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## **LEARNING AND MICROLENDING**

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## ABSTRACT

### Learning and Microlending\*

For many self-employed poor in the developing world, entrepreneurship involves experimenting with new technologies and learning about oneself. This paper explores the (positive and normative) implications of learning for the practice of lending to the poor. The optimal lending contract rationalizes several common aspects of microlending schemes, such as "mandatory saving requirements", "progressive lending" and "group funds". Joint liability contracts are, however, not necessarily optimal. Among the poorest borrowers the model predicts excessively high retention rates, the contemporaneous holding of borrowing and savings at unfavorable interest rates as well as the failure to undertake profitable and easily available investment opportunities, such as accepting larger loans to scale-up business. Further testable predictions can be used to interpret and guide the design of controlled field experiments to evaluate microlending schemes.

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*“Each of us has much more hidden inside us than we have had a chance to explore. Unless we create an environment that enables us to discover the limits of our potential, we will never know what we have inside of us.”*

Muhammad Yunus, Founder of Grameen’s Bank

## 1 Introduction

For many self-employed poor in the developing world, entrepreneurship involves experimenting with new technologies and learning about oneself. Microfinance practitioners, for instance, emphasize the provision of credit to start small businesses as a way of “transforming people’s minds” and empowering clients through “self-confidence enhancement”. Similarly, it is believed that giving access to small, uncollateralized, loans, allows poor women to acquire experience in “non-traditional” roles. In order for these statements to make sense, however, prospective borrowers must be uncertain about their abilities before trying out a new venture.<sup>1</sup>

This paper explores the (positive and normative) implications of learning about oneself for the practice of lending to the poor. In doing so, it explains several aspects of the behavior of micro-entrepreneurs that are still poorly understood, and rationalizes several common, yet overlooked, aspects of microlending schemes such as mandatory saving requirements, “stepped” lending and “group funds”.<sup>2</sup>

More specifically, we embed a simple experimentation problem into a two-period lending relationship with moral hazard. A prospective borrower, who has little or no wealth and is protected by limited liability, privately learns her “natural” predisposition towards entrepreneurship (effort costs) after beginning a project in period one. Because of moral hazard, some rents are required to induce the agent to successfully complete the project. These rents, in turn, give

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<sup>1</sup>Borrowers uncertainty over their own abilities is consistent with evidence reported in Ross and Savanti (2005), Hashemi (2007) and Karlan and Valdivia (2006).

<sup>2</sup>CGAP defines *Mandatory* (or *Compulsory*) *Savings* as “Savings payments that are required as part of loan terms or as a requirement for membership”, specifying that “The amount, timing, and level of access to these deposits are determined by the policies of the institution rather than by the client”. The ACCION network, defines *Stepped* (or *Progressive*) *lending* as “the process by which borrowers who repay loans on time are eligible for increasingly larger loans”.

a reason to the agent with high effort costs to seek further funding in period two even when this is socially inefficient.

It is useful to distinguish borrowers with respect to their initial confidence (ex-ante prior about being suited for entrepreneurship). Individuals with low levels of confidence are credit constrained. If they had enough personal wealth, they would start a project on their own, but are unable to obtain funds from lenders. For individual with higher confidence, however, the following contract achieves the first best allocation. It specifies that the first period loan is repaid in two installments, in period one and period two. The contract also requires the borrower to save a pre-specified amount of money. The borrower can default on her loan obligations but, if she does so, she loses her savings and will not be able to borrow in the future.

When initial borrower's confidence is lowest among those who can borrow, further loans are awarded only to borrowers that signal trustworthiness by holding savings balances in excess of mandatory ones. These borrowers, therefore, hold contemporaneous borrowing and saving balances at unfavorable interest rates. The reason why the interest rate on borrowing is higher than the interest rate paid on savings is simple: the clients who borrow in both periods, cross-subsidize the experimentation, learning and savings of those borrowers that drop-out.

Since the borrower is initially unsure about her own abilities, there is a natural tendency to "start small", in order to economize on the costs of learning and experimenting. Conditional on continuing borrowing, loan size will increase. The model therefore provides a natural framework to study "stepped" lending. The promise of larger loans in the future, however, has a double effect. On the one hand, it disciplines borrowers in the current period; on the other hand, it makes it more difficult to screen out borrowers with high effort costs. For these reasons, the second period loan can be lower than its optimal size, implying credit constraints and limited scaling-up. When this happens, moreover, there is over-investment in period one. In this case, the optimal contract earns more money by "exploiting" the client's eagerness to repay to keep her savings. A direct implication of this result is that, for borrowers with intermediate levels of initial confidence, there always exists a larger first period loan at the same interest rate that gives higher first period utility and consumption to the borrower, gives non-negative profits to the lender and, yet, is rejected by the client.<sup>3</sup>

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<sup>3</sup>Banerjee and Duflo (2007) and Ross and Savanti (2005), among others, have documented how, often, the poor reject the offer of larger loans and do not expand their business.

The model can be extended to consider a group of borrowers that learn each other's type. We show that, while group lending is always beneficial, joint liability is not necessarily optimal. The optimal contract rationalizes other aspects of group lending, such as "group funds" and "group savings".<sup>4</sup>

A simple extension introduces a distinction between effort costs, that make it harder for the borrower to successfully complete the project, and psychological (or emotional) costs, that make self-employment unattractive for certain borrowers. The optimal contract, then, induces excessively high retention rates, especially among the poorest clients.<sup>5</sup> In order to save on the rents necessary to screen out those borrowers that bear the highest emotional costs from the project, the second best contract fails to screen out the intermediate types; i.e., those clients that have low emotional costs but high effort costs. The model, therefore, emphasizes the possibility that a significant proportion of microfinance clients will (and *should*) not run larger business.

This paper is related to several strands in the literature. First of all, it relates to the theoretical literature in microfinance (e.g., Morduch (1999), Ghatak and Guinnane (1999) and Rai and Sjöström (2004)). We show that a simple framework that emphasizes initial uncertainty over borrowers type allows to think about *multiple contractual aspects* of microlending schemes at the same time. In contrast, most of the theoretical literature on microcredit has focussed on explaining the (apparent) success of joint liability contracts, neglecting the fact that joint liability is neither the most common elements of microcredit contracts nor has been shown to be the most critical element for success.<sup>6</sup> This paper, instead, shifts attention to the dynamic aspects of microlending schemes - saving requirements and stepped lending - and on how those contractual elements *interact* with joint liability. The dynamic elements of microlending schemes have received relatively little theoretical attention in the literature. However, a recent paper by Ghosh and Van Tassel (2008) analyzes a two-period model with moral hazard in which first period outcomes are used to create collateral for the sec-

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<sup>4</sup>While joint liability contracts have received most of the attention in the literature, mandatory saving requirements, stepped lending and group funds, alongside with frequent repayment schedules and lending to women, appear to be much more common in practice (see, e.g., Morduch (1999), Armendáriz de Aghion and Morduch (2000), Hermes and Lensink (2007), and Dowla and Alamgir (2003)).

<sup>5</sup>Banerjee and Duflo (2008) and de Mel, McKenzie, and Woodruff (2008) discuss evidence consistent with these findings.

<sup>6</sup>In this respect, we share a similar motivational background with Baland and Somanathan (2008) and Fischer (2008).

ond period. Their paper shares with our paper the cross-subsidization between periods that is central to some of our results, but does not focus on learning and adverse selection, nor discusses the resulting implications for stepped and group lending.<sup>7</sup>

While we apply our model to the study of lending to the small businesses of the poor in the developing world, the framework can be applied to understand financing in other contexts. In particular, the optimal contract looks similar to common contractual practices in venture capital, such as “stage financing”, “staging with milestones”, as well as “breach of contract” and “liquidation” fees.<sup>8</sup>

The remaining of the paper is organized as follows. Section 2 presents the model, derives the optimal behavior of a self-financing agent, characterizes investment behavior and consumption paths. Section 3 derives indirect mechanisms that achieve first best. Section 4 considers loans of variable size. Group lending is analyzed in Section 5. Section 6 presents an extension with three types, while Section 7 focuses on the main implications of limited commitment on the borrower’s side. Section 8 discusses how to test the model empirically as well as its implications for the interpretation and design of controlled field experiment on microlending schemes. Finally, Section 9 offers some concluding remarks. The proofs are in the Appendix.

## 2 Model

### 2.1 Set Up

There is an agent that lives for two periods,  $\tau = 1, 2$ . There is a discount rate  $\delta \in [0, \infty)$  across the two periods. In each period the agent has the opportunity to undertake a project that needs an initial capital investment of 1 and yields return  $r$  when completed. A project that is not completed fails and yields 0.

The agent has no assets, is protected by limited liability, and needs to borrow 1 unit of capital in order to start the project. The credit market is competitive, that is, lenders make zero profits in expectation.

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<sup>7</sup>We share our emphasis on experimentation and learning with Giné and Klonner (2007). The two papers, however, differ substantially in focus, modeling approach and application. Other dynamic models of microlending are Armendáriz de Aghion and Morduch (2000), Alexander-Tedeschi (2006) and Jain and Mansuri (2003).

<sup>8</sup>See, e.g., Neher (1999), Bergemann and Hege (1998), Bergemann and Hege (2005), Qian and Xu (1998), Chan, Siegel, and Thakor (1990), and Manso (2007).

To complete the project the agent needs to appropriately invest the unit of capital and to exert effort. The agent can divert a share  $\psi \leq 1$  of the initial investment for private consumption. If she does so, the project fails. The parameter  $\psi$  reflects the difficulty in monitoring investments by the lender.

With respect to effort, there are two types of agents: good agents  $G$  and bad agents  $B$ . The cost of effort for a good agent is  $e_G = 0$ , and is  $e_B = e > 0$  if the agent is bad. Heterogeneity in the cost of effort across agents captures differences in the “natural” predisposition towards entrepreneurship as well as in the opportunity cost of time subtracted from non-market activities (e.g. taking good care of children and other relatives, collecting wood and water for the household, etc...).

Initially both the agent and the lenders are uninformed about the type of the agent and have a common prior about the probability of the agent being a good type  $\rho$ . The agent privately learns her type upon starting the project in period 1. After having learned her type, she decides whether to exert effort and whether to divert the capital.

Whenever effort is exerted and investment is not diverted, the project succeeds and yields  $r$ . We assume that the output is contractible: when the project succeeds, the agent repays loans out of revenue  $r$ . If the agent does not borrow, she takes her outside option  $u > 0$ .

We make the following parametric assumptions:

**Assumption 1**  $\max\{1, e\} < r - \psi$ .

**Assumption 2**  $r - 1 < u + e$ .

**Assumption 3**  $u < \psi$ .

The first Assumption has two implications. First,  $r - 1 > \psi$  implies that the project generates enough revenues to solve moral hazard in investment by the good type. Second,  $r > \psi + e$  implies that, once the project is started and the initial outlay of 1 unit of capital is sunk, it is optimal to continue with the project regardless of the agent’s type.

The second Assumption implies that it is not optimal to invest if the agent is (known to be) bad: the opportunity costs of investment  $1 + u$  is higher than revenues  $r$ , net of effort costs  $e$ .

Finally, since the agent can always divert funds and keep  $\psi$ , the third Assumption implies that an agent always prefers to borrow instead of taking her outside option  $u$ .

The timing of events is postponed until Section 2.3.

## 2.2 Optimal Experimentation by a Self-Financed Agent

Let us first consider the benchmark case in which the agent has enough wealth so that she does not need to borrow. In this case the agent is the residual claimant of the project: there are no incentive problems and therefore the *first best* investment plan is chosen.

Once she has started the project in period 1, the agent exerts effort and completes the project regardless of her type, since  $r > \psi + e$ . In period 2 she invests and completes the project again only if she has learned that she is the good type. Otherwise, if she has learned that she is the bad type, she prefers to take her outside option, since  $u > r - 1 - e$ . Therefore, the first best allocation is the following. Conditional on starting the project in period 1, the agent completes the project regardless of her type. In period 2, she undertakes and completes a project only if she has learned she is the good type. Otherwise, she takes her outside option.

Investment in period 1 can be thought of as *experimentation*: its costs are borne in period 1 while part of the benefits are realized in period 2. After the agent has learned her type she will be able to make an informed decision (i.e., there is a positive value of information). The costs of experimentation,  $C_1$ , are given by the difference between the opportunity cost  $u$  and the expected surplus created by the project in period 1, i.e.,  $(r - 1) - (1 - \rho)e$ . The benefits of experimentation, instead, are given by the value of better decision-making in period 2. With probability  $\rho$ , the information gathered through experimentation leads the agent to start a project, instead of taking the outside option. With probability  $(1 - \rho)$ , instead, the agent learns she is a bad type and takes her outside option. In this case, the information gathered through experimentation does not change her decision. The value of information is therefore given by  $I = \rho(r - 1 - u)$ . Experimentation is optimal if its costs are lower than its benefits, i.e., if  $-C_1 + \delta I \geq 0$ . Rearranging terms gives the following Lemma.

**Lemma 1** *If the agent does not need to borrow, experimentation (investment in*

period 1) is optimal if and only if  $\delta \geq \delta_E$ , where

$$\delta_E \equiv \frac{u + e(1 - \rho) - (r - 1)}{\rho(r - 1 - u)}. \quad (1)$$

As in standard experimentation problems, starting the project in period 1 becomes profitable if the future is sufficiently important, i.e., if the discount factor is high enough. This Lemma also shows that the agent starts the project in period 1 if she is sufficiently confident about being a good type (high  $\rho$ ), if the opportunity costs are not too high (low  $u$ ) and if the project yields high returns (high  $r - 1$ ).

### 2.3 Contracts and Investment

We now turn to the case when the agent borrows from a competitive credit market to start a project. Even absent any incentive problem, starting a project in period 1 may not be the optimal choice, as we showed in Section 2.2. When this is the case, lending is not profitable either.

In this Section we derive optimal financial contracts that maximize the expected utility of the agent and guarantee non-negative expected profits to lenders, subject to the incentive compatibility constraints induced by moral hazard and adverse selection.

Lenders offer (and commit) to two-period contracts of the following form. A lender finances the project in period 1. Immediately after the agent learns her type, she sends a message about her type  $m \in \{Good, Bad\}$  to the lender.<sup>9</sup> According to the message, the lender gives her a pre-specified contract that spells out agent's actions in period 1, as well as a re-financing policy in period 2 and transfers to the agent in period 1 conditional on the outcome of the project in period 1. The contract also specifies transfers to the agent in period 2 which are conditional on output realizations in periods 1 and 2. We assume that in the beginning of period 2 the agent cannot change her lender, but cannot be forced into a relationship, i.e., she can always take her outside option  $u$ .

The timing of events and structure of the contract are summarized in Figure 1.

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<sup>9</sup>There is no loss in generality in restricting attention to messages about the type of the agent. Since lenders have full commitment power, Revelation Principle applies.

– INSERT FIGURE 1 HERE –

We say that an allocation can be *implemented* if there exist a two-period contract that gives appropriate incentives to the agent and satisfy the lender's zero-profit constraint.

The combination of incentive problems induced by private learning (adverse selection) as well as non-contractible investment and effort costs (moral hazard) imply that implementing first best is costly.

First, by Assumption 2, it is not optimal to lend to the bad type in period 2. The contract has to induce the bad type to take her outside option and the good type to invest in period 2. Since, however, rents equal to  $\psi > u$  are necessary to induce a good agent to complete the project in period 2, a bad agent always has incentives to seek financing in period 2 as well (adverse selection).

Second, in period 1 it is optimal to complete the project even when the type is bad. This is because, at that stage, the initial outlay of 1 unit of capital is sunk (Assumption 1). Inducing both types of agents to complete the project is, again, costly. It is necessary to compensate the bad agent for her effort costs  $e$  (moral hazard) and this gives to the good type an incentive to pretend to be a bad type.

The required rents might be so high that it may not be possible to implement the first best. Other than first best, we show in the Appendix that the only allocation that is implemented is the one in which the bad agent does not complete the project in period 1 and, consequently, does not get funds in period 2, while the good agent completes the project in each period. We name this allocation *second best*. Implementing the second best is appropriate when solving the moral hazard problem of the bad type in period 1 is too costly. This happens when  $\delta$  is relatively low. When  $\delta$  is relatively high, the second best is not cheaper to implement, since moral hazard in period 1 is already solved by the rents required to separate the two types in period 2.

The next Proposition characterizes the (constrained) optimal allocation, i.e., when implementing the first best and the second best is feasible.

**Proposition 1** *There are thresholds  $\bar{\delta}^{FB}(\rho)$ ,  $\underline{\delta}^{FB}(\rho)$  and  $\underline{\delta}^{SB}(\rho)$  such that:*

1. *If  $\delta \geq \delta_\psi = \frac{\psi+e}{\psi-u}$ , first best is implemented if  $\delta \leq \bar{\delta}^{FB} \equiv \frac{r-1}{(1-\rho)(\psi-u)-\rho(r-1-\psi)}$ ;*

2. If, instead,  $\delta < \delta_\psi$ , first best is implemented if  $\underline{\delta} \geq \underline{\delta}^{FB} \equiv \frac{\psi+e-(r-1)}{\rho(r-1-u)}$ , while second best is implemented if  $\delta \in [\underline{\delta}^{SB}, \underline{\delta}^{FB})$ ; and
3. no project is financed for other parameter values.

**Proof.** See Appendix A. ■

– INSERT FIGURE 2 HERE –

Figure 2 illustrates Proposition 1. Let us consider the cases in which first best can be implemented. When  $\delta \leq \delta_\psi$ , the binding agency problem is moral hazard in period 1. Denoting by  $T_G^*$  and  $T_B^*$  the discounted values of the minimal incentive compatible transfers, the optimal contract pays

$$T_B^* = \psi + e \quad \text{and} \quad T_G^* = \psi + e + \delta u \quad (2)$$

to the good and bad type, respectively. The logic for when experimentation is optimal is the same as in (1), with the difference that the cost of experimenting is higher by an amount equal to  $\Delta_{MH} = \psi - u + \rho e$ . This is because the contract needs to pay rents  $\psi + e$  to both types, which are in excess of the expected opportunity costs when moral hazard is not an issue, i.e.  $u + (1 - \rho)e$ . While the rents due to moral hazard increase the costs of experimentation, they do not change the nature of the trade-off involved.

When, instead,  $\delta \geq \delta_\psi$ , the binding agency problem is solving adverse selection in period 2. The discounted value of the minimal transfer is now given by

$$T_B^* = \delta(\psi - u) \quad \text{and} \quad T_G^* = \delta\psi \quad (3)$$

to the good and bad type, respectively. The logic of the trade-off involved in experimentation, however, is now reversed. In period 2, the lender needs to pay  $(\psi - u)$  to prevent the bad type from obtaining a project. If the profits generated by the good type,  $\rho(r - 1 - \psi)$ , are higher than  $(1 - \rho)(\psi - u)$ , the project in the second period generates enough surplus to separate the two types. When this is the case, implementing first best possible for any  $\delta$ . When, however,  $\rho(r - 1 - \psi) < (1 - \rho)(\psi - u)$ , the profits generated in period 1 are necessary to separate the two types in period 2. Those profits are given by  $r - 1$ , since  $\delta \geq \delta_\psi = \frac{\psi+e}{\psi-u}$  implies that no further transfer is required to solve for period 1 moral hazard. When  $\delta$  increases, however, from the perspective of period 2 the

value of those rents decreases, and experimentation becomes *more*, rather than less, costly.

An agent is *credit constrained* if she is unable to borrow to start a project that she would otherwise self-finance, had she enough money. Figure 2 shows that individuals with intermediate levels of initial confidence are credit constrained: for *any* discount factor  $\delta$ , there is always a range of initial levels of confidence in which agents are credit constrained.

The interplay of adverse selection and moral hazard constraints implies that access to credit in period 1 is non-monotonic in the discount factor  $\delta$  and outside option  $u$ . At relatively low levels of  $\delta$ , inducing repayment in period 1 is the relevant constraint. An increase in the discount factor makes the punishment from not repaying (exclusion from future borrowing) more severe and relaxes the relevant moral hazard constraint. This makes borrowing in period 1 easier. At relatively high levels of  $\delta$ , preventing the bad type from seeking funds in period 2 is the binding constraint. The rents that have to be given to solve this adverse selection problem,  $\delta(\psi - u)$ , are increasing in  $\delta$ . When  $\delta$  is high, therefore, borrowing becomes harder. When, instead,  $\rho(r - 1 - \psi) > (1 - \rho)(\psi - u)$ , there is enough surplus in period 2 to solve for the adverse selection of the bad type.

A similar role is played by the outside option. When the key agency problem to solve is moral hazard in period 1, i.e., when  $\delta$  is low, a higher outside option reduces the future costs of being denied access to credit in period 2, and therefore reduces the severity of the punishment available to lenders. This obviously makes lending more difficult. When the key agency problem to be solved is keeping the bad type out of the market in period 2, instead, a higher outside option reduces the attractiveness of seeking funds and therefore reduces the amount of rents that need to be paid to bad agents in period 2. This makes lending easier.<sup>10,11</sup>

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<sup>10</sup>The remaining comparative statics have the expected sign. An agent is more likely to obtain credit in period 1 the more profitable the project is (higher  $r - 1$ ) and the easier it is to monitor the appropriate use of funds (lower  $\psi$ ). The cost of effort  $e$  affects the likelihood of obtaining credit in period 1 only at intermediate levels of  $\delta$ , i.e., when inducing first period repayment from the bad type is the relevant constraint.

<sup>11</sup>When the agent has an initial amount of wealth  $w < 1$ , the model is equivalent to the one analyzed above in the case in which the initial investment required to start the project is equal to  $1 - w$ . Higher wealth, therefore, makes financing easier. Contractual arrangements for borrowers with higher  $w$  and a given confidence  $\rho$ , are equivalent to those in the baseline model for borrowers with higher confidence  $\rho$ .

### 3 Indirect Mechanism: Borrowing and Saving

#### 3.1 The Optimal Contract

Because of the linear structure of the model, each allocation can be implemented by (infinitely) many contracts. In order to pin down the exact contract, we introduce a refinement: we consider a small degree of risk aversion in the utility function of the agent.<sup>12</sup>

From an ex-ante perspective, since the agent is risk averse, the optimal contract minimizes the spread in expected utility across the two types, subject to the incentive compatibility constraints. Let us denote by  $F(\rho)$  the (expected) monetary profits generated by implementing the first best allocation, i.e.,  $F(\rho) = (r - 1) + \delta\rho(r - 1)$ , and by  $M(\rho)$  the monetary profits after paying minimal transfers, i.e.,  $M(\rho) = F(\rho) - \rho T_G^* - (1 - \rho)T_B^*$ , where  $T_G^*$  and  $T_B^*$  are given, depending on  $\delta$ , by 2 and 3. We then have

**Proposition 2** *The optimal contract achieves perfect consumption smoothing for the bad type, i.e.,  $c_1^B = \frac{F(\rho)+(1-\rho)\delta u}{1+\delta}$  and  $c_2^B = \frac{F(\rho)+(1-\rho)\delta u}{1+\delta} - u$ .*

*For the good type, the optimal contract achieves perfect consumption smoothing  $c_1^G = c_2^G = \frac{F(\rho)+(1-\rho)\delta u}{1+\delta}$  if  $M(\rho) \geq \psi$ . Otherwise, the optimal contract implies  $c_1^G = M(\rho) < c_2^G = \psi$ .*

**Proof.** See Appendix. ■

The optimal contract provides full consumption insurance to the borrower against “bad” realizations of her entrepreneurial talent. The contract also provides perfect consumption smoothing across the two periods for the bad type, but might fail to achieve perfect consumption smoothing for the good type. Because of the moral hazard rents that need to be paid on the period 2 project given to her, the good type achieves perfect consumption smoothing only if the ex-ante expected surplus generated by the venture is large enough,  $M(\rho) > \psi$ .

We can now turn our attention to the form of optimal contracts. A contract is an  $\mathcal{N}$ -tuple,  $C = \{D_1, D_2, B_2, S^C, S^V, i_s, i_b\}$ , defined as follows. The agent borrows 1 unit of capital at the beginning of period 1 at an interest rate  $1 + i_b$ .

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<sup>12</sup>In the limit case in which the utility function is close to risk neutrality, the solution found in the first step of the Proof of Lemma 2 is (approximately) correct. Qualitatively, results are unchanged for larger degrees of risk aversion.

The contract specifies that this loan is repaid in two installments,  $D_1$  and  $D_2$ , in periods 1 and 2 respectively. At the same time, the contract requires the agent to save an amount  $S^C$  on which the lender pays an interest rates  $1 + i_s$ . We assume that the borrower can default on  $D_1$  and/or  $D_2$ , but if she does so, she loses her savings. In other words, we assume that the lender can “force” a minimum amount of savings  $S^C$ . We focus on the contract  $C^*$  that minimizes enforcement requirements, i.e.,  $S^C = D_1 + \delta D_2$ .

In period 2, the borrower can apply for further funding. The contract specifies that she will obtain further funding if she has not defaulted on  $D_1$  and  $D_2$ , and if she has a saving balance at least equal to  $S^V$  to be pledged as collateral. If the agent seeks and obtains funding in period 2, she borrows 1 unit of capital and is expected to repay  $B_2$  on that loan. If she defaults, she loses her savings.

In sum, the agent borrows 1 unit of capital at the beginning of period 1 and learns her type. The optimal contract always induces investment and no default. If the agent is a good type, she will save  $S^V$  and obtain further funding in period 2. If, instead, she learns to be a bad type, she saves  $S^C$ . First period consumptions are given by

$$c_1^G = r - D_1 - S^V \text{ and } c_1^B = r - D_1 - S^C \quad (4)$$

for the good and bad type, respectively. Similarly, second period consumptions are given by

$$c_2^G = r - D_2 - B_2 + (1 + i_s)S^V \text{ and } c_2^B = (1 + i_s)S^C - D_2. \quad (5)$$

There is no loss in generality in restricting attention to contracts in which the lender offers “competitive” interest rates on savings, i.e.,  $1 + i_s = \frac{1}{\delta}$ . Moreover, the contract described above implicitly defines the borrowing interest rate as  $1 + i_b = \frac{D_2}{1 - D_1}$ .<sup>13</sup>

We distinguish two cases, depending on whether the optimal contract  $C^*$  achieves perfect consumption smoothing or not (see Proposition 2). The following proposition characterizes the optimal contract.

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<sup>13</sup>Interest rates are only defined across the two periods. After borrowing 1 unit of capital and repaying  $D_1$  at the end of the first period, the outstanding loan is equal to  $1 - D_1$  with associated repayment  $D_2$ .

**Proposition 3** (*If feasible*) *The contract  $C^*$  that implements first best is characterized by:*

$$C^* = \begin{bmatrix} S^{C^*} \\ D_1^* \\ D_2^* \\ B_2^* \end{bmatrix} = \begin{bmatrix} r - F(\rho) - \delta\rho u \\ \delta \left( \frac{F(\rho) - (1+\delta\rho)u}{1+\delta} \right) \\ S^{C^*} - \frac{D_1^*}{\delta} \\ r - u \end{bmatrix}$$

When perfect consumption smoothing is feasible, saving  $S^{C^*}$  guarantees access to second period loan, i.e.,  $S^{V^*} = S^{C^*}$ . When perfect consumption is not feasible, i.e., if  $M(\rho) < \psi$ , access to second period loan requires  $S^{V^*} = S^{C^*} + \delta \left( (\psi - u) - \frac{F(\rho) - (1+\delta\rho)u}{1+\delta} \right)$ .

In the optimal contract, compulsory savings  $S^{C^*}$  decrease with  $\rho$ , while first period installment  $D_1^*$  increases with  $\rho$ . Better clients are not requested to save too much, and repay an increasing fraction of their loans in period 1. Furthermore, it is possible to show that the interest rate on borrowing,  $1 + i_b^* = \frac{D_2^*}{1 - D_1^*}$ , decreases in  $\rho$  and is larger than the interest rate on savings,  $1 + i_b^* > 1 + i_s$ . This happens because, since the contract minimizes savings and the surplus generated by the bad type is smaller than the rents she is paid, expected zero profits for the lenders imply that the good type cross-subsidizes the bad type.

### 3.2 Interpretation

Savings requirements are a common feature of most microlending schemes. For instance, Grameen, BRAC and ASA (the three largest MFIs in Bangladesh) have collected compulsory regular savings from their clients from the very start of their programs (see, e.g., Dowla and Alamgir (2003)). All of the five major microfinance institutions described by Morduch (1999) use combinations of borrowing and savings, while only two of them use joint liability. In recent years, many MFIs have also started offering more flexible savings products (see, e.g., Ashraf, Karlan, Gons, and Yin (2003)).

In the model, savings accomplish three conceptually distinct roles. First, savings are used to achieve the desired levels of *consumption smoothing*. By saving part of the loan disbursed in the first period, the contract creates alternative sources of income in excess of  $u$  in period 2. Second, savings are used to provide incentives to repay first period loans (solving moral hazard). If the borrower does not repay her loan, she will not be able to access her savings in period 2. Finally,

savings are used to create collateral and act as signalling device to allocate loans to trustworthy borrowers in period 2 and maintain portfolio quality.

In practice, there is an important distinction between *mandatory* (or compulsory) vs. *voluntary* savings. The former are payments that are required for participation in the scheme and are part of loan terms. They are often required in place of collateral and the amount, timing, and access to these deposits are determined by the policies of the institution rather than by the client.<sup>14</sup> Voluntary savings, instead, are a more recent evolution, and have the objective of meeting individual clients' demand for tiny savings with deposits made at weekly meetings.<sup>15</sup>

The model highlights a key difference between compulsory and voluntary savings. When initial confidence and outside opportunities are sufficiently high, the optimal contract achieves perfect consumption smoothing and compulsory savings  $S^{C*}$  create enough collateral to solve the selection problem in the period 2. When this is not the case, however, higher savings  $S^{V*} > S^{C*}$  signal that the client should be trusted for a second period loan. It is natural to interpret those higher savings as voluntary: if they were mandatory they would not be a signal.

The optimal contract, therefore, induces contemporaneous borrowing and savings (in excess of mandatory ones) at unfavorable interest rates for some borrowers. This observation matches the evidence reported in Basu (2008).<sup>16</sup> Among clients of FINCA, mostly women who own and operate small informal businesses in the cities of Lima and Ayacucho, all borrowers are required to maintain a savings account on which the (risk adjusted) interest rate is lower than the interest rate on borrowing. A significant proportion of borrowers maintain savings that are above the required minimum. Furthermore, as predicted by the model, this behavior is most common in Ayacucho, where incomes are relatively low and access to credit is mostly limited to moneylenders who charge high

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<sup>14</sup>Clients may be allowed to withdraw at the end of the loan term; after a predetermined number of weeks, months or years; or when they terminate their memberships. Historically, those savings requirements were collected with the explicit view that the money would act as a de-facto lump sum ‘pension’ when a client leaves the organization.

<sup>15</sup>Small deposits during weekly meetings make it easier for the lender to enforce minimum savings requirements.

<sup>16</sup>Which is based on unpublished fieldwork by Dean Karlan.

interest rates.<sup>17,18</sup>

Different contracts can implement the desired consumption path, and the contract  $C^*$  might not be feasible. An alternative contractual structure simply pays an “exit”, or “liquidation”, fee to clients that do not undertake a project in period 2. In practice, such schemes might not be used by formal lenders because they are subject to gaming by “bogus” clients interested in collecting exit fees. Informal sources of insurance, however, de facto implement this type of schemes.

## 4 Variable Scale

### 4.1 Setup

This section extends the model assuming that the returns to the project  $r$  depend on the capital invested  $k$ . Specifically,  $r(k)$  is given by

$$r(k) = \begin{cases} f(k) & \text{if } k \geq \underline{k} \\ 0 & \text{otherwise} \end{cases}$$

where  $f(k)$  is increasing and concave. The technology of production therefore implies an initial non-convexity, so that a client cannot learn her type by starting an arbitrarily small project. As before, to complete the project the agent needs to appropriately invest capital and exert effort. The agent can now steal a share  $\psi$  of the investment  $k$ , that is,  $\psi k$ ; while the effort cost is equal to  $ek$  for the bad type and to zero for the good type. Denoting by  $k^*$  the level of capital that maximizes  $f(k) - k$ , i.e.,  $k^*$  as implicitly defined by  $f'(k^*) = 1$ , let us assume that  $k^* > \underline{k}$ .

We keep the same assumptions as in the previous Section. When applied to this context, the assumptions become:

**Assumption 4**  $k \cdot \max\{1, e\} < f(k) - \psi k$ , if  $k \in [\underline{k}, k^*]$ .

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<sup>17</sup>Banerjee and Mullainathan (2007) and Basu (2008) rationalizes the evidence building on time-inconsistent preferences. Baland, Guirkinger, and Mali (2007), instead, offer a non-behavioral explanations based on signalling. In contrast to these contributions, we provide a supply driven, rather than demand driven, explanation.

<sup>18</sup>The model fits other observed practices. Because of the requirement that members save (little amounts each week), longer membership is correlated with higher savings (Dowla and Alamgir (2003)). Moreover, collateral requirements are increasing for subsequent loans. For instance, in the case of BRAC, the program requires 5% of the disbursed amount for the first loan, 10% for the second, 15% for the third and 20% for the fourth and beyond.

**Assumption 5**  $f(k) - k < u + ek$  for all  $k \geq \underline{k}$ .

**Assumption 6**  $u < \psi \underline{k}$ .

Conditional on starting a project, the optimal investment level  $k_\tau^*$  chosen by a self-financed agent in period  $\tau \in \{1, 2\}$  are implicitly given by

$$f'(k_1^*) = 1 + (1 - \rho)e \text{ and } k_2^* = k^*.$$

It immediately follows that  $k_2^* > k_1^*$ : the model captures a natural tendency for the project to grow. Because of effort costs that are increasing in the amount of capital invested, the optimal investment path requires to “start small”, in order to economize on the learning costs, and then increase the project size once the agent is confident that she is a good type.

## 4.2 Constrained optimal choice of projects

We now turn to the determination of the optimal size profile when the agent has no wealth and is subject to the moral hazard and adverse selection problems described above. Competition among lenders assures that equilibrium contracts maximize the borrowers’ utility subject to the zero profit constraint for the lender (and all relevant incentive compatibility constraints). Since the structure of the parametric configurations under which it is possible to implement the first best investment path  $k_1^*$  and  $k_2^*$  is very similar to the one described in Proposition 1, when the size of projects was fixed, we relegate its formal presentation to the Appendix.

When the first best investment path cannot be financed, the optimal contract either induces a bad type not to invest in period 1 (as in Section 2) or it distorts the size of the project in the two periods, still inducing both types to invest in period 1. We focus on the latter case, in which the model delivers predictions for the size of the projects financed in the two periods. The next Proposition characterizes such a distortion.

**Proposition 4** *When first-best investment levels  $k_1^*$  and  $k_2^*$  cannot be financed and the contract induces investment from both types in period 1, there exists a threshold  $\tilde{\delta}_\psi(\rho)$  such that the optimal sizes of projects in periods 1 and 2,  $k_1$  and  $k_2$ , are given by:*

- $k_1 < k_1^*$  and  $k_2 = k_2^*$  if  $\delta < \tilde{\delta}_\psi$ ;

- $k_1 > k_1^*$  and  $k_2 < k_2^*$  if  $\delta > \tilde{\delta}_\psi$ .

The model captures two facets of the practice of “stepped” lending (i.e., the fact that solvent borrowers become eligible for larger loans as time goes by). On the one hand, the promise of larger loans in the future induces appropriate investment and repayment on current loans. On the other hand, the promise of larger loans in the future makes it harder for the lender to screen out bad types.<sup>19</sup>

When the discount factor  $\delta$  is sufficiently low, the contract has to provide rents to the borrower to exert effort in period 1, and repay the loan. In these circumstances it might be necessary to reduce the effort costs associated with the first period project by reducing the size of the project,  $k_1 < k_1^*$ . When this is the case, there is no need to distort the size of the project in period 2.

Conversely, when the discount factor  $\delta$  is sufficiently high, the contract needs to pay rents to the bad type to solve the adverse selection problem. Since these rents are increasing in the size of the project in period 2, it might be necessary to reduce the size of the project,  $k_2 < k_2^*$ .

More interestingly, in the latter case, the optimal contract implies that the first period loan is larger than the ex-ante optimal one,  $k_1 > k_1^*$ . In order to solve the adverse selection in period 2, the contract exploits the eagerness to repay of the bad type. This observation is in line with concerns about microlending schemes inducing excessive anxieties and emotional stress on clients (see, e.g., Rahman (1999)). From an ex-ante perspective, however, such contracts are aiming at creating as much surplus as possible in order to “subsidize” exploration and learning.

These observations have implications for interpreting the lack of growth in the businesses of microlending clients. First, the model directly implies scaling-up in project size is particularly limited. This will be especially true when outside opportunities are low, and the rents that are required to solve for period 2 adverse selection are high. Second, as shown in the next proposition, the model provides a natural lens to interpret why so often the poor fail to undertake more profitable and easily available investment opportunities.

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<sup>19</sup>See, e.g., Morduch (1999) for a discussion of these issues in the context of microlending schemes.

**Proposition 5** Denote  $E(c_1)$  the expected consumption in period 1 and  $i_b$  the interest rate on borrowing. There exist  $\rho'$ ,  $\delta'$  and another allocation  $(k'_1, k_2^*)$  such that given these  $\rho'$  and  $\delta'$ ,

- Both  $(k_1^*, k_2^*)$  and  $(k'_1, k_2^*)$  can be financed;
- $k'_1 > k_1^*$ ;
- $E(c'_1) > E(c_1^*)$ ;
- $i'_b = i_b^*$ .

In a context that is particularly related to our model, Ross and Savanti (2005) report that clients of microfinance institutions often refuse larger loans to scale-up their business (see also Rahman (1999)). They underline the role of the confidence acquired by clients with their first investments as a key determinant of the willingness to borrow a larger amount in the future. Proposition 5 says that if the optimal contract can implement the first best investment profile, but the borrower has sufficiently low initial confidence, there always exist contracts that offer larger loans (and consumption) in period 1 at the same interest rate, and, yet, are rejected by the borrower. Since, through the zero profit constraint, the borrower eventually ends up paying all the costs of learning her type, she might prefer to “start small” before “scaling-up” if she does not feel confident enough about the project.<sup>20</sup>

## 5 Group Lending

### 5.1 Set Up

Group lending and joint liability have been the focus of most theoretical literature on microlending schemes (see, e.g., Ghatak and Guinnane (1999) for an early review). This section extends the basic model to analyze the optimal contract offered to a group of borrowers.

Conceptually, group lending and joint liability are two distinct aspects of the contractual relationship between the MFI and the clients. Group lending simply

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<sup>20</sup>The result suggests that from the mere observation of contractual terms, it might be difficult to infer whether a borrower is turning down the offer of a bigger loan because of, say, time-inconsistent preferences, or whether she is simply choosing the loan size that best fits her expectations.

refers to the fact that most of the activities related to the administration of the loan are executed in groups. For instance, at the weekly meeting, clients repay installments, deposit saving requirements, discuss the accounting of the group, and so on. Joint liability refers, instead, to a particular contractual aspect: namely, if one of the members of the group defaults, the other members are jointly liable for her debt obligations.

Let consider the case in which the MFI lends to a group of two agents. The two agents are initially uninformed about their types, which can be arbitrarily correlated with each other. The model is as in Section 2, under the assumption that, upon starting a project, agents perfectly learn both their own and each other's type.

We look for the optimal mechanism that implements the first best allocation for both agents. In particular, we allow for *cross-reporting*: each agent reports both her type and the type of the other group member. After having learned their respective types, the two agents, however, can coordinate their messages. We initially consider the extreme case in which the two agents cannot transfer rents among themselves at the reporting stage, and relegate to the end of this section a discussion of the implications of relaxing this assumption.

Let  $T_G^*$  and  $T_B^*$  be the discounted value of the minimal transfers in the individual contract, in which each agent only reports about her type, as defined above. Similarly, let  $T_{ij}^*$  be the discounted value of the minimal transfer to type  $i \in \{G, B\}$  when the other group member is type  $j \in \{G, B\}$ . The following Proposition characterizes the optimal group lending contract.

**Proposition 6** *In the optimal group lending contract,  $T_{GG}^* = T_G^*$  and  $T_{BB}^* = T_B^*$ . If  $\delta \geq \delta_\psi$ , then  $T_{GB}^* = T_G^*$  and  $T_{BG}^* < T_{BB}^*$ . If, instead,  $\delta < \delta_\psi$ , then  $T_{BG}^* = T_B^*$  and  $T_{GB}^* < T_{GG}^*$ .*

**Proof.** See Appendix. ■

Similarly to other theoretical work in the area, group lending exploits the superior information that borrowers have about each other. Since, for at least one realization of borrowers' profiles, the transfers which are necessary to implement first best are reduced, group lending in the basic framework *always* expands access to credit. The optimal contract exploits "disagreement": the rents to be

paid to implement first best are smaller when the realizations of type for the two borrowers differ.<sup>21</sup>

## 5.2 Interpretation

The optimal transfers derived above have a very natural interpretation. When  $\delta < \delta_\psi$ , the payoff of a good type is lower when she is paired with a bad type. In this case, the optimal contract displays *joint liability*. When, instead,  $\delta > \delta_\psi$ , it is the bad type that receives lower transfers if her partner turns out to be a good type. In this case, the optimal contract displays a “group fund”, in which group savings are used to finance the project of the good type.

The model, therefore, shows that, while group lending is always beneficial, joint liability is not necessarily optimal. This is important in light of the evidence that relatively few MFIs use joint liability. For instance, only 16% of MFIs in Hermes and Lensink (2007) sample and only two of the five microfinance institutions surveyed in Morduch (1999) use joint liability. In recent years, the industry is witnessing a further move away from joint liability contracts to individual contracts.

While *group funds* have received far less attention than *joint liability*, they appear to be of substantial practical relevance. For instance, many programs, such as Grameen’s, display “emergency funds” or “group taxes”. These funds are typically used by the group under unanimous consent, and amount to a form of compulsory saving. In other programs, more explicit provisions on compulsory savings replicate similar arrangements. Often, in fact, compulsory savings cannot be withdrawn without the unanimous consent of the group, and come to act as a form of (group) collateral. Unanimous consent imply that poor performing members might lose part of their savings, if they are either excluded by group, or if their savings are used to finance others members’ projects.

The model suggests that group lending is only necessary at the early stages of the life of the group: once borrowers have learned (and revealed) their types and / or accumulated enough collateral, there is no further need for linking clients through joint liability. Therefore, in line with the experimental evidence in Giné and Karlan (2008), the model predicts that after removing group lending

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<sup>21</sup>The lender does better when the types are negatively correlated. In practice, however, (ex-ante) negatively correlated agents might be too different and therefore less likely to learn each other type. Also, if clients have some private information on the correlation of their types they will have incentives to form homogenous groups.

from groups started with such contractual arrangement, no large effect should be found on repayment and investment behavior.<sup>22</sup>

### 5.3 Group Lending: Further Discussion

#### *Group Lending and Variable Scale*

Anecdotal evidence suggests the existence of a negative correlation between joint liability and loan size, both in time and in the cross-section (Morduch (1999)). Similarly, it has been argued that joint liability contracts might limit scaling-up. In principle, the negative correlation could be explained by i) a causal negative relationship between joint liability and loan size, or ii) underlying omitted variables that codetermine both loan size and the use of joint liability contracts.

With variable scale, the model predicts that group lending changes the constrained optimal size of projects. As implementing any project becomes cheaper, the first-best projects  $k_1^*$  and  $k_2^*$  can be financed for a larger range of parameters. When first best investment levels cannot be financed, the distortions are smaller than with individual contracts.

**Proposition 7** *With group lending contracts, the first best projects  $k_1^*$  and  $k_2^*$  can be implemented for a larger range of parameters. Moreover, the constrained optimal projects are less distorted:*

- $k_1^* > k_1^{GL} > k_1^{IN}$  and  $k_2^{GL} = k_2^{IN} = k_2^*$  if  $\delta < \tilde{\delta}_\psi$  [Joint Liability];
- $k_1^* \leq k_1^{GL} < k_1^{IN}$  and  $k_2^{IN} < k_2^{GL} < k_2^*$  if  $\delta > \tilde{\delta}_\psi$  [Group Funds].

Combining the results on loan size of section 4 with those on group lending derived here, the model predicts a negative relationship between joint liability and loan size. First of all, joint liability is useful for those agents that would otherwise be credit constrained, i.e. those with relatively low  $\rho$ . Those agents also tend to borrow less and run smaller projects. Therefore if  $\rho$  is unobservable to the econometrician, in a cross-section of microfinance clients, the model implies a negative correlation between joint liability and loan size.

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<sup>22</sup>Their evidence is not consistent with theories of joint liability based on peer monitoring and moral hazard. It is, however, consistent with screening stories, à la Ghatak (1999).

Joint liability, arises when  $\delta \leq \tilde{\delta}_\psi = \frac{(\psi+e)k_1}{\psi k_2 - u}$ . Therefore, joint liability is relatively more likely for those cases in which  $k_1$  is relatively high and  $k_2$  is relatively low. Joint liability, therefore, is relatively more likely when the growth in project size,  $g(\rho) = \frac{k_2(\rho)}{k_1(\rho)}$  is small: a negative correlation between joint liability and *scaling-up* obtains.

However, in empirical specifications in which selection is controlled for, either by observing  $\rho$  or by experimental design, the model predicts that group lending in general allow to finance larger loans (either in period 1 through joint liability, or in period 2 through group funds). These predictions are consistent with the findings in Giné and Karlan (2008).

### *Group Lending under Collusion*

Joint liability has also come recently under scrutiny for its costs in terms of lack of flexibility, especially with respect to the timing of payments and consumption smoothing (see, e.g., Karlan and Mullainathan (2007)). An interesting trade-off between joint liability and flexibility emerges in a setting in which agents desire to smooth consumption and can transfer, subject to some transaction costs, rents to each other at the reporting stage.<sup>23</sup> In particular, if agents' capacity to transfer rents to each other is limited by the money that the contract leaves them in period 1, the optimal collusion-proof contract pays all rents in period 2. This logic naturally provides an additional reason to have compulsory savings. Group lending and compulsory savings are thus complementary.

If borrowers desire to smooth consumption across the two periods, the group lending contract might require lower consumption smoothing in order to prevent collusion. When  $\delta \leq \frac{e}{\psi - u}$ , joint liability might emerge in those cases in which the optimal individual contract induces perfect consumption smoothing. The model would then imply that clients in joint liability contracts enjoy less flexibility in consumption than similar borrowers in individual contracts.<sup>24</sup>

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<sup>23</sup>It can be shown that if agents have access to a perfect technology to transfer rents among themselves (i.e. at the collusion stage they maximize the sum of their utilities), the optimal group lending contract does not improve upon individual contracts at all.

<sup>24</sup>Note, however, that agents could collude by borrowing from a moneylender against future income streams. The monopolistic interest rate charged by the moneylender would be a natural formulation to think about the transaction costs associated with transfers in the collusion stage. Moreover, this remark confirms the intuition that borrowers use credit from moneylenders and other informal sources to achieve greater flexibility in consumption.

## 6 Emotional Stress and Excessive Retention

In order to take into account the impact of subjective factors related to the emotional (and even physical) well-being of the poor, this section introduces emotional costs associated with the management of a project. In particular, we distinguish between effort costs, that make it harder for the borrower to successfully complete the project, from psychological (or emotional) costs, that make the management of the project unattractive for certain borrowers.

While some personal characteristics that make entrepreneurship unsuitable for certain individual, such as attitude towards risk, might be known to the individual before starting a project, others might be discovered only later on, once the individual is involved with the management of her project. For instance, the emotional stress associated with the discrepancy between expectations and achievement, anxieties and tensions resulting from newly adopted non-traditional roles, the negative peer pressure induced by pitting borrowers against one another as a substitute for collateral to keep repayment rates high, and even (domestic) violence resulting from modified power structures, mostly within the household, have been pointed out as important factors in affecting women attitude and involvement in micro-businesses.

Specifically, let us assume that, once the project in period 1 has been started, the borrower's can take one of three type realizations. With probability  $\rho$ , the agent turns out to be a good type, while with probability  $\lambda$ , the agent is a bad type. Both types are as above. With probability  $1 - \rho - \lambda$ , finally, the agent is a depressed,  $D$ , type. We assume that, as for the bad type, the depressed type has an effort cost equal to  $e$ . Moreover, the depressed type experiences a private emotional cost  $s$  from the mere fact of being given a project to manage.

We assume

**Assumption 7**  $\psi < u + s$ .

The assumption implies that the depressed type will not seek funds for a project in period 2, since the emotional costs associated with a project,  $s$ , are larger than the net benefits of undertaking the project,  $\psi - u$ . From the point of view of the investor, therefore, the depressed type will naturally self-select out of entrepreneurship in period 2. When  $\delta \leq \delta_\psi$ , we know that the constraints associated with selection in period 2 are not binding, and therefore the analysis and

results are as in the baseline model. We therefore focus on the more interesting case  $\delta \geq \delta_\psi$ .

The *3-types first best* allocation is now defined as follows. Conditional on starting a project in period 1, the agent invests and repays the first period loan regardless of her type. In period 2, only a *G* type starts a new project, invests and repays it. In proving Proposition 8 below, we show that, when  $\delta \geq \delta_\psi$ , the following (constrained efficient) allocation can also be implemented in equilibrium. Following a first period in which the agent invests and repays regardless of her type, in period 2 the *D* type does not start a project, the *B* type starts a project and does not repay, and the *G* type starts a project, invests and repays. We label this allocation the *3-types second best*.

More precisely, we have

**Proposition 8** *When  $\delta \geq \delta_\psi$ , there exist thresholds  $\tilde{\lambda}$ ,  $\tilde{\delta}_{SB}(\rho)$  and  $\bar{\delta}_{FB}(\rho)$  such that:*

1. *the first best is implemented in the same region described in Proposition 1, i.e. if  $\rho \geq \bar{\delta}_{FB}^{-1}$ ,*
2. *the second best is implemented if  $\lambda < \tilde{\lambda}$  and  $\rho \in [\tilde{\delta}_{SB}^{-1}, \bar{\delta}_{FB}^{-1}]$ , and*
3. *no project is financed for other parameter values.*

**Proof.** See Appendix. ■

The main implication of the proposition is that, when the probability of emotional stress,  $(1 - \lambda - \rho)$ , is sufficiently high (i.e., for a given  $\rho$ ,  $\lambda$  is low), an intermediate region in which the second best allocation is implemented emerges. The allocation deviates from the first best since it allows the refinancing of the bad type, which is unprofitable: the scheme induces excessively high retention rates among clients. As in the baseline model, in order to prevent the bad type from undertaking a project in period 2 and implement first best, the scheme needs to pay rents in the form of a cash transfer to those clients that do not undertake a project in period 2. Since the transfer is paid in cash, the depressed type has an incentive to pretend to be a bad type and claim the transfer. This, in turn, increases the rents necessary to implement the allocation, and makes financing more difficult. When the probability of a depressed type is high enough, the

scheme saves rents by giving a project to the bad type in period 2, therefore avoiding the cash payment to the depressed type.<sup>25</sup>

An implication of this result is that excessively high retention rates are more likely among the relatively poorer borrowers. First, a significant body of literature has established that emotional stress resulting from poverty can lead to the development and/or maintenance of common mental health problems such as anxieties and depression (see, e.g., references discussed in Rahman (1999)). In other words, the realization of the depressed type is relatively more frequent among the poorest segments of the population. Second, a lower outside option  $u$  in period 2, which is likely to be correlated with poverty, expands the region in which the second best is implemented and shrinks the region in which first best is implemented.

The fact that excessively high retention rates might emerge among the poorest clients as part of the optimal lending scheme, suggests that a (potentially significant) fraction of *continuing* microcredit clients might not be suited for entrepreneurship. Because of their high effort costs, they might be reluctant to scale-up and commit substantial physical and emotional resources to the growth of their business (see, e.g., Banerjee and Duflo (2007)). This implication is consistent with the evidence reported in de Mel, McKenzie, and Woodruff (2008), according to which around 2/3rds of small, self-employed, entrepreneurs in their sample, have characteristics which makes them more similar to wage workers than to larger firms owners.

## 7 Borrower's Limited Commitment

This section discusses the contractual implications of allowing the possibility that the agent can leave the lender and sign contracts with alternative lenders in period 2.<sup>26</sup>

### *Default Rates and Retention of Clients under Contract $C^*$*

The contract  $C^*$  described in Section 3, by minimizing compulsory saving requirements  $S^C$ , economizes on enforcement and achieves zero default rates in equilibrium. These two desirable properties, however, come at the cost of high

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<sup>25</sup>The scheme, however, does not induce the bad type to repay, by paying  $\delta(\psi + e)$ , in order to economize on the rents paid to the good type.

<sup>26</sup>The formal statements and proofs of all results discussed in this section were included in a previous version of the paper, and are available upon request.

interest rates on second period loans:  $B_2 = r - u > 1$ . This is problematic if borrowers can leave the scheme in period 2 and obtain better loans in the credit market.

An alternative “refinement”, therefore, would minimize payments associated with second period loans,  $B_2$ . From the consumption equations (5), it immediately follows that  $B_2 < r - u$ , requires  $\frac{S^G}{\delta_s} - D_2^G < \frac{S^B}{\delta_s} - D_2^B$ . A simple way to achieve this is to allow the bad type to default, i.e. set  $D_2^B = 0$ . This logic suggests that, in practice, it might not be optimal for microlending institutions to insist on achieving 100% repayment rates, as this might jeopardize retention of the very best clients after having invested in them.

#### *Contractual and Welfare Implications of Limited Commitment*

In considering whether to leave the program in period 2 and borrow from an alternative lender, the agent takes into account the contractual terms offered in the market. Let us assume that outside lenders, in period 2, compete offering one period contracts to the agent. Denote with  $B_2^o$  the debt associated with those loans.

The consumption path associated with the optimal contract  $C^*$  always gives incentives for the good type to leave the program in period 2, unless  $c_2^G \geq r - 1$ . Competing lenders, in fact, can always separate the two types and offer period 2 loans with associated repayment  $B_2^o = 1$  to the good type. To see why this is the case, note that, when contract  $C^*$  does not achieve perfect consumption smoothing, the two types save different amounts and can be easily separated. When the contract achieves perfect consumption smoothing, instead, the bad type has no incentive to leave the scheme, since  $c_2^B > \psi$ , which is the maximum consumption she would obtain from outside lenders. The good type, however, has incentives to leave the contract unless  $c_2^G \geq r - 1$ .<sup>27</sup>

Since, as noted above, the first best allocation separates types in period 2, it always allows outside lenders to offer  $B_2^o = 1$  to the good type only. In order to avoid the good type leaving, in period 2 the contract has to satisfy the further constraint  $c_2^G \geq r - 1$ . When the contract has to satisfy this constraint, it is straightforward to show that, relative to the full commitment case, agents with relatively low  $\rho$  cannot borrow and are credit constrained. Competition among lenders, therefore, might decrease access to credit for the very poorest.<sup>28</sup>

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<sup>27</sup>Note that common contractual provisions according to which the borrower loses her savings if she leaves the scheme do not solve the problem.

<sup>28</sup>Conversely, lower mobility and lack of alternative employment opportunities might explain

Repeating the analysis at the end of Section 2, the optimal contract under limited commitment requires higher compulsory savings  $S^C$ , since the good type has to be given higher consumption in period 2, and therefore allows for substantially lower consumption smoothing and might even fail to provide perfect insurance against the type realization.<sup>29</sup>

### *Demand for Moneylenders*

Clients that cannot commit to the contract and are, consequently, unable to borrow from competitive lenders, might enter lending relationships with informal moneylenders.<sup>30</sup> Those “informal” lenders enjoy an advantage over “formal” lender in ensuring borrower’s commitment to the contract. They do so by relying on common social networks that provide adequate means of enforcement and sanctions. Since access to the social network is a scarce resource, the benefits of better monitoring and enforcement comes at the cost of some degree of monopoly power of the lender.

If monopolistic moneylenders enjoy the ability to ensure agent’s commitment to the contract (but otherwise face the same moral hazard and adverse selection problems as any other lender), the model delivers empirical predictions on the “demand” for moneylenders. The main implications are that clients with sufficiently low  $\rho$  borrow from moneylenders. Therefore, in a cross-section of borrowers, clients of moneylenders are charged higher interest rates and have higher drop-outs. For the same borrower, however, moneylenders offer more flexible contracts with better consumption smoothing properties. These observations appear to be in line with evidence in Karlan and Mullainathan (2007).<sup>31</sup>

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the success of microlending schemes with women. Interestingly, not all program started originally by focusing on woman, but most ended up with portfolios with more than 90% clients being woman.

<sup>29</sup>The lender might try to prevent the good type from leaving the scheme by offering contracts that render the two types indistinguishable from the point of view of other lenders. One way to do this is to form a group and give projects to both types in period 2. The lender, then, commit to a policy in which, if one member of the group leaves the scheme, the group is dissolved and further lending denied to all members of the group.

<sup>30</sup>A common characteristic of credit markets in developing countries is the extensive role played by “informal” sources of credit. Examples of this sort of informal transactions are loans made by moneylenders, traders, landlords, and family.

<sup>31</sup>The availability of independent means of savings for the borrower is also a concern if the agent is able to save the money she steals from the project,  $\psi$ . Since clients with higher  $\rho$  have better access to loans in the secondary credit market, private savings introduce a countervailing force that makes it harder to lend to clients with “intermediate” levels of confidence  $\rho$ .

## 8 Empirical Implications: Predictions and Tests

This section describes empirical implications and tests of the model. It is divided into two parts. First, it discusses how to test some of the predictions of the model using retrospective data. Second, it describes how the model can be used to interpret and guide the design of controlled field experiments to evaluate various contractual provisions in microlending schemes.

### *Testing the Model with Retrospective Data*

From an empirical point of view, it is important to distinguish two cases, depending on whether (a proxy for) initial confidence  $\rho$  is observed, or not.

Assume first that the level of initial confidence,  $\rho$ , is unobserved by the econometrician. Since  $\rho$  is the common determinant for several observable variables and patterns of behavior, the model implies correlation patterns that can be tested with retrospective data (see Panel A in Table 1). First, low  $\rho$  correlates with various contractual elements, such as higher interest rates, a steeper repayment profile, limited scaling-up and imperfect consumption smoothing. Low  $\rho$  also correlates with other behavioral aspects, such as higher drop-out and default rates and with borrowing from informal moneylenders. Finally, low  $\rho$  also correlates with certain aspects of behavior among the poor that are still not well understood, such as i) the contemporaneous holding of borrowing and savings at unfavorable terms, ii) the failure to scale-up and rejection of larger (first period) loans, and iii) excessively high retention rates among microfinance clients. It is worth pointing out that *all* those patterns are more likely to happen for those borrowers with the lowest  $\rho$  among those that undertake a project in period 1. In a cross-section of borrowers, therefore, the model predicts that these patterns of behavior will be clustered together. Some of the implied correlations are shared with other models of the credit market. Others, however, are not. For instance, the predictions related to contemporaneous holding of borrowing and savings at unfavorable interest rates is more specific to our model and is supported by empirical evidence.<sup>32</sup>

The model suggests a distinction between individual characteristics, and the (ex-ante) beliefs about those characteristics. In the model, initial confidence  $\rho$  and the realization of types are two different variables that can be disentangled. Variables such as default rates and borrowing behavior do not depend on  $\rho$ , once

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<sup>32</sup>See footnote 17 above.

the realization of type, or initial selection, are controlled for. Contractual terms (such as interest rates, saving requirements and loan size), however, do depend on  $\rho$  even *after* controlling for the realization of the type.<sup>33</sup>

For a given contract, therefore, certain behavioral patterns can be used as proxy for unobservable individual characteristics. Controlling for those behavioral patterns, then, allows to establish more accurately how contractual terms respond to initial confidence and how much they impact other outcomes of interest. For example, consider establishing the effects of contractual terms, such as interest rates, on borrower's business dynamics and growth. Suppose the econometrician estimates the following regression on a sample of borrowers,

$$SU_j = \alpha + \beta i_{b_j} + \mu_j,$$

where  $i_{b_j}$  is the interest rate paid on loans,  $SU_j$  a measure of borrower's business scaling-up, and  $\mu_j = \rho_j + \varepsilon_j$  is the error term. The error term is composed of two elements:  $\rho$  is the initial confidence, observed by the borrower and the lender, but not by the econometrician, and  $\varepsilon_j$  is a standard *i.i.d.* error term. The model predicts  $\beta > 0$ .

However, had the econometrician access to savings data, and observed contemporaneous borrowing and savings in excess of mandatory ones,  $CS_j$ , the model predicts that the regression

$$SU_j = \alpha + \widehat{\beta} i_{b_j} + \gamma CS_j + \mu_j$$

gives  $\gamma < 0$ , and  $0 \leq \widehat{\beta} < \beta$ . This is because  $CS_j$  is a proxy for the *realization* of the "type" of the client, and indicates a good predisposition towards entrepreneurship.

In certain circumstances, direct measures of, or proxy variables for, the initial confidence,  $\rho$ , might be available to the econometrician. For instance, data from entrepreneurial psychology surveys focussing on 'psychological' or 'attitudinal' variables (such as ambition, work centrality, optimism, tenacity, passion for work,

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<sup>33</sup>An example of how such a distinction can be used to interpret existing evidence, is given by Figure 2 in Giné and Karlan (2008). If center members are relatively homogenous from an ex-ante perspective, the variable "Number of times clients had difficulty in repaying, centre average" is a reasonable proxy for  $1 - \rho$ . The model is then consistent with the fact that this variable does not differ across treatment and control groups for baseline clients, but it is higher for control groups with group liability among new clients, since group lending allows to select individuals with lower  $\rho$ .

etc...) could be used to construct proxies for  $\rho$  for first time clients, and for the realization of the type for mature clients. The model then predicts differential effects of these variables on behavior, depending on borrower's tenure.

#### *Microlending Schemes: Design of Field Experiments*

The model can be used to interpret (and guide) controlled field experiment to evaluate and explore the interactions between the *multiple* elements of microlending schemes. A field experiment such as Giné and Karlan (2008), for example, allows to disentangle the selection from the causal impact of group lending on loan size. In a cross section of individuals, the model predicts that clients with individual liability contracts receive larger loans. In the regression

$$LS_j = \alpha + \beta IL_j + \mu_j$$

where  $LS_j$  is a measure of loan size,  $IL_j$  is a dummy for individual liability and  $\mu_j = \rho_j + \varepsilon_j$  is the error term described above, the model predicts  $\beta > 0$ , since  $\rho$  positively correlates with  $IL_j$ . With experimental data, however,  $IL_j$  becomes orthogonal to  $\rho$ , and the model predicts  $\beta < 0$ . This latter prediction is consistent with the findings in Giné and Karlan (2008) on the effects of group lending on loan size for the baseline clients.<sup>34</sup>

Another experiment can be done to evaluate how saving requirements and group lending interact. The model suggests that those two contractual feature are complements. To test this prediction, borrowers could be randomly assigned to one “control” group, with group lending and standard saving requirements, or to three “treatment” groups. A first treatment group would keep similar saving requirements, under an individual contract. A second and third treatment groups, instead, would have lower saving requirements, and either group, or individual contracts. Such an experiment would be a first step towards unpacking how the multiple components of the contractual package interact.

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<sup>34</sup>The model rationalizes most of the findings in Giné and Karlan (2008), including those on the causal impact of group lending on first period loan size (see, also, Section 5). More broadly, the model suggests that contracts can be made more flexible as time and learning unfold, provided borrowers have accumulated enough savings (see, also, the experiment in Field and Pande (2008)). These predictions could be easily tested.

## 9 Conclusion

This paper studied a two-period lending relationship with moral hazard in which an agent with no wealth (and protected by limited liability) privately learns her entrepreneurial talent upon borrowing in a competitive credit market to start a project.

If the borrower is sufficiently poor, the optimal lending contract is similar to microfinance schemes observed in practice. In particular, it always displays compulsory savings requirements and step lending. While group lending is always beneficial, joint liability may or may not be optimal. The model also rationalizes group funds.

The paper shows that taking into account self-discovery helps explaining certain aspects of micro-entrepreneurs that are still poorly understood, such as the failure of the poor to undertake more profitable and easily available investment opportunities, the contemporaneous holding of borrowing and savings at unfavorable interest rates as well as the excessive retention of clients into microlending schemes.

Exploration of unknown activities lies at the heart of our model. The analysis in this paper is highly stylized, and would benefit from being extended in several directions in the future. The learning process in the model is highly simplistic, and some important psychological elements related to entrepreneurship are, at best, modeled in a very reduced form. Both areas deserve closer theoretical scrutiny. This research will eventually lead to the formulation of better lending and savings products for micro-entrepreneurs in the developing world.

## 10 Appendix: Proofs

**Lemma 2** *There are thresholds  $\rho_u$ ,  $\rho^{FB} < \rho_u$ ,  $\bar{\delta}^{FB}(\rho)$  and  $\underline{\delta}^{FB}(\rho)$ , with  $\frac{\partial \bar{\delta}^{FB}(\rho)}{\partial \rho} > 0$ ;  $\frac{\partial \underline{\delta}^{FB}(\rho)}{\partial \rho} < 0$  and  $\bar{\delta}^{FB}(\rho^{FB}) = \underline{\delta}^{FB}(\rho^{FB})$  such that first best can be implemented in two regions:*

- i) if  $\rho \geq \rho_u$  and  $\delta \geq \underline{\delta}^{FB}$ ;
- ii) or if  $\rho < \rho_u$  and  $\delta \in [\underline{\delta}^{FB}, \bar{\delta}^{FB}]$ .

**Proof of Lemma 2.** *First step.* Find the cost-minimizing contract, that is, the contract that implements the first best with the least possible transfer. Denote  $t_{i,\tau}$ ,  $i = G, B$ ,  $\tau = 1, 2$ , the transfer that the type  $i$  receives in period  $\tau$ , and  $T_i = t_{i,1} + \delta t_{i,2}$  the total transfer to the type  $i$ . Let us show that the cost-minimizing contract takes the following form:

$$\begin{cases} T_G^* = \psi + e + \delta u & , \text{ if } \delta \leq \delta_\psi \text{ and} \\ T_B^* = \psi + e & , \text{ if } \delta \geq \delta_\psi \end{cases}$$

The agent can deviate in period 1 by i) not reporting her true type, and/or ii) diverting the investment (and not exerting the effort). The contract has to satisfy the following constraints:

for the good type :

$$\begin{aligned} T_G &\geq \psi + \delta u && IC_{G,1} \\ T_G &\geq T_B + \delta u && TT_G \\ t_{G,2} &\geq \psi && IC_{G,2} \\ t_{G,i} &\geq 0 && LL_{G,i} \end{aligned}$$

for the bad type :

$$\begin{aligned} T_B + \delta u &\geq \psi + e + \delta u && IC_{B,1} \\ T_B + \delta u &\geq t_{G,1} + \delta \max\{t_{G,2} - e, \psi\} && TT_B \\ t_{B,i} &\geq 0 && LL_{B,i} \end{aligned}$$

Rewrite  $TT_B$  as

$$\begin{aligned} T_B + \delta u &\geq T_G + \delta \max\{-e, \psi - t_{G,2}\} && TT_B \\ &= T_G - \delta \min\{e, t_{G,2} - \psi\} \end{aligned}$$

From  $IC_{B,1}$  it follows that  $T_B \geq \psi + e > \psi$  and therefore  $TT_G$  implies  $IC_{G,1}$ . Also note that cash constraints for the bad type never bind. We don't need

him to produce in period 2 and so we can distribute  $T_B$  into  $t_{B,1}$  and  $t_{B,2}$  as we like.

There are two cases.

- Case 1:  $t_{G,2} \leq e + \psi \Rightarrow \min\{e, t_{G,2} - \psi\} = t_{G,2} - \psi$ . Setting  $t_{G,2} = \psi$  never hurts and it could even be beneficial to relax the cash constraint  $t_{G,1} \geq 0$ . Then  $\min\{e, t_{G,2} - \psi\} = t_{G,2} - \psi = 0$  and we have the following system of constraints:

$$\begin{aligned} T_G &\geq \delta\psi && \Leftrightarrow t_{G,2} = \psi \\ T_G &\geq T_B + \delta u && TT_G \\ T_B &\geq \psi + e && IC_{B,1} \\ T_B &\geq T_G - \delta u && TT_B \end{aligned}$$

From  $TT_G$  and  $TT_B$ ,  $T_G = T_B + \delta u$  and therefore there are two possible cases:

- Case 1a:  $IC_{B,1}$  binds and thus  $T_B = \psi + e$  and thus  $T_G = \psi + e + \delta u$ . We need only to check that  $T_G \geq \delta\psi$ , that is,

$$\psi + e + \delta u \geq \delta\psi \Leftrightarrow \delta \leq \delta_\psi \equiv \frac{\psi + e}{\psi - u}.$$

- Case 1b:  $T_G = \delta\psi$  and thus  $T_B = \delta(\psi - u)$ .  $IC_{B,1}$  is satisfied iff  $\delta \geq \delta_\psi$ .

- Case 2:  $t_{G,2} \geq \psi + e \Rightarrow \min\{e, t_{G,2} - \psi\} = e$  and so we have the following system of constraints:

$$\begin{aligned} T_G &\geq \delta(\psi + e) && \Leftrightarrow t_{G,2} \geq \psi + e \\ T_G &\geq T_B + \delta u && TT_G \\ T_B &\geq \psi + e && IC_{B,1} \\ T_B &\geq T_G - \delta(u + e) && TT_B \end{aligned}$$

Let us show why we cannot do better than in Case 1.

If  $T_G \geq \delta(\psi + e)$  binds, so  $T_G = \delta(\psi + e)$ , from  $TT_B$ ,  $T_B \geq \delta(\psi - u)$  and this is clearly worse than case 1b).

If  $T_G \geq \delta(\psi + e)$  does not bind, then  $T_B = \psi + e$  and  $T_G = \psi + e + \delta u$  as in case 1a).

Finally, note that  $T_G \geq \delta(\psi + e)$  binds at  $\psi + e + \delta u = \delta(\psi + e)$ , that is, at  $\delta = \frac{\psi+e}{\psi+e-u} < \delta_\psi$ .

*Second step.* Now plug cost-minimizing contract (1) into the zero profit constraint

$$(r - 1)(1 + \delta\rho) \geq \rho T_G^* + (1 - \rho)T_B^*.$$

Region 1. When  $\delta \leq \delta_\psi$ ,  $T_B^* = \psi + e$  and  $T_G^* = \psi + e + \delta u$

$$(r - 1)(1 + \delta\rho) \geq \psi + e + \rho\delta u$$

which is satisfied if and only if  $\delta \geq \underline{\delta}^{FB} \equiv \frac{\psi+e-(r-1)}{\rho(r-1-u)}$ .

Region 2. When  $\delta \geq \delta_\psi$ ,  $T_B^* = \delta(\psi - u)$  and  $T_G^* = \delta\psi$

$$(r - 1)(1 + \delta\rho) \geq \delta\psi - \delta(1 - \rho)u$$

which is satisfied if and only if  $\delta \leq \bar{\delta}^{FB} \equiv \frac{r-1}{\psi-\rho(r-1)-(1-\rho)u}$  when  $\rho < \rho_u \equiv \frac{\psi-u}{r-1-u}$  and always otherwise.

Finally, denote  $\rho^{FB} \equiv \frac{(\psi+e)\psi-(r-1)(\psi+u)}{(\psi+e)(r-1-u)}$  such  $\rho$  at which  $\underline{\delta}^{FB} = \bar{\delta}^{FB} = \delta_\psi$ . ■

**Lemma 3** *Second best can be implemented in three regions:*

- $\delta \geq \underline{\delta}_1^{SB} \equiv \frac{1-\rho(r-\psi)}{\rho(r-1-u)}$  if  $\delta \leq \frac{\psi}{\psi-u}$  (region 1);
- $\delta \geq \underline{\delta}_2^{SB} \equiv \frac{1-\rho r}{\rho(r-1-\psi)}$  if  $\delta \in \left[ \frac{\psi}{\psi-u}, \delta_\psi \right]$  (region 2);
- $\delta \leq \bar{\delta}^{SB} \equiv \frac{\rho r + (\psi+e)(1-\rho)-1}{\psi-\rho(r-1)-(1-\rho)u}$  if  $\rho < \rho_u$  and  $\delta \geq \delta_\psi$  and always if  $\rho > \rho_u$  and  $\delta \geq \delta_\psi$  (region 3).

At  $\rho^{SB} \equiv \frac{\psi-u}{2\psi r - ur - \psi^2 - e\psi - \psi + er - e}$ ,  $\underline{\delta}_2^{SB} = \bar{\delta}^{SB} = \delta_\psi$ . At  $\rho < \rho^{SB}$ , second best cannot be implemented for any  $\delta$ .

**Proof of Lemma 3.** The proof proceeds as in the proof of Lemma 2 once the appropriate incentive constraints are considered. For the sake of brevity, we omit analytical derivations. ■

**Proof of Proposition 1.** First best is implemented whenever possible; when it is impossible, second best is implemented; otherwise, there is no investment.

From Lemma 2 and Lemma 3 we know when each regime can be implemented. We are left to show the region where only second best can be implemented.

In region 3 as defined in Lemma 3 where the second best can be implemented the first best can be implemented as well since  $\bar{\delta}^{SB} \leq \bar{\delta}^{FB}$ .

Define  $\underline{\delta}^{SB}$  as the frontier of the region below  $\delta_\psi$  where the second best can be implemented, that is, lower bound of regions 1 and 2 in Lemma 3:

$$\underline{\delta}^{SB} \equiv \max \left\{ \min \left\{ \underline{\delta}_1^{SB}, \underline{\delta}_2^{SB}, \delta_\psi \right\}, 0 \right\}.$$

$(\underline{\delta}^{SB} = 0 \text{ for } \rho \geq \frac{1}{r-\psi}).$   
 $\underline{\delta}^{SB}$  and  $\underline{\delta}^{FB}$  intersect because i) at  $\rho = 1$ ,  $\underline{\delta}^{FB} > \underline{\delta}^{SB} = 0$  and ii) since  $\bar{\delta}^{SB} < \bar{\delta}^{FB}$  for  $\rho < 1$ ,  $\rho^{SB} > \rho^{FB}$  and therefore, at  $\rho = \rho^{SB}$ ,  $\underline{\delta}^{SB} = \delta_\psi > \underline{\delta}^{FB}$ . Denote the intersection  $\rho_r$  (it can be checked that it is unique). For  $\rho > \rho_r$ ,  $\underline{\delta}^{FB} > \underline{\delta}^{SB}$ . Therefore, for  $\delta \in [\underline{\delta}^{SB}, \underline{\delta}^{FB})$  first best cannot be implemented while the second best can be implemented. ■

**Proof of Proposition 2.** Suppose the agent has a strictly concave utility function given by  $U(c)$ . The contract maximizes the expected utility of the agent subject to the constraint that the discounted values of consumption equal the discounted values of the minimal incentive compatible transfers,  $T_G^*$  and  $T_B^*$ , derived in the Proof of Proposition 1.

From an ex-ante perspective, since the agent is risk averse, the optimal contract minimizes the spread in expected utility across the two types, subject to the incentive compatibility constraints. This implies keeping  $T_H^* - T_L^* = \delta u$ , and “redistributing”  $M(\rho)$  equally to both types.<sup>35</sup> The (approximation to the) optimal contract, therefore, solves the following problem.

$$\begin{aligned} \max \quad & \rho [U(c_1^G) + \delta U(c_2^G)] + (1 - \rho) [U(c_1^B) - e + \delta U(u + c_2^B)] \quad (\text{OC}) \\ \text{s.t.} \quad & \begin{cases} c_2^G \geq \psi, \\ c_1^G + \delta c_2^G \leq T_G^* + M, \\ c_1^B + \delta c_2^B \leq T_B^* + M \end{cases} \end{aligned}$$

By inspection of problem (OC), it is obvious that i) the second and third constraints are binding, and, ii) the solution is separable, in the sense that  $c_t^G$  is independent of  $c_t^B$ , for  $t \in \{1, 2\}$ . For the  $B$  type  $c_1^B = \frac{T_B^* + M(\rho) + \delta u}{1+\delta}$  and  $c_2^B = \frac{T_B^* + M(\rho) + \delta u}{1+\delta} - u$ . For the  $G$  type  $c_1^G = \frac{T_G^* + M(\rho)}{1+\delta}$  and  $c_2^G = \frac{T_G^* + M(\rho)}{1+\delta}$  if  $\frac{T_G^* + M(\rho)}{1+\delta} > \psi$ ,

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<sup>35</sup>It is easy to check that this satisfies all incentives constraints.

and otherwise  $c_1^G = M(\rho)$  and  $c_2^G = \psi$ .

So, if the constraint  $c_2^G \geq \psi$  is not binding, since  $T_B^* + M(\rho) = F(\rho)$  and  $T_G^* + M(\rho) = F(\rho) + \delta u$ , we immediately obtain

$$c_1^B = c_2^B + u = \frac{F(\rho) + (1 - \rho)\delta u}{1 + \delta} \text{ and } c_1^G = c_2^G = \frac{F(\rho) + (1 - \rho)\delta u}{1 + \delta}.$$

If, instead,  $\frac{F(\rho) + (1 - \rho)\delta u}{1 + \delta} < \psi$ , the constraint will be binding. The condition can be rewritten as  $\delta > \frac{r - 1 - \psi}{(\psi - u) - \rho(r - 1 - u)}$ . ■

**Proof of Proposition 3.** The solution to the system of equations defined by (4) and (5) determines the optimal contract. Equations in (4) immediately imply that under perfect consumption smoothing, since  $c_1^G = c_1^B$ , it must be that  $S^{V*} = S^{C*}$ .

Second, equations in (5), together with  $c_2^G = \max\{\frac{F(\rho) + \delta u}{1 + \delta}; \psi\}$  (note that  $\frac{F(\rho) + \delta u}{1 + \delta} > \psi$  if  $M(\rho) > \psi$ ), imply  $B_2 = r - u$ .

Finally, the minimum saving requirement  $S^C = D_1 + \delta D_2$ , gives  $D_2$ . ■

The next Proposition corresponds to the first part of Proposition 1 and characterizes the parametric configuration under which the first-best project sizes  $k_1^*$  and  $k_2^*$  can be financed when project size is variable.

**Proposition 9** *There are thresholds  $\tilde{\rho}_u$ ,  $\tilde{\rho}^{FB} < \tilde{\rho}_u$ ,  $\tilde{\delta}^{FB}(\rho)$  and  $\underline{\tilde{\delta}}^{FB}(\rho)$ , with  $\frac{\partial \tilde{\delta}^{FB}(\rho)}{\partial \rho} > 0$ ;  $\frac{\partial \underline{\tilde{\delta}}^{FB}(\rho)}{\partial \rho} < 0$  (at least, for low  $\rho$ ) and  $\tilde{\delta}^{FB}(\tilde{\rho}^{FB}) = \underline{\tilde{\delta}}^{FB}(\tilde{\rho}^{FB})$  such that first best projects  $k_1^*$  and  $k_2^*$  can be implemented in two regions:*

- if  $\rho \geq \tilde{\rho}_u$  and  $\delta \geq \tilde{\delta}^{FB}$ ;
- or if  $\rho < \tilde{\rho}_u$  and  $\delta \in \left[\underline{\tilde{\delta}}^{FB}, \tilde{\delta}^{FB}\right]$ .

**Proof of Proposition 9.** We proceed in the same way as in the proof of Proposition 1: first, we find the cheapest transfers implement the first best, and then, we find when these transfers allow the lenders to have non-negative profits.

The transfers that implement the first best have to satisfy the following conditions:

$$\begin{aligned} \overline{IC}_1 : \bar{t}_1 + \delta \bar{t}_2 &\geq \psi k_1^* + \delta u \\ \underline{IC}_1 : \underline{t}_1 + \delta (\underline{t}_2 + u) &\geq (\psi + e) k_1^* + \delta u \\ \overline{TT} : \bar{t}_1 + \delta \bar{t}_2 &\geq \underline{t}_1 + \delta (\underline{t}_2 + u) \\ \underline{TT} : \underline{t}_1 - ek_1^* + \delta (\underline{t}_2 + u) &\geq \bar{t}_1 - ek_1^* + \delta \max\{\bar{t}_2 - ek_2^*, \psi k_2^*\} \\ MH_2 : \bar{t}_2 &\geq \psi k_2^* \end{aligned}$$

Analogously to the proof of Proposition 1, the cost-minimizing transfers are the following:

$$\begin{cases} \bar{T}^* = (\psi + e) k_1^* + \delta u & , \text{ if } \delta \leq \tilde{\delta}_\psi^* \\ \underline{T}^* = (\psi + e) k_1^* & \\ \begin{cases} \bar{T}^* = \delta \psi k_2^* & , \text{ if } \delta \geq \tilde{\delta}_\psi^* \\ \underline{T}^* = \delta (\psi k_2^* - u) & \end{cases} & \end{cases} \quad (6)$$

where  $\tilde{\delta}_\psi^* = \frac{(\psi+e)k_1^*}{\psi k_2^* - u}$ .

The zero profit constraint is

$$ZP : \rho \bar{T}^* + (1 - \rho) \underline{T}^* \leq f(k_1^*) - k_1^* + \rho \delta (f(k_2^*) - k_2^*)$$

and when transfers (6) are plugged it becomes

$$\begin{aligned} ZP_1 : & (\psi + e) k_1^* + \rho \delta u \leq f(k_1^*) - k_1^* + \rho \delta (f(k_2^*) - k_2^*), & \text{if } \delta \leq \tilde{\delta}_\psi^* \\ ZP_2 : & \delta \psi k_2^* - (1 - \rho) \delta u \leq f(k_1^*) - k_1^* + \rho \delta (f(k_2^*) - k_2^*), & \text{if } \delta \geq \tilde{\delta}_\psi^* \end{aligned}$$

When  $\delta \leq \tilde{\delta}_\psi^*$ ,  $ZP_1$  is satisfied if and only if

$$\delta \geq \underline{\delta}^{FB} \equiv \frac{(\psi + e + 1) k_1^* - f(k_1^*)}{\rho (f(k_2^*) - k_2^* - u)}.$$

When  $\delta \geq \tilde{\delta}_\psi^*$ , the relevant constraint is  $ZP_2$

$$f(k_1^*) - k_1^* + \delta [\rho (f(k_2^*) - k_2^*) - \psi k_2^* + (1 - \rho) u] \geq 0.$$

It is satisfied for any  $\delta$  if

$$\rho \geq \tilde{\rho}_u \equiv \frac{\psi k_2^* - u}{f(k_2^*) - k_2^* - u}$$

and for

$$\delta \leq \tilde{\delta}^{FB} \equiv \frac{f(k_1^*) - k_1^*}{\psi k_2^* - (1 - \rho) u + \rho (f(k_2^*) - k_2^*)}$$

otherwise.

Finally, denote  $\tilde{\rho}^{FB}$  as the solution to (assume it exists and unique)

$$\rho = \frac{[(\psi + e + 1) k_1^* - f(k_1^*)] [\psi k_2^* - u]}{[f(k_2^*) - k_2^* - u] (\psi + e) k_1^*}.$$

■

### Proof of Proposition 4.

When the first best cannot be financed, the problem is the following

$$\begin{aligned} \max_{k_1, k_2, \bar{T}, \underline{T}} W &= \rho \bar{T} + (1 - \rho)(\underline{T} - k_1 e + \delta u) \\ \text{s.t.} \\ ZP : \rho \bar{T} + (1 - \rho) \underline{T} &= f(k_1) - k_1 + \rho \delta (f(k_2) - k_2) \\ \bar{T} \text{ and } \underline{T} &\text{ are (6) (without stars)} \end{aligned}$$

We used the fact that when the first-best cannot be financed, the transfers have to be cost-minimizing given the desired projects  $k_1$  and  $k_2$ . Also, the zero-profit constraint is binding.

Plug the transfers from  $ZP$  into  $W$  and replace transfers as in (6) to rewrite the problem as

$$\begin{aligned} \max_{k_1, k_2} W &= f(k_1) - k_1 + \rho \delta (f(k_2) - k_2) + (1 - \rho)(-ek_1 + \delta u) \\ \text{s.t.} \\ ZP_1 : (\psi + e)k_1 + \rho \delta u &\leq f(k_1) - k_1 + \rho \delta (f(k_2) - k_2), \text{ if } \delta \leq \tilde{\delta}_\psi \\ ZP_2 : \delta \psi k_2 - (1 - \rho)\delta u &\leq f(k_1) - k_1 + \rho \delta (f(k_2) - k_2), \text{ if } \delta \geq \tilde{\delta}_\psi \end{aligned} \tag{7}$$

Suppose we are in the first case. Write the Lagrangian

$$\begin{aligned} \max_{k_1, k_2, \lambda_1} \mathcal{L}_1 &= f(k_1) - k_1 + \rho \delta (f(k_2) - k_2) + (1 - \rho)(-ek_1 + \delta u) \\ &\quad - \lambda_1 [(\psi + e)k_1 + \rho \delta u - (f(k_1) - k_1 + \rho \delta (f(k_2) - k_2))] \end{aligned}$$

As  $ZP_1$  is binding (and first best cannot be financed),  $\lambda_1 > 0$ . The first-order conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}_1}{\partial k_1} &= (f'(k_1) - 1)(1 + \lambda_1) - (1 - \rho)e - \lambda_1(\psi + e) = 0 \\ \frac{\partial \mathcal{L}_1}{\partial k_2} &= \rho \delta (f'(k_2) - 1)(1 + \lambda_1) = 0 \end{aligned}$$

Then,  $k_2 = k_2^*$  and since  $\frac{(1-\rho)e+\lambda_1(\psi+e)}{(1+\lambda_1)} > (1 - \rho)e$ ,  $k_1 < k_1^*$ .

If we are in the second case the Lagrangian is

$$\begin{aligned} \max_{k_1, k_2, \lambda_2} \mathcal{L}_2 &= f(k_1) - k_1 + \rho \delta (f(k_2) - k_2) + (1 - \rho)(-ek_1 + \delta u) \\ &\quad - \lambda_2 [\delta \psi k_2 - (1 - \rho)\delta u - (f(k_1) - k_1 + \rho \delta (f(k_2) - k_2))] \end{aligned}$$

As  $ZP_2$  is binding (and first best cannot be financed),  $\lambda_2 > 0$ . The first-order conditions are

$$\begin{aligned}\frac{\partial \mathcal{L}_2}{\partial k_1} &= (f'(k_1) - 1)(1 + \lambda_2) - (1 - \rho)e = 0 \\ \frac{\partial \mathcal{L}_2}{\partial k_2} &= \rho\delta(f'(k_2) - 1)(1 + \lambda_2) - \lambda_2\delta\psi = 0\end{aligned}$$

Then, since  $0 < \frac{(1-\rho)e}{1+\lambda_2} < (1 - \rho)e$ ,  $k_2^* > k_1 > k_1^*$  and since  $\frac{\lambda_2\psi}{\rho(1+\lambda_2)} > 0$ ,  $k_2 < k_2^*$ .

Finally, the threshold  $\tilde{\delta}_\psi(\rho)$  is implicitly defined by the equation  $\frac{k_1^{reg2}(\rho, \delta)}{\psi k_2^{reg2}(\rho, \delta) - u} = \frac{k_1^{reg1}(\rho, \delta)}{\psi k_2^{reg1}(\rho, \delta) - u}$  where  $k_\tau^{reg1}$  and  $k_\tau^{reg2}$  are the (distorted) project sizes in the two regimes studied above, for  $\tau = \{1, 2\}$ . ■

**Proof of Proposition 5.** Take any  $k'_1$  such that  $k_1^* < k'_1 \leq \arg \max\{f(k) - k\}$ . The region where  $(k'_1, k_2^*)$  can be financed has the same form and it is found in the same way where the first-best  $(k_1^*, k_2^*)$  can be financed (see Proposition 9). Denote  $\tilde{\delta}'_\psi = \frac{(\psi+e)k'_1}{\psi k_2^* - u}$ .

Take  $\rho'$  and  $\delta'$  such that  $\delta' = \tilde{\delta} = \underline{\delta}'$  where  $\underline{\delta}'$  is defined as  $\tilde{\delta}^{FB}$  replacing  $k_1^*$  for  $k'_1$ . At these  $\rho'$  and  $\delta'$  both  $(k_1^*, k_2^*)$  and  $(k'_1, k_2^*)$  can be financed. Note that  $\tilde{\delta}_\psi^* < \delta' < \tilde{\delta}'_\psi$ .

Let us compute the expected consumption in period 1 (we apply an analogue to Proposition 2):

$$\begin{aligned}E(c_1^*) &= (1 - \rho) \frac{\delta' \psi k_2^*}{1 + \delta'}; \\ E(c'_1) &= (1 - \rho) \frac{(\psi + e) k'_1 + \delta' u}{1 + \delta'} + \rho ((\psi + e) k'_1 - \delta' \psi k_2^* + \delta' u)\end{aligned}$$

and their difference is

$$E(c'_1) - E(c_1^*) = ((\psi + e) k'_1 - \delta' \psi k_2^* + \delta' u) \frac{1 + \delta' \rho'}{1 + \delta'} > 0.$$

The comparison for the interest rates follows the same logic, and is therefore omitted. ■

**Proof of Proposition 6.** As it is customary in this type of environments, the optimal mechanism punishes any contradicting reports across the two agents. This immediately implies that individual truthtelling constraints can be ignored. From the point of view of the principal, the coalition of agents becomes a single player that can take upon 3 types  $\langle ij \rangle \in \{GG, BB, \{GB, BG\}\}$ .

Omitting individual truthtelling constraints, the following are the constraints left. First, there are limited liability constraints:  $t_{ij,\tau} \geq 0$ , for  $i, j \in \{B, G\}$  and  $\tau \in \{1, 2\}$ .

For a coalition with two good types,  $\langle ij \rangle = GG$ , we have the following constraints

$$\begin{aligned} T_{GG} &\geq \psi + \delta u & IC_{GG,1} \\ T_{GG} &\geq \begin{cases} \min\{T_{BG} + \delta u, T_{GB}\} \\ T_{BB} + \delta u \end{cases} & TT_{GG} \\ t_{GG,2} &\geq \psi & IC_{GG,2} \end{aligned}$$

For a coalition with two bad types,  $\langle ij \rangle = BB$ , we have the following constraints

$$\begin{aligned} T_{BB} + \delta u &\geq \psi + e + \delta u & IC_{BB,1} \\ T_{BB} + \delta u &\geq \begin{cases} \min\{T_{BG} + \delta u, \tilde{T}_{GB}\} \\ \tilde{T}_{GG} \end{cases} & TT_{GG} \end{aligned}$$

where  $\tilde{T} = t_{.} + \delta \max\{\psi, t_{.2} - e\}$ .

Finally, for a “mixed” coalition,  $\langle ij \rangle \in \{GB, BG\}$ , we have the following set of constraints

$$\begin{aligned} T_{GB} &\geq \psi + \delta u & IC_{GB,1} \\ T_{BG} + \delta u &\geq \psi + e + \delta u & IC_{BG,1} \\ \begin{cases} T_{GB} \geq T_{GG} \\ T_{BG} + \delta u \geq \tilde{T}_{GG} \end{cases} && TT_{GB} / T_{BG} \\ \max\{T_{GB}, T_{BG} + \delta u\} &\geq T_{BB} + \delta u \\ t_{GB,2} &\geq \psi & IC_{GB,2} \end{aligned}$$

The solution to this problem mimics the solution for the single agent case. First, the cheapest set of transfers that implements first best allocation is found. Second, the resulting transfers are plugged into the zero profit constraint to find the region where first best can be implemented.

There are two cases, depending on whether  $\delta \geq \delta_\psi$  or  $\delta < \delta_\psi$ .

**Case 1:**  $\delta \geq \delta_\psi$ .

First, it is easy to see that  $IC_{Gj,2}$ , for  $j \in \{B, G\}$ , must be binding. This implies  $\tilde{T} = \delta\psi$  (and since  $\delta \geq \delta_\psi$ ,  $t_{.1} = 0$ ). Setting  $T_{BB} + \delta u = T_{GG} = \delta\psi$  must be the cheapest way to satisfy the respective constraints. By the same logic,  $T_{GB} = T_{GG} = \delta\psi$ . Finally, the last constraint to be checked is  $T_{BG} + \delta u \geq \psi + e + \delta u$ , which implies  $T_{BG} = \psi + e < \delta(\psi - u)$ .

**Case 2:**  $\delta \leq \tilde{\delta}_\psi$

First, note that there is no loss in generality in setting  $IC_{Gj,2}$ , for  $j \in \{B, G\}$ , binding. This immediately implies  $\tilde{T} = t_{.,1} + \delta\psi$ . Setting  $T_{BB} = T_{BG} = \psi + e$  must be the cheapest way to satisfy the respective constraints. By the same logic,  $T_{GG} = \psi + e + \delta u$ . Finally, the last constraint to be checked is  $T_{GB} \geq \psi + \delta u$ , which implies  $T_{GB} = \psi + \delta u < \psi + e + \delta u$ .

This concludes the proof of the proposition. ■

**Proof of Proposition 7.** It can be shown in a way similar to the proof of Proposition 6 (group lending, fixed size of projects) that for given projects  $k_1$  and  $k_2$ , the cost minimizing transfers are

$$\begin{cases} T_{GG} = (\psi + e)k_1 + \delta u \\ T_{GB} = \psi k_1 + \delta u \\ T_{BB} = T_{BG} + (\psi + e)k_1 \end{cases}, \text{ if } \delta \leq \tilde{\delta}_\psi$$

$$\begin{cases} T_{GG} = T_{GB} = \delta\psi k_2 \\ T_{BG} = (\psi + e)k_1 \\ T_{BB} = \delta(\psi k_2 - u) \end{cases}, \text{ if } \delta \geq \tilde{\delta}_\psi$$

The problem (7) becomes

$$\max_{k_1, k_2} W = f(k_1) - k_1 + \rho\delta(f(k_2) - k_2) + (1 - \rho)(-ek_1 + \delta u)$$

s.t.

$$ZP_1 : (\psi + e)k_1 - \rho(1 - \rho)ek_1 + \rho\delta u \leq f(k_1) - k_1 + \rho\delta(f(k_2) - k_2), \text{ if } \delta \leq \tilde{\delta}_\psi$$

$$ZP_2 : \delta\psi k_2 - \rho(1 - \rho)[\delta\psi k_2 - (\psi + e)k_1] - (1 - \rho)\delta u \leq f(k_1) - k_1 + \rho\delta(f(k_2) - k_2), \text{ if } \delta \geq \tilde{\delta}_\psi$$

To find the region when the first-best  $k_1^*$  and  $k_2^*$  proceed as in the proof of Proposition 9. To find distortions, write a Lagrangian for each of the two cases as in the proof of Proposition 4.

When  $\delta \leq \tilde{\delta}_\psi$ ,

$$\frac{\partial \mathcal{L}_1}{\partial k_1} = (f'(k_1) - 1)(1 + \lambda_1) - (1 - \rho)e - \lambda_1(\psi + e) + \lambda_1\rho(1 - \rho)e = 0$$

$$\frac{\partial \mathcal{L}_1}{\partial k_2} = \rho\delta(f'(k_2) - 1)(1 + \lambda_1) = 0$$

Then,  $k_2^{GL} = k_2^*$  and since  $(1 - \rho)e < \frac{(1-\rho)e+\lambda_1(\psi+e)}{(1+\lambda_1)} < \frac{(1-\rho)e+\lambda_1(\psi+e)-\lambda_1\rho(1-\rho)e}{(1+\lambda_1)}$ ,  $k_1^* > k_1^{GL} > k_1^{IN}$ .

When  $\delta \geq \tilde{\delta}_\psi$ ,

$$\begin{aligned}\frac{\partial \mathcal{L}_2}{\partial k_1} &= (f'(k_1) - 1)(1 + \lambda_2) - (1 - \rho)e - \lambda_2\rho(1 - \rho)(\psi + e) = 0 \\ \frac{\partial \mathcal{L}_2}{\partial k_2} &= \rho\delta(f'(k_2) - 1)(1 + \lambda_2) - \lambda_2\delta\psi + \lambda_2\rho(1 - \rho)\delta\psi = 0\end{aligned}$$

Then, since  $\frac{(1-\rho)e+\lambda_2\rho(1-\rho)(\psi+e)}{1+\lambda_2} > \frac{(1-\rho)e}{1+\lambda_2}$ ,  $k_1^{GL} < k_1^{IN}$  (note that  $k_1^{GL} \leq k_1^*$ ) and since  $\frac{\lambda_2\psi}{\rho(1+\lambda_2)} > \frac{\lambda_2\psi-\lambda_2\rho(1-\rho)\psi}{\rho(1+\lambda_2)} > 0$ ,  $k_2^{IN} < k_2^{GL} < k_2^*$ . ■

**Proof of Proposition 8.** The incentive compatibility and truth telling constraints for the three types are respectively given by:

For the  $G$  type :

$$\begin{array}{ll} T_G \geq \psi + \delta u & IC_{G,1} \\ T_G \geq T_D + \delta u & TT_{GD} \\ T_G \geq T_B + \delta(1 - \mathbf{I}_B)u & TT_{GB} \\ t_{G,2} \geq \psi & IC_{G,2} \\ t_{G,i} \geq 0 & LL_{G,i} \end{array}$$

For the  $D$  type :

$$\begin{array}{ll} T_D + \delta u - s \geq \psi + e + \delta u - s & IC_{D,1} \\ T_D + \delta u - s \geq t_{G,1} - s + \delta(\max\{t_{G,2} - e, \psi\} - s) & TT_{DG} \\ T_D + \delta u - s \geq t_{B,1} - s + \delta \mathbf{I}_B(\max\{t_{B,2} - e, \psi\} - s) + (1 - \mathbf{I}_B)\delta u & TT_{DB} \\ t_{D,i} \geq 0 & LL_{D,i} \end{array}$$

For the  $B$  type :

$$\begin{array}{ll} T_B + \delta(\mathbf{I}_B\psi + (1 - \mathbf{I}_B)u) \geq \psi + e + \delta u & IC_{B,1} \\ T_B + \delta(\mathbf{I}_B\psi + (1 - \mathbf{I}_B)u) \geq t_{G,1} + \delta(\max\{t_{G,2} - e, \psi\}) & TT_{BG} \\ T_B + \delta u \geq t_{D,1} + \delta u & TT_{BD} \\ t_{B,i} \geq 0 & LL_{B,i} \end{array}$$

where  $\mathbf{I}_B = 1$  if the  $B$  type gets a project in period 2 and  $\mathbf{I}_B = 0$  otherwise. It is straightforward to show that a  $D$  type is never given a loan in the second period. Similarly, since we focus on the case in which  $\delta \geq \tilde{\delta}_\psi$ , we do not consider allocations in which some types do not repay the loan in period 1. As in the proof of proposition 1, those allocations would be implemented only for  $\delta$  sufficiently low. We therefore focus on allocations in which: i) the  $G$  type always get a loan and repays it in periods 1 and 2, ii) the  $D$  type repays the loan in period 1 but does not get a loan in period 2. Depending on the investment of the  $B$  type,

there are only three allocations that satisfy the two properties:

*Allocation A1: the B type does not get a loan in period 2,*

*Allocation A2: the B type does get a loan in period 2 and repays it,*

*Allocation A3: the B type does get a loan, but does not repay it.*

As in the proof of proposition 1, we first compute the minimum transfers required to implement each allocation, and derive the set of parameters in which each allocation can be implemented. Once this is done, the allocation implemented in equilibrium immediately follows from the fact that allocation *A1*, the first best, is implemented whenever feasible; allocation *A2* is implemented whenever it is feasible but *A1* is not; and finally *A3* is implemented if *A1* and *A2* are not feasible.

Following the reasoning in the proof of Lemma 2, it is easy to show that the minimal total transfers to type  $i \in \{B, D, G\}$  required to implement allocation  $a \in \{1, 2, 3\}$ ,  $T_{i,a}^*$ , are given by:

$$A1: \begin{cases} T_{G,1}^* = \delta\psi \\ T_{B,1}^* = \delta(\psi - u) \\ T_{D,1}^* = \delta(\psi - u) \end{cases}, \quad A2: \begin{cases} T_{G,2}^* = \delta(\psi + e) \\ T_{B,2}^* = \delta(\psi + e) \\ T_{D,2}^* = \psi + e \end{cases}, \quad A3: \begin{cases} T_{G,3}^* = \delta\psi \\ T_{B,3}^* = 0 \\ T_{D,3}^* = \psi + e \end{cases}$$

It is immediate to see that allocation *A1*, the first best, can be implemented in the same region as in Proposition 1. Similarly, it is easy to show that allocation *A2* cannot be implemented in the region under consideration.<sup>36</sup> Finally, allocation *A3* can be implemented if

$$\rho\delta\psi + (1 - \lambda - \rho)(\psi + e) \leq (1 + \rho\delta)(r - 1) - \delta\lambda. \quad (8)$$

Define  $\rho_\lambda = \frac{\lambda}{r-1-\psi}$  and  $\rho_e = (1 - \lambda) - \frac{r-1}{e+\psi}$ . Allocation *A3* can always be implemented if  $\rho \geq \max\{\rho_e, \rho_\lambda\}$  and never if  $\rho \leq \min\{\rho_e, \rho_\lambda\}$ . If  $\rho_e > \rho_\lambda$ , *A3* can be implemented if  $\delta \geq \delta_{A3}(\rho, \lambda)$ ; with  $\delta_{A3}$  implicitly defined by (8). If, instead,  $\rho_e < \rho_\lambda$ , *A3* can be implemented if  $\delta \leq \delta_{A3}(\rho, \lambda)$ .

In either cases,  $\rho_\lambda < \rho_u = \frac{\psi-u}{r-1-u}$  is a necessary and sufficient condition for the existence of a region in which *A3* is feasible but *A1* is not. Such a region is defined by  $\rho \in [\delta_{A3}^{-1}, \delta_{A1}^{-1}]$  and  $\delta \geq \delta_A^*$ , where  $\delta_A^*$  is implicitly defined by  $\delta_{A3}^{-1} = \delta_{A1}^{-1}$  and  $\delta_{A1} = \bar{\delta}_{FB}$ .

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<sup>36</sup>To see why this is the case, note that *A2* can only be implemented if  $(\psi + e)[(1 - \beta - \rho) + \delta(\beta + \rho)] \leq (r - 1)(1 + \delta(\rho + \beta))$ , i.e., if  $\delta \leq \delta_{A2} = \frac{(\psi+e)}{(\psi+e)-(r-1)} - \frac{1}{(\beta+\rho)} < \delta_\psi$ .

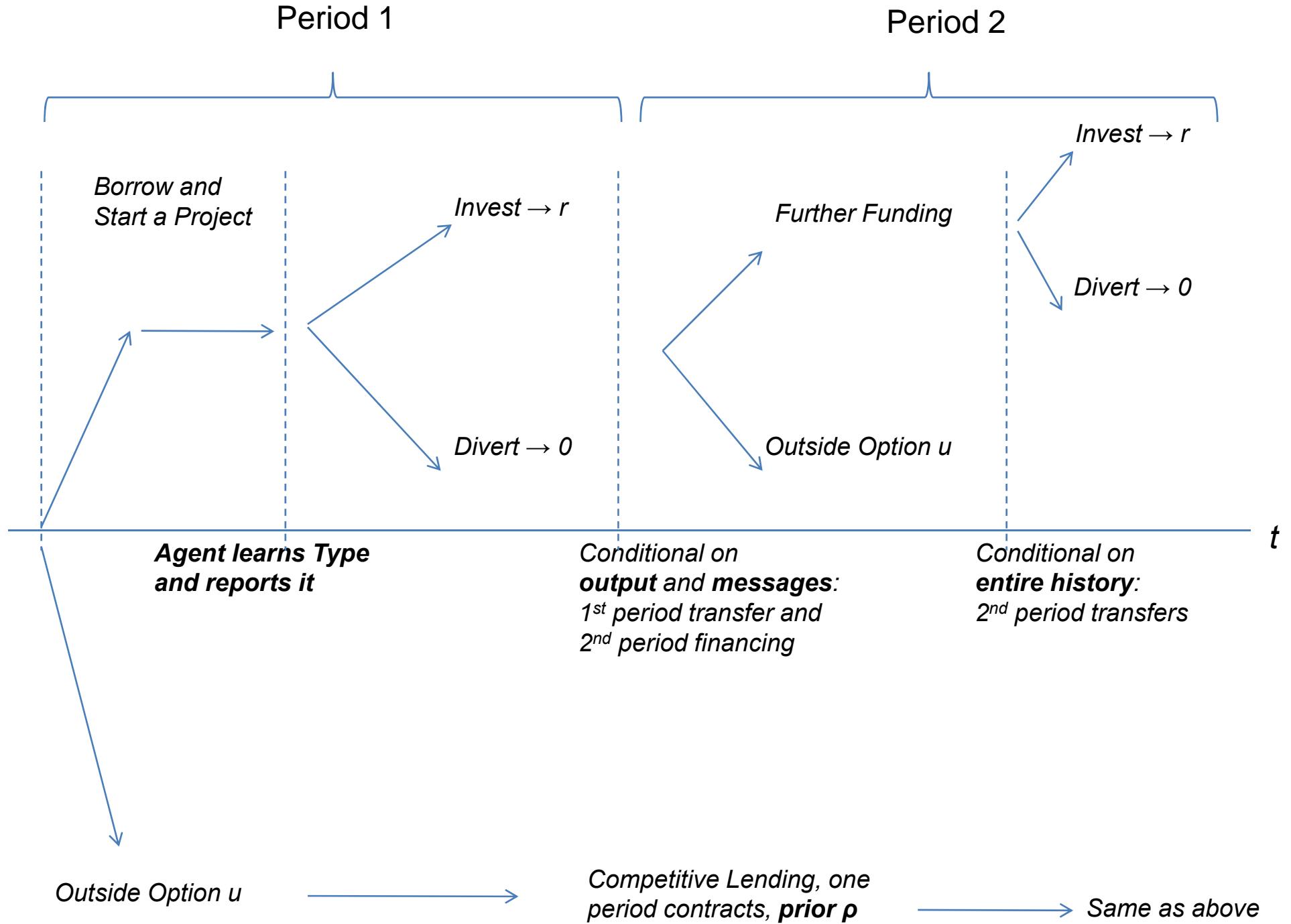
Finally, the comparative statics with respect to  $u$  directly follows from the fact that  $\delta_{A3}^{-1}$  is independent of  $u$ , while  $\frac{\partial \delta_{A1}^{-1}}{\partial u} = -\frac{\delta(r-1-\psi)+(r-1)}{\delta(u-r+1)^2} < 0$ . ■

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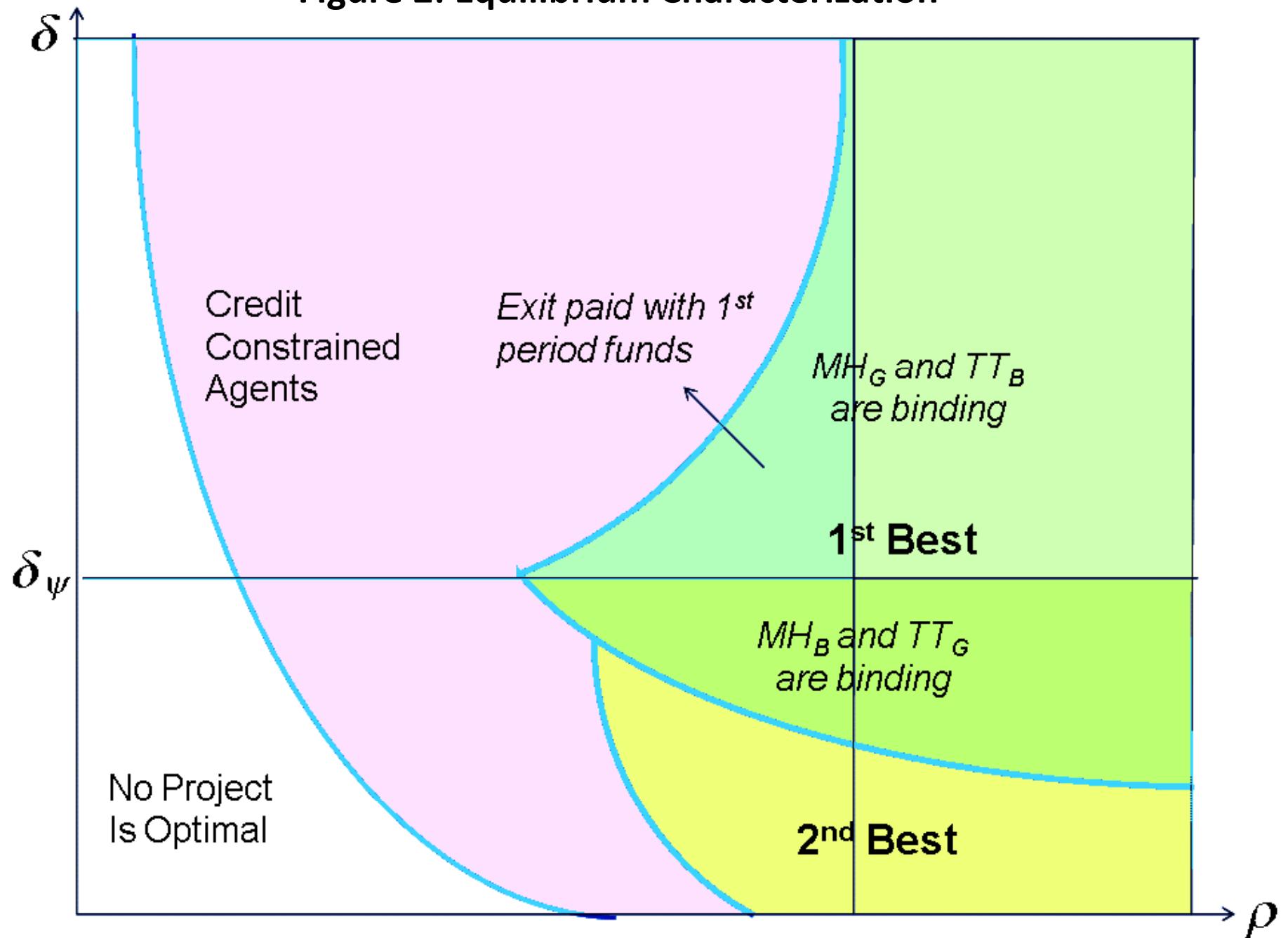
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**Figure 2: Equilibrium Characterization**



**Table 1: Summary of Main Findings and Predictions**

<b>Panel A: Predictions on Behaviour</b>	<b>Panel B: Implications for Micro-Lending Schemes</b>		
<b>Low confidence <math>\rho</math> and wealth correlate with:</b>	<b>Contractual Elements</b>	<b>Sign / Relation.</b>	<b>Mechanism</b>
- High interest rates,	Compulsory Savings and Group Lending	(+) corr. complements	Selection, Low confidence $\rho$ Collusion
- High drop-out / default rates	Group Lending and Cons. Smoothing	(-) corr. $\downarrow$ , causal	Selection, Low confidence $\rho$ Collusion
- Incomplete Consumption Smoothing	Joint Liability, Loan Size and Scaling-Up	(-) corr. $\uparrow$ , causal	Selection, Low confidence $\rho$ Cross-reporting
- Flat Repayment Profiles	Group Lending and Tenure	(-) corr.	Learning
- Limited Scaling-up	Savings and Tenure	(+) corr.	Selection, Low confidence $\rho$
- Contemporaneous Borrowing and Saving at unfavourable interest rates,	Collateral Requirements and Tenure	(+) corr.	Adverse Selection
- Rejection of Larger Loans			
- Excessive retention of bad clients			
- Loans from moneylenders			
<b>→ behavioral patterns clustered / (+) correlated with each other</b>			