

DISCUSSION PAPER SERIES

No. 6857

DYNAMIC MORAL HAZARD AND PROJECT COMPLETION

Robin Mason and Juuso Välimäki

INDUSTRIAL ORGANIZATION



Centre for **E**conomic **P**olicy **R**esearch

www.cepr.org

Available online at:

www.cepr.org/pubs/dps/DP6857.asp

DYNAMIC MORAL HAZARD AND PROJECT COMPLETION

Robin Mason, University of Southampton and CEPR
Juuso Välimäki, Helsinki School of Economics, University of Southampton and
HECER

Discussion Paper No. 6857
June 2008

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

This Discussion Paper is issued under the auspices of the Centre's research programme in **INDUSTRIAL ORGANIZATION**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as a private educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions. Institutional (core) finance for the Centre has been provided through major grants from the Economic and Social Research Council, under which an ESRC Resource Centre operates within CEPR; the Esmée Fairbairn Charitable Trust; and the Bank of England. These organizations do not give prior review to the Centre's publications, nor do they necessarily endorse the views expressed therein.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Robin Mason and Juuso Välimäki

CEPR Discussion Paper No. 6857

June 2008

ABSTRACT

Dynamic Moral Hazard and Project Completion*

We analyse a simple model of dynamic moral hazard in which there is a clear and tractable trade-off; between static and dynamic incentives. In our model, a principal wants an agent to complete a project. The agent undertakes unobservable effort, which affects in each period the probability that the project is completed. The principal pays only on completion of the project. We characterise the contracts that the principal sets, with and without commitment. We show that with full commitment, the contract involves the agent's value and wage declining over time, in order to give the agent incentives to exert effort.

JEL Classification: C73, D82 and J31

Keywords: continuous time, moral hazard, principal-agent model and project completion

Robin Mason
Department of Economics
University of Southampton
Highfield,
Southampton
S017 1BJ
Email: robin.mason@soton.ac.uk

Juuso Välimäki
Helsinki School of Economics
Arkadiankatu 7
FI-0100 Helsinki
FINLAND
Email: juuso.valimaki@hse.fi

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=145252

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=148707

* We are grateful to Matti Liski for many helpful comments.

Submitted 29 May 2008

1 Introduction

It has been estimated that approximately 90% of American companies use, to some extent at least, agency firms to find workers; see Fernandez-Mateo (2003). According to Finlay and Coverdill (2000), between 13% and 20% of firms use private employment agencies “frequently” to find a wide variety of workers. The proportion is higher when searching for senior executives. Recent surveys in the pharmaceutical sector estimate almost two-thirds of senior executive hires involve a ‘headhunter’. Typically, it takes a number of months to find and recruit a candidate; for example, the same pharmaceutical source states that the average length of time taken to fill a post is between four and six months from the start of the search.. Payment to headhunters takes two forms. Contingency headhunters are paid only when they place a candidate successfully (a typical fee is 20–35% of the candidate’s first year’s salary). Retained headhunters receive an initial payment, some payment during search, and a bonus payment on success. See Finlay and Coverdill (2000). There is risk attached to search: between 15% and 20% of searches in the pharmaceutical sector fail to fill a post.¹

Residential real estate accounts for a large share of wealth—33% in U.S. in 2005, according to Merlo, Ortalo-Magné, and Rust (2006), and 26% in the UK in 1999, according to the Office of National Statistics. The value of residential house sales in England and Wales between July 2006 and July 2007 was of the order of 10% of the UK’s GDP.² According to the Office of Fair Trading (2004), over nine out of ten people buying and selling a home in England and Wales use a real estate agent. In the UK, the majority of residential properties are marketed under sole agency agreements: a single agent is appointed by a seller to market their property. (The details of the UK market are given in Merlo and Ortalo-Magné (2004) and Merlo, Ortalo-Magné, and Rust (2006).) The median time in the UK to find a buyer who eventually buys the property is 137 days. (Once an offer is accepted, it still takes on average 80–90 days until the transfer is completed.) The

¹Pharmafocus (2007) provides documentation for this market.

²The UK’s GDP in 2006 was £1.93 trillion, according to the ONS. The UK Land Registry reports that roughly 105,000 houses were sold each month over the period July 2006—July 2007, at an average price of around £180,000 each. See Land Registry (2007).

real estate agent can affect buyer arrival rates, through exerting marketing effort. But there is also exogenous risk (such as general market conditions) that affect the time to sale. In most agreements, the real estate agent is paid a proportion of the final sale price on completion.

In both of these examples, a principal hires an agent to complete a project. The principal gains no benefit until the project is completed. The agent can affect the probability of project completion by exerting effort. The principal's objective is to provide the agent with dynamic incentives to exert effort. The task of this paper is to analyse the dynamic incentives that arise in these settings and the contracts that are written as a result.

Dynamics matter for both sides. For the agent, its myopic incentives are to equate the marginal cost of effort with the marginal return. But if it fails to complete the project today, it has a further chance tomorrow. We assume that the agent has an increasing marginal cost of effort. As a result, the forward-looking agent reduces its current effort, substituting towards future effort—it smooths its effort over time. Hence this dynamic factor, all other things equal, tends to reduce the agent's current effort towards project completion. Similarly, the principal's myopic incentives trade off the marginal costs of inducing greater agent effort (through higher payments) with the marginal benefits. But the principal also knows that the project can be completed tomorrow; all other things equal, this tends to lower the payment that the principal pays today for project completion. On the other hand, the principal also realises that the agent faces dynamic incentives; this factor, on its own, tends to increase the payment to the agent.

Our modelling approach allows us to resolve these different incentives to arrive at analytical conclusions. We can do this with some degree of generality; for example, we allow for the principal and agent to have different discount rates. This then allows us isolate the different channels that are at work in the model. A number of features lie behind our approach. First, we look at pure project completion: the principal cares only about final success and receives no interim benefit from the agent's efforts. Secondly, both the principal and the agent are risk neutral. Insurance and consumption smoothing play no part in our analysis; this helps to make the dynamics much clearer. Thirdly, in our

model, the agent's participation constraint is not binding; consequently, its continuation value is strictly positive while employed by the principal. (This arises because of a limited liability constraint.) The principal uses the level and dynamics of the agent's continuation value to generate incentives for effort. Finally, we deal with the continuous-time limit of the problem. As noted by Sannikov (forthcoming), this can lead to derivations that are much more tractable than those in discrete time models.

We start by comparing two stationary problems, in which the principal pays a sequentially rational wage (i.e., has no ability to commit to a contract to pay the agent); and in which the principal can commit to pay a constant wage to the agent. We call these the stationary equilibrium solution and the stationary commitment solution respectively.

Given our set-up, the principal pays only on completion of the project: the agent receives no interim payments. In the stationary commitment case, a higher payment today implies a higher payment tomorrow. As a result, the principal encourages the agent to smooth its effort across time: to substitute from current to future effort. Hence this dynamic incentive decreases the agent's current effort. In contrast, in the stationary sequentially rational equilibrium, a change in the wage offer currently offered by the principal has no effect on future wage offers. As a result, there is no dynamic incentive effect to consider. Consequently, the constant commitment wage is lower than the wage offer in the stationary equilibrium. We also give a brief discussion of nonstationary sequentially rational wage contracts. In these equilibria, the current wage offer by the principal is supported by a threat of reversion to the stationary equilibrium wage level.

This comparison between the stationary problems gives an immediate intuition for what the full commitment solution looks like: we show that it must involve the agent's wage and effort falling over time. This is the only way in which the principal can resolve in its favour the trade-off between static incentives (which call for a high current wage) and dynamic incentives (which call for lower future wages). We establish this result for all discount rates, regardless of whether the principal or the agent has the higher discount rate. Further, we show that there are two cases. When the principal is less patient than the agent, the agent's wage and effort, must converge to zero—this is the only way in the

which the principal can induce high current effort. On the other hand, when the principal is more patient than the agent, the agent's wage and effort converge to strictly positive levels. In this case, the principal can rely on the agent's impatience to provide incentives for current effort. It is clear that solutions where the effort level converges to zero cannot be supported as sequentially rational solutions of the game and, as a result, we see that the principal gains from her ability to commit.

We can also say something about the principal's use of deadlines to provide incentives. We can show that when the principal commits to a constant wage and can employ only one agent, a deadline is not used. But it is clear that the best outcome for the principal is to use a sequence of agents, each for a single period. So suppose that the principal can fire an agent and then find a replacement with some probability: replacement agents arrive according to a Poisson process. We show that the principal's optimal deadline decreases with the arrival rate of replacement agents.

We conclude the analysis by considering how the main results might change when project quality matters. (For most of the paper, we assume that the completed project yields a fixed and verifiable benefit to the principal.) The issue is complicated; but we provide at least one example in which our results hold even with this complication.

At first glance, our results look similar to those in papers that look at unemployment insurance: see e.g., Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). In these papers, a government must make payments to an unemployed worker to provide a minimum level of expected discounted utility to the worker. The worker can exert effort to find a job; the government wants to minimise the total cost of providing unemployment insurance. Shavell and Weiss (1979) show that the optimal benefit payments to the unemployed worker should decrease over time. Hopenhayn and Nicolini (1997) establish that the government can improve things by imposing a tax on the individual when it finds work.

Our result—that the principal's optimal payment under full commitment decreases over time—looks similar to the unemployment insurance literature—of decreasing unemployment benefits over time. Despite this similarity, our results are quite different.

Perhaps the easiest way to see this is to note that both Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) require that the agent (worker) is risk averse. Without this assumption, neither paper can establish a decreasing profile of payments. In contrast, we have a risk neutral agent. We ensure that the principal does not simply sell the project to the agent by imposing limited liability. This constraint is satisfied implicitly in the unemployment insurance work (and is used also by e.g., Sannikov (2007)). In the unemployment insurance papers, the need to smooth over time the consumption of the risk-averse worker constrains the incentives that can be offered through unemployment benefits. In this paper, the risk neutral agent smooths its effort over time; the principal sets a declining wage to counteract this incentive.

We also argue that our paper identifies much more clearly the intertemporal incentives in this type of dynamic moral hazard problem. We show explicitly how continuation values affect current choices. By allowing for different discount rates between the principal and the agent, we can close off different channels in the model in order to highlight their effects. The simplicity of our set-up allows us to consider issues—such as deadlines and project quality—that are not dealt with in the unemployment insurance papers.

Our work is, of course, related to the broader literature on dynamic moral hazard problems: particularly the more recent work on continuous-time models. This literature has demonstrated in considerable generality the benefits to the principal of being able to condition contracts on the intertemporal performance of the agent. By doing so, the principal can relax the agent's incentive compatibility constraints. See e.g., Malcomson and Spinnewyn (1988) and Laffont and Martimort (2002). More recently, Sannikov (2007), Sannikov (forthcoming) and Willams (2006) have analysed principal-agent problems in continuous time. For example, in Sannikov (forthcoming), an agent controls the drift of a diffusion process, the realisation of which in each period affects the principal's payoff. When the agent's action is unobserved, Sannikov characterises the optimal contract quite generally, in terms of the drift and volatility of the agent's continuation value in the contract.

An immediate difference between this paper and e.g., Sannikov (forthcoming) is that

we concentrate on project completion. We think this case is of independent interest for a number of different economic applications. But we also think that our setting, while less general in some respect than Sannikov’s, serves to make very clear the intertemporal incentives at work. In Sannikov’s models, the principal cares about the output of the project at all points in time; and makes payment to the agent in each period to provide incentives. A limited liability constraint (the agent’s consumption must be non-negative) means that the agent’s value is non-negative. When the risk neutral agent’s value is low, the agent receives zero consumption from the principal, and its value drifts upwards. In other words, its consumption is ‘back-loaded’. When the risk neutral agent’s value is high, the agent receives a higher consumption (set to satisfy the agent’s participation constraint), and its value drifts downwards. Its consumption is then ‘front-loaded’. In either case, the principal exposes the agent to as much risk as possible (subject to the limited liability constraint).³

In our model, the agent’s value drifts in only one direction: downwards. This is a direct consequence of the project completion setting. This same setting allows us to analyse the full time path of wages and actions, which also drift downwards. We can deal with different discount rates for the principal and the agent, leading to a distinction in terms of the asymptotic behaviour of the agent’s value, wage and effort. The agent is exposed to risk—no payments are made until the project is completed. In our view, this set-up allows a particularly clear demonstration of the dynamics of the optimal contract.

The rest of the paper is structured as follows. Section 2 lays out the basic model. Section 3 looks at the sequentially rational solution in which the principal has no commitment ability. Section 4 looks at the situation when the principal commits to a wage that is constant over time. The contrast between this and the (stationary) sequentially rational solution gives a strong intuition for the properties of the wage that the principal sets when it has full commitment power (and so can commit to a non-constant wage). The latter is analysed in section 5. Section 6 looks at the issue of deadlines; section 7

³In Sannikov (2007) and Sannikov (forthcoming), only the case of a risk averse agent is dealt with. The dynamics of the contract are complicated by the need for the principal to offer insurance to the agent, as well as by income effects. It is straightforward to derive the solution for a risk neutral agent. This case is also covered in DeMarzo and Sannikov (2006), but they allow for savings by the agent.

considers the issues that arise when the agent can affect the quality of the completed project. Our overall conclusions are stated in section 8. An appendix contains longer proofs.

2 The Model

Consider a continuous-time model where an agent must exert effort in any period in order to have a positive probability of success in a project. Assume that the effort choices of the agent are unobservable but the success of the project is verifiable; hence payments can be contingent only on the event of success or no success.

The principal and the agent are risk neutral. The agent is credit constrained so that payments from principal to agent must be non-negative in all periods. (Otherwise the solution to the contracting problem would be trivial: sell the project to the agent.) In fact, the agent could be allowed to be risk averse. The key assumption for our analysis is that the agent's value is positive, which here is a result of limited liability.

The instantaneous probability of success when the agent exerts the effort level a within a time interval of length Δt is $a\Delta t$ and the cost of such effort is $c(a)\Delta t$. We make the following assumption about the cost function.

Assumption 1 1. $c'(a) \geq 0, c''(a) \geq 0, c'''(a) \geq 0$ for all $a \geq 0$.

2. $c(0) = 0$ and $\lim_{a \rightarrow \infty} c'(a) = \infty$.

Because of limited liability, we can restrict attention to contracts where the principal pays $w(t) \geq 0$ to the agent if a success takes place in time period t and nothing if there is no success: it is clearly not optimal to make any payment to the agent before project completion. Success is worth $v \geq 0$ to the principal. Both the principal and the agent discount, with discount rates of r_P and r_A respectively.

We consider several models of contracting between the principal and the agent. We solve first for the sequentially rational wage offers. We then consider the case where the principal must choose a stationary wage at the beginning of the game. We show that the

sequentially rational wage exceeds the stationary commitment level. We then show that under full commitment, any stationary wage profile is dominated by a non-stationary, non-increasing one. Finally, we consider the use of deadlines for providing incentives.

3 Sequentially rational wage offers

3.1 Stationary wages

We start by supposing that the principal offers a spot wage contract for each period to the agent (or has the power to offer a temporary bonus for immediate success). We consider wage proposals of the form

$$w(s) = \begin{cases} w & \text{for } s \in [t, t + \Delta t), \\ \tilde{w} & \text{for } s \geq t + \Delta t. \end{cases}$$

There is no loss of generality in this, since we are looking for a stationary wage level. Wages in a stationary equilibrium are the same in all periods regardless of past wage offers. The crucial feature of this wage proposal is that, in principle, the current wage w can be different from the future wage \tilde{w} . In a stationary equilibrium, the two will obviously coincide.

The agent's problem can be characterized by a dynamic programming equation. Let the agent's value function from time $t + \Delta t$ onwards be \tilde{W} ; let its value function at time t be W . Then

$$W = \max_a \{a\Delta t w - c(a)\Delta t + e^{-r_A \Delta t} (1 - a\Delta t)\tilde{W}\}.$$

This can be rewritten as a continuous-time Hamilton-Jacobi-Bellman (HJB) equation:

$$W - \tilde{W} = \max_a \{(a(w - \tilde{W}) - c(a) - r_A \tilde{W})\Delta t\} \tag{1}$$

for sufficiently small Δt .

The first-order condition (which is necessary and sufficient by convexity of c) is

$$c'(a) = w - \tilde{W}.$$

Note that \tilde{W} is an ‘opportunity cost’ of exerting effort. By exerting effort, the agent increases the probability of project completion; if the project completes, the agent loses the continuation value \tilde{W} .

Denote the solution to this first-order condition by $a(w; \tilde{w})$. Note that if $w > \tilde{w}$, then $a(w; \tilde{w}) > a(w; w) > a(\tilde{w}; \tilde{w})$. The implicit function theorem implies that

$$\frac{\partial a(w; \tilde{w})}{\partial w} = \frac{1}{c''(a(w; \tilde{w}))}. \quad (2)$$

In a stationarity equilibrium, the sequentially rational wage will be constant: $w = \tilde{w}$. Hence $W = \tilde{W}$, so that the agent’s first-order condition can be written as

$$w - c'(a(w; w)) - \left(\frac{a(w; w)c'(a(w; w)) - c(a(w; w))}{r_A} \right) = 0. \quad (3)$$

This first-order condition has two components. The first, $w - c'(\cdot)$, relates to the myopic incentives that the agent faces, equating the wage to its marginal cost of effort. The second, $-(ac'(\cdot) - c(\cdot))/r_A$, describes the dynamic incentives. Since $c(\cdot)$ is convex, this term is non-positive. Consequently, this component leads the agent to decrease its current effort and substitute towards effort in the future—to smooth its effort profile. When r_A is very large (the agent discounts the future entirely), only the myopic incentives matter. When r_A is very small (no discounting), only dynamic incentives matter; the agent then exerts very low effort.

Let the principal’s value function at time t be denoted V . The principal’s dynamic programming equation is

$$V = \max_w \{a(w; \tilde{w})\Delta t(v - w) + e^{-r_P\Delta t}(1 - a(w; \tilde{w})\Delta t)\tilde{V}\}.$$

\tilde{V} is independent of w and hence the first-order condition is

$$w = v - \frac{a(w; w)(a(w; w) + r_P)}{r_P} \frac{1}{\frac{\partial a(w; \tilde{w})}{\partial w}}.$$

(By assumption 1, this is both necessary and sufficient for an optimum.) This equation shows the principal's balance of myopic and dynamic incentives. When r_P is very large (so that the principal discounts the future entirely), the first-order condition reduces to the myopic marginal equality:

$$(v - w) \frac{\partial a(w; \tilde{w})}{\partial w} - a(w; \tilde{w}) = 0.$$

When r_P is very small (no discounting), the first-order condition is dominated by dynamic incentives and the principal sets a zero wage.

Substitution gives

$$w = v - \frac{a(w; w)(a(w; w) + r_P)}{r_P} c''(a(w; w)) \quad (4)$$

which along with equation (3) gives the sequentially rational wage w^S and the agent's effort level a^S .

Equations (3) and (4) are reaction functions for the dynamic game; their intersection point determines the sequentially rational equilibrium. The equations give relationships between the wage w and effort a , which can also be interpreted in terms of the demand and supply of effort. The agent's supply of effort, given by equation (3), is an upward-sloping curve in (a, w) space: the agent requires a higher wage in order to put in more current effort. The principal's (inverse) demand for effort, given by equation (4), is downward-sloping: the higher the effort put in, the more likely it is that the principal has to pay the wage, and so the lower the wage that the principal wants to set. Equilibrium is determined by the unique intersection point of the reaction functions: an effort level a^S and wage level w^S .⁴

⁴Assumption 1 ensures that equation (3) yields an upward-sloping curve and equation (4) a downward-sloping curve.

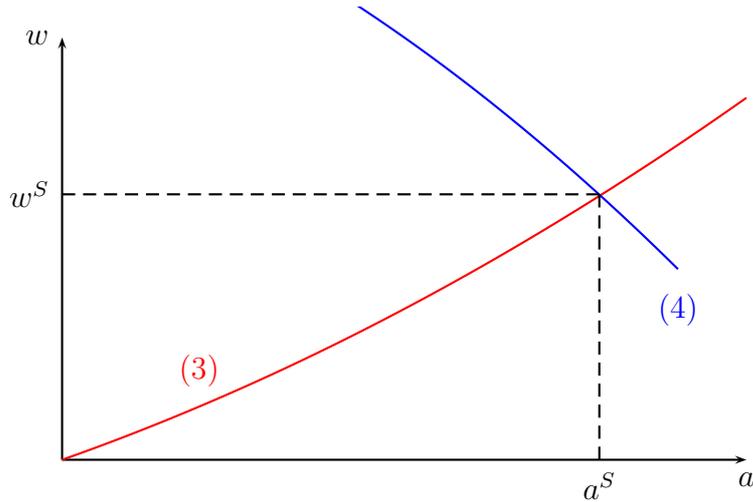


Figure 1: The sequentially rational solution with quadratic costs

The solution is illustrated in figure 1, which plots equations (3) and (4) for the case of quadratic costs: $c(a) = \gamma a^2$, where $\gamma > 0$. In this example, equation (3) gives

$$w(a) = \frac{\gamma a^2 + 2\gamma r_A a}{r_A}$$

and equation (4) gives

$$w(a) = v - \frac{2a\gamma(a + r_P)}{r_P}.$$

3.2 Non-stationary equilibrium wages

In this subsection, we discuss other sequentially rational equilibria of the game. Since the game is a dynamic game with frequent interactions, reasoning along the lines of the Folk-Theorem can be used to demonstrate the existence of other equilibria. A particular feature of these equilibria is that the agent cannot be punished since her action is completely unobservable. As a result, all continuation payoffs are contingent only on the wage offers given by the principal. We start the characterization of such equilibria by showing that the stationary sequential equilibrium wage gives the lowest possible equilibrium payoff to the principal.

Proposition 1 *Let \underline{V} be the lowest equilibrium payoff to the principal, and let V^S be the*

principal's payoff in the stationary sequentially rational equilibrium. Then $\underline{V} = V^S$.

Proof. Standard arguments establish that all equilibria in the current game can be supported by optimal penal codes in the sense of Abreu (1988). Consider then an arbitrary wage offer and continuation payoffs $V^e(w)$ following the equilibrium wage choice w . Denote the agent's equilibrium continuation value following the equilibrium wage offer w by $U^e(w)$. Let $a(w; U)$ denote the agent's effort choice at current wage w and continuation payoff U .

We have

$$V^e = \max_w \{a(w; U^e(w))\Delta t(v - w) + e^{-r_P \Delta t}(1 - a(w; U^e(w))\Delta t)V^e(w)\}$$

Since $V^e(w) \geq \underline{V}$ for all w by the definition of \underline{V} and the stationarity of the game (i.e., all continuation games are strategically equivalent to the entire game), we have for the worst equilibrium

$$\underline{V} = \max_w \{a(w; \underline{U})\Delta t(v - w) + e^{-r_P \Delta t}(1 - a(w; \underline{U})\Delta t)\underline{V}\},$$

where \underline{U} is the continuation payoff to the agent in the equilibrium with the worst payoff for the principal. But this is simply the principal's problem in the stationary equilibrium.

□

This proposition allows for a quick check whether a proposed wage path is consistent with equilibrium in this model. All one has to do is to calculate the payoff to the principal and check whether the continuation value ever stops below V^S . All the wage paths generating higher continuation values are consistent with equilibrium, whereas paths where the continuation values fall below this threshold are not. To see how to support the non-stationary equilibrium wage profiles, consider an arbitrary path of wages $\hat{w}(t)$ with the property that the continuation values $\hat{V}(t)$ to the principal along this path stay above V^S .⁵ Consider strategies of the form $w(t) = \hat{w}(t)$ if $w(s) = \hat{w}(s)$ for all $s < t$, $w(t) = w^S$

⁵We give an informal discussion of how to construct these equilibria to avoid some notational difficulties arising from our use of a continuous time model. The details of the argument for the discrete time model

if $w(s) \neq \widehat{w}(s)$ for some $s < t$. Since the principal gains from any deviation from the proposed path $\widehat{w}(t)$ for a duration of Δt only and then suffers a capital loss of $\widehat{V}(t) - V^S > 0$, it is never optimal to deviate.

In section 5, we show that under some circumstances, the optimal commitment wage profile decreases towards zero. It is clear that these wage profiles are not consistent with sequential rationality and hence the power to commit has real bite in the model.

4 Commitment to a single wage offer

We now contrast sequentially rational equilibria with the alternative case in which the principal commits to a wage w for the duration of the game and the agent maximizes utility by choosing effort optimally in each period.

The agent's problem is characterized by the dynamic programming equation:

$$W = \max_a \{a\Delta t w - c(a)\Delta t + e^{-r_A \Delta t} (1 - a\Delta t)W\}$$

where, as before, W denotes the agent's value function at time t . Letting $\Delta t \rightarrow 0$ (the continuous-time limit) and rearranging, we obtain the HJB equation:

$$r_A W = \max_a \{aw - c(a) - aW\}.$$

The agent's first-order condition (which is necessary and sufficient by convexity of c) is

$$c'(a) = w - W.$$

Substituting W from the first-order condition into the HJB equation gives:

$$W = \frac{ac'(a) - c(a)}{r_A}; \tag{5}$$

for short time intervals between decisions are standard and available from the authors upon request.

finally, this gives

$$w - c'(a) - \left(\frac{ac'(a) - c(a)}{r_A} \right) = 0 \quad (6)$$

which determines the optimal effort level $a(w)$, as a function of w , in this case. As before, the two parts of this first-order condition give the myopic $(w - c'(a))$ and dynamic $(-(ac'(a) - c(a))/r_A)$ incentives of the agent. (In fact, equation (6) has the same form as equation (3), although of course the equilibrium wage and action will be different.) As before, dynamic considerations lead the agent to substitute away from current effort towards future effort, as a result of the convexity of the agent's effort cost.

The implicit function theorem implies that

$$\frac{da}{dw} \equiv a'(w) = \frac{r_A}{(a + r_A)c''(a)} > 0. \quad (7)$$

Notice that the agent's current effort is *less* elastic in this case than in the sequentially rational solution. This is because a change in the constant commitment wage increases both the current and future wages. An agent faced with a higher future wage has a higher continuation value, and is therefore less willing to supply effort now.

Consider next the principal's optimization problem. Given the effort level $a(w)$, the value to the principal is

$$V(a, w) = \frac{a(w)(v - w)}{a(w) + r_P}.$$

Assumption 1 ensures that the necessary and sufficient optimality condition is

$$V'(w) = \frac{d}{dw}V(a(w), w) = \frac{\partial V(a, w)}{\partial a} \frac{da}{dw} + \frac{\partial V}{\partial w} = 0.$$

Hence (simplifying) we have:

$$w = v - \frac{a(w)(a(w) + r_P)}{r_P} \frac{1}{a'(w)}. \quad (8)$$

Substituting for $a'(w)$ gives

$$w = v - \frac{a(w)(a(w) + r_A)(a(w) + r_P)}{r_A r_P} c''(a(w)), \quad (9)$$

which, along with equation (6), can be solved for the equilibrium effort level a^C and the optimal wage w^C .

Equations (6) and (9) are the reaction functions for the dynamic game with commitment to a constant wage. As in the sequentially rational solution, the agent's reaction function is an upward-sloping curve in (a, w) space; the principal's reaction is downward-sloping. Equilibrium is determined by the unique intersection point of the reaction functions: an effort level a^C and wage level w^C .

The solution is illustrated in figure 2, which plots equations (6) and (9) for the case of quadratic costs: $c(a) = \gamma a^2$, where $\gamma > 0$. In this example, equation (6) gives

$$w(a) = \frac{\gamma a^2 + 2\gamma r_A a}{r_A}$$

and equation (9) gives

$$w(a) = v - \frac{2a\gamma(a + r_A)(a + r_P)}{r_A r_P}.$$

Comparison of equations (2) and (7) shows that the agent's current effort is *more* elastic in the sequentially rational case, because a marginal change in the current wage does not (necessarily) raise all future wages as well. In terms of reaction functions, the agent's reaction function is the same in the sequentially rational and constant commitment cases.

The principal's reaction is higher in the sequentially rational case: the principal is willing to pay a higher wage, for any given effort level. This is illustrated in figure 2 (for the quadratic cost case), which shows the upward shift in the principal's reaction function.

Consequently, the following proposition follows immediately.

Proposition 2 *In the sequentially rational solution, both the wage and the effort level*

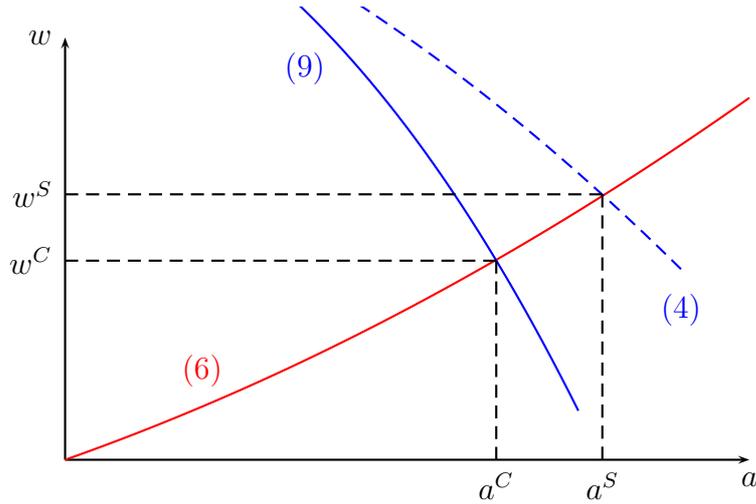


Figure 2: The constant commitment and sequentially rational solutions with quadratic costs

are higher than in the constant commitment case: $w^S \geq w^C$ and $a^S \geq a^C$.

In some applications, it may be a good idea to assume that the wage level must be constant (for example due to menu costs). In others, wages are naturally thought to be flexible over time. For those cases, the commitment equilibrium also gives a lower bound for the maximal payoff to the principal in sequentially rational equilibria. Since the continuation payoff in the commitment equilibrium is above the stationary equilibrium payoff level, these commitment wages can be supported as equilibrium wages with non-stationary wage offers.⁶

4.1 Comparative statics of equilibrium

The comparative statics of the equilibrium action and wage in the constant commitment case can now be investigated: how the action and wage depend on the separate discount rates r_P and r_A . Two cases are of particular interest:

1. $r_P = r_A = +\infty$: both the principal and the agent are myopic, ignoring all continuation values and playing the game as if it were one-shot.

⁶The non-stationarity arises here off the equilibrium path.

2. $r_P < r_A = +\infty$: the agent is myopic, but the principal is not.

These two cases allow us to identify the dynamic incentives in the model, by shutting down various channels in turn. The second case, with $r_A = +\infty$, also has a useful interpretation, as a case where the principal operates for an infinite number of periods, employing a sequence of different agents each for one period. This case will be useful when analysing wages with deadlines in section 6.

In the myopic case, with $r_P = r_A = +\infty$, the agent's and principal's continuation values are equal to zero. The agent's first-order condition is then

$$w = c'(a). \tag{10}$$

The principal's first-order condition is

$$w = v - ac''(a). \tag{11}$$

Equations (10) and (11) define the myopic wage w^M and effort a^M .

It is straightforward to show that the myopic effort a^M is greater than the effort in the constant commitment case a^C . The comparison with the constant commitment wage w^C is more difficult. Figure 3 (using quadratic costs) illustrates why. Equation (10) defines a curve in (a, w) space that lies below the curve defined by equation (6). That is, in the dynamic problem, the agent requires a higher wage to exert the same effort level as in the static situation. Clearly, this is due to the continuation value that is present in the dynamic problem. Equation (11) defines a curve in (a, w) space that lies above the curve defined by equation (9): the dynamic principal offers a lower wage than the static principal, for any given effort level. The reason is the same: the prospect of continuation value in the dynamic problem leads the principal to lower the wage. Both shifts lead to a lower effort level in the dynamic problem; but have an ambiguous effect on the wage.

Now consider the case $r_P < r_A = +\infty$. The agent is myopic, and so its first-order condition is

$$c'(a(w)) = w.$$

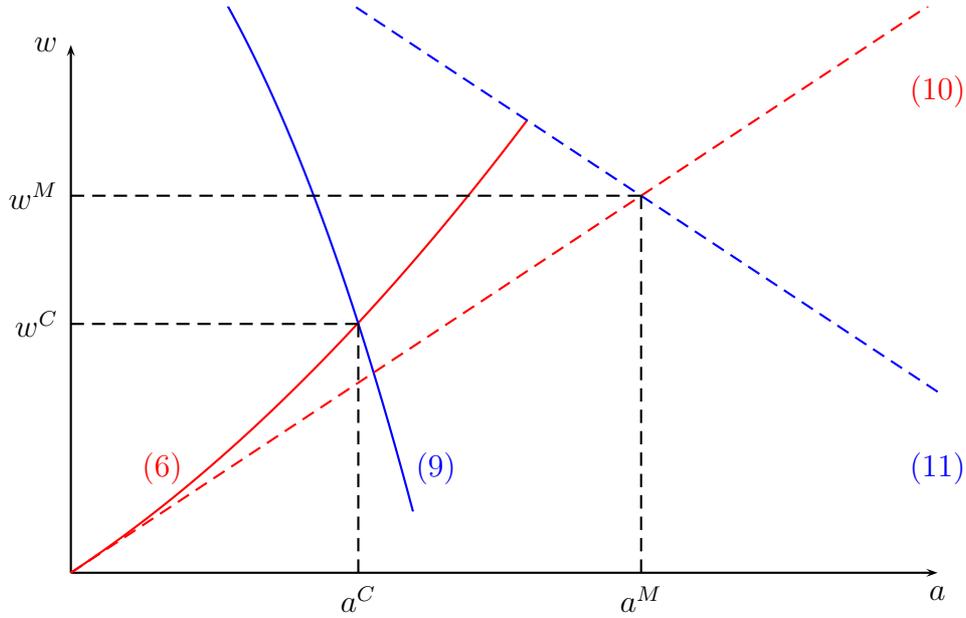


Figure 3: The constant commitment and myopic solutions

The principal's first-order condition is

$$w = v - \frac{a(w)(a(w) + r_P)}{r_P} c''(a(w)). \quad (12)$$

Let the effort level in this case be $a^{R,\infty}$ and the wage level $w^{R,\infty}$. (The notation will become clearer in section 6.) As in the previous case, the effect of increasing r_A to infinity on the effort level is easy to establish, but the effect on wage is ambiguous. An increase in the agent's discount rate always increases the equilibrium effort level. This occurs because the agent's reaction function shifts downwards, while the principal's reaction function shifts upwards.

The shifts are illustrated in figure 4 for the quadratic cost case. The figure also summarises the different cases that we have considered. The principal's reaction functions are labelled 'P', subscripted with the values of the discount rates. The agent's reaction functions are labelled 'A'. The sequentially rational solution is labelled 'S'; the constant commitment solution with r_P and r_A finite is labelled 'C'; the myopic case (with $r_P = r_A = \infty$) is labelled 'M'; and the agent replacement case (with $r_P < r_A = \infty$) is labelled

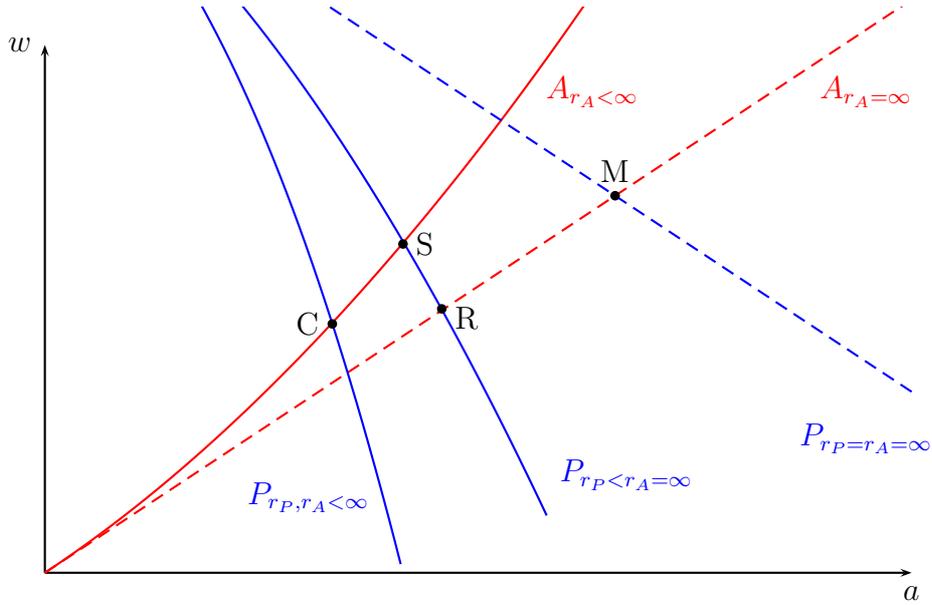


Figure 4: Summary of the cases

‘R’.

The figure shows that we can make the following general statements.

Proposition 3 1. *The agent’s effort is least in the case with a constant wage; higher in the sequentially rational solution; higher still when only the agent is myopic; and highest when both the principal and agent are myopic. That is, $a^C \leq a^S \leq a^{R,\infty} \leq a^M$.*

2. *The wage paid by the principal is lower in the constant wage case than in the sequentially rational solution. The wage paid to a myopic agent by a non-myopic principal is lower than both the sequentially rational wage and the wage paid when both are myopic. That is, $w^C \leq w^S$; $w^{R,\infty} \leq w^S$; and $w^{R,\infty} \leq w^M$.*

Only a little more can be said about equilibrium wages in particular cases. For example, suppose that costs are quadratic: $c(a) = \gamma a^2$. We can then establish the following.

Proposition 4 *Suppose that the cost function is quadratic: $c(a) = \gamma a^2$, with $\gamma > 0$. In this case $w^C < w^M$.*

Proof. With quadratic costs, $a^M = v/4\gamma$ and $w^M = v/2$. The proof works by determining the action levels in equations (6) and (9) that result by setting $w^C = w^M = v/2$. We establish that the action level from equation (9) is less than the action level from equation (6). This then necessarily means that $w^C < w^M$. From equation (6),

$$\frac{v}{2} = 2\gamma a + \frac{\gamma a^2}{r_A}; \quad (13)$$

from equation (9),

$$\frac{v}{2} = 2\gamma a + \frac{2\gamma a^2(r_A + r_P + a)}{r_A r_P}. \quad (14)$$

It is clear, therefore, that the action level in equation (13) is greater than the action level in equation (14). This concludes the proof. \square

Even with quadratic costs, no clear comparison can be made between e.g., w^C and $w^{R,\infty}$. Numerical solution with particular values for v, r_A and r_P shows that w^C can be greater or less than $w^{R,\infty}$, depending on the size of γ , the cost parameter. Outside of the quadratic cost case, the ordering wage levels in the different cases can be changed by altering the degree of convexity of the cost function and the discount rates. The effect of these changes is to alter the balance between the myopic and dynamic incentives in the model. (Diagrammatically, these changes affect the extent to which the principal's and agent's reaction functions shift as the comparative statics are done.)

5 Commitment to non-stationary wages

The analysis so far shows that the optimal commitment wage is not stationary in general. If the optimal commitment wage is stationary, then it must be at the level given in section 4. In section 3, we considered deviations for Δt units of time from a stationary wage. Since the analysis shows that it is optimal for the principal to offer a different wage if continuation wages are at w^C , we conclude immediately that the optimal commitment path of wages cannot be constant.

In order to derive the optimal commitment wage schedule, we use the method developed by Spear and Srivastava (1987) and Phelan and Townsend (1991) and write the optimal contract in terms of the agent's continuation value as the state variable. Consider an arbitrary reward function $w(t)$. The agent's HJB equation is given by:

$$r_A W(t) = \max_a \{a(w(t) - W(t)) - c(a) + \dot{W}(t)\}. \quad (15)$$

The agent's first-order condition is

$$w(t) = W(t) + c'(a(t)). \quad (16)$$

(Again, convexity of $c(\cdot)$ ensures that this is necessary and sufficient for an optimum.)

Substituting into the HJB equation gives

$$\dot{W}(t) = r_A W(t) - (a(t)c'(a(t)) - c(a(t))). \quad (17)$$

Equations (16) and (17) are constraints on the principal's problem.

The principal's HJB equation is

$$r_P V(W) = \max_a \left\{ a(t)(v - V(W) - W - c'(a(t))) + (r_A W - (a(t)c'(a(t)) - c(a(t))))V'(W) \right\} \quad (18)$$

where equation (16) has to be used to substitute for the wage and equation (17) for \dot{W} . The principal's first-order condition is

$$v - V(W) - W - (c'(a(t)) + a(t)c''(a(t))) - a(t)c''(a(t))V'(W) = 0; \quad (19)$$

from the properties of the cost function (see assumption 1), this is necessary and sufficient

for an optimum. Differentiating this first-order condition with respect to time gives

$$\begin{aligned} & -V'(W)\dot{W} - \dot{W} - (2c''(a(t)) + a(t)c'''(a(t)))\dot{a}(t) \\ & - (c''(a(t)) + a(t)c'''(a(t)))V'(W)\dot{a}(t) - a(t)c''(a(t))V''(W)\dot{W} = 0. \end{aligned} \quad (20)$$

The envelope theorem on equation (18) gives

$$-a(t)(V'(W) + 1) - (r_P - r_A)V'(W) + V''(W)\dot{W} = 0. \quad (21)$$

Combining equations (17), (20) and (21) gives

$$\begin{aligned} & (c''(a(t)) + (V'(W) + 1)(c''(a(t)) + a(t)c'''(a(t))))\dot{a}(t) \\ & = -(V'(W) + 1)(r_A W - (a(t)c'(a(t)) - c(a(t)))) \\ & \quad + a^2(t)c''(a(t))(r_P - r_A)V'(W). \end{aligned} \quad (22)$$

Equations (17) and (22) determine the dynamics of the system.

There are two further optimality conditions. The principal is free to choose the level of the agent's value at the start of the program. The initial value W_0 is determined by the (necessary) condition $V'(W_0) = 0$. The second optimality condition is the transversality condition that the system converges to a steady state in which $\dot{a} = \dot{W} = 0$ and a and W are bounded.

We are now able to characterise the dynamics of the optimal commitment contract.

Proposition 5 *In the full commitment solution, the agent's continuation value $W(t)$, the optimal wage profile $w(t)$, and the agent's effort level $a(t)$ are all decreasing over time. If $r_P \geq r_A$, then the continuation value, wage and effort levels converge to zero: $\lim_{t \rightarrow \infty} W(t) = \lim_{t \rightarrow \infty} w(t) = \lim_{t \rightarrow \infty} a(t) = 0$. If $r_P < r_A$, then the wage and effort levels converge to strictly positive levels. The initial effort level, and hence all levels, are below the myopic effort: $a(t) \leq a^M$.*

Figures 5 and 6 illustrate the proof of the proposition (which, given its length, is in

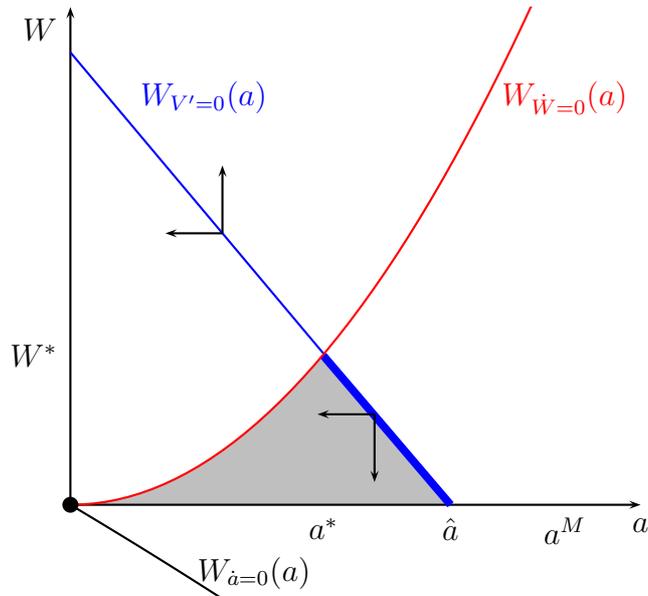


Figure 5: Phase diagram for the problem with full commitment with $r_P \geq r_A$

the appendix). In the shaded area in the figures, \dot{a} and \dot{W} are non-positive. The optimal initial point must lie on the portion of the $W_{V'=0}(a)$ curve that is in bold. Any optimal path from that portion of the curve must move into the shaded area; hence \dot{a} and \dot{W} must be non-positive along the entire path. Possible steady states are marked with a dot; clearly, they must lie on the curve along which $\dot{W} = 0$. If $r_P \geq r_A$, then only one steady state exists: the origin, with $a = W = 0$. Otherwise, two steady states exist (as shown in the figure). The optimal path converges to the steady state with $a = W = 0$, if $r_P \geq r_A$. Otherwise, it converges to a steady state with strictly positive levels of a and W . The latter is consistent with the previous result that in the limit, as $r_A \rightarrow \infty$, the equilibrium effort level converges to the (positive) level $a^{R,\infty}$.

These results are intuitive. Because the agent has increasing marginal cost of effort, it looks to smooth its effort over time: to substitute away from current effort toward future effort. Limited liability means that the agent earns positive rents from the contract. The principal uses the dynamics of these rents to provide the agent with incentives to exert effort. In particular, the full commitment contract ensures that the agent's continuation value is decreasing in equilibrium. By these means, the principal gives the agent incentives to exert current effort. The continuation value is driven downwards by a decreasing wage;

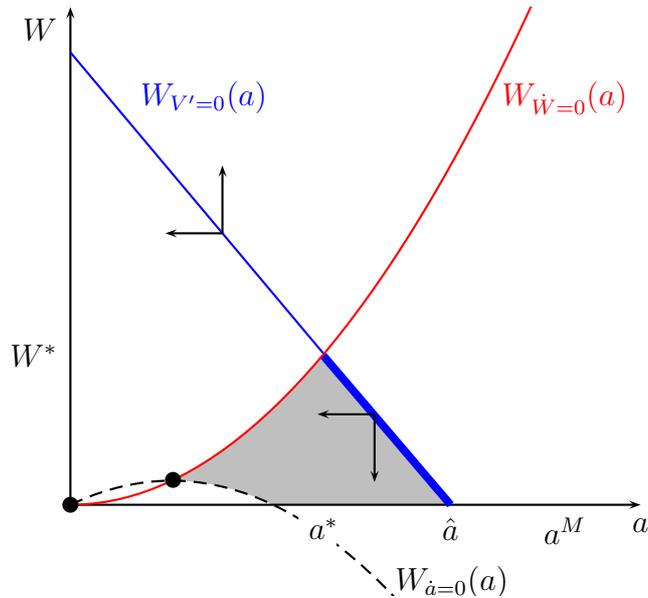


Figure 6: Phase diagram for the problem with full commitment with $r_P < r_A$

the agent's effort also decreases over time. When the agent is more patient than the principal, the principal has to drive the agent's value, wage and hence effort down to zero in the long-run in order to generate incentives for effort. But when the agent is less patient, the principal can rely on the agent's impatience to generate incentives. In this case, the long-run value, wage and effort are all strictly positive. In all cases, the principal induces less effort from a forward-looking agent: equilibrium effort is less than the myopic level.

6 Wages with a deadline

Consider now the case where the principal can commit to a constant wage w until time T . In general, this wage policy is not optimal, although it captures in a stark way a key aspect of the full commitment policy: a declining wage. In this section, we analyse whether such a policy can ever be optimal.

The agent's HJB equation is as before:

$$r_A W = \max_a \{a(w - W) - c(a) + \dot{W}\};$$

but note now that the wage w is not a function of time. The agent's first-order condition is also unchanged:

$$w = W + c'(a(t)).$$

Differentiating this first-order condition with respect to time gives

$$c''(a)\dot{a} = -\dot{W}. \tag{23}$$

The agent's continuation value at T must be zero: $W(T) = 0$. Since the agent's value is non-negative, it must be that $\dot{W} \leq 0$. Hence equation (23) implies that $\dot{a} \geq 0$: the agent's effort increases over time when the principal sets a constant wage and a deadline. The effort level at the terminal time must equal the myopic level, for the given wage. We use this fact in the proof of the following proposition (which, since it is lengthy, is given in the appendix).

Proposition 6 *If $r_P = r_A$, then when committing to pay a constant wage to a single agent, the principal does not use a finite deadline.*

Proposition 6 shows that the principal will not use a deadline—with a constant wage and equal discount rates for the principal and the agent, at least. On the other hand, the optimal situation for the principal is to employ an infinite sequence of agents each for a single period—the benchmark analysed in section 3. To span these two cases, suppose that the principal can search for a replacement agent, but only after it has fired its current agent. Search takes time: a replacement agent is found according to a Poisson process with parameter λ . The model considered in proposition 6 sets $\lambda = 0$: there is no prospect of replacement. The model in section 3 has $\lambda = +\infty$: replacement can occur infinitely often. (This explains the notation in that section, where $w^{R,\infty}$ is the wage when replacement occurs infinitely often.)

The principal's optimisation problem is

$$V(\lambda) \equiv \max_{w,T} \left\{ \int_0^T \exp\{-r_P t - A_{w,T}(t)\} a_{w,T}(t) (v - w) dt + \exp\{-r_P T\} \frac{\lambda}{r_P + \lambda} V(\lambda) \right\}. \quad (24)$$

Equation (24) makes explicit the dependence of the principal's value $V(\lambda)$ on the arrival rate λ of replacement agents. We are interested in characterising the dependence of the optimal choice of $T(\lambda)$ on the arrival rate λ .

Proposition 7 *$T(\lambda)$ is non-increasing in λ .*

The proof of the proposition is fairly mechanical and is given in the appendix. The proposition shows the (expected) result that the principal uses a shorter deadline when it is easier to replace the agent. In the limit, of course, when agents can be replaced infinitely often ($\lambda \rightarrow \infty$), the principal is in the ideal position of using a sequence of agents, each for an interval $dt \rightarrow 0$.

7 Project quality

In the analysis so far, the benefit to the principal from a completed project is fixed and verifiable. More generally, we might suppose that the agent is able to affect the quality, and hence the benefit to the principal, of the completed project.

We shall not attempt a general analysis of this issue here. Instead, we outline a variant of our model where project quality has no effect the overall conclusions. Suppose that an agent can affect the probability of project completion by exerting effort, in the same way as in previous sections. But now, the quality of a completed project is drawn at random from a distribution $F(v)$ where $v \geq 0$, which is common knowledge.

At this point, there are two possibilities, which (it will turn out) are equivalent. The first is that the realised project quality cannot be verified. (For example, the agent may be a headhunter and the completed project a candidate for an executive position in the principal's firm. The fit of the candidate for the principal's post may be very difficult to

establish to a third-party.) Hence the principal cannot condition payment on the realised project quality.

The second possibility is that the project quality is verifiable. This then raises the possibility that the principal pays only when the realised project quality is at least as great as the current completion wage. But it is easy to see that this cannot be optimal. The agent's expected payment on completing at time t would be $\mathbb{E}[w|v \geq w]$ where the expectation is taken with respect to the quality distribution $F(\cdot)$. The principal gains $\mathbb{E}[v - w|v \geq w]$. But the principal could offer this level of payment unconditionally (and hence present the agent with the same effort incentives); and then accept all completed projects. The principal would gain $\mathbb{E}[v|v \leq w]$ from this.

So, the principal will continue to pay only on project completion. The problem is unaltered by this modification: the principal's benefit v is replaced by an expected benefit—that is all. Hence our previous analysis continues to hold.

The crucial feature of this example is that the agent's effort affects only the arrival rate of project completion, but not the realised quality. If project quality is affected (stochastically) by effort levels, then the principal will adjust the wage in order to affect both the quality and rate of completion of the project. We leave this and other related issues to further work.

8 Conclusions

We have developed a model of dynamic moral hazard involving project completion which has allowed us to identify clearly the intertemporal incentives involved. The contrast between the sequentially rational solution and the contract with commitment to a constant wage points immediately to the form of the full commitment contract. It involves a completion payment that decreases over time. In this way, the principal decreases the agent's continuation value over time; by doing so, the principal counteracts the agent's incentive to smooth its effort over time by substituting from current effort to future effort.

Given the simplicity of the current model, we believe a number of interesting extensions

could be considered. In the example of selling real estate, the principal has an additional instrument at her disposal: the required sales price. By changing the required price, the principal also changes the marginal impact of additional effort by the agent (since the probability of completing the sale changes). This is an example of a setting where the quality of the project is verifiable, and can be adjusted along the project.

In this paper, we have assumed that the outside option of the agent is exogenously fixed (at zero). Another extension of the model would analyze changes in the continuation values, and the resulting changes in the optimal contracts, when the outside option of the agent arises endogenously as in a matching model. The division of surplus between the principal and the agent would be of particular interest in such markets.

Appendix

Proof of Proposition 5

The proof uses a phase diagram in (a, W) space. Three aspects need to be analysed: the dynamics of a and W ; and the sign of $V'(\cdot)$ in equilibrium.

The sign of \dot{W} is, from equation (17), determined by the sign of $r_A W - (ac'(a) - c(a))$. This defines an upward-sloping function in (a, W) space, given by

$$W_{\dot{W}=0}(a) \equiv \frac{ac'(a) - c(a)}{r_A}$$

so that for $W > (<)W_{\dot{W}=0}(a)$, $\dot{W} > (<)0$. Note that $W_{\dot{W}=0} = 0$.

To determine the sign of $V'(\cdot)$ in equilibrium, manipulate the principal's Bellman equation and first-order condition to give

$$(r_A W - ac'(a) + c(a) + (a + r_P)ac''(a))V'(W) = r_P(v - W - c'(a)) - (a + r_P)ac''(a).$$

Consider the left-hand side of this equation. From assumption 1, $r_A W - ac'(a) + c(a) + (a + r_P)ac''(a) \geq 0$ for all non-negative values of a and W . Hence the sign of $V'(W)$ is

determined by the sign of $r_P(v - W - c'(a)) - (a + r_P)ac''(a)$. This defines a function in (a, W) space, given by

$$W_{V'=0}(a) \equiv v - c'(a) - \left(\frac{a + r_P}{r_P} \right) ac''(a).$$

This is a downward-sloping function, with an intercept $W_{V'=0}(0) = v$; it hits the horizontal axis at an effort level \hat{a} strictly less than the myopic level a^M (defined by $v - c'(a^M) - a^M c''(a^M) = 0$). For values of (a, W) lying below (above) this function, $V'(W)$ is positive (negative); along the function, $V'(W) = 0$. The function $W_{\dot{W}=0}(a)$ is, therefore, split into two portions by the function $W_{V'=0}(a)$; call the intersection point of the two functions (a^*, W^*) .

Now consider the dynamics of a , determined by equation (22). When $V'(\cdot) = 0$ (in particular, at the optimal initial choice of W), the term on the left-hand side, $c''(a(t)) + (V'(W)+1)(c''(a(t)) + a(t)c'''(a(t)))$, is non-negative (using assumption 1). The right-hand side is equal to

$$-(r_A W - (a(t)c'(a(t)) - c(a(t))) + a^2 c''(a)),$$

which by assumption 1 is negative for all non-negative values of a and W . Hence $\dot{a} \leq 0$ at the optimal initial choice of W .

The function defined by $\dot{a} = 0$ is

$$W_{\dot{a}=0}(a) \equiv \frac{ac'(a) - c(a)}{r_A} - \frac{ac''(a)}{r_A} \left(a + (r_P - r_A) \frac{V'(W)}{V'(W) + 1} \right).$$

Note that when $V' = 0$,

$$W_{\dot{a}=0}(a) \equiv \frac{ac'(a) - c(a) - a^2 c''(a)}{r_A} \leq 0$$

from assumption 1. Hence the function $W_{\dot{a}=0}(a)$ crosses the function $W_{V'=0}(a)$ at a point below the horizontal axis. If $r_P \geq r_A$, then $W_{\dot{a}=0}(a) \leq 0$ for all $a \in [0, \hat{a}]$. If $r_P < r_A$, then $W_{\dot{a}=0}(a) > W_{\dot{W}=0}(a)$ for sufficiently small a . Since $W_{\dot{a}=0}(a)$ is continuous in a , the $W_{\dot{a}=0}(a)$ curve must therefore cross the $W_{\dot{W}=0}(a)$ at a value of a that is strictly less than

a^* .

We can now determine the dynamics of a and W . The region of particular interest for the analysis is defined as follows. Let

$$\mathcal{W}(a) \equiv \{W \in \mathbb{R}_+ | W \leq W_{\dot{W}=0}(a) \text{ and } W \leq W_{V'=0}(a) \text{ and } W \geq W_{\dot{a}=0}(a)\}$$

for $a \in [0, \hat{a}]$. Let

$$\mathcal{E} \equiv [0, \hat{a}] \tilde{A} - - - \mathcal{W}(a).$$

For $(a, W) \in \mathcal{E}$, both \dot{a} and \dot{W} are non-positive. (The region \mathcal{E} is illustrated as the shaded regions in figures 5 and 6.) If $r_P \geq r_A$, then \mathcal{E} is defined as the (lower) area between the curves $W_{\dot{W}=0}(a)$ and $W_{V'=0}(a)$. If $r_P < r_A$, then \mathcal{E} is further defined by the curve $W_{\dot{a}=0}(a)$.

An initial choice of W on the portion of the $W_{V'=0}(a)$ curve above the $W_{\dot{W}=0}(a)$ cannot be optimal. The reason is that the dynamics from this point involve $\dot{W} \geq 0$. Hence any path from such a point cannot converge to a steady state, and by transversality cannot be optimal.

Hence the optimal initial choice of W must lie on the portion of the $W_{V'=0}(a)$ curve below the $W_{\dot{W}=0}(a)$. (Note that this must involve an initial effort level less than \hat{a} , and hence less than the myopic level a^M .) The initial dynamics from such a point are $\dot{W} \leq 0$ and $\dot{a} \leq 0$. The resulting path lies in the region \mathcal{E} , and hence $\dot{W} \leq 0$ and $\dot{a} \leq 0$ along all parts of an optimal path. The dynamics of $w(t)$ then follows from (16).

If $r_P \geq r_A$, then assumption 1 ensures that the function $W_{\dot{a}=0}(a)$ is negative for all values of $a \in [0, \hat{a}]$. Hence the only feasible steady state is $a = W = 0$. If $r_P < r_A$, then there is a second steady state with strictly positive $a \in (0, a^*)$ and $W \in (0, W^*)$; the optimal path converges to this steady state.

Proof of Proposition 6

The proof has three steps.

1. For any given wage, the expected discounted costs of the agent's efforts are higher

when the agent is employed in perpetuity than when a finite deadline is used. Let a_w be the agent's effort with no deadline (since the wage is constant, the effort is constant). $a_{w,T}(t)$ is the agent's effort with a deadline T ; because of the deadline, this effort varies with time. Then we shall show that

$$\begin{aligned} C_{w,\infty} &\equiv \int_0^\infty \exp\{-r_A t - a_w t\} c(a_w) dt = \frac{c(a_w)}{r_A + a_w} \\ &\geq C_{w,T} \equiv \int_0^T \exp\{-r_A t - A_{w,T}(t)\} c(a_{w,T}(t)) dt \end{aligned}$$

for finite T , where

$$A_{w,T}(t) \equiv \int_0^t a_{w,T}(s) ds.$$

The proof of this step is by contradiction: suppose not. The expected discounted cost $C_{w,T}$ is continuous in T and $\lim_{T \rightarrow \infty} C_{w,T} = C_{w,\infty}$. Hence if $C_{w,\infty}$ is not larger than $C_{w,T}$ for all finite T , then there must be some $T^* < \infty$ such that $C_{w,T}$ is maximised at $T = T^*$. By continuity, for some cost level $C \equiv C_{w,T^*} - \Delta$, for a small, positive Δ , there exist two times $T_1 < T^* < T_2$ such that $C_{w,T_1} = C_{w,T_2} = C$. At $T = T_1$, $C_{w,T}$ must be increasing in T . In the limit, as $\Delta \rightarrow 0$, this means that $rC_{w,T_1} > c(a_{w,T_1}(0))$. At $T = T_2$, $C_{w,T}$ must be decreasing in T . In the limit, as $\Delta \rightarrow 0$, this means that $rC_{w,T_2} < c(a_{w,T_2}(0))$. Since, by construction, $C_{w,T_1} = C_{w,T_2}$, this means that $c(a_{w,T_2}(0)) > c(a_{w,T_1}(0))$, or $a_{w,T_2}(0) > a_{w,T_1}(0)$. But we have established that $a_{w,T}(T) = a^M$ for any T . Equation (23) then implies that if $T_2 > T_1$, then $a_{w,T_2}(0) < a_{w,T_1}(0)$. Hence $C_{w,\infty} \geq C_{w,T}$ for any finite T .

2. For any given wage, the agent's expected discounted payoff when it is employed in perpetuity is higher than when a finite deadline is used:

$$\begin{aligned} R_{w,\infty} &\equiv \int_0^\infty \exp\{-r_A t - a_w t\} a_w w dt = \frac{a_w w}{r_A + a_w} \\ &\geq R_{w,T} \equiv \int_0^T \exp\{-r_A t - A_{w,T}(t)\} a_{w,T}(t) w dt. \end{aligned}$$

This must hold because $R_{w,\infty} - C_{w,\infty} \geq R_{w,T} - C_{w,T}$ and $C_{w,\infty} \geq C_{w,T}$. The first

statement holds because, when faced with an infinite deadline, the agent can always choose its effort as if it were facing a finite deadline.

3. The principal's expected discounted payoff when it employs the agent in perpetuity is higher than when a finite deadline is used. To see this, note first that the principal's expected discounted payoff is equal to the agent's expected discounted payoff, multiplied by a constant factor:

$$\int_0^\infty \exp\{-r_P t - a_w t\} a_w (v - w) dt = \frac{a_w (v - w)}{r_P + a_w} = \frac{v - w}{w} R_{w, \infty},$$

$$\int_0^T \exp\{-r_P t - A_{w, T}(t)\} a_{w, T}(t) (v - w) dt = \frac{v - w}{w} R_{w, T}.$$

In these equalities, we use the condition that $r_A = r_P$. Secondly, step 2 established $R_{w, \infty} \geq R_{w, T}$.

Proof of Proposition 7

Write the principal's optimisation problem as

$$\max_{w, T} \{ \alpha(w, T)(v - w) + \exp\{-r_P T\} g(\lambda) \},$$

where

$$\alpha(w, T) \equiv \int_0^T \exp\{-r_P t - A_{w, T}(t)\} a_{w, T}(t) dt,$$

$$g(\lambda) \equiv \frac{\lambda}{r_P + \lambda} V(\lambda).$$

Since $V(\lambda)$ must be non-decreasing in λ , $g(\lambda)$ is non-decreasing in λ . The first-order conditions for interior solutions are

$$\alpha_w (v - w) - \alpha = 0, \tag{25}$$

$$\alpha_T (v - w) - r_P \exp\{-r_P T\} g(\lambda) = 0 \tag{26}$$

where subscripts denote partial derivatives and the arguments of α have been omitted for brevity. (Note the proposition 6 established that $\alpha_T > 0$; it is straightforward to establish that $\alpha_w > 0$ also.)

The comparative statics of the optimal controls can be established using Cramer's rule. The determinant of the Hessian is

$$H \equiv \begin{vmatrix} \alpha_{ww}(v-w) - 2\alpha_w & \alpha_{wT}(v-w) - \alpha_T \\ \alpha_{wT}(v-w) - \alpha_T & \alpha_{TT}(v-w) + r^2 \exp\{-r_P T\}g(\lambda) \end{vmatrix}.$$

From the second-order condition for interior solutions, $H \geq 0$. By Cramer's rule,

$$\begin{aligned} \frac{dT(\lambda)}{d\lambda} &= \frac{1}{H} \begin{vmatrix} \alpha_{ww}(v-w) - 2\alpha_w & 0 \\ \alpha_{wT}(v-w) - \alpha_T & r_P \exp\{-r_P T\}g'(\lambda) \end{vmatrix} \\ &= \frac{1}{H} (\alpha_{ww}(v-w) - 2\alpha_w) r_P \exp\{-r_P T\}g'(\lambda) \leq 0, \end{aligned}$$

where the inequality follows from the second-order conditions and $g'(\lambda) \geq 0$.

References

- ABREU, D. (1988): "On the Theory of Infinitely Repeated Games with Discounting," *Econometrica*, 56(2), 383–396.
- DEMARZO, P., AND Y. SANNIKOV (2006): "Optimal Security Design and Dynamic Capital Structure in a Continuous-Time Agency Model," *Journal of Finance*, 61, 2681–2724.
- FERNANDEZ-MATEO, I. (2003): "How Free Are Free agents? Relationships and Wages in a Triadic Labor Market," Available at http://www.london.edu/assets/documents/Isabel_paper.pdf.
- FINLAY, W., AND J. COVERDILL (2000): "Risk, Opportunism, and Structural Holes: How Headhunters Manage Clients and Earn Fees," *Work and Occupations*, 27, 377–405.

- HOPENHAYN, H., AND J. P. NICOLINI (1997): “Optimal Unemployment Insurance,” *Journal of Political Economy*, 105, 412–438.
- LAFFONT, J.-J., AND D. MARTIMORT (2002): *The Theory of Incentives: the Principal-Agent Model*. Princeton University Press.
- LAND REGISTRY (2007): “House Price Index,” Discussion paper.
- MALCOMSON, J., AND F. SPINNEWYN (1988): “The Multi-Period Principal-Agent Problem,” *Review of Economic Studies*, 55, 391–408.
- MERLO, A., AND F. ORTALO-MAGNÉ (2004): “Bargaining over Residential Properties: Evidence from England,” *Journal of Urban Economics*, 56, 192–216.
- MERLO, A., F. ORTALO-MAGNÉ, AND J. RUST (2006): “Bargaining and Price Determination in the Residential Real Estate Market,” Available at http://gemini.econ.umd.edu/jrust/research/nsf_pro_rev.pdf.
- OFFICE OF FAIR TRADING (2004): “Estate Agency Market in England and Wales,” Discussion Paper OFT693, Office of Fair Trading.
- PHARMAFOCUS (2007): “Finding top people for the top job,” Internet Publication, http://www.pharmafocus.com/cda/focusH/1,2109,22-0-0-0-focus_feature_detail-0-491515,00.html.
- PHELAN, C., AND R. TOWNSEND (1991): “Computing Multi-Period, Information-Constrained Optima,” *Review of Economic Studies*, 58, 853–881.
- SANNIKOV, Y. (2007): “Games with Imperfectly Observable Actions in Continuous Time,” *Econometrica*, 75(5), 1285–1329.
- (forthcoming): “A Continuous-Time Version of the Principal-Agent Problem,” *Review of Economic Studies*.
- SHAVELL, S., AND L. WEISS (1979): “The Optimal Payment of Unemployment Insurance Benefits over Time,” *Journal of Political Economy*, 87(6), 1437–1362.

SPEAR, S., AND S. SRIVASTAVA (1987): “On Repeated Moral Hazard with Discounting,”
Review of Economic Studies, 54, 599–617.

WILLAMS, N. (2006): “On Dynamic Principal-Agent Problems in Continuous Time,”
Available at <http://www.princeton.edu/~noahw/pa1.pdf>.