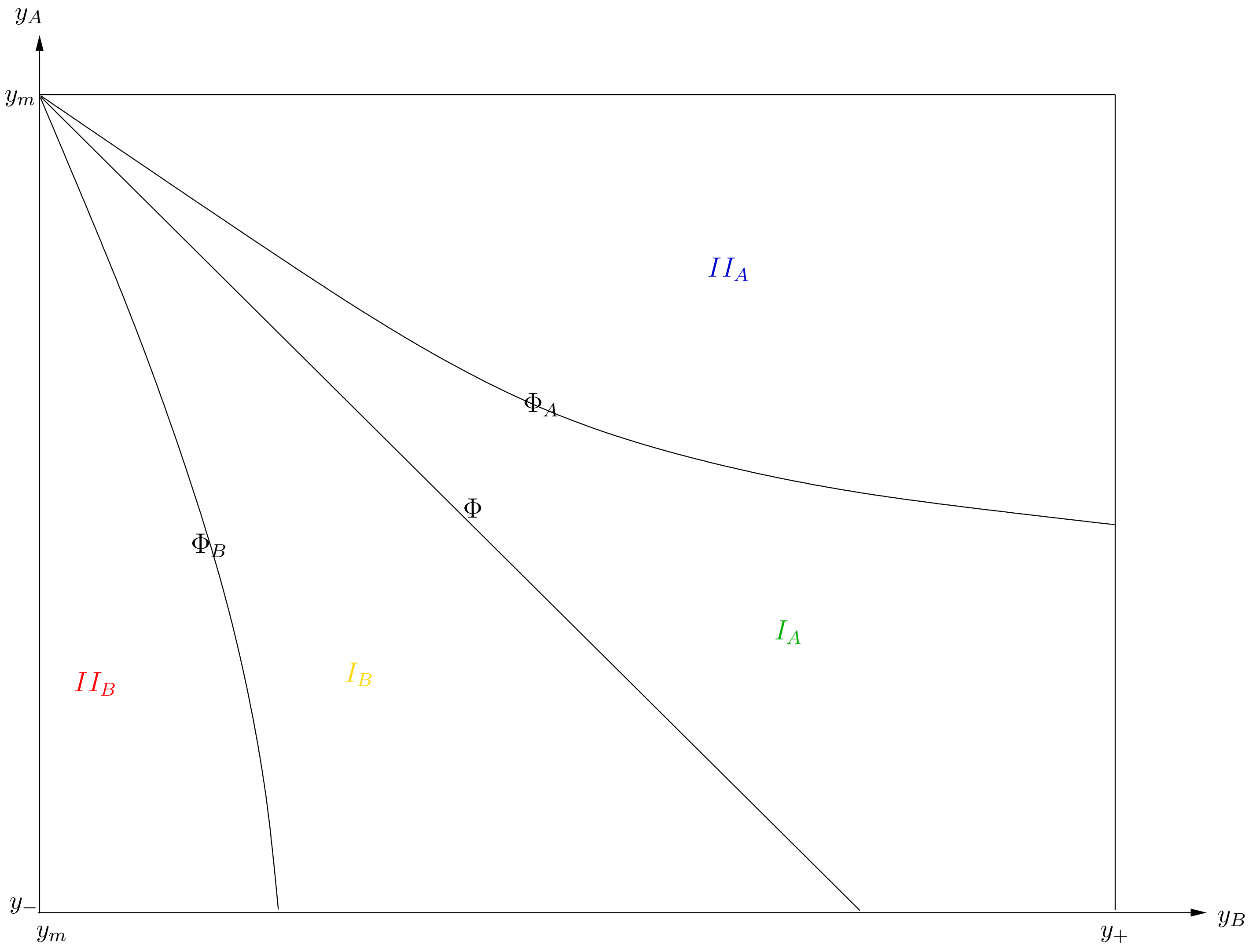

Figure 4: Partition of equilibria

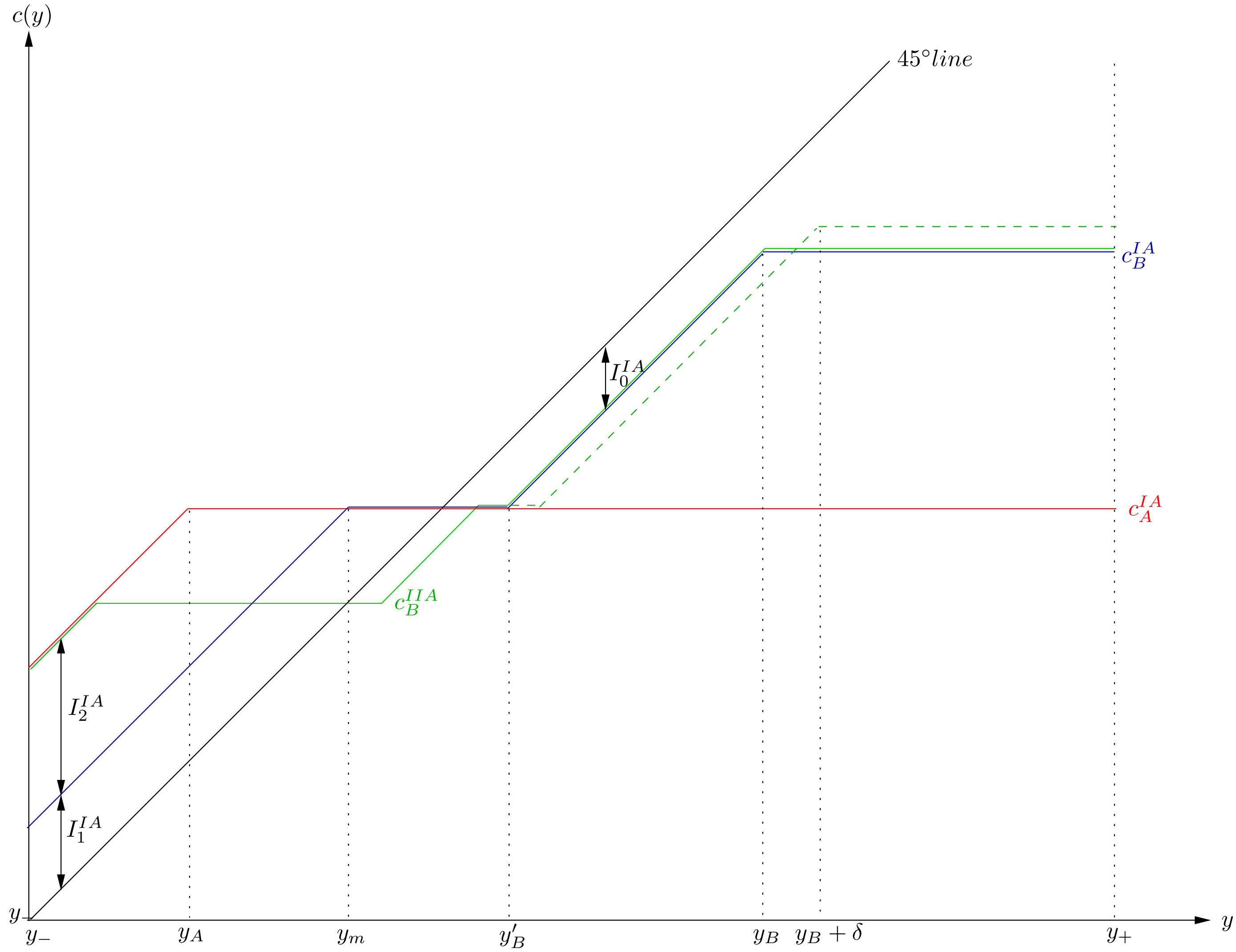

Figure 5: Construction of Φ_A