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ABSTRACT

Fiscal Sustainability and Demographics - Should We Save or Work More?*

Approaching demographic shifts are raising concerns about fiscal sustainability in most OECD countries. A widespread view based on the tax-smoothing idea is that a prior consolidation of public finances is required to cope with the predicted trend deterioration in the primary budget balance. Both positive aspects in assessing the order of magnitude of sustainability problems and normative aspects of formulating policy strategies are addressed. It is argued that the smoothing argument cannot unconditionally be applied to the demographic problem. It is important to distinguish between increases in the dependency ratio driven by changes in fertility and longevity. For the former the smoothing argument may be appropriate, but not for the latter. In the case of longevity, a trade-off between consolidation and increasing retirement ages becomes relevant, and there are strong arguments why the latter should be pursued by e.g. linking retirement ages to longevity.

JEL Classification: E60, H50 and J11
Keywords: fertility, fiscal sustainability, longevity and tax smoothing

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1 Introduction

Projected increases in dependency ratios raise fundamental questions on the fiscal sustainability of public policies in a number of countries[1]. The reason is simply that the number of net-beneficiaries is increasing relative to the number of net-contributors. The main drivers of the changing demographic structure are an echo effect of swings in the fertility rate (the baby-boom effect) and the increase in longevity (the ageing effect).

The fiscal sustainability problem will show up in a systematic tendency to budget deficits and therefore growing public debt levels. These trends have implied that many countries are including considerations of fiscal sustainability in their fiscal policy planning. For EU countries, it is now part of the Stability and Growth Pact that member countries, in their stability and convergence programmes, shall assess the long-term sustainability of fiscal policy (see e.g. Beetsma and Oksanen (2008)). The monitoring of fiscal balances in the medium to long run has thus gained importance in policy planning[2], and some countries have undertaken reforms while others are pushing reform needs ahead of them.

A common theme in the debate is that reforms should go in the direction of consolidation of public finances. That is, prior to the demographic changes, governments should consolidate public finances so as to reduce debt levels to make it possible to accommodate future deficits. This is reflected in reports from the IMF (2004), OECD (2002) and the European Commission (2006a,b), and in work by Auerbach (2008), Hauner et al. (2007), McKissack and Cornley (2005) among others. These views are quite clear in the following two policy statements:

> ".....there will as noted be an ageing bulge in the years around 2035, where the dependency ratio peaks. With a pay-as-you-go financing, the ageing problem will be financed by those alive and paying taxes around 2035. With the alternative tax-smoothing policy the financing of the ageing bulge will be distributed across all generations (Danish Government, 2000, p 173).

and

[1] European Commission (2006 a,b) presents calculations showing a fiscal sustainability problem for most EU countries. Board of Trustees (2008) gives similar assessments for the US.

[2] A number of countries have adopted numerical fiscal rules, see European Commission (2006) and OECD (2007). In many cases, these rules include targets for budget surpluses or public debt levels to ensure a sufficient consolidation of public finances to prepare for projected demographic shifts.

A current high level of public saving is basically motivated by the need to ensure a more equal distribution of consumption possibilities across generations" (Swedish Government, 2008, p 170).

The theoretical rationale for these viewpoints is based on the tax smoothing argument advanced by Barro (1979). The point is that tax distortions are minimized by keeping tax rates constant, and therefore taxes should be set at a level consistent with long run revenue requirements. Temporal variations in expenditures should be allowed to affect the budget balance. Applying this reasoning to the demographic problems leads to the immediate conclusion that a once-and-for-all strategy is needed to adjust public policies so as to ensure that the intertemporal budget constraint is fulfilled. Since the underlying trend is a deterioration of the public budget, this implies that a sufficient consolidation should be ensured prior to the demographic changes taking full effect. This interpretation is implied when the metric on the sustainability problem (European Commission (2006a)) given in terms of the needed permanent budget improvement (as a share of GDP) is turned into a normative target for public finances.

The aim of this paper is to consider this smoothing or consolidation argument. It is argued that the Barro-smoothing argument cannot readily be applied to any net-expenditure variations independently of their cause. In particular, considering the demographic changes, it is important to distinguish between increases in the dependency ratio driven by changes in fertility and longevity. Policy debates rarely make an explicit distinction between the two. It is well-known that the peak in fertility in the 1940s and 1950s and the subsequent drop have caused a baby-boom effect, and this is now becoming quite visible in many countries because large generations are approaching retirement at the same time as smaller generations are entering the labour market. These variations in fertility and the implied effects on the dependency ratio are irreversible, but in a demographic sense, they constitute a temporary phenomenon (presuming that fertility has reached a new long-run level). Yet, the adjustment to these changes is not trivial. Increases in longevity, on the other hand, constitute a more permanent change (ruling out epidemics causing steep increases in mortality)[3], which causes persistent changes in the dependency ratio. Most countries have experienced and are forecasted to experience further increases in longevity[4]. Changes in longevity differ from changes in fertility also

[3] Oeeppen and Vaubel (2002) have shown that the long-run trend is a continuous increase in longevity.

[4] While increases in longevity earlier was driven by a decrease in child mortality, it has in recent decades been driven by a shift in mortality rates for higher age groups.

in the sense that they have a direct welfare consequence for individuals, and increasing longevity is a fundamental indication of improved living standards[5]. Increasing longevity is thus in itself a welfare improvement, but it raises the challenge for public finances of how to cope with the implied increase in the dependency ratio. The policy issues raised by increases in longevity are therefore fundamentally different from those caused by variations in fertility.

This paper explores some basic aspects of the interaction between demographic shifts and public finances when the public policy package involves age dependent entitlements which are (mainly) tax financed. All OECD countries have tax financed public pension and health care systems, albeit the level and composition differ. Since most of the tax revenue is collected based on labour market income (either when it is earned or used), it follows that public finances are very sensitive to the age distribution of the population between the work-age population and the retired. First, the paper considers the positive aspects related to assessing fiscal sustainability problems arising under a given policy package. Particular focus will be on how standard metrics of fiscal sustainability problems are affected by demographic changes as well as growth. Second, the paper considers the normative aspects of how to adjust the public policy package to changes in demographics. An important point here is that there is a fundamental difference between changes induced by variations in fertility and changes induced by changes in longevity.

There is by now a rather large literature with explicit analyses of fiscal sustainability problems arising from demographic changes in various countries[6]. These analyses are usually rather complicated since it is important to capture country specific aspects including policy rules and institutions. While this is important to make credible evaluations in the country specific cases, it also tends to make the outcome of such computations a "black box". This applies not least to the interpretation of the results of sensitivity analyses involving e.g. effects of changing assumptions concerning growth rates or the demographic developments. This paper offers a simple OLG framework which captures the essence of the much larger simulation models, but allows a transparent analysis of the main aspects.

There is a small theoretical literature that has considered the issue of whether a consolidation or pre-funding strategy is optimal. Flodén (2005) shows for the case where

[5] This is reflected by the fact that longevity has a weight of 1/3 in the Human development index calculated by the UNDP.

[6] For EU countries in European Commission (2006), for Denmark in Danish Welfare Commission (2006), for Norway in Frederiksen et al. (2006), and for Sweden in Petterson et al. (2006).

households are infinitely lived (longevity is thus not an issue) that a change in the dependency ratio should be addressed mainly by a consolidation of public finances; that is, pre-funding. In this case, all generations come to contribute equally to the financing needed to ensure fiscal sustainability. This result is driven by a consumption smoothing or sharing argument; that is, the burden is distributed (almost) equally across generations. This model can be interpreted as dealing with changes in the demographic composition driven by changes in fertility. De la Croix et al. (2004) also consider the optimal allocation across generations when there is a drop in fertility. They find that the capital intensity increases while it is ambiguous whether the retirement age should increase or decrease. Andersen (2008 a,b) considers the role of increases in longevity focussing on both intergenerational distribution and risk sharing arguments. It is shown that optimal policies call for retirement ages to adjust to longevity.

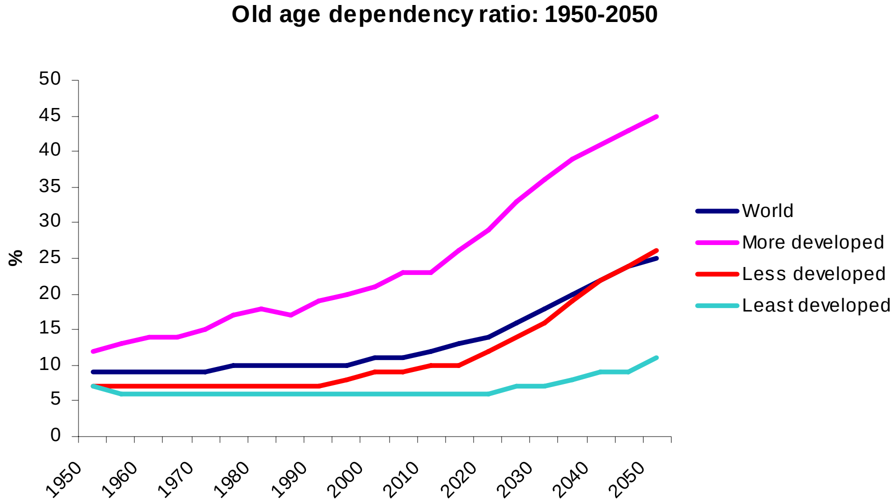
The paper is organised as follows: A few basic facts on projected demographic shifts and their interrelationship with public policies are presented in section 2. The issue of computations of fiscal sustainability indicators for a given policy package is discussed in section 3 with outset in a simple OLG-model, and some basic aspects related to fiscal sustainability are explored in section 4. Section 5 considers the question of whether demographic shifts should be addressed by pre-funding or some other adjustment of policy. Section 6 considers the normative aspect of how to adjust public policies to changes in the demographic composition, and section 7 offers a few concluding comments.

2 Demographics and public finances

2.1 Demographic trends

Substantial demographic changes unfold at a global level. Fertility has been declining at the same time as longevity has been and is expected to keep increasing mainly due to drop in mortality rates at higher ages. As a result, the global population will keep increasing at the same time as the age composition is changing significantly. In short, the world population is getting older. Population forecasts (see e.g. UN (2006)) clearly point to an upward trend in the dependency ratio, although there are some variations across regions and countries, cf. figure 1.

Figure 1: Old-age dependency ratio: 1950-2050

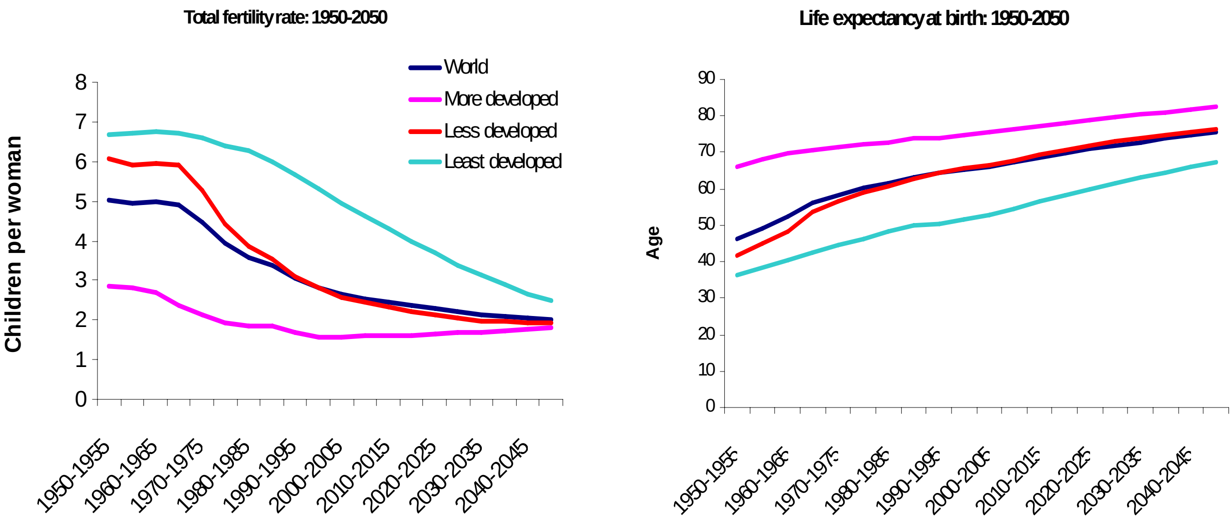


Note: Age group 65+ relative to age group 15-64, based on medium variant projection, less developed countries is exclusive of least developed countries.

Source: World Population Prospects: The 2006 revision, Population Database, Population Division, UN.

There are two main drivers behind the changes in the dependency ratio, namely, fertility and longevity. Fertility has been on a downward trend after peaking in the middle of the 20th century. Currently, it is stabilized at a level close to two, see figure 2. This will give an echo effect on the dependency ratio when large generations reach retirement age, and new smaller generations enter the working age groups (the baby-boom effect).

Figure 2: Fertility rates and longevity: 1950-2050



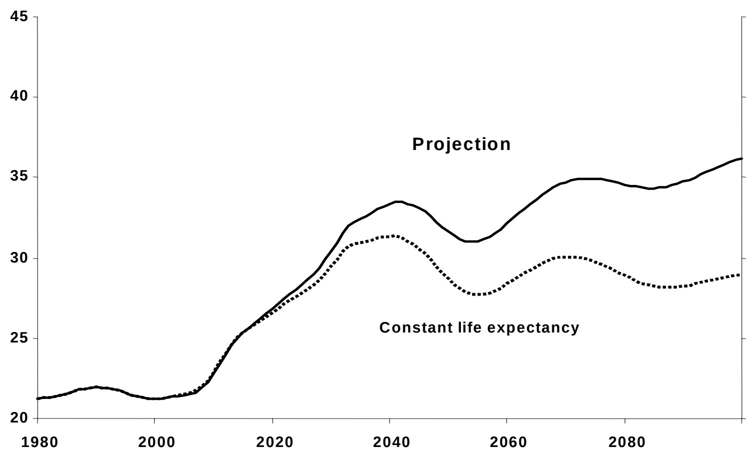
Source: As figure 1.

Note that for the more developed countries the fact that the fertility rate has been rather stable and is projected to remain so implies that the child dependency ratio is constant. According to the UN projection, the child dependency ratio in more developed countries is currently 27 %, and it will be at this level for the whole forecast period to 2050. For other regions, we will see some decline in the child dependency ratio.

Longevity has been on an upward trend with increases amounting to 0.1 to 0.2 years per year; i.e. longevity increases by one year between cohorts with an age difference of 6 to 12 years, cf. figure 2. The increase in longevity is striking and constitutes a major welfare improvement. Considering the development in historical perspective, it is noteworthy that life expectancy has been on an upward trend which does not seem to level off. An obvious question is by how much longevity can increase, but so far we do not seem to have reached the limit (see Oeppen and Vaubel (2002)).

An example of a decomposition of the dependency ratio in the component attributed to the baby-boom effect and longevity is given in figure 3. In this figure mortality rates are frozen at the 2003 level, implying that the figure only shows the importance of future increases in longevity and not those which have been realized in the past; i.e. the role of longevity is undervalued. The figure brings out quite clearly that the trend decline in fertility tends to generate a hump-shaped pattern for the dependency ratio, while the projected increases in longevity cause the dependency ratio to remain "high".

Figure 3: Dependency ratio - decomposition in fertility and longevity component, Denmark

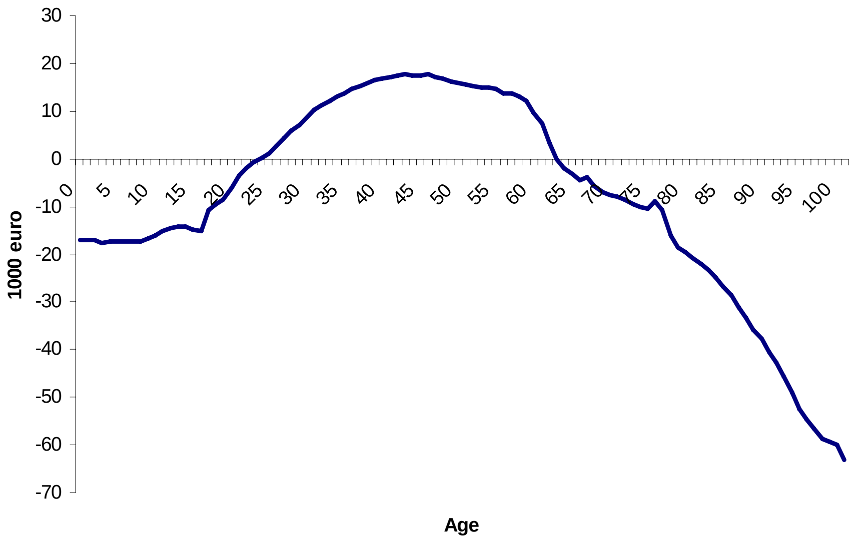


Source: Danish Welfare Commission, 2006

2.2 Public policies and age structure

All OECD countries have tax financed education, public pension and health care systems, albeit the level and composition differ. Most of the tax revenue is collected based on labour market income (either when it is earned or used). It follows that there is a strong age component with respect to when the average person benefits from and contributes to public policies. The average person will be a net beneficiary as young and old, and a net-contributor in working ages. Figure 5 illustrates this age dependency for the case of Denmark, and similar relations are found for other countries[7].

Figure 5: Age dependent net contributions to the public sector, Denmark



Note: The figure shows the net-contribution to public finances defined as all types of tax payments minus transfers and individualized public consumption. Numbers apply to 2003.

Source: Danish Welfare Commission (2006).

This age dependency has the structure of a Pay-As-You-Go (PAYG) system in the sense that the contributions (=tax payments) by the working age population finance the services provided to the young and old[8]. Clearly, this is not in a formal sense a PAYG system since the public sector is not required to run a balanced budget. The government may run deficits or surpluses to smooth adjustments across time and generations. The

[7] While the profile is similar across countries, the amplitude depends on the relative size of the public sector. In e.g. Scandinavian countries with a relatively large public sector, the net amount involved is large.

[8] For a survey of the literature on the political economy of social security, see e.g. Galasso and Profeta (2002) and Mulligan and Xala-I-Martin (2004).

possibilities of doing so are constrained by the intertemporal budget constraint. This is precisely where the issue of fiscal sustainability enters. Combining demographic developments in figure 1 with the age profile in figure 5 raises the obvious question; will it be possible to maintain current public policies or will expenditures outrun revenues eventually causing a need for policy reform? To address this issue, the next section presents a stylized OLG model capturing the essence of age dependency shown in figure 4 and allowing this to be combined with demographic changes to assess fiscal sustainability.

3 An illustrative OLG-framework

Consider an economy with access to an international capital market offering a risk-free asset with return r, which, for simplicity, is assumed exogenous (constant) throughout time (small open economy assumption). To simplify, the model is deterministic[9].

3.1 Individuals and demographics

Each period, N_t individuals are born, and they all live through two life phases[10]; one denoted young and the other old. The first phase has a given length normalized to unity, while the second has a length L (=longevity)[11][12]. The productivity of work is the same for all workers (young and old)[13] and equal to the wage rate (competitive markets), and is denoted y_t. (≤ 1). Denote the generation being young in period t as generation t. Individuals work and consume both as young and old. The retirement age is denoted R.

The utility of a representative generation t individual/household is

$$V_t(c_{1t}, c_{2t+1}, R_{t+1}, L_{t+1}) = u(c_{1t}, e_{t+1}) + \frac{1}{1+\theta}\left[L_{t+1}u(c_{2t+1}, h_{t+1}) - R_{t+1}v\left(\frac{R_{t+1}}{L_{t+1}}\right)\right]$$

[9] For an analysis of longevity risk, see e.g. Andersen (2008b).

[10] Childhood is disregarded to simplify and since there is no clear trend in the child dependency ratio, cf. section 2.

[11] This can, to a first approximation, be taken to match observed mortality rates which are constant (and low) up to a certain age, from which they are increasing with age.

[12] Note that this approach allows for generations having different longevity at the same time as it avoids complications from aggregating over generations with different survival rates and ages alive at the same time.

[13] It would be straightforward to allow for a productivity difference between young and old. If the productivity of old is some constant fraction of the productivity of the young, it would change nothing qualitatively in the analysis.

where c_{1t} and c_{2t+1} are the consumption flows as young and old in period t and $t+1$, respectively. e_t denotes public services provided to the young (education) and h services provided to the old (health care), cf. below. θ is the subjective time preference and v gives the disutility of work. Standard assumptions apply, that is $u' > 0, u'' \leq 0$ and $v' > 0, v'' > 0, \lim_{R \longrightarrow L} v' = \infty$, and hence $V'_{c_1} > 0, V'_{c_2} > 0, V'_R < 0$, and $V'_L \lesseqgtr 0$. Note that the specification makes disutility of work depend on longevity, and the formulation is consistent with so-called "healthy ageing", implying that the marginal disutility from later retirement depends on the retirement age relative to longevity.[14][15]

3.2 The public sector

Consider a public sector which has three main activities, namely, service provision to the young (e_t), service provision to the old (h_t) and pensions provided to the retired (p_t). The pension is paid as a life long income support from retirement at age R_t (i.e. it is received for a period of duration $L_t - R_t$). For the two service components, it is assumed that output is measured by expenditures (national account convention). The activities of the public sector are financed by a proportional income tax levied on all income (at the rate τ_t). The "policy package" at any point in time $t+i$ is thus given by $\{e_{t+i}, h_{t+i}, p_{t+i}, R_{t+i}, \tau_{t+i}\}_{i=0}^{\infty}$

The primary balance of the public sector in period t can be written

$$B_{t+i} = N_{t+i} T_{1t+i} + N_{t+i-1} L_{t+i} T_{2t+i} \tag{1}$$

where T_{1t+i} denotes the net contribution of each young to the public sector in period $t+i$, i.e.

$$T_{1t+i} = \tau_{t+i} y_{1t+i} - e_{t+i}$$

[14] Consider that a given individual with longevity can attain a given desired consumption flow as old by retiring at a certain age, implying a given disutility from later retirement. Suppose now that longevity increases, then the present formulation implies that the same consumption flow can be maintained over the now longer longevity, by retiring at an age increasing proportionally to the increase in longevity, and with an unchanged disutility from still working at this age. In this sense, the ability to work has improved with the increase in longevity.

[15] The marginal disutility from later retirement is

$$\frac{\partial}{\partial R_{t+1}} \left[R_{t+1} v \left(\frac{R_{t+1}}{L_{t+1}} \right) \right] \equiv \eta \left(\frac{R_{t+1}}{L_{t+1}} \right) = v \left(\frac{R_{t+1}}{L_{t+1}} \right) + \frac{R_{t+1}}{L_{t+1}} v' \left(\frac{R_{t+1}}{L_{t+1}} \right) > 0$$

and T_{2t+i} denotes the flow of net contribution of each old to the public sector in period $t+i$, i.e.

$$T_{2t+i} = \widehat{R}_{t+i}\tau_{t+i}y_{t+i} - (1-\widehat{R}_{t+i})p_{t+i} - h_{t+i}$$

where

$$\widehat{R}_{t+i} \equiv \frac{R_{t+i}}{L_{t+i}}$$

is the relative retirement age; i.e. retirement age relative to longevity.

If there is an operational intergenerational transfer of resources as depicted in figure 4, we have $T_{1t+i} < 0$ and $T_{2t+i} > 0$. In the following, it is assumed that the age/cohort specific net-contributions fulfil these constraints.

The dependency ratio is defined as

$$D_{t+i} \equiv \frac{L_{t+i}N_{t+i-1}}{N_{t+i}} = \frac{L_{t+i}}{1+n_{t+i}}$$

and it is driven by both fertility and mortality (longevity). The dependency ratio is increasing in longevity and falling in the fertility rate. It follows that D is declining when the population is growing, and increasing when it is decreasing.

It turns out to be useful to define variables relative to the total gross income of young cohorts [16] $Y_{t+i} \equiv N_{t+i}y_{t+i}$, implying that (1) can now be written

$$b_{t+i} \equiv \frac{B_{t+i}}{Y_{t+i}} = \frac{T_{1t+i}}{y_{t+i}} + D_{t+i}\frac{T_{2t+i}}{y_{t+i}} \qquad (2)$$

Expression (2) captures the standard effect that for given net contributions of the young ($\frac{T_{1t+i}}{y_{t+i}} > 0$) and net benefits of the old ($\frac{T_{2t+i}}{y_{t+i}} < 0$), the budget balance deteriorates if the dependency ratio increases.

3.3 Fiscal sustainability

The intertemporal budget constraint requires that the present value of revenue collected can cover the present value of expenditures plus initial debt, and it can be written[17]

[16] Note this is not equivalent to GDP, cf. appendix A.

[17] Notice that debt dynamics are determined by

$$D_t = (1+r)D_{t-1} - B_t$$

$$\sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^i B_{t+i} \geq D_t \tag{3}$$

Expression (3) says that the present value of the primary budget balance should equal the initial debt level. Defining variables relative to Y_{t+i} , and assuming that the growth rate of Y is exogenous and constant over time[18] (=g), it follows that (3) can be written

$$\sum_{i=0}^{\infty} \left(\frac{1}{1+r_g}\right)^i b_{t+i} \geq d_t$$

where $b_{t+i} \equiv \frac{B_{t+i}}{Y_{t+i}}$, $d_{t+i} \equiv \frac{D_{t+i}}{Y_{t+i}}$, and $r_g \equiv \frac{1+g}{1+r} - 1 > 0$ is the growth corrected discount rate.

The sustainability question is whether the intertemporal budget constraint is fulfilled given the path for revenues and expenditures and thus the primary balance implied by given policies (and the discount rate, growth rate, and the initial debt level) and projected future developments. Here we focus on demographics as the exogenous factor influencing public finances. Nothing is ensuring that a given "public policy package" characterized by $\{e_{t+i}, h_{t+i}, p_{t+i}, R_{t+i}, \tau_{t+i}\}_{i=0}^{\infty}$ is satisfying the intertemporal budget constraint, and therefore is fiscally sustainable. Define by s_t the permanent change in the primary budget balance (relative to Y) undertaken with effect from period t and for all future periods (that is, a permanent budget change), ensuring that the intertemporal budget constraint holds exactly[19]. The variable s_t is an indicator of fiscal sustainability: if $s_t > 0$, a permanent budget improvement is needed, and therefore current policies are not fiscally sustainable; i.e. they do not fulfil the intertemporal budget constraint, and a policy reform changing the elements of the policy package will eventually be necessary. Oppositely, if s_t is positive, the current policy is fiscally sustainable and even leaves room for either permanent tax decreases or expenditure increases being consistent with satisfying the intertemporal budget constraint. Formally, we have that s_t is defined as the solution to

$$\sum_{i=0}^{\infty} \left(\frac{1}{1+r_g}\right)^i b_{t+i} + s_t \sum_{i=0}^{\infty} \left(\frac{1}{1+r_g}\right)^i = d_t$$

implying that the sustainability indicator can be written

$$s_t = \frac{r_g}{1+r_g} \left[-\sum_{i=0}^{\infty} \left(\frac{1}{1+r_g}\right)^i b_{t+i} + d_t \right] \tag{4}$$

[18] Since $Y_{t+i} = N_{t+i} y_{t+i}$, it follows that $g = n + k$, where n is population growth, and k is productivity growth.

[19] The European Commission denotes this indicator by S2, cf. European Commission (2006a).

4 Basic arithmetics of fiscal sustainability

Two aspects are in particular important when considering the sustainability indicator (s), namely the role of demographics and growth. These issues are considered in the following.

4.1 Policy package

To assess fiscal sustainability, it is necessary to define the policy package which applies to all periods, i.e. $\{e_{t+j}, h_{t+j}, p_{t+j}, R_{t+j}, \tau_{t+j}\}_{j=0}^{\infty}$. Specifically, it is assumed that the policy package is defined by the following policy rules

$$p_{t+i} = py_{t+i} \quad (5)$$

$$e_{t+i} = ey_{t+i} \quad (6)$$

$$h_{t+i} = hy_{t+i} \quad (7)$$

and a given retirement age (R) and tax rate (τ). For pensions and welfare services, it follows that they are proportional to current income. The first part of this assumption implies a fixed relation between pensions and income for those working; i.e. gains from growth are distributed equally among active and passive members of society. This may be interpreted as a distributional assumption or constraint, namely, that the income distribution between income from work and pension remains unchanged. The second part implies that expenditures on welfare services are evolving proportionally to income; that is, the expenditures to produce a given level of services evolve proportionally to the general income level in the economy[20]. This reflects that welfare services are labour intensive and that public wages follow private wages very closely.[21] The assumption underlying (6) and (7) may be interpreted as a public service constraint, namely, that service provision should be of unchanged real standards[22]. Notice that all parameters are assumed to be time-independent; i.e. given rules for the public policy package are considered. These assumptions give, to a first approximation, a simple version of the "public policy package" underlying figure 5.

[20]This captures the basic Baumol-effect; that is, if there are productivity increases in the private sector but none in the public (service) sector.

[21]This tendency is reinforced if the income elasticity in demand for welfare service is high (Wagner effect). OECD (2006) uses a unitary elasticity as implied by the formulation here as a benchmark capturing both the Baumol and the Wagner effect.

[22]For a further discussion, see Andersen and Pedersen (2006).

It is now possible to calculate the present value of expected future primary budget balances and therefore the sustainability indicator. Given the properties of the policy package made above and using (2), we have

$$b_{t+i} = \tau - e + D_{t+i}(\widehat{R}_{t+i}\tau - (1 - \widehat{R}_{t+i})p + h) \tag{8}$$

and hence the present value is

$$\sum_{i=0}^{\infty}\left(\frac{1}{1+r_g}\right)^i b_{t+i} = \sum_{i=0}^{\infty}\left(\frac{1}{1+r_g}\right)^i \left[\tau - e + D_{t+i}(\widehat{R}_{t+i}\tau - (1 - \widehat{R}_{t+i})p + h)\right]$$

Inserting in (4), we find that the sustainability indicator is

$$s_t = \frac{r_g}{1+r_g}\left[-\sum_{i=0}^{\infty}\left(\frac{1}{1+r_g}\right)^i \Big(\tau - e + D_{t+i}(\widehat{R}_{t+i}\tau - (1 - \widehat{R}_{t+i})p + h)\Big) + d_t\right] \tag{9}$$

4.2 Demographics

A simple but basic implication is that the fiscal sustainability metric (9) is affected differently by changes in fertility and longevity, even if these changes have the same effect on the dependency ratio. To see this note that fiscal sustainability is worsened by a higher dependency ratio but improved by a higher relative retirement age, i.e.[23]

$$\begin{aligned} \frac{\partial b_{t+i}}{\partial D_{t+i}} &= -\Big(\widehat{R}_{t+i}\tau - (1 - \widehat{R}_{t+i})p + h\Big) > 0 \\ \frac{\partial b_{t+i}}{\partial \widehat{R}_{t+i}} &= -D_{t+i}(\tau + p) < 0 \end{aligned}$$

Hence, the direct effect of an increase in the dependency ratio is that fiscal sustainability is worsened (s increases). This holds irrespective of whether the change is driven by a fall in fertility or an increase in longevity. However, an increase in longevity has the additional effect of decreasing the effective retirement age (for a given statutory retirement age), which also contributes to worsen fiscal sustainability. The intuition for this result is thus that there are two effects, namely, the balance between net-contributors

[23] The sign follows from observing that we have that $\tau > e$ and $\tau + \widehat{R}_{t+i}p - (p+h) < 0$ and hence

$$\widehat{R}_{t+i}(e+p) - (p+h) < e + \widehat{R}_{t+i}p - (p+h) < 0$$

and net-beneficiaries captured by the dependency ratio, and the fraction of time one is a net-beneficiary captured by the effective retirement age.

As an illustration, the fiscal sustainability indicator can be computed under the assumption that longevity evolves as $L_{t+i} = L_t(1+l)^i$ and the fertility rate as $1 + n_{t+i} = (1+n_t)(1+n)^i$, in which case the dependency ratio can be written

$$D_{t+i} = D_t \frac{(1+l)^i}{(1+n)^i}$$

Under these assumptions, it follows that the sustainability indicator is given as

$$s_t = \frac{r_g}{1+r_g}\left[-\sum_{i=0}^{\infty}\left(\frac{1}{1+r_g}\right)^i\left(\tau - e + D_t\frac{(1+l)^i}{(1+n)^i}(p+h-\frac{R}{L_0\,(1+l)^i}(\tau+p))\right)\right]$$

It follows that a higher rate of increase in longevity (l) worsens sustainability, while a higher fertility rate improves sustainability, i.e.

$$\frac{\partial s_t}{\partial l} = \frac{r_g}{1+r_g}\left[\sum_{i=0}^{\infty}\left(\frac{1}{1+r_g}\right)^i\left(\frac{D_t i\,(1+l)^{i-1}\,(p+h)}{(1+n)^i}\right)\right] > 0$$

and

$$\frac{\partial s_t}{\partial n} = \frac{r_g}{1+r_g}\left[\sum_{i=0}^{\infty}\left(\frac{1}{1+r_g}\right)^i i(1+n)^{i-1}\left(\frac{R(e+p) - D_t\,(1+l)^i\,(p+h))}{((1+n)^i)^2}\right)\right] < 0$$

It may be argued that the sustainability problem caused by increasing longevity derives from the assumption that retirement ages are invariant. That this is not so is easily seen since a policy package $(e, h, p, \tau, \widehat{R})$ is not fiscally sustainable if D_{t+i} is increasing. To see this, assume that the period t budget is in balance, i.e.

$$b_t = \tau - e + D_t(\widehat{R}\tau - (1-\widehat{R})p + h) = 0$$

and that there is an operative social contract, i.e. $\frac{T_{1t}}{y_t} = \tau - e > 0$ and $\frac{T_{2t}}{y_t} = \widehat{R}\tau - (1 - \widehat{R})p - h < 0$. Then

$$b_{t+j} = \tau - e + D_{t+j}(\widehat{R}\tau - (1-\widehat{R})p + h) < 0 \text{ for } j : D_{t+j} > D_t$$

Hence, if $D_{t+j+1} \geq D_{t+j}$ for all $j > 0$, it follows that $s_t > 0$.

Notice that this result is not depending on the specific elements of the social contract but arises as a simple consequence of any policy package implying net transfers from young to old (see Andersen (2008b)). Hence, ensuring that the retirement age is proportional to longevity is not sufficient to eliminate problems of fiscal sustainability. Moreover, this result holds even if the scheme only includes pensions (i.e. $h = 0$).

4.3 Growth effects

An issue which often comes up in policy debates is whether a problem of fiscal sustainability can be solved by increasing the growth rate. The intuition is that by growing the pie, the financing problem is eased. This is not obvious. First, observe that the "distributional" and "service" constraints underlying the policy rules (5), and (6) and (7) imply that a higher growth rate translates not only into proportionally higher tax revenues but also proportionally higher expenditures. Hence, the budget balance relative to income, cf. (8), is not directly affected by the growth rate. Clearly, there may be reasons why the budget balance depends directly on the growth rate. Two important reasons are that pensions (and other transfers) are not fully indexed to current wages, and that not all tax bases vary proportionally with productivity (see Appendix B).

Under the small open economy assumption that the interest rate is exogenous, there is an additional effect of growth running via a change in the growth corrected discount rate (r_g). To see the importance of this channel, consider first the simple case where the dependency ratio and the relative retirement age are constant, i.e. $D_{t+i} = D$ and $\widehat{R}_{t+i} = \widehat{R}$ for all i. In this case the budget balance (b) is time invariant, and we have that the sustainability indicator is

$$\begin{aligned} s_t &= \frac{r_g}{1+r_g}\left[-\sum_{i=0}^{\infty}\left(\frac{1}{1+r_g}\right)^i\left(\tau - e + D(\widehat{R}\tau - (1-\widehat{R})p + h)\right) + d_t\right] \\ &= \tau - e + D(\widehat{R}\tau - (1-\widehat{R})p + h) + \frac{r_g}{1+r_g}d_t \qquad (10) \end{aligned}$$

i.e. the sustainability indicator is simply equal to the underlying deficit on the primary balance plus a term to cover the initial debt level. Notice that an increase in the discount rate causes the sustainability factor to increase, i.e. $\frac{\partial s_t}{\partial r_g} = \frac{1}{(1+r_g)^2}d_t > 0$. It is an implication that a higher growth rate works to diminish the importance of the initial debt level for the fiscal sustainability metric s. The intuition is that the debt servicing becomes smaller with a higher growth rate. In the following the initial debt level is assumed to be zero ($d_t = 0$) to simplify. Hence, basically growth has a neutral effect on the sustainability factor if demographic factors are stationary.

Consider now a case where the budget balance is time dependent due to demographic changes. In this case the growth rate will affect the sustainability metric even if initial debt is zero ($d_t = 0$). Actually, under plausible assumptions there is the surprising finding that the sustainability problem worsens with higher growth[24]. To see how the discount

[24] This is not a theoretical curiosity since it is the case for fiscal sustainability calculations made for

rate affects the sustainability indicator, it is useful to rewrite (4) as (assuming $d_t = 0$)

$$s_t = -\sum_{i=0}^{\infty} v_i b_{t+i}$$

where

$$v_i = \frac{r_g}{1+r_g}\left(\frac{1}{1+r_g}\right)^i \quad ; \quad \sum_{i=0}^{N} v_i = 1$$

i.e. the sustainability indicator is a weighted average of the budget balance in all future periods. The weight to the budget balance in any period $t+i$, v_i depends only on the growth corrected rate of return, and we have

$$\frac{\partial v_i}{\partial r_g} = \frac{1}{(1+r_g)^2}\left(\frac{1}{1+r_g}\right)^i [1 - r_g i]$$

Hence,

$$\frac{\partial v_i}{\partial r_g} \lesseqgtr 0 \quad for \quad i \gtreqless \widetilde{i} \equiv \frac{1}{r_g}$$

i.e. a higher discount rate (r_g) increases the weight to budget balances in the near future ($i < \widetilde{i}$) but decreases the weight to budget balances in the more far future ($i > \widetilde{i}$).[25] The effect of a change in the discount rate on the sustainability indicator (for a given budget profile) is therefore in general ambiguous since it depends on the underlying profile for the budget balances. If the budget profile displays a tendency towards deteriorating budget balances, it follows that a lower discount rate implied by higher growth may increase the sustainability problem. To see this, consider the following example where the budget balance is given as

$$b_{t+i} = b + a(1+z)^i$$

In this case, the sustainability indicator becomes

$$\begin{aligned} s_t &= x + \frac{r_g}{1+r_g}\left[-\sum_{i=0}^{\infty}\left(\frac{1}{1+r_g}(1+z)\right)^i\right] \\ &= x + \frac{r_g}{(1+z)-(1+r_g)} \end{aligned}$$

and it follows that

$$\frac{\partial s_t}{\partial r_g} = -\frac{z}{((1+z)-(1+r_g))^2}$$

e.g. Denmark and Norway, see Andersen and Pedersen (2006) and Frederiksen et al. (2005).

[25] Observe that if $b_{t+i} = b$ for all i, it follows that $s_t = -b$, and thus independent of the growth rate.

Hence,

$$sign \frac{\partial s_t}{\partial r_g} = sign\ z$$

i.e. if there is an underlying tendency to a deterioration in the budget balance ($z < 0$), then a higher discount rate (r_g) will reduce the sustainability problem. The intuition is that a higher discount rate implies less weight to future periods with a worse budget balance. Since a higher growth rate leads to a decrease in the discount rate, it follows that it worsens the sustainability problem. The sustainability metric distributes the financing requirements over time. Since a higher growth rate means increasing expenditures in the future, and since current generations are contributing to the financing, it follows that the sustainability indicator increases.

4.4 Numerical illustration

To illustrate the results from above, consider a situation where there, over 25 period horizon, is a demographic transition resulting in a higher dependency ratio[26]. Table 1 shows calculations[27] of the sustainability indicator (s) for a case where the change in the dependency ratio is driven by fertility (decreasing) or by longevity (increasing), and for different levels of the discount rate (r_g). The parameters are set such that the budget initially is in balance (and initial debt = 0). The underlying change in the dependency ratio is the same; that is, an increase from 0.25 to either 0.4 or 0.5.

Table 1: Sustainability indicators (s) under different demographic paths and growth rates.

[26] Note that the formula for the sustainability indicator is worked out from basic budget effects which can be based on an annual period length, and it is thus not necessary to impose the straightjacket implied by the standard interpretation of the period length in OLG models.

[27] The numerical illustrations are made for the following parameter choices: $\tau = 0,25, e = 0,2, h = 0,15$, and $p = 0,35$. Parameter choices imply that initial primary balance is zero, and initial debt is assumed equal to zero. For the case where the dependency ratio increases from 0.25 to 0.4 over a 25 period horizon, $l = 0,018978$ and $n = -0.01862$, and for the case with an increase to $0,5$, $l = 0,028114$ and $n = -0,02735$.

r_g	Dependency ratio			
	From 0.25 to 0.4		From 0.25 to 0.5	
r_g	Fertility	Longevity	Fertility	Longevity
0.5	2.3	6.2	3.7	10.4
1.0	2.2	5.8	3.5	9.7
1.5	2.0	5.5	3.3	9.0

The table indicates that it has a significant quantitative importance whether a given change in the dependency ratio is caused by changes in fertility or longevity. In the latter case, the sustainability factor is more than twice that in the former case. Moreover, the table shows that the sustainability indicator is strongly dependent on the assumed discount rate and therefore the underlying growth rate. The higher the growth rate, the larger the sustainability problem for all of the demographic scenarios considered.

Another way of illustrating the importance of increasing longevity is to compare a situation with a given statutory retirement/pension age (R constant) and one where the relative retirement age is constant ($\widehat{R} = \frac{R}{L}$ constant). Obviously, this only matters if demographic shifts are caused by changes in longevity, and table 2 shows the sustainability indicator in these two cases for the same scenarios as in table 1. It is seen that the sustainability problem is significantly different between a case with a constant retirement age and a constant relative retirement age[28].

Table 2: Sustainability factor under different demographic paths: Constant retirement age vs. constant relative retirement age with increasing longevity

	Dependency ratio			
	From 0.25 to 0.4		From 0.25 to 0.4	
r_g	R constant	$\widehat{R}$ constant	R constant	$\widehat{R}$ constant
0.5	6.2	2.3	10.4	3.6
1.0	5.8	2.2	9.7	3.4
1.5	5.5	2.1	9	3.2

This shows that there is a clear trade-off in policy strategies between a pre-funding strategy and an adjustment strategy. Linking retirement age to longevity is an adjustment strategy in the sense that it implies that retirement ages are continuously changed alongside changes in longevity. In contrast, the pre-funding is a once-and-for-all strategy

[28] In the stylized model considered here, demographic changes induced by changes in fertility have the same effects on fiscal sustainability as changes in longevity when the relative retirement age is constant.

in the sense of a permanent (and thus time invariant) change in the budget balance to ensure sufficient consolidation of public finances ahead of future demographic shifts. The table shows that under an adjustment strategy linking retirement ages to longevity, the needed consolidation is significantly reduced. Or phrased differently, the more retirement ages are increased alongside increases in longevity, the smaller is the need to consolidate public finances. The pros and cons of these different strategies are discussed below.

5 Sustainability problems: Save or adjust?

In policy debates, it is often presumed that sustainability problems should be solved by a consolidation of public finances. The reasoning usually relies on a tax-smoothing argument (Barro (1979)) that tax rates should be kept constant, and variations in expenditures should be accommodated via variations in the budget balance. This logic is immediately carried over to demographic shifts if the positive metric of the sustainability problem (s) is turned into a normative target for the permanent budget improvement. Since demographic changes are unfolding over time, it is an implication of the latter policy strategy that it implies pre-funding of public finances; that is, fiscal finances have to be consolidated sufficiently to ensure that the trend decline in the budget balance can be made consistent with fiscal sustainability. This follows readily by observing that for a given policy package, we have from (8)

$$b_{t+j} \leq b_{t+j-1} \text{ if } L_{t+j} \geq L_{t+j-1}$$

the budget balance after the adjustment in accordance with the sustainability indicator is implying a budget balance equal to $\widetilde{b}_{t+j} = b_{t+j} + s_t$ and therefore

$$\widetilde{b}_{t+j} \leq \widetilde{b}_{t+j-1} \text{ if } L_{t+j} \geq L_{t+j-1}$$

Since fiscal sustainability requires

$$0 = -\sum_{i=0}^{\infty} v_i \widetilde{b}_{t+i}$$

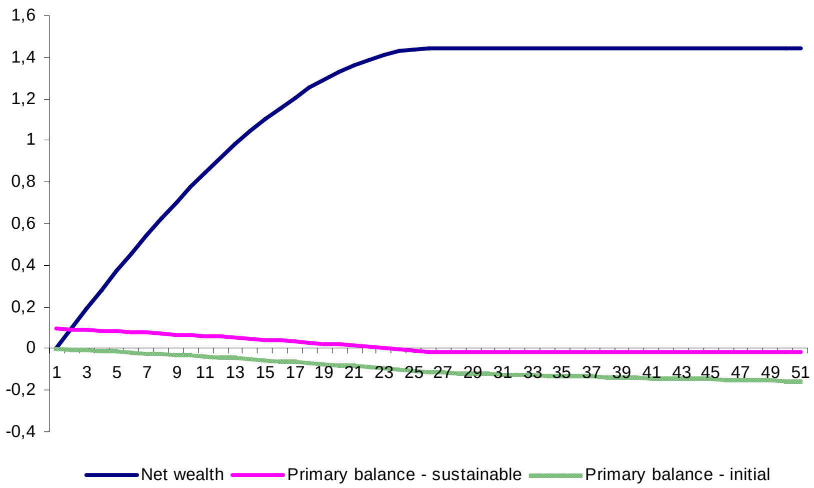
it follows that $\widetilde{b}_{t+i} > 0$ for $i < i^c$ and $\widetilde{b}_{t+i} < 0$ for $i > i^c$. Therefore budget surpluses in initial periods are contributing to the financing needs arising in future periods.

If the dependency ratio is expected to stabilize at some new level D^*, implying that a budget balance will be $b^* < 0$, it follows that the consolidation prior to attaining this steady state should ensure a net-wealth NW^* determined by $\frac{r_g}{1+r_g}NW^* = -b^*$ (follows from (10)). A similar type of result is known from the literature on optimal debt policy focussing on either capital income taxation or prudence motives (see e.g. van der Ploeg (2008)).

Numerical illustration

The pre-funding implication of a policy which implies a permanent increase in taxes to eliminate the sustainability problem is shown in figure 6 for the case corresponding to table 1, where the dependency ratio increases from 0.25 to 0.5 over 25 periods. It is seen that the sustainable path implies a parallel upward shift in the primary budget (=sustainability factor s), implying initial periods with budget surpluses followed by a systematic tendency to deficits in the future. The profile of the budget balance reflects the assumed trend increase in the dependency ratio. The figure also shows how public sector financial net worth develops (initial value =0), and it is seen that the policy implies a rather large wealth accumulation.

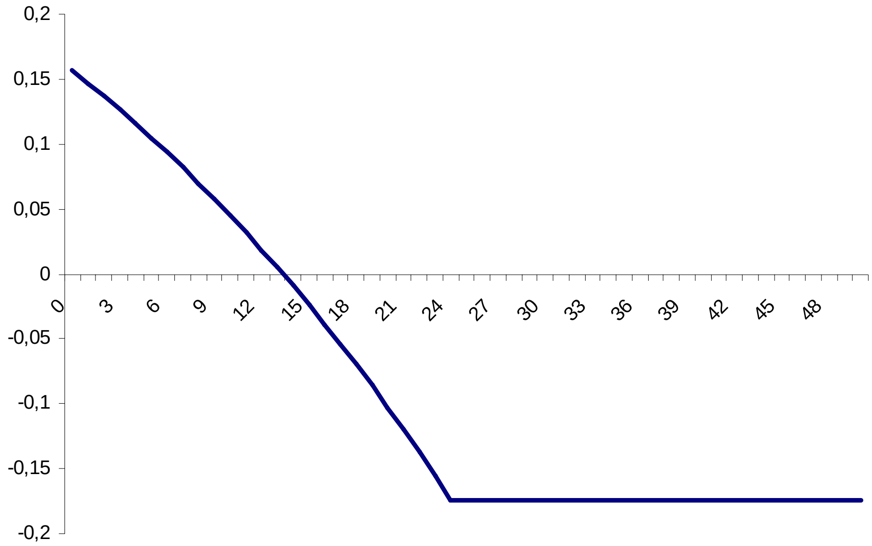
Figure 6: Primary balance and public sector net-worth scenario A.



It is an implication of this profile that current generations with relatively low longevity contribute to the future financial obligation caused by future generations enjoying longer longevity. It may be questioned whether this is a just policy (see next section). To illustrate this, compare two policy scenarios. Scenario A is the once-and-for-all strategy with an immediate and permanent change in the tax rate to ensure fiscal sustainability. Scenario B is the adjustment strategy with a constant relative retirement age (i.e. future generations enjoying longer longevity will also have to retire later), and the remaining gap for fiscal sustainability is closed by an immediate and permanent tax increase. Figure

6 shows the difference in net contributions of different cohorts between scenarios A and B. It is seen that scenario A relative to B implies that generations in the near future are making larger net contributions and vice versa for generations in the far future. In short, scenario A compared to B implies a redistribution from current generations with (relatively) short longevity to future generations with (relatively) long longevity.

Figure 7: Difference in net-contributions for different cohorts - Scenario A vs B



6 Optimal policy - intergenerational distribution

Turning to the normative issue of how to cope with demographic shifts, consider a social planner respecting individual preference orderings including the subjective time preference. Assuming utilitarian preferences is not unproblematic but useful in clarifying the essential trade-offs.[29] The problem of the planner is to allocate across time and thus generations, respecting the intertemporal resource constraint[30]. To simplify, the following disregards growth ($g = 0$, therefore the normalization $y \equiv 1$ is used).

[29] See e.g. Padilla (2002) for a discussion of some of the problems arising when analyzing intergenerational questions, assuming that future generations hold the same utility function, and that utilities can be discounted.

[30] Beetsma and Oksanen (2008) takes a different approach in assessing the normative aspects by considering a criterion of actuarial neutrality between generations. This essentially implies that each generation is fully funded but within the public system. The present approach differs in allowing for intergenerational redistribution.

The objective of the social planner is thus to choose an allocation which maximizes

$$\Psi_t = \left[N_{t-1} \left[L_t u(c_{2t}, h_t) - R_t v \left(\frac{R_t}{L_t} \right) \right] \right]$$
$$+ N_t \left[u(c_{1t}, e_t) + \frac{1}{1+\theta} \left[L_{t+1} u(c_{2t+1}, h_{t+1}) - R_{t+1} v \left(\frac{R_{t+1}}{L_{t+1}} \right) \right] \right]$$
$$+ \frac{N_{t+1}}{1+\theta} \left[u(c_{1t+1}, e_{t+1}) + \frac{1}{1+\theta} \left[L_{t+2} u(c_{2t+2}, h_{t+1}) - R_{t+1} v \left(\frac{R_{t+2}}{L_{t+2}} \right) \right] \right] + ++$$

The objective function can in more condensed form be written as

$$\Psi_t = \Theta(\mathbf{x}_t) + \frac{1}{1+\theta} \Theta(\mathbf{x}_{t+1}) + \left(\frac{1}{1+\theta} \right)^2 \Theta(\mathbf{x}_{t+2}) + + \tag{11}$$

where the vector $\mathbf{x}_t$ is defined as $\mathbf{x}_t \equiv (c_{1t}, c_{2t}, e_t, h_t, R_t, L_t)$, and the S-function gives the utility generated to young and old alive in a given period, i.e.

$$\begin{aligned} \Theta(\mathbf{x}_{t+i}) &\equiv N_{t+i} u(c_{1t+i}, e_{t+i}) + N_{t+i-1} \left[L_{t+i} u(c_{2t+i}, h_{t+i}) - R_{t+i} v \left(\frac{R_{t+i}}{L_{t+i}} \right) \right] \\ &= N_{t+i} \left[u(c_{1t+i}, e_{t+i}) + D_{t+i} \left[u(c_{2t+i}, h_{t+i}) - \widehat{R}_{t+i} v \left(\widehat{R}_{t+i} \right) \right] \right] \end{aligned}$$

with the properties $\Theta'_{c_1} > 0, \Theta'_{c_2} > 0, \Theta'_e > 0, \Theta'_h > 0, \Theta'_R < 0$, and $\Theta'_L \lesseqgtr 0$.

The planner is operating under the intertemporal budget constraint that

$$\sum_{i=0}^{\infty} \frac{(N_{t+i} + N_{t+i-1} R_{t+i})}{(1+r)^i} y = \sum_{i=0}^{\infty} \frac{(N_{t+i} (c_{1t+i} + e_{t+i}) + N_{t+i-1} L_{t+i} (c_{2t+i} + h_{t+i})}{(1+r)^i}$$

or

$$\sum_{i=0}^{\infty} \frac{N_{t+i} \left(1 + D_{t+i} \widehat{R}_{t+i} \right)}{(1+r)^i} y = \sum_{i=0}^{\infty} N_{t+i} \frac{(c_{1t+i} + e_{t+i} + D_{t+i} (c_{2t+i} + h_{t+i})}{(1+r)^i}$$

The first order conditions (second order conditions) are

$$\begin{aligned} N_{t+i} u_c(c_{1t+i}, e_{t+i}) &= \lambda N_{t+i} \\ N_{t+i} u_e(c_{1t+i}, e_{t+i}) &= \lambda N_{t+i} \\ N_{t+i} D_{t+i} u_c(c_{2t+i}, h_{t+i}) &= \lambda N_{t+i} D_{t+i} \\ N_{t+i} D_{t+i} u_h(c_{2t+i}, h_{t+i}) &= \lambda N_{t+i} D_{t+i} \\ N_{t+i} D_{t+i} \eta(\widehat{R}_{t+i}) &= \lambda N_{t+i} D_{t+i} y \end{aligned}$$

where $\eta(\widehat{R}_{t+i}) \equiv v \left(\widehat{R}_{t+i} \right) + \widehat{R}_{t+i} v_{\widehat{R}} \left(\widehat{R}_{t+i} \right)$ denotes the marginal disutility from later retirement.

These first order conditions determine the social optimal allocation across time and thus generations. They imply that the marginal utilities of consumption of private and public goods should be the same across time and generations. Within periods we have

$$u_c(c_{1t+i}, e_{t+i}) = u_e(c_{1t+i}, e_{t+i}) = u_c(c_{2t+i}, h_{t+i}) = u_h(c_{2t+i}, h_{t+i}) = \eta(\widehat{R}_{t+i}) \qquad (12)$$

and across periods the marginal utilities of consumption are the same

$$u_c(c_{1t+i}, e_{t+i}) = u_c(c_{1t+i+1}, e_{t+i+1}) \qquad (13)$$

$$u_e(c_{1t+i}, e_{t+i}) = u_e(c_{1t+i+1}, e_{t+i+1}) \qquad (14)$$

$$u_c(c_{2t+i}, h_{t+i}) = u_c(c_{2t+i+1}, h_{t+i+1}) \qquad (15)$$

$$u_h(c_{2t+i}, h_{t+i}) = u_h(c_{2t+i+1}, h_{t+i+1}) \qquad (16)$$

and finally and importantly the marginal disutility from later retirement is the same, i.e.

$$\eta(\widehat{R}_{t+i}) = \eta(\widehat{R}_{t+i+}) \qquad (17)$$

These first order conditions support a policy of the form outlined in section 4 with the important proviso that the relative retirement rate is constant across generations which may have different longevity, i.e.

$$\widehat{R}_{t+i} = \widehat{R}_{t+i+1} = \widehat{R} \text{ for all } i$$

Hence, consumption equalization or smoothing also implies equalization or smoothing of the relative retirement age ($\widehat{R}$).

Two important implications follow directly from these conditions. First, variations in fertility have to affect all generations alike. Therefore, if a trend decline in fertility causes a trend increase in the dependency ratio, it follows that there is a case for making a once and for all adjustment of the welfare package so as to ensure fiscal sustainability. This policy will inevitable implying pre-funding. That is, current generations will have to contribute to the financing of increasing expenditures in the future. This can be phrased in the way that if current generations fail to get enough children, they will have to make financial contributions to future generations. Second, if the dependency ratio is increasing due to increases in longevity, it is part of the optimal policy to adjust eligibility rules such that they are proportional to longevity. That is, if there is a trend increase in longevity, it is an implication that future generations enjoying longer longevity will have to retire later.

However, as was noted above, this is not in itself sufficient to ensure fiscal sustainability, and some pre-funding may still be needed.

Interestingly, the abovementioned reasoning is found rather explicitly in the report from the UK Pensions Commission

> Over the long run, fairness between generations suggests that average pension ages should tend to rise proportionately in line with life expectancy, with each generation facing the same proportion of life contributing to and receiving state pensions (UK Pensions Commission, 2005, p.4).

The specific formulation of both the individual utility function and the social welfare function can be discussed, but the main points that fertility and longevity have different implications for the optimal allocation, and that the retirement age is depending positively on the retirement age, generalize beyond the specific formulations adopted here (see Appendix C).

7 Concluding remarks

Dependency ratios are affected both by changes in fertility and longevity, but these two drivers have different implications for fiscal sustainability. This means in the positive sense that a given change in the dependency ratio does not leave the same fiscal sustainability problem independently of the drivers of the changes in the dependency ratio. Moreover, from a normative perspective there is also an important difference since a change in longevity has a direct utility effect whereas a change in fertility (under standard assumption) does not. Policy debates often take it for granted that fiscal sustainability problems are to be solved by a consolidation or pre-funding strategy, but this paper questions the appropriateness of this policy response if the main driver of the increase in the dependency ratio is a trend increase in longevity. This change should foremost be addressed by an increase in the retirement age.

Demographic projections are uncertain, and therefore the pre-funding strategy also raises the question of how the savings level should be adapted to the underlying risks. Is there a case for pre-cautionary savings to accommodate changes in demographic forecasts (see e.g. van der Ploeg (2008)), or will the savings targets have to be revised when new forecasts indicate significant changes in the dependency ratio? In contrast, by making pension and retirement ages contingent on life expectancy, an automatic adjustment mechanism is introduced which will cope with an important element of risk in demographic projections.

An additional aspect related to the pre-funding strategy is political economy consideration. The pre-funding policy implies systematic budget surpluses and possible building up of public net-wealth (or very low debt levels) at the same time as it keeps a very strict policy allowing no improvements (better services or lower taxes). Is this a time-consistent policy? While this may not be that problematic at initial high debt levels, it may turn so when debt levels become smaller (in international and historic comparison), and especially if public wealth is being accumulated[31].

Appendix A: Measurement relative to GDP

In the text all budget items are measured relative to the gross income of young cohorts [32] $Y_{t+i} \equiv N_{t+i}y_{t+i}$. However, in practice the conventional measure is relative to GDP. In the present model, GDP is given as

$$\begin{aligned} G_{t+i} &= N_{t+i}y_{t+i} + N_{t+i-1}L_{t+i}y_{t+i} \\ &= N_{t+i}y_{t+i}\left(1 + D_{t+i}\widehat{R}_{t+i}\right) \end{aligned}$$

Hence, from(1) we get

$$b_{gt+i} \equiv \frac{B_{t+i}}{G_{t+i}} = \frac{T_{1t+i} + D_{t+i}T_{2t+i}}{1 + D_{t+i}\widehat{R}_{t+i}}$$

From () we now have

$$\sum_{i=0}^{\infty}\left(\frac{1}{1+r_g}\right)^i b_{gt+i} = d_{gt}$$

where $b_{t+i} \equiv \frac{B_{t+i}}{G_{t+i}}$, $d_{t+i} \equiv \frac{D_{t+i}}{Y_{t+i}}$, and $r_g \equiv \frac{1+g_g}{1+r}$ and where g_g is the growth rate of GDP

[31] Norway is an example of a country where a substantial pre-funding is taking place based on oil and gas revenue. The situation is thus different to the extent that the pre-funding is based on "extra income". Moreover, time-consistency problems have been attempted solved by the establishment of the oil fund (accumulation takes place outside public records in traditional sense) and the rule (handlingsreglen) for the use of the money. For Norway this has worked reasonably well so far, although there have been some deviations from the rule. The US experience in recent years can also be taken to show that initial favourable public finances contributing to coping with fiscal sustainability problems were not time-consistent. Denmark has partially followed a pre-funding strategy and is currently facing a strong pressure to expanded public consumption with reference to the fact that public finances are in good shape!

[32] Note this is not equivalent to GDP. GDP is given as $GDP_{t+i} = N_{t+i}y_{t+i} + N_{t+i-1}R_{t+i}y_{t+i} = N_{t+i}(1 + D_{t+i}\widehat{R}_{t+i})y_{t+i} = Y_{t+i}\ (1 + D_{t+i}\widehat{R}_{t+i}) > Y_{t+i}$.

(G_t). Note that the growth rate of GDP is given as

$$g_t = n + k + m$$

where n is the growth rate of N, k is the growth rate of y (technological progress), and m is the growth rate of $\left(1 + D_{t+i}\widehat{R}_{t+i}\right)$. Note that the later growth rate is increasing in the dependency ratio and the relative retirement age. Note that since

$$b_{gt+i} = \frac{b_{t+i}}{1 + D_{t+i}\widehat{R}_{t+i}}$$

it follows that all measures reported in terms of Y easily can be transformed to measures in terms of G.

Appendix B: Growth and the budget balance

This appendix considers two reasons why the budget balance may depend directly on the growth rate.

I: Indexation of transfers

In (5) it is assumed that pensions (=transfers) are linked to current wages; that is, they are fully indexed by current wages. Only few countries have such an explicit indexation, and in many cases there are no formal indexation, indexation to prices, or indexation to past wages (or a mixture of wages and prices). It is straightforward that less than full current indexation will imply that the growth rate affects the budget balances. Consider as an example the case where pensions are indexed to past wages (i.e. pensions are related to wages as young), i.e.

$$p_{t+i} = p y_{t+i-1}$$

In this case we have that

$$\begin{aligned} b_{t+i} &= \tau - e + D_{t+i}(\widehat{R}_{t+i}\tau - (1 - \widehat{R}_{t+i})p\frac{y_{t+i-1}}{y_{t+i}} + h) \\ &= \tau - e + D_{t+i}(\widehat{R}_{t+i}\tau - \frac{(1 - \widehat{R}_{t+i})}{1+g}p + h) \end{aligned}$$

i.e. a higher growth rate improves the budget balance (relative to Y). Through this channel, higher growth will work to improve fiscal sustainability.

II. Tax bases

In the base model the tax base is current income, and therefore it is automatically proportional to the growth rate. However, some tax bases depend on lagged income. One example is capital income taxation. Assume that there is a capital income tax at the rate η on the real return on savings r. Savings by the young are given as

$$s_{t+i} = sy_{t+i}$$

and hence the tax revenue accruing in period $t+i+1$ from the capital income tax is ηrsy_{t+i}. In this case the budget balance reads

$$\begin{aligned} b_{t+i} &= \tau - e + D_{t+i}(\widehat{R}_{t+i}\tau + \eta rs\frac{y_{t+i-1}}{y_{t+i}} - (1-\widehat{R}_{t+i})p + h) \\ b_{t+i} &= \tau - e + D_{t+i}(\widehat{R}_{t+i}\tau + \frac{\eta}{1+g}rs - (1-\widehat{R}_{t+i})p + h) \end{aligned}$$

i.e. a higher growth rate worsens the budget balance (relative to Y). Through this channel, higher growth will work to deteriorate fiscal sustainability.

Appendix C

Consider a more general formulation of the disutility from work when old. The current formulation is $Lv(\frac{R}{L})$. A more general formulation is

$$v(R, L)$$

where $v_R > 0, v_{RR} > 0$. This implies that the social welfare function reads

$$\begin{aligned} \Theta(\mathbf{x}_{t+i}) &\equiv N_{t+i}u(c_{1t+i}, e_{t+i}) + N_{t+i-1}\left[L_{t+i}u(c_{2t+i}, h_{t+i}) - v\left(R_{t+i}, L_{t+i}\right)\right] \\ &= N_{t+i}\left[u(c_{1t+i}, e_{t+i}) + D_{t+i}\left[u(c_{2t+i}, h_{t+i}) - \frac{v\left(R_{t+i}, L_{t+i}\right)}{L_{t+i}}\right]\right] \end{aligned}$$

The foc condition determining retirement age reads in this case

$$\frac{v_R\left(R_{t+i}, L_{t+i}\right)}{L_{t+i}} = \frac{v_R\left(R_{t+i+1}, L_{t+i+1}\right)}{L_{t+i+1}} \quad \text{for all } i$$

Note that

$$\frac{\partial}{\partial R_{t+i}}\frac{v_R\left(R_{t+i}, L_{t+i}\right)}{L_{t+i}} = \frac{v_{RR}\left(R_{t+i}, L_{t+i}\right)}{L_{t+i}} > 0$$

$$\frac{\partial}{\partial L_{t+i}}\left[\frac{v_R\left(R_{t+i}, L_{t+i}\right)}{L_{t+i}}\right] = \frac{v_{RL}\left(R_{t+i}, L_{t+i}\right)L_{t+i} - v_R\left(R_{t+i}, L_{t+i}\right)}{\left(L_{t+i}\right)^2} < 0 \text{ for } v_{RL} < 0$$

Hence, if $V_{RL} < 0$, then the optimal policy implies that those generations with longer longevity should retire later

$$R_{t+i} > R_{t+i-1} \text{ for } L_{t+i} > L_{t+i}$$

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