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ABSTRACT

Voting on Parametric Reforms of the Pay-As-You-Go Pension System*

We assess the political support for parametric reforms of the Pay-As-You-Go pension system following a downward fertility shock. Using a continuous time overlapping generations model, we show that, for a large class of utility functions, the majority of the population favor a cut in pension benefits over an increase in the contribution rate. Our framework also allows us to evaluate the political support for raising the retirement age and to determine how the timing of the different reforms affect their political support.

JEL Classification: D72 and H55
Keywords: fertility shock, parametric reforms and Pay-as-you-go

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1 Introduction

Our societies are becoming older and older. As reported by Gruber and Wise (1998), the ratio of individuals above 65 over individuals aged between 20 and 64 will approximately double from year 2000 to 2050 in most developed countries. The sources of population aging lie in two demographic phenomena: rising life expectancy and declining fertility. For almost eighty years, fertility rates have been declining in both Europe and the US. This drop has been quite dramatic, falling from around 3 children per woman in 1920 or 1930 to the current levels of 1.2 to 2 children per woman, depending on the country. This decline has not however been steady over time. In the post world war II period, fertility rates have been increasing suddenly in all industrialized countries (baby boom) and have started to fall again in the mid 60s. Since the beginning of the 80s, fertility rates has been more or less stable (Boldrin et al. (2005)).

This demographic trend puts an increasing pressure on the financial viability of Pay-As-You-Go (PAYG) systems: for given parameters of the system, there are not enough contributors to finance the pensions of the baby-boomers. These systems thus need to be reformed. We consider in this paper parametric reforms of the PAYG system, that consist in adjusting its key parameters. To deal with population aging, one can raise the contribution rate, decrease the level of benefits and/or increase the retirement age. As argued by Cremer and Pestieau (2000), this problem is mainly political. Any reform induces some conflicts of interest between generations. Old workers and retirees prefer to raise taxes rather than to cut benefits whereas young workers have opposite preferences. The retirees favor an increase in the retirement age, as they benefit from this change but do not support its cost, whereas some workers may oppose it. In these circumstances, how will these conflicts be resolved? How will the burden of the demographic transition be shared among the different generations?

Our aim is to provide some elements of answer to these questions. We consider a society that faces a downward fertility shock in a given period and needs to adjust its PAYG system to maintain its financial viability. Individuals are identical in all respects, except for their age. During working life, they contribute to the pension system. After retirement, they receive a pension benefit. We assume no savings, so that pensions is the only source of income for a retiree. We also allow for the possibility for the government to raise a lump-sum tax on all individuals, both workers and retirees.

Section 3 determines the preferences of the citizens with respect to the pension system in the steady state before the demographic shock. The optimal contribution rate of the workers is increasing with age and the retirees want the tax rate to be as high as possible. This result was first established by Browning (1975) and is by now standard in the literature on the political economy of pensions (see for example Sjoblom (1985)). We also determine the optimal retirement age among different age groups. We show that it is constant until a threshold age and then equal to the age of the individual considered.

Section 4 addresses the demographic transition. In a first step, we determine the values of the income taxes needed to balance the government budget constraint when no parameter of the pension system is altered. As the ratio of the number of workers to the number of retirees is decreasing over time, these taxes should increase with time. We next consider reforms of the PAYG system and first compare a marginal increase in the contribution rate with a marginal cut in benefits. We show that, when the utility function exhibits Non Increasing Absolute Risk Aversion and the coefficient of relative risk aversion is below 1, the majority of the voters prefer to cut benefits rather than to increase the contribution rate. The intuition behind this result is that, even though it yields a decrease in lifetime income, a drop in benefit leads to a repartition of lifetime income along the life-cycle that is more in line with the individuals' preferences.

The previous results concerned the reform of the payroll tax rate or the benefit level. What about raising the retirement age? It turns out that the political support among the workers for this reform is decreasing with age. More precise analytical results on the political support for this reform turns out to be difficult to obtain. We thus rely on numerical simulations. These simulations suggest that increasing the retirement age may receive a strong political support and be majority preferred to the other two reforms described before.

The reforms just described were assumed to take place contemporaneously to the shock. We envisage in a last section the possibility to defer the reform to a later date. We find that delaying the reform has two effects, of opposite direction. On the one hand, it changes the age of the individuals indifferent between implementing the reform or not. As time goes on, older people starts favoring a cut in benefits. This makes the political support for reducing pension benefits increase over time. On the other hand, those people who support this reform are the younger ones. Due to population ageing, their proportion is decreasing with time, leading to a lower political support. The way the political support for a reform evolves over time thus depends on the comparison of these two effects.

Related Literature

There exists a large literature, starting from Browning (1975) and surveyed in Galasso and Profeta (2002), that study analytically the majority voting determination of PAYG pension systems in the steady state.

Most of the studies that address the problem of social security reform are based on calibrated quantitative models in which the welfare effects of different reforms are calculated (see, among others, Conesa and Krueger (1999), Conesa and Garriga (2008a, 2009b), De Nardi et al. (1999), Nishiyama and Smetters (2007)). Moreover, these articles often focus on the transition to a fully funded system.

To our knowledge, the only papers that approach this question from an analytical standpoint are Breyer and Stolte (2001) and Gonzalez-Eiras and Niepelt (2008). Breyer and Stolte show that, when taxation is distortionary (endogenous labor supply), pensioners share the burden of the demographic change with the workers, meaning that pension benefits are decreased. Gonzalez-Eiras and Niepelt consider a two-period dynamic probabilistic voting model with logarithmic utility. Their main findings are that population ageing should lead to higher contribution tax rates and a larger share of pensions in GDP.

2 The model

2.1 Timeline and demographics

The model is a continuous time overlapping-generations model where each agent lives for an interval of size l. His life is divided between working life, from birth to age a, and retirement life, from age a to death at age l.

At a given point in time t, retirees comprehend the group of people older than a, those born from $t-l$ to $t-a$, and workers are those younger that a, those born from $t-a$ to t. We denote N_t^r the number of retirees and N_t^w the number of workers, both at time t.

2.2 Wages and taxes

All workers receive the same wage, normalized to one. There are two taxes: a linear contribution rate τ to finance a PAYG pension system and a lump-sum income tax to the general government budget. The proceeds of the income tax are used to complement the financing of pension benefits when the PAYG's budget is in deficit. The remaining revenues are, then, rebated lump-sum.[1] This assumption accounts for the supplementary role that general government revenues play on the financing of pensions. It will also be useful to allow some imbalancement in PAYG budget before reform takes place, enabling the model to study the timing of the reform. The income tax paid by the agents is denoted by T. Consumption at date t is thus $c_t^w = 1-\tau_t - T_t$ for a worker and $c_t^r = b_t - T_t$ for a retiree where b_t stands for pension benefits.

2.3 Preferences

The unique heterogeneity among agents is their birth date. The monetary disutility from working is assumed constant[2] and denoted by γ. Instantaneous utility of

[1] One could alternatively assume that the income tax is used to finance a public good, which provision is reduced in case of deficit on PAYG's budget.

[2] It is more common in the literature to assume that the disutility from working is increasing with age. Although this is likely to be true for manual workers, it is not so likely for intellectual workers: professors, administrators, ... One could also argue that the utility of leisure is decreasing with age: free time is worth more when younger. What matters for retirement decision is the

consumption $u(c)$ is assumed continuous, twice differentiable and strictly concave.

Assuming that the agents do not discount future consumption, the life-cycle utility of a worker and a retiree aged x at time t are respectively given by

$$U_t^w(x) \equiv \int_t^{t+a-x} \left(u\left(1-\tau_v-T_v\right)-\gamma\right) dv + \int_{t+a-x}^{t+l-x} u\left(b_v-T_v\right) dv \tag{1}$$

and

$$U_t^r(x) \equiv \int_t^{t+l-x} u\left(b_v-T_v\right) dv. \tag{2}$$

2.4 Resource constraint

The Government Budget Constraint (GBC) at any time t is

$$N_t^r b_t = N_t^w \tau_t + \left(N_t^w + N_t^r\right) T_t. \tag{3}$$

Equation (3) states that PAYG's benefits (Left Hand Side) must be completely financed from the proceeds of PAYG's contribution rate plus transfers from the general government budget at each time t (Right Hand Side). Note that workers pay PAYG's contribution rate (τ_t) and income tax (T_t) while retirees pay only the latter.

Assumption 1 *No deficit on PAYG budget in the initial steady state.*

This assumption implies that there is no need to supplement PAYG revenues with transfers from the general government budget: $T_t = 0$. The government budget constraint in the steady state before the fertility shock is thus

$$N_t^r b_t = N_t^w \tau_t. \tag{4}$$

Equation (5) below gives the well known formula that pension benefit is the product of the contribution rate times the inverse of the dependency rate[3] when the PAYG's

difference in utility levels when working with respect to when retired, and this difference may be increasing, constant or decreasing depending on the which effect dominates: the (possibly) increasing disutility from work or the decreasing utility of leisure. This paper takes no side in this discussion and assumes that the parameter γ that captures this difference is constant.

[3]The dependency rate is defined as the ratio of the number of retirees to the number of workers, N_t^r/N_t^w.

budget is balanced:

$$b_t = \tau_t \rho_t (n, a, l). \tag{5}$$

It makes explicit the fact that the dependency rate (and its inverse denoted by ρ) depends on fertility rate (n), total lifetime (l) and on the size of working life (a). Fertility rate and total lifetime are exogenous demographic parameters while the size of working life is an endogenous parameter of the PAYG system.

3 Initial steady state

In the initial steady state, the rate of population growth is constant and equal to n. The PAYG system in this steady state is characterized by the triplet $(\tau_{ss}, b_{ss}, a_{ss})$, chosen through the political process. We make no specific assumption about this political process. We study in the next section the reform of the system following a negative fertility shock, *for any given initial system* $(\tau_{ss}, b_{ss}, a_{ss})$.[4] But before, we study the preferences of the different individuals on the contribution rate and the benefit level in the initial steady state.

3.1 Preferences over the contribution rate

Using (1) and (2), the maximization problem of workers and retirees are given by the following equations where x stands for their age. The pension benefit given by the steady state resource constraint (5) is already substituted into the objective function. Retirees' preferences over the PAYG contribution rate are given by the following program:

$$\max_{\tau} U^r_{ss}(x) \equiv \int_t^{t+l-x} u(\rho_{ss}\tau)\, dv. \tag{6}$$

They vote for the maximal contribution rate since they do not contribute to PAYG anymore.

Workers' preferences over the PAYG contribution rate are given by the maxi-

[4] The only restriction we impose is that these parameters are fixed at a level which is optimal for some individual. In other words, they must belong to the Pareto set.

mization problem:

$$\max_{\tau} U_{ss}^{w}(x) \equiv \int_{t}^{t+a-x} \left(u\left(1-\tau\right)-\gamma\right) dv + \int_{t+a-x}^{t+l-x} u\left(\rho_{ss}\tau\right) dv \tag{7}$$

The relationship of optimal tax rates with respect to age is described in the following proposition.

Proposition 1 *The preferred contribution rate is strictly increasing in worker's age and it is maximal for retirees:*

$$\tau_{ss}^{*}(x) \begin{cases} \text{is strictly increasing in } x & , \text{ if } x \in [0, a_{ss}) \\ = 1 & , \text{ if } x \in [a_{ss}, l]. \end{cases}$$

Proof. In the steady state, everything inside the integrals is constant. Thus the problem reduces to

$$\max_{\tau} \left(a_{ss}-x\right)\left(u\left(1-\tau\right)-\gamma\right)+\left(l-a_{ss}\right)u\left(\rho_{ss}\tau\right)$$

which first-order condition is

$$-\left(a_{ss}-x\right)u'\left(1-\tau_{ss}^{*}\right)+\left(l-a_{ss}\right)\rho_{ss}u'\left(\rho_{ss}\tau_{ss}^{*}\right)=0. \tag{8}$$

Applying the implicit function theorem gives

$$\frac{d\tau_{ss}^{*}}{dx}=-\frac{u'\left(1-\tau_{ss}^{*}\right)}{\left(a_{ss}-x\right)u''\left(1-\tau_{ss}^{*}\right)+\left(l-a_{ss}\right)\rho_{ss}^{2}u''\left(\rho_{ss}\tau_{ss}^{*}\right)}>0.$$

■

This proposition is a generalization of the result already established in the literature for the three-periods overlapping generations case.[5] Older workers maximize over a reduced horizon, since they do not take into account their past contributions in their optimization problem. Another way to state this is to say that PAYG's return increases with worker's age, since its benefit is the same for all agents, but its cost is decreasing with age.

The profile of optimal contribution rates is represented on the figure below.

Note, for further reference, that the first-order condition for the contribution rate can be written as follows:

$$\frac{u'\left(1-\tau_{ss}^{*}\right)}{u'\left(\rho_{ss}\tau_{ss}^{*}\right)}=\frac{l-a_{ss}}{a_{ss}-x}\rho_{ss}. \tag{9}$$

[5] See Browning (1975) or Sjoblom (1985).

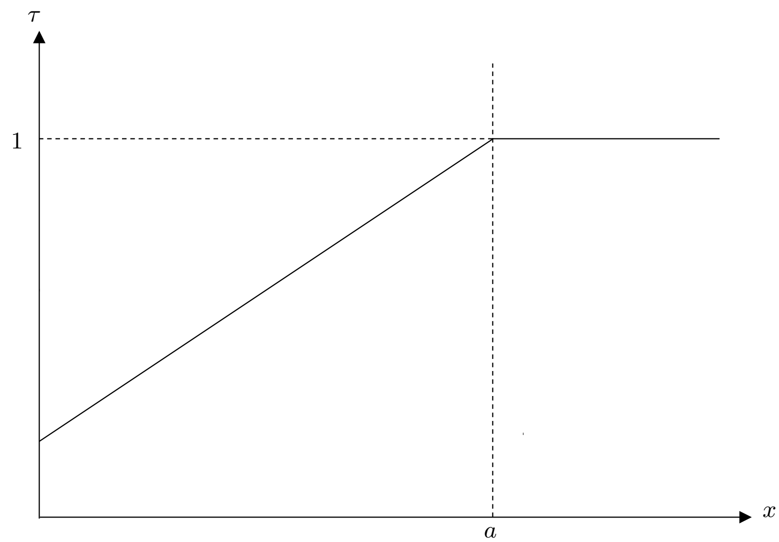

Figure 1: Preferences over the contribution rate by age cohort

This equation makes explicit the dependence of τ^* on worker's age. Moreover, it allows us to prove the following lemma, that will be useful later on.

Lemma 1 *In the steady state before the shock, all individuals optimally choose to consume more during retirement than during working life:* $1 - \tau^*_{ss} < \rho_{ss}\tau^*_{ss}$.

Proof. By definition ρ_{ss} is the inverse of the dependency rate, thus it is equal to the rate of workers per retiree:

$$\rho_{ss}(a) \equiv \frac{N^w}{N^r} = \frac{\int_{t-a}^{t} e^{ns} ds}{\int_{t-l}^{t-a} e^{ns} ds} = \frac{1 - e^{-na}}{e^{-na} - e^{-nl}} = e^{nl}\frac{e^{na} - 1}{e^{nl} - e^{na}}. \tag{10}$$

It attains its lower bound, $a_{ss}/(l - a_{ss})$, when $n = 0$. Using (9), this implies that

$$\frac{u'(1 - \tau^*_{ss})}{u'(\rho_{ss}\tau^*_{ss})} > \frac{a_{ss}}{a_{ss} - x} \geq 1.$$

■

The result in this lemma may appear counterfactual. In reality, people tend to consume more during working life. This is however a direct consequence of the no discounting assumption. If the agents were to discount future consumption, we could well obtain the opposite conclusion.

3.2 Preferences over the retirement age

Each agent chooses the retirement age $a_{ss}^{*}(x)$ that maximizes their remaining lifetime utility. This choice implies that the agent will be a worker, if $x < a_{ss}^{*}$, or a retiree, if $x \geq a_{ss}^{*}$. When making his choice, the agent must take into account that this retirement age will be mandatory to everyone in the PAYG system. A higher retirement age yields a smaller dependency ratio, this is summarized in the following lemma.

Lemma 2 *$\rho_{ss}(a)$ is strictly increasing in a, with $\rho_{ss}(0) = 0$ and $\lim_{a\to l} \rho_{ss}(a) = +\infty$.*

Proof. Inspecting (10), it is clear that $\rho_{ss}(0) = 0$ and $\lim_{a\to l} \rho_{ss}(a) = +\infty$. To prove that $\rho(a)$ is increasing in a, we compute

$$\frac{\partial \rho_{ss}(a)}{\partial a} = ne^{na}\left(\frac{e^{nl} + \rho_{ss}}{e^{nl} - e^{na}}\right) > 0.$$

■

If an agent aged x prefers to be retired, then he maximizes (6) with respect to a, subject to $a \leq x$. The solution to this problem is $a_{ss}^{*}(x) = x$. In words, he chooses the highest retirement age such that he is retired. The pension benefit is increasing with a because ρ_{ss} is increasing with a. Choosing $a_{ss}^{*} > x$ would violate the maximization constraint. On the other hand, choosing $a_{ss}^{*} < x$ would decrease the pension benefit since there would be more retirees to divide PAYG revenues and less workers to finance it.

If an agent aged x prefers to be a worker, then he maximizes (7) with respect to a, subject to $a > x$. The first-order condition of problem (7), taking into account the impact of a higher retirement age on the inverse of the dependency rate, is given by:

$$u(1-\tau_{ss}) - \gamma - u(b_{ss}) + (l - a_{ss}^{*})\, u'(b_{ss})\, \tau_{ss} \frac{\partial \rho_{ss}}{\partial a} = 0. \tag{11}$$

The first two terms capture the effect of a marginal increase in working life and the third term corresponds to the marginal decrease in retirement life. The last term captures the impact of a higher retirement age on the value of pension benefits. This term is not usually acknowledged in the endogenous retirement literature (Crawford and Lilien (1981), Sheshinski (1978)) because this latter deals with individual choice on retirement age, not extensible to other agents in the pension system. Here, the retirement age chosen by majority voting is applicable to all agents within the pension system. Lacomba and Lagos (2007) name this effect of a higher retirement age on the value of pension benefits as the macro effect. This effect is positive, implying that $u(1-\tau_{ss})-\gamma<u\left(b_{ss}\right)$. In words, agents get an increase in utility when they retire.

The results concerning the preferences over the retirement age in the steady state before the shock are summarized in the proposition below.

Proposition 2 *For a given payroll tax rate, the preferred retirement age of individuals younger than a_{ss}^{*} is a_{ss}^{*}. The preferred retirement age of an individual aged x, with $x\geq a_{ss}^{*}$ is x.*

The profile of preferred retirement ages is represented on the figure below.

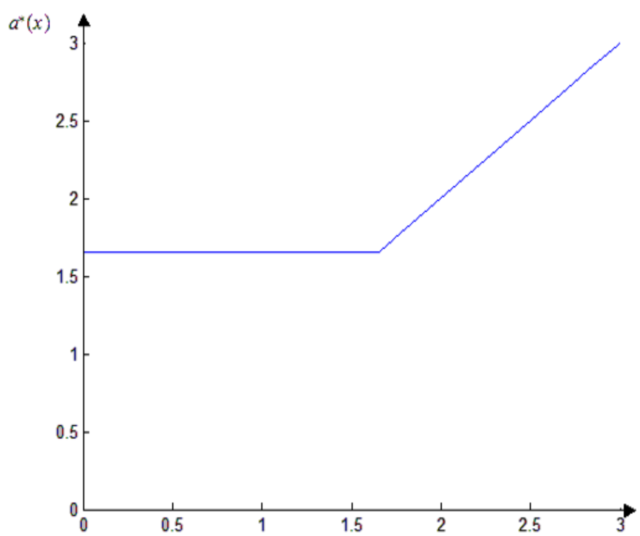

Figure 2: Optimal retirement ages according to age

4 Adjustment to fertility shock

4.1 Demographic transition

This section studies the political economy of PAYG's adjustment to a negative fertility shock. It assumes that instantaneous population growth rate is reduced from n to m at time t_o.

Assumption 2 *Instantaneous population growth rate is*
$$\begin{cases} n & , \text{for } t \in (\ -\infty, t_o) \\ m & , \text{for } t \in [\ t_o, +\infty) \end{cases} \quad \text{with } n > m.$$

This shock produces a long transition period and the new steady state is achieved only at time $t_o + l$.[6]

Lemma 3 *A decrease in the fertility rate creates a transition period of length l. During this demographic transition, ρ is strictly decreasing (the dependency rate is strictly increasing): $\dot{\rho}_t < 0$, for $t \in (t_o, t_o + l)$.*

Proof. See appendix. ■

Corollary 3 *If no reform is undertaken, then PAYG's per capita deficit is forever positive and strictly increasing during demographic transition: $T_t(\emptyset) > 0$, for $t > t_o$; $\dot{T}_t(\emptyset) > 0$, for $t \in (t_o, t_o + l)$, where $\emptyset$ stands for no reform.*

Proof. Solving (3) for T_t and using the equalities $(N_t^w + N_t^r)/N_t^r = 1 + \rho_t$ and $(N_t^w + N_t^r)/N_t^w = (1 + \rho_t)/\rho_t$ gives

$$T_t = \frac{1}{1 + \rho_t} b_t - \frac{\rho_t}{1 + \rho_t} \tau_t. \tag{12}$$

Substituting the steady state pension and contribution rate gives

$$T_t(\emptyset) = \frac{\rho_{ss} - \rho_t}{1 + \rho_t} \tau_{ss}. \tag{13}$$

Therefore, using lemma 3 ($\rho_{ss} > \rho_t$), we obtain that $T_t(\emptyset) > 0$, $\forall t > t_o$.

[6] If one assumes a life expectancy of 80 years, this implies that the transition would take 80 years to accomplish.

Computing $\dot{T}_t(\emptyset)$:

$$\dot{T}_t(\emptyset) = -\frac{\dot{\rho}_t(1+\rho_t) + (\rho_{ss} - \rho_t)\dot{\rho}_t}{(1+\rho_t)^2}\tau_{ss} > 0$$

because $\dot{\rho}_t < 0$ during transition and $\rho_{ss} > \rho_t$. ■

The lemma and corollary above are direct consequences of the fact that there are fewer workers to support retirees' pensions.

Figure 3 illustrates the adjustment path of ρ_t and T_t during the demographic transition.[7]

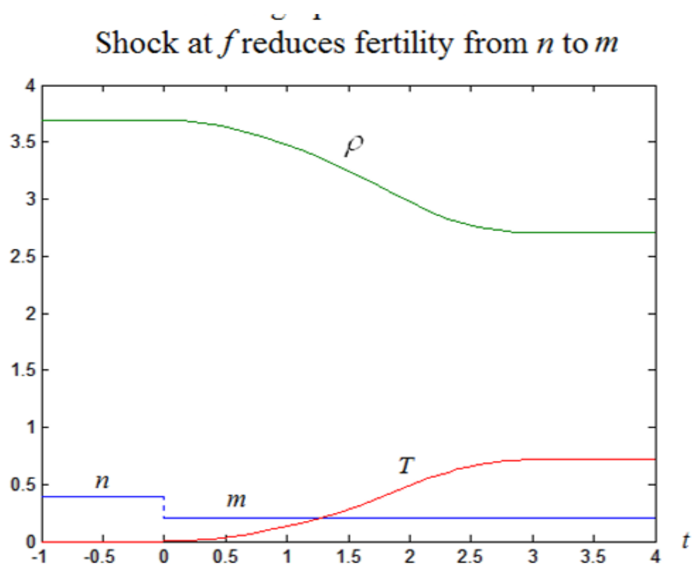

Figure 3: Demographic transition

The results above shed some initial light on the problem of reforming PAYG during demographic transition. If we want a reform that eliminates PAYG's deficit at a given time t_1, then immediately after this reform (at $t_1 + \epsilon$) the deficit would reappear. Conversely, if we want a reform that eliminates PAYG's deficit within a given time interval $[t_1, t_2]$, then PAYG budget necessarily would produce a superavit in the beginning of the interval and a deficit at its end.

[7]All graphs use the parameter values $a = 2$, $l = 3$ to simulate a 40-year working life followed by a 20-year retirement period (the unitary interval is equivalent to 20 years). Instantaneous population growth rates are $n = 0.396$ and $m = 0.199$, equivalent to annual growth rates of 2% and 1% respectively ($log(1.02^{20}) = 0.396$ and $log(1.01^{20}) = 0.199$). The value of T_t is multiplied by 10 in the graph.

4.2 The reforms set

To partially, or fully, counter the effects of an increasing dependency rate ($\dot{\rho}_t < 0$) agents may agree on reforming PAYG. The reforms' set include the full set of parametric reforms: increasing the contribution rate, increasing the retirement age and decreasing pension benefits.

Assumption 3 *The reforms' set is*

Reform	*Reform's content*
$\emptyset$	*No reform*
A	*Increase retirement* ***A****ge* ($\uparrow a$)
B	*Reduce pension* ***B****enefits* ($\downarrow b$)
C	*Increase* ***C****ontribution rates* ($\uparrow \tau$)

Reform $\emptyset$ is the default outcome when agents do not agree on which reform to implement. In this case, PAYG's parameters remain the same and everyone, including retirees, support its deficit. Reform B reduces pension benefits for current and future retirees, it is equivalent to introducing a tax on pensions. Reforms A and C correspond respectively to an increase of the retirement age and of the contribution rate.

Notice that the steady state problem had only two choice variables (a, τ) while the third PAYG parameter (b) was given by the budget-balanced condition (assumption 1). Now, there is one additional choice variable (b) since PAYG's budget might be unbalanced.

Reforms A, B or C reduce PAYG's deficit. Nevertheless, the deficit starts growing again until the next steady state (corollary 3). The PAYG's deficit plays two roles in this model. First, as already mentioned before, it allows some imbalancement in PAYG budget before reform takes place, enabling the model to study the timing of the reform. Second, if we required the PAYG budget to be balanced at each time t, then it would be necessary to permanently and continuously reform PAYG during the demographic transition.

4.3 Preferences over small reforms

In this section, we study marginal reforms of the pension system at time t_0.

4.3.1 Reducing pension benefits or increasing the contribution rate?

We first compare reforms B and C. The first reform consists in decreasing the benefit level from b_{ss} to $b_{ss} - db$ at time t_0 where db represents a marginal increment; the second reform consists in raising the payroll tax rate from τ_{ss} to $\tau_{ss} + d\tau$.

In the following proposition, we argue that any worker either prefers to increase the contribution rate or decrease the level of benefits, compared to the no reform case.

Lemma 4 *A worker of age x at t_0 either wants to decrease pension benefits or increase the payroll tax rate marginally. He cannot favor or reject both reforms at the same time.*

Proof. From (1), the lifetime utility of a worker aged x at t_0 under no reform is:

$$U_{t_0}^{w}(x) = \int_{t_0}^{t_0+a_{ss}-x} \left(u\left(1-\tau_{ss}-T_v\right)-\gamma\right)dv + \int_{t_0+a_{ss}-x}^{t_0+l-x} u\left(b_{ss}-T_v\right)dv.$$

The effect of a marginal increase of the payroll tax rate on his utility is

$$\frac{dU}{d\tau} = \int_{t_0}^{t_0+a_{ss}-x} \left(-1-\frac{\partial T_v}{\partial \tau}\right)u'(1-\tau_{ss}-T_v)dv + \int_{t_0+a_{ss}-x}^{t_0+l-x} -\frac{\partial T_v}{\partial \tau}u'\left(b_{ss}-T_v\right)dv.$$

From (12):

$$\frac{\partial T_v}{\partial \tau} = -\frac{\rho_v}{1+\rho_v},$$

and thus

$$\frac{dU}{d\tau} = \int_{t_0}^{t_0+a_{ss}-x} \left(-\frac{1}{1+\rho_v}\right)u'(1-\tau_{ss}-T_v)dv + \int_{t_0+a_{ss}-x}^{t_0+l-x} \frac{\rho_v}{1+\rho_v}u'\left(b_{ss}-T_v\right)dv. \quad (14)$$

To determine the impact of a marginal decrease in pension benefits, we note that, from (12),

$$\frac{\partial T_v}{\partial b} = \frac{1}{1+\rho_v},$$

and compute:

$$\begin{aligned}
\frac{dU}{db} &= \int_{t_0}^{t_0+a_{ss}-x} \left(-\frac{\partial T_v}{\partial b}\right)u'(1-\tau_{ss}-T_v)dv + \int_{t_0+a_{ss}-x}^{t_0+l-x} \left(1+\frac{\partial T_v}{\partial b}\right)u'\left(b_{ss}-T_v\right)dv \\
&= \int_{t_0}^{t_0+a_{ss}-x} \left(-\frac{1}{1+\rho_v}\right)u'(1-\tau_{ss}-T_v)dv + \int_{t_0+a_{ss}-x}^{t_0+l-x} \frac{\rho_v}{1+\rho_v}u'\left(b_{ss}-T_v\right)dv.
\end{aligned}$$

The two expressions for $dU/d\tau$ and dU/db are identical. Hence the result. ■

Before stating the main results of this section, we prove two further lemmas.

Lemma 5 *(i)*

$$\partial(\frac{1}{1+\rho_t}u'(1-\tau_{ss}-T_t))/\partial t > 0;$$

(ii) Denote $R_r(x) = -xu''(x)/u'(x)$ the coefficient of relative risk aversion, then

$$\partial(\frac{\rho_t}{1+\rho_t}u'(b_{ss}-T_t))/\partial t \begin{cases} >0 & , \text{ if } R_r(.)>1 \\ =0 & , \text{ if } R_r(.)=1 \\ <0 & , \text{ if } R_r(.)<1 \end{cases}.$$

Proof.

Use (12) to substitute $1-\tau_{ss}-T_t = 1-\tau_{ss}(1+\rho_{ss})/(1+\rho_t)$ into $u'(1-\tau_{ss}-T_t)$. Differentiating,

$$\begin{aligned} & \frac{\partial(1/(1+\rho_t))u'(1-\tau_{ss}-T_t)}{\partial t} \\ = \quad & -\frac{\dot{\rho}_t}{(1+\rho_t)^2}\left(u'(1-\frac{1+\rho_{ss}}{1+\rho_t}\tau_{ss}) - \frac{1}{1+\rho_t}(1+\rho_{ss})\tau_{ss}u''(1-\frac{1+\rho_{ss}}{1+\rho_t}\tau_{ss})\right) > 0. \end{aligned}$$

Use (12) to substitute $b_{ss} - T_t = (\rho_t/(1+\rho_t))\tau_{ss}(1+\rho_{ss})$ into $u'(b_{ss}-T_t)$. Differentiating,

$$\begin{aligned} & \frac{\partial(\rho_t/(1+\rho_t))u'(b_{ss}-T_t)}{\partial t} \\ = \quad & \frac{\dot{\rho}_t}{(1+\rho_t)^2}\left(u'(\frac{\rho_t}{1+\rho_t}\tau_{ss}(1+\rho_{ss})) + \frac{\rho_t}{1+\rho_t}\tau_{ss}(1+\rho_{ss})u''(\frac{\rho_t}{1+\rho_t}\tau_{ss}(1+\rho_{ss}))\right). \end{aligned}$$

The expression above is positive (negative) if the coefficient of relative risk aversion is greater (lower) than 1.[8] ■

In the next lemma, we get more insight into the workers' preferences toward the reform.

Lemma 6 *The marginal benefit of increasing the contribution rate is increasing with age if the coefficient of relative risk aversion is lower than 1.*

[8] Admittedly, relative risk aversion is not the proper concept to use as there is no uncertainty in the problem under study. In fact, one should note that the coefficient of relative risk aversion corresponds to the elasticity of the marginal utility of consumption, which is more accurate in our framework. Abusing language, we shall continue to name this parameter relative risk aversion in the remainder of the paper.

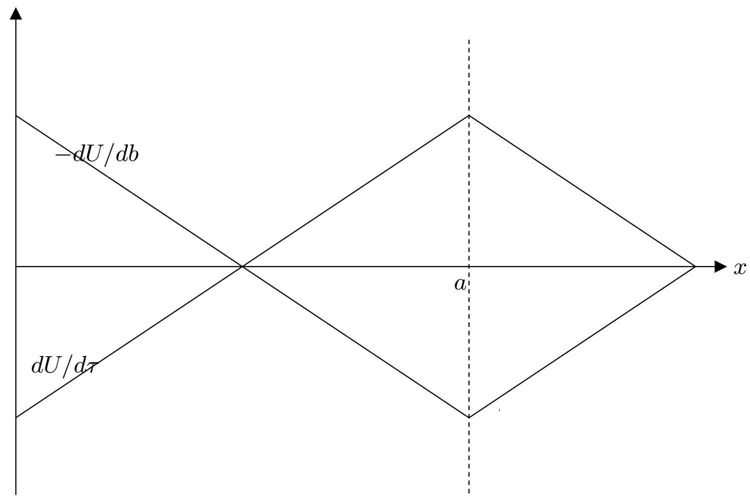

Figure 4: Welfare effects of reforms B and C

Proof.

Differentiating $dU/d\tau$ (given in (14)) with respect to x, we obtain

$$\begin{aligned}
\frac{d^2U}{dxd\tau} &= \frac{d}{dx}\left(\int_{t_0}^{t_0+a_{ss}-x}(-\frac{1}{1+\rho_v})u'(1-\tau_{ss}-T_v)dv + \int_{t_0+a_{ss}-x}^{t_0+l-x}\frac{\rho_v}{1+\rho_v}u'\left(b_{ss}-T_v\right)dv\right) \quad (15)\\
&= \frac{1}{1+\rho_{t_0+a_{ss}-x}}u'(1-\tau_{ss}-T_{t_0+a-x})\\
&\quad+\frac{\rho_{t_0+a_{ss}-x}}{1+\rho_{t_0+a_{ss}-x}}u'\left(b_{ss}-T_{t_0+a_{ss}-x}\right)-\frac{\rho_{t_0+l-x}}{1+\rho_{t_0+l-x}}u'\left(b_{ss}-T_{t_0+l-x}\right).
\end{aligned}$$

By lemma 5, the difference between the two last terms is positive for a relative risk aversion lower than 1. This condition is thus *sufficient* for having $d^2U/dxd\tau > 0$. ■

Understanding that the retirees always benefit from an increase in the tax rate and, conversely, are hurt by a cut in benefits, we can draw the preferences for small reforms B and C of all the living individuals at time t_0.

It is clear from figure 4 that the preferences of the majority of the voters depend on the relative position of the median age individual and the individual indifferent between the two reforms. Denote by x_τ the age of the voters whose optimal payroll tax rate in the steady state before the demographic shock correspond to the equilibrium tax rate in the steady state. We determine in the next proposition the preferences of individuals x_τ over reforms B and C for a given class of utility functions.

Proposition 4 *Suppose that the utility function exhibits Non Increasing Absolute Risk Aversion (NIARA). Then, if the coefficient of relative risk aversion is below 1, individuals x_τ favor a marginal decrease in pension benefits.*

Proof.

We write (14) for the individual aged x_τ:

$$\frac{dU}{d\tau} = \int_{t_0}^{t_0+a_{ss}-x_\tau} -\frac{1}{1+\rho_v} u'(1-\tau_{ss}-T_v)dv + \int_{t_0+a_{ss}-x_\tau}^{t_0+l-x_\tau} \frac{\rho_v}{1+\rho_v} u'\left(b_{ss}-T_v\right)dv.$$

Then we compute:

$$\begin{aligned} \frac{\partial}{\partial T}\left(\frac{u'(1-\tau_{ss}-T)}{u'\left(b_{ss}-T\right)}\right) &= \frac{-u''(1-\tau_{ss}-T)u'\left(b_{ss}-T\right)+u'(1-\tau_{ss}-T)u''\left(b_{ss}-T\right)}{u'\left(b_{ss}-T\right)^2} \\ &\geq 0 \text{ iff } -\frac{u''(1-\tau_{ss}-T)}{u'(1-\tau_{ss}-T)} \geq -\frac{u''\left(b_{ss}-T\right)}{u'\left(b_{ss}-T\right)}. \end{aligned}$$

Recall that $1-\tau_{ss} < b_{ss}$ (lemma 1). Therefore if the utility function is NIARA, $u'(1-\tau_{ss}-T)/u'\left(b_{ss}-T\right)$ is increasing with T. This implies in particular:

$$\frac{u'(1-\tau_{ss}-T)}{u'\left(b_{ss}-T\right)} \geq \frac{u'(1-\tau_{ss})}{u'\left(b_{ss}\right)}.$$

As x_τ obtains his optimal contribution rate in the steady state, we have from (9):

$$\frac{u'\left(1-\tau_{ss}\right)}{u'\left(b_{ss}\right)} = \frac{(l-a_{ss})}{(a_{ss}-x_\tau)}\rho_{ss}$$

and therefore

$$\frac{u'(1-\tau_{ss}-T)}{u'\left(b_{ss}-T\right)} \geq \frac{(l-a_{ss})}{(a_{ss}-x_\tau)}\rho_{ss},\ \forall T.$$

Coming back to the expression for $dU/d\tau$ and substituting the previous relationship, we have

$$\begin{aligned} \frac{dU}{d\tau} &\leq \frac{(l-a_{ss})}{(a_{ss}-x_\tau)} \int_{t_0}^{t_0+a_{ss}-x_\tau} -\frac{\rho_{ss}}{1+\rho_v} u'\left(b_{ss}-T_v\right)dv + \int_{t_0+a_{ss}-x_\tau}^{t_0+l-x_\tau} \frac{\rho_v}{1+\rho_v} u'\left(b_{ss}-T_v\right)dv \\ &< \frac{(l-a_{ss})}{(a_{ss}-x_\tau)} \int_{t_0}^{t_0+a_{ss}-x_\tau} -\frac{\rho_v}{1+\rho_v} u'\left(b_{ss}-T_v\right)dv + \int_{t_0+a_{ss}-x_\tau}^{t_0+l-x_\tau} \frac{\rho_v}{1+\rho_v} u'\left(b_{ss}-T_v\right)dv, \end{aligned}$$

where we have used the fact that $\rho_{ss} > \rho_v$ (lemma 3).

Using lemma 5, we have that, when relative risk aversion is lower than 1,

$$\begin{aligned} -\frac{\rho_v}{1+\rho_v}u'\left(b_{ss}-T_v\right) &< -\frac{\rho_{t+a_{ss}-x_\tau}}{1+\rho_{t+a_{ss}-x_\tau}}u'\left(b_{ss}-T_v\right), \forall v < t+a_{ss}-x_\tau \\ \frac{\rho_v}{1+\rho_v}u'\left(b_{ss}-T_v\right) &< \frac{\rho_{t+a_{ss}-x_\tau}}{1+\rho_{t+a_{ss}-x_\tau}}u'\left(b_{ss}-T_v\right), \forall v > t+a_{ss}-x_\tau \end{aligned}$$

and thus

$$\begin{aligned} \frac{dU}{d\tau} &< \frac{(l-a_{ss})}{(a_{ss}-x_\tau)}\frac{(a_{ss}-x_\tau)}{(l-a_{ss})}\left(-\frac{\rho_{t+a_{ss}-x_\tau}}{1+\rho_{t+a_{ss}-x_\tau}}u'(b_{ss}-T_{\rho_{t+a_{ss}-x_\tau}})\right) \\ &+ \frac{\rho_{t+a_{ss}-x_\tau}}{1+\rho_{t+a_{ss}-x_\tau}}u'(b_{ss}-T_{\rho_{t+a_{ss}-x_\tau}}) = 0. \end{aligned}$$

■

This result is somewhat surprising. It says that, for a large class of utility functions (that includes in particular the commonly used isoelastic utility function), the majority of the voters prefer to cut benefits rather than to increase the payroll tax rate. This latter reform however generates a larger lifetime income than the former. To see this, consider a marginal increase in τ of size $d\tau$. An individual aged x incur a loss of income equal to

$$d\tau(a_{ss}-x)$$

and a benefit

$$d\tau\int_{t_0}^{t_0+l-x}\frac{\partial T_v}{\partial\tau}dv = d\tau\int_{t_0}^{t_0+l-x}\frac{\rho_v}{1+\rho_v}dv,$$

generating a net loss of income

$$d\tau\left(a_{ss}-x-\int_{t_0}^{t_0+l-x}\frac{\rho_v}{1+\rho_v}dv\right).$$

On the other hand, when pension benefits are cut by an amount db, the net loss of income is

$$db\left(l-a_{ss}-\int_{t_0}^{t_0+l-x}\frac{1}{1+\rho_v}dv\right).$$

With positive population growth, the lower bound on ρ_v is $a_{ss}/(l-a_{ss})$. Therefore

$$d\tau\left(a_{ss}-x-\int_{t_0}^{t_0+l-x}\frac{\rho_v}{1+\rho_v}dv\right) < d\tau\left(\frac{a_{ss}}{l}-1\right)x \leq 0$$

and

$$db\left(l - a_{ss} - \int_{t_0}^{t_0+l-x} \frac{1}{1+\rho_v} dv\right) > db(l - a_{ss})x \geq 0.$$

In words, reform C generates an increase in lifetime income whereas reform B leads to lower income. This does not however imply that individuals will favor such a reform over the status-quo, as proposition 4 demonstrates. The cost of this reform is that it may allocate consumption at different dates in an undesirable way for the individual. There is thus a trade-off between the increase in life-cycle income and the distortion of the consumption pattern. The way this trade-off is resolved depends crucially on the shape of the utility function.

Note finally that proposition 4 holds when the coefficient of relative risk aversion is below 1. For a larger relative risk aversion, it may well be that the majority of the voters prefer reform C. In this case, the "higher income" effect of reform C would dominate the misallocation of income over the life-cycle it induces.

We now specialize our result to the case where the "decisive" voter in the steady state x_τ is the median age individual.

Corollary 5 *Suppose that x_τ is equal to the median age at time t_0. Then, if the coefficient of relative risk aversion is lower than 1, the majority of the population have the same preferences toward the reform as x_τ.*

Proof. If a marginal increase in τ is beneficial to x_τ, then, by lemma (6), it is also beneficial to every older worker. Observing that the retirees are always favorable to an increase in τ compared to the no reform case, we can conclude that a majority of the voters benefit from a marginal increase in the tax rate.

Conversely, if $\partial U/\partial\tau$ is negative for x_τ, it is also negative for all individuals younger than x_τ, meaning that a marginal reduction of the pension benefit is beneficial to a majority of the voters. ■

4.3.2 Increasing the retirement age

We now turn to the reform of the retirement age. Obviously, the retirees always favor an increase in the retirement age, as it allows to decrease the tax T at no cost for them.

Reform A increases the retirement age from a_{ss} to $a_{ss}+da$ while keeping the contribution rate and pension benefits at their steady state level. To assess the effect of a marginal increase in a at time t_0 on the life-cycle utility of a worker aged x, we differentiate (1) with respect to a:

$$\begin{aligned}\frac{dU}{da} &= u(1-\tau_{ss}-T_{t_0+a_{ss}-x})-\gamma-u\left(b_{ss}-T_{t_0+a_{ss}-x}\right)\\ &\quad-\left(\int_{t_0}^{t_0+a_{ss}-x}\frac{\partial T_v}{\partial a}u'(1-\tau_{ss}-T_v)dv+\int_{t_0+a_{ss}-x}^{t_0+l-x}\frac{\partial T_v}{\partial a}u'\left(b_{ss}-T_v\right)dv\right).\end{aligned} \tag{16}$$

As far as the workers are concerned, the cost of increasing the retirement age, for τ and b given, is $u\left(1-\tau_{ss}-T\right)-\gamma-u\left(b_{ss}-T\right)$. On the other hand, the benefit corresponds to the decrease in T. By differentiating (16) with respect to x, we evaluate how the net benefit for a worker of increasing the retirement age varies with age:

$$\begin{aligned}\frac{d^2U}{dxda} &= (\dot{T}_{t_0+a_{ss}-x}+\frac{\partial T_{t_0+a_{ss}-x}}{\partial a})(u'(1-\tau_{ss}-T_{t_0+a_{ss}-x})-u'\left(b_{ss}-T_{t_0+a_{ss}-x}\right))\\ &\quad+\frac{\partial T_{t_0+l-x}}{\partial a}u'\left(b_{ss}-T_{t_0+l-x}\right).\end{aligned} \tag{17}$$

We know from lemma 1 that $1-\tau_{ss}<b_{ss}$. Therefore the first term on the right hand side has the same sign as $\dot{T}_{t_0+a_{ss}-x}+\partial T_{t_0+a_{ss}-x}/\partial a$. We prove in the appendix that it is negative. Observing that $\partial T_t/\partial a<0$ and thus that the last term is negative, we can conclude that $d^2U/dxda$ is negative. This leads to the next lemma.

Lemma 7 *The marginal benefit of postponing retirement is decreasing with age.*

We now want to sign expression (16). Consider an individual close to retirement at time t_0 (x close to a_{ss}); for this individual, (16) becomes:

$$\begin{aligned}\frac{dU}{da} &= u(1-\tau_{ss}-T_{t_0})-\gamma-u\left(b_{ss}-T_{t_0}\right)-\int_{t_0}^{t_0+l-a_{ss}}\frac{\partial T_v}{\partial a}u'\left(b_{ss}-T_v\right)dv\\ &= u(1-\tau_{ss})-\gamma-u\left(b_{ss}\right)-\int_{t_0}^{t_0+l-a_{ss}}\frac{\partial T_v}{\partial a}u'\left(b_{ss}-T_v\right)dv,\end{aligned}$$

where we have used the fact that $T_{t_0}=0$.

We saw in section 3.2 that all the workers have the same optimal retirement age in the steady state before the shock. It is given by (11). Using this condition yields:

$$\frac{dU}{da} = -\left(l - a_{ss}\right) u'\left(b_{ss}\right) \tau_{ss} \frac{\partial \rho_{ss}}{\partial a} - \int_{t_0}^{t_0+l-a_{ss}} \frac{\partial T_v}{\partial a} u'\left(b_{ss} - T_v\right) dv.$$

From (12):

$$T_t = \tau_{ss} \frac{\rho_{ss} - \rho_t}{1 + \rho_t},$$

and thus

$$\frac{\partial T_t}{\partial a} = -\tau_{ss} \frac{\partial \rho_t}{\partial a} \frac{1 + \rho_{ss}}{(1 + \rho_t)^2}.$$

Differentiating (19) gives:

$$\begin{aligned} \frac{\partial \rho_t}{\partial a} &= n \frac{e^{n(t-a_{ss})}}{e^{n(t-a_{ss})} - e^{n(t-l)}} (1 + \rho_t) \\ &= n \frac{1}{1 - e^{n(a_{ss}-l)}} (1 + \rho_t), \end{aligned}$$

which implies

$$\frac{\partial T_t}{\partial a} = -\tau_{ss} n \frac{1}{1 - e^{n(a_{ss}-l)}} \frac{1 + \rho_{ss}}{(1 + \rho_t)}.$$

It thus appears that $\partial T_t / \partial a$ is increasing with ρ_t. As ρ_t is decreasing over time, we have:

$$0 > \frac{\partial T_{t_0}}{\partial a} > \frac{\partial T_{t_0+l-a}}{\partial a}.$$

Moreover, T_t is increasing over time. Therefore

$$0 < u'\left(b_{ss} - T_{t_0}\right) < u'\left(b_{ss} - T_{t_0+l-a_{ss}}\right).$$

These two inequalities imply that

$$\int_{t_0}^{t_0+l-a_{ss}} \frac{\partial T_v}{\partial a} u'\left(b_{ss} - T_v\right) dv > \left(l - a_{ss}\right) \frac{\partial T_{t_0}}{\partial a} u'\left(b_{ss} - T_{t_0}\right).$$

Observing that $T_{t_0} = 0$ and

$$\frac{\partial T_{t_0}}{\partial a} = -\tau_{ss} \frac{\partial \rho_{ss}}{\partial a} \frac{1}{(1 + \rho_{ss})},$$

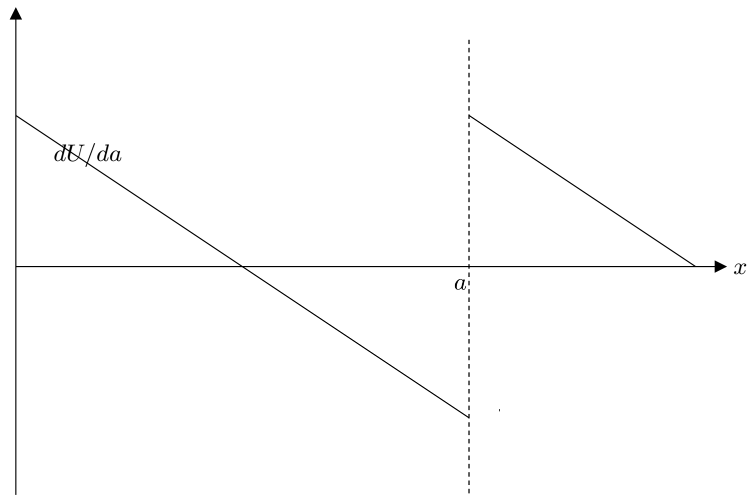

Figure 5: Welfare effects of reform A

we thus have

$$\frac{dU}{da} < -\left(l - a_{ss}\right) u'\left(b_{ss}\right) \tau_{ss} \frac{\partial \rho_{ss}}{\partial a} + (l - a_{ss}) u'\left(b_{ss}\right) \tau_{ss} \frac{\partial \rho_{ss}}{\partial a} \frac{1}{(1 + \rho_{ss})} < 0.$$

It should be emphasized that we have used expression (19) for developing our argument and this expression is only valid for $t \leq t_0 + a_{ss}$. Therefore this argument holds true as long as $l - a_{ss} < a_{ss}$, which means that individuals spend more time working than on retirement. Under this assumption, we obtain the following proposition.[9]

Proposition 6 *Assume $l - a_{ss} < a_{ss}$. Then the individuals close to retirement at t_0 are made worse-off by a marginal increase in the retirement age.*

This proposition, combined with lemma 7, implies that the preferences toward reform A of the living individuals at date t_0 can be represented as on figure 5.

We rely on numerical simulations (section 5) to get more insights about the political support for reform A and the comparison between reforms A, B and C.

[9] It can be shown that, when relative risk aversion is lower than 1, this result extends to the case $t > t_0 + a_{ss}$.

4.3.3 Discussion of the results

Assuming that the age of the agent indifferent to reforms C and A is the same, $x^C = x^A$, allows us to summarize the preference ranking over reforms in table 2.

Table 2. Preference ranking over reforms by age group: $x^C = x^A$

Age subscript	Age cohort	Preference ranking
y	Young workers	$\{A, B\} \succ_y \emptyset \succ_y C$
o	Old workers	$C \succ_o \emptyset \succ_o \{A, B\}$
r	Retirees	$\{A, C\} \succ_r \emptyset \succ_r B$

Table 2 shows that the main conflict about reform options is not between workers and retirees, but between young and old workers. Their preference rankings are exactly the opposite of each other. This opposition between young and old workers puts retirees in a convenient situation: they may ally with young workers to approve reform A or with old workers to approve reform C. We will see in section 5 (numerical simulations) that this indeed makes reform A very likely to be approved by a majority of voters.

Another fact worth mentioning is the difference in preferences between steady state and transition problems. Use (7) to compute

$$\frac{dU_{ss}^w}{d\tau} = \int_{t_0}^{t_0+a_{ss}-x} -u'(1-\tau_{ss})dv + \int_{t_0+a_{ss}-x}^{t_0+l-x} \rho_{ss} u'\left(b_{ss}\right) dv$$

Differentiating with respect to worker's age yields

$$\frac{d^2 U_{ss}^w}{dx d\tau} = u'(1-\tau_{ss})$$

which is just another way to demonstrate that, in steady state, a younger worker supports the cost of an increase in contribution rates for a longer time span, $u'(1-\tau_{ss})$, and gets the same benefit as an older worker. There are two differences between this and (15). First, the cost of increasing the contribution rate during transition is still supported for a longer time span, however younger workers benefit from the deficit reduction during their working life: the first term of (15) is multiplied by $1/(1+\rho_{t+a_{ss}-x})$ which is the fraction of retirees in the population. Second, younger workers get a slightly smaller benefit of increasing the contribution rate since they have to wait longer to retire and pension benefit is decreasing with time: the difference between the last two terms of (15) is positive for relative risk aversion smaller than one (note however that this difference becomes zero at the next steady state).

Doing the same thing for retirement age gives

$$\frac{dU_{ss}^w}{da} = u(1-\tau_{ss}) - \gamma - u\left(b_{ss}\right)$$

and

$$\frac{d^2U_{ss}^w}{dxda} = 0$$

which reflects the fact that all workers desire the same retirement age in steady state. This contrasts with (17) which is negative and shows that younger workers benefit more from an increase in retirement age during the demographic transition.

4.4 Timing of the reform

So far, the reform was assumed to take place at date t_0. We now consider the possibility that it happens at a later date $t > t_0$. Let us define x_t^i, $i = A, B, C$, as the age of the workers indifferent between implementing reform i at date t and no reform; x_t^C is implicitly determined by solving

$$\frac{dU}{d\tau} = \int\limits_{t}^{t+a_{ss}-x_t^C} -\frac{1}{1+\rho_v}u'(1-\tau_{ss}-T_v)dv + \int\limits_{t+a_{ss}-x_t^C}^{t+l-x_t^C} \frac{\rho_v}{1+\rho_v}u'\left(b_{ss}-T_v\right)dv = 0. \quad (18)$$

Differentiating this equation with respect to t, we can determine how x_t^C changes when reform C is postponed marginally. The following lemma states a sufficient condition on the utility function for this variable to be increasing with the timing of the reform.

Lemma 8 *x_t^C is increasing with t if the coefficient of relative risk aversion is lower than 1.*

Proof. The differentiation of (18) leads to

$$\dot{x}_t^C = \frac{(1/(1+\rho_t))u'(1-\tau_{ss}-T_t) - (1/(1+\rho_{t+a_{ss}-x_t^C}))u'(1-\tau_{ss}-T_{t+a_{ss}-x_t^C})}{-d^2U/dxd\tau}$$
$$+\frac{-(\rho_{t+a_{ss}-x_t^C}/(1+\rho_{t+a_{ss}-x_t^C}))u'(b_{ss}-T_{t+a_{ss}-x_t^C}) + (\rho_{t+l-x_t^C}/(1+\rho_{t+l-x_t^C}))u'(b_{ss}-T_{t+l-x_t^C})}{-d^2U/dxd\tau} > 0.$$

Lemma 5 implies that the numerator is negative and lemma 6 implies that the denominator is also negative, both when relative risk aversion is smaller than 1. Note again that this is just a *sufficient* condition. ■

People who support reform B are those younger than x_t^C, representing a proportion $F_t(x_t^C)$ of the total population. Symmetrically, by using lemma 4, the proportion of people supporting C is $1-F_t(x_t^C)$. The variation with time of the political support for these reforms is thus given by

$$\frac{\partial}{\partial t}F_t(x_t^C) = \dot{x}_t^C F_t'(x_t^C) + \dot{F}_t(x_t^C).$$

From lemma 8, the first term on the right hand side is positive, meaning that older individuals support reform B when time goes on. However the second term is negative: due to proportion of people younger than a given age is declining over time. These two effects are thus of opposite direction. The evolution over time of the political support for reforms B and C depends on which of the two dominates.

We now turn to reform A. Individuals x_t^A are defined implicitly by the condition

$$\begin{aligned}\frac{dU}{da} &= u(1-\tau_{ss}-T_{t+a_{ss}-x_t^A}) - \gamma - u(b_{ss}-T_{t+a_{ss}-x_t^A}) \\ &\quad - \left(\int_t^{t+a_{ss}-x_t^A} \frac{\partial T_v}{\partial a} u'(1-\tau_{ss}-T_v)dv + \int_{t+a_{ss}-x_t^A}^{t+l-x_t^A} \frac{\partial T_v}{\partial a} u'\left(b_{ss}-T_v\right)dv \right) = 0.\end{aligned}$$

Differentiating this expression, we obtain:

$$\begin{aligned}\dot{x}_t^A &= -\dot{T}_{t+a_{ss}-x}(u'(1-\tau_{ss}-T_{t+a_{ss}-x_t^A}) - u'(b_{ss}-T_{t+a_{ss}-x_t^A})) \\ &\quad + \frac{\partial T_t}{\partial a}u'(1-\tau_{ss}-T_t) - \frac{\partial T_{t+a_{ss}-x_t^A}}{\partial a}u'(1-\tau_{ss}-T_{t+a_{ss}-x_t^A}) \\ &\quad + \frac{\partial T_{t+a_{ss}-x_t^A}}{\partial a}u'(b_{ss}-T_{t+a_{ss}-x_t^A}) - \frac{\partial T_{t+l-x_t^A}}{\partial a}u'(b_{ss}-T_{t+l-x_t^A}).\end{aligned}$$

It can be shown that the sum of the first three terms on the right hand side is negative. However, one can also show that the sum of the last two terms is positive. Therefore the sign of $\dot{x}_t^A$ is ambiguous.

5 Numerical simulations

We have performed several numerical simulations with a isoelastic utility function. In the theoretical setting we have investigated so far, the discount rate was assumed to be 0 and results were demonstrated for a coefficient of relative risk aversion lower than 1. The first aim of these simulations consists in checking the robustness of our results when individuals discount the future and relative risk aversion is larger than 1. It turns out that the result in proposition 4 continues to hold in this case: reform B is majority preferred to reform C, unless the discount rate is very high and/or relative risk aversion is much larger than 1. This is illustrated in the figure below, in which we have drawn the effect of reforms B and C on the median voter's utility. In the region with a + (resp. -), the reform is beneficial (resp. detrimental) to the median voter.

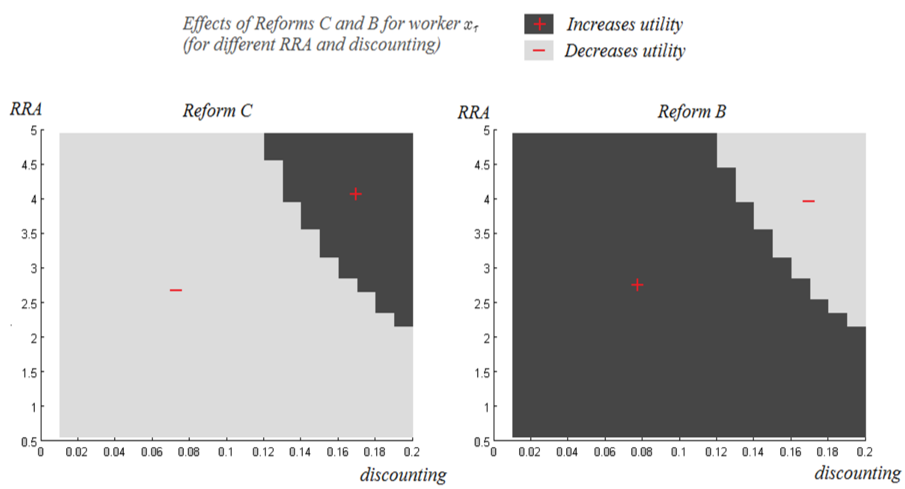

Figure 6: Effect of reforms B and C on the median voter

As far as reform A is concerned, we were not able to obtain unambiguous analytical results. The numerical examples suggest that the political support for this reform is very high, this reform being majority preferred to reforms A and B in all the simulations. Moreover, when relative risk aversion is lower than 1 and there is no discounting, the political support for reforms A and B increases over time whereas the political support for C is declining. This is illustrated on the graph below, that shows the evolution of the political support for the three possible reforms over time.

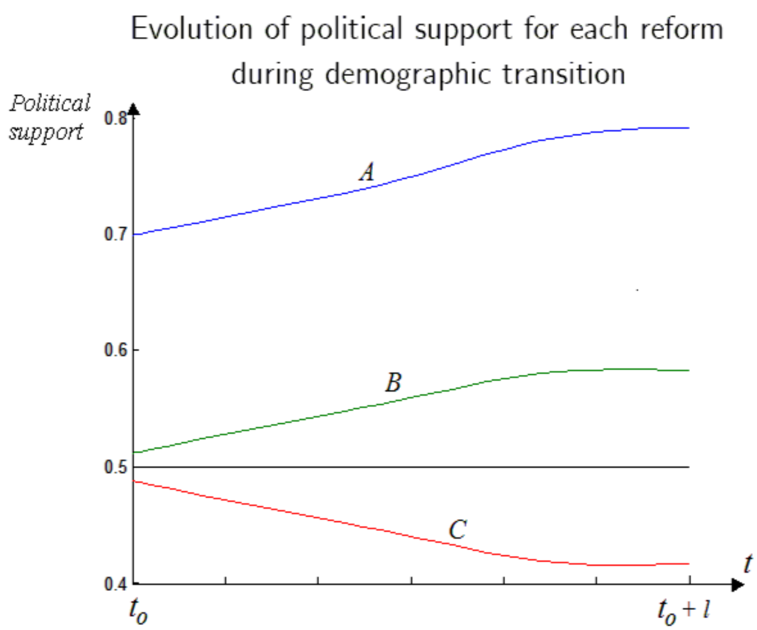

Figure 7: Evolution of political support over time

6 Conclusion

This paper contributes to a better understanding of the political support for different parametric reforms of the PAYG pension system, when confronted to a downward fertility shock.

Its first contribution is about the effects of a downward fertility shock. This shock produces a long lasting demographic transition that takes a whole lifetime to accomplish. During this transition, PAYG's deficit is strictly increasing and creates the need of repeatedly reforming PAYG until the next steady state. An important second point is that the fertility shock reduces everybody's welfare (including retirees, who also support PAYG's deficit) and the reforms imply an additional loss for some age group. This might help explaining why it is so difficult to carry out these reforms.

The comparison of steady state and transition problems shows that their natures are quite different. In steady state, workers disagree on contribution rates and agree on retirement age (propositions 1 and 2). This agreement on retirement age no longer holds during demographic transition, since younger workers benefit more from an increase in a than older workers. The disagreement over increasing the contribution rate is also different from steady state's problem and it is probably reduced.

Concerning preferences over reforms, we have shown that the majority of the population is more likely to approve reform B (cut in benefits) than reform C (increase in the contribution rate). However, numerical simulations suggest that these two reforms are dominated (in the sense of receiving less political support) by reform A (increase in the retirement age). This latter reform indeed garners the votes of a large fraction of the workers and of all the retirees. In these numerical simulations, we also observe that the political support for reforms A and B (resp. C) increases (resp. decreases) over time.

This analysis relies on a number of simplifying assumptions, that should be relaxed in future research. Two extensions deserve a special attention. First, we assume no savings and this affects the preferences of the agents toward the reform. Besides adding an important ingredient to the picture, the introduction of savings would allow us to consider the move, at least partial, to a fully funded system. Second, taxation of labor income does not create distortions in this model. One can guess that the political support for increasing the payroll tax rate would be undermined if such tax distortions were taken into account.

Technical appendix

A Proof of lemma 3

Assumption 2 implies that the number of agents born at t is

$$\begin{cases} e^{nt} & \text{, from } -\infty \text{ to } t_o \\ e^{nt_o}e^{m(t-t_o)} & \text{, from } t_o \text{ to } +\infty. \end{cases}$$

a) Beginning of transition: $t \in [t_0, t_0 + a]$

$$\rho_{begin}(t) \equiv \frac{\int_{t-a}^{t_o} e^{nv}dv + e^{(n-m)t_o}\int_{t_o}^{t} e^{mv}dv}{\int_{t-l}^{t-a} e^{nv}dv} = \frac{e^{nt_o-nt} - e^{-na} + \frac{n}{m}e^{nt_o-nt}\left(e^{m(t-t_o)} - 1\right)}{e^{-na} - e^{-nl}}. \tag{19}$$

To prove that ρ_{begin} is decreasing:

$$\dot{\rho}_{begin}(t) = \frac{e^{nt_o-nt}\left(\left(\frac{nm-n^2}{m}\right)\left(e^{m(t-t_o)} - 1\right) - n\right)}{e^{-na} - e^{-nl}} < 0.$$

b) End of transition: $t \in [t_0 + a, t_0 + l]$

$$\begin{aligned} \rho_{end}(t) &\equiv \frac{e^{(n-m)t_o}\int_{t-a}^{t} e^{mv}dv}{\int_{t-l}^{t_o} e^{nv}dv + e^{(n-m)t_o}\int_{t_o}^{t-a} e^{mv}dv} \\ &= \frac{e^{nt_o}\left(1 - e^{-ma}\right)}{\frac{m}{n}\left(e^{nt_o-mt+mt_o} - e^{n(t-l)-mt+mt_o}\right) + e^{nt_o}\left(e^{-ma} - e^{-mt+mt_o}\right)}. \end{aligned}$$

To prove that ρ_{end} is decreasing:

$$\dot{\rho}_{end}(t) = \frac{-e^{nt_o}\left(1 - e^{-ma}\right)\left(\frac{m}{n}\left(-me^{nt_o-mt+mt_o} - (n-m)e^{n(t-l)-mt+mt_o}\right) + me^{nt_o-mt+mt_o}\right)}{\left(\frac{m}{n}\left(e^{nt_o-mt+mt_o} - e^{n(t-l)-mt+mt_o}\right) + e^{nt_o}\left(e^{-ma} - 1\right)\right)^2}.$$

Multiplying by n/m gives

$$\frac{-e^{nt_o}\left(1 - e^{-ma}\right)(n-m)e^{mt_o-mt}\left(e^{nt_o} - e^{n(t-l)}\right)}{\frac{n}{m}\left(\frac{m}{n}\left(e^{nt_o-mt+mt_o} - e^{n(t-l)-mt+mt_o}\right) + e^{nt_o}\left(e^{-ma} - 1\right)\right)^2} < 0.$$

c) New steady-state

$$\rho_{nss}(t) \equiv \frac{e^{(n-m)t_o} \int\limits_{t-a}^{t} e^{mv} dv}{e^{(n-m)t_o} \int\limits_{t-l}^{t-a} e^{mv} dv} = \frac{\left(\frac{e^{mt}-e^{m(t-a)}}{m}\right)}{\left(\frac{e^{m(t-a)}-e^{m(t-l)}}{m}\right)} = \frac{1-\frac{1}{e^{ma}}}{\frac{1}{e^{ma}}-\frac{1}{e^{ml}}}.$$

B Proof of lemma 7

A sufficient condition for $d^2U/dxda < 0$ is $\dot{T}_{t_0+a_{ss}-x} + \partial T_{t_0+a_{ss}-x}/\partial a < 0$. From (12),

$$T_t = \frac{1}{1+\rho_t} b_t - \frac{\rho_t}{1+\rho_t} \tau_t.$$

Substitute the steady state pension and contribution rate to obtain

$$T_t = \frac{\rho_{ss} - \rho_t}{1+\rho_t} \tau_{ss}.$$

Then compute the derivatives (notice that ρ_{ss} is constant)

$$\begin{aligned}
\dot{T}_t &= \frac{-\dot{\rho}_t (1+\rho_t) - (\rho_{ss} - \rho_t) \dot{\rho}_t}{(1+\rho_t)^2} \tau_{ss} = -\dot{\rho}_t \left(\frac{1+\rho_{ss}}{(1+\rho_t)^2} \tau_{ss} \right), \\
\frac{\partial T_t}{\partial a} &= \frac{-(\partial \rho_t/\partial a)(1+\rho_t) - (\rho_{ss} - \rho_t)(\partial \rho_t/\partial a)}{(1+\rho_t)^2} \tau_{ss} = -\frac{\partial \rho_t}{\partial a} \left(\frac{1+\rho_{ss}}{(1+\rho_t)^2} \tau_{ss} \right).
\end{aligned}$$

It follows that $\dot{T}_{t_0+a_{ss}-x} + \partial T_{t_0+a_{ss}-x}/\partial a < 0$ iff $\dot{\rho}_{t_0+a_{ss}-x} + \partial \rho_{t_0+a_{ss}-x}/\partial a > 0$. Using (19), we have

$$\frac{\partial \rho_t}{\partial a} = ne^{-na} e^{-n(t-t_o)} \frac{1 + (n/m)(e^{m(t-t_o)} - 1)}{(e^{-na} - e^{-nl})^2} - ne^{-na} \frac{e^{-nl}}{(e^{-na} - e^{-nl})^2}$$

and

$$\dot{\rho}_t = \frac{\partial \rho_{begin}}{\partial t} = -ne^{-n(t-t_o)} \frac{1 + (n/m)(e^{m(t-t_o)} - 1)}{e^{-na} - e^{-nl}} + ne^{-n(t-t_o)} \frac{e^{m(t-t_o)}}{e^{-na} - e^{-nl}}.$$

Adding these two expressions and after some manipulations, we obtain:

$$\begin{aligned}
&\dot{\rho}_t + \frac{\partial \rho_t}{\partial a} \\
&= \frac{ne^{-nl} e^{-n(t-t_o)}}{(e^{-na} - e^{-nl})^2} \left((\frac{n}{m} - 1)(e^{m(t-t_o)} - 1) + (e^{nl} e^{m(t-t_o)} - e^{n(t-t_o)}) e^{-na} \right) > 0.
\end{aligned}$$

The first term within square brackets is clearly positive as $n > m$. The second term is positive even for $m = 0$. In this case, the expression within parenthesis becomes $e^{nl} - e^{n(t-t_o)}$ and this is positive since the upper bound for the difference $t - t_0$ is a.

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