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ABSTRACT

The Design of the University System*

This paper compares the organisation of the university sector under private provision with the structure which would be chosen by a welfare maximising government. It studies a general equilibrium model where universities carry out research and teach students. To attend university, and earn higher incomes in the labour market, students pay a tuition fee, and each university chooses its tuition fee to maximise the amount of resources it can devote to research. Research bestows an externality on society because it increases labour market earnings. Government intervention needs to balance labour market efficiency considerations -- which would tend to equalise the number of students attending each university -- with considerations of efficiency on the production side, which suggest that the most productive universities should teach more students and carry out more research. We find that government concentrates research more than the private market would, but less than it would like to do if it had perfect information about the productivity of universities. It also allows fewer universities than would operate in a private system.

JEL Classification: H420, I210 and I280
Keywords: higher education and university sector

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The Design of the University System

Non technical summary

The paper studies the optimal organization of the university sector. It views it as “producer” of two goods – tertiary education and “blue sky” research. Both have important effects on society: earnings, both in graduate jobs and in the unskilled labour market, depend on the number of graduates, and the productivity of each worker is affected by the amount of research carried out by the university sector.

We study first the outcome of the unfettered interaction of independent universities, each intent on maximizing the amount and the quality of the research it carries out. We find that “better” universities, that is universities which can produce a given amount of research using fewer resources, employ more staff, teach more students, and produce more research: this is often found to be the case in empirical cross-section studies of universities, and typically attributed to the presence of economies to scale and of scope in production, whereas our model shows that it can be observed even though the production technology displays neither.

Compared to an ideal distribution of resources to the university sector, a system of unfettered private provision distributes research and teaching suboptimally among universities. Specifically, resources are too dispersed, that is there are too many small universities and too few large ones, and research, which should ideally be concentrated in the most productive institutions only, is spread too thinly among all teaching universities. In particularly, “teaching only universities” which would feature in the ideally designed sector, do not emerge in an unfettered system. Finally, too few students are enrolled in the sector overall.

While the comparison with the ideal system is interesting, in practical terms a more meaningful comparison is with the system that a benevolent government would be able to design, taking into account that the universities' have objectives which are not necessarily aligned with its own. This is assumed to be the maximisation of the total after-tax income of all citizens, reduced by the additional costs incurred by those that do attend university, and increased by non-tangible benefits of research, even tough, as shown in the final part of the paper, it is straightforward to allow for redistributive concerns on the government's part.

In this case, the system that the government would put in place is somewhat a compromise, sharing some of the features of the unfettered private and of the ideal system. In this case, there are fewer universities, and the larger ones, while smaller than ideal, are larger than they would be with unfettered private provision. Research is more concentrated though not to the extreme extent of creating “teaching only universities” as the government would ideally like to do.

The paper is concluded by showing that many of the simplifying assumptions used to obtain an analytically tractable model can be relaxed with no consequence on the qualitative nature of the results.

1 Introduction.

This paper studies the organisation of the university sector. The dearth of theoretical analyses devoted to this topic, relative, for example, to public sector procurement, health, and primary and secondary school systems, is surprising, in view of the wealth of peculiar features characterising it and its immediate relevance to the daily life of many researchers in this area.

In the real world, universities supply both tertiary education and research. This is often attributed (Becker, 1975 and 1979) to economies of scope between teaching and research, so that teachers (respectively researchers) are more productive if they also do some research (respectively teaching). The empirical evidence for this is flimsy[1] and we therefore refrain from making our analysis hinge on it: in particular we assume that research and teaching are perfect substitutes in the university production function and allow in principle "research only" and "teaching only" universities. We do not posit any comparative advantage in these two activities either: universities that are "better at research" are also equally "better at teaching". In the model, universities are the only institutions that can impart tertiary education and their payoff is the amount of research that they do, interpreted here as "blue sky" research.[2] In our paper the complementarity between teaching and research and the existence of economies of scale and of scope in the university sector production function are *derived* as features of the equilibrium arising from the interaction between universities and the government, not assumed exogenously.

Students attend university to increase their labour market earnings; they differ in the overall utility they derive from attending university, and they are imperfectly mobile in the sense that there are some friction in the students' capacity to choose university. Two universities offering identical services to students need not therefore charge the same price. The government[3] maximises a standard welfare function; universities use their information advantage to recruit students as a means of raising funds for research. The link between

[1] For a survey of empirical results see Hattie and Marsh, 1996. For detailed discussions of the functional forms chosen for the universities production function and of output measures for teaching and research see Agasisti and Dal Bianco, 2007.

[2] Of course, universities also engage in more applied research: we take this to be externally funded, directed towards specific projects, and view it more as commercial activity. In this area of activity, competition with private providers is usually present, and we assume that if universities do engage in such research, they make no economic profits, so that we can disregard it in what follows.

[3] We refer to the financing or regulatory agency, as "the government", although, conceivably, the views of an international organisation could also be influent for a developing country.

teaching and research emerges in our model as a consequence of market or information imperfections: indeed, if the government had no information disadvantage vis-à-vis the universities, it would separate teaching and research, allocating each independently of the other, in particular creating teaching only universities and concentrating research in few highly productive institutions. In the realistic case of asymmetric information, the complementarity between teaching and research emerges endogenously as a feature of the equilibrium, rather than being due to exogenously given characteristics of the technology.

Research bestows an externality on society. We capture this with the catch-all assumption that individual earnings, and hence the gross domestic product, depend positively on the amount of research carried out in society. On the other hand research uses up resources which are diverted from alternative uses. Therefore it is not necessarily the case that research is underprovided. Whether or not it is depends in general on the balance between technology – how much the nation's scientific and cultural state affects the production of goods and services –, the direction research takes – how much of what researchers do spills over to the rest of society –, and on the subjective preferences of the government.[4] The study of the balance between aggregate costs and benefits is outside the scope of paper: we concentrate instead on the *distribution* of a given amount of total research and teaching among different institutions.

The gist of our results can be summarized by stating that the optimal government policy is to concentrate research and teaching in the most productive universities. While the location of research is a matter of indifference, the imperfect geographical mobility of the students implies that the same is not true of teaching: concentrating teaching in some institutions means that some students cannot attend university who would have benefited from doing so and would have been able under unfettered private provision. There is, that is, a trade-off between concentrating teaching and research in the most productive universities and ensuring that the students who benefit most attend university, irrespective of their location. This trade-off is drawn in sharp focus by the perfect information case: in this case, and only in this case, the government creates "teaching only" universities, allocating the total amount of research exclusively to the most productive institutions, and making all students pay for

[4] While a mathematical theorem may eventually help improve computer software used in designing robots, a chemical discovery may allow the development of more effective drugs, reducing the number of days lost due to illness and advances in game theory may lead to improved incentive mechanisms used by organizations to select and motivate staff, other research activities could instead be viewed as an end in itself, academics indulging in their hobbies, with no expected current or long term benefit to society.

research, including those attending teaching only universities. This separation is however not possible when the government has an information disadvantage vis-à-vis the universities. In this case, just as with fully private provision with no government intervention, all universities do both teaching and research, and while the government would like to concentrate research in the most productive institutions, it does so only to a lesser extent: the most productive universities carry out more research than they would in a fully private system, but less than they would if the government could observe each university's productivity; there are fewer research active universities than in a private system, but more than the government would allow if it had no information disadvantage. And, again like under fully private provision and unlike the symmetric information regime, there are no teaching only universities. We note finally that "research only" universities can only exist in a private system where the government offers all institutions a lump sum subsidy, unrelated to teaching or research performance, and are sub-optimal.

The paper is organised as follows. We present the model in Section 2: the universities in Section 2.1, the students and the labour market in 2.2. In Section 3 we study a private university system unencumbered by government intervention, and in Section 4 derive the government optimal policy. In Section 5 these two systems are compared against the common benchmark of the system a perfectly informed government would design. Finally, in Section 6 we argue that the analysis is robust to relaxation of some of the assumptions: we indicate how distributional concerns can be taken into account in Section 6.1; we introduce some student mobility in Section 6.2, the dependence of their willingness to pay on the research output of the university in Section 6.3, and the endogenous determination of a university's productivity in Section 6.4. Section 7 is a brief conclusion. The proof of all mathematical results are gathered in the Appendix.

2 The model

2.1 Universities.

We study an economy with a continuum of separated local education markets and a single economy-wide labour market. The local education markets can be thought of as different towns or counties. In the global labour market there are two types of jobs, skilled and unskilled: to obtain a skilled job it is necessary to have a university education. This is obtained in the local education markets, in each of which there is a single potential university, which has monopoly power:[5] it is available to all local residents, and only to them.[6] Potential universities differ in the value of an exogenously given[7] productivity parameter, $\theta \in \left(0, \bar{\theta}\right]$, with $\bar{\theta} > 1$. The distribution of θ in the economy follows a differentiable function $F(\theta)$, with density $f(\theta) = F'(\theta) > 0$ for $\theta \in \left(0, \bar{\theta}\right)$, and monotonic hazard rate, $\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right) > 0$. The total number of universities, $F\left(\bar{\theta}\right)$, is normalised without loss of generality to 1.

Universities can engage in research and teaching,[8] and to do so they must build lecture theatres, laboratories, libraries and so on, and employ "professors". Let $n > 0$ be the number of professors employed, then the university of type θ can choose a combination of $r \geqslant 0$, the amount of research carried out, and $t \geqslant 0$, the number of students taught, satisfying $n = \tilde{n}(t, r, \theta)$ with $\tilde{n}_r(t, n, \theta) > 0$, $\tilde{n}_t(t, n, \theta) > 0$, $\tilde{n}_\theta(t, n, \theta) < 0$: the first two imply that research and teaching both require professors, and the third defines θ as a positive measure of productivity: a university with a higher θ can do the same amount of research and teaching with fewer professors. We choose the following functional shape:

$$n = \frac{t + r}{\theta}. \tag{1}$$

(1) implies that teaching and research are perfect substitutes as outputs,[9] and

[5] Our focus in this paper is not on competition among universities, which has also received limited attention in the literature: among the few contribution, in Del Rey (2001), universities choose the amount of research and funding is positively related to the number of students. De Fraja and Iossa (2002) show that, if students are sufficiently mobile, competition among universities causes the emergence of "elite" institutions, which carry out more research and teach the best students. Gautier and Wauthy (2007) focus instead on the internal incentives structure of competing universities: specifically, they found that in multi-department universities, redistribution of resources can determine free-riding of teaching effort, but well designed internal financial rules can induce multi-department universities to perform better than independent departments.

[6] This is a simple way of capturing the assumption that students are not infinitely mobile: if this were the case, one university, the best, would attract *all* students, and carry out *all* research. Introspection suggests that this is not optimal for tertiary education, (whereas it may be for sporting academies), and Section 6.2 hints that allowing (imperfect) students mobility does not alter the qualitative features of the analysis.

[7] The stylised model of Section 6.4 illustrates how a university's value of θ can be determined endogenously as an equilibrium variable.

[8] That is, universities are multi-product firms, as recognised by Cohn et al. 1989. Further outputs that have been suggested, such as the transfer of knowledge (Johnes et al., 2005), or the production of human capital (Rothschild and White (1995)), could clearly be incorporated by extending the concept of teaching.

[9] Though there may be some complementarities, today's highly specialised nature of scien-

in a linear relationship with the number of professors: there are no economies of scale or scope. All non-staff resources are assumed to be proportional to staff numbers, and therefore can be fully captured by the parameter θ. The normalisation of $\frac{1}{\theta}$ as the number of staff needed to produce one combined unit of outputs will prove convenient when solving the government optimisation problem.

Universities receive income from students, who pay a tuition fee of $p \in \mathbb{R}$ each (not restricted to be non-negative), and possibly from the government, in the form of a grant $g \in \mathbb{R}$ (which again can be negative and therefore a tax). A university's costs are the salaries associated with their professors, and so its budget constraint is

$$pt + g - yn = 0, \tag{2}$$

where y is the salary paid to a professor, endogenously determined by a competitive labour market for skilled workers (see below).

We posit that the objective function of universities is the maximisation of the amount of research they do. This is natural in a world where universities are by and large managed by academics whose vocation is research, and tallies with the empirical regularity that academics' reward and progress are more closely linked to success in research than in teaching (see, among others, Hammond *et al.* (1969) and Tuckman *et al.* (1976)). In our model, therefore, universities view teaching as a source of income, a way to fund research, not an activity that increases their utility directly. This of course does not mean that universities do not care about teaching: a university may care passionately about the quality of its teaching exactly in the same sense as a firm cares passionately about the quality of its products: both are means of furthering the organisation's objective, not ends in themselves.[10] Note also that in our set-up the concept of research can be plausibly extended: "research" can be defined as any academic activity which bestows an externality on society by benefitting individuals or organisations who cannot be made to pay for it.[11]

tific research suggests that they are at best limited, and that resources, *viz* academic time, devoted to teaching are in fact resources subtracted to research.

[10] Analogous to the is the common assumption that nominally non-profit hospitals do in fact strive to maximise profit, which they do not distribute to shareholders as dividends, but use instead to further their aims, from excellence in research, to treating patients who are unable to pay; see Danzon (1982) and Dranove and White (1994) Dranove1994. The link is in fact arguably stronger in universities, as they may find than good teaching attracts good students who in turn may attract good staff and improves research prowess.

[11] Thus for example universities may subsidise doctoral supervision, or offer scholarship and financial aid to students from deprived backgrounds: these activities are undertaken by

We also assume that there is an upper limit to the amount of research that can be carried out in each university: let $r_{\max}$ be this bound. This constraint is only necessary when the government has perfect information, and therefore we think of it as very "high", in a sense made precise in Assumption 4.

2.2 Students and the labour market.

Each local education market serves a population of potential students, with measure normalised to 1. They all can obtain basic education, available in each local labour markets at no cost, which guarantees an unskilled job, with income

$$y_U(R,T). \tag{3}$$

$R \geqslant 0$ measures the "state of technology in the society", the value of all research undertaken in the university sector, and $T \in [0,1]$ the total number of graduates in the economy. These are defined as follows: denote by $r(\theta)$ and $t(\theta)$ the *average* amount of research carried out by the universities of type θ, and the *average*[12] number of students in the universities of type θ. We have:

$$R = \int_0^{\bar{\theta}} r(\theta) f(\theta) \, d\theta, \tag{4}$$

$$T = \int_0^{\bar{\theta}} t(\theta) f(\theta) \, d\theta. \tag{5}$$

The positive externality of research implies that $\frac{dy_U(R,T)}{dR} > 0$: workers are more productive if more research is carried out in society.

In each local market, the potential university may choose to become active. If it does so, it sets a tuition fee which students must pay to attend university, obtain a degree and subsequently work in the skilled labour market. All students who attend university graduate and subsequently work in the skilled labour market, receiving income:[13]

$$y(R,T). \tag{6}$$

universities because they increase their payoff, even though – by definition – they do not generate enough revenue to cover their cost: to the extent that they generate benefits to (parts of) society, for example by increasing future research activities or enhancing diversity and offering role-models to able individuals in deprived neighbourhoods, they fit this extended definition of "research".

[12] In the equilibrium we consider, all type θ universities make identical choices, and so $r(\theta)$ and $t(\theta)$ are the amount of research and the number of students in each type θ university.

[13] While there is some evidence of different external effects on the earnings of skilled and unskilled workers (eg Moretti, 2004), the small size of this difference justifies the simplification of the algebra which can be achieved by positing the same function $w(R)$ in the two labour markets.

(3) and (6) imply that the labour markets are "global": the earnings associated to a job depend only on aggregate variables, R and T, not on where the workers has obtained education or on the location of the job.

There are two possible destinations in the skilled labour market: graduates can work as academics in one of the local universities, or they can be employed as graduates in the "business" sector. If N is the number of academics, from (1) we have:

$$N = \int_0^{\bar{\theta}} \frac{r(\theta) + t(\theta)}{\theta} f(\theta)\, d\theta. \tag{7}$$

Income and job satisfaction are the same in the "business sector" and in academia, and the former absorbs all the graduates that are not required to work as academics. This implicitly assumes that the labour academic market is "small" relative to the rest of the economy, so that the academic salary is the same as in the rest of the skilled labour market, and independent of the size of the sector. Relaxing this assumption would imply writing the functions y and y_U as $y(R, T - N)$ and $y_U(R, T - N)$.

It is convenient to assume that the externality is the same for both markets, and that the earnings functions are separable.

Assumption 1 *$y(R,T) = x(T) + w(R)$, and $y_U(R,T) = x_U(T) + w(R)$.*

Individuals differ in the cost of the effort they need to exert if they go to university. This is denoted by $a \in [a_{\min}, a_{\max})$, distributed in each local market according to a distribution function $\Phi(a)$ with density $\phi(a) = \Phi'(a)$ and monotonic hazard rate $\frac{d}{da}\left(\frac{\Phi(a)}{\phi(a)}\right) > 0$. A student of type a who attends university bears a cost $\alpha(a)$, with $\alpha'(a) > 0$. The payoff of an unskilled worker is his income, whereas for a graduate the payoff is the difference between the labour market income, net of tuition fees, and the cost of effort while at university.

In a local market where the university is of type θ and charges a tuition fee $p(\theta)$, a potential student of type a takes this tuition fee, and the earnings in the graduate, $y(R,T)$, and unskilled labour market, $y_U(R,T)$, as fixed, and chooses to attend university and work in the skilled labour market if and only if[14]

$$x(T) + w(R) - p(\theta) - \alpha(a) \geqslant x_U(T) + w(R).$$

[14] We maintain throughout, for the sake of definiteness, the assumption that indifferent students attend university: since they have measure 0 in $[a_{\min}, a_{\max}]$, this entails no loss of generality.

Define $\Delta(T)$ as $x(T) - x_U(T) > 0$, and so we have that there is a threshold value of a, given by

$$\alpha^{-1}(\Delta(T) - p(\theta)), \tag{8}$$

such that all individuals with a equal or lower than (8), and only them, attend university and work in the skilled labour market.

In order to ensure that some – but not all – potential students do in fact choose to attend university, we impose some natural "boundary" assumptions on $x(T)$ and $x_U(T)$.

Assumption 2 *(i)* $x(1) - \alpha(a_{\max}) < x_U(1)$*;*
(ii) $x(0) - \alpha(a_{\min}) > x_U(0)$*.*

That is, (i) if there are very few unskilled workers, they can obtain a higher utility than the students with the highest cost of effort would obtain if they attended university for free, and (ii) if there are very few graduates, there are also at least some potential students whose cost of effort is low enough for them to prefer to attend university, if the tuition fee were 0. Our model can accommodate disparate assumptions regarding the relative shape of the functions $x(T)$ and $x_U(T)$. For example, if demand and supply determine earnings in each market, it would plausibly be $x'_U(T) > 0$ and $x'(T) < 0$. With skill driven technical progress and a sufficiently strong human capital externality, $x'_U(T) > 0$ and $x'(T) > 0$ would hold. Assuming instead perfectly international labour mobility would imply $x'_U(T) = x'(T) = 0$.

Note that, for simplicity, a denotes only the cost of attending university. Allowing earnings also to depend on a would have no qualitative effects on the analysis of the potential students' choices as long as $x(T) - \alpha$ decreases with a more rapidly that $x_U(T)$ does, but would unnecessarily complicate the analysis, because university salary costs would in this case not be straightforward to calculate as they are in the present set-up.

Another simplification is that the students' utility and hence their willingness to pay for a university education does not depend on the type of the university attended. Section 6.3 suggests a stylised but plausible way of relaxing this restriction, which would also add realism to the tuition fee structure of the university sector. We do not introduce this in the main part of the paper, because doing so would complicate the algebra considerably.

Finally, the model is essentially static. This implies, for example, that labour market earnings are affected only by current research, not by the accumulated stock of knowledge, and that there are no intergenerational transfers:

the outcome of research, the payment of tuition fees and the earnings in the labour market all happen at the same time. It would be clearly straightforward to redefine the model as a dynamic one, and concentrate on a steady state where new research carried out in a period balance the reduction of research stock due to obsolescence, and where students tuition is financed either by parents or by loans secured on their future income. In the absence of perfect capital markets, differential borrowing cost among households can be included in the cost of attending university, a, which could thus measure a combination of the utility cost of effort and the interest payments on loans financing university attendance.

3 Equilibrium with no government

In this section the university sector is private, and the government intervention is limited to at most a lump sum subsidy, independent of anything a university does. In each local education market, the local potential university decides whether or not to become active, and if so, the tuition fee to charge. The number of students[15] taught by a university of type θ which chooses price $p(\theta)$ is:

$$t(\theta) = \Phi\left(\alpha^{-1}\left(\Delta(T) - p(\theta)\right)\right). \tag{9}$$

Universities charge the same price to all students, for example, because they cannot observe their type, and have no instruments to provide incentives for truth-telling.

Let us define the function $z_k : [0,1] \longrightarrow \mathbb{R}$, with $k \in [0,1]$, which plays a key role in the rest of the analysis:

$$z_k(t) = \alpha\left(\Phi^{-1}(t)\right) + kt\frac{\alpha'\left(\Phi^{-1}(t)\right)}{\phi\left(\Phi^{-1}(t)\right)}. \tag{10}$$

z_k can be called the adjusted marginal teaching cost. To interpret it, note that if a university wants to add a further student to those already enrolled, it needs to enrol a type $\Phi^{-1}(t)$ student. The latter needs to be enticed with a fee reduction equivalent to the cost of her effort, $\alpha\left(\Phi^{-1}(t)\right)$. The second component of the RHS of (10), for $k = 1$, is the lost fee due to the fact that already enrolled students also pay the lower fee: to enrol an extra student, the

[15] In this set up, where students have perfect information about their type, there is no scope for selection or screening (that is excluding students who are willing to enrol at the current price). Gary-Bobo and Trannoy 2007 is a model where fee charging universities also select students.

fee must be reduced by $\alpha'\left(\Phi^{-1}(t)\right)$. The number of the inframarginal students who benefit from the fee reduction, relative to the number of newly enrolled type $\Phi^{-1}(t)$ students, is the hazard rate, $\frac{\Phi\left(\Phi^{-1}(t)\right)}{\phi(\Phi^{-1}(t))}$, which equals $\frac{t}{\phi(\Phi^{-1}(t))}$. This component of the marginal cost is offset when there is a social benefit to the fact that attendance to university becomes cheaper. We can take $k \in [0,1]$, as an inverse measure of the social benefit of the fee reduction: when $k = 0$, the lower revenues of the university are exactly compensated by the lower fees paid by the students, and the marginal cost of an extra student is unaffected by the fact that the university suffers a loss of revenue loss. In the intermediate case, $k \in (0,1)$, the university's revenue loss is only partly compensated by gains elsewhere; as we see in the next Section, this is natural if the decision maker strictly prefers the universities to be funded by students fees than by general taxation, for example because of distributional, deadweight or administrative costs of taxation.

Assumption 3 *For every* $a \in (a_{\min}, a_{\max})$, $-\frac{\phi(a)}{\Phi(a)} < \frac{\alpha''(a)}{\alpha'(a)}$.

This is true if $\alpha''(a) \geqslant 0$, so that $\alpha'(a)$ is non-decreasing. It is also true if $\alpha'(a)$ is decreasing, provided that it does not decrease too fast.

Lemma 1 *If Assumption 3 holds,* $z_k(t)$ *is monotonically strictly increasing for every* $k \in (0,1]$.

The proof of all the results in the paper is in the Appendix. We can now establish our first result.

Proposition 1 *If Assumption 3 holds, the university of type* θ *sets the following tuition fee:*

$$p(\theta) = \Delta(T) - \alpha\left(\Phi^{-1}\left(z_1^{-1}\left(\Delta(T) - \frac{y(R,T)}{\theta}\right)\right)\right), \tag{11}$$

and enrols

$$t(\theta) = z_1^{-1}\left(\Delta(T) - \frac{y(R,T)}{\theta}\right) \tag{12}$$

students, provided the above is non-negative.

The number of students is set in (12) at a level such that the revenue which can be extracted from an additional student – her additional income $\Delta(T)$ –, reduced by $z(t(\theta))$ – the lost revenue due to the fee reduction necessary to induce this additional student – equals the university's cost of teaching this

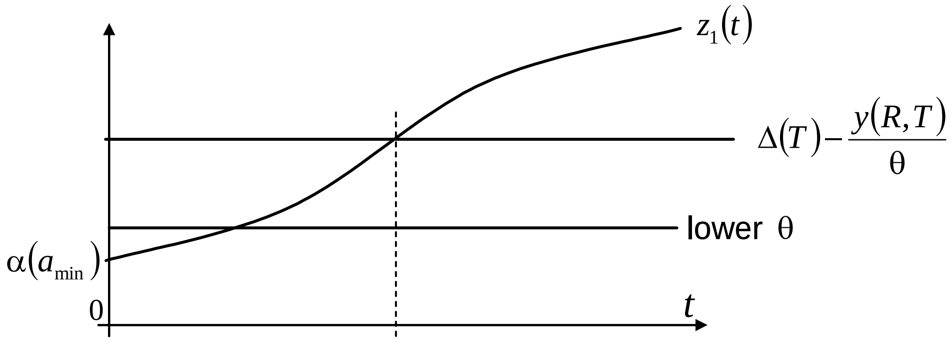

Figure 1: The determination of the number of students in a university.

student, $\frac{y(R,T)}{\theta}$. The university acts as a profit maximising monopolist, and uses the profit to finance research. The equilibrium number of students is illustrated in Figure 1. The increasing line is $z_1(t)$. The abscissa of its intersection with the horizontal line $\Delta(T) - \frac{y(R,T)}{\theta}$ gives the number of students (see (A6) in the Appendix). In Figure 1, for lower values of θ, the horizontal curve is also lower, implying that fewer students are enrolled. For values of θ such that the horizontal line intersects the curve $z_1(t)$ at $t = 0$, or lower, the university is unable to recruit any students at a price which covers its cost: this value of θ is given in Corollary 2.

The next Corollary gives the relationship between productivity and size and fees.

Corollary 1 *$\frac{dt}{d\theta} > 0$, $\frac{dp}{d\theta} < 0$, $\frac{dr}{d\theta} > 0$, and $\frac{dn}{d\theta} > 0$.*

More productive universities do therefore teach more students, charge a lower price to attract them, employ more academics and carry out more research. These relationships between size output and efficiency are often observed in empirical studies,[16] and Corollary 1 shows that they need not necessarily imply that there are economies of scale and scope in the technology, but they could be due instead to an underlying unobserved parameter: universities which employ more staff have lower measured unit costs, both in teaching and

[16] Cohn et al (1989) study a sample of 1887 US colleges and universities in 1981-82, through a multi-product cost approach: they find economies of scale and scope. De Groot, et al (1991) and Glass, et al (1995) extended Cohn's research focusing respectively on the sensitivity of the cost functions estimates to different output measures and on flexible cost function. See also, for a stochastic frontier analysis and DEA, Johnes and Johnes,(1995) and McMillan and Chan (2006).

in research, even though each university employs the same linear technology with no economies of scale and scope.

The above analysis clearly holds if the university of type θ is able to operate. We turn next to the question of which universities are in fact active. For university of type θ to be active, it must be that, at its optimal number of students, it can make positive revenues to pay for its research. This is the case if:

$$(p(t(\theta))t(\theta)+g)\frac{\theta}{y(R,T)}-t(\theta)\geqslant 0. \tag{13}$$

Note that, for an active university, the optimal number of students is independent of the grant g. This has the following immediate consequence.

Corollary 2 *If $g \geqslant 0$, a university of type θ enrols students and carries out research if and only if*

$$\theta > \frac{y(R,T)}{\Delta(T)-\alpha(a_{\min})}. \tag{14}$$

The interpretation is natural: the teaching cost of a student is $\frac{y(R,T)}{\theta}$. For the university to want to teach at least one student it must be worth for at least the students with the lowest a to pay for this cost, and the willingness to pay for tuition of this student is the increase in her labour market earnings as a consequence of her having a degree, $\Delta(T)$, reduced by the utility cost of attending university, $\alpha(a_{\min})$.

Consider research now. Clearly, if $g=0$, a university can do research if and only if it can make positive revenues from its teaching. If $g>0$, then universities with type below (14) can do some research by enrolling no students and spending all their grant on research: they are research only institutions, but they are the least productive among the active universities. Conversely, if $g<0$, then some universities are prevented from becoming active who would be able to cover teaching cost, and the smallest universities teach a strictly positive number of students and do no research.

To close the model, we need to derive R and T. They are obtained from (4) and (5) as the simultaneous solution in T and R of the system:

$$R=\int_{\frac{y(R,T)}{\Delta(T)-\alpha(a_{\min})}}^{\bar{\theta}}\left((p(t(\theta))t(\theta)+g)\frac{\theta}{y(R,T)}-t(\theta)\right)f(\theta)\,d\theta, \tag{15}$$

$$T=\int_{\frac{y(R,T)}{\Delta(T)-\alpha(a_{\min})}}^{\bar{\theta}}t(\theta)f(\theta)\,d\theta, \tag{16}$$

where of course $p(\theta)$ and $t(\theta)$ are given in Proposition 1.

4 Government intervention.

We now assume that the government intervenes actively in the higher education sector. To the extent that private and public universities do not differ in efficiency or in objective function, which seems plausible and reflects anecdotal evidence, there is no reason in our model to distinguish between them: what matters in the model is whether or not the government imposes constraints on the university sector, and that, if it does, the type ownership of a university does not alter the nature of the constraints it must respect. The crucial assumption in the paper is that universities have both a trade-off between teaching and research which leans more towards research than the government's, and an informational advantage over the government. This tallies with our perception of universities, be they public or private. Formally, the government maximises total utility in society, and universities know their own θ and how much r they carry out, while the government cannot observe either.

The government designs the university policy:[17] in our model, this is simply a pair of functions $\{p(t), g(t)\}$, offered to all potential universities: the government commits to linking the number of students enrolled, t, with the tuition fee a university is allowed to charge, $p(t)$, and with the lump sum grant awarded to the university, $g(t)$ (both $p(t)$ and $g(t)$ can be negative). Faced with this policy, each university can choose freely the number of students it wishes to enrol, receiving the corresponding government grant $g(t)$, and charging the corresponding tuition fee $p(t)$. The government's grant to universities is funded by general taxation; to keep things simple, we model taxation as a lump sum tax h, the same for all individuals. Raising one unit of resources in tax has an exogenously given cost $(1+\lambda) > 1$. As Section 6.1 shows, in addition to the standard administrative and distortionary costs of taxation, λ also captures the government's preference for redistribution.

To determine the government's optimal policy, we take a standard revelation approach. The government asks each university to report its own type, and commits to imposing a vector of variables as functions of the reported type, which the university must adhere to: by the revelation principle, the government cannot improve on the payoff it can obtain by restricting its choices to the set of mechanisms with the property that no university has an incentive to mis-report its type.

With this perspective, a policy is a triple, $\{t(\theta), p(\theta), g(\theta)\}_{\theta\in(0,\bar{\theta}]}$, the

[17] Beath et.al. (2005) take the funding mechanism as given and concentrate on the effects on incentives for teaching and research quality of different public funding schemes.

number of students, the tuition fee and the government grant as a function of the reported type. The employment at university of type θ is given by $n(\theta) = \frac{p(\theta)t(\theta)+g(\theta)}{y(R,T)}$. We include *both* t and p as policy variables, thus allowing, potentially, the number of students enrolled in a university to be different from the number of individuals who, given the tuition fee, would prefer to graduate. Clearly, it cannot exceed it, and so we must impose the constraint

$$\Phi^{-1}(t(\theta)) \leqslant \alpha^{-1}(\Delta(T) - p(\theta)), \qquad \theta \in [\underline{\theta}, \bar{\theta}]. \tag{17}$$

(17) says that the type of the marginal student must be no greater than the type of the student who is indifferent between going and not going to university. As we show below, (17) is in fact binding at the government's optimal policy: because of the shadow cost of public funds, it is always preferable for the government to raise funds through tuition fees than through taxes. If this were not the case, the number of university places would need to be rationed and some individuals willing to pay the tuition fee prevented from attending.

We also impose a cap on the amount of taxation that can be imposed on each individual: at the very least is seems natural to require that no one is required to pay more than their maximum potential income, net of tuition fees:

$$h \leqslant \max\{y(R,T) - p(\theta), y_U(R,T)\} = y_U(R,T).$$

The last equality follows from the fact that, since $\alpha(a) \geqslant 0$ for $a \in [a_{\min}, a_{\max}]$, a university that charged more than $\Delta(T)$ would not enrol any students. Combining the above with the government's budget constraint,

$$h = (1+\lambda)\int_0^{\bar{\theta}} g(\theta) f(\theta)\, d\theta,$$

we obtain:

$$(1+\lambda)\int_0^{\bar{\theta}} g(\theta) f(\theta)\, d\theta \leqslant y_U(R,T). \tag{18}$$

To set up the government's problem as an optimal control (Leonard and van Long, 1992) in a suitable way, we introduce auxiliary variables R, the total amount of research, and T, the total number of students, subject to definitional constraints (4) and (5). $\underline{\theta}$, the cut-off point type of university, such that those above operate, those below do not, is also determined endogenously, as the "initial time" (Leonard and van Long, 1992, p 222 ff). Finally, $r(\theta)$ too is treated as a variable chosen by the government, of course subject, as explained above, to the incentive compatibility constraint that all universities prefer to reveal their type truthfully. We now derive this constraint in Proposition 2. Note first that the utility of a university who has reported type θ is

$$r(\theta) = [p(\theta)t(\theta) + g(\theta)]\frac{\theta}{y(R,T)} - t(\theta). \tag{19}$$

Proposition 2 *The incentive compatibility constraint and the participation constraint are given, for $\theta \in [\underline{\theta}, \bar{\theta}]$, by:*

$$\dot{r}(\theta) = \frac{p(\theta)t(\theta) + g(\theta)}{y(R,T)}, \qquad r(\underline{\theta}) = 0, \qquad r(\bar{\theta}) \text{ free,} \tag{20}$$

$$\dot{t}(\theta) \geqslant 0. \tag{21}$$

A further constraint is that each university satisfies its budget constraint:

$$\frac{y(R,T)}{\theta}(r(\theta) + t(\theta)) - g(\theta) - p(\theta)t(\theta) = 0, \qquad \theta \in [\underline{\theta}, \bar{\theta}]. \tag{22}$$

The number of students in a local market must be non-negative

$$t(\theta) \geqslant 0, \qquad \theta \in [\underline{\theta}, \bar{\theta}], \tag{23}$$

and cannot exceed 1 either: by virtue of Assumption 2, if the number of students from a local education market were 1, then the total utility form individuals in that market could be increased simply by stopping the students with the highest cost of effort a, from attending university, and so a situation were there are some $\theta \in [0, \bar{\theta}]$ were $t(\theta) = 1$ cannot happen at the optimum, and the constraint $t(\theta) \leqslant 1$ can be omitted.

The instruments described and the constraints derived, we finally present the government's objective function. This is the total after tax income of the population, net of the disutility costs borne by those who attend university, plus the non-monetary value of research measured by ωR, with $\omega \geqslant 0$.[18] ω depends largely on the government preferences, and captures aspects such as the national pride at the award of Nobel prizes and other formal or informal recognition, and is added to the direct, productivity enhancing effect of research, given by $w'(R)$.

Proposition 3 *The government's objective function can be written as:*

$$\int_{\underline{\theta}}^{\bar{\theta}} \left((\Delta(T) - p(\theta))t(\theta) - \int_{a_{\min}}^{\Phi^{-1}(t(\theta))} \alpha(a)\phi(a)\,da - (1+\lambda)g(\theta) \right) f(\theta)\,d\theta$$
$$+ w(R) + x_U(T) + \omega R. \tag{24}$$

[18] Formally this is identical to including a proportion ω of the universities payoff in the computation of social welfare, just as a share of a regulated firm's profit is typically included.

To derive the Proposition intuitively, note that the total utility of the potential students in a type θ local education market is given by

$$(\Delta(T) - p(\theta))\, t(\theta) - \int_{a_{\min}}^{\Phi^{-1}(t(\theta))} \alpha(a)\, \phi(a)\, da + w(R) + x_U(T) - \tau. \qquad (25)$$

This has a natural interpretation: before tax, all potential students receive utility $x_U(T) + w(R) - h$ at least; of the potential students, $t(\theta)$ do go to university, and receive an *additional* income, net of tuition fee, equal to $\Delta(T) - p(\theta)$: this is the first term in (25). On top of this, those whose type is a incur a disutility of effort $\alpha(a)$, and so their aggregate disutility is given by the second term in (25). Integrate over θ and add ωR, the direct benefit of research, to obtain (24).

In the language of optimal control analysis, the problem can be written as a free initial time optimal control problem with R and T as parameters; the integral constraint (4), (5) and (18) are re-written as state constraints (Leonard and van Long, 1992, p 190), with $r_0(\theta)$, $t_0(\theta)$ and $g_0(\theta)$ as auxiliary variables:

$$\dot{r}_0(\theta) = r(\theta)\, f(\theta), \qquad r_0(\underline{\theta}) = 0, \qquad r_0(\bar{\theta}) = R; \qquad (26)$$

$$\dot{g}_0(\theta) = g(\theta)\, f(\theta), \qquad g_0(\underline{\theta}) = 0, \qquad y_U(R,T) - g_0(\bar{\theta}) \geqslant 0; \qquad (27)$$

$$\dot{t}_0(\theta) = t(\theta)\, f(\theta), \qquad t_0(\underline{\theta}) = 0, \qquad t_0(\bar{\theta}) = T. \qquad (28)$$

And so the government's problem can finally be stated as:

$$\max_{\substack{p(\theta),t(\theta),\\ r(\theta),g(\theta),\\ R,T,\underline{\theta}}} \int_{\underline{\theta}}^{\bar{\theta}} \left((\Delta(T) - p(\theta))\, t(\theta) - \int_{a_{\min}}^{\Phi^{-1}(t(\theta))} \alpha(a)\, \phi(a)\, da - (1+\lambda)\, g(\theta) \right) f(\theta)\, d\theta$$

$$+ w(R) + x_U(T) + \omega R, \qquad (29)$$

s.t. (20), (21), (22), (17), (23) (26), (27) and (28)

Proposition 4 describes the government's optimal policy. Let

$$\begin{aligned}\sigma(R,T,N;\theta) = \Delta(T) &- \frac{y(R,T)}{\theta}\left(1 - \frac{(1-F(\theta))}{f(\theta)\,\theta}\right) \\ &- \left(w'(R)\left(\frac{1}{1+\lambda} - N\right) - \frac{\omega}{1+\lambda}\right)\frac{\int_{\underline{\theta}}^{\bar{\theta}} \tilde{\theta} f(\tilde{\theta})\, d\tilde{\theta}}{f(\theta)\,\theta^2} \\ &+ x_U'(T)\left(\frac{1}{1+\lambda} - T\right) - x'(T)(T-N), \qquad (30)\end{aligned}$$

and let the vector $(R, T, \underline{\theta}, N)$ simultaneously satisfy the following conditions:

$$T = \int_{\underline{\theta}}^{\bar{\theta}} z_{\frac{\lambda}{1+\lambda}}^{-1} \left(\sigma\left(R, T, N; \theta\right)\right) f\left(\theta\right) d\theta, \tag{31}$$

$$\alpha\left(a_{\min}\right) = \sigma\left(R, T, N; \underline{\theta}\right), \tag{32}$$

$$R = \int_{\underline{\theta}}^{\bar{\theta}} \theta \int_{\underline{\theta}}^{\theta} \frac{z_{\frac{\lambda}{1+\lambda}}^{-1} \left(\sigma\left(R, T, N; \tilde{\theta}\right)\right)}{\tilde{\theta}^2} d\tilde{\theta} f\left(\theta\right) d\theta, \tag{33}$$

$$N = \int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{z_{\frac{\lambda}{1+\lambda}}^{-1} \left(\sigma\left(R, T, N; \theta\right)\right)}{\theta} + \int_{\underline{\theta}}^{\theta} \frac{z_{\frac{\lambda}{1+\lambda}}^{-1} \left(\sigma\left(R, T, N; \tilde{\theta}\right)\right)}{\tilde{\theta}^2} d\tilde{\theta} \right) f\left(\theta\right) d\theta. \tag{34}$$

We said informally before that we require $r_{\max}$ to be "high". Formally we now posit:

Assumption 4 $r_{\max} > \bar{\theta} \int_{\underline{\theta}}^{\bar{\theta}} \frac{z_{\frac{\lambda}{1+\lambda}}^{-1} \left(\sigma\left(R,T,N;\tilde{\theta}\right)\right)}{\tilde{\theta}^2} d\tilde{\theta}$.

This implies that at the solution of (29), no university is constrained by the requirement $r(\theta) \leqslant r_{\max}$, and this constraint can be ignored in this problem.

Proposition 4 *Let Assumption 4 hold, and let*

$$\frac{w'(R)}{y(R,T)} \left(\frac{1}{1+\lambda} - N \right) + \frac{\omega}{1+\lambda} > \frac{1 - F(\theta)}{\int_{\theta}^{\bar{\theta}} \tilde{\theta} f\left(\tilde{\theta}\right) d\tilde{\theta} + \frac{f(\theta)^2 \theta^2}{f'(\theta)\theta + 2f(\theta)}}, \tag{35}$$

$$\frac{f'(\theta)\theta + 2f(\theta)}{f(\theta)^2 \theta^2} \int_{\theta}^{\bar{\theta}} \tilde{\theta} f\left(\tilde{\theta}\right) d\tilde{\theta} > -1, \tag{36}$$

for every $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$. If constraint (18) is slack at the solution to problem (29), we have:

$$t(\theta) = z_{\frac{\lambda}{1+\lambda}}^{-1} \left(\sigma\left(R, T, N; \theta\right)\right), \tag{37}$$

$$p(\theta) = \Delta(T) - \alpha\left(\Phi^{-1}\left(z_{\frac{\lambda}{1+\lambda}}^{-1} \left(\sigma\left(R, T, N; \theta\right)\right)\right)\right), \tag{38}$$

$$r(\theta) = \theta \int_{\underline{\theta}}^{\theta} \frac{z_{\frac{\lambda}{1+\lambda}}^{-1} \left(\sigma\left(R, T, N; \tilde{\theta}\right)\right)}{\tilde{\theta}^2} d\tilde{\theta}. \tag{39}$$

The requirement that constraint (18) be slack means that the total tax needed to finance the preferred level of tertiary education is not so high as to require more than the total income that the economy can obtain with no university sector, in which case all workers are unskilled. This is a plausible requirement.

Begin to note that more productive universities have more students.

Corollary 3 *If the solution to Problem (29) is such that (37) holds, then (21) is satisfied and so* $\dot{t}(\theta) \geqslant 0$.

They also do more research, (from (20) or (39)). As we saw in Section 3, this was also the case with unencumbered private provision. The explanation, however, is slightly different: with no government intervention, a more productive university needs to teach more students to be able to carry out more research. The government, when it control the sector, asks more productive universities to teach more students because they are more productive: precisely for the same reason it also asks them to carry out more research. This different angle is brought in starker relief in the next Section, which studies the case in which the government is not constrained by its information disadvantage, and is therefore able to separate the allocation of teaching and research. With limited information, in this Section, the government can do so only to a limited extent.

We end this Section by showing how the above mechanism can be implemented in practice. Totally differentiate (22), to obtain:[19]

$$\dot{g}(\theta) + \frac{d(p(\theta)\,t(\theta))}{d\theta} = y\dot{t}(\theta) \tag{40}$$

From (40) we can derive the relationship between $t(\theta)$ and the total funding available to a university. This is done in Figure 2. It illustrates the relationships between the number of students $t(\theta)$, in the south-west quadrant, and the total funding, TF, in the north-west quadrant, and the type of the university. Joining the two via the 45 degree line in the south-east quadrant, we obtain, in the north-east quadrant, a relationship between the number of students and the total funding available to a university. If the government offers this relationship to all universities, allowing each to choose any point on the curve, then each university will select the combination of students given by (37) and total funding which will allow it carry out the amount of research given by (39). Note that

[19] To derive (40), differentiate (22) with respect to θ:

$$\frac{y(R,T)}{\theta}\left(\dot{r}(\theta) + \dot{t}(\theta)\right) - \frac{y(R,T)}{\theta^2}\left(r(\theta) + t(\theta)\right) - \dot{g}(\theta) - \frac{d(p(\theta)\,t(\theta))}{d\theta} = 0,$$

and substitute (20) and (22).

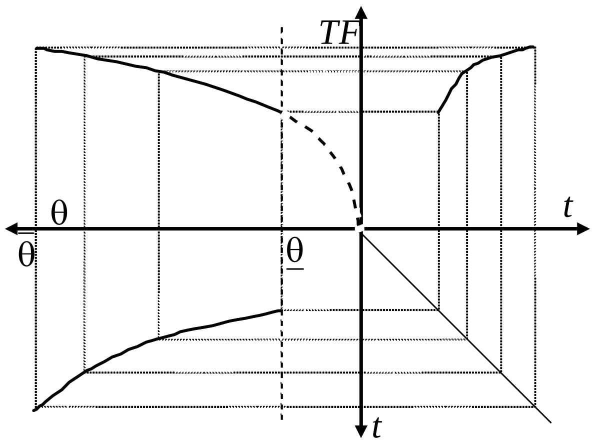

Figure 2: The implementation of the optimal policy

$t(\theta)$ is increasing, and since the total funding also is (from (40)), then so is the relationship between number of students and total funding, as depicted.[20]

5 The systems compared.

To interpret the solution obtained in Proposition 4, it is helpful to compare it with the policy the government would choose if it knew perfectly the type of each university (we also refer to this as the first best) described in the next proposition. Begin by considering the perfect information case: technically, we set $\delta = 0$ and $\eta(\theta) = 0$ in (A23) the proof. Let

$$\sigma_p(R, T, N; \theta) = \Delta(T) - \frac{y(R,T)}{\theta} + x'_U(T)\left(\frac{1}{1+\lambda} - T\right) + x'(T)(T - N), \quad (41)$$

and let the vector $(R, T, \underline{\theta}, N)$ simultaneously satisfy the following conditions

$$T = \int_{\underline{\theta}}^{\bar{\theta}} z^{-1}_{\frac{\lambda}{1+\lambda}}(\sigma_p(R, T, N; \theta)) f(\theta)\, d\theta, \quad (42)$$

$$\alpha(a_{\min}) = z^{-1}_{\frac{\lambda}{1+\lambda}}(\sigma_p(R, T, N; \theta)), \quad (43)$$

[20] If, moreover, the relationship is concave, then the policy can be implemented by offering universities a "subsidy per student"-"lump-sum grant" combination: details can be derived from the analysis in Laffont and Tirole 1993.

$$N=r_{\max}\int_{\frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R,T)}{w'(R)}}^{\bar{\theta}}\frac{f(\theta)}{\theta}d\theta+\int_{\underline{\theta}}^{\bar{\theta}}\frac{z_{\frac{\lambda}{1+\lambda}}^{-1}\left(\sigma_p\left(R,T,N;\theta\right)\right)}{\theta^2}f(\theta)\,d\theta, \tag{44}$$

$$R=\left(1-F\left(\frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R,T)}{w'(R)}\right)\right)r_{\max}. \tag{45}$$

Proposition 5 *The optimal policy of the government with perfect information satisfies:*

$$t(\theta)=z_{\frac{\lambda}{1+\lambda}}^{-1}\left(\sigma_p\left(R,T,N;\theta\right)\right), \tag{46}$$

$$p(\theta)=\Delta(T)-\alpha\left(\Phi^{-1}\left(z_{\frac{\lambda}{1+\lambda}}^{-1}\left(\sigma_p\left(R,T,N;\theta\right)\right)\right)\right), \tag{47}$$

$$r(\theta)=\begin{cases} r_{\max} & \textit{for} \quad \theta\geqslant\frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R,T)}{w'(R)} \\ 0 & \textit{for} \quad \theta\in\left[\underline{\theta},\frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R,T)}{w'(R)}\right) \end{cases}. \tag{48}$$

The determination of the number of students in (46) has similar qualitative features to the corresponding expression for the case of asymmetric information, (37). The expression for research, on the other hand, is radically different in (37) and (48): while Assumption 4 ensured that the upper boundary on research would not be binding in the asymmetric information case, this is not possible in this case, since the government need not provide incentives for research, but can simply command and control the activities of the universities. Notice that the hypothesis of an exogenous upper bound on research expenditure, implies a cost function where the marginal cost is 0 up to this bound, and increases to $+\infty$ beyond; with less extreme forms of decreasing returns to research expenditure, the principle would remain that research is concentrated in the most productive universities. Unlike the private market case and the case of imperfect information, with perfect information there are "teaching only" universities. Universities with θ in $\left[\underline{\theta},\frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R,T)}{w'(R)}\right)$ enrol students but carry out no research. From Proposition 5, we can determine the government subsidy $g(\theta)$:

$$g(\theta)=\begin{cases} \frac{r_{\max}}{\theta}y(R,t)-\frac{\alpha'\left(\Phi^{-1}(t)\right)}{\phi(\Phi^{-1}(t))}t(\theta)^2 & \text{for} \quad \theta\geqslant\frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R,T)}{w'(R)} \\ -\frac{\alpha'\left(\Phi^{-1}(t)\right)}{\phi(\Phi^{-1}(t))}t(\theta)^2 & \text{for} \quad \theta\in\left[\underline{\theta},\frac{1+\lambda}{1-(1+\lambda)N}\frac{y(R,T)}{w'(R)}\right) \end{cases}.$$

This implies that the "teaching only" universities receive more in fees that they pay out in salaries, and their surplus is transferred to the high θ universities which do research.

The situation is depicted in Figure 3, for the special but illuminating case in which the total research, R, is the same across regimes. The solid curve

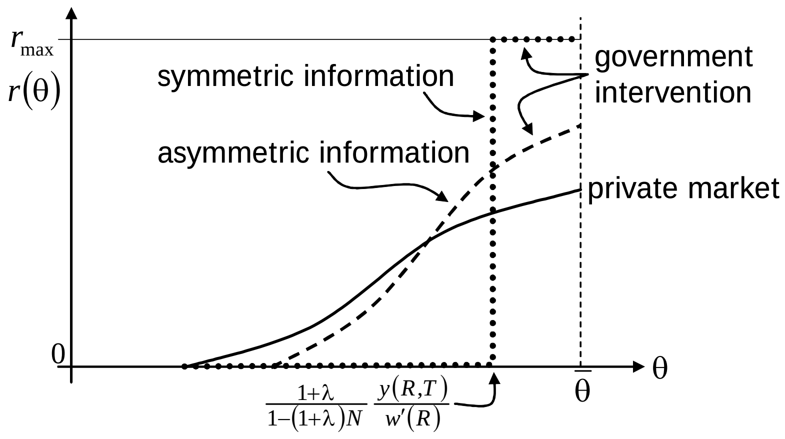

Figure 3: The amount of research with private and government provision.

indicates the amount of research with private provision as a function of the productivity of the university, the dashed curve with government intervention with imperfect information; as shown below, the dashed line is above the solid one for high θ and vice versa as depicted. By contrast, the dotted line plots the amount of research carried out by universities when the government can perfectly observe each university's productivity: there is a threshold value of θ such that universities with higher productivity carry out the maximum amount of research technological feasible, those with lower productivity do no no research at all: these are "teaching only" institutions. With perfect information, the government can separate teaching and research: the former is chosen on the basis of efficiency consideration alone, in each local market to the point where marginal benefits equal marginal costs, adjusted by the externality that graduates in one market bestow on the global market captured by the last two terms in (41). On the other hand, the total amount of research, R, is chosen at the optimally global level (see (45)), given by the condition that amount such that global marginal benefits equal global marginal costs. This total amount of research is allocated to the university sector in the most cost effective way, that is by asking the most productive universities to do as much research as they can.

Consider next the number of students. The following result is helpful in the comparison between unencumbered private provision and government intervention in the private sector.

Lemma 2 *(i) For every $k \in [0,1]$, $z_k(0) = \alpha(a_{\min})$. (ii) Let $k_1 > k_0$; then,* *for every $t > 0$, we have $z_{k_1}(t) > z_{k_0}(t)$. (iii) If moreover we have $\alpha''(a) \geqslant 0$, we also have $z'_{k_1}(t) > z'_{k_0}(t)$.*

As a notational convention, we use the superscript pr, PI and AI to refer to the equilibrium variables in the three regimes, small case pr for no government intervention in an unfettered private market, and capital case for government intervention, with perfect (PI) and asymmetric (AI) information.

Corollary 4 *If $\Delta'(T) < 0$, there exists a threshold value of λ, λ_0 such that for $\lambda > \lambda_0$ we have $t^{pr}(\theta) > \max\left\{t^{AI}(\theta), t^{PI}(\theta)\right\}$, and $T^{pr} > \max\left\{T^{AI}(\theta), T^{PI}(\theta)\right\}$.*

That is, when the shadow cost of public funds is sufficiently high, the government *taxes* university attendance. The intuition is straightforward: for given distortionary and administrative cost of taxation, a high λ entails higher redistributive cost of taxation, given that students' earnings (net of tuition fees) are higher than unskilled workers. The government may therefore prefer to transfer resources from the former to the latter, and, given the restriction to a uniform head tax, it can do so by increasing the tuition fee and reducing the head tax. The increase in the cost of tuition also reduces the number of students.

While this effect is interesting, the focus of the paper is more microeconomic: how should teaching and research be allocated to the nation's universities, actual or potential? A simple way of ensuring that the redistributive effect of the total number of graduates does not interfere with the design of the optimal university system is to assume that their number does not affect earnings.

Assumption 5 *For every $T \in [0, 1]$, $x'_U(T) = x'(T) = 0$.*

This may be due to the fact that international labour markets are perfectly mobile. Even when Assumption 5 holds, the amount of research is in general different in the three regimes, and no a priori ranking can be determined. This is because even though research does bestow an externality, it is not necessarily underprovided by an unfettered private university sector: the first best value of R (the preferred amount of research when the government has perfect information) can be higher or lower than the amount of research carried out in a fully private market. To see this, suppose that research has little social benefit: $w'(R)$, the effect on aggregate income, and ω, the non-monetary benefit of research are both small. As long as students are willing to pay for university tuition, universities will engage in research with the income raised, whereas a welfare maximising government would want to choose a lower total level of research.[21]

The comparison between regimes, the public and private, with the perfect information as the benchmark, can therefore conceptually be divided into two parts: the difference in the global level of research and teaching, and the difference in the distribution of these activities. To concentrate on the more microeconomic aspect of the distribution of teaching and research across institutions, in the rest of this section, we compare the situations where the total amount of research, R, is the same across regimes.

Assumption 6 *$r_{\max}$ and ω are such that the total amount of research is the same in the three regimes considered.*

It is in general possible for the parameters $r_{\max}$ and ω to satisfy Assumption 6. Let $\hat{R}$ be the amount of research in the unfettered private market. $\hat{R}$ is independent of both $r_{\max}$ and ω, and, with Assumption 4, the government policy with asymmetric information is independent of $r_{\max}$, and so ω can be adjusted to ensure that the government chooses $\hat{R}$ in this case. Similarly for $r_{\max}$: once ω is fixed, $r_{\max}$ appears only in the symmetric information regime, and so it too can be adjusted to ensure that $\hat{R}$ is chosen in this regime as well.

The following summarises the comparison.

Corollary 5 *Let Assumptions 4, 5 and 6 hold. Then:*

1. *the universities active with unfettered private provision are the same a perfectly informed government would allow to operate; fewer universities are active with asymmetric information;*

2. *with unfettered private provision, each active university has fewer students than it would have with a perfectly informed government;*

3. *relative to perfect information, the government information disadvantage reduces the number of students at each university except the most productive.*

The diagram of Figure 4 illustrates the situation. The Corollary implies that, compared with private provision, government intervention concentrates students in the most productive institutions: the higher (lower) productivity

[21] Formally, given the assumption that $w''(R) < 0$, $\frac{y(R,T)}{w'(R)}$ is increasing, and the optimal R is 0 if $\frac{1+\lambda}{1-(1+\lambda)N} \frac{y(0,T)}{w'(0)} > 1$, see (45).

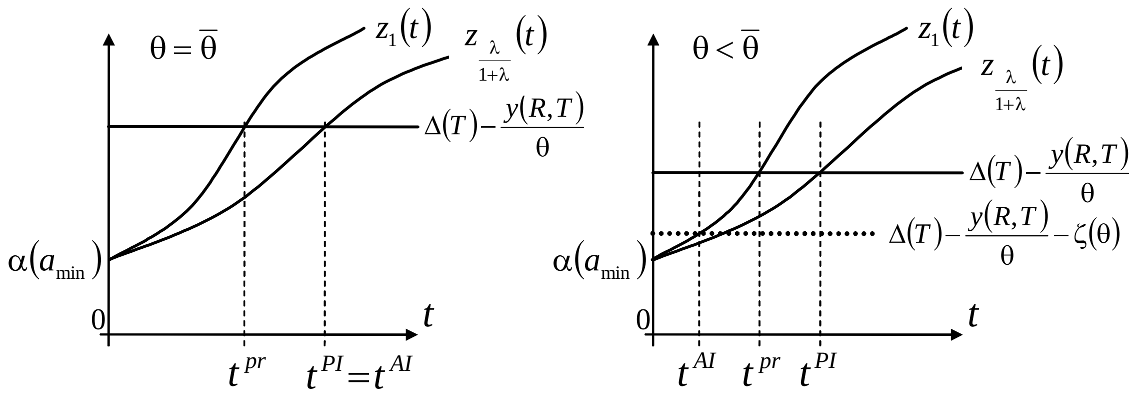

Figure 4: The Determination of the number of students.

institutions have more (fewer) students than they would in a private system. The horizontal axis measures the number of students. This, for university θ, is given by the intersection of the increasing line $z_{\frac{\lambda}{1+\lambda}}(t)$, with under government provision, and $z_1(t)$ for the private market, with the appropriate horizontal line:

$$\Delta(T) - \frac{y(R,T)}{\theta} \qquad \text{for the private market or with perfect information,}$$
$$\Delta(T) - \frac{y(R,T)}{\theta} + \zeta(\theta) \qquad \text{with imperfect government information,}$$

where $\zeta(\theta)$, defined in A27 in the Proof of Corollary 3, in an increasing function.

The LHS of Figure 4 shows the determination of students numbers at the highest possible value of θ. Assumption (5) holds, and the amount of research is the same in the three regimes, and so therefore is the expression $\Delta(T) - \frac{y(R,T)}{\theta}$. It follows that the horizontal curve is the same in all three regimes, and hence the number of students is lower with private provision than with government intervention. This is natural, given that private universities derive no benefit from the lower fee paid by the inframarginal students as a consequence of the enrolment of an additional student. Graphically, Lemma 2 implies that, as k increases from 0 to 1, the curve z swings anticlockwise around the point $(0, \alpha(a_{\min}))$, and since $1 = \lim_{\lambda \to +\infty} \frac{\lambda}{1+\lambda}$, the curve z is lower under government provision for every finite value of λ. In this case we have $t^{pr} < t^{AI} = t^{PI}$. $t^{AI} = t^{PI}$ is the standard "efficiency at the top" result. Point 2 in the Corollary, $t^{pr} < t^{PI}$, is illustrated in the LHS diagram in Figure 4.

The RHS of Figure 4 considers a value of θ lower than $\bar{\theta}$. For such θ, the horizontal curve is lower, for all three regimes, than with $\theta = \bar{\theta}$, but is "more lower", as it were, when the government has imperfect information, as depicted by the dotted horizontal line: this is shown in Corollary 3. As the RHS diagram shows, for sufficiently low θ, we have that $t^{AI} < t^{pr}$ and both of them are smaller than t^{PI}.

Graphically, the reason is that the horizontal line is lowered by the term $\zeta(\theta)$ given in (A27) in the Appendix. Because research is cheaper in more productive universities, the government prefers them to carry out more research. To give them the incentive to do so, it rewards them with a combination of a larger total income *and* a bigger number of students. A less productive university, which would like to receive the higher total income promised to a productive one, is however deterred from doing so by the linked increase in the number of students: since it is less productive, they are more expensive than they would for a more productive university, and the extra total income is not sufficient to cover the cost of teaching these additional students. The term $\zeta(\theta)$ can therefore be interpreted as the information cost incurred by the government: the balance between student fees and lump-sum grant is skewed towards student fees, so as to make less productive universities less willing to expand their student intake. This of course creates an information externality among universities in different local education markets: whether some students attend university or not depends on the cost conditions in the rest of the university sector, even though the cost of providing university education is fully determined at the local level.

As θ decreases further, the intersection of the horizontal lines with the curves $z_1(t)$ and $z_{\frac{\lambda}{1+\lambda}}(t)$ move towards the vertical axis. The value of θ for which this intersection reaches the axis is the type of the least productive university that teaches any students: clearly this happens for the same value when provision is via an unfettered private market and in the first best, and for a lower value of θ when the government has imperfect information.

Figure 5 sketches the possible relationship between θ and the number of students, in the three regimes, and is analogous to Figure 3 for research. The dotted line, depicting the perfect information case, coincides with the dashed one, the asymmetric information case, at $\theta = \bar{\theta}$ (efficiency at the top), and with the solid one, the private market case, at $\theta = \underline{\theta}$, because $z_1(0) = z_{\frac{\lambda}{1+\lambda}}(0)$.

Recall that this picture is drawn for the special case when the total amount of research R is the same in the three regimes. If this is not the case, the horizontal curve in Figure 4 would have a different position in each regime, and the comparison would have to be made taking into account of the different position of this curve. In general, however, note that this horizontal curve shifts

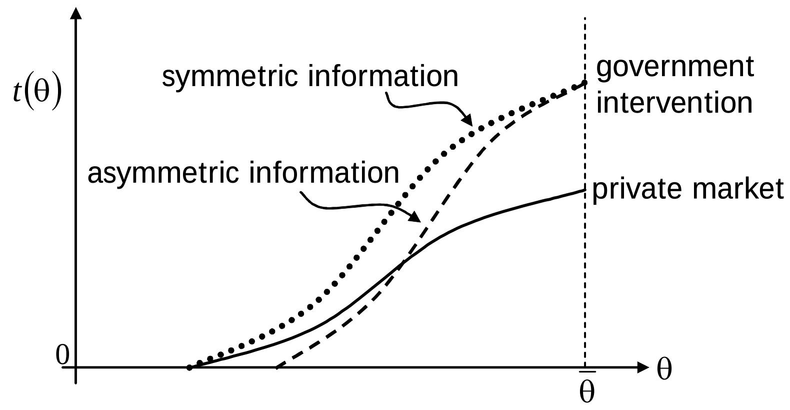

Figure 5: The number of students with private and government provision.

down when R is higher: this naturally reflects the trade-off between teaching and research.

6 Extensions of the model.

We have studied a taxation and (higher) education general equilibrium model, with a continuum of different local markets, a continuum of students in each market, and information, teaching and research interaction among all the local markets mediated by a global labour market. To keep the analysis at a manageable level, we have introduced a number of simplifying assumptions, which are difficult to modify while keeping the analysis tractable. In this section, we try to illustrate, however, that our results would survive unscathed in a richer set-up where some of the assumptions are relaxed.

6.1 Concave utility function

In normative analysis, it is customary to assume concavity of the utility function, to reflect the government's preference for redistribution. In the paper, consumers' utility function is instead assumed linear in income.[22] Conversely, we have also assumed that the shadow cost of public funds is strictly positive and endogenously given. Both assumptions, we show here, can be relaxed at once, to obtain a richer, and intractable, general equilibrium model, where the shadow cost of public funds is determined *endogenously* and is higher when the

[22] This is not restricitve for the positive analysis of Section 3, since with a single good, any strictly increasing utility function can be monotonically transformed into a linear one.

utility function is "more concave". In our simplified model, therefore, λ can be used as a good proxy for the government preference for redistribution.

To proceed in this case, let the utility function of a type a consumer be given by $U(y-\alpha(a))$, with $U'(\cdot)>0$, $U''(\cdot)<0$. To keep things simple, let there be a single value of θ (that is, the support of f is degenerate): in this case, clearly, $T=t$ and $R=r$, and the government's problem becomes one of complete information which can be written as:

$$\max_{p,t,g,r}\int_{a_{\min}}^{\Phi^{-1}(t)} U(x(t)+w(r)-p-\alpha(a)-g)\,\phi(a)\,da+(1-t)\,U(x_U(t)+w(r)-g), \tag{49}$$

$$\text{s.t.: } g+pt-\frac{x(t)+w(r)}{\theta}(r+t)=0, \tag{50}$$

$$\Delta(t)-p-\alpha\left(\Phi^{-1}(t)\right)=0. \tag{51}$$

We have dropped the argument (θ) of the functions p, t, g, and r chosen by the government, omitted the constraint $t>0$, and used the government budget constraint $g=h$. We have also assumed, to keep things simple, that $x'(t)=x'_U(t)=0$.

Proposition 6 *If $U''(\cdot)<0$, at the solution of problem (49)-(51), the Lagrange multiplier of constraint (50) is given by*

$$\frac{U'\left(x(t)+w(r)-p-\alpha\left(\Phi^{-1}(t)\right)-g\right)}{\frac{\int_{a_{\min}}^{\Phi^{-1}(t)} U'(x(t)+w(r)-p-\alpha(a)-h)\phi(a)da}{t}}-1 \tag{52}$$

where p, t, g, and r are the optimal tuition fee, number of students, lump-sum grant and research effort.

The denominator of the first term is the average marginal utility of consumption for all the individuals who become students, and the numerator is the marginal utility of consumption of the marginal student. Since the average student's income, net of utility effort cost, is higher for the average than for the marginal student, if $U''(\cdot)<0$ for at least some of the students, then the first term is greater than 1, implying a strictly positive shadow cost of public funds, even in the absence of distortionary and administrative costs of taxation.

6.2 Students mobility

We have so far assumed that a university's potential demand is fixed and given by all university age individuals at the university location, and only them, because students can only study at their local institution. In reality, students are not perfectly immobile. While students mobility is not the focus of this paper, it is worth hinting at how the hypothesis of no mobility can be relaxed in a natural way.

Students may choose to study at a university other than the one at their location because the price on offer elsewhere is sufficiently preferable to outweigh the disutility of moving to go to university.[23]

Suppose therefore that some students are prepared to trade moving costs off lower tuition and leave their local education market to attend a university in different one. As the price gap between the fee charged by the university in local education market θ, $p(\theta)$, and the average tuition fee in the sector, say $\bar{p}$, widens, more and more potential students want to attend university θ. A reduced form, compact, manner of accounting for this is to let the distribution of potential students of a university charging tuition p be described by

$$\Phi_c(a; p, \bar{p}),$$

with density $\phi_c(a; p, \bar{p}) = \Phi_c'(a; p, \bar{p})$, and monotonic hazard rate $\frac{\partial}{\partial a}\left(\frac{\Phi_c(a;p,\bar{p})}{\phi_c(a;p,\bar{p})}\right) > 0$. Students preference for lower fees is captured by the assumption that if $p_1 > p_0$, then $\Phi_c(a; p_1, \bar{p})$ first order stochastically dominates $\Phi_c(a; p_0, \bar{p})$. Repeating for the present case the analysis carried out in Section 3, we would find that a university of type θ charging tuition fee $p(\theta)$ will enrol a number of student $t(\theta)$ satisfying:

$$t(\theta) = \Phi_c\left(\alpha^{-1}\left(\Delta(T) - p(\theta)\right), p(\theta), \bar{p}\right).$$

The relationship between t and p is now given by (denote, with slight abuse of notation, by $\Phi_c^{-1}(t; p, \bar{p})$ the inverse of $\Phi_c(a; p, \bar{p})$ for given p and $\bar{p}$):

$$\frac{dp}{dt} = -\frac{1}{\frac{\phi\left(\Phi_c^{-1}(t;p,\bar{p})\right)}{\alpha'\left(\Phi_c^{-1}(t;p,\bar{p})\right)} - \frac{\partial \Phi_c(t;p,\bar{p})}{\partial p}} < 0,$$

since $\frac{\partial \Phi_c(t;p,\bar{p})}{\partial p} < 0$. The analytical tractability of being able to define the function z_k independently of p is now lost, as the first order condition for the choice of t now becomes:

$$\left(-\frac{t}{\frac{\phi\left(\Phi_c^{-1}(t;p,\bar{p})\right)}{\alpha'\left(\Phi_c^{-1}(t;p,\bar{p})\right)} - \frac{\partial \Phi_c(t;p,\bar{p})}{\partial p}} + \Delta(T) - \alpha\left(\Phi_c^{-1}(t; p, \bar{p})\right)\right)\frac{\theta}{y(R,T)} = 1.$$

[23] Or, to the extent that students *prefer* to leave home to go to university, for some the price available at their own location outweighs the utility cost of staying put.

However, since the qualitative features of the function z_k are unchanged in this more general set-up, we would expect to obtain similar results as before. A possible interesting by-product of this extension could be the result that universities that have more students also have more able students: this might follow from the fact that the link is severed between the number of students and the ability level of the lowest ability student in a given university.

6.3 Quality dependant willingness to pay.

In Section 6.2 students move to a different location in response to price differences. In practice, of course, students go away from home to university in response to other variables, principally quality. In the model developed so far, there is no scope for this, as the *type* of the university attended by a student does not affect directly her utility. This is unrealistic, and has the unrealistic implication that more productive universities charge lower tuition fees.

A natural way to make the university type directly affect a student's utility is to let θ be an argument of the function α; write $\alpha(a,\theta)$, with $\alpha_\theta(a,\theta) < 0$: studying at a better university is easier, and so has a lower utility cost.[24]. The analysis of the paper can be repeated almost verbatim with the convention that $\alpha^{-1}(x,\theta)$ denotes the inverse image of $x \in [0,1]$ into $[a_{\min}, a_{\max})$, for fixed θ. The function $z_k(t)$ given in (10) is now replaced by

$$z_k(t,\theta) = \alpha\left(\Phi^{-1}(t),\theta\right) + kt\frac{\alpha'\left(\Phi^{-1}(t),\theta\right)}{\phi\left(\Phi^{-1}(t)\right)},$$

where, with another obvious of notation, $\alpha'(a,\theta) = \frac{\partial\alpha(a,\theta)}{\partial a}$. Lemma 1 continues to hold, *mutatis mutandis* for the new notation. The university first order condition for the choice of r, (A4) in the Appendix, is unchanged and so the relation between t and θ remains positive: more productive universities enrol more students. On the other hand, the relationship between productivity and prices is given by the sign of $\frac{dp}{d\theta}$. To calculate it, expand $p(t(\theta)) = \Delta(T) - \alpha\left(\Phi^{-1}(t(\theta)),\theta\right)$:

$$\frac{dp}{d\theta} = -\frac{\alpha'\left(\Phi^{-1}(t(\theta))\right)}{\phi\left(\Phi^{-1}(t(\theta))\right)}\frac{dt}{d\theta} - \alpha_\theta\left(\Phi^{-1}(t(\theta)),\theta\right).$$

The first term is negative, which gives Corollary 1 when $\alpha_\theta(\cdot) = 0$. In the case considered here, $\alpha_\theta(a,\theta) < 0$, and, if it is sufficiently large in absolute value, $\frac{dp}{d\theta}$

[24] Assuming that θ is an argument of y so that graduates from a better university earn more would have similar effects on the student's willingness to pay to attend university of type θ, but with the disadvantage that the salary cost incurred by a university would depend on the university attended by its staff, complicating conceptually the analysis.

can become positive in a range of values of θ, yielding the realistic conclusion that universities with higher productivity not only carry out more research and teach more students, but also charge higher fees.

6.4 Endogenous determination of θ.

We have so far assumed that the value of θ characterising a university, its productivity parameter, is exogenously given. This is the standard assumption in the regulation and procurement literature (Baron and Myerson 1982, Laffont and Tirole 1993), which inspired the methodology of this paper. In the current set-up, the productivity of a university depends in large measure on its human capital, rather than exogenously given technology and environmental conditions.

A good starting point to model formally the relationship between θ and a university's human capital is the observation that the parameter characterising a worker is in practice highly correlated with success in carrying out research activity: those for whom studying at university is relatively less costly are also likely to have a more productive academic career, and universities that appoint staff with lower a will, *ceteris paribus*, be more productive. In this subsection we capture in a highly stylised way the idea that differences in the productivity of universities are linked to differences in the distribution of a of their staff.

Time is divided in periods. Universities are long lived. Workers live two periods: they receive education during their first period of life, are hired by employers after completing education at the beginning of their second period of life, and cannot change job until they retire. In each period a university trains workers in their first period of life, and employs skilled workers in their second period of life. Within each period, the production and education processes take place after the labour markets have cleared.

The markets for skilled labour opens at the beginning of each period, and closes before research and teaching begin. Whilst the labour market is open, each university receives a stream of workers which are randomly drawn from $[a_{\min}, a_{\max}]$ according to the distribution of skilled workers determined by the education process of the previous period. Assume that a university cannot turn away individual workers, but must hire all those who come along, until it decides to stop hiring altogether. Assume also that, before commencing to teach and carry out research, a university can dismiss the last hired of its workers. These are clearly all ad hoc hypothesis, and we stress that the aim of this subsection is only to hint at what the building blocks of a model of endogenous determination of θ could be.

Workers have a reservation wage, what they can earn in the "business" skilled labour market, $y(R, T)$. Workers, universities and employers are all assumed to predict correctly the values that T and R will take. In equilibrium a university's salary offer will be exactly $y(R, T)$. The set of workers hired at any moment of the hiring process obey a distribution F^A, which is a monotonically increasing differentiable function from $[a_{\min}, a_{\max}]$ into $\mathbb{R}_+$, such that $F^A(a_{\min}) = 0$. Let $\mathcal{A}$ be the space of all such functions, and let Θ be a function:

$$\begin{aligned} \Theta &: \mathcal{A} \longrightarrow \left[0, \bar{\theta}\right], \\ \Theta &: F^A \longmapsto \theta. \end{aligned} \tag{53}$$

In words, if a university distribution of staff abilities in period t is $F^A(a)$, then its productivity parameter in that period is $\Theta\left(F^A\right)$. To fix ideas, Θ may be thought of as mapping a distribution into its mean, but one can easily think of reasons why size, variance, skewness, minimum and maximum all matter, and so a generic functional form gives more flexibility at no cost. A natural, though not strictly necessary, requirement is that if F^{A1} first order stochastically dominates F^{A2} (with $F^{A1}(a_{\max}) = F^{A2}(a_{\max})$), then $\Theta\left(F^{A1}\right) > \Theta\left(F^{A2}\right)$.

Each university knows the slope of the relationship between n, the number of staff hired, and the productivity parameter θ which will prevail in equilibrium, $n(\theta)$: given the large number of universities, this is deterministic, both in a private market and with government intervention. At each moment during the hiring process, the pair (θ, n) describing the number of staff hired, n, and the value of θ of the university given the distribution of its staff, has followed a path in the $\left[0, \bar{\theta}\right] \times \mathbb{R}_+$ space. This path associates the size of the university, n, to the value of the productivity the university had when the distribution of its academics was given by F^A, with $F^A(a_{\max}) = n$ and $\Theta\left(F^A\right) = \theta$. The current (n, θ) pair also determines the possible pairs (n, θ) which, with positive probability, may characterise the university in the future. Let $\mathcal{P}(n, \theta)$ denote the set of all such pairs.

Our assumption is that a university stops hiring when the intersection between $n(\theta)$ and the interior of the set $\mathcal{P}(n, \theta)$ is empty, and subsequently reduce its work force to its preferred point on the intersection set of its past path and $n(\theta)$.

Figure 6 illustrates the idea. It depicts the (θ, n) Cartesian plane. A university's first employee determines its initial quality, the point where the squiggly line meets the horizontal axis. Subsequent hires determine different values of θ,

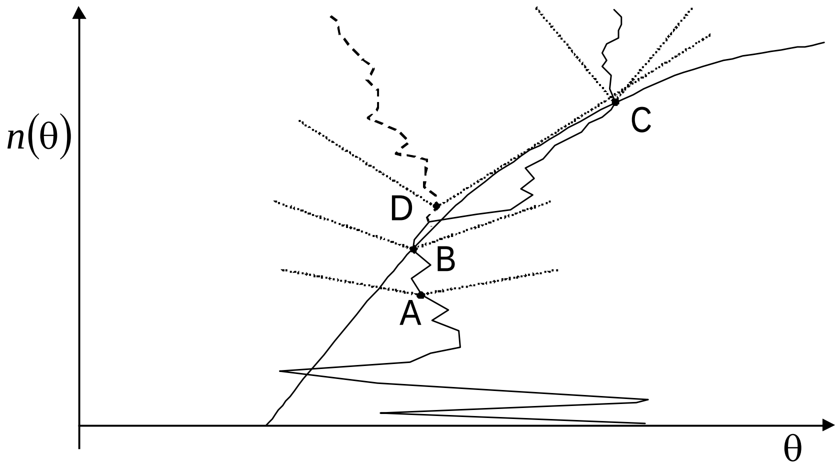

Figure 6: The endogenous determination of θ

depicted by the path. A university that has reached a point like A will rationally continue to hire staff, because the (θ, n) combination represented by point A will not be "allowed" by the government or the private market. Consider next a university that has reached a point like B (say (θ^B, n^B). This combination will be possible (in the specific sense that universities of type θ^B will be given the incentive to choose to employ $n(\theta) = n^B$ academics). However, a university that has reached point B will not stop hiring. The reason is that it would be possible, if the university had a streak of high quality hires, to reach a better point, which will again be feasible, such as point C, along the solid path: the set $\mathcal{P}(n, \theta)$ (represented by the "cone" centred in B of the points which could theoretically be reached) does contain preferred point on the $n(\theta)$ curve. At point C, on the other hand, this is not the case, and a university that has reached point C will stop hiring. Finally, at a point like D, along the dashed path, the university will dismiss some academics, and return to point B.

This model could be further enriched to include, plausibly, some history dependence. A fully dynamic model may link the value of θ in one period to its past values, even though in each period hires are random. This could be due to better laboratories or structures inherited from the past: a shortcut to capture this formally is to replace the function (53) with

$$\Theta : [0, \bar{\theta}] \times \mathcal{A} \longrightarrow [0, \bar{\theta}],$$
$$\Theta : (\theta_{t-1}, F^A) \longmapsto \theta_t.$$

In addition, skilled workers with a high a may prefer to work at a university which had a high θ in the previous period. In this case, the distribution of graduates seeking employment at a university could itself be affected by θ, and would be a differentiable function from $\left[0, \bar{\theta}\right] \times [a_{\min}, a_{\max}]$ into $\mathbb{R}_+$, monotonically increasing in the second argument and such that $F^A(\theta, a_{\min}) = 0$ for every $\theta \in \left[0, \bar{\theta}\right]$.

The assumption which must be maintained to retain the qualitative structure of the model studied in Sections 2-5 is that a university cannot affect the quality of its staff by altering the salary it offers, or by selectively rejecting applicants with low a.[25]

7 Concluding remarks

This paper studies how a utilitarian government should intervene in the university sector. Intervention is beneficial because of the existence of an externality in research, which implies that, even though the total amount of research which would be carried out in an unfettered private is set at the optimal level, its *distribution across* universities is suboptimal. Specifically, the private market "selects" into existence the same universities which should be selected in the first best, but that too few students are enrolled relative to the first best. Moreover, research is too thinly spread: at the first best, research is highly concentrated into the most productive institutions, and the less productive universities carry out no research: students at these "teaching only" universities subsidise research carried out elsewhere.

The information disadvantage of the government vis-à-vis the universities implies that there are fewer universities than in the first best, and with a fully private private sector, the qualitative features of a public provision system are nearer to the private market than to the symmetric information, first best optimal system would be. It is only in the latter that research only universities exist: both with private provision and in the optimal second best public university system all universities do both research and teaching.

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[25] Work is in progress (Carrillo-Tudela and De Fraja 2008) to incorporate the above sketched into a rigourous search model, where graduates have preferences over the quality of the institution they join.

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Appendix

Proof of Lemma 1. Differentiate (10), writing $(\cdot)$ for $\left(\Phi^{-1}(t)\right)$:

$$
\begin{aligned}
z_k'(t) = \frac{dz_k}{dt} &= \frac{\alpha'(\cdot)}{\phi(\cdot)} + kt\frac{\alpha''(\cdot) - \alpha'(\cdot)\frac{\phi'(\cdot)}{\phi(\cdot)}}{\phi(\cdot)^2} + k\frac{\alpha'(\cdot)}{\phi(\cdot)} \\
&= \frac{k\alpha'(\cdot)\,t}{\phi(\cdot)^2}\left(\frac{\phi(\cdot)}{t}\frac{1+k}{k} + \left(\frac{\alpha''(\cdot)}{\alpha'(\cdot)} - \frac{\phi'(\cdot)}{\phi(\cdot)}\right)\right).
\end{aligned}
$$

This is positive if

$$
kt\frac{\alpha'(\cdot)}{\phi(\cdot)^2}\left(\frac{\frac{1+k}{k}\phi(\cdot)}{t} + \left(\frac{\alpha''(\cdot)}{\alpha'(\cdot)} - \frac{\phi'(\cdot)}{\phi(\cdot)}\right)\right) > 0.
$$

That is if (recall that $\alpha'(\cdot)$ is positive):

$$
\frac{\alpha''(\cdot)}{\alpha'(\cdot)} > \frac{\phi'(\cdot)}{\phi(\cdot)} - \frac{\frac{1+k}{k}\phi(\cdot)}{t}. \tag{A1}
$$

Now notice that, given the assumption of a monotonic hazard rate for $\Phi(\cdot)$, we have:

$$
\frac{d}{dx}\left(\frac{\Phi(x)}{\phi(x)}\right) = 1 - \frac{\Phi(x)\,\phi'(x)}{\phi(x)^2} > 0,
$$

and therefore:

$$
\frac{\phi'(x)}{\phi(x)} < \frac{\phi(x)}{\Phi(x)};
$$

when evaluated at $x = \Phi^{-1}(t)$, this becomes:

$$
\frac{\phi'\left(\Phi^{-1}(t)\right)}{\phi\left(\Phi^{-1}(t)\right)} < \frac{\phi\left(\Phi^{-1}(t)\right)}{t}.
$$

Add the term $\frac{1+k}{k}\frac{\phi\left(\Phi^{-1}(t)\right)}{t}$ to both sides of the above to obtain:

$$
\frac{\phi'\left(\Phi^{-1}(t)\right)}{\phi\left(\Phi^{-1}(t)\right)} - \frac{\frac{1+k}{k}\phi\left(\Phi^{-1}(t)\right)}{t} < -\frac{\phi\left(\Phi^{-1}(t)\right)}{kt} < \frac{\alpha''\left(\Phi^{-1}(t)\right)}{\alpha'\left(\Phi^{-1}(t)\right)} \tag{A2}
$$

The last inequality sign follows from Assumption 3. Thus (A1) holds and the Lemma is established. ■

Proof of Proposition 1. Substituting (1) into (2), we can write:

$$
pt + g - y(R,T)\frac{t+r}{\theta} = 0.
$$

That is

$$
r = (pt+g)\frac{\theta}{y(R,T)} - t.
$$

Inverting (9), we obtain the price that a university must charge in order to enrol $t(\theta)$ students:

$$
p(t) = \Delta(T) - \alpha\left(\Phi^{-1}(t)\right). \tag{A3}
$$

A university chooses t to maximise the amount of research it does, and therefore, if an internal solution exists, it satisfies the first order condition:

$$r'(t) = (p'(t)\,t + p(t))\,\frac{\theta}{y(R,T)} - 1 = 0. \tag{A4}$$

Where the dependence of p and r on t is made explicit. From (A3) we obtain:

$$\frac{dp}{dt} = -\frac{\alpha'\left(\Phi^{-1}(t)\right)}{\phi\left(\Phi^{-1}(t)\right)} < 0. \tag{A5}$$

And so (A4) is

$$\left(-\frac{\alpha'\left(\Phi^{-1}(t)\right)}{\phi\left(\Phi^{-1}(t)\right)}t + \Delta(T) - \alpha\left(\Phi^{-1}(t)\right)\right)\frac{\theta}{y(R,T)} - 1 = (\Delta(T) - z_1(t))\,\frac{\theta}{y(R,T)} - 1 = 0.$$

The second order condition is

$$r''(t) = -\frac{\theta}{y(R,T)}z_1'(t) < 0,$$

which, as shown in the Lemma, is satisfied if Assumption 3 holds. By (A4), at the optimum choice of r:

$$z_1(t) = \Delta(T) - \frac{y(R,T)}{\theta}, \tag{A6}$$

from which (12) is derived. (11) is immediate from (A3), and the proof of the proposition is complete. ■

Proof of Corollary 1. The first assertion follows from (12), noting that z_k^{-1} is increasing. The second from the first and (A5). For the third, develop $\frac{dr}{d\theta}$:

$$\frac{dr}{d\theta} = \left((\Delta(T) - z_1(t))\,\frac{\theta}{y(R,T)} - 1\right)\frac{dt}{d\theta} + \frac{p(t(\theta))\,t(\theta) + g}{y(R,T)}. \tag{A7}$$

The above is positive because the first term vanishes by the first order condition on the choice of r, (A4). Finally,

$$\frac{dn}{d\theta} = \frac{\frac{dt}{d\theta} + \frac{dr}{d\theta}}{\theta} - \frac{t(\theta) + r(\theta)}{\theta^2}.$$

Using (A7) and $t(\theta) + r(\theta) = \frac{p(t(\theta))t(\theta)+g}{y(R,T)}\theta$, this simplifies to $\frac{1}{\theta}\frac{dt}{d\theta} > 0$. ■

Proof of Corollary 2. The preferred value of t is given by the intersection of the increasing function $z_1(t)$ with the horizontal line $\Delta(T) - \frac{y(R,T)}{\theta}$ (see (A6)). This intersection occurs for a positive value of t if $z_1(0) < \Delta(T) - \frac{y(R,T)}{\theta}$:

$$\alpha\left(\Phi^{-1}(0)\right) = \alpha(a_{\min}) < \Delta(T) - \frac{y(R,T)}{\theta}.$$

If $g \geqslant 0$, a university with θ satisfying (14) can pay for the research it wants to carry out. This establishes the Corollary. ■

Proof of Proposition 2. Let the government policy be $\{t(\theta), p(\theta), g(\theta)\}$. By choosing to report type $\hat{\theta} \in [0,1]$, university of type θ receives a price for tuition $p\left(\hat{\theta}\right)$, a grant $g\left(\hat{\theta}\right)$, and is required to teach $t\left(\hat{\theta}\right)$ students. Given the market salary for its staff, $y(R,T)$, it employs:

$$\frac{p\left(\hat{\theta}\right) t\left(\hat{\theta}\right) + g\left(\hat{\theta}\right)}{y(R,T)} \tag{A8}$$

academics, which will enable it to carry out an amount of research u such that:

$$\frac{p\left(\hat{\theta}\right) t\left(\hat{\theta}\right) + g\left(\hat{\theta}\right)}{y(R,T)} = \frac{u + t\left(\hat{\theta}\right)}{\theta}.$$

Hence the utility of a university of type θ for reporting $\hat{\theta}$ is

$$\varphi\left(\theta, \hat{\theta}\right) = \frac{p\left(\hat{\theta}\right) t\left(\hat{\theta}\right) + g\left(\hat{\theta}\right)}{y(R,T)}\theta - t\left(\hat{\theta}\right).$$

The revelation principle requires that the above is maximised at $\hat{\theta} = \theta$. The first order condition for the choice of $\hat{\theta}$ is:

$$\left.\frac{\partial\varphi\left(\theta, \hat{\theta}\right)}{\partial\hat{\theta}}\right|_{\hat{\theta}=\theta} = \left.\frac{\partial\left(\frac{p(\hat{\theta})t(\hat{\theta})+g(\hat{\theta})}{y(R,T)}\theta - t\left(\hat{\theta}\right)\right)}{\partial\hat{\theta}}\right|_{\hat{\theta}=\theta} = 0, \tag{A9}$$

which gives:

$$\left(p(\theta)\dot{t}(\theta) + \dot{p}(\theta)t(\theta) + \dot{g}(\theta)\right)\frac{\theta}{y(R,T)} - \dot{t}(\theta) = 0. \tag{A10}$$

Next differentiate $r(\theta)$ given in (19),

$$\dot{r}(\theta) = \left[p(\theta)\dot{t}(\theta) + \dot{p}(\theta)t(\theta) + \dot{g}(\theta)\right]\frac{\theta}{y(R,T)} + \frac{p(\theta)t(\theta) + g(\theta)}{y(R,T)} - \dot{t}(\theta),$$

and substitute (A10) in it to obtain (20). $r(\underline{\theta})$ cannot clearly be below 0, and the government would not want to set it strictly above 0. Now (21): following Laffont and Tirole (1993, p 121), for a policy to be incentive compatible it must be that $\frac{\partial^2\varphi(\theta,\hat{\theta})}{\partial\theta\partial\hat{\theta}} \geqslant 0$. We have

$$\frac{\partial^2\varphi\left(\theta, \hat{\theta}\right)}{\partial\theta\partial\hat{\theta}} = \frac{\partial\left(\frac{p(\hat{\theta})t(\hat{\theta})+g(\hat{\theta})}{y(R,T)}\right)}{\partial\hat{\theta}} = \frac{\partial\left(\frac{r(\hat{\theta})+t(\hat{\theta})}{\hat{\theta}}\right)}{\partial\hat{\theta}} \geqslant 0,$$

$$\frac{1}{\hat{\theta}}\left(\dot{r}\left(\hat{\theta}\right) + \dot{t}\left(\hat{\theta}\right) - \frac{r\left(\hat{\theta}\right) + t\left(\hat{\theta}\right)}{\hat{\theta}}\right) \geqslant 0.$$

substitute $\frac{r(\hat{\theta})+t(\hat{\theta})}{\hat{\theta}} = \frac{p(\hat{\theta})t(\hat{\theta})+g(\hat{\theta})}{y(R,T)} = \dot{r}\left(\hat{\theta}\right)$ from (20), to obtain (21). ■

Proof of Proposition 3. Consider local labour market θ. The total pre-tax utility of the potential students is:

$$\int_{a_{\min}}^{\Phi^{-1}(t(\theta))} \left(x\left(T\right)+w\left(R\right)-p\left(\theta\right)-\alpha\left(a\right)-h\right)\phi\left(a\right)da+\left(1-t\left(\theta\right)\right)\left(x_U\left(T\right)+w\left(R\right)-h\right)$$

where the first term is the total utility of the individuals who go to university, and the second the total utility of those who work in the unskilled labour market. Rearrange to obtain (25) integrating for $\theta > \underline{\theta}$, using the fact that $(1+\lambda)\int_{\underline{\theta}}^{\bar{\theta}} g\left(\theta\right) f\left(\theta\right) d\theta = h$ (the total tax paid equals the total value of the subsidies given by the government to the university sector increased by the deadweight loss costs of taxation per unit of tax raised) and rearranging gives (24). ■

Proof of Proposition 4. Begin by constructing the Lagrangean for (29):

$$\begin{aligned}\mathcal{L} = &\left(\left(\Delta(T)-p\left(\theta\right)\right)t\left(\theta\right)-\int_{a_{\min}}^{\Phi^{-1}(t(\theta))}\alpha\left(a\right)\phi\left(a\right)da-\left(1+\lambda\right)g\left(\theta\right)\right)f\left(\theta\right)\\ &+\mu\left(\theta\right)\delta\frac{p\left(\theta\right)t\left(\theta\right)+g\left(\theta\right)}{y\left(R,T\right)}+\beta\left(\theta\right)\left[\frac{y\left(R,T\right)}{\theta}\left(r\left(\theta\right)+t\left(\theta\right)\right)-g\left(\theta\right)-p\left(\theta\right)t\left(\theta\right)\right]\\ &+\tau\left(\theta\right)\left[\Delta(T)-\alpha\left(\Phi^{-1}\left(t\left(\theta\right)\right)\right)-p\left(\theta\right)\right]+\eta\left(\theta\right)t\left(\theta\right)+\left(\gamma g\left(\theta\right)+\rho r\left(\theta\right)+\xi t\left(\theta\right)\right)f\left(\theta\right)\end{aligned} \tag{A11}$$

Where $\beta\left(\theta\right)$, $\tau\left(\theta\right)$, $\eta\left(\theta\right)$, are the Lagrange multipliers for constraints (22), (17), (23), respectively, and $\mu\left(\theta\right)$, γ, ρ, and ξ are the Pontagryin multipliers for the state variables in constraints (20), (27), (26), (28) respectively. To simplify the analysis of the perfect information case, presented in Proposition 5, we have multiplied the incentive compatibility constraint (20) by an indicator $\delta \in \{0,1\}$, with $\delta = 1$ for the imperfect information case, and $\delta = 0$ for the case in which the government can costlessly observe the type of the university. The first order conditions are:

$$-\frac{\partial\mathcal{L}}{\partial r\left(\theta\right)} = \delta\dot{\mu}\left(\theta\right) = -\beta\left(\theta\right)\frac{y\left(R,T\right)}{\theta}-\rho f\left(\theta\right), \tag{A12}$$

$$\frac{\partial\mathcal{L}}{\partial g\left(\theta\right)} = \frac{\delta\mu\left(\theta\right)}{y\left(R,T\right)}-\beta\left(\theta\right)-\left(1+\lambda-\gamma\right)f\left(\theta\right)=0, \tag{A13}$$

$$\frac{\partial\mathcal{L}}{\partial p\left(\theta\right)} = -t\left(\theta\right)f\left(\theta\right)+\frac{\delta\mu\left(\theta\right)t\left(\theta\right)}{y\left(R,T\right)}-\beta\left(\theta\right)t\left(\theta\right)-\tau\left(\theta\right)=0, \tag{A14}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial t\left(\theta\right)} = &\left[\Delta(T)-p\left(\theta\right)-\alpha\left(\Phi^{-1}\left(t\left(\theta\right)\right)\right)\right]f\left(\theta\right)+\frac{\delta\mu\left(\theta\right)p\left(\theta\right)}{y\left(R,T\right)}\\ &+\beta\left(\theta\right)\left(\frac{y\left(R,T\right)}{\theta}-p\left(\theta\right)\right)-\tau\left(\theta\right)\frac{\alpha'\left(\Phi^{-1}\left(t\left(\theta\right)\right)\right)}{\phi\left(\Phi^{-1}\left(t\left(\theta\right)\right)\right)}+\xi f\left(\theta\right)+\eta\left(\theta\right)=0,\end{aligned} \tag{A15}$$

and, for T, R, and $\underline{\theta}$ (Leonard and van Long, 1992, Theorem 7.11.1, p 255):

$$\xi = x_U'(T) + \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial \mathcal{L}(\theta)}{\partial T} d\theta, \tag{A16}$$

$$\rho = w'(R) + \omega + \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial \mathcal{L}(\theta)}{\partial R} d\theta, \tag{A17}$$

$$\mathcal{L}(\underline{\theta}) = 0, \tag{A18}$$

Derive $\beta(\theta)$ from (A13):

$$\beta(\theta) = \frac{\delta\mu(\theta)}{y(R,T)} - (1 + \lambda - \gamma) f(\theta), \tag{A19}$$

and substitute it into (A12):

$$\delta\dot{\mu}(\theta) = -\frac{\delta\mu(\theta)}{\theta} + (1 + \lambda - \gamma) y(R,T) \frac{f(\theta)}{\theta} - \rho f(\theta).$$

When $\delta = 1$, the two differential equations

$$\begin{aligned} \dot{\mu}(\theta) &= -\frac{\mu(\theta)}{\theta} + (1+\lambda) y(R,T) \frac{f(\theta)}{\theta} - \rho f(\theta) \qquad \mu(\underline{\theta}) \text{ free} \qquad \mu\left(\bar{\theta}\right) = 0, \\ \dot{r}(\theta) &= \frac{p(\theta) t(\theta) + g(\theta)}{y(R,T)} \qquad\qquad r(\underline{\theta}) = 0 \qquad r\left(\bar{\theta}\right) \text{ free} \end{aligned}$$

determine the state variable $r(\theta)$ and the multiplier $\mu(\theta)$.

$$\mu(\theta) = -\frac{\rho \int_{\theta}^{\bar{\theta}} \tilde{\theta} f\left(\tilde{\theta}\right) d\tilde{\theta} - (1 - F(\theta))(1 + \lambda - \gamma) y(R,T)}{\theta} \tag{A20}$$

next substitute (A19) into (A14), to obtain:

$$\tau(\theta) = (\lambda - \gamma) t(\theta) f(\theta). \tag{A21}$$

If constraint (18) is slack, then $g\left(\bar{\theta}\right) < y_U(R,T)$ in (27), and the multiplier associated to it, γ, must be 0. A positive γ would correspond to an increase in the shadow cost of public funds. Then (A21) implies that $\tau(\theta) > 0$ if $\lambda > 0$ and $t(\theta) > 0$, and so (17) holds as an equality: $p(\theta) = \Delta(T) - \alpha\left(\Phi^{-1}(t(\theta))\right)$. Substitute this, $\beta(\theta)$ from (A19), $\gamma = 0$ and $\tau(\theta)$ from (A21) into (A15) and re-arrange:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial t(\theta)} &= \frac{\delta\mu(\theta) p(\theta)}{y(R,T)} + \left(\frac{\delta\mu(\theta)}{y(R,T)} - (1+\lambda) f(\theta)\right) \frac{y(R,T)}{\theta} - \\ &\left(\frac{\delta\mu(\theta)}{y(R,T)} - (1+\lambda) f(\theta)\right) p(\theta) - \lambda t(\theta) f(\theta) \frac{\alpha'\left(\Phi^{-1}(t(\theta))\right)}{\phi\left(\Phi^{-1}(t(\theta))\right)} + \xi f(\theta) + \eta(\theta) = 0. \end{aligned}$$

that is

$$\alpha\left(\Phi^{-1}(t(\theta))\right) = \frac{\delta\mu(\theta)}{(1+\lambda) f(\theta)\theta} - \frac{y(R,T)}{\theta} + \Delta(T) - \frac{\lambda}{1+\lambda} \frac{\alpha'\left(\Phi^{-1}(t(\theta))\right) t(\theta)}{\phi\left(\Phi^{-1}(t(\theta))\right)} + \frac{\xi}{1+\lambda} + \eta(\theta) \tag{A22}$$

substitute $\mu(\theta)$ from (A20) to obtain:

$$z_{\frac{\lambda}{1+\lambda}}(t(\theta)) = \Delta(T) - \frac{y(R,T)}{\theta} + \delta\left(\frac{-\frac{\rho\int_{\theta}^{\bar{\theta}}\tilde{\theta}f(\tilde{\theta})d\tilde{\theta}-(1-F(\theta))(1+\lambda)y(R,T)}{\theta}}{(1+\lambda)f(\theta)\theta}\right) + \frac{\xi}{1+\lambda} + \eta(\theta). \tag{A23}$$

Lemma A3 *Let*

$$\frac{\rho}{(1+\lambda)y(R,T)} > \frac{1-F(\theta)}{\int_{\theta}^{\bar{\theta}}\tilde{\theta}f\left(\tilde{\theta}\right)d\tilde{\theta} + \frac{f(\theta)^2\theta^2}{f'(\theta)\theta+2f(\theta)}}$$

for every $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$. *Then the RHS of (A23) is increasing in* θ.

Proof. The statement is true for $\delta = 0$. Let therefore $\delta = 1$. The derivative of the RHS of (A23) is given by:

$$\frac{1}{\theta}\left(\frac{\rho}{1+\lambda}\left(1+\frac{f'(\theta)\theta+2f(\theta)}{f(\theta)^2\theta^2}\int_{\theta}^{\bar{\theta}}\tilde{\theta}f\left(\tilde{\theta}\right)d\tilde{\theta}\right) - y(R,T)\frac{f'(\theta)\theta+2f(\theta)}{f(\theta)^2\theta^2}(1-F(\theta))\right).$$

Now if $f'(\theta)\theta+2f(\theta) \geqslant 0$, the coefficient of $\frac{\rho}{1+\lambda}$ is positive; if $f'(\theta)\theta+2f(\theta) < 0$, the assumption that $\frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right) < 0$, implies $f'(\theta) > -\frac{f(\theta)^2}{1-F(\theta)}$ and so $1+\frac{f'(\theta)\theta+2f(\theta)}{f(\theta)^2\theta^2}\int_{\theta}^{\bar{\theta}}\tilde{\theta}f\left(\tilde{\theta}\right)d\tilde{\theta} > 1+\frac{-\frac{f(\theta)^2}{1-F(\theta)}\theta+2f(\theta)}{f(\theta)^2\theta^2}\int_{\theta}^{\bar{\theta}}\tilde{\theta}f\left(\tilde{\theta}\right)d\tilde{\theta}$, which is positive by (36). Therefore we can write (7) as

$$\frac{1}{\theta}\frac{\rho}{1+\lambda}\frac{f'(\theta)\theta+2f(\theta)}{f(\theta)^2\theta^2}(1-F(\theta))\left(\frac{1}{\psi(\theta)} - \frac{y(R,T)}{\frac{\rho}{(1+\lambda)}}\right)$$

and the Lemma is established as all terms are positive. ■

Because of the complementarity slackness, if $t(\theta)$ is positive, then $\eta(\theta) = 0$. Since z_k is increasing, given that the expression on the RHS of (A23) is increasing in θ, then $t(\theta)$ is itself increasing, which implies that (21) holds, and moreover that there is a threshold value of θ, call it θ_0, such that $t(\theta) > 0$ if and only if $\theta > \theta_0$.The threshold θ_0 is given by the solution in θ of

$$\alpha(a_{\min}) = \Delta(T) - \frac{y(R,T)}{\theta} - \frac{\frac{\int_{\theta}^{\bar{\theta}}\tilde{\theta}f(\tilde{\theta})d\tilde{\theta}}{f(\theta)\theta^2}\rho - \xi}{1+\lambda} + \frac{(1-F(\theta))}{f(\theta)\theta^2}y(R,T).$$

To continue with the proof, we need to derive the values of the multipliers ξ and ρ. Expand the first order condition for ξ, (A16):

$$\xi = x'_U(T) - (1+\lambda)\int_{\underline{\theta}}^{\bar{\theta}}\left(\frac{r(\theta)+t(\theta)}{\theta}\right)f(\theta)\,d\theta y_T(R,T) + \Delta'(T)(1+\lambda)\int_{\underline{\theta}}^{\bar{\theta}}t(\theta)f(\theta)\,d\theta.$$

Now recall the definition of N given in (7), and we can write:

$$\frac{\xi}{1+\lambda} = x'_U(T)\left(\frac{1}{1+\lambda} - T\right) + x'(T)(T-N). \tag{A24}$$

Similarly for R: expanding (A17) we have

$$\rho = w'(R)\left(1-(1+\lambda)\int_{\underline{\theta}}^{\bar{\theta}} \frac{r(\theta)+t(\theta)}{\theta} f(\theta)\,d\theta\right)+\omega,$$

$$\frac{\rho}{1+\lambda} = w'(R)\left(\frac{1}{1+\lambda}-N\right)+\frac{\omega}{1+\lambda}. \tag{A25}$$

Now substitute (A24) and (A25) into (A23), to obtain:

$$\begin{aligned} z_{\frac{\lambda}{1+\lambda}}(t(\theta)) &= \Delta(T) - \frac{y(R,T)}{\theta}\left(1-\delta\frac{(1-F(\theta))}{f(\theta)\,\theta}\right) \\ &\quad - \delta\left(w'(R)\left(\frac{1}{1+\lambda}-N\right)+\frac{\omega}{1+\lambda}\right)\frac{\int_{\theta}^{\bar{\theta}} \tilde{\theta} f\left(\tilde{\theta}\right) d\tilde{\theta}}{f(\theta)\,\theta^2} \\ &\quad + x_U'(T)\left(\frac{1}{1+\lambda}-T\right)+x'(T)(T-N). \end{aligned}$$

Which, for $\delta = 1$ is (30). Now we want to establish that the lowest θ determined by (31)-(34) is also the value of θ such that $t(\theta)=0$ and $t(\theta)>0$ in a right neighbourhood. Expand (A18). At $\underline{\theta}$, the terms in the square brackets in (A11) and the term $\eta(\underline{\theta})\,t(\underline{\theta})$ are all 0 because of the slackness complementarity constraints. Also 0 is the term $\rho r(\underline{\theta}) f(\underline{\theta})$, because $r(\underline{\theta})=0$, And so:

$$\begin{aligned} \mathcal{L}(\underline{\theta}) &= \left((\Delta(T)-p(\underline{\theta}))\,t(\underline{\theta}) - \int_{a_{\min}}^{\Phi^{-1}(t(\underline{\theta}))} \alpha(a)\,\phi(a)\,da-(1+\lambda)\,g(\underline{\theta})\right) f(\underline{\theta}) \\ &+\delta\mu(\underline{\theta})\,\frac{p(\underline{\theta})\,t(\underline{\theta})+g(\underline{\theta})}{y(R,T)}+\xi t(\underline{\theta})\,f(\underline{\theta})=0. \end{aligned}$$

Since $r(\underline{\theta})=0$,

$$g(\underline{\theta}) = \left(\frac{y(R,T)}{\underline{\theta}}-p(\underline{\theta})\right) t(\underline{\theta})$$

and so

$$\begin{aligned} \mathcal{L} &= \left((\Delta(T)-p(\underline{\theta}))\,t(\underline{\theta}) - \int_{a_{\min}}^{\Phi^{-1}(t(\underline{\theta}))} \alpha(a)\,\phi(a)\,da-(1+\lambda)\left(\frac{y(R,T)}{\underline{\theta}}-p(\underline{\theta})\right) t(\underline{\theta})\right) f(\underline{\theta}) \\ &+\frac{\delta\mu(\underline{\theta})}{\underline{\theta}}t(\underline{\theta})+\xi t(\underline{\theta})\,f(\underline{\theta})=0. \end{aligned}$$

$$\begin{aligned} \mathcal{L} = &\left[\Delta(T)-p(\underline{\theta})-\frac{\int_{a_{\min}}^{\Phi^{-1}(t(\underline{\theta}))} \alpha(a)\,\phi(a)\,da}{t(\underline{\theta})}-(1+\lambda)\left(\frac{y(R,T)}{\underline{\theta}}-p(\underline{\theta})\right)\right. \\ &\left.+\frac{\delta\mu(\underline{\theta})}{f(\underline{\theta})\,\underline{\theta}}+\xi\right] t(\underline{\theta})\,f(\underline{\theta})=0 \end{aligned} \tag{A26}$$

write (A22) (with $\eta(\theta)=0$) as

$$\frac{\delta\mu(\theta)}{f(\theta)\,\theta}+\xi = \left(z_{\frac{\lambda}{1+\lambda}}(t(\theta))+\frac{y(R,T)}{\theta}-\Delta(T)\right)(1+\lambda),$$

and so (A26) becomes:

$$\mathcal{L} = \left(z_\lambda \left(t \left(\underline{\theta} \right) \right) t \left(\underline{\theta} \right) - \int_{a_{\min}}^{\Phi^{-1}(t(\underline{\theta}))} \alpha \left(a \right) \phi \left(a \right) da \right) f \left(\underline{\theta} \right) = 0.$$

This is 0 at $t\left(\underline{\theta}\right) = 0$. Moreover

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial t \left(\underline{\theta} \right)} &= \left(z_\lambda' \left(t \left(\underline{\theta} \right) \right) t \left(\underline{\theta} \right) + z_\lambda \left(t \left(\underline{\theta} \right) \right) - \alpha \left(\Phi^{-1} \left(t \left(\underline{\theta} \right) \right) \right) \right) f \left(\underline{\theta} \right) \\ &= \left[z_\lambda' \left(t \left(\underline{\theta} \right) \right) t \left(\underline{\theta} \right) + t \left(\underline{\theta} \right) \lambda \frac{\alpha' \left(\Phi^{-1} \left(t \left(\underline{\theta} \right) \right) \right)}{\phi \left(\Phi^{-1} \left(t \left(\underline{\theta} \right) \right) \right)} \right] f \left(\underline{\theta} \right) \end{aligned}$$

is positive. That is, whenever $L\left(t\left(\theta\right)\right) = 0$, $\frac{\partial \mathcal{L}}{\partial t(\underline{\theta})}$ is increasing in $t\left(\theta\right)$, and therefore there can only be one value where this happens, which is $t\left(\underline{\theta}\right) = 0$. What remains to be established are (38) and (39). The first follows from (17). To derive (39), start from the following equality:

$$r\left(\theta\right) = \left[p\left(\theta\right) t\left(\theta\right) + g\left(\theta\right)\right] \frac{\theta}{y\left(R,T\right)} - t\left(\theta\right) = \int_{\underline{\theta}}^{\theta} \frac{p\left(\tilde{\theta}\right) t\left(\tilde{\theta}\right) + g\left(\tilde{\theta}\right)}{y\left(R,T\right)} d\tilde{\theta},$$

and write it as:

$$g\left(\theta\right)\theta = \int_{\underline{\theta}}^{\theta} g\left(\tilde{\theta}\right) d\tilde{\theta} + \int_{\underline{\theta}}^{\theta} p\left(\tilde{\theta}\right) t\left(\tilde{\theta}\right) d\tilde{\theta} + t\left(\theta\right) y\left(R,T\right) - p\left(\theta\right) t\left(\theta\right)\theta.$$

Differentiate both sides with respect to θ, and divide by θ:

$$\dot{g}\left(\theta\right) = \frac{\dot{t}\left(\theta\right)}{\theta} y\left(R,T\right) - \frac{d\left(p\left(\theta\right) t\left(\theta\right)\right)}{d\theta}.$$

Integrate both sides in the above to get

$$g\left(\theta\right) = \left(\frac{t\left(\theta\right)}{\theta} + \int_{\underline{\theta}}^{\theta} \frac{t\left(\tilde{\theta}\right)}{\tilde{\theta}^2} d\tilde{\theta} \right) y\left(R,T\right) - p\left(\theta\right) t\left(\theta\right),$$

and therefore:

$$\begin{aligned} r\left(\theta\right) &= \left[p\left(\theta\right) t\left(\theta\right) + \left(\frac{t\left(\theta\right)}{\theta} + \int_{\underline{\theta}}^{\theta} \frac{t\left(\tilde{\theta}\right)}{\tilde{\theta}^2} d\tilde{\theta} \right) y\left(R,T\right) - p\left(\theta\right) t\left(\theta\right) \right] \frac{\theta}{y\left(R,T\right)} - t\left(\theta\right) \\ &= \theta \int_{\underline{\theta}}^{\theta} \frac{t\left(\tilde{\theta}\right)}{\tilde{\theta}^2} d\tilde{\theta}. \end{aligned}$$

From which (39) is derived. Integrate it to obtain (33). Similarly, integration of $t\left(\theta\right)$ and $\frac{r(\theta)+t(\theta)}{\theta}$ gives the values of T and N in (31) and (34); to complete the proof we only have to verify that (21) holds at the solution. This is done in the proof of Corollary 3. ■

Proof of Corollary 3. Let

$$\zeta(\theta) = \frac{y(R,T)(1-F(\theta)) - \left[w'(R)\left(\frac{1}{1+\lambda} - N\right) - \frac{\omega}{1+\lambda}\right]\int_{\theta}^{\bar{\theta}} \tilde{\theta} f\left(\tilde{\theta}\right) d\tilde{\theta}}{f(\theta)\,\theta^2}. \tag{A27}$$

Note that $\zeta(\theta)$, is 0 at $\theta = \bar{\theta}$ and increases in θ. From (30), $\sigma(\cdot)$ is increasing in θ, and so we have

$$\frac{dt}{d\theta} = \frac{\frac{\partial\sigma(\cdot)}{\partial\theta}}{z'_{\frac{\lambda}{1+\lambda}}(t)} > 0.$$

■

Proof of Proposition 5. Impose $\delta = 0$ and $\eta(\theta) = 0$ in the proof of Proposition 4. (A13) becomes:

$$-\beta(\theta) - (1+\lambda) f(\theta) = 0 \tag{A28}$$

As before, we have set $\gamma = 0$. Because of the possibility that the optimum is a corner solution, (A12) must be replaced by

$$\begin{aligned} r = 0 \text{ if } \frac{\partial\mathcal{L}}{\partial r(\theta)} &= (1+\lambda) f(\theta)\frac{y(R,T)}{\theta} - \rho f(\theta) < 0 \\ \frac{\partial\mathcal{L}}{\partial r(\theta)} &= (1+\lambda) f(\theta)\frac{y(R,T)}{\theta} - \rho f(\theta) = 0 \text{ for } r \in (0, r_{\max}) \\ r = r_{\max} \text{ if } \frac{\partial\mathcal{L}}{\partial r(\theta)} &= (1+\lambda) f(\theta)\frac{y(R,T)}{\theta} - \rho f(\theta) > 0 \end{aligned}$$

Using (A28), this implies that

$$\begin{aligned} r = 0 \text{ if } \theta &< \frac{1+\lambda}{\rho} y(R,T), \\ r = r_{\max} \text{ if } \theta &> \frac{1+\lambda}{\rho} y(R,T). \end{aligned}$$

That is the solution is "bang-bang". (A14) becomes

$$\begin{aligned} -t(\theta) f(\theta) - \beta(\theta) t(\theta) - \tau(\theta) &= 0, \\ \tau(\theta) &= \lambda f(\theta) t(\theta), \end{aligned}$$

as before. (A23) in turn becomes:

$$z_{\frac{\lambda}{1+\lambda}}(t(\theta)) = \Delta(T) - \frac{y(R,T)}{\theta} + \frac{\xi}{1+\lambda},$$

and the last university active is given by the solution in θ of

$$\alpha(a_{\min}) = \Delta(T) - \frac{y(R,T)}{\theta} + \frac{\xi}{1+\lambda}.$$

The multipliers ρ and ξ are still given by (A25) and (A24), and substituting the value of ρ into the above, the Proposition is obtained. ■

Proof of Lemma 2. (i) follows immediately from (A6). (ii). Take $z_{k_1}(t) - z_{k_0}(t) = (k_1 - k_0)\, t \frac{\alpha'(\Phi^{-1}(t))}{\phi(\Phi^{-1}(t))}$. (iii) To establish the last statement, write:

$$
\begin{aligned}
\frac{dz_k'(t)}{dk} &= \frac{\alpha'(\cdot)\, t}{\phi(\cdot)^2}\left(\frac{\phi(\cdot)}{t}\frac{1+k}{k} + \left(\frac{\alpha''(\cdot)}{\alpha'(\cdot)} - \frac{\phi'(\cdot)}{\phi(\cdot)}\right)\right) - \frac{k\alpha'(\cdot)\, t}{\phi(\cdot)^2}\frac{\phi(\cdot)}{t}\frac{1}{k^2} \\
&= \frac{\alpha'(\cdot)\, t}{\phi(\cdot)^2}\left(\frac{\phi(\cdot)}{t} + \left(\frac{\alpha''(\cdot)}{\alpha'(\cdot)} - \frac{\phi'(\cdot)}{\phi(\cdot)}\right)\right) > 0. \qquad \text{(A29)}
\end{aligned}
$$

(A29) is equivalent to (A2), evaluated at $\frac{1+k}{k} = 1$. which implies $k \to +\infty$, and the penultimate term in (A2) becomes 0. ■

Proof of Lemma 4. If $\lambda \to +\infty$, then $z_{\frac{\lambda}{1+\lambda}}(t)$ tends to $z_1(t)$, that is, the increasing functions on the LHS of (A6) and (A23) coincide with private and government provision. The RHS, however, is lower with government provision: therefore, for every active university, there are fewer students under public control. Moreover, in this case, the same argument used for the private provision case illustrates that the "last" university in the market has higher type with government intervention, and the statement follows. ■

Proof of Proposition 6. From the Lagrangen L, with β and τ the multipliers for constraints (50) and (51) (omit for brevity the dependence of x and x_U on t):

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial t} =& \frac{1}{\phi(\Phi^{-1}(t))} U(x + w(r) - p - \alpha(a) - g)\, \phi\left(\Phi^{-1}(t)\right) - U(x_U + w(r) - g) \\
& - \beta\left(\frac{x + w(r)}{\theta} - p\right) - \tau \frac{\alpha'\left(\Phi^{-1}(t)\right)}{\phi(\Phi^{-1}(t))} \qquad \text{(A30)}\\
\frac{\partial \mathcal{L}}{\partial p} =& -\int_{a_{\min}}^{\Phi^{-1}(t)} U'(x + w(r) - p - \alpha(a) - g)\, \phi(a)\, da + \beta t - \tau \qquad \text{(A31)}\\
\frac{\partial \mathcal{L}}{\partial g} =& -\int_{a_{\min}}^{\Phi^{-1}(t)} U'(x + w(r) - p - \alpha(a) - g)\, \phi(a)\, da - (1-t)\, U'(x_U + w(r) - g) + \beta \qquad \text{(A32)}\\
\frac{\partial \mathcal{L}}{\partial r} =& \Bigg(\int_{a_{\min}}^{\Phi^{-1}(t)} U'(x + w(r) - p - \alpha(a) - g)\, \phi(a)\, da + (1-t)\, U'(x_U + w(r) - g) \\
& + \frac{r+t}{\theta}\Bigg) w'(r) - \beta \frac{x + w(r)}{\theta} \qquad \text{(A33)}
\end{aligned}
$$

Note that, by the indifference condition (51), $U(x + w(r) - p - \alpha(a) - g) = U(x_U + w(r) - g)$, and so (A30) simplifies to

$$
\frac{\partial \mathcal{L}}{\partial t} = -\beta\left(\frac{x + w(r)}{\theta} - p\right) + \tau \frac{\alpha'\left(\Phi^{-1}(t)\right)}{\phi(\Phi^{-1}(t))}
$$

Define $u(t)$ to be the expression at the denominator of (52), so the integral in (A31)-

(A33) is $u(t)t$. Setting the first order conditions to 0, and rearrange to get:

$$\beta = u(t)t + (1-t)U'(\cdot), \tag{A34}$$
$$\tau = \beta t - u(t)t = t(1-t)(U'(\cdot) - u(t)),$$
$$\left(\alpha\left(\Phi^{-1}(t)\right) + \frac{(1-t)(U'(\cdot)-u(t))}{u(t)t + (1-t)U'(\cdot)} t \frac{\alpha'\left(\Phi^{-1}(t)\right)}{\phi\left(\Phi^{-1}(t)\right)}\right) = \Delta(T) - \frac{x+w(r)}{\theta}.$$

This corresponds to (46): notice that the left hand side is the function z with

$$k = \frac{(1-t)(U'(\cdot)-u(t))}{u(t)t + (1-t)U'(\cdot)}$$

Recall that if λ is the shadow cost of public funds, then the function z_k is $z_{\frac{\lambda}{1+\lambda}}$. Solve $\frac{(1-t)\left(U'(\cdot)-u(t)\right)}{u(t)t+(1-t)U'(\cdot)} = \frac{\lambda}{1+\lambda}$ to obtain:

$$\lambda = \frac{U'(\cdot) - u(t)}{u(t)}.$$

This establishes the Proposition. ■