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ABSTRACT

A Comparison of Debt and Primary-deficit Constraints*

This paper compares constraints on the public debt with constraints on the primary deficit. The analysis takes into account how an optimizing government reacts to the different constraints when deciding on a spending and borrowing plan. We find that the economy behaves similarly under both constraints, although for our benchmark calibration welfare is higher under the debt constraint. Further, the debt constraint is more robust against changes in the interest rate. Our results lend support to the enhanced focus on the public debt after the recent reform of the Stability and Growth Pact.

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1 Introduction

The recent reform of the European Stability and Growth Pact (SGP) has shifted some of the emphasis from public deficits towards public debt sustainability.[1] The reform ties in closely with the ongoing discussion about the choice between constraints on the public debt versus constraints on the public deficit (see e.g. Fatás et al., 2003, and Wyplosz 2005).

The existing literature provides no clear answer to this question. Common arguments in favour of debt constraints are that they reward fiscal discipline in good times, because there is a future benefit to reducing the debt when the economy is doing well; that they provide a better measure of the sustainability of the government's finances; and that they avoid inefficient budget cuts and tax increases that would be sub-optimal from a longer-term perspective when the economy falls victim to sudden (unexpected) adverse developments.[2] However, constraints on the public debt have the disadvantage that the current government, which faces the constraint, may at best only be partly responsible for the current debt level, while if the reference level of the debt substantially exceeds the actual debt level, the government may be tempted to spend resources on wasteful projects (by contrast a deficit rule binds more directly).

This paper compares the consequences of constraints on the public debt with constraints on the primary deficit.[3] Although the SGP imposes a constraint on the total deficit in European countries, we concentrate on the primary deficit, since this constraint is not affected by the level of the public debt in an economy, while the total deficit rises with the public debt via the interest payments on the debt.[4] In effect, a constraint on the total deficit would become tighter as the public debt increases.

Our framework is a simple open economy framework with income shocks and potentially myopic governments as a source of excessive spending. The government's impatience, which we use as a proxy for modelling finite planning horizons of governments in office, provides a rationale for imposing fiscal constraints either on the debt or on the primary deficit.

We find that the internalization of the penalties for the violation of the debt and primary-deficit constraints induces the government to resort to precautionary fiscal behavior. Thus, the constraints will not only affect fiscal policy in cases when they are actually binding, but will also have an effect on government expenditures and public debt when they are not binding. Debt constraints smooth public spending after an income

[1]For a review of the reform of the SGP and its consequences, see Buti et al. (2005), Morris et al. (2006) and Beetsma and Debrun (2007).

[2]The Netherlands and Germany provide examples in this respect. Not so long ago, the SGP forced these countries to make budget cuts, thereby reinforcing the adverse economic circumstances they were already facing.

[3]The paper limits itself to these two alternatives and does not aim at finding the generally optimal fiscal rule. It also does not analyze the optimal interaction between monetary and fiscal policy under constraints, as do Lambertini (2006) and Pappa and Vassilatos (2007).

[4]Assuming a primary-deficit constraint is also analytically more convenient .

shock. Further, the appropriate debt constraint is more robust to changes in the interest rate than is the appropriate deficit constraint. This suggests that a debt constraint might be easier to implement in practice, because it would be politically difficult to make frequent and large adjustments to the constraints. These results support the greater emphasis on debt of the SGP after its recent reform.

Variations in the model parameters yield further insights. A more myopic government implies a rise in average debt under a primary-deficit constraint, but a fall in average debt under a debt constraint. The latter is the result of precautionary behavior: by being close to the reference debt level, the government would run the risk of front-loading severe spending cuts when income is hit by a bad shock. This consequence is worse for a more myopic government in view of its preferred time profile of public spending. Hence, in the presence of a debt constraint such a government accumulates *less* debt on average. Further, also an increase in the variance of the endowment shocks or an increase in their persistence produces lower debt on average under a debt constraint (in order to maintain a larger safety margin relative to the reference debt level), but higher average debt under a primary-deficit constraint. Higher average debt corresponds to a higher average primary surplus, which implies a larger safety margin to the reference deficit level, as desired by the government. An increase in income persistence in turn increases the relative attractiveness of a primary deficit constraint because any effort exerted to adhere to the constraint now when income is hit by a bad shock has an extra pay off in the future, in that it will be easier to adhere to the constraint also in the future when income is also more likely to be low. Finally, an increase in risk aversion produces lower average debt under a debt constraint, but higher average debt under a primary-deficit constraint.

The rest of the paper is organized as follows. Section 2 describes the model and characterizes the equilibrium for the non-myopic government. In Section 3 we solve the model for the myopic government under each type of fiscal constraint. Section 4 derives the social welfare criterion. Next, Section 5 performs a numerical evaluation of the model calibrated to the EU situation and checks the robustness of the results for changes in the parameters. Finally, Section 6 concludes the paper. Derivations and technicalities are relegated to the Appendix. Sections F - H of the Appendix are available on http://www1.fee.uva.nl/toe/content/people/beetsma.shtm or available upon request from the authors.

2 The model

In this section we present a model of a small open economy. It is specified in real terms, such that neither money nor monetary policy is modelled here. Nevertheless, we interpret this country as a member of a monetary union, where its fiscal policy is constrained by unionwide constraints (as is the case for the countries that have adopted the Euro). Further, we assume that the constraints are credibly enforced.

2.1 The private sector

There exists a continuum of infinitely lived and identical households of total mass one. Their utility increases in consumption C_t and government spending G_t. The objective of a representative household is given by

$$\mathrm{E}_t \sum_{s=t}^{\infty} \beta_w^{s-t} \left[v(C_s) + u(G_s) \right], \tag{1}$$

where $\beta_w \in (0,1)$ denotes the discount factor, and E_t the expectations operator conditional on information at time t. The functions u and v are further assumed to be increasing in C and G, strictly concave, twice continuously differentiable, and to satisfy the usual Inada conditions.

In each period households are endowed with Y_t and with wealth in the form of one period risk-free government bonds, B_t, and foreign bonds, F_t. Households have to pay taxes on their endowment Y_t. As a result, the household flow budget constraint reads

$$C_t + B_{t+1} + F_{t+1} \leq (1 + r_{t-1}^H) B_t + (1 + r_{t-1}) F_t + (1 - \tau_t^y)\, Y_t, \tag{2}$$

where r_t denotes the exogenous real interest rate on foreign bonds, r_t^H the real interest rate on domestic bonds and τ_t^y the (income) tax rate. Maximizing (1) subject to no-Ponzi game conditions for domestic and foreign borrowing, $\lim_{t\to\infty} B_t \prod_{i=1}^{t} (1 + r_{i-1}^H)^{-1} \geq 0$ and $\lim_{t\to\infty} F_t \prod_{i=1}^{t} (1 + r_{i-1})^{-1} \geq 0$, and to (2) leads to the first-order conditions

$$\frac{v'(C_t)}{\mathrm{E}_t v'(C_{t+1})} = \beta_w (1 + r_t), \tag{3}$$

$$r_t^H = r_t, \tag{4}$$

and the transversality conditions $\lim_{t\to\infty} \mathrm{E}_0 F_t \prod_{i=1}^{t} (1 + r_{i-1})^{-1} = 0$ and $\lim_{t\to\infty} \mathrm{E}_0 B_t \prod_{i=1}^{t} (1 + r_{i-1}^H)^{-1} = 0$. We assume that the endowment follows an exogenous stochastic process

$$Y_t - \overline{Y} = \rho \left(Y_{t-1} - \overline{Y} \right) + \sigma_\varepsilon \varepsilon_t, \qquad \varepsilon_t \sim N(0,1), \tag{5}$$

where ε_t is independently, identically and normally distributed with mean zero and unit variance ($\mathrm{E}_t \varepsilon_s = 0$ and $\mathrm{E}_t \varepsilon_s^2 = 1$ for $s > t$). In addition, ρ satisfies $0 \leq \rho \leq 1$ and $\sigma_\varepsilon \geq 0$ is a known parameter.

2.2 The government

The government issues bonds, raises tax revenues $T_t = \tau_t^y Y_t$ and purchases goods G_t from the households. Without any further constraints, its period-by-period budget constraint reads $B_{t+1} = (1 + r_t) B_t + G_t - T_t$, where we have used that $r_t^H = r_t$. The government potentially discounts future periods at a higher rate than society does. More specifically, in the spirit of Grossmann and van Huyck (1998), we allow its discount factor β_g to deviate from the households' discount factor by the factor α:

$$\beta_g \equiv \alpha \beta_w, \text{ where } 0 < \alpha \leq 1.$$

The government then chooses the sequences $\{G_s\}_{s=t}^{\infty}$ and $\{B_s\}_{s=t}^{\infty}$, to maximize its objective function, which takes the form

$$\mathrm{E}_t \sum_{s=t}^{\infty} \beta_g^{s-t} \left[v(C_s) + u\left(G_s\right) \right]. \tag{6}$$

We can interpret the case of $\beta_g = \alpha\beta_w$ as a situation in which the government has each period a constant probability α of being in office, during which it receives a utility flow of $v(C_s) + u\left(G_s\right)$, and a probability of $(1 - \alpha)$ of not being in office, during which it receives a zero utility flow. The case of $\alpha < 1$ implies excessive borrowing and, hence, a potential role for borrowing constraints. In this regard we make two observations. First, a stable equilibrium might not exist in this case. Secondly, the equilibrium allocation might be inefficient compared to the case of an unbiased government (or a social planner) that shares the private sector's rate of time preference.

We consider two types of fiscal constraints that are intended to reduce the incentive for excessive borrowing, namely a constraint on the public debt and a constraint on the primary deficit. Such constraints do not come as a free lunch. Even if the government is not myopic, the constraints may sometimes lead to losses due to the possibility that unexpected, adverse macroeconomic shocks cause the constraints to become binding. In that case, it is simply the macroeconomic uncertainty rather than the opportunistic behavior of the government that gives rise to potential sanctions.

We assume that under a *debt* constraint, the government pays a fine if the stock of debt, B_t, exceeds some reference value, B^c, while no fine has to be paid otherwise. The government's period-t budget constraint is then modified into:

$$B_{t+1} = \left(1 + r_{t-1}\right) B_t + G_t - T_t + k^B \left(B_t - B^c\right) I_B \left[B_t; B^c\right], \tag{7}$$

where the parameter $k^B > 0$ captures the tightness of the constraint and $I_B \left[B_t; B^c\right]$ is an indicator function, such that $I_B \left[B_t; B^c\right] = 1$, if $B_t > B^c$, and $I_B \left[B_t; B^c\right] = 0$, otherwise.

The other constraint is expressed in terms of the primary deficit, which is defined as

$$D_{t+1} = B_{t+1} - \left(1 + r_{t-1}\right) B_t. \tag{8}$$

Under a primary deficit constraint, the government pays a fine if the primary deficit D_t exceeds some reference value D^c, while it pays no fine otherwise. The government's period-t budget constraint under this type of constraint becomes

$$B_{t+1} = \left(1 + r_{t-1}\right) B_t + G_t - T_t + k^D \left(D_t - D^c\right) I_D \left[D_t; D^c\right],$$

where $I_D \left[D_t; D^c\right] = 1$, if $D_t > D^c$, and $I_D \left[D_t; D^c\right] = 0$, otherwise. Further, parameter $k^D > 0$ captures the tightness of the constraint when D_t exceeds D^c. Using (8) we can rewrite the budget constraint as

$$D_{t+1} = G_t - T_t + k^D \left(D_t - D^c\right) I_D \left[D_t; D^c\right]. \tag{9}$$

Finally, the government taxes income at the constant and exogenously given rate τ^y, such that its tax revenues are

$$T_t = \tau^y Y_t. \tag{10}$$

Hence, given that taxes are exogenously determined, there is one degree of freedom left for the government, which is its choice of $\{G_s\}_{s=t}^{\infty}$. The debt path follows residually. We restrict our attention to fiscal constraints that induce the government plan to be consistent with the household plan in equilibrium. Thus, the parameters of the constraints $\left(k^B, B^c\right)$ or $\left(k^D, D^c\right)$ are always chosen to satisfy $\lim_{s\to\infty} \mathrm{E}_t B_s \prod_{i=1}^{s}(1+r_{i-1}^H)^{-1} = 0$, implying that fiscal solvency is guaranteed.

2.3 Equilibrium

We assume that the real interest rate on (foreign and domestic) bonds is exogenously determined and constant at a level r. Given that the tax revenues are exogenous, the optimal decisions of the private sector and the government are independent of each other. Domestic demand consists of private sector consumption, C_t, and government consumption, G_t. Income can further be invested in internationally traded assets, F_t, such that the resource constraint reads $Y_t = C_t + G_t + F_{t+1} - (1+r)F_t + k_t$, where $k_t = k^B\left(B_t - B^c\right) I_B\left[B_t; B^c\right]$ in the case of a debt constraint and $k_t = k^D\left(D_t - D^c\right) I_D\left[D_t; D^c\right]$ in the case of a primary deficit constraint.

Definition I *For a given endowment sequence $\{Y_t\}_{t=0}^{\infty}$, a constant tax rate τ^y, a constant interest rate r, and initial values F_0 and B_0, a rational expectations equilibrium consists of a set of sequences $\{C_t, G_t, B_t, F_t\}_{t=0}^{\infty}$ satisfying the utility-maximizing plan of the private sector, i.e. (3), and the transversality condition $\lim_{t\to\infty} E_0(1+r)^{-t}B_t = 0$; the plan of the government that maximizes (6), subject to (7) or (9); and the intertemporal resource constraint $F_0 = E_0 \sum_{t=0}^{\infty}(1+r)^{-t}\left(Y_t - C_t - G_t - k_t\right)$.*

To allow for the existence of a non-explosive equilibrium, we assume that the exogenous interest rate r satisfies $(1+r)\beta_w = 1$. We restrict our attention to rational expectations equilibria that are Markov perfect, such that the equilibrium decision rules and, hence, the outcomes depend on the current state of the economy only.

2.4 Equilibrium under a non-myopic government

To provide a benchmark for the subsequent analysis, we briefly examine the case, in which the government discounts future events at the same rate as households, $\beta_g = \beta_w$. It should be noted that the plan of the non-myopic government is consistent with the first best allocation that would be chosen by a social planner, since its objective equals household welfare and taxes are non-distorting. The government maximizes $\mathrm{E}_t \sum_{s=t}^{\infty} \beta_w^{s-t}\left[v(C_s) + u\left(G_s\right)\right]$ subject to its budget constraint and given tax revenues, T_s. Hence, the government's behavior in any period $s \geq t$ can be described by

$$\mathrm{E}_t u'(G_s) = \beta_w (1+r) \mathrm{E}_t u'(G_{s+1}). \tag{11}$$

Since $\beta_w = 1/(1+r)$, expenditures will therefore satisfy $u'(G_t) = \mathrm{E}_t u'(G_{t+j})$, $\forall\ j \geq 0$, implying a random walk behavior of government expenditures, G.[5] In equilibrium, the household first-order condition (3) must further be satisfied, which implies with $\beta_w = 1/(1+r)$ that $v'(C_t) = \mathrm{E}_t v'(C_{t+j})$, $\forall j \geq 0$. Thus, private consumption C also exhibits a unit root. Further, in equilibrium the intertemporal resource constraint, the intertemporal government budget constraint and the household's transversality condition must hold, implying

$$(1+r)B_0 = \mathrm{E}_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t (T_t - G_t). \tag{12}$$

Given these conditions, one can easily solve for the set of equilibrium sequences $\{C_t, G_t, B_t, F_t\}_{t=0}^{\infty}$ given $\{Y_t\}_{t=0}^{\infty}$ and the initial values B_0 and F_0 (see Appendix A). The state space solution form is linear and given by

$$B_{t+1} = B_t - \tau^y \frac{1-\rho}{1+r-\rho}\left(Y_t - \overline{Y}\right), \tag{13}$$

$$G_t = \tau^y Y_t \frac{r}{1+r-\rho} + \tau^y \overline{Y} \frac{1-\rho}{1+r-\rho} - rB_t, \tag{14}$$

$$F_{t+1} = F_t + \frac{1-\rho}{1-\rho+r}\left(Y_t - \overline{Y}\right), \tag{15}$$

$$C_t = (1-\tau^y)\frac{r}{r-\rho+1} Y_t + (1-\tau^y)\frac{1-\rho}{1+r-\rho}\overline{Y} + r\left(B_t + F_t\right). \tag{16}$$

Due to the unit root(s), the unconditional means are undetermined.

3 A myopic government

We maintain the assumption that $\beta_w = 1/(1+r)$, but now we assume $\beta_g < \beta_w$. In this case, the government tends to pre-draw government expenditure and will start borrowing from the private sector. We solve for the sequences of government expenditures and debt under our two types of fiscal constraints, which are intended to limit public borrowing.

3.1 Debt constraints

We start with the case in which the government faces a debt constraint. Under this constraint, the government maximizes (6) subject to (7). To simplify the original problem, which exhibits a discontinuity, we approximate the indicator function $I[B_t; B^c]$ with a

[5] If $G_t \neq \mathrm{E}_t G_{t+j}$, for $j > 0$, unconditional higher order moments of G_t and G_{t+j} must also differ to satisfy $u'(G_t) = \mathrm{E}_t u'(G_{t+j})$. This would introduce a non-recursive element in equilibrium, such that the state would not follow a Markov-process.

transition function, which allows us to apply standard local approximation (perturbation) methods. In particular, we apply the logistic function[6]

$$L_t^B \equiv L\left(B_t; \gamma, B^c\right) = \frac{1}{1+\exp\left(-\gamma\left(B_t - B^c\right)\right)}, \quad \gamma > 0. \tag{17}$$

When $\gamma \longrightarrow \infty$, $L\left(B_t; \gamma, B^c\right) \longrightarrow I\left[B_t; B^c\right]$. Hence, for high values of γ, the logistic function will be a good approximation to the indicator function. This alters the intertemporal budget constraint of the government under a debt constraint in the following way:

$$B_{t+1} = (1+r)\, B_t + G_t - T_t + k^B\left(B_t - B^c\right) L_t^B. \tag{18}$$

The myopic government maximizes (6) subject to (18) by choosing $\{G_t, B_{t+1}\}_{t=0}^{\infty}$. The first-order conditions can be combined to the following condition:

$$u'\left(G_t\right) = \beta_g E_t \left\{\left[1 + r + k^B L_{t+1}^B + \gamma k^B \left(B_{t+1} - B^c\right)\left(L_{t+1}^B - \left(L_{t+1}^B\right)^2\right)\right] u'\left(G_{t+1}\right)\right\}. \tag{19}$$

We see that the debt constraint affects the "effective" cost of issuing debt with the additional term $k^B L_{t+1}^B + \gamma k^B\left(B_{t+1} - B^c\right)(L_{t+1}^B - (L_{t+1}^B)^2)$. The Euler equation contains two additional elements compared to the first order condition of the unconstrained problem (11). The first element, $k^B L_{t+1}^B$, measures the additional marginal costs of each unit of debt that exceeds the reference value B^c. The second element, $\gamma k^B\left(B_{t+1} - B^c\right)\left(L_{t+1}^B - (L_{t+1}^B)^2\right)$, is the marginal effect of an increase in B_{t+1} on this marginal cost, multiplied by the factor $\left(B_{t+1} - B^c\right)$. In a sense, this term measures how an increase in debt raises the "probability" of hitting the constraint, reflecting the government's internalization of the fiscal constraint.

The set of conditions that characterize the equilibrium sequences $\{G_t, B_{t+1}\}_{t=0}^{\infty}$ under the debt constraint are given by (18), (19), and the terminal condition (see Appendix B)

$$\left(1 + r + k^B L_0^B\right) B_0 = \mathrm{E}_0 \sum_{t=0}^{\infty} \frac{\tau^y Y_t - G_t + k^B B^c L_t^B}{\prod_{v=1}^{t}\left(1 + r + k^B L_v^B\right)}, \tag{20}$$

given an initial value B_0.

3.2 Primary-deficit constraints

For the case of a primary deficit constraint, we apply an analogous approximation to the original indicator function using

$$L_t^D \equiv L\left(D_t; \gamma, D^c\right) = \frac{1}{1+\exp\left(-\gamma\left(D_t - D^c\right)\right)}, \quad \gamma > 0. \tag{21}$$

Rewriting the budget constraint in terms of the deficit, we can summarize the government's problem as the maximization of (6) subject to

$$D_{t+1} = G_t - T_t + k^D\left(D_t - D^c\right) L_t^D, \tag{22}$$

[6]The logistic function has been applied to various nonlinear models, see, for example, Bayoumi et al. (1995).

by choosing $\{G_t, D_{t+1}\}_{t=0}^{\infty}$. Using that the problem is continuous and recursive, the government's choice for D_{t+1} is characterized by the following first-order condition

$$u'(G_t) = \beta_g k^D E_t \left\{ \left[L_{t+1}^D + (D_{t+1} - D^c)\,\gamma \left(L_{t+1}^D - \left(L_{t+1}^D\right)^2 \right) \right] u'(G_{t+1}) \right\}. \qquad (23)$$

The set of conditions that characterize the equilibrium sequences $\{G_t, D_{t+1}\}_{t=0}^{\infty}$ under the deficit constraint are given by (22), (23), and the terminal condition (see Appendix C)

$$(1+r)\,B_0 = \mathrm{E}_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \frac{\tau^y Y_t - G_t + k^D D^c L_t^D}{1 - k^D L_t^D}, \qquad (24)$$

given an initial value B_0.

3.3 The solution

In order to solve the model when fiscal constraints are imposed and to evaluate welfare departures from the solution under a non-myopic government, we use a perturbation method following Judd (1998), Collard and Juillard (2001), and Schmitt-Grohé and Uribe (2004). For a generic variable X_t, we define its *deterministic steady state value* as its value when shocks are completely absent and denote this value $\overline{X}$. Further, we define (1) its *stochastic steady state value* as its unconditional expectation, $\widehat{X} \equiv \mathrm{E}[X_t]$, and (2) deviations from its deterministic steady state value as $\widetilde{X}_t \equiv X_t - \overline{X}$.

The values of government expenditures $\overline{G}$, debt $\overline{B}$ and deficit $\overline{D}$ in a deterministic steady state under the two fiscal constraints are found by solving the systems formed by the government budget constraint and the Euler equation under the respective constraints (see Appendices D and E). The values $\overline{G}$, $\overline{B}$ and $\overline{D}$ will generally differ, given that the two systems are generally different. Nevertheless, as we shall discuss in Section 5, we will always calibrate the parameters k^B and k^D and the reference values under the fiscal constraints, B^c and D^c, so as to obtain the same deterministic steady states under the two constraints.

3.3.1 The solution under a debt constraint

In the case where the myopic government faces a debt constraint, the solution has to satisfy the set of conditions (18), (19), and (20). In order to solve that system, we apply a second order Taylor expansion evaluated at the deterministic steady state. To this end, we postulate the existence of two auxiliary functions that describe the system's evolution as a function of the observable state variables $(B_t, Y_t, \sigma_\varepsilon)$ in period t. The format of the general solution for G_t and B_{t+1} is given by

$$G_t = f(B_t, Y_t, \sigma_\varepsilon), \qquad (25)$$

$$B_{t+1} = h(B_t, Y_t, \sigma_\varepsilon). \qquad (26)$$

Eliminating G_t with (18), (19) can be written as $0 = E_t[g(B_{t+2}, B_{t+1}, B_t, Y_{t+1}, Y_t, \sigma_\varepsilon)]$. The latter condition can, by using (26) and $B_{t+2} = h\left(h(B_t, Y_t, \sigma_\varepsilon), \rho Y_t + (1-\rho)\overline{Y} + \sigma_\varepsilon \varepsilon_{t+1}, \sigma_\varepsilon\right)$, be rewritten as a function of the state variables $B_t, Y_t, \sigma_\varepsilon$ and ε_{t+1} and the function $h(\cdot)$:

$$\mathrm{E}_t\left\{g\left(\begin{array}{c} h\left(h\left(B_t, Y_t, \sigma_\varepsilon\right), \rho Y_t+(1-\rho)\overline{Y}+\sigma_\varepsilon\varepsilon_{t+1}, \sigma_\varepsilon\right), \\ h\left(B_t, Y_t, \sigma_\varepsilon\right), B_t, \rho Y_t+(1-\rho)\overline{Y}+\sigma_\varepsilon\varepsilon_{t+1}, Y_t, \sigma_\varepsilon \end{array}\right)\right\}=0. \tag{27}$$

We then identify the approximated solution for debt (26) by taking first- and second-order Taylor expansions of (27). Appendix F describes in detail the derivation of the second-order approximation. The unconditional expectation of debt under the debt constraint will be denoted by $\widehat{B^B}$ and the unconditional variance of B by $\mathrm{Var}(B_t) \equiv \mathrm{E}(B_t - \widehat{B^B})^2$. The approximated solution of G_t is then derived using (18) and the solution for B_t. The unconditional expectation of government expenditure under the debt constraint will be denoted by $\widehat{G^B}$.

3.3.2 The solution under primary deficit constraint

We use an analogous procedure for the case of the primary deficit constraint to solve for the sequences of D_t and G_t satisfying the conditions (22), (23), and (24). The general solution form for G_t and D_t is given by

$$G_t = j\left(D_t, Y_t, \sigma_\varepsilon\right), \tag{28}$$

$$D_{t+1} = l\left(D_t, Y_t, \sigma_\varepsilon\right), \tag{29}$$

Eliminating G_t in (23) with (22), we can write $0 = E_t[i\left(D_{t+2}, D_{t+1}, D_t, Y_{t+1}, Y_t, \sigma_\varepsilon\right)]$. Using (29) and $D_{t+2} = l\left(l\left(D_t, Y_t, \sigma_\varepsilon\right), \rho Y_t + (1-\rho)\overline{Y} + \sigma_\varepsilon\varepsilon_{t+1}, \sigma_\varepsilon\right)$, we get

$$\mathrm{E}_t\left\{i\left(\begin{array}{c} l\left(l\left(D_t, Y_t, \sigma_\varepsilon\right), \rho Y_t+(1-\rho)\overline{Y}+\sigma_\varepsilon\varepsilon_{t+1}, \sigma_\varepsilon\right), \\ l\left(D_t, Y_t, \sigma_\varepsilon\right), D_t, \rho Y_t+(1-\rho)\overline{Y}+\sigma_\varepsilon\varepsilon_{t+1}, Y_t, \sigma_\varepsilon \end{array}\right)\right\}=0. \tag{30}$$

The approximated solution for the deficit (29) is found by taking first- and second-order Taylor expansions of (30) (derived in Appendix G). The means of D_t and G_t under the debt constraint will be denoted by $\widehat{D^D}$ and $\widehat{G^D}$, respectively.

4 Social welfare

To assess social welfare under the different fiscal constraints, we refer to household utility (1). Given that the consumption decision is independent of fiscal policy, we restrict our welfare analysis to the consequences of the constraints for government expenditures only. Specifically, we apply a second-order Taylor expansion of (1) at the deterministic steady state and use the solutions for the mean and the unconditional variance of government expenditures, $\widehat{G}$ and $\mathrm{Var}(G)$, respectively, to compute welfare under both types of constraints (see Appendix H). Hence, the welfare effects of government policy are measured by:

$$\begin{aligned} U_w^z &\approx \sum_{t=0}^{\infty}\beta_w^t\left[u\left(\overline{G}\right)+u'\left(\overline{G}\right)*\left(\widehat{G}^z-\overline{G}\right)+\frac{1}{2}u''\left(\overline{G}\right)*\mathrm{Var}\left(G^z\right)\right] \\ &\approx \frac{u\left(\overline{G}\right)+u'\left(\overline{G}\right)*\left(\widehat{G}^z-\overline{G}\right)+\frac{1}{2}u''\left(\overline{G}\right)*\mathrm{Var}\left(G^z\right)}{1-\beta_w}, \end{aligned} \tag{31}$$

where z denotes the type of fiscal constraint that is imposed, B (on debt) or D (on the primary deficit). Based on (31), we compare the welfare effects of both constraints.

5 Numerical evaluation

In this section we perform a numerical evaluation of the model and study public debt, government expenditures and welfare under the two fiscal constraints. We also analyze the consequences of changes in both the "deep" parameters and the policy parameters.

5.1 Calibration

We calibrate the model using average values for eleven members of the Eurozone, which we denote by "Euro-11".[7] The calibration is based on annual data from the OECD Economic Outlook (OECD, 2006, n^o. 79) for 1970-2006.

Table 1 summarizes the calibration of our benchmark specification. It describes the parameters, lists the values chosen for them and provides the motivation for these choices. For simplicity, we normalize average income, $\overline{Y}$, to 100 and set the income tax rate, τ^y, to 1. This choice is immaterial given that private sector behavior is not affected by public policy in any case. Initial public debt, B_0, is set at 63.14, which is the average debt/GDP ratios over all observations in our panel. The initial amount of foreign debt, F_0, is set to 0. The real interest rate, r, is fixed at 0.0262, which is the average ex-post long-term real interest rate over all observations in our panel. The income persistence parameter ρ is set at 0.264. It is obtained by taking log-deviations of real GDP from its trend for each country,[8] and estimating an AR-1 process on these constructed series for the Euro-11 over the period 1970-2006. We then take the non-weighted average of the estimated AR-1 coefficients, which provides us with our choice of ρ. The variance of the income shocks, $\sigma_\varepsilon^2 = 38.821$, is the non-weighted average for the Euro-11 of the sum of the squared residuals of the estimated AR-1 income process. As the benchmark value for α we choose 0.933, which implies $\beta_g = 0.910$. The term $1/(1-\alpha)$ can also be interpreted as a measure of the expected planning horizon of the politician (see Grossman and van Huyck, 1998, and Kumhof and Yakadina, 2007). Our benchmark value of α corresponds to an expected planning horizon of the government of 15 periods (the same value as in the calibration by Kumhof and Yakadina, 2007).

Further, utility from government expenditure is specified as

$$u(G) \equiv \frac{G^{1-\frac{1}{\mu}}}{1-\frac{1}{\mu}}, \tag{32}$$

where μ is the constant elasticity of intertemporal substitution. For the benchmark

[7]These countries are Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, The Netherlands, Portugal and Spain.

[8]We use the Hodrick-Prescott filter with smoothing parameter $\lambda = 100$.

parametrization we set μ equal to 0.7.[9] Finally, the parameter γ of the logistic function is set at 300. This value enables a good approximation of the indicator function in (7) and (9), as argued in Franses and Van Dijk (2000).

As regards to the policy parameters, we set k^B and k^D each at 100, implying very high penalties for violating the constraints. Then, we search for those reference values (B^c and D^c) for the debt, respectively primary deficit, constraints that yield as the deterministic steady state the level of the public debt that satisfies the intertemporal government budget constraints (20) and (24), i.e. $\overline{B^B} = \overline{B^D} = B_0 = 63.140$. Specifically, this results into a value of $B^c = 63.144$ under our benchmark parameter setting. For the primary deficit constraint, under the baseline parameter setting, the intertemporal government budget constraint (24) holds only when the reference value D^c is not higher than -1.649, implying that the reference value corresponds to a surplus.

5.2 Results

We are now ready to discuss our numerical results. First, we present our findings for the benchmark case. Next, we perform a sensitivity analysis in which we investigate how changes in the parameters affect welfare and the dynamics of the economy.

5.2.1 The benchmark case

Stochastic steady state Table 2 provides the results for our benchmark calibration. Making use of (8), Table 2 also reports the value $D^{Bc} = -1.653$ of the primary deficit that corresponds to the reference value B^c, and the level of the debt $B^{Dc} = 62.98$ that corresponds to the reference deficit level D^c. We separate the outcomes into four blocks. The first block presents the outcomes for a non-myopic government (i.e. $\beta_g = \beta_w$). Even though we consider a tax rate of 1 and initial net foreign debt of zero, because initial public debt is positive, $\overline{G}$ is smaller than $\overline{Y}$ and the associated long-run level of private consumption is positive ($\overline{C} = 1.65$). The corresponding primary surplus is then equal to the value of consumption.

The second and third blocks of Table 2 report the outcomes under a myopic government subject to a debt, respectively primary deficit, constraint. By construction, the deterministic steady states of debt under both types of fiscal constraints ($\overline{B^B}$ and $\overline{B^D}$) are equal to the initial value of the public debt B_0, leading to identical deterministic steady state levels of government expenditure under the myopic and non-myopic governments. Because the equilibrium outcomes of debt and deficit are exceeded by their respective reference values B^c and D^c and because we only approximate the true constraint, the calculated

[9] There is no consensus on the most appropriate value of μ. The value chosen here provides a coefficient of constant relative risk aversion close to 1.43, which falls within the range of values used in the literature. In particular, our benchmark value for μ is close to the value of 1.391 estimated for the Euro Area by Smets and Wouters (2003) and to the value of 1.5 assumed for the U.S. by Aiyagari and McGrattan (1998). In our sensitivity analyses we vary μ to check the robustness of our results for different degrees of relative risk aversion $1/\mu$.

equilibrium values for the fines, $k^B(B_t - B^c) L_B[B_t; B^c]$ and $k^D(D_t - D^c) L_D[D_t; D^c]$, respectively, are negative. Therefore, we add these negative fines to the deterministic and stochastic steady state values of government spending before reporting them. This way, we avoid the creation of resources "out of nothing" and we can compare welfare across the various cases that we consider.

Impulse responses Besides comparing steady state outcomes, we also investigate the responses of our variables to an income shock (i.e., we study the transition path to the steady state after an income shock). For the non-myopic government we do this using the system of linear dynamic equations (13) to (16), thus assuming that this government is not subject to any fiscal constraint. This yields the socially optimal solution, i.e. the solution that is optimal from the households' perspective. It can be used as a benchmark for comparison with the other solutions. To find the solutions for the myopic government, we employ our second-order approximations.

We calculate the impulse responses to a transitory negative income shock of one percent of GDP in period t ($\sigma_\varepsilon \varepsilon_t = -1$), assuming that the economy is initially in its deterministic steady state and assuming no further shocks. Hence, $Y_t = 99$. The impulse responses for income, public debt, government expenditure and the primary deficit for the non-myopic government, the myopic government under a debt constraint and the myopic government under a primary deficit constraint are displayed in Figures 1, 2 and 3, respectively. The dashed lines represent the deterministic steady states of the variables, whereas the solid lines show the impulse responses to the income shock. For the case of a non-myopic government we plot in addition the dynamics of the net foreign debt and of private consumption.

Under the non-myopic government (Figure 1), the negative income shock causes an immediate fall in the level of government expenditure in period t, which remains constant from then onwards. The fall in government spending is smaller than the initial drop in tax revenues and additional debt is accumulated until it reaches a new steady state level. This debt build up is consistent with the temporary decline in the primary surplus. However, the primary surplus converges to a new steady state level that is higher than before, consistent with government solvency (12). Private consumption remains constant throughout at its original steady state level. That is because the government and households can freely borrow or lend on the international capital market at a given interest rate, so that C_t is chosen independently of G_t. Therefore, the additional issuance of government bonds due to the shock in tax revenues combined with the constant level of private consumption implies an accumulation of foreign debt (a negative impulse response of F_t in Figure 1).[10]

For the myopic government facing a debt constraint (Figure 2), the negative income shock leads to a sharper decline in government expenditure in period t than for the non-myopic government. The reason is that the myopic government wants to avoid building up

[10]Since all private income is taxed away and taxes fall below government spending in the short run, foreign debt must rise.

debt and risking to violate its debt constraint. Under the debt constraint, the government abstains from excessive borrowing and will reduce its expenditures even more to meet the budget constraint. Over time, tax revenue and government expenditure converge almost back to the stochastic steady state level, which deviates only marginally from the initial, deterministic steady state. Hence, in the long run public debt and the primary deficit are only affected to a small extent.

For the myopic government under the primary deficit constraint (Figure 3), the negative income shock leads to a pattern of government expenditure that is similar to that under the debt constraint but with a slightly stronger contraction in period t. Again this pattern is generated by the desire to avoid violating the fiscal constraint. In effect, the primary deficit remains almost constant and close to D^c.

The results described so far provide the benchmark for the sensitivity analysis that we perform next. The benchmark setting is almost neutral in terms of welfare for the two types of fiscal constraint as we can observe from the fourth block of Table 2. There, we report the percentage difference $G^{pd} \equiv [(G_w^B - G_w^D)/G_w^D] * 100$ between the permanently constant public spending levels G_w^B and G_w^D that produce the social welfare levels U_w^B and U_w^D, respectively.[11] The permanent difference in spending is very small ($G^{pd} = 2.89E-07\%$), which is hardly surprising in view of the simplicity of the model and the absence of any further distortion.

5.2.2 Sensitivity analysis

In this section, we investigate how changes in the deep parameters affect welfare under the two types of fiscal constraints. In all instances we adjust B^c and D^c to keep the deterministic steady state of debt $\overline{B}$ equal to its benchmark level B_0. We report the results in differences from the benchmark outcomes in Table 2.

The government's discount factor (β_g) Because the introduction of our fiscal constraints is motivated by government myopia and the consequent lack of fiscal discipline, we start the sensitivity analysis by investigating the consequences of an increase in the degree of government myopia (i.e., β_g falls), while keeping all the other parameters at their benchmark values.

Table 3 reports the results under the two fiscal constraints for different values of β_g. Under a debt constraint, when the government becomes more myopic, the public debt $\widehat{B^B}$ falls, while under a primary deficit constraint the public debt $\widehat{B^D}$ increases as β_g falls. The intuition behind these outcomes is the following. Consider first the primary deficit constraint. When the government is more myopic, it has a stronger incentive to shift spending from the future to the present and, hence, to borrow. Hence, in the steady state debt will be higher and the primary deficit constraint has to be tightened in order to

[11]Precisely, using (32) we compute G_w^z $(z = B, D)$ from $(G_w^z)^{1-\frac{1}{\mu}} / \left[\left(1-\frac{1}{\mu}\right)(1-\beta_w)\right] = U_w^z$. This approach is followed in, for example, Jensen (2002) and Beetsma and Jensen (2005).

induce the government to run higher primary surpluses to service the interest payments on its debt.

Under a debt constraint a similar mechanism implies an increase in the steady state level of the debt. However, there is a mechanism pushing steady state debt in the opposite direction. With a direct constraint on the debt, a more myopic government has a stronger incentive to limit debt to protect itself against bad income shocks that would force it to cut current spending in the very short run. Hence, a more myopic government increases its precautionary saving.

The outcomes for this case can be summarized as:

Result I *Ceteris paribus, an increase in government myopia (a fall in β_g) leads to a fall (rise) in the stochastic steady state level of the public debt under a debt (primary deficit) constraint. Welfare under a debt constraint rises relative to welfare under a primary-deficit constraint.*

The interest rate (r) Next, we investigate the consequences of a change in the interest rate r. Changes in the interest rate affect the steady state outcomes under both the non-myopic and myopic government.

Table 4 reports the outcomes. In all cases, the lower is the interest rate, the lower is the steady state level of consumption $\overline{C}$ and the higher is the steady state level of government spending $\overline{G}$. A lower interest rate reduces interest income earned by the private sector on its assets, which thus reduces its private consumption. Lower interest spending also allows the government to raise debt without violating the public budget constraint. Hence, the mean debt level under both constraints rises when r falls.

In order to keep the deterministic steady state value of the public debt at B_0 when the interest rate falls, the reference value D^c under the primary deficit constraint has to rise substantially (corresponding to a much lower surplus than under the benchmark), while the reference value B^c under the debt constraint needs to change only marginally. To see the intuition, notice that the deterministic steady state debt and primary deficit are linked as $\overline{B} = -(1/r)\,\overline{D}$ ($-\overline{D}$ is the constant primary surplus that pays off an initial debt $\overline{B}$). A given fall in r thus requires a relatively large increase in $\overline{D}$ to maintain $\overline{B} = B_0$. In turn, this rather large shift in $\overline{D}$ translates into a large shift in the reference value D^c.

We summarize this as:

Result II *The reference value D^c under the primary-deficit constraint is more sensitive to changes in the interest rate than the reference value B^c under the debt constraint.*

Making frequent and large changes in reference values of fiscal constraints may be politically problematic (every change in the fiscal constraint is likely to re-open a political debate on what should be the appropriate design of the constraint).[12] Result II suggests

[12] For a more formal discussion of the political issues involved in the implementation of fiscal constraints, see Krogstrup and Wyplosz (2006), Debrun and Kumar (2007), and Ribeiro and Beetsma (2008).

that from this perspective, a debt constraint becomes more attractive relative to a deficit constraint.

The variance (σ_ε^2) and persistence (ρ) of the income shock To see how the relative performance of the two fiscal constraints depends on the volatility of the business cycle, we vary σ_ε^2, keeping all other parameters at their benchmark values. Table 5 shows the outcomes of this variation. Of course, changes in the variance of the income shock do not affect the deterministic steady state outcomes, which are therefore not repeated in Table 5. However, they do affect the stochastic steady state.

Under the debt constraint, a rise in uncertainty induces the government to reduce debt on average, so that it can absorb the shocks that are now larger on average (in absolute terms) without hitting the reference debt level. By contrast, under the primary deficit constraint, a higher σ_ε^2 implies an increase in the steady state public debt. The reason is that the latter corresponds to a higher stochastic steady state surplus, which is the result of a desire to maintain a larger safety margin relative to its reference level. Further, under both constraints, welfare (not reported in Table 5) falls as a result of the higher ex ante uncertainty about the available resources.

Table 6 shows the results of varying the persistence of the income process, ρ. A rise in ρ implies a larger unconditional variance of income, for given σ_ε^2. Hence, qualitatively speaking the consequences for public debt and welfare of an increase in ρ are the same as those of an increase in σ_ε^2. Interestingly, though, an increase in ρ makes a primary deficit constraint relatively more attractive from a welfare point of view. Any effort to adhere to the primary deficit constraint after a bad income shock now makes it easier to adhere to the constraint in the future when income is also likely to be low. Hence, more discipline now has an extra pay off in the future and the overall discounted cost of adhering to the constraint falls. This dampens the welfare loss of higher income persistence under a primary deficit constraint relative to that under a debt constraint.

Result III *Ceteris paribus, both an increase in income uncertainty (a higher σ_ε^2) and an increase in income persistence (a higher ρ), lower the expected debt level under the debt constraint and raise the expected debt level under the primary-deficit constraint. An increase in each of the variables also produces a fall in social welfare under both constraints. Finally, an increase in σ_ε^2 (in ρ) reduces (increases) the relative attractiveness of a primary deficit constraint.*

The coefficient of relative risk aversion ($1/\mu$): Table 7 shows that the average debt level $\widehat{B^B}$ under the debt constraint increases when relative risk aversion $1/\mu$ falls. The myopic government prefers to keep a smaller margin relative to the reference debt level, because it cares less about the higher risk that the debt constraint becomes binding and spending has to be cut. Obviously, the higher average debt level is accompanied by lower average public spending in the steady state. By contrast, the average debt level $\widehat{B^D}$

under the primary-deficit constraint falls as risk aversion falls, the reason being that the government now prefers to maintain a smaller safety margin to the reference deficit level. Hence, the average primary surplus falls, implying a lower average debt level. The main results of an increase in μ are summarized as:

Result IV *Ceteris paribus, a fall in relative risk aversion (a rise in μ), raises (lowers) the average public debt level under the debt (primary-deficit) constraint. Further, the permanent spending difference G^{pd} corresponding to the welfare difference under the two constraints falls.*

6 Conclusion

This paper has compared primary-deficit constraints with debt constraints in a simple macroeconomic model with a myopic government. In our framework and based on our calibration, a debt constraint yields higher welfare than a primary-deficit constraint if both constraints are selected to lead to the same sustainable steady state debt level. This finding is reinforced when government myopia is stronger and income shocks have a higher variance. An increase in income persistence in turn increases the relative attractiveness of a primary deficit constraint because any effort exerted to adhere to the constraint now when income is hit by a bad shock has an extra pay off in the future, in that it will be easier to adhere to the constraint also in the future when income is also more likely to be low. Moreover, a debt constraint is more robust to changes in the interest rate, possibly making it relatively more attractive from a political perspective.

Our analysis lends support to the recent reform of the Stability and Growth Pact in 2005 (see Beetsma and Debrun, 2007, for a discussion), which has shifted (although for other reasons) some of the focus more explicitly towards constraining the public debt.

Our analysis also suggests several avenues for further research. Throughout the paper we have assumed an exogenous and constant interest rate. An interesting extension would be to allow for an endogenous interest rate or for the presence of debt default risk. We have also not included public capital in our model. This would allow us to study golden rules as a third alternative fiscal constraint.

7 Tables and figures

Table 1: Benchmark parameter calibration

Par.	Value	Description and computation
Deep parameters		
$\overline{Y}$	100	*Average Income*: normalized value.
τ^y	1	*Income tax*: for convenience set equal to unity.
B_0	63.14	*Initial domestic debt*: average debt ratio for the Euro-11 over 1970-2006.
F_0	0	*Initial net foreign debt*: calibrated to zero.
r	0.02617585	*Long-term real interest rate*: average ex-post real interest rate for the Euro-11 over 1970-2006.
ρ	0.264	*Income persistence*: average AR(1) coefficient from the estimation of an AR(1) process on the log-deviations of income from its trend for the Euro-11 over 1970-2006.
σ_ϵ^2	38.821	*Variance of the income shock*: average (over all observations) of the squared residuals of the above income regression.
α	0.933	Parameter that determines the ratio between the myopic government discount factor β_g and the social discount factor β_w, such that $\beta_g = \alpha\beta_w$.
μ	0.7	*Constant elasticity of intertemporal substitution*: provides a coefficient of relative risk aversion close to Aiyagari and McGrattan (1998) and Smets and Wouters (2003).
γ	300	*Smoothness of the logistic function*: value to approximate the indicator function.
Policy parameters		
k^B	100	*Tightness of the debt-based sanction*: value implies a very tight restriction.
k^D	100	*Tightness of the primary deficit-based sanction*: value implies a very tight restriction.
B^c	63.14425	*Reference value for the debt constraint*: provides a deterministic steady state of debt equal to B_0 and satisfies the transversality condition (tvc).
D^c	-1.64864435	*Reference value for the primary deficit constraint*: provides a deterministic steady state of debt equal to B_0 and satisfies the tvc.

Source: OECD (2006) and own calculations.

Table 2: Results for the benchmark parameter calibration

Variables	Result	Description
Non-Myopic Government		
β_w	0.974492	Social discount factor.
$\overline{G}$	98.347257	Deterministic steady state of government spending.
$\overline{C}$	1.652743	Deterministic steady state of private consumption.
$\overline{D}$	-1.652743	Deterministic steady state of primary deficit.
Myopic Government with Debt Constraint		
B^c	63.144250	Reference value of debt constraint.
D^{Bc}	-1.652854	Primary deficit when debt equals its reference value under the debt constraint.
$\overline{G^B}$	98.347257	Deterministic steady state of government spending.
$\overline{D^B}$	-1.652743	Deterministic steady state of primary deficit.
$\overline{B^B}$	63.140000	Deterministic steady state of public debt.
$\widehat{G^B}$	98.347261	Stochatic steady state of government spending.
$\widehat{D^B}$	-1.652743	Stochatic steady state of primary deficit.
$\widehat{B^B}$	63.139955	Stochastic steady state of public debt.
Myopic Government with Primary Deficit Constraint		
B^{Dc}	62.983415	Debt when primary deficit equals its reference value under the primary-deficit constraint.
D^c	-1.648644	Reference value of primary deficit constraint.
$\overline{G^D}$	98.347257	Deterministic steady state of government spending.
$\overline{D^D}$	-1.652743	Deterministic steady state of primary deficit.
$\overline{B^D}$	63.140000	Deterministic steady state of public debt.
$\widehat{G^D}$	98.347261	Stochastic steady state of government spending.
$\widehat{D^D}$	-1.652785	Stochastic steady state of primary deficit.
$\widehat{B^D}$	63.141598	Stochastic steady state of public debt.
Welfare		
G^{pd}	2.89E-07	Percentage difference between the permanent constant levels of public consumption that produce U_w^B and U_w^D given by (31).

Table 3: Results for different β_g in deviations ($\times$E-06) from the benchmark values*

Variables[a,b]	$\beta_g = 0.65$	$\beta_g = 0.81$	$\beta_g = 0.86$	$\beta_g = 0.89$
B^c	-65.500	-20.000	-9.000	-3.000
D^{Bc}	1.715	0.524	0.236	0.079
$\overline{G^B}$	-0.015	-0.015	-0.019	-0.020
$\overline{D^B}$	-0.015	-0.015	-0.019	-0.020
$\overline{B^B}$	0.587	0.584	0.715	0.759
$\widehat{G^B}$	27.801	7.088	3.158	1.172
$\widehat{D^B}$	-0.015	-0.015	-0.019	-0.020
$\widehat{B^B}$	-15.568	-4.490	-1.685	-0.170
B^{Dc}	2381.585	741.141	349.559	135.621
D^c	-62.340	-19.400	-9.150	-3.550
$\overline{G^D}$	-0.006	-0.002	0.003	-0.008
$\overline{D^D}$	-0.006	-0.002	0.003	-0.008
$\overline{B^D}$	0.247	0.087	-0.114	0.320
$\widehat{G^D}$	26.271	6.697	2.998	1.115
$\widehat{D^D}$	-15.368	-4.819	-2.274	-0.890
$\widehat{B^D}$	587.115	184.109	86.878	34.015
G^{pd}	1.558	0.399	0.164	0.058

Notes: * Deviations are expressed in %-points of average output, except for G^{pd}, which is the difference relative to its baseline value.
[a] Recall that $\beta_g = \alpha\beta_w$.
[b] The variables in the first column are defined as in Table 2.
[c] In the other columns, we vary the value of β_g while keeping the other parameters at their benchmark values indicated in Table 1.

Table 4: Results for different interest rates (r) in deviations from the benchmark values*

Var.^{a,b,c}	$r = 0.002$	$r = 0.014$	$r = 0.041$	$r = 0.056$	$r = 0.071$	$r = 0.090$
$\overline{G}$	1.53	0.77	-0.94	-1.88	-2.83	-4.03
$\overline{C}$	-1.53	-0.77	0.94	1.88	2.83	4.03
$\overline{D}$	1.53	0.77	-0.94	-1.88	-2.83	-4.03
B^c	-3.00E-06	-1.00E-06	3.00E-06	5.00E-06	7.30E-06	1.02E-05
D^{Bc}	1.53	0.77	-0.94	-1.88	-2.83	-4.03
$\overline{G^B}$	1.53	0.77	-0.94	-1.88	-2.83	-4.03
$\overline{D^B}$	1.53	0.77	-0.94	-1.88	-2.83	-4.03
$\overline{B^B}$	6.81E-07	8.54E-07	7.40E-07	4.52E-07	4.61E-07	4.57E-07
$\widehat{G^B}$	1.53	0.77	-0.94	-1.88	-2.83	-4.03
$\widehat{D^B}$	1.53	0.77	-0.94	-1.88	-2.83	-4.03
$\widehat{B^B}$	2.09E-06	1.57E-06	-1.58E-07	-1.38E-06	-2.34E-06	-3.61E-06
B^{Dc}	-1.89	-0.14	0.06	0.08	0.10	0.11
D^c	1.53	0.77	-0.94	-1.88	-2.83	-4.03
$\overline{G^D}$	1.53	0.77	-0.94	-1.88	-2.83	-4.03
$\overline{D^D}$	1.53	0.77	-0.94	-1.88	-2.83	-4.03
$\overline{B^D}$	2.91E-08	-1.45E-07	8.01E-08	3.00E-08	-2.71E-08	-1.80E-07
$\widehat{G^D}$	1.53	0.77	-0.94	-1.88	-2.83	-4.03
$\widehat{D^D}$	1.53	0.77	-0.94	-1.88	-2.83	-4.03
$\widehat{B^D}$	1.87E-02	1.34E-03	-5.58E-04	-8.21E-04	-9.73E-04	-1.09E-03
G^{pd}	-3.59E-08	-3.19E-08	-5.76E-09	2.08E-08	3.30E-08	4.02E-08

Notes: * Deviations are expressed in %-points of average output, except for G^{pd}, which is the difference relative to its baseline value.

[a] We vary β_w such that we keep $\beta_w (1 + r) = 1$.

[b] The variables in the first column are defined as in Table 2.

[c] In the other columns, in addition, we vary the value of the interest rate (r) while keeping the other parameters at their benchmark values indicated in Table 1.

Table 5: Results for different σ_ϵ^2 in deviations (×E-06) from the benchmark values*

Var.[a,b]	σ_ϵ^2 =1	$\sigma_\epsilon^2 = 12$	$\sigma_\epsilon^2 = 24$	$\sigma_\epsilon^2 = 50$	$\sigma_\epsilon^2 = 75$	$\sigma_\epsilon^2 = 100$
$\widehat{G^B}$	-3.96	-3.69	-2.57	2.80	11.74	24.40
$\widehat{D^B}$	0.00	0.00	0.00	0.00	0.00	0.00
$\widehat{B^B}$	44.39	40.34	27.63	-29.57	-122.85	-253.67
$\widehat{G^D}$	-3.73	-3.47	-2.42	2.64	11.06	22.98
$\widehat{D^D}$	41.81	38.00	26.03	-27.85	-115.72	-238.95
$\widehat{B^D}$	-1597.44	-1451.63	-994.34	1064.07	4421.03	9128.71
G^{pd}	-0.28	-0.25	-0.17	0.18	0.73	1.51

Notes: * Deviations are expressed in %-points of average output, except for G^{pd}, which is the difference relative to its baseline value. [a] The variables in the first column are defined as in Table 2. [b] In the other columns, in addition, we vary the value of σ_ϵ^2 while keeping the other parameters at their benchmark values indicated in Table 1.

Table 6: Results for different ρ in deviations (×E-08) from the benchmark values*

Var.[a,b]	$\rho = 0.01$	$\rho = 0.4$	$\rho = 0.6$	$\rho = 0.8$	$\rho = 0.99$
$\widehat{G^B}$	-32.50	12.55	27.14	38.48	47.15
$\widehat{D^B}$	0.00	0.00	0.00	0.00	0.00
$\widehat{B^B}$	25.45	-9.83	-21.26	-30.15	-36.94
$\widehat{G^D}$	-30.63	11.83	25.58	36.27	44.44
$\widehat{D^D}$	24.15	-9.33	-20.17	-28.60	-35.04
$\widehat{B^D}$	-922.49	356.38	770.45	1092.55	1338.62
G^{pd}	1.48	-0.57	-1.24	-1.79	-3.17

Notes: * Deviations are expressed in %-points of average output, except for G^{pd}, which is the difference relative to its baseline value.
[a] The variables in the first column are defined as in Table 2.
[b] In the other columns, in addition, we vary the value of ρ while keeping the other parameters at their benchmark values indicated in Table 1.

Table 7: Results for different μ in deviations ($\times$E-06) from the benchmark values*

Var.^{a,b}	$\mu = 0.3$	$\mu = 0.5$	$\mu = 1.01$	$\mu = 2$	$\mu = 3$	$\mu = 4$
$\widehat{G^B}$	11.98	2.79	-1.67	-3.06	-3.42	-3.57
$\widehat{D^B}$	0.00	0.00	0.00	0.00	0.00	0.00
$\widehat{B^B}$	-140.85	-32.46	19.21	34.83	38.74	40.42
$\widehat{G^D}$	11.28	2.63	-1.57	-2.88	-3.22	-3.36
$\widehat{D^D}$	-132.67	-30.57	18.10	32.81	36.49	38.07
$\widehat{B^D}$	5068.52	1167.97	-691.30	-1253.38	-1393.87	-1454.48
G^{pd}	0.89	0.21	-0.12	-0.22	-0.24	-0.25

Notes: * Deviations are in %-points of average output, except for G^{pd}, which is the difference relative to its baseline value. [a] The variables in the first column are defined as in Table 2. [b] In the other columns, in addition, we vary the value of μ while keeping the other parameters at their benchmark values indicated in Table 1.

Figure 1: Impulse responses for non-myopic government with benchmark parameter values

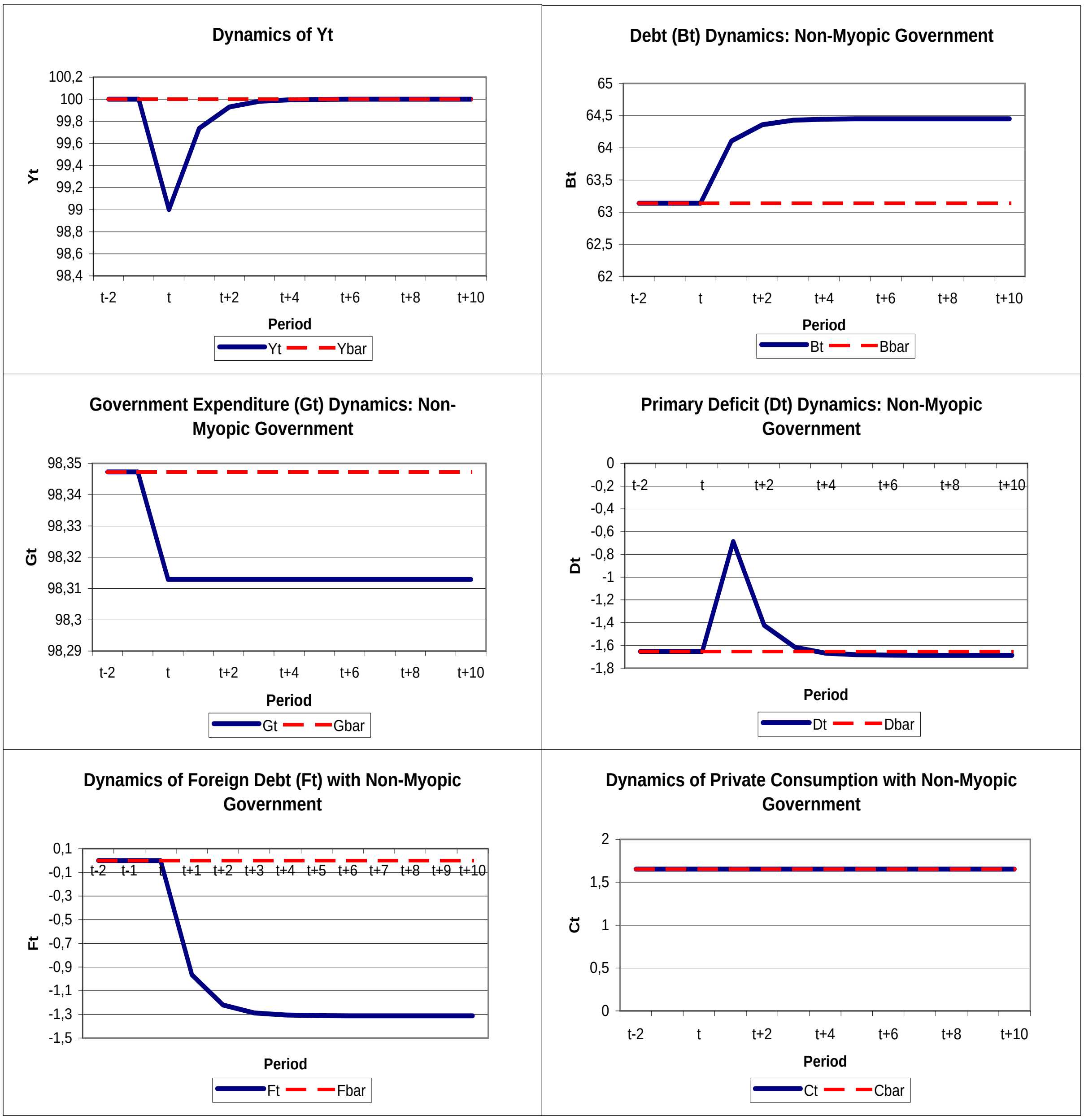


Figure 2: Impulse responses for myopic government under debt constraint with benchmark parameter values

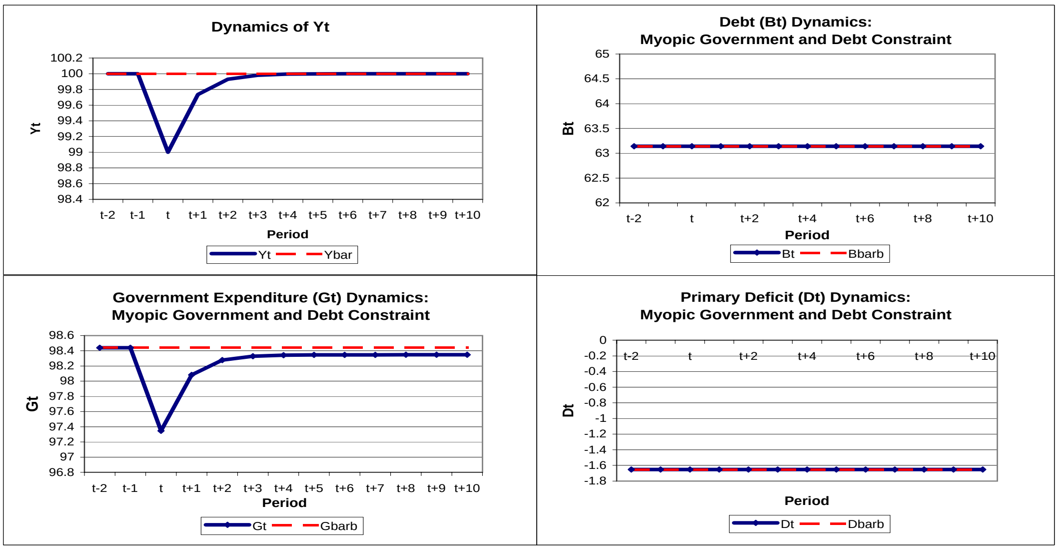
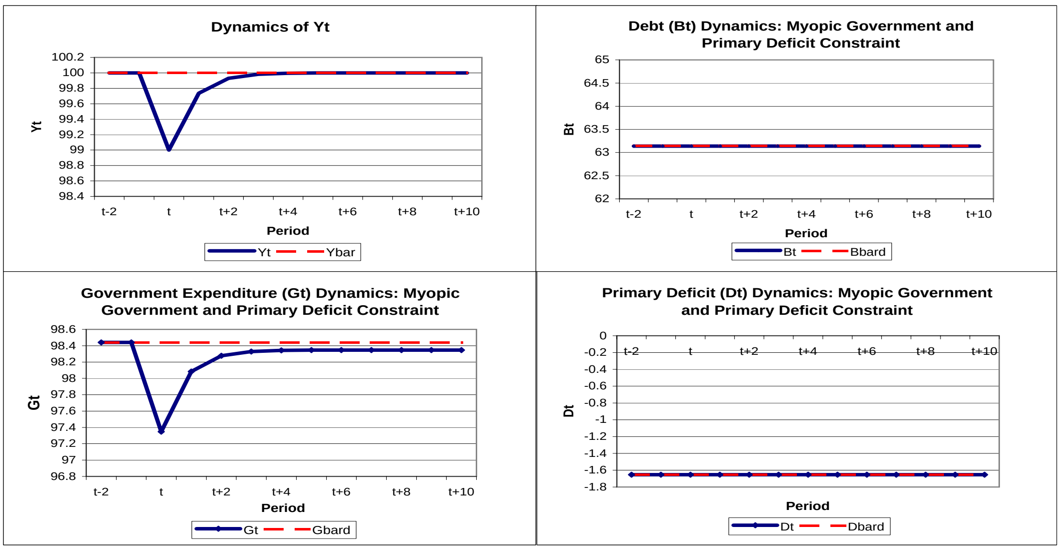


Figure 3: Impulse responses for myopic government under primary deficit constraint with benchmark parameter values



References

[1] Aiyagari, S. R., and E.R. McGrattan (1998). The Optimum Quantity of Debt, *Journal of Monetary Economics* 42, 447-469.

[2] Bayoumi, T., M. Goldstein, and G. Woglom (1995). Do Credit Markets Discipline Sovereign Borrowers? Evidence from U.S. States, *Journal of Money, Credit and Banking* 27, 4, 1046-1059.

[3] Beetsma, R., and X. Debrun (2007). The New Stability and Growth Pact: A First Assessment, *European Economic Review* 51, 2, 453-478.

[4] Beetsma, R., and H. Jensen (2005). Monetary and Fiscal Policy Interactions in a Micro-Founded Model of a Monetary Union, *Journal of International Economics* 67, 320-352.

[5] Buti, M., S. Eijffinger, and D. Franco (2005). The Stability Pact Pains : a Forward-Looking Assessment of the Reform Debate, *CentER Working Paper*, n^o. 2005-101, Tilburg University, August.

[6] Collard, F., and M. Juillard (2001). A High-Order Taylor Expansion Approach to Simulation of Stochastic Forward-Looking Models with an Application to a Nonlinear Phillips Curve Model, *Computational Economics* 17, 125-139.

[7] Debrun, X., and M.S. Kumar (2007). The Discipline-Enhancing Role of Fiscal Institutions: Theory and Empirical Evidence, *IMF Working Paper*, WP/07/171.

[8] Fatás, A., A. Hughes Hallett, A. Sibert, R. Strauch, and J. von Hagen (2003). Monitoring European Integration, *CEPR Occasional Paper*, n^o.13, London.

[9] Franses, P.H., and D. van Dijk (2000). *Nonlinear Time Series Models in Empirical Finance*, Cambridge University Press: Cambridge.

[10] Grossman, H., and J.B. van Huyck (1988). Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation, *American Economic Review* 78, 5, 1088-1097.

[11] Jensen, H. (2002). Targeting Nominal Income Growth or Inflation? *American Economic Review* 92, 4, 928-956.

[12] Judd, K., (1998). *Numerical Methods in economics,* MIT Press: Cambridge.

[13] Krogstrup, S., and C. Wyplosz (2006). A Common Pool Theory of Deficit Bias Correction, *CEPR Discussion Paper* n^o. 5866, August.

[14] Kumhof, M., and I. Yakadina (2007). Politically Optimal Fiscal Policy, *IMF Working Paper*, WP/07/68.

[15] Lambertini, L. (2006). Optimal Fiscal Policy in a Monetary Union, *Mimeo*, Claremont McKenna College, available at: www.claremontmckenna.edu/econ/llambertini/.

[16] Morris, R., H. Ongena, and L. Schuknecht (2006). The Reform and Implementation of the Stability and Growth Pact, *ECB Occasional Paper*, n^o. 47, Frankfurt.

[17] OECD (2006). *Economic Outlook*, June.

[18] Pappa, E., and V. Vassilatos (2007). The Unbearable Tightness of Being in a Monetary Union: Fiscal Restrictions and Regional Stability, *European Economic Review* 51, 6, 1492-1513.

[19] Ribeiro, M.P., and R. Beetsma (2008). The Political Economy of Structural Reforms under a Deficit Restriction, *Journal of Macroeconomics* 30, 179-198.

[20] Schmitt-Grohé, S., and M. Uribe (2004). Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function, *Journal of Economics Dynamic and Control* 28, 4, 755-775.

[21] Smets, F., and R. Wouters (2003). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area, *Journal of the European Economic Association* 1, 5, 1123-1175.

[22] Wyplosz, C. (2005). Fiscal Policy: Rules or Institutions? *National Institute Economic Review*, n^o.191, January.

Appendices

A Solution for the non-myopic government

For a given endowment sequence $\{Y_t\}_{t=0}^{\infty}$, constant tax rate τ^y, and initial values F_0 and B_0, a rational expectations equilibrium under a non-myopic government consists of a set of sequences $\{C_t, G_t, B_{t+1}, F_{t+1}\}_{t=0}^{\infty}$ satisfying

$$\begin{aligned}
u'(G_t) &= \mathrm{E}_t u'(G_{t+1}), \\
(1+r)B_t &= \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left(\tau^y Y_s - G_s\right), \\
v'(C_t) &= \mathrm{E}_t v'(C_{t+1}), \\
-(1+r)F_t &= \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left(Y_s - C_s - G_s\right).
\end{aligned}$$

The first two conditions feature two unknowns, G_t and B_{t+1}. Hence, one can in principle separately solve for G_t and B_{t+1} and subsequently for C_t and F_{t+1}, using the last two conditions and the solution for G_t.

Since we restrict the equilibrium to be recursive, we know that the condition $u'(G_t) = \mathrm{E}_t u'(G_{t+1})$ implies that the conditional expectations of $E_t G_{t+j}$ are constant for all $j \geq 0$ and thus equal to G_t, $E_t G_{t+j} = G_t$, $\forall j \geq 0$. Similarly, $v'(C_t) = \mathrm{E}_t v'(C_{t+1})$, implies that $E_t C_{t+j} = C_t$, $\forall j \geq 0$.

We, firstly, aim to derive solutions for G_t and B_{t+1}, i.e., to identify the state space representation of G_t and B_{t+1}. We start with the subset of equilibrium conditions

$$\begin{aligned}
u'(G_t) &= \mathrm{E}_t u'(G_{t+1}) \\
(1+r)B_t &= \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left(\tau^y Y_s - G_s\right),
\end{aligned}$$

given B_0. Taking expectations conditional on the information in period t, the intertemporal budget constraint implies

$$\mathrm{E}_t \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} G_s = \tau^y Y_t + \mathrm{E}_t \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \tau^y Y_s - (1+r)B_t.$$

Using $\mathrm{E}_t G_{t+j} = G_t$, $\forall j > 0$, and $\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} = 1/\left(1 - \frac{1}{1+r}\right) = \frac{r+1}{r}$ given that $1+r > 1$, leads to

$$\frac{r+1}{r} G_t = \tau^y Y_t + \mathrm{E}_t \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \tau^y Y_s - (1+r)B_t.$$

To determine the period t expectation of Y_s for $s \geq t$, we use that Y_t follows the stochastic process

$$Y_t - \overline{Y} = \rho\left(Y_{t-1} - \overline{Y}\right) + \sigma_{\varepsilon}\varepsilon_t.$$

Expectations conditional on the information in period t are thus

$$\mathrm{E}_t Y_s = (1-\rho)\overline{Y} + \rho E_t Y_{s-1}.$$

To write $\mathrm{E}_t Y_s$ as a function of Y_t, we iterate backwards to get $E_t Y_s = (1-\rho)\overline{Y} + \rho(1-\rho)\overline{Y} + \ldots + \rho^{(s-t)-1}(1-\rho)\overline{Y} + \rho^{(s-t)} E_t Y_t$, or, concisely

$$\mathrm{E}_t Y_s = (1-\rho)\overline{Y} \sum_{k=0}^{(s-t)-1} \rho^k + \rho^{(s-t)} Y_t.$$

Eliminating $E_t Y_s$ in $\frac{r+1}{r} G_t = \tau^y Y_t + \tau^y \mathrm{E}_t \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} Y_s - (1+r) B_t$ then gives

$$\begin{aligned}
\frac{r+1}{r} G_t &= \tau^y Y_t + \tau^y \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left((1-\rho)\overline{Y} \sum_{k=0}^{(s-t)-1} \rho^k + \rho^{(s-t)} Y_t \right) - (1+r) B_t \\
&= \left\{ \begin{array}{c} \tau^y Y_t + \tau^y \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (1-\rho)\overline{Y} \sum_{k=0}^{(s-t)-1} \rho^k + \tau^y \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \rho^{(s-t)} Y_t \\ -(1+r) B_t \end{array} \right\} \\
&= \tau^y Y_t \left(1 + \frac{\rho}{r-\rho+1}\right) + \left[(1-\rho)\overline{Y}\tau^y \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \sum_{k=0}^{(s-t)-1} \rho^k \right] - (1+r) B_t.
\end{aligned}$$

Using $\sum_{k=0}^{(s-t)-1} \rho^k = \left(\frac{1}{1-\rho} - \frac{\rho^{s-t}}{1-\rho}\right)$ for any $\rho \neq 1$, the term in the square bracket can further be simplified to

$$\begin{aligned}
&\tau^y (1-\rho)\overline{Y} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \sum_{k=0}^{(s-t)-1} \rho^k \\
&= \tau^y (1-\rho)\overline{Y} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left(\frac{1}{1-\rho} - \frac{\rho^{s-t}}{1-\rho}\right) \\
&= \tau^y \overline{Y} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left(1 - \rho^{s-t}\right).
\end{aligned}$$

By using $\sum_{k=0}^{\infty} a^{k+1} = a/(1-a)$ for any $|a| < 1$, such that $\sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} = 1/r$ and $\sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \rho^{s-t} = \frac{\rho}{r-\rho+1}$, the last expression becomes

$$\begin{aligned}
&\tau^y \overline{Y} \left(\frac{1}{r} - \frac{\rho}{r-\rho+1}\right) \\
&= \tau^y \overline{Y} \frac{(1-\rho)(1+r)}{r(1+r-\rho)}.
\end{aligned}$$

Hence, we end up with the following state space solution for G_t :

$$\begin{aligned}
\frac{r+1}{r} G_t &= \tau^y Y_t \left(1 + \frac{\rho}{r-\rho+1}\right) + \tau^y \overline{Y} \frac{(1-\rho)(1+r)}{r(1+r-\rho)} - (1+r) B_t \\
&= \tau^y Y_t \frac{r+1}{r-\rho+1} + \tau^y \overline{Y} \frac{(1-\rho)(1+r)}{r(1+r-\rho)} - (1+r) B_t \\
&\Rightarrow G_t = \tau^y Y_t \frac{r}{1+r-\rho} + \tau^y \overline{Y} \frac{1-\rho}{1+r-\rho} - r B_t.
\end{aligned}$$

The solution for B_{t+1} can then simply be found by eliminating G_t in the period-by-period budget constraint

$$\begin{aligned} B_{t+1} &= (1+r)\,B_t + G_t - \tau^y Y_t \\ &= (1+r)\,B_t + \left(\tau^y Y_t \frac{r}{1+r-\rho} + \tau^y \overline{Y}\frac{1-\rho}{1+r-\rho} - rB_t\right) - \tau^y Y_t \\ &= B_t - \tau^y \frac{1-\rho}{1+r-\rho}\left(Y_t - \overline{Y}\right), \end{aligned}$$

which discloses the unit root in B_t and thus in G_t. Hence, for a given initial value B_0 and a given endowment sequence $\{Y_t\}_{t=0}^{\infty}$ we obtain a unique set of sequences $\{G_t, B_{t+1}\}_{t=0}^{\infty}$. Due to the unit root, there are infinitely many solutions for the unconditional expectations for G and B, which satisfy

$$\overline{G} = \tau^y \overline{Y} - r\overline{B}.$$

We can now derive the solutions for C_t and F_{t+1}, for which we use the remaining conditions (taking the solution for G_t as given)

$$\begin{aligned} v'(C_t) &= \mathrm{E}_t v'(C_{t+1}) \\ -(1+r)F_t &= \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(Y_s - C_s - G_s\right), \end{aligned}$$

given $F_0 = 0$. Taking expectations conditional on the information in period t, the intertemporal resource constraint leads to

$$\begin{aligned} \mathrm{E}_t \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_s &= \mathrm{E}_t \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(Y_s - G_s\right) + (1+r)F_t \\ \Rightarrow \frac{1+r}{r}C_t &= \mathrm{E}_t \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Y_s - \frac{1+r}{r}G_t + (1+r)F_t \end{aligned}$$

where we used $\mathrm{E}_t C_{t+j} = C_t$, $\forall j > 0$, and $\mathrm{E}_t G_{t+j} = G_t$, $\forall j > 0$. Using that

$$\mathrm{E}_t \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Y_s = Y_t \frac{r+1}{r-\rho+1} + \overline{Y}\frac{(1-\rho)\,(1+r)}{r\,(1+r-\rho)},$$

holds (see above), consumption C_t can be written as a function of Y_t, G_t, and F_t :

$$\begin{aligned} \frac{1+r}{r}C_t &= Y_t \frac{r+1}{r-\rho+1} + \overline{Y}\frac{(1-\rho)\,(1+r)}{r\,(1+r-\rho)} - \frac{1+r}{r}G_t + (1+r)F_t \\ &\Rightarrow \ C_t = Y_t \frac{r}{r-\rho+1} + \overline{Y}\frac{1-\rho}{1+r-\rho} - G_t + rF_t. \end{aligned}$$

Plugging in the solution for G_t, $G_t = \tau^y Y_t \frac{r}{1+r-\rho} + \tau^y \overline{Y}\frac{1-\rho}{1+r-\rho} - rB_t$, we get the following state space solution for consumption

$$\begin{aligned} C_t &= Y_t \frac{r}{r-\rho+1} + \overline{Y}\frac{(1-\rho)}{(1+r-\rho)} - \left(\tau^y Y_t \frac{r}{1+r-\rho} + \tau^y \overline{Y}\frac{1-\rho}{1+r-\rho} - rB_t\right) + rF_t \\ &= (1-\tau^y)\frac{r}{r-\rho+1}Y_t + (1-\tau^y)\frac{1-\rho}{1+r-\rho}\overline{Y} + rB_t + rF_t. \end{aligned}$$

Using the period-by-period resource constraint, $F_{t+1} = (1+r)F_t + Y_t - C_t - G_t$, and the solutions for C_t and G_t, we can compute the solution for F_t with

$$\begin{aligned} F_{t+1} &= (1+r)F_t + Y_t \\ &\quad - \left((1-\tau^y) \frac{r}{r-\rho+1} Y_t + (1-\tau^y) \frac{1-\rho}{1+r-\rho} \overline{Y} + rB_t + rF_t \right) \\ &\quad - \left(\tau^y Y_t \frac{r}{1+r-\rho} + \tau^y \overline{Y} \frac{1-\rho}{1+r-\rho} - rB_t \right) \\ &= F_t + \frac{1-\rho}{1-\rho+r} \left(Y_t - \overline{Y} \right), \end{aligned}$$

and the initial value $F_0 = 0$. The unconditional means of F and C, which satisfy $-r\overline{F} = \overline{Y} - \overline{C} - \overline{G}$ are not determined.

To summarize, the solution $X_t = \Gamma_X (B_t, F_t, Y_t)$ for $X_t \in (G_t, C_t, B_{t+1}, F_{t+1})$ is represented by (13) to (16) given B_0 and F_0, with unconditional means satisfying

$$\overline{G} = \tau^y \overline{Y} - r\overline{B} \quad \text{and} \quad \overline{C} = (1-\tau^y) \overline{Y} + r\overline{F} + r\overline{B}$$

B Intertemporal budget constraint under a debt constraint

Substitute (10) and $I\left[B_s; \overline{B}\right]$ by L_s^B in (7) and rewrite it for $s = t$:

$$\begin{aligned} B_{t+1} &= (1+r) B_t + G_t - \tau^y Y_t + k^B (B_t - B^c) L_t^B \Leftrightarrow \\ \left(1 + r + k^B L_t^B\right) B_t &= \tau^y Y_t - G_t + k^B B^c L_t^B + B_{t+1}. \end{aligned}$$

Iterating this process forward one period, we can jot down this last expression like:

$$B_{t+1} = \frac{\tau^y Y_{t+1} - G_{t+1} + k^B B^c L_{t+1}^B}{\left(1 + r + k^B L_{t+1}^B\right)} + \frac{B_{t+2}}{\left(1 + r + k^B L_{t+1}^B\right)}.$$

So, the intertemporal budget constraint under a debt constraint is equal to:

$$\left(1 + r + k^B L_t^B\right) B_t = \left\{ \begin{array}{c} \tau^y Y_t - G_t + k^B B^c L_t^B + \frac{\tau^y Y_{t+1} - G_{t+1} + k^B B^c L_{t+1}^B}{\left(1+r+k^B L_{t+1}^B\right)} \\ + \frac{\tau^y Y_{t+2} - G_{t+2} + k^B B^c L_{t+2}^B}{\left(1+r+k^B L_{t+1}^B\right)\left(1+r+k^B L_{t+2}^B\right)} + \ldots \end{array} \right\},$$

that can be represented by:

$$\begin{aligned} \left(1 + r + k^B L_t^B\right) B_t &= \left[\begin{array}{c} \tau^y Y_t - G_t + k^B B^c L_t^B + \frac{\tau^y Y_{t+1} - G_{t+1} + k^B B^c L_{t+1}^B}{\left(1+r+k^B L_{t+1}^B\right)} \\ + \frac{\tau^y Y_{t+2} - G_{t+2} + k^B B^c L_{t+2}^B}{\prod_{v=t+1}^{t+2} (1+r+k^B L_v^B)} + \ldots + \frac{\tau^y Y_{t+T} - G_{t+T} + k^B B^c L_{t+T}^B}{\prod_{v=t+1}^{t+T} (1+r+k^B L_v^B)} \\ + \lim_{T\to\infty} \frac{B_{t+T+1}}{\prod_{v=t+1}^{t+T} (1+r+k^B L_v^B)} \end{array} \right] \Leftrightarrow \\ \left(1 + r + k^B L_t^B\right) B_t &= \sum_{s=t}^{\infty} \frac{\tau^y Y_s - G_s + k^B B^c L_s^B}{\prod_{v=t+1}^{s} (1 + r + k^B L_v^B)} + \lim_{T\to\infty} \frac{B_{t+T+1}}{\prod_{v=t+1}^{t+T} (1 + r + k^B L_v^B)}, \end{aligned}$$

where we define $\prod_{v=t+1}^{t}\left(1+r+k^B L_v^B\right)\equiv 1$. In equilibrium that constraint is then equal to (20). Moreover, for the deterministic steady state, the transversality condition imposes:

$$\left(1+r+k^B L_0^B\right)B_0-\frac{1+r+k^B\overline{L^B}}{r+k^B\overline{L^B}}\left(\tau^y\overline{Y}-\overline{G}+k^B B^c\overline{L^B}\right)=0.$$

Note that if $B_0=\overline{B}$, as we assume in our numerical analysis, then the last equation simplifies as follows:

$$\begin{aligned}\left(1+r+k^B\overline{L^B}\right)\overline{B}-\frac{1+r+k^B\overline{L^B}}{r+k^B\overline{L^B}}\left(\tau^y\overline{Y}-\overline{G}+k^B B^c\overline{L^B}\right) &= 0\Leftrightarrow\\ \left(r+k^B\overline{L^B}\right)\overline{B} &= \tau^y\overline{Y}-\overline{G}+k^B B^c\overline{L^B}\Leftrightarrow\\ r\overline{B}+k^B\left(\overline{B}-B^c\right)\overline{L^B} &= \tau^y\overline{Y}-\overline{G}.\end{aligned}$$

For large values of γ and thus good approximations of the indicator function, this simplifies to:

$$r\overline{B}=\tau^y\overline{Y}-\overline{G},$$

which is the usual constraint that should hold in steady state.

C Intertemporal budget constraint under a primary deficit constraint

Substituting (8), $I\left[D_t>D^c\right]$ by L_t^D, and (10) in (22):

$$\begin{aligned}B_{t+1} &= (1+r)B_t+G_t-\tau^y Y_t+k^D\left(D_t-D^c\right)L_t^D\Leftrightarrow\\ B_{t+1} &= (1+r)B_t+G_t-\tau^y Y_t+k^D\left(B_{t+1}-(1+r)B_t-D^c\right)L_t^D\Leftrightarrow\\ (1+r)\left(1-k^D L_t^D\right)B_t &= \tau^y Y_t-G_t+k^D D^c L_t^D+\left(1-k^D L_t^D\right)B_{t+1}\Leftrightarrow\\ (1+r)B_t &= \frac{\tau^y Y_t-G_t+k^D D^c L_t^D}{1-k^D L_t^D}+B_{t+1}.\end{aligned}$$

Iterating this process forward one period, we can rewrite this last expression like:

$$B_{t+1}=\frac{\tau^y Y_{t+1}-G_{t+1}+k^D D^c L_{t+1}^D}{(1+r)\left(1-k^D L_{t+1}^D\right)}+\frac{B_{t+2}}{1+r}.$$

So, the intertemporal budget constraint under a debt constraint is equal to:

$$(1+r)B_t=\left\{\begin{array}{c}\frac{\tau^y Y_t-G_t+k^D D^c L_t^D}{1-k^D L_t^D}+\frac{\tau^y Y_{t+1}-G_{t+1}+k^D D^c L_{t+1}^D}{(1+r)\left(1-k^D L_{t+1}^D\right)}+\\ \frac{\tau^y Y_{t+2}-G_{t+2}+k^D D^c L_{t+2}^D}{(1+r)^2\left(1-k^D L_{t+2}^D\right)}+\frac{\tau^y Y_{t+3}-G_{t+3}+k^D D^c L_{t+3}^D}{(1+r)^3\left(1-k^D L_{t+3}^D\right)}+\ldots+\\ \frac{\tau^y Y_{t+T}-G_{t+T}+k^D D^c L_{t+T}^D}{(1+r)^T\left(1-k^D L_{t+T}^D\right)}+\lim_{T\to\infty}\frac{B_{t+T+1}}{(1+r)^{T+1}}\end{array}\right\},$$

which in equilibrium becomes (24). Finally, for the deterministic steady state, the transversality condition imposes:

$$(1+r)\,B_0 - \frac{1+r}{r}\left(\frac{\tau^y\overline{Y} - \overline{G} + k^D D^c \overline{L^D}}{1-k^D\overline{L^D}}\right) = 0.$$

Note that if $B_0 = \overline{B}$, as we assume in our numerical analysis, then the last equation simplifies as follows:

$$\begin{aligned}(1+r)\,\overline{B} - \frac{1+r}{r}\left(\frac{\tau^y\overline{Y} - \overline{G} + k^D D^c \overline{L^D}}{1-k^D\overline{L^D}}\right) &= 0 \Leftrightarrow \\ r\left(1-k^D\overline{L^D}\right)\overline{B} &= \tau^y\overline{Y} - \overline{G} + k^D D^c \overline{L^D}.\end{aligned}$$

For large values of γ and thus good approximations of the indicator function, this simplifies to:

$$r\overline{B} = \tau^y\overline{Y} - \overline{G},$$

which is the usual constraint that should hold in steady state.

D Deterministic steady state under a debt constraint

Under the debt constraint the equilibrium conditions are given by (18) and (19). Hence, a deterministic steady state has to satisfy the combination

$$\begin{aligned}1 &= \beta_g\left[1 + r + k^B\overline{L^B} + \gamma k^B\left(\overline{B} - B^c\right)\overline{L^B}(1-\overline{L^B})\right],\\ 0 &= r\overline{B} + \overline{G} - \tau^y\overline{Y} + k^B\left(\overline{B} - B^c\right)\overline{L^B}\end{aligned}$$

and thus

$$\begin{aligned}\beta_g^{-1} - (1+r) &= k^B\overline{L^B}\left(1 + \gamma\left(\overline{B} - B^c\right)(1-\overline{L^B})\right),\\ \overline{L^B} &= \frac{\tau^y\overline{Y} - r\overline{B} - \overline{G}}{k^B\left(\overline{B} - B^c\right)}. \qquad (33)\end{aligned}$$

Eliminating $\overline{L^B}$ gives

$$\begin{aligned}\beta_g^{-1} - (1+r) &= k^B\frac{\tau^y\overline{Y} - r\overline{B} - \overline{G}}{k^B\left(\overline{B} - B^c\right)}\left(1 + \gamma\left(\overline{B} - B^c\right)\left(1 - \frac{\tau^y\overline{Y} - r\overline{B} - \overline{G}}{k^B\left(\overline{B} - B^c\right)}\right)\right)\\ &= \frac{\tau^y\overline{Y} - r\overline{B} - \overline{G}}{\overline{B} - B^c}\left(1 + \gamma\left(\overline{B} - B^c\right) - \gamma\frac{\tau^y\overline{Y} - r\overline{B} - \overline{G}}{k^B}\right). \qquad (34)\end{aligned}$$

Or, instead, using (17) to eliminate $\overline{G}$, in the previous system of equations, we obtain:

$$\beta_g^{-1} - (1+r) = \frac{k^B\left\{\left[1+\exp\left(-\gamma\left(\overline{B} - B^c\right)\right)\right] * \left[1 + \gamma\left(\overline{B} - B^c\right)\right] - \gamma\left(\overline{B} - B^c\right)\right\}}{\left[1+\exp\left(-\gamma\left(\overline{B} - B^c\right)\right)\right]^2}.$$

This equation determines $\overline{B}$. We can substitute it into (33) to find then $\overline{G}$. Moreover, if we pass the denominator multiplying the left-hand side of the previous equation we get:

$$\left[\beta_g^{-1} - (1+r)\right]*\left[1+\exp\left(-\gamma\left(\overline{B}-B^c\right)\right)\right]^2 = k^B\left\{1+\exp\left(-\gamma\left(\overline{B}-B^c\right)\right)*\left[1+\gamma\left(\overline{B}-B^c\right)\right]\right\}.$$

The derivative of the left-hand side of this expression is:

$$\left[\beta_g^{-1} - (1+r)\right]*2\left[1+\exp\left(-\gamma\left(\overline{B}-B^c\right)\right)\right]*\left[-\gamma\exp\left(-\gamma\left(\overline{B}-B^c\right)\right)\right] < 0,$$

if $\beta_g < \beta_w$, $\gamma >> 0$ and $\overline{B} < B^c$. Further, the derivative of the right-hand side is:

$$k^B\left\{-\gamma\exp\left(-\gamma\left(\overline{B}-B^c\right)\right)*\left[1+\gamma\left(\overline{B}-B^c\right)\right]+\gamma\exp\left(-\gamma\left(\overline{B}-B^c\right)\right)\right\} > 0,$$

using the same assumptions as before. Therefore, this steady state is unique.

E Deterministic steady state under deficit constraint

Under the primary deficit constraint the equilibrium conditions are given by (22) and (23). Hence, a deterministic steady state has to satisfy

$$\begin{aligned} 1 &= \beta_g k^D\left[\overline{L^D}+\gamma\left(\overline{D}-D^c\right)\overline{L^D}(1-\overline{L^D})\right], \\ \overline{D} &= \overline{G}-\tau^y\overline{Y}+k^D\left(\overline{D}-D^c\right)\overline{L^D} \end{aligned}$$

and thus

$$\beta_g^{-1} = k^D\overline{L^D}\left(1+\gamma\left(\overline{D}-D^c\right)(1-\overline{L^D})\right), \tag{35}$$

$$\overline{L^D} = \frac{\tau^y\overline{Y}+\overline{D}-\overline{G}}{k^D\left(\overline{D}-D^c\right)}. \tag{36}$$

Eliminating $\overline{L^D}$ gives

$$\beta_g^{-1} = k^D\frac{\tau^y\overline{Y}+\overline{D}-\overline{G}}{k^D\left(\overline{D}-D^c\right)}\left(1+\gamma\left(\overline{D}-D^c\right)\left(1-\frac{\tau^y\overline{Y}+\overline{D}-\overline{G}}{k^D\left(\overline{D}-D^c\right)}\right)\right) \Leftrightarrow \tag{37}$$

$$\beta_g^{-1} = \frac{\tau^y\overline{Y}+\overline{D}-\overline{G}}{\overline{D}-D^c}\left(1+\gamma\left(\overline{D}-D^c\right)-\gamma\frac{\tau^y\overline{Y}+\overline{D}-\overline{G}}{k^D}\right). \tag{38}$$

Instead, if we use (21) in (36) and eliminate $\overline{G}$ in that system of equations, we arrive at:

$$\beta_g^{-1} = \frac{k^D\left\{\left[1+\exp\left(-\gamma\left(\overline{D}-D^c\right)\right)\right]*\left[1+\gamma\left(\overline{D}-D^c\right)\right]-\gamma\left(\overline{D}-D^c\right)\right\}}{\left[1+\exp\left(-\gamma\left(\overline{D}-D^c\right)\right)\right]^2}.$$

This determines $\overline{D}$. We can substitute the solution into (36) to find $\overline{G}$. Moreover, If we pass the denominator multiplying the left-hand side of the previous equation we get:

$$\beta_g^{-1}\left[1+\exp\left(-\gamma\left(\overline{D}-D^c\right)\right)\right]^2 = k^D\left\{1+\exp\left(-\gamma\left(\overline{D}-D^c\right)\right)*\left[1+\gamma\left(\overline{D}-D^c\right)\right]\right\}.$$

The derivative of the left-hand side of this expression is:

$$\beta_g^{-1} * 2\left[1 + \exp\left(-\gamma\left(\overline{D} - D^c\right)\right)\right] * \left[-\gamma \exp\left(-\gamma\left(\overline{D} - D^c\right)\right)\right] < 0,$$

if $\gamma >> 0$ and $\overline{D} < D^c$. Further, the derivative of the right-hand side is:

$$k^D\left\{-\gamma \exp\left(-\gamma\left(\overline{D} - D^c\right)\right) * \left[1 + \gamma\left(\overline{D} - D^c\right)\right] + \gamma \exp\left(-\gamma\left(\overline{D} - D^c\right)\right)\right\} > 0,$$

using the same assumptions as before. Therefore, this steady state is unique.