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ABSTRACT

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JEL Classification: D44, D82 and D83
Keywords: descending auction, Dutch auction, first price sealed bid auction and information acquisition

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Information acquisition during a descending auction*

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Abstract

If bidders can acquire information during the auction the descending auction is no longer equivalent to a first-price-sealed-bid auction. Revenue equivalence does not hold. The incentive to acquire information can even be larger in a descending auction than in an ascending auction.

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1 Introduction

It is textbook wisdom that the descending auction and the first-price-sealed-bid auction are strategically equivalent. In a first price sealed bid (FPSB) auction every bidder makes an offer, the highest bid wins the auction and the price the winner has to pay is equal to her bid. In the descending or Dutch auction a clock price is lowered until the first bidder accepts the clock price. So each bidder has to decide when to accept the clock price. This

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choice is only relevant if no one else has stopped the auction before. The reasoning in this decision process is similar to the decision which bid to make in an FPSB auction. From this observation authors have concluded that both auction formats are identical. Paul Klemperer in his overview article is representative in stating that "... the equivalence of descending and first-price sealed-bid auctions is completely general in single-unit auctions" (Klemperer, 2004, p.14).[1]

Here it will be shown that this conclusion no longer holds if bidders can acquire information during the auction. Bidders in an FPSB auction might well refrain from acquiring costly information, as the additional information does not increase the expected profit from the auction enough to justify the costs. In a descending auction, however, bidders might wait until the clock price is sufficiently low and only acquire information in that case. By waiting they can assure that information will only be bought if it is worthwhile.

This work contributes to the literature on information acquisition during auctions (Compte and Jehiel, 2007; Rasmusen, 2006; Rezende, 2005;).[2] While Rasmusen (2006) analyzes the incentive to acquire information in the framework of an eBay-style auction, the other two papers compare the ascending or English auction with a second price sealed bid (SPSB) auction. Compte and Jehiel (2007) show in a model where bidders are either fully informed or completely uninformed that the ascending auction leads to more acquisition of information and to a higher revenue for the auctioneer. The reason why more information is acquired is that bidders can in an ascending auction observe how many competitors are still active. This allows them to condition their information acquisition on the degree of competitiveness, i.e. they only buy information if there are few remaining competitors. This is

[1]Similar Milgrom (1989): "What this argument shows is that when the Dutch and sealed bid auctions are each modeled in 'strategic form games' the games are identical." Krishna (2002) in his textbook on auction theory discusses the strategic equivalence between the descending auction and the FPSB auction in the introduction. In the remainder of his book, only the FPSB auction is considered.

[2]Some authors have analyzed the incentives to acquire information before the auction (or the mechanism) has started, see Bergemann and Välimäki (2002) and Persico (2000) among others.

different in the descending auction analyzed here, as exit does not occur. The descending auction ends as soon as someone accepts the clock price. Here, information acquisition occurs because bidders can wait until the clock price has reached a relevant region, where information acquisition becomes worthwhile. Furthermore, we show that, different from Compte and Jehiel (2007), even in the case of two bidders the incentive to acquire information is larger in an ascending auction than in an SPSB auction.

The paper contributes to the growing literature on market design by motivating the use of the descending auction, which due to its apparent similarity with the FPSB auction seems to have been somewhat neglected in the literature.[3] Take the example of a license auction, where the bidders are representatives of a company, who typically have limited discretion over the price they are allowed to offer. Suppose the management has given them a limit price above which they cannot quote. Now in an FPSB auction the best the representative can do is to offer this limit price. In a descending auction, the same representative might wait and see if the auction is still open when the clock price is say 10% above her limit price. She might then approach her manager arguing that with a 10% increase of her limit she could get the license for sure. As arguing with the manager for an increase in the limit price is costly, the representative might only do so if she can guarantee the manager that by increasing the limit price the license will be won. In this example an English auction might fare worse. Once the clock price has reached the limit price of the agent, she cannot be sure that by getting an increase from her manager she will win the license. Thus she might abstain from costly negotiations with her manager. It is this difference in the incentives to acquire information which will be exploited in section 4 where we provide an example where information will be acquired with positive probability in a descending auction, but not in an ascending auction.

This result also links our work with the seminal paper by Vickrey (1968).

[3]For example, Ausubel and Cramton (2006) in their work for practitioners on "Dynamic auctions in procurement" do not even consider the Dutch auction as a potential dynamic auction.

Vickrey first observed that the FPSB auction "is isomorphic with the Dutch auction". He then went on to "inquire whether there is not some sealed bid procedure that would be logically isomorphic to the progressive auction", which led him to introduce the SPSB auction. When comparing both auction formats he makes a strong case for the SPSB or ascending auction, arguing that in these auction formats "the bidder can confine his efforts and attention to an appraisal of the value the article would have in his own hands". The result discussed in the previous paragraph shows that the ascending auction and thus the SPSB auction might give less incentives to inquire about the own valuation compared to a descending auction.

There is some piece of evidence which is in accordance with the theoretical result of this paper. Lucking-Reiley (1999) conducted field experiments with Dutch and FPSB internet auctions of collectibles and demonstrated that the former produced significantly more revenue. With the model in this paper one could argue that if people are uncertain about their true valuation, the Dutch auction provides more incentives to make them undertake the necessary steps to evaluate their willingness to pay, once the clock price is in a relevant region.[4]

In the next section we provide an example before analyzing the general case in section 3. In section 4 the differences with respect to information acquisition in a descending auction compared to an ascending auction are discussed. We conclude in section 5.

2 An example

Suppose there are two bidders $i \in \{1, 2\}$ with expected valuations $\hat{v}_i$ being uniformly distributed on the unit interval ($\hat{v}_i \in [0, 1]$ with distribution

[4]Other explanations have also been brought forward: Carare and Rothkopf (2005) analyze a model where bidders incur costs if they delay bidding. In Katok and Kwasnica (2007) bidders are impatient in that they prefer to leave the auction earlier. Both papers come to the conclusion that in longer lasting Dutch auctions bidders might bid more aggressively. In a framework of non-expected utility, differences between the two auction formats have also been obtained by Karni (1988) and Grimm and Schmidt (2000).

$F(\hat{v}_i) = \hat{v}_i$). Both bidders can at costs c learn their true valuation, which is either $v_i = \hat{v}_i - \Delta$ or $v_i = \hat{v}_i + \Delta$, each with probability $\frac{1}{2}$. It holds that $0 < \Delta < 1$.

In an FPSB auction, both bidders make a bid and the highest bid wins the auction at this price. If both bidders do not acquire information, both will bid according to the bidding function $\beta(\hat{v}) = \frac{1}{2}\hat{v}$ (see e.g. Krishna, 2002, p. 17). As the probability of winning for a bidder with expected valuation $\hat{v}$ is $F(\hat{v}) = \hat{v}$, her expected profit is given by:

$$\pi_{fp}^{ni}(\hat{v}) = F(\hat{v})(\hat{v} - \beta(\hat{v})) = \frac{1}{2}\hat{v}^2 \tag{1}$$

If this bidder were to acquire information, she will learn her true valuation. Assuming that the other bidder does not acquire information, her new bidding function is then:

$$\begin{array}{ll} \text{if} \quad v = \hat{v} - \Delta & \rightarrow \beta(v) = \max\{\frac{1}{2}v, 0\} \\ \text{if} \quad v = \hat{v} + \Delta & \rightarrow \beta(v) = \min\{\frac{1}{2}v, \frac{1}{2}\} \end{array}$$

Note that by bidding 0 (or $\frac{1}{2}$ respectively), the bidder loses (respectively wins) the auction for sure. The expected profit of a bidder with expected valuation $\hat{v} \in [\Delta, 1 - \Delta]$ who acquires information is therefore given by:

$$\begin{aligned} \pi_{fp}^{i}(\hat{v}) &= \tfrac{1}{2}F(\hat{v} - \Delta)(\hat{v} - \Delta - \beta(\hat{v} - \Delta)) + \tfrac{1}{2}F(\hat{v} + \Delta)(\hat{v} + \Delta - \beta(\hat{v} + \Delta)) \\ &= \tfrac{1}{4}(\hat{v} - \Delta)^2 + \tfrac{1}{4}(\hat{v} + \Delta)^2 = \tfrac{1}{2}(\hat{v}^2 + \Delta^2) \end{aligned} \tag{2}$$

Thus the value of information is $\frac{1}{2}\Delta^2$. We show in the Appendix that the value of information for a bidder with expected valuation $\hat{v} < \Delta$ or $\hat{v} > 1 - \Delta$ is even less. Thus we can conclude that if $c > \frac{1}{2}\Delta^2$ an equilibrium exists where no bidder acquires information.

In a descending auction, a clock price is lowered and the first bidder to accept is granted the good at this clock price. If no one acquires information, then as a consequence of the strategic equivalence of the FPSB auction and the descending auction in that case, both bidders will again 'bid' according to $\beta(\hat{v}) = \frac{1}{2}\hat{v}$, i.e. they will accept the clock price if the clock price is equal to $\beta(\hat{v})$. Now, assuming the other bidder remains uninformed we analyze the incentive to acquire information for a bidder with expected valuation $\hat{v}$. Suppose this bidder were to acquire information and learns that her true valuation is $v = \hat{v} + \Delta$. Then if the clock price is larger than $\beta(\hat{v} + \Delta) = \frac{1}{2}(\hat{v} + \Delta)$, it will be optimal for her to wait and accept the clock price once it reaches $\frac{1}{2}(\hat{v} + \Delta)$. The reason is that with a true valuation of $v = \hat{v} + \Delta$ she is facing exactly the same decision problem as an uninformed bidder, who does not acquire information, with an expected valuation of $\hat{v} + \Delta$.[5] From this reasoning it follows that this bidder will not acquire information before the clock price has reached $\frac{1}{2}(\hat{v} + \Delta)$. The other bidder might end the auction before in which case she could have saved on her information costs. So suppose the bidder considers acquiring information at clock price $p = \frac{1}{2}(\hat{v}+\Delta)$. At this stage her expected profit without acquiring information would be:

$$\pi_d^{ni}(\hat{v}|p = \frac{1}{2}(\hat{v} + \Delta)) = \frac{F(\hat{v})}{F(\hat{v} + \Delta)}(\hat{v} - \beta(\hat{v})) = \frac{1}{2(\hat{v} + \Delta)}\hat{v}^2 \tag{3}$$

Note that the probability of winning without acquiring information is $\frac{F(\hat{v})}{F(\hat{v}+\Delta)}$, i.e. the probability that the other bidder has expected valuation less than $\hat{v}$, given that her expected valuation is lower than $\hat{v} + \Delta$.

Next consider the incentive to acquire information. With $\hat{v} \in [\Delta, 1 - \Delta]$ the expected profit is given by:

$$\begin{aligned} \pi_d^i(\hat{v}|p = \tfrac{1}{2}(\hat{v} + \Delta)) &= \tfrac{1}{2}\tfrac{F(\hat{v}-\Delta)}{F(\hat{v}+\Delta)}(\hat{v} - \Delta - \beta(\hat{v} - \Delta)) + \tfrac{1}{2}(\hat{v} + \Delta - \beta(\hat{v} + \Delta)) \\ &= \tfrac{\hat{v}^2+\Delta^2}{2(\hat{v}+\Delta)} \end{aligned} \tag{4}$$

which is equal to the expression in equation (2) divided by $F(\hat{v}+\Delta)$. So the value of buying information at this clock price is equal to $\frac{1}{2}\Delta^2/(\hat{v}+\Delta)$ which is larger than $\frac{1}{2}\Delta^2$ if $\hat{v}$ is small.

[5] It follows that in case the bidder learns that her true valuation is $v = \hat{v} + \Delta$ when the clock price is already below $\beta(\hat{v} + \Delta)$, she will accept the clock price immediately.

This shows the first effect a descending auction has on the value of information. A bidder can wait in order to acquire information only when it becomes relevant. Thus from an ex ante point of view the costs of information are not given by c but by (less than) $F(\hat{v}+\Delta)c$, as they only need to be paid if no one else has ended the auction before, which occurs with probability $F(\hat{v}+\Delta)$.

There is a second effect which increases the value of information in a descending auction compared to an FPSB auction. Recall that the bidding function is defined such that at clock price $\beta(\hat{v}+\Delta)$ a bidder with valuation $\hat{v}+\Delta$ is just indifferent between accepting the clock price and staying in the auction. Thus an uninformed bidder with expected valuation $\hat{v}$ will at clock price $\beta(\hat{v}+\Delta)$ strictly prefer to wait somewhat longer to save on expected information costs.

As an extreme case, consider a bidder with expected valuation of zero who buys information when the clock price has reached $p=\epsilon$ with ϵ small. With probability $\frac{1}{2}$ she will learn that she has high valuation of Δ and she will then accept the clock price immediately. So the value of information is $\frac{1}{2}(\Delta-\epsilon)+\frac{1}{2}0 \simeq \frac{1}{2}\Delta$. As it turns out, this also determines the maximum costs for which information will be acquired with positive probability. We summarize these results as an observation:

Observation 1
In case of two bidders with uniformly distributed valuations, if $\frac{1}{2}\Delta > c > \frac{1}{2}\Delta^2$, an equilibrium without information acquisition does exist in a first-price-sealed-bid auction, but not in a descending auction.

Having shown the result in a simple case with uniform distribution, we now turn to the analysis of the equilibrium of the descending auction in a general model.

3 The descending auction: equilibrium analysis

3.1 General properties

There are n bidders with expected valuations $\hat{v}_i$ ($i \in \{1, 2, ..., n\}$), which are private information. The expected valuations $\hat{v}_i$ are independently and individually distributed according to a common distribution $F(\hat{v})$. Bidders can at costs c learn their true valuation v, with $v = \hat{v} + x \in [\hat{v}_i - \Delta, \hat{v}_i + \Delta]$. x is a random variable with distribution $L(x)$, $x \in [-\Delta, \Delta]$. $L(x)$ is the probability that the true valuation of a bidder with expected valuation $\hat{v}_i$ is smaller than or equal to $\hat{v}_i + x$. It holds that $\int_{-\Delta}^{\Delta} x dL(x) = 0$. The other bidders cannot observe whether the bidder has learned her true valuation or not.

A strategy of a bidder in a descending auction takes the following form: As the clock price is decreasing, and given that acceptance by any bidder implies that the auction ends, a bidder has to specify whether she will acquire information at some clock price or rather accept a clock price without acquiring information. Denote by $b(\hat{v})$ an information acquisition function, which is defined such that a bidder with expected valuation $\hat{v}$ acquires information when the clock price reaches $b(\hat{v})$ (and the auction has not ended yet). In case the bidder will never acquire information, set $b(\hat{v}) = 0$. A bidder with expected valuation $\hat{v}$ who acquires information learns her true valuation v. Denote by $\beta^l(v, \hat{v})$ the bidding function of such a bidder, i.e. she will accept the clock price once it reaches $\beta^l(v, \hat{v})$. It is clear that $\beta^l(v, \hat{v}) \leq b(\hat{v})$. Finally, denote by $\hat{\beta}(\hat{v})$ the bidding function of those types with expected valuation $\hat{v}$ who end the auction at clock price $\hat{\beta}(\hat{v})$ without acquiring information.

To describe the equilibrium properties, it turns out to be useful to define two additional functions. First, let $\beta(v)$ be the function which is such that given the equilibrium bidding strategies by all bidders, if a single bidder were to learn her true valuation v before the auction begins, then this bidder would accept the clock price at $\beta(v)$. Second, denote by $H(v)$ the probability for a single bidder that in equilibrium the clock price will reach $\beta(v)$, assuming that this bidder does not accept the clock price before. As a bidder with valuation v does not find it in her interest to 'mimic' a bidder with valuation v', it must hold that:

$$v = \text{argmax}_{v'} \quad H(v')(v - \beta(v')) \tag{5}$$

We are now in a position to state properties of the bidding function and the information acquisition function.

Proposition 1

Let x^* be defined uniquely by $x^* = E[x|x \geq x^*] - c/(1 - L(x^*))$.

(i) The bidding function of a bidder who remains uninformed is given by

$$\hat{\beta}(\hat{v}) = \beta(\hat{v})$$

(ii) If a bidder with expected valuation $\hat{v}$ acquires information, she will do so when the clock price has reached

$$b(\hat{v}) = \beta(\hat{v} + \Delta - c/(1 - L(x^*)))$$

(iii) If she learns that her true valuation is $v = \hat{v} + x$ with $x \geq x^*$ she will accept the clock price immediately, i.e.

$$\text{if} \quad v - \hat{v} \geq x^* \;\Rightarrow\; \beta^l(v, \hat{v}) = b(\hat{v})$$

(iv) If she learns that her true valuation is $\hat{v} + x$ with $x < x^*$ she will accept the clock price at $\beta(v)$, i.e.

$$\text{if} \quad v - \hat{v} < x^* \;\Rightarrow\; \beta^l(v, \hat{v}) = \beta(v)$$

Proof:

(i) is obvious. Concerning (iv), it is also obvious that a bidder who learns that her true valuation is v at some clock price larger than $\beta(v)$ will accept the clock price in the descending auction once it reaches $\beta(v)$. Similarly, concerning (iii): A bidder who learns her true valuation will accept the clock price immediately, if the valuation is such that a bidder with this true valuation would have accepted the clock price already before. It remains to show (ii), i.e. we have to determine when a bidder with expected valuation $\hat{v}$ will acquire information. From the reasoning in the example it follows that a bidder will not acquire information before the clock price has reached $\beta(\hat{v}+\Delta)$. Suppose the clock price in the auction has reached $p = \beta(\hat{v}+\Delta)$ and a bidder with expected valuation $\hat{v}$ plans to acquire information when the clock price reaches $\beta(\hat{v}+x')$ for some x' with $-\Delta \leq x' \leq \Delta$. If at this point this bidder learns that her true valuation is $v = \hat{v} + x$ with $x \geq x'$ she will accept the clock price immediately (result (iii)). If she learns that $x < x'$ she will wait until $p = \beta(\hat{v} + x)$ before accepting the clock price if the auction has not ended before (result (iv)). Thus her expected profit is given by:

$$\frac{H(\hat{v}+x')}{H(\hat{v}+\Delta)}\left(\int_{x'}^{\Delta}(\hat{v}+x-\beta(\hat{v}+x'))dL(x)+\int_{-\Delta}^{x'}\frac{H(\hat{v}+x)}{H(\hat{v}+x')}(\hat{v}+x-\beta(\hat{v}+x))dL(x)-c\right) \tag{6}$$

The first term is the probability of reaching the clock price $\beta(\hat{v}+x')$, while the other terms are the expected profit conditional on reaching this clock price minus the costs of information. Maximizing this expression with respect to x' entails two terms for the borders of the integrals, which cancel each other out. Thus the maximization problem is equivalent to maximizing the expression

$$H(\hat{v}+x')\left((1-L(\bar{x}))\left(\hat{v}+E[x|x\geq\bar{x}]-\beta(\hat{v}+x')\right)-c\right)$$

with respect to x', where $\bar{x}$ has to be set equal to the optimal x' after the optimization. This expression has the same structure as the maximization problem in (5), so the optimal x' is given by $x^* = E[x|x \geq x^*] - c/(1-L(x^*))$.

Reformulating this expression we get: $c = \int_{x^*}^{\Delta}(x - x^*)L(x)dx$. As the right-hand side is monotonously decreasing in x^*, this expression determines x^* uniquely. □

Remark: If $x \in \{-\Delta, \Delta\}$, each with probability $\frac{1}{2}$ (as in the example), a bidder with expected valuation $\hat{v}$ will acquire information (if she acquires information) when the clock price reaches $\beta(\hat{v}+\Delta-2c)$, so that $x^* = \Delta - 2c$.

It may appear surprising that independent of the number of bidders, independent of the distribution function, and independent of who acquires information in equilibrium, a bidder with expected valuation $\hat{v}$ will acquire information (if she acquires information), when the clock price in the auction has reached a level which a bidder with valuation $\hat{v} + E[x|x \geq x^*] - c/(1 - L(x^*))$ will accept. But note first that all the factors just mentioned influence the bidding function $\beta(v)$. And second, this property only refers to the question when to acquire information, and not whether. Considering when to acquire information, a bidder will incur costs of c in any case. Now if information turns out to be negative, i.e. the true valuation is relatively low ($v = \hat{v} + x$ with $x < x^*$) then she will accept the clock price at $\beta(v)$, independent of when exactly she acquired information. So the case which is of relevance for the decision when to acquire information is $x \geq x^*$, which occurs with probability $(1 - L(x^*))$. The relevant valuation is thus $(1 - L(x^*))(\hat{v} + E[x|x \geq x^*]) - c$. The relevant payment, which also has to be paid with probability $(1 - L(x^*))$ only, is $(1 - L(x^*))\beta(\hat{v} + x^*)$. This leads to the result that the bidder will behave as if her valuation is $\hat{v} + E[x|x \geq x^*] - c/(1 - L(x^*))$.

The next step is to determine which type will acquire information in equilibrium. For this purpose, the value of information, once the clock price reaches $\beta(\hat{v} + x^*)$ has to be calculated. This value of information is given by the expression in brackets in (6), minus the expected profit in case the bidder does not acquire information. The calculation is done in the Appendix, where we provide a simple expression for verifying whether a type should acquire information or not. Here we proceed by calculating an explicit equilibrium in a special case, in order to discuss the efficiency and revenue properties of the equilibrium.

3.2 Efficiency and revenue

To discuss properties of the equilibrium we return to the case analysed in the example, i.e. $x \in \{-\Delta, \Delta\}$, each with probability $\frac{1}{2}$, and $\hat{v}$ being uniformly distributed between zero and one.

We calculate an equilibrium were the costs of acquiring information are sufficiently low such that all types acquire information.[6] As every bidder with expected valuation $\hat{v}$ acquires information once (and if) the price has reached $\beta(\hat{v} + \Delta - 2c)$ such a bidder behaves in case she learns that her true valuation is $v = \hat{v} - \Delta$ like a type-$(\hat{v} - \Delta)$ bidder, while in case her true valuation is $v = \hat{v} + \Delta$ she behaves like a type-$(\hat{v} + \Delta - 2c)$ bidder. $H(v)$ is the probability that the other $n - 1$ bidders will accept a clock price lower than or equal to $\beta(v)$. So we can write $H(v) = G(v)^{n-1}$, where the relevant type density $g(v)$ is given by:

$$g(v) = \begin{cases} \frac{1}{2} & -\Delta \leq v < \Delta - 2c \\ 1 & \Delta - 2c \leq v < 1 - \Delta \\ \frac{1}{2} & 1 - \Delta \leq v \leq 1 + \Delta - 2c \end{cases} \tag{7}$$

with corresponding distribution:

$$G(v) = \begin{cases} \frac{1}{2}(v + \Delta) & -\Delta \leq v < \Delta - 2c \\ v + c & \Delta - 2c \leq v < 1 - \Delta \\ \frac{1}{2}(v + 1 - \Delta + 2c) & 1 - \Delta \leq v \leq 1 + \Delta - 2c \end{cases} \tag{8}$$

$f(\hat{v})$ and $g(v)$ are shown in Figure 1.

[6] It turned out to be numerically not tractable to determine an equilibrium where only some types acquire information. This is due to the effect that an equilibrium does not exist with a single cutoff point, i.e. which is such that types with expected valuation smaller (larger) than this cutoff point do (do not) acquire information.

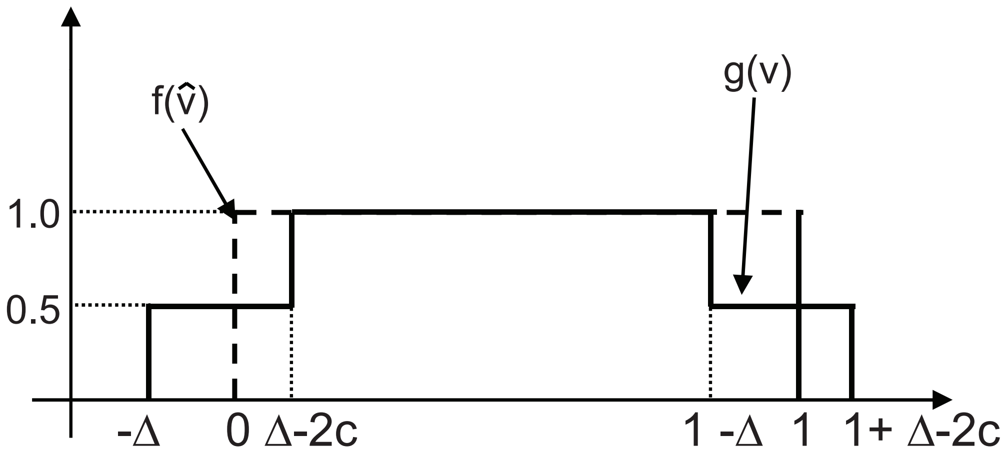

Figure 1: *Expected valuation density function (dotted line) and the according density function for the descending auction (solid line) when all types acquire information*

The bidding function in the descending auction can then be calculated to give (see e.g. Krishna, 2002, p. 19):

$$\beta(v) = v - \frac{1}{H(v)} \int_{-\Delta}^{v} H(s)ds. \tag{9}$$

To analyze the effects of information acquisition on efficiency and revenue, we compare three different scenarios:
(i) The first benchmark is a FPSB auction where no bidder acquires information.
(ii) The second benchmark is a FPSB auction under the assumption that all bidders acquire information.
(iii) These benchmarks are compared to a descending auction with parameter values such that in equilibrium all types ($\hat{v}$) acquire information if the clock price reaches $\beta(\hat{v} + \Delta - 2c)$.

Efficiency:

With respect to efficiency, the different effects are analyzed first for the case of two bidders only.

(i) In case no one acquires information, the bidder with the highest expected valuation $\hat{v}^{(1)}$ wins the auction. The resulting allocation is inefficient, if the true valuation of this bidder is $v^{(1)} = \hat{v}^{(1)} - \Delta$, and at the same time the other bidder has expected valuation $\hat{v}^{(2)} > \hat{v}^{(1)} - 2\Delta$ and a true valuation of $v^{(2)} = \hat{v}^{(2)} + \Delta$. The efficiency loss can be calculated to give

$$\int_0^1 2F(\hat{v}^{(1)})f(\hat{v}^{(1)}) \int_{\hat{v}^{(1)}-2\Delta}^{\hat{v}^{(1)}} \frac{1}{4}[2\Delta - (\hat{v}^{(1)} - \hat{v}^{(2)})]f(\hat{v}^{(2)})d\hat{v}^{(2)}d\hat{v}^{(1)}$$

which is equal to

$$\int_0^1 2\hat{v}^{(1)}\frac{1}{4}(2\Delta^2)d\hat{v}^{(1)} = \frac{1}{2}\Delta^2$$

(ii) If every bidder acquires information, the allocation will be efficient, i.e. the bidder with the highest true valuation wins the contest. However, an inefficiency arises due to the costs of information acquisition, which in this case are equal to $2c$, as both bidders acquire information.

(iii) In a descending auction, there are two types of inefficiencies. First, information will be acquired by at least the bidder with the highest expected valuation. The other bidder will acquire information only if $v^{(1)} = \hat{v}^{(1)} - \Delta$, which occurs with probability $\frac{1}{2}$, and at the same time $\hat{v}^{(2)} + \Delta - 2c > \hat{v}^{(1)} - \Delta$. So the expected information acquisition costs are equal to

$$c + \frac{1}{2}c\int_0^1 2F(\hat{v}^{(1)})f(\hat{v}^{(1)}) \int_{\hat{v}^{(1)}-2\Delta+2c}^{\hat{v}^{(1)}} \frac{1}{2}f(\hat{v}^{(2)})d\hat{v}^{(2)}d\hat{v}^{(1)} = c(1 + \Delta - c)$$

In addition there is an efficiency loss due to a possibly inefficient allocation. This occurs if the bidder with the highest expected valuation has a true valuation of $v^{(1)} = \hat{v}^{(1)} - \Delta$, and if the other bidder has an expected valuation such that $\hat{v}^{(2)} + \Delta - 2c < \hat{v}^{(1)} - \Delta$ and $\hat{v}^{(2)} + \Delta > \hat{v}^{(1)} - \Delta$. In that case, if the second bidder has a true valuation of $v^{(2)} = \hat{v}^{(2)} + \Delta$, the resulting allocation will be inefficient. Formally, this inefficiency is calculated to give:

$$\int_0^1 2F(\hat{v}^{(1)})f(\hat{v}^{(1)}) \int_{\hat{v}^{(1)}-2\Delta}^{\hat{v}^{(1)}-2(\Delta-c)} \frac{1}{4}[2\Delta - (\hat{v}^{(1)} - \hat{v}^{(2)})]f(\hat{v}^{(2)})d\hat{v}^{(2)}d\hat{v}^{(1)} = \frac{1}{2}c^2$$

Thus the overall efficiency loss is equal to $c(1 + \Delta - \frac{1}{2}c)$.

Comparing these three cases, it turns out that if c is small compared to Δ acquiring information leads to a higher efficiency. Because in the descending auction not all bidders will acquire information, the efficiency loss due to inefficient information acquisition is reduced.

If the number of bidders (n) increases, the efficiency loss in case (i) converges to Δ, as the bidder with the highest expected valuation $\hat{v}^{(1)}$ has a true valuation of $\hat{v}^{(1)} - \Delta$ with probability $\frac{1}{2}$. If there are very many bidders, the probability that a bidder with a slightly lower expected valuation than $\hat{v}^{(1)}$ has a true valuation of $\hat{v}^{(2)} + \Delta$ converges toward one. In case (ii), as all bidders acquire information, the inefficiency is equal to nc. In the descending auction, an inefficiency arises if the bidder with the largest highest expected valuation learns that her true valuation is equal to $\hat{v} - \Delta$. In the limit of infinitely many bidders there will be no efficiency loss due to misallocation, but with probability $\frac{1}{2}$ more than one bidder acquires information. The probability that exactly two (three, four, ...) bidders acquire information is given by $(\frac{1}{2})^2$ $((\frac{1}{2})^3$, $\frac{1}{2})^4$, ...). So in the limit the overall expected costs of information acquisition are given by

$$c\sum_1^{\infty}(\frac{1}{2})^n n = c\frac{1}{2}/(1 - \frac{1}{2})^2 = 2c$$

Revenue:

The key to understanding the following (numerical) results with respect to the revenue in the different scenarios is the observation that if all bidders were to acquire information before the auction, then the resulting type distribution will be a mean-preserving spread of the original distribution. Such a mean preserving spread has two consequences: First, as the distribution is less concentrated, bidders will compete less intensely, i.e. the bidding function has for every given valuation a lower value in an auction with full information compared to an auction without information acquisition. Second, a mean preserving spread leads to a shift of probability mass to the extremes which makes it more likely that a bidder has a higher valuation. If there is only a small number of bidders, then the first effect dominates. If the number of bidders is large, however, the latter effect is stronger, thus leading to an increase in revenue. Now, in a descending auction where all types acquire information (if the clock price is sufficiently low and the auction has not yet ended), the resulting consequence for the distribution is a mean preserving spread plus an additional shift in mass to lower values, because types who learn that they have a higher true valuation do not behave as a $\hat{v} + \Delta$ type, but as a $\hat{v} + \Delta - 2c$ type (see Figure 1). As this weakens both effects just mentioned, a general revenue ranking is not possible. In Table 1 below numerical calculations for the three scenarios are shown.

The revenue is calculated as follows (see Krishna, 2002, p.21): With a bidding function of $\beta(v)$, the expected payment per type is given by $H(v)\beta(v)$. The expected revenue is given by $nE[H(v)\beta(v)]$. With the expression of β given in (9), the expected revenue in the descending auction can be calculated to give:

$$n \int_{-\Delta}^{1+\Delta-2c} (1 - H(v))vf(v)^{n-1}dv$$

In case (i) and (ii), the function $H(v)$ and the boundaries of integral have to be adapted accordingly. If no one acquires information (scenario (i)), this expression is equal to $\frac{n-1}{n+1}$. In the other two cases, this expression has to be solved numerically.

The table shows that for a small number of bidders, not acquiring information leads to a higher revenue, in particular if Δ is large. However, for $n = 5$ and Δ small, the revenue is larger if information is acquired.

n	Δ	c	(i) FPSB (without info)	(ii) FPSB (with info)	(iii) Descending auction
2	0.05	0.0005	0.333	0.331	0.330
2	0.1	0.005	0.333	0.324	0.320
2	0.1	0.001	0.333	0.324	0.323
2	0.2	0.009	0.333	0.299	0.292
2	0.4	0.03	0.333	0.216	0.200
5	0.05	0.0005	0.667	0.822	0.821
5	0.1	0.005	0.667	0.790	0.788
5	0.1	0.001	0.667	0.790	0.783
5	0.2	0.009	0.667	0.671	0.669
5	0.4	0.03	0.667	0.283	0.323

Table 1: Expected revenue

4 Comparing the descending with the ascending auction

In this section the two standard auction formats, the ascending and the descending auction, are compared with respect to the incentive to acquire information during the respective auctions. First, however, some observations on the ascending auction are made.

In an ascending auction, bidders will stay in the auction until the clock price has reached their expected valuation. It is quite obvious that bidders with low expected valuation have less incentive to acquire information as the probability that the others have an expected valuation even larger is quite high. On the other hand, high valuation types might be interested in acquiring information once the clock price is sufficiently high.[7] Details are given in the Appendix.

A general comparison between the two auction formats descending and ascending auction is not possible. However, the reasoning behind observation 1 and the preceding paragraph shows that systematic differences exist. In the descending auction the bidders with low expected valuation have a higher

[7] In Compte and Jehiel (2007) bidders have a larger incentive to acquire information in an ascending auction compared to an second price sealed bid (SPSB) auction as they can wait and see whether the number of bidders drops to two. In the Appendix it is shown that even with two bidders to start with the incentives are larger in an ascending auction than in an SPSB auction.

incentive to acquire information while in the ascending auction the bidders with high expected valuation have the larger incentive. This difference allows us to provide an example where information will only be acquired in a descending auction but not in an ascending auction.

For this we model the distribution of types such that even for bidders with a relatively high valuation there is always a chance that someone else has a larger expected valuation. In our opinion this is a realistic assumption, as in a real auctions no one can ever be sure that she has the highest valuation. Formally, it is assumed that there are two bidders with expected valuations $\hat{v}_i$, $i \in \{1, 2\}$, independently distributed on $[0, \infty)$ with distribution $F(\hat{v})$. The distribution function has the following property: Conditional on the expected valuation being larger than $\hat{v}$, the probability that the expected valuation is smaller than $\hat{v} + 2\Delta$ is equal to or smaller than λ, with $\lambda < \frac{1}{2}$. Formally:

$$\forall \hat{v} \in [0, \infty) \qquad \frac{F(\hat{v} + 2\Delta) - F(\hat{v})}{1 - F(\hat{v})} \leq \lambda \tag{10}$$

An example for a distribution function which satisfies this property is $F(\hat{v}) = 1 - e^{-\alpha\hat{v}}$ with $\alpha = \frac{-ln(\lambda)}{2\Delta}$.

We again consider the (special) case of section 2, where by acquiring information a bidder learns whether her true valuation is either $v = \hat{v} - \Delta$ or $v = \hat{v} + \Delta$, each with probability $\frac{1}{2}$.

Proposition 2

If the distribution satisfies the property of equation (10), then if $\lambda\Delta < c < \frac{1}{2}\Delta$, information will be acquired with positive probability in a descending auction, but not in an ascending auction.

Proof: For the descending auction, consider the incentives to acquire information for the lowest type, i.e. $\hat{v} = 0$. At $p = \epsilon$ with ϵ small, this type would by acquiring information learn with probability $\frac{1}{2}$ that her true valuation is Δ. In that case she will end the auction immediately by accepting the clock price. In the limit of $\epsilon \to 0$, the value of information is thus $\frac{1}{2}\Delta$, which proves the claim for the descending auction.

Next consider the incentive to acquire information in the ascending auction. Suppose that only two bidders are active[8] and that the other bidder does not acquire information. It is intuitively clear (and discussed in more detail in the Appendix) that if a bidder decides to acquire information, she will do so when the auction has reached some clock price p between $\hat{v}-\Delta$ and $\hat{v}$. Call this clock price x. The bidder will then exit the auction if she learns that her true valuation is $v = \hat{v} - \Delta$. She will remain in the auction until the clock price reaches $\hat{v} + \Delta$ if she learns that her true valuation is $v = \hat{v} + \Delta$. Thus the expected profit when acquiring information (and assuming that the other bidder does not acquire information) is:

$$\pi_a^i(\hat{v}|p = x) = \frac{1}{2}\frac{F(\hat{v} + \Delta) - F(x)}{1 - F(x)}(\hat{v} + \Delta - E[\hat{v}_{-i}|x \leq \hat{v}_{-i} \leq \hat{v} + \Delta]) \quad (11)$$

The fraction is the probability that the bidder will win the auction, which is equal to the probability that the other bidder has an expected valuation smaller than $\hat{v} + \Delta$ (given that her expected valuation is larger than x). The term in brackets is the expected profit in case of winning. The whole expression is multiplied by $\frac{1}{2}$ which is the probability of having a high true valuation (i.e. $v = \hat{v} + \Delta$). With the assumption in (10), and by using that $E[\hat{v}_{-i}|x \leq \hat{v}_{-i} \leq \hat{v}+\Delta] \geq x \geq \hat{v} - \Delta$, an upper bound for the expected profit can be obtained:

$$\pi_a^i(\hat{v}|p = x) \leq \lambda\Delta \quad (12)$$

Thus the value of information, which is obtained by subtracting the expected profit if no information is acquired from $\pi_a^i(v|p = x)$ is also smaller than $\lambda\Delta$. Therefore, in the ascending auction no information will be acquired if $c \geq \lambda\Delta$. □

[8]By showing that a bidder has no incentive to acquire information if only two bidders remain, she will have even less incentive to acquire information if more bidders are still active.

5 Conclusion

If bidders care only about the price, a descending auction and a first-price-sealed-bid auction are strategically equivalent, thus delivering the same winners and prices in equilibrium. If however bidders can acquire information during the auction, the descending auction might induce more bidders to acquire information, as bidders can condition their decision whether to buy information or not on the price level reached. In particular, lower valuation types might wait until the clock price is sufficiently low (and the auction has not ended yet) before they acquire information.[9]

While in the descending auction the bidders with low valuation have an incentive to acquire information once the clock price is sufficiently low and the auction has not stopped before, in an ascending auction the bidders with a high valuation have an incentive to acquire information if the clock price is sufficiently high and the auction has not stopped before. An example is given where only in the descending auction information will be acquired with positive probability.

From an auction design perspective the descending auction has with respect to information acquisition one advantage compared to all other auction formats. By waiting until the clock price is sufficiently low, bidders are assured that in case the new information turns out to be positive, they can win the auction with probability one. I.e. information pays out directly. In any sealed bid auction and also in the ascending auction (at least under real circumstances where no one can be sure that she has the highest valuation) there is always a chance that other bidders have a higher valuation and thus bid larger prices.

[9] As pointed out to me by Johannes Münster, a similar argument could be made if bidders have to pay a fee in case they make a bid, i.e. in case they accept the clock price (rather than paying a fee for participating in the auction).

6 Appendix

6.1 Analysis of boundary types in an FPSB auction

We show that boundary types have less incentive to acquire information in an FPSB auction. The expected profit of a bidder with expected valuation $\hat{v} \in [1-\Delta, 1]$ who acquires information is given by:

$$\pi^i_{fp}(v) = \frac{1}{2}F(\hat{v}-\Delta)(\hat{v}-\Delta-\beta(\hat{v}-\Delta))+\frac{1}{2}(\hat{v}+\Delta-\frac{1}{2}) = \frac{1}{4}(\hat{v}-\Delta)^2+\frac{1}{2}(\hat{v}+\Delta-\frac{1}{2})$$

The value of information is thus

$$\frac{1}{4}(\hat{v}-\Delta)^2+\frac{1}{2}(\hat{v}+\Delta-\frac{1}{2})-\frac{1}{2}\hat{v}^2$$

which after some calculations is equal to $\frac{1}{2}\Delta^2 - \frac{1}{4}[(1-\hat{v})^2+\Delta^2]$ which is smaller than $\frac{1}{2}\Delta^2$.

The expected profit of a bidder with expected valuation $\hat{v} \in [0, \Delta]$ who acquires information is given by:

$$\pi^i_{fp}(\hat{v}) = \frac{1}{2}0+\frac{1}{2}F(\hat{v}+\Delta)(\hat{v}+\Delta-\beta(\hat{v}+\Delta)) = \frac{1}{4}(\hat{v}+\Delta)^2$$

The value of information is thus

$$\frac{1}{4}(\hat{v}+\Delta)^2-\frac{1}{2}\hat{v}^2 = \frac{1}{2}\Delta^2-\frac{1}{4}(\Delta-\hat{v})^2$$

which is also smaller than $\frac{1}{2}\Delta^2$.

6.2 Comparing the ascending auction with an SPSB auction

We consider the scenario described in section 2, i.e. a bidder can by incurring costs of c learn whether her true valuation is either $v = \hat{v}-\Delta$ or $v = \hat{v}+\Delta$.

The following observation, which will be proven below, shows that even with two bidders the incentive to acquire information can be larger in an ascending auction than in a second-price-sealed-bid (SPSB) auction.

Observation 2

In case of two bidders with uniformly distributed valuations, if $\frac{1}{2}\Delta > c > \frac{1}{2}\Delta^2$*, information will not be acquired in a second-price-sealed-bid auction, while with positive probability it will be acquired in an ascending auction.*

Proof: In an SPSB auction it is a dominant strategy to bid one's own valuation. The expected profit of a bidder with expected valuation $\hat{v}$ is given by $\hat{v} \times (\hat{v} - \frac{1}{2}\hat{v})$, where $\hat{v}$ is the probability of winning, and $\frac{1}{2}\hat{v}$ is the expected price, i.e. the expected bid of the other bidder, conditional on winning. Acquiring information leads for $\hat{v} \in [\Delta, 1-\Delta]$ to an expected profit of $\frac{1}{4}(\hat{v}+\Delta)^2 + \frac{1}{4}(\hat{v}-\Delta)^2$, so that the value of information is equal to $\frac{1}{2}\Delta^2$.[10] This shows the first part of observation 2.

Next consider the ascending auction, where a bidder with expected valuation $\hat{v} \in [0, 1-\Delta]$ expects the other bidder not to acquire information. In case the clock price has reached $\hat{v}$, if she does not acquire information she will exit the auction at this stage, thus her profit would be zero. If she acquires information instead, her expected profit net of information costs is given by

$$\pi_a^i(\hat{v}|p = \hat{v}) = \frac{1}{2}\frac{\Delta}{1-\hat{v}}\frac{1}{2}\Delta$$

where the first term is the probability of having valuation $v = \hat{v} + \Delta$, the second term is the probability of winning the auction given that the clock price has already reached $p = \hat{v}$, and the third term is the expected profit in case of winning, i.e. $\hat{v} + \Delta - E[\hat{v}_{-i}|\hat{v} \leq \hat{v}_{-i} < \hat{v} + \Delta]$. It follows that those with the largest valuation have the highest incentive to acquire information. In case $\hat{v} \in [1-\Delta, 1]$ the calculation has to be modified, however, as such a bidder who learns that her true valuation is $v = \hat{v} + \Delta$ will win the auction

[10] Bidders with expected valuation below Δ or above $1-\Delta$ have even less incentive to acquire information.

for sure. Her expected profit if she acquires information is given by

$$\pi_a^i(\hat{v}|p=\hat{v}) = \frac{1}{2}(\hat{v}+\Delta-E[\hat{v}_{-i}|\hat{v} \leq \hat{v}_{-i} \leq 1]) = \frac{1}{2}(\hat{v}+\Delta-\frac{1}{2}(1+\hat{v})) = \frac{1}{4}(\hat{v}+2\Delta-1)$$

This term is largest for $\hat{v} = 1$, from which the claim in observation 2 follows. □

In the proof it was assumed that if a bidder intends to acquire information she will do so once the clock price has reached her expected valuation, i.e. $p = \hat{v}$. It can be shown that this is only optimal if the valuation is very large. Bidder with smaller expected valuation $\hat{v}$ who acquire information will do so at a clock price larger than $\hat{v} - \Delta$ but smaller than $\hat{v}$. The exit option, i.e. to exit the auction if valuation turns out to be low, is also of value. Thus there is an incentive to acquire information before the expected valuation is reached. This is different to the result in Compte and Jehiel (2007): They show that if more than two bidders compete, a bidder might wait beyond $p = \hat{v}$ and only acquire information if the number of active bidders is reduced to two.

6.3 Determination of the types which acquire information

Here we provide a condition to determine the types in a descending auction who acquire information in equilibrium. This condition is obtained by calculating the value of information of a bidder when the clock price reaches $\beta(\hat{v} + x^*)$. Formally, this is given by the expression in brackets in (6), minus the expected profit in case the bidder does not acquire information. This gives:

$$\begin{aligned}&(1 - L(x^*))\,[\hat{v} + E[x|x \geq x^*] - \beta(\hat{v} + x^*)] + \int_{-\Delta}^{x^*} \tfrac{H(\hat{v}+x)}{H(\hat{v}+x^*)}(\hat{v} + x - \beta(\hat{v} + x))dL(x)\\ &-c - \tfrac{H(\hat{v})}{H(\hat{v}+x^*)}(\hat{v} - \beta(\hat{v}))\end{aligned} \quad (13)$$

By using the expression of $\beta(v)$ generalized from equation (9) and the definition of x^*, the value of information can be calculated to be equal to:

$$(1-L(x^*))\int_{-\Delta}^{\hat{v}+x^*}\frac{H(s)}{H(\hat{v}+x^*)}ds+\int_{-\Delta}^{x^*}\frac{H(\hat{v}+x)}{H(\hat{v}+x^*)}\int_{-\Delta}^{\hat{v}+x}\frac{H(s)}{H(\hat{v}+x^*)}dsdL(x)-\frac{H(\hat{v})}{H(\hat{v}+x^*)}\int_{-\Delta}^{\hat{v}}\frac{H(s)}{H(\hat{v})}ds$$

which after further calculations gives:

$$\frac{1}{H(\hat{v}+x^*)}\left[(1-L(x^*))\int_{\hat{v}}^{\hat{v}+x^*}H(s)ds-\int_{-\Delta}^{x^*}\left(\int_{\hat{v}+x}^{\hat{v}}H(s)ds\right)dL(x)\right] \quad (14)$$

A bidder with valuation $\hat{v}$ will (will not) acquire information if expression (14) is positive (respectively negative).

In the example used in the analysis with $x \in \{-\Delta, \Delta\}$ this expression simplifies to give:

$$\frac{1}{H(\hat{v}+x^*)}\left[\frac{1}{2}\int_{\hat{v}}^{\hat{v}+\Delta}H(s)ds-\frac{1}{2}\int_{\hat{v}-\Delta}^{\hat{v}}H(s)ds\right] \quad (15)$$

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