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ABSTRACT

Monetary Policy Under Uncertainty in an Estimated Model with Labour Market Frictions*

We study the design of monetary policy in an estimated model with sticky prices, search and matching frictions, and staggered nominal wage bargaining. We find that the estimated natural rate of unemployment is consistent with the NBER description of the U.S. business cycle, and that the inflation/unemployment trade-off facing monetary policymakers is quantitatively important. We also show that parameter uncertainty has a limited effect on the performance or design of monetary policy, while natural rate uncertainty has more sizeable effects. Nevertheless, policy rules that respond to the output or unemployment gaps are more efficient than rules responding to output or unemployment growth rates, also in the presence of uncertainty about the natural rates.

JEL Classification: E24, E32, E52 and J64
Keywords: labour market search, monetary policy, natural rate uncertainty, parameter uncertainty and unemployment

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1 Introduction

In recent years, monetary business cycle models with monopolistic competition and staggered price setting have been widely used to study the implications of alternative specifications of monetary policy. One shortcoming of these models, however, is that they typically do not include a very detailed description of the labor market, and are therefore not suited to discuss the relationship between monetary policy and unemployment. In the labor market literature, on the other hand, search and matching models with equilibrium unemployment have been fairly successful in explaining aggregate labor market fluctuations. Such labor market specifications have recently been extended to monetary business cycle models, originally by Trigari (2004, 2006) and Walsh (2005b), and thus present a natural alternative to the standard monetary framework.

Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003) have demonstrated that nominal wage rigidities are a crucial ingredient when explaining U.S. business cycles, using monetary business cycle models without search and matching frictions. Within a similar model, Levin, Onatski, Williams, and Williams (2005) have shown that wage rigidities account for the main welfare cost of business cycle fluctuations, and that a monetary policy rule that responds only to nominal wage inflation performs almost as well as the welfare-optimizing policy. However, these results are very sensitive to the precise form of wage rigidities, suggesting that the specification of the labor market has important consequences for monetary policy.

The aim of this paper is to better understand the importance of labor market frictions and the evolution of labor market variables for the design of monetary policy. We study a micro-founded macroeconometric model with sticky prices, search and matching frictions on the labor market, and staggered nominal wage bargaining, following Gertler and Trigari (2006) and Gertler, Sala, and Trigari (2007). Compared with the models of Christiano et al. (2005), Smets and Wouters (2003), and Levin et al. (2005), our model includes a more realistic description of the labor market, featuring equilibrium unemployment, and wage rigidities are not subject to the Barro (1977) critique. It is therefore a natural laboratory for studying issues related to monetary policy and the labor market. In addition, Gertler et al. (2007) show that this new framework fits U.S. data well.

Using this model we study the behavior of the natural rate of unemployment and the implied unemployment (and output) gap(s), and we quantify the trade-offs facing the monetary authorities. We also analyze the design of monetary policy in the estimated model and the effects of parameter and natural rate uncertainty on optimized monetary policy rules.

In contrast to the existing literature on monetary policy in models with search and matching frictions, for instance, Blanchard and Galí (2006) and Thomas (2007), we use a quantitative framework and we study the implications for monetary policy of uncertainty concerning parameters and the natural rates of unemployment and output. While many authors have studied robust monetary policy with parameter and model uncertainty, for example, Levin, Wieland, and Williams (1999, 2003), Leitemo and Söderström (2005), Levin et al. (2005), Batini, Justiniano, Levine, and Pearlman (2006), and Edge, Laubach, and Williams (2007b), to our knowledge no one has considered uncertainty in a model with equilibrium unemployment.

Our analysis proceeds in the following steps. We first develop our model (in Section 2) and estimate it on U.S. data using Bayesian techniques (in Section 3). This part of the paper follows closely Gertler et al. (2007). We show that the estimated model fits U.S. data very well, also for the rate of unemployment and the degree of labor market tightness, variables that were not used when estimating the model.

We then discuss some properties of the model that are important for the design of monetary policy (see Section 4). In particular, we study the behavior of the estimated natural rates of output and unemployment and the implied output and unemployment gaps. We find that the implied path for the natural rate of unemployment is similar to estimates obtained with very different methodologies, for instance, by Staiger, Stock, and Watson (1997, 2002) or Orphanides and Williams (2002), and that the estimated unemployment and output gaps coincide closely with the standard view of the U.S. business cycle (for example, contractions dated by the National Bureau of Economic Research). This feature of the model is in stark contrast with other estimated macroeconometric models, e.g., Levin et al. (2005) or Edge, Kiley, and Laforte (2007a). We also discuss the trade-offs facing monetary policymakers in terms of inflation and unemployment stability, showing that complete inflation stabilization is very costly in terms of unemployment volatility, mainly due to shocks to price markups, but also to the bargaining power of workers and (in the presence of wage rigidities) technology.

Finally, we study the design of monetary policy in our framework, assuming that the central bank aims at minimizing a loss function that is consistent with the mandate of the U.S. Federal Reserve (see Section 5). In particular, we compare the performance of standard monetary policy rules that respond to the rate of inflation and the output gap with rules that in addition to inflation respond to the unemployment gap, the output growth rate, or the change in the unemployment rate. We also study the effects of uncertainty concerning model parameters and the natural rates of output and unemployment on the appropriate conduct of monetary policy. We show that the optimized monetary policy rules are superinertial, that is, the interest rate should respond to the lagged interest rate with a coefficient larger than one. Parameter uncertainty has little effect on the performance of monetary policy rules, while uncertainty concerning the natural rates has a more sizeable effect, especially for the rule responding to the unemployment gap. Finally, we show that monetary policy rules that respond to the output or unemployment gaps dominate rules responding to the growth rates of output and unemployment, also when the central bank faces uncertainty about the natural rates.

2 The model

The model is based on Gertler, Sala, and Trigari (2007) and is a monetary Dynamic Stochastic General Equilibrium (DSGE) framework with habit formation, investment adjustment costs, variable capital utilization, and nominal price and wage rigidities. The model also includes growth in the form of a non-stationary productivity shock, as in Altig, Christiano, Eichenbaum, and Lindé (2005). In contrast to conventional DSGE models, the labor market involves search and matching in the spirit of Mortensen and Pissarides (1994) and others, and nominal wage rigidity in the form of staggered Nash bargaining as in Gertler and Trigari

(2006).

We here provide a sketch of the model; for more details, see Gertler et al. (2007). There are three types of agents in the model: households, wholesale firms, and retail firms. Following Merz (1995) we assume a representative family in order to introduce complete consumption insurance. Production takes place at competitive wholesale firms that hire workers and negotiate wage contracts. Monopolistically competitive retail firms buy goods from wholesalers, repackage them as final goods, and set prices on a staggered basis.

2.1 Households

There is a representative household with a continuum of members of measure unity. At each time t a measure n_t of household members are employed and a measure $1 - n_t$ are unemployed. Household members are assumed to pool their labor income to insure themselves against income fluctuations. The household consumes final goods, saves in one-period nominal government bonds, and accumulates physical capital through investment. It transforms physical capital to effective capital by choosing the capital utilization rate, and then rents effective capital to firms.

The household thus chooses consumption c_t, bond holdings B_t, the rate of capital utilization ν_t, investment i_t, and physical capital k_t^p to maximize the utility function

$$\mathrm{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \varepsilon_{t+s}^b \log\left(c_{t+s} - hc_{t+s-1}\right) \right\}, \tag{1}$$

where β is a discount factor, h measures the degree of habits in consumption preferences, and ε_t^b is a preference shock with mean unity.[1]

The capital utilization rate ν_t transforms physical capital into effective capital according to

$$k_t = \nu_t k_{t-1}^p, \tag{2}$$

which is rented to wholesale firms at the rate r_t^k. The cost of capital utilization per unit of physical capital is given by $\mathcal{A}(\nu_t)$, and we assume that $\nu_t = 1$ in steady state, $\mathcal{A}(1) = 0$ and $\mathcal{A}'(1)/\mathcal{A}''(1) = \eta_\nu$, as in Christiano et al. (2005) and others.

Physical capital accumulates according to

$$k_t^p = (1-\delta)k_{t-1}^p + \varepsilon_t^i \left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\right)\right] i_t, \tag{3}$$

where δ is the rate of depreciation, ε_t^i is an investment-specific technology shock with mean unity, and $\mathcal{S}(\cdot)$ is an adjustment cost function which satisfies $\mathcal{S}(\gamma_z) = \mathcal{S}'(\gamma_z) = 0$ and $\mathcal{S}''(\gamma_z) = \eta_k > 0$, where γ_z is the steady-state growth rate.

Let p_t be the nominal price level, r_t the one-period nominal interest rate, w_t the real wage, b_t the flow value of unemployment (including unemployment benefits), Π_t lump-sum

[1] As in Gertler et al. (2007), we do not allow for variation in hours on the intensive margin, for two reasons. First, most of the cyclical variation in hours in the U.S. is on the extensive margin. Second, earlier estimates confirmed that the intensive margin was unimportant to the cyclical variation, as the estimated Frisch elasticity was close to zero, in line with the microeconomic evidence.

profits, and T_t lump-sum transfers. The household's budget constraint is then given by

$$c_t + i_t + \frac{B_t}{p_t r_t} = w_t n_t + (1 - n_t) b_t + r_t^k \nu_t k_{t-1}^p + \Pi_t + T_t - \mathcal{A}(\nu_t) k_{t-1}^p + \frac{B_{t-1}}{p_t}. \tag{4}$$

The first-order conditions with respect to c_t, B_t, ν_t, i_t, and k_t^p imply relationships that jointly determine consumption, capital utilization, investment, and Tobin's Q (see Gertler et al. (2007) for detail).

2.2 Wholesale firms

There is a continuum of wholesale firms measured on the unit interval. Each firm i produces output $y_t(i)$ using capital $k_t(i)$ and labor $n_t(i)$ according to the Cobb-Douglas production function

$$y_t(i) = k_t(i)^\alpha \left[z_t n_t(i) \right]^{1-\alpha}, \tag{5}$$

where z_t is a common labor-augmenting productivity factor, whose growth rate $\varepsilon_t^z = z_t / z_{t-1}$ follows a stationary exogenous process with steady-state value ε^z which corresponds to the economy's steady-state (gross) growth rate γ_z. Thus, technology is non-stationary in levels but stationary in growth rates. We assume that capital is perfectly mobile across firms and that there is a competitive rental market for capital.

To attract new workers wholesale firms need to post vacancies $v_t(i)$. The total number of vacancies and employed workers are then equal to $v_t = \int_0^1 v_t(i) di$ and $n_t = \int_0^1 n_t(i) di$. All unemployed workers are assumed to look for a job, and unemployed workers who find a match go to work immediately within the period. Accordingly, the pool of unemployed workers is given by

$$u_t = 1 - n_{t-1}. \tag{6}$$

The number of new hires is determined by the number of searchers and vacancies according to a matching function

$$m_t = \sigma_m u_t^\sigma v_t^{1-\sigma}. \tag{7}$$

The probability that a firm fills a vacancy is then given by $q_t = m_t / v_t$, and the probability that a worker finds a job is $s_t = m_t / u_t$.

It is useful to define the hiring rate $x_t(i)$ as the ratio of new hires $q_t v_t(i)$ to the existing workforce $n_{t-1}(i)$:

$$x_t(i) = \frac{q_t v_t(i)}{n_{t-1}(i)}, \tag{8}$$

where the law of large numbers implies that the firm knows $x_t(i)$ with certainty at time t, as it knows the likelihood q_t that each vacancy will be filled. Therefore, we can treat the hiring rate as the firm's control variable.

Firms exogenously separate from a fraction $1 - \rho$ of their existing workforce $n_{t-1}(i)$ in each period, and workers who lose their jobs are not allowed to search until the next period.

The total workforce is then the sum of the number of surviving workers and new hires:

$$n_t(i) = \rho n_{t-1}(i) + x_t(i) n_{t-1}(i). \tag{9}$$

Let p_t^w denote the relative price of intermediate goods and $\beta \mathrm{E}_t \Lambda_{t,t+1}$ be the firm's discount rate, where $\Lambda_{t,t+s} = \lambda_{t+s}/\lambda_t$ and λ_t is the marginal utility of consumption at time t. Then the value of firm i, $F_t(i)$, is given by

$$F_t(i) = p_t^w y_t(i) - w_t(i) n_t(i) - \frac{\kappa_t}{2} x_t(i)^2 n_{t-1}(i) - r_t^k k_t(i) + \beta \mathrm{E}_t \left\{ \Lambda_{t,t+1} F_{t+1}(i) \right\}, \tag{10}$$

where $(\kappa_t/2) x_t(i)^2 n_{t-1}(i)$ is a quadratic labor adjustment cost. In order to maintain a balanced steady-state growth path, this adjustment cost is allowed to drift proportionately with productivity, so[2]

$$\kappa_t = \kappa z_t. \tag{11}$$

The firm maximizes its value by choosing the hiring rate $x_t(i)$ and its capital stock $k_t(i)$, given its existing employment stock $n_{t-1}(i)$, the rental rate on capital r_t^k, and the current and expected path of wages $w_t(i)$. The first-order condition for capital is given by

$$r_t^k = p_t^w \alpha z_t^{1-\alpha} \widetilde{k}_t^{\alpha - 1}, \tag{12}$$

where $\widetilde{k}_t$ is the capital/employment ratio, which is the same across firms due to Cobb-Douglas technology and perfect capital mobility.

The optimal hiring decision yields

$$\kappa_t x_t(i) = J_t(i), \tag{13}$$

where

$$J_t(i) = p_t^w a_t - w_t(i) - \beta \mathrm{E}_t \left\{ \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(i)^2 \right\} + \beta \mathrm{E}_t \left\{ \Lambda_{t,t+1} \left[\rho + x_{t+1}(i) \right] J_{t+1}(i) \right\}, \tag{14}$$

where a_t denotes the current marginal product of labor, which is also equal across firms. The hiring condition (13) equates the cost of having another worker at time t, $\kappa_t x_t(i)$, to its value defined after hiring decisions at time t have been made and adjustment costs are sunk, $J_t(i)$.

Combining equations yields a forward looking difference equation for the hiring rate:

$$\kappa_t x_t(i) = p_t^w a_t - w_t(i) + \beta \mathrm{E}_t \left\{ \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(i)^2 \right\} + \rho \beta \mathrm{E}_t \left\{ \Lambda_{t,t+1} \kappa_{t+1} x_{t+1}(i) \right\}. \tag{15}$$

The hiring rate thus depends on the discounted stream of earnings and the saving on adjustment costs. Observe that the only firm-specific variable affecting the hiring rate is the wage.

[2] A constant adjustment cost would become relatively less important as the economy grows.

2.3 Workers

Let $V_t(i)$ be the value to a worker of employment at firm i, and let U_t be the value of unemployment. These values are defined after hiring decisions at time t have been made and are measured in units of consumption goods. The value of employment is given by

$$V_t(i) = w_t(i) + \beta \mathrm{E}_t \left\{ \Lambda_{t,t+1} \left[\rho V_{t+1}(i) + (1-\rho) U_{t+1} \right] \right\}. \tag{16}$$

To construct the value of unemployment, denote by $V_{x,t}$ the average value of employment conditional on being a new worker, given by[3]

$$V_{x,t} = \int_0^1 \left[V_t(i) \frac{x_t(i) n_{t-1}(i)}{x_t n_{t-1}} \right] di. \tag{17}$$

Then U_t can be expressed as

$$U_t = b_t + \beta \mathrm{E}_t \left\{ \Lambda_{t,t+1} \left[s_{t+1} V_{x,t+1} + (1 - s_{t+1}) U_{t+1} \right] \right\}, \tag{18}$$

where, as before, s_t is the probability of finding a job, and

$$b_t = b k_t^p \tag{19}$$

is the flow value of unemployment (measured in units of consumption goods). The flow value is assumed to grow proportionately with the physical capital stock in order to maintain balanced growth.

Finally, the worker surplus at firm i, $H_t(i)$, and the average worker surplus conditional on being a new hire, $H_{x,t}$, are given by

$$H_t(i) = V_t(i) - U_t, \tag{20}$$
$$H_{x,t} = V_{x,t} - U_t. \tag{21}$$

It follows that

$$H_t(i) = w_t(i) - b_t + \beta \mathrm{E}_t \left\{ \Lambda_{t,t+1} \left[\rho H_{t+1}(i) - s_{t+1} H_{x,t+1} \right] \right\}. \tag{22}$$

2.4 Wage bargaining

Firms and workers are not able to negotiate their wage contract in every period, but wage bargaining is assumed to be staggered over time, as in Gertler and Trigari (2006). As in Gertler et al. (2007), firms and workers bargain over nominal wages. In each period, each firm faces a fixed probability $1 - \lambda_w$ of being able to renegotiate the wage. The fraction λ_w of firms that cannot renegotiate the wage instead index the nominal wage to past inflation according to

$$w_t^n(i) = \overline{\gamma}_w \pi_{t-1}^{\gamma_w} w_{t-1}^n(i), \tag{23}$$

[3] One technical aspect is that there is no steady-state distribution of employment shares across firms. One solution to this issue would be to take averages integrating over the distribution of wages across workers, which is well-defined in the steady state. However, as this would not affect the log-linearized equilibrium, we choose to sidestep this issue here in order to keep the exposition simple. See Gertler and Trigari (2006) for details.

where $\pi_t = p_t/p_{t-1}$ is the gross rate of inflation, $\overline{\gamma}_w = \gamma_z \pi^{1-\gamma_w}$, and $\gamma_w \in [0,1]$ measures the degree of indexing.

Let w_t^{n*} denote the nominal wage of a firm-worker pair that renegotiates at t. Given constant returns to scale, all sets of renegotiating firms and workers set the same wage. The firm negotiates with the marginal worker over the surplus from the marginal match. Assuming Nash bargaining, the contract wage w_t^{n*} is chosen to solve

$$\max H_t(i)^{\eta_t} J_t(i)^{1-\eta_t}, \tag{24}$$

subject to

$$w_{t+j}^n(i) = \begin{cases} \overline{\gamma}_w w_{t+j-1}^n(i) \pi_{t+j-1}^{\gamma_w} & \text{with probability } \lambda_w \\ w_{t+j}^{n*} & \text{with probability } 1-\lambda_w. \end{cases} \tag{25}$$

The variable $\eta_t \in [0,1]$ reflects the worker's relative bargaining power, and is assumed to evolve according to

$$\eta_t = \eta \varepsilon_t^\eta, \tag{26}$$

where ε_t^η is a shock with mean unity that implies a disturbance to the wage equation.

The first-order condition for the Nash bargaining solution is given by

$$\chi_t(i) J_t(i) = [1 - \chi_t(i)] H_t(i), \tag{27}$$

where

$$\chi_t(i) = \frac{\eta_t}{\eta_t + (1-\eta_t)\mu_t(i)/\epsilon_t} \tag{28}$$

is the (horizon-adjusted) effective bargaining power of workers,

$$\mu_t(i) = 1 + \beta \lambda_w \mathrm{E}_t \left\{ \Lambda_{t,t+1} [\rho + x_{t+1}(i)] \frac{p_t}{p_{t+1}} \overline{\gamma}_w \pi_t^{\gamma_w} \mu_{t+1}(i) \right\} \tag{29}$$

is the firm's cumulative discount factor, and

$$\epsilon_t = 1 + \beta \rho \lambda_w \mathrm{E}_t \left\{ \Lambda_{t,t+1} \frac{p_t}{p_{t+1}} \overline{\gamma}_w \pi_t^{\gamma_w} \epsilon_{t+1} \right\} \tag{30}$$

is the worker's cumulative discount factor.[4]

As in Gertler and Trigari (2006), the bargaining solution gives a difference equation for the real wage $w_t^* = w_t^{n*}/p_t$ as

$$\epsilon_t w_t^* = w_t^o(i) + \rho \beta \lambda_w \mathrm{E}_t \left\{ \Lambda_{t,t+1} \epsilon_{t+1} w_{t+1}^* \right\}, \tag{31}$$

[4] Firms and workers have different horizons when contracting. The firm cares about the impact of the contract wage on existing workers as well as on new workers expected to join the firm under the terms of the current contract. Workers, on the other hand, care only about the impact of the contract wage on the expected job tenure.

where $w_t^o(i)$ can be interpreted as the real target wage, and is given by

$$\begin{aligned} w_t^o(i) &= \chi\left[p_t^w a_t + \beta \mathrm{E}_t\left\{\Lambda_{t,t+1}\frac{\kappa_{t+1}}{2}x_{t+1}(i)^2\right\}\right] \\ &\quad +(1-\chi)\left[b_t + \beta\mathrm{E}_t\left\{\Lambda_{t,t+1}s_{t+1}H_{x,t+1}\right\}\right] + \Phi_t(i), \end{aligned} \tag{32}$$

and where $\Phi_t(i)$ is due to a "horizon effect." Due to the staggered wage bargaining, the contract wage w_t^* depends not only on the current target wage, but on the expected sequence of future target wages. The target wage, in turn, is a convex combination of what a worker contributes to the match (the marginal product of labor plus the saving on adjustment costs) and what the worker loses by accepting a job (the flow value of unemployment plus the expected discounted gain of moving from unemployment this period to employment next period), where the weights depend on the worker's average effective bargaining power. The additional effect, captured by $\Phi_t(i)$, reflects the impact of shifts in the effective bargaining power.

Finally, the average nominal wage is given by

$$w_t^n = \int_0^1 \left[w_t^n(i)\frac{n_t(i)}{n_t}\right] di. \tag{33}$$

2.5 Retailers

There is a continuum of monopolistically competitive retailers indexed by j on the unit interval. These buy intermediate goods from the wholesale firms, differentiate them with a technology that transforms one unit of intermediate goods into one unit of retail goods, and sell them to households. Retailers set prices on a staggered basis.

Following Smets and Wouters (2007), we assume that each firm's elasticity depends inversely on its relative market share, as in Kimball (1995), who generalizes the standard Dixit-Stiglitz aggregator. Thus, letting $y_t(j)$ be the quantity of output sold by retailer j and $p_t(j)$ the nominal price, final goods, denoted y_t, are a composite of individual retail goods following

$$\int_0^1 \mathcal{G}\left(\frac{y_t(j)}{y_t}, \varepsilon_t^p\right) dj = 1, \tag{34}$$

where the function $\mathcal{G}(\cdot)$ is increasing and strictly concave with $\mathcal{G}(1) = 1$, and ε_t^p is a shock that influences the elasticity of demand.

We assume that prices are staggered as in Calvo (1983), but with indexing as in Christiano et al. (2005) and Smets and Wouters (2003). Thus, each retailer faces a fixed probability $1 - \lambda_p$ of reoptimizing its price in a given period, in which case it sets its price to p_t^* to maximize the expected discounted stream of future profits. All firms that reoptimize set the same price. Firms that do not reoptimize instead index their price to past inflation following

$$p_t(j) = \overline{\gamma}_p \pi_{t-1}^{\gamma_p} p_{t-1}(j), \tag{35}$$

where $\overline{\gamma}_p = \pi^{1-\gamma_p}$ is an adjustment for steady-state inflation.

It is possible to show that the optimal price p_t^* depends on the expected discounted stream of the retailers' nominal marginal cost given by $p_t p_t^w$. Using the hiring condition (15), real marginal cost is given by

$$p_t^w = \frac{1}{a_t}\left[w_t(i) + \kappa_t x_t(i) - \beta \mathrm{E}_t\left\{\Lambda_{t,t+1}\frac{\kappa_{t+1}}{2}x_{t+1}(i)^2\right\} - \rho\beta\mathrm{E}_t\left\{\Lambda_{t,t+1}\kappa_{t+1}x_{t+1}(i)\right\}\right], \quad (36)$$

so real marginal cost depends on unit labor cost adjusted for the labor adjustment cost.

2.6 The government sector

The government sets government spending g_t according to

$$g_t = \left[1 - \frac{1}{\varepsilon_t^g}\right] y_t, \quad (37)$$

where ε_t^g follows an exogenous process.

The central bank sets the short-term nominal interest rate r_t according to the Taylor rule

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho_s}\left[\left(\frac{\mathrm{E}_t\pi_{t+1}}{\pi}\right)^{r_\pi}\left(\frac{y_t}{y_t^n}\right)^{r_y}\right]^{1-\rho_s}\varepsilon_t^r, \quad (38)$$

where y_t^n is the natural (or flexible-price) level of output and ε_t^r is a monetary policy shock. Following much of the literature on estimated DSGE models, we define the natural level of output as the level of output in the equilibrium with flexible prices and wages and without shocks to the price markup and the bargaining power of workers.[5] Associated with the natural level of output, there is also a natural rate of unemployment, denoted u_t^n.

2.7 Resource constraint and model summary

Finally, the resource constraint implies that output is equal to the sum of consumption, investment, government spending, and adjustment and utilization costs:

$$y_t = c_t + i_t + g_t + \frac{\kappa_t}{2}\int_0^1 \left[x_t(i)^2 n_{t-1}(i)\right] di + \mathcal{A}(\nu_t)k_{t-1}^p. \quad (39)$$

The complete model consists of 28 equations for the 28 endogenous variables. There are also seven exogenous disturbances: to technology, investment, preferences, the price markup, workers' bargaining power, government spending, and monetary policy. The technology shock follows a unit-root process, while the remaining six shocks are stationary. In particular, technology growth and the other six shocks follow

$$\log\left(\varepsilon_t^j\right) = (1-\rho_j)\log\left(\varepsilon^j\right) + \rho_j\log\left(\varepsilon_{t-1}^j\right) + \zeta_t^j, \quad (40)$$

for $j = z, i, b, p, \eta, g, r$, where $\varepsilon^i = \varepsilon^b = \varepsilon^\eta = \varepsilon^r = 1$, and where ζ_t^j are mean-zero innovations with constant variances σ_j^2. We log-linearize the model around its deterministic steady state with balanced growth, allowing for the fact that output, investment, consumption, and the real wage are non-stationary. The derivation of the steady state and the log-linearized system of equations are available in Gertler et al. (2007).

[5] See, for instance, Smets and Wouters (2003, 2007), Levin et al. (2005), and Edge, Kiley, and Laforte (2007a). This definition deviates from that of Woodford (2003) and Blanchard and Galí (2007); see Section 4.2 for a detailed discussion.

2.8 The role of labor market frictions

To clarify the role of labor market frictions in our model, we compare with a model that shares exactly the same structure except for the treatment of the labor market. The search frictions and staggered Nash bargaining are replaced by the mechanism in Erceg, Henderson, and Levin (2000), where monopolistically competitive workers set wages on a staggered basis. This model is representative of the latest vintage of DSGE models used for policy analysis and we refer to it as the EHL model.

In their log-linearized versions, inflation dynamics in both models is determined by the behavior of real marginal cost, according to a New Keynesian Phillips curve given by

$$\widehat{\pi}_t = \iota_b \widehat{\pi}_{t-1} + \iota_o \left(\widehat{p}^w_t + \varepsilon^p_t \right) + \iota_f \mathrm{E}_t \widehat{\pi}_{t+1}, \tag{41}$$

where hats denote log deviations from steady state, and $\iota_b, \iota_o, \iota_f$ are determined by the parameters in the firms' price-setting problem. The two models differ in their specification of real marginal cost, $\widehat{p}^w_t$. In the EHL model this is given by the real wage normalized by the marginal product of labor, with labor adjusting at the intensive margin. In our model, in contrast, firms employ workers at long-term employment relationships by paying a labor adjustment cost. Thus, as seen in equation (36) marginal cost has two components: the real wage and a component related to the adjustment cost, both normalized by the marginal product of labor. The adjustment cost component is the cost of hiring an additional worker at t, reflected by the second term in squared brackets, net of saved future adjustment costs, reflected by the third term, and future hiring costs, reflected by the fourth term.

Due to staggered wage determination, the aggregate wage in both models depends on the lagged wage as well as the expected future wage according to a difference equation of the form

$$\widehat{w}_t = \gamma_b \left(\widehat{w}_{t-1} - \widehat{\pi}_t + \gamma \widehat{\pi}_{t-1} - \widehat{\varepsilon}^z_t \right) + \gamma_o \left(\widehat{w}^o_t + \varepsilon^w_t \right) + \gamma_f \left(\widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \gamma \widehat{\pi}_t + \widehat{\varepsilon}^z_{t+1} \right), \tag{42}$$

where $\gamma_b, \gamma_o, \gamma_f$, and the structural interpretation of the shock ε^w_t depend on the wage-setting or wage-bargaining problem. Here, the two models differ in terms of the driving force of aggregate wages, $\widehat{w}^o_t$: in the EHL model it is given by the marginal rate of substitution between consumption and leisure, while in our model it is the period-by-period Nash bargained wage (adjusted for the horizon effect).

In principle, our model implies a Phillips curve relating inflation to unemployment. For simplicity we do not derive it explicitly but only use it to describe numerically the trade-offs faced by a central bank wanting to stabilize inflation and/or unemployment. Labor market frictions and institutions influence the trade-off faced by the central bank by influencing the elasticity of real marginal cost, both the wage and labor adjustment cost component, to movements in unemployment, and more generally in labor market activity.

We consider below the effects of labor market institutions on the trade-off, for instance, the role of unemployment insurance (modelled as the flow value of unemployment) or the implications of a labor market with lower average job turnover. In contrast, labor market frictions and institutions in the EHL model are solely reflected in the market power of workers. While the EHL model is silent on the trade-off between inflation and unemployment studied below, estimated versions of the two models imply a quantitatively similar trade-off between inflation and the output gap. Thus, the main advantage of our model with search frictions is that it allows us to discuss the relationship between inflation and unemployment and the role of labor market institutions.

3 Estimation

As in Gertler et al. (2007), we estimate the log-linearized version of the model on quarterly U.S. data from 1960Q1 to 2005Q1 for seven variables: (1) output growth: the quarterly growth rate of per capita real GDP; (2) consumption growth: the quarterly growth rate of per capita real personal consumption expenditures of nondurables; (3) investment growth: the quarterly growth rate of per capita real investment; (4) employment: hours of all persons in the non-farm business sector divided by population, multiplied by the ratio of total employment to employment in the non-farm business sector; (5) real wage growth: the quarterly growth rate of compensation per hour in the non-farm business sector; (6) inflation: the quarterly growth rate of the GDP deflator; and (7) the nominal interest rate: the quarterly average of the federal funds rate.[6]

The model contains 22 structural parameters, not including the parameters that characterize the seven exogenous shocks. We calibrate three of the five labor market parameters: the average monthly separation rate $1-\rho$ is set to 0.105, the match elasticity with respect to unemployment, σ, to 0.5, and the labor adjustment cost parameter κ such that the average job finding rate is $s = 0.95$. We also calibrate five "conventional" parameters using standard values: the discount factor β is set to 0.99, the capital depreciation rate δ to 0.025, the capital share α in the Cobb-Douglas production function is set to 0.33, and the average ratio of government spending to output g/y to 0.2. Finally, we calibrate the sensitivity of the firm's elasticity of demand with respect to shifts in its market share, the Kimball aggregator parameter, denoted ξ, to 10.

We estimate the two labor market parameters $\widetilde{b}$, which determines the relative flow value of unemployment,[7] and η, the average bargaining power of workers. We also estimate the elasticity of the utilization rate to the rental rate of capital, η_ν;[8] the elasticity of the capital adjustment cost function, η_k; the habit parameter h; the steady-state price markup ε^p; the wage and price rigidity parameters λ_w and λ_p; the wage and price indexing parameters γ_w and γ_p; and the Taylor rule parameters r_π, r_y, and ρ_s. In addition, we estimate the autoregressive parameters of all the exogenous disturbances, as well as their respective standard deviations.

We estimate the model with Bayesian methods (see An and Schorfheide, 2007, for an

[6] We use growth rates for the non-stationary variables (output, consumption, investment, and the real wage, which are non-stationary also in the theoretical model) and we write the measurement equation of the Kalman filter to match the seven observable series with their model counterparts. All data were obtained from the FRED data base of the Federal Reserve Bank of St. Louis.

[7] The relative flow value of unemployment is defined as

$$\widetilde{b} = \frac{b\,(k/z)}{p^w\,(a/z) + \beta\,(\kappa/2)\,x^2}.$$

[8] Following Smets and Wouters (2007), we define ψ_ν such that $\eta_\nu = (1-\psi_\nu)\,/\psi_\nu$ and estimate ψ_ν.

overview). Letting $\boldsymbol{\theta}$ denote the vector of structural parameters to be estimated and $\mathbf{Y}$ the data sample, we combine the likelihood function, $L(\boldsymbol{\theta}, \mathbf{Y})$, with priors for the parameters to be estimated, $p(\boldsymbol{\theta})$, to obtain the posterior distribution: $L(\boldsymbol{\theta}, \mathbf{Y})p(\boldsymbol{\theta})$. Draws from the posterior distribution are generated with the Random-Walk Metropolis-Hastings algorithm.

Tables 1 and 2 report the prior distribution of the parameters along with the median and the 5th and 95th percentiles of the posterior distribution. The choice of calibration and priors and the resulting parameter estimates are discussed in detail in Gertler et al. (2007).

4 Model properties

We now discuss some properties of the estimated model that are particularly important for monetary policy. First, we discuss the fit of the estimated model, both for the variables used in estimation and for unemployment and labor market tightness, which were not used when estimating the model. We then turn to the estimated behavior of the natural rates of unemployment and output, and the estimated unemployment and output gaps (the percent deviation of unemployment and output from their natural rates) over the sample. Finally, we discuss the trade-offs facing the central bank.

4.1 Empirical fit

To illustrate the fit of the estimated model, Figures 1–3 show autocovariance functions for three blocks of the variables used in the estimation: aggregate demand variables (output, consumption, and investment, all in terms of growth rates) in Figure 1; labor market variables (output growth, employment, and real wage growth) in Figure 2; and monetary policy variables (output growth, inflation, and the federal funds rate) in Figure 3.

The solid lines are autocovariances of U.S. data, while the dashed lines are 5th and 95th percentiles from the posterior distribution.[9] We see that the autocovariance functions of U.S. data fall within the 90% interval of the empirical distribution at most leads and lags. Thus, the estimated model captures well the covariance structure of U.S. data.

Next we study the implied behavior of the unemployment rate and the degree of labor market tightness, two variables that were *not* used in the estimation. Figure 4 show U.S. data and model estimates of the unemployment rate and the degree of labor market tightness over the sample.[10] The estimated model matches remarkably well the movements over the sample period.[11] Figure 5 shows the autocovariance function of unemployment with the variables

[9] To construct these intervals, we draw 500 times from the posterior parameter distribution, and simulate 100 samples of 160 observations (as in the data sample) for each draw. Thus, the intervals are determined by both parameter and sampling uncertainty. These autocovariances are discussed in more detail by Gertler et al. (2007).

[10] The unemployment series is civilian unemployment divided by the civilian non-institutional population over 16, obtained from the Bureau of Labor Statistics, adjusted to have a sample mean of 6 percent. The series for vacancies used to calculate labor market tightness is the help-wanted index constructed by the Conference Board. The unemployment rate in the log-linearized model is measured as the percent deviation from steady state. For simplicity we have translated the unemployment rate to percentage points, assuming a steady-state rate of unemployment of 6 percent (similar to the sample mean of the U.S. unemployment rate).

[11] In the model unemployment is inversely proportional to employment (total hours), which was used for estimation. Thus, the good fit of unemployment reflects the fact that unemployment and total hours tend to

used for estimation. Again, the estimated probability intervals include the covariances of U.S. data at almost all leads and lags.

4.2 Natural rates and gaps

We now study the behavior of the estimated natural rate of unemployment, as well as the unemployment and output gaps. These gaps are important measures of the degree of slack in the economy, and therefore important indicators for monetary policy.

We begin by recalling that the natural rates of output and unemployment in our model are defined as the level of output and the rate of unemployment in the equilibrium with flexible prices and wages, and without shocks to the price markup or the bargaining power of workers. As noted earlier, this definition follows much of the literature on estimated DSGE models, for instance Smets and Wouters (2003, 2007); Levin et al. (2005); and Edge et al. (2007a), but it is in contrast to Woodford (2003, Ch. 6) and Blanchard and Galí (2007) who include also time-variation in price and wage markups (or tax wedges) in the natural level of output. Woodford (2003) shows that in this case monetary policy should not aim at stabilizing the gap between the actual and natural levels of output, but the gap between the actual level and the efficient level, defined as the equilibrium where also the steady-state distortions due to monopolistic competition are removed. Thus, monetary policy should accommodate movements in the efficient level of output (due to shifts in technology, tastes, and government spending), but lean against shocks to the price or wage markups. In our estimated model, the natural rates only enter through the monetary policy rule, while below we will also assume that the central bank aims at stabilizing unemployment around the natural rate. To be consistent with optimal monetary policy we therefore prefer to exclude the shocks to the price markup and the bargaining power from our definition of the natural rates. We note, however, that including these shocks would increase the volatility of the natural rates relative to those reported below.[12]

Figure 6 shows the estimated path for the rate of unemployment (discussed earlier) and the natural rate of unemployment (with a 90% probability interval), while Figure 7 reports the unemployment and output gaps (with probability intervals) over the sample period.[13] According to our model, the natural rate of unemployment has moved gradually over the sample period, trending upwards in the 1970s and early 1980s and then falling slowly in the 1990s. This pattern is similar to alternative measures, for instance those constructed by Staiger, Stock, and Watson (1997, 2002) or Orphanides and Williams (2002), and the picture that emerges from Figures 6 and 7 is consistent with official accounts of the U.S. business

comove closely, implying that most fluctuations in employment are on the extensive, rather than the intensive, margin.

[12]Our definition is not inconsistent with the original definitions of Wicksell (1898), Frisch (1936), Phelps (1968), or Friedman (1968), or with the concept of a NAIRU (non-accelerating-inflation rate of unemployment). In future work, we plan to investigate further the properties of alternative definitions of the natural rates in our framework.

[13]The unemployment gap is defined as the percentage point deviation of the unemployment rate from the natural rate, $u_t - u_t^n$, assuming a steady-state rate of unemployment of 6 percent, while the output gap is defined as the percent deviation of the level of output from the natural level, $(y_t - y_t^n)/y_t^n$. The probability intervals were constructed taking into account both parameter uncertainty and Kalman filter uncertainty, see Hamilton (1994, Ch. 13.7).

cycle. According to the NBER Business Cycle Dating Committee, our sample includes six contractions, indicated by shaded areas in Figures 6–7. Each of these contractions coincides with a sharp increase in the actual rate of unemployment above the natural rate and therefore an increase in the estimated unemployment gap and a fall in the estimated output gap.

Thus, our model is able to generate business cycle fluctuations that are similar to alternative accounts, in contrast to the estimated business cycles of Levin et al. (2005) or Edge et al. (2007a).[14] In particular, as stressed by Walsh (2005a), the model estimated by Levin et al. (2005) interprets the decrease in economic activity during the Volcker disinflation in the early 1980s as a large fall in the natural level of output and a positive output gap, rather than a drop in actual output below the natural level. Our estimates instead suggest that the natural rate of unemployment increased only marginally in this period, while the actual unemployment rate increased substantially. Thus, our model interprets this period as a large increase in unemployment above the natural rate and a fall in output below the natural level.

Figure 8 shows the contribution of four sets of shocks to the estimated unemployment gap: the non-stationary technology shock, the monetary policy shock, the two "supply shocks" (the shocks to the price markup and the bargaining power), and the three "demand shocks" (the preference, investment, and government spending shocks). The estimated model suggests that the high rates of unemployment in the 1970s and the low rates in the late 1990s and early 2000s were mainly due to shocks to "supply" and "demand," not to technology or monetary policy. In contrast, the Volcker recession in the early 1980s was to a large extent due to monetary policy, but also to the technology shock. Again, this pattern at least partially coincides with the traditional view of U.S. business cycles.

From the two gaps in Figure 7 we can also identify an Okun's law relationship in terms of a negative correlation between the unemployment gap and the output gap. The unconditional correlation between the two gaps is -0.88, and estimating the Okun's law regression

$$x_t^y = a + b x_t^u + e_t, \tag{43}$$

where x_t^y is the output gap and x_t^u the unemployment gap, would give an unconditional slope coefficient of -1.185. Thus, an unemployment rate one percent above the natural rate tends to coincide with a negative output gap of -1.2%.

4.3 Monetary policy trade-offs

We now illustrate the existence and the nature of the trade-off faced by the central bank in conducting monetary policy. In order to do so, we consider a central bank that aims at stabilizing inflation and the unemployment gap and we allow for varying weights on the two objectives. In particular, we consider two extreme policies: strict inflation targeting, that is, when the central bank only aims at stabilizing inflation, and strict unemployment targeting, that is, the central bank only cares about stabilizing unemployment around its natural rate. We also consider intermediate cases in which the central bank attaches some positive weight on both targets. We report the standard deviations of the annualized rate of inflation and the

[14] Our business cycle estimates are also similar to those of the FRB/US model, as reported by Edge et al. (2007a).

unemployment gap when the central bank implements the optimal policy with commitment as the relative weight on inflation varies from one to zero, that is, the efficient policy frontier.

Our aim with this exercise is threefold: first, to understand whether a policy trade-off exists; second, to identify the shocks that are responsible for this trade-off; and third, to understand how the structure of the labor market affects the nature of the trade-off.

Figure 9 displays the policy frontier in the model first when all shocks are included (in Panel (a)), then considering one shock at a time (in Panels (b) and (c)). Panel (a) shows that there is a marked trade-off between inflation and unemployment volatility. In order to achieve full price stability, the unemployment gap (the percent deviation from unemployment from the natural rate) must be extremely volatile, with a standard deviation around 30 percentage points. In the other extreme, full unemployment stability (a zero unemployment gap) would be possible at the cost of a standard deviation of the inflation rate around 3 percent.

The remaining two Panels in Figure 9 reveal that most of this trade-off originates from the price markup shock, while technology and wage bargaining shocks contribute to a smaller extent, and demand shocks (to preferences, investment, and government spending) barely create a trade-off at all.

It is well-known that the presence of wage rigidities is an important determinant of the monetary policy trade-off, as first demonstrated by Erceg et al. (2000), and more recently stressed by Blanchard and Galí (2007). Figure 10 aims at quantifying the importance of wage rigidities in our framework, by showing the trade-offs when wages are flexible. As shown in Panel (a), wage rigidities have a large effect on the trade-off: with flexible wages, complete inflation stabilization is achieved with a standard deviation of the unemployment gap around 4 percent, compared with 30 with staggered wages. Conversely, strict unemployment targeting implies a standard deviation of inflation of 2.1 rather than 3 percent. Panels (b) and (c) reveal that with flexible wages, price markup and wage bargaining shocks have a much smaller effect than with staggered wages, while technology and demand shocks do not create a trade-off at all when wages are flexible.

Figure 11 examines how the policy trade-offs are affected by the structure of the labor market. We first explore the effect of variations in the average relative flow benefit from being unemployed, $\widetilde{b}$. Panel (a) reports the policy trade-offs for two values of $\widetilde{b}$: the estimated value of 0.722 and a lower value of 0.4, which is typically assumed when $\widetilde{b}$ is interpreted as only unemployment insurance (see Shimer, 2005). As shown in the figure, a reduction in $\widetilde{b}$ improves the policy trade-off. For example, the strict inflation targeting policy implies a standard deviation of unemployment of about 19 percent with the lower value of $\widetilde{b}$, rather than 30 in the benchmark case. The improved policy trade-off is a consequence of the higher elasticity of marginal cost to movements in unemployment, which in turn implies a higher elasticity of inflation to unemployment. In this case, smaller changes in unemployment are necessary to obtain a given change in inflation. There are two reasons why marginal costs are more responsive to changes in unemployment with a lower value for $\widetilde{b}$. As discussed above, equation (36) shows that marginal cost depends on the real wage as well as the labor adjustment costs. First, Hagedorn and Manovskii (2006) have recently emphasized that a high value of the relative unemployment benefit stabilizes the workers' outside option in bargaining and, through this channel, the Nash-bargained wage. Thus, a reduction in $\widetilde{b}$ increases the responsiveness of real target and aggregate wages to labor market fluctuations, in particular to unemployment fluctuations.[15] In addition, a smaller value of $\widetilde{b}$ is associated with a larger value of the parameter κ, which measures the size of the labor adjustment costs.[16] The larger is κ, the larger will be the elasticity of the labor adjustment cost component of marginal cost to movements in hiring rates.

We then explore the effect of a proportional reduction in both the average job finding and job separation rates, that is a reduction in the average turnover rate. In particular, we reduce the average job finding rate s from 0.95 to 0.5 and the job destruction rate $1-\rho$ from 0.105 to 0.055. Panel (b) of Figure 11 shows that an economy characterized by a lower turnover faces an improved policy trade-off. Under the full inflation stabilization policy the standard deviation of unemployment decreases from 30 percent to a value little above 19. First, as before, the experiment increases the size of the labor adjustment costs κ implied by our calibration, leading to a larger elasticity of marginal costs to the hiring rate. Second, the decrease in the average job finding probability reduces the spillover effect that average wages have on the contract wage by influencing the workers' outside option in bargaining through their effect on the average value of employment next period conditional on finding a job next period (see Gertler and Trigari, 2006). This makes target and aggregate wages more responsive to labor market fluctuations. Both effects work in the direction of increasing the elasticity of marginal cost to unemployment and the elasticity of inflation to unemployment.

5 The design of monetary policy

We now turn to the design of monetary policy in the estimated model. For this purpose, we will consider a central bank with a mandate similar to that of the U.S. Federal Reserve as specified in the Federal Reserve Act, that is, "maximum employment, stable prices, and moderate long-term interest rates." We formalize this mandate with the intertemporal loss function[17]

$$\mathcal{L}_t = (1-\widehat{\beta})\mathrm{E}_t \sum_{j=0}^{\infty} \widehat{\beta}^j \left[\overline{\pi}_{t+j}^2 + \lambda_u \left(x_{t+j}^u\right)^2 + \lambda_r \overline{r}_{t+j}^2\right], \tag{44}$$

where $\overline{\pi}_t$ is the annualized rate of inflation (four times the quarterly rate); $x_t^u = u_t - u_t^n$ is the unemployment gap, that is, the percentage point deviation of the rate of unemployment from

[15]While a reduction in λ_w or a reduction in $\widetilde{b}$ have similar implications for the elasticity of real wages and the inflation/unemployment trade-off, a positive λ_w is necessary for the existence of a trade-off after efficient shocks (see Figure 10), whereas there is a trade-off also when $\widetilde{b} = 0$.

[16]We set the parameter κ to target an average job finding rate of 0.95. For a given job finding rate, the reduction in $\widetilde{b}$ increases the average surplus from a match and thus the average firm's value of hiring an additional worker. Thus, on average, the marginal cost of hiring an additional worker must also increase, leading to a higher implied value for κ.

[17]Much of the recent literature on optimal monetary policy studies the welfare-maximizing policy where the objective is to maximize an approximation to the household's utility function. This has been done either using numerical methods in larger-scale models (for instance, Schmitt-Grohé and Uribe, 2004; or Levin et al., 2005), or using smaller models where it is possible to analytically derive an approximation of household utility (recent examples include Blanchard and Galí, 2006; Thomas, 2007; and Edge et al., 2007b). Unfortunately, our model does not aggregate well in its non-linear form, so we are unable to derive a utility-based welfare criterion or apply the numerical methods to analyze welfare-maximizing policy.

the natural rate; $\overline{r}_t$ is the annualized federal funds rate; and $\widehat{\beta}$ is the central bank discount factor. Thus the central bank strives at minimizing the volatility of inflation around a target level normalized to zero ("stable prices"), unemployment around the natural rate ("maximum employment"), and the federal funds rate ("moderate long-term interest rates").[18]

We calibrate the weights (λ_u, λ_r) in the loss function so that the volatility of the unemployment gap and the federal funds rate relative to that of inflation under the unconstrained optimal policy match those in the estimated model. This gives $\lambda_u = 0.833$ and $\lambda_r = 0.08$, implying that a one percentage point deviation of the unemployment rate from the natural rate is equivalent in terms of loss to a deviation of inflation from target of $\sqrt{0.833} = 0.91$ percent.

We will focus on optimized rules for monetary policy of the form advocated by Taylor (1993). However, as a benchmark we will use the unconstrained optimal policy with commitment, calculated using the algorithms developed by Dennis (2007), and setting the central bank discount factor to $\widehat{\beta} = 0.99$. This policy implies standard deviations of inflation, the unemployment gap, and the federal funds rate of 2.19, 1.55, and 2.61, respectively. The estimated rule instead implies standard deviations of 2.37, 1.67, and 2.85, while the standard deviations of inflation and the federal funds rate in U.S. data over our sample period are 2.44 and 3.10, respectively.

5.1 Optimized monetary policy rules

We study the performance of four different rules for monetary policy. The first rule is a standard Taylor rule, where the central bank sets the interest rate as a function of the rate of inflation, the output gap, and the lagged interest rate. In terms of the log-linearized model, this rule is specified as

$$\widehat{r}_t = r_\pi \widehat{\pi}_t + r_y x_t^y + \rho_s \widehat{r}_{t-1}, \tag{45}$$

where hats denote log deviations from steady state, and $x_t^y = (y_t - y_t^n)/y_t^n$ is the percent deviation of output from its natural level. This rule is similar to our estimated rule above, the only difference being that the estimated rule responds to the one-period ahead expectation of inflation, $\mathrm{E}_t \widehat{\pi}_{t+1}$.

The second rule includes the unemployment gap instead of the output gap, so

$$\widehat{r}_t = r_\pi \widehat{\pi}_t + r_y x_t^u + \rho_s \widehat{r}_{t-1}, \tag{46}$$

[18] We approximate the objective of moderate long-term interest rates with federal funds rate volatility, as such volatility may lead to increased term premia and therefore higher long-term interest rates (see Tinsley, 1999). Woodford (2003) instead shows that the welfare-maximizing policy aims at reducing interest rate volatility when there are money transaction frictions or when the central bank wants to avoid the zero-lower bound of nominal interest rates. Note that as $\widehat{\beta}$ approaches 1, the loss function (44) approaches the unconditional expectation of $\mathcal{L}$, that is,

$$\lim_{\widehat{\beta} \to 1} \mathcal{L}_t = \mathrm{E}\mathcal{L} = \mathrm{Var}(\overline{\pi}_t) + \lambda_u \mathrm{Var}(x_t^u) + \lambda_r \mathrm{Var}(\overline{r}_t),$$

where $\mathrm{Var}(\cdot)$ denotes the unconditional variance. We use this specification to optimize and evaluate the monetary policy rules below.

which is similar to that used by Orphanides and Williams (2002) in an estimated two-equation model of inflation and unemployment.

These two rules that respond to the output or unemployment gap rely heavily on the central bank's estimate of the natural rates of output and unemployment. Therefore they may be difficult to implement in practice, and they may be inefficient if the central bank does not have perfect information about the natural rates. We therefore also study two rules that respond to the growth rate of output and unemployment:

$$\widehat{r}_t = r_\pi \widehat{\pi}_t + r_y \Delta \log y_t + \rho_s \widehat{r}_{t-1}, \tag{47}$$

$$\widehat{r}_t = r_\pi \widehat{\pi}_t + r_y \Delta \widehat{u}_t + \rho_s \widehat{r}_{t-1}, \tag{48}$$

which do not rely on the natural rates. Such rules (with $\rho_s = 1$) are shown by Orphanides and Williams (2002) to be robust against natural rate misperceptions.[19]

We first study the performance of optimized versions of our four rules with the benchmark parameterization of the model. We will then introduce uncertainty about parameter values and the natural rates, and study the effect of such uncertainty on the performance of the optimized benchmark rules, as well as optimized rules taking uncertainty into account.

Table 3 shows the optimized coefficients in the four rules. The optimized rules are all "superinertial," that is, with a coefficient on the lagged interest rate larger than one. As first discussed by Rotemberg and Woodford (1999), this is due to the forward-looking nature of the model: the optimal policy is to offset movements in inflation so that the price level returns towards its initial level. With a monetary policy rule, this can be achieved by threatening to increase (or decrease) the interest rate exponentially if shocks to inflation are not offset in the future. Forward-looking agents foresee this threat and adjust appropriately so that the central bank never needs to carry through its threat.

Table 4 shows how these rules perform in terms of standard deviations of inflation, the output and unemployment gaps, and the interest rate, comparing with the fully optimal policy. Relative to the optimal policy, the rules that respond to the output or unemployment gaps tend to overstabilize unemployment (and output) and understabilize inflation. Thus, these rules suffer from a "stabilization bias" similar to that of discretionary policy.[20] To quantify the inefficiency of each optimized rule, we calculate an "unemployment equivalent," which measures the permanent percentage point deviation of the unemployment rate from a natural rate of 6 percent that is equivalent in terms of loss to moving from the fully optimal policy to the optimized rule. In other words, the unemployment equivalents in Table 4

[19]We also experimented with rules that include both the output and unemployment gaps, the level of output and unemployment (that is, the deviation from steady state), the degree of labor market tightness, and the rate of wage inflation. Level rules perform very similarly to rules that respond to the deviation from the natural rates. Rules responding to labor market tightness perform very similarly to those that respond to output or unemployment. Rules that include both output and unemployment and/or the rate of wage inflation give very small improvements compared with rules with only one real variable. This last result is in contrast to Levin et al. (2005) who show that rules that respond to wage inflation are very efficient and a rule with only wage inflation performs almost as well as the welfare-maximizing rule. Our results suggest that wage inflation is important in their framework because it obtains a large weight in the welfare criterion, not because responding to wage inflation is beneficial for macroeconomic stability in general.

[20]See Dennis and Söderström (2006) for a discussion and quantification of the stabilization bias of discretionary monetary policy.

represent the permanent increase of unemployment that the central bank would be willing to accept in order to implement the fully optimal policy rather than the optimized rule.[21]

As shown in the rightmost column of Table 4, the unemployment equivalent is 0.11 and 0.15 percent for the two gap rules. Interestingly, the rule responding to the output gap is slightly more efficient than the unemployment gap rule. Even more surprisingly, the output gap rule is more efficient in stabilizing the unemployment gap than is the rule that responds directly to the unemployment gap. The differences between the two rules are small, however, and depend to some extent on the parameterization of the loss function: using a larger weight on the unemployment gap eventually reverses the ranking of the two rules, and also implies that unemployment is more stable with the unemployment gap rule.

Table 4 also reveals that the two difference rules perform substantially worse than the gap rules, and the rule that responds to output growth is particularly inefficient, with high volatility in the unemployment (and output) gap and the interest rate. As a consequence, the unemployment equivalents for these two rules are 1.02 and 0.56, respectively. Thus, in the case where the central bank has perfect information about the natural rates of output and unemployment, responding to the output and unemployment gaps is vastly superior to rules that respond to output or unemployment growth.

5.2 Introducing parameter and natural rate uncertainty

We now introduce uncertainty about the parameters and the natural rates of output and unemployment. We first evaluate the performance of the optimized rules in Table 3 under uncertainty by calculating the expected loss over 5,000 draws from the posterior distribution, keeping the rule coefficients fixed at their optimized values. To evaluate the costs of uncertainty, we report unemployment equivalents measuring the cost of parameter uncertainty, that is, the outcome relative to the case with the corresponding rule but without uncertainty reported in Table 4.

Table 5 reports the average standard deviations and loss as well as the 5th and 95th percentiles of their distribution for the four rules optimized with the benchmark parameters. In this first case, although parameter uncertainty also implies uncertainty about the natural rates, we assume that the central bank correctly perceives the natural rates of output and unemployment, and thus is able to respond to the correct output and unemployment gaps.

We first see that parameter uncertainty has fairly small effects on the performance of the benchmark policy rules. With the rules responding to the output or unemployment gaps the effects are on average equivalent to a permanent unemployment rate 0.07 percentage points above the natural rate. However, the range of outcomes is fairly wide, from -0.3 to 0.5. Uncertainty thus introduces risk for the policymaker: the outcome could be better than in the benchmark model, but also substantially worse. The difference rules tend to perform even worse on average than in the benchmark case without uncertainty: the average unemployment

[21]To calculate the unemployment equivalent, note that the loss function (44) implies that a permanent unemployment gap of x percent gives a loss of $(1-\widehat{\beta})\sum_{j=0}^{\infty}\widehat{\beta}^j\lambda_u x^2 = \lambda_u x^2$. Denoting by $\mathcal{L}_R$ the loss under the optimized rule and by $\mathcal{L}^*$ the loss under the optimal policy, the permanent unemployment gap that would be equivalent to moving from the fully optimal policy to the optimized rule is given by $x = \sqrt{\mathcal{L}_R/\lambda_u} - \sqrt{\mathcal{L}^*/\lambda_u}$. (See also Jensen, 2002).

equivalent is 0.15 percent, with ranges from −0.4 to 0.9. Thus, not only are difference rules inefficient in the benchmark model, they are also more sensitive to uncertainty, both on average and in terms of risk.

However, the difference rules are by definition insensitive to uncertainty about the natural rates, while such uncertainty may make gap rules inefficient. For instance, Orphanides and Williams (2002) argue that uncertainty about the natural rates of interest and unemployment is so large as to undermine the efficiency of monetary policy rules responding to the natural rates. Instead, they argue that a difference rule, where the central bank sets the change in the interest rate as a function of the rate of inflation and the change in the rate of unemployment, is robust to natural rate uncertainty, as it does not rely on estimates of the natural rates. Furthermore, in their framework such a rule performs well relative to gap rules also when there is no natural rate uncertainty.

To evaluate the importance of natural rate uncertainty in our model, we proceed in two steps. First, we follow Edge et al. (2007b) and assume that the central bank responds to deviations of output or unemployment from misperceived natural rates, which are constructed using the benchmark (median) parameter values. Thus, although the true parameters are drawn randomly from the posterior distribution, the gap rules for monetary policy instead respond to the gaps in terms of the benchmark natural rates. Edge et al. (2007b) show that such natural rate misperceptions have large effects on the performance of output gap rules, which tend to be dominated by rules that do not include the natural rates.[22]

Table 6 reports how the gap rules optimized for the benchmark model perform when we allow for such natural rate misperceptions. Comparing with Table 5, there are almost no effects of natural rate misperceptions in our model: the volatility of all variables are unchanged, and the average unemployment equivalents are almost identical to the case where the central bank can observe the correct natural rates.

As a second source of natural rate uncertainty, we follow Rudebusch (2001), Orphanides and Williams (2002), Svensson and Woodford (2003), and others in introducing persistent errors in the measurement of the output and unemployment gaps. Thus we assume that the central bank observes noisy indicators of the two gaps, given by

$$\widetilde{x}_t^y = x_t^y + \varepsilon_t^y, \tag{49}$$

$$\widetilde{x}_t^u = x_t^u + \varepsilon_t^u, \tag{50}$$

where ε_t^y and ε_t^u are measurement errors that follow

$$\varepsilon_t^y = \rho_y \varepsilon_{t-1}^y + \zeta_t^y, \tag{51}$$

$$\varepsilon_t^u = \rho_u \varepsilon_{t-1}^u + \zeta_t^u, \tag{52}$$

where ζ_t^y and ζ_t^u are i.i.d. innovations with mean zero and standard deviation σ_y, σ_u. To calibrate the processes of the measurement errors, we set $\rho_y = \rho_u = 0.9$, in line with the estimates of Rudebusch (2001) and Orphanides and Williams (2002), and we set σ_y, σ_u such

[22] Edge et al. (2007b) use a fairly small estimated model to study monetary policy rules that are constructed to maximize household welfare, thus also capturing the effects of parameter uncertainty on the welfare criterion.

that the model with the benchmark rules implies a signal-to-noise ratio of 0.75. Thus, 75% of the variance of the noisy gap indicators is due to the variance of the true gaps. This implies $\sigma_y = 0.52$ and $\sigma_u = 0.46$.

Table 7 reports the performance of the optimized benchmark rules when there are persistent errors in the observed gaps, first with constant parameters and then with parameter uncertainty, again represented by 5,000 draws from the posterior distribution. The unemployment equivalents are now calculated relative to the case with perfectly observable output and unemployment gaps, either without or with parameter uncertainty. The effect of such uncertainty is considerably larger than that of the natural rate misperceptions in Table 6. Without parameter uncertainty, the deterioration in the performance of the output and unemployment gap rules are equivalent to a permanent unemployment gap of 0.09 or 0.22 percentage points, respectively. With parameter uncertainty, the average unemployment equivalents are 0.17 and 0.29 percentage points, respectively, with a wide range of outcomes. The rule responding to the unemployment gap is thus more sensitive than the output gap rule to persistent measurement errors in the unemployment gap.

Nevertheless, the gap rules with imperfectly observable output and unemployment gaps still dominate the rules responding to the output and unemployment growth rates in Tables 3 and 4: the difference is equivalent to a permanent unemployment gap of 0.8–0.9 percentage points for the output rule and 0.2–0.3 percentage points for the unemployment rule. For the output growth rule to dominate the output gap rule we need substantially larger measurement errors ($\sigma_y > 1.73$), implying a signal-to-noise ratio of 0.52. For the unemployment rules, the required increase in noise is slightly smaller ($\sigma_u > 0.63$), implying a signal-to-noise ratio of 0.67.

Our finding that parameter uncertainty has little effect on the appropriate monetary policy is similar to the results of Levin et al. (2005) and Edge et al. (2007b), using estimated DSGE models. However, in contrast to Edge et al. (2007b), our model also implies that natural rate misperceptions of the first type considered has small effects. We suspect that this is a consequence of the high precision with which we estimate the natural rates. Figure 7 shows that the probability intervals around the estimated unemployment and output gaps are fairly narrow, and these intervals depend mainly on filtering uncertainty. Thus parameter uncertainty has a very small effect on the precision of the gap estimates.

The results concerning the persistent measurement errors in the output and unemployment gaps are more in line with the existing literature; see, for instance, Rudebusch (2001) and Orphanides and Williams (2002). But the amount of uncertainty considered here is not sufficient to invalidate the use of output or unemployment gaps as indicators for monetary policy in favor of rules responding to output growth or the change in unemployment.

5.3 Optimized monetary policy under uncertainty

Finally, having evaluated how the monetary policy rules estimated for the benchmark model perform in the presence of uncertainty about the parameters and the natural rates, we now re-optimize the policy rules to take into account such uncertainty. The results are reported in Table 8.

Comparing with the rules optimized for the benchmark model in Table 3, we first see that parameter uncertainty has a small effect on the optimized parameters. The gap rules respond slightly more to inflation and less to the output and unemployment gaps, while the difference rules respond slightly less to inflation and more to output and unemployment. But overall the effects are very small, which is not surprising as we have already shown that parameter uncertainty has a negligible effect on the performance of the benchmark rules. The unemployment equivalents relative to the model with the benchmark rule and parameter uncertainty are all below one hundredth of a percent.

We note that the qualitative effects of parameter uncertainty are only partly consistent with the traditional Brainard (1967) argument that parameter uncertainty should make optimal policy less responsive: in our case parameter uncertainty makes policy respond more aggressively to some variables and less aggressively to others. As shown by Söderström (2002), Walsh (2003) and others, Brainard's result does not generalize to uncertainty concerning all parameters in the model. Instead, in general the effects of parameter uncertainty on optimal policy are ambiguous.

Introducing natural rate misperceptions (assuming that the central bank uses the median parameter to construct the natural rates) also has little effect on the optimized rules. Indeed, the rules are almost identical to those with parameter uncertainty but with the correct natural rates.

Finally, introducing persistent errors in the measurement of the output and unemployment gaps does have a quantitative effect on the optimized rules. In particular, the coefficients on inflation are considerably larger compared with the benchmark model, especially for the rule responding to the unemployment gap, while the coefficient on the output gap is slightly smaller (the coefficient on the unemployment gap is not affected). Nevertheless, the unemployment equivalent relative to the benchmark rule is small also for the unemployment gap rule, at 0.05 percent, so the gains from explicitly taking natural rate uncertainty into account seem limited.

6 Concluding remarks

We have used an estimated model with sticky prices, search and matching frictions on the labor market, and staggered nominal wage bargaining to discuss the properties of the natural rate of unemployment, the unemployment and output gaps, the inflation/unemployment trade-off facing monetary policymakers, and the implications for the design of monetary policy. The estimated path for the natural rate of unemployment is very similar to other estimates, for instance, by Staiger et al. (1997, 2002), and the implied unemployment and output gaps fit remarkably well with the standard view of U.S. business cycle. This feature of the model is in stark contrast with other estimated DSGE models, e.g., Levin et al. (2005) and Edge et al. (2007a).

Our model implies that the trade-off in terms of inflation and unemployment volatility facing the central bank is mainly driven by price markup shocks, and to some extent by technology and wage bargaining shocks. This trade-off is worsened significantly by the presence of wage rigidities.

Using a central bank loss function that is consistent with the mandate of the Federal Reserve, we showed that optimized monetary policy rules are superinertial, that is, the interest rate should respond to the lagged interest rate with a coefficient larger than one. Parameter uncertainty was shown to have small effects on the performance of monetary policy rules and the optimized rules themselves, in parallel with the findings of Levin et al. (2005). Finally, our analysis of uncertainty about the natural rates of output and unemployment generated mixed results. Assuming that the central bank uses median parameter values to construct the natural rates while the true parameters vary within the estimated posterior distribution does not have any effects on the performance of the benchmark policy rules, nor on the optimized rule coefficients. In contrast, introducing persistent errors in the measurement of the output and unemployment gaps lead to a significant deterioration of the performance of the benchmark rules, especially for a rule that responds to the unemployment gap. Nevertheless, the rules that respond to the output or unemployment gaps continue to dominate rules responding to the growth rates of output and unemployment also under uncertainty about the natural rates.

In future work, we intend to extend this analysis in two directions. First, while the current paper only studies local model uncertainty, around a given benchmark model, we plan to study more global model uncertainty, taking into account several competing models of the labor market. Second, our estimates of the natural rates of output and unemployment and the implied output and unemployment gaps depends on the exact definition of the natural rates. We therefore plan to further investigate the properties of alternative definitions of the natural rates in our framework.

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Table 1: Prior and posterior distribution of structural parameters

		Prior distribution	Posterior distribution Median	5%	95%
Utilization rate elasticity	ψ_ν	Beta (0.5,0.1)	0.686	0.603	0.761
Capital adjustment cost elasticity	η_k	Normal (4,1.5)	2.423	1.639	3.457
Habit parameter	h	Beta (0.5,0.1)	0.724	0.672	0.773
Bargaining power parameter	η	Beta (0.5,0.1)	0.913	0.868	0.946
Relative flow value of unemployment	$\widetilde{b}$	Beta (0.5,0.1)	0.722	0.656	0.790
Calvo wage parameter	λ_w	Beta (0.75,0.1)	0.718	0.656	0.782
Calvo price parameter	λ_p	Beta (0.66,0.1)	0.851	0.804	0.887
Wage indexing parameter	γ_w	Uniform (0,1)	0.806	0.689	0.915
Price indexing parameter	γ_p	Uniform (0,1)	0.013	0.001	0.055
Steady-state price markup	ε^p	Normal (1.15,0.05)	1.407	1.360	1.455
Taylor rule response to inflation	r_π	Normal (1.7,0.3)	2.036	1.916	2.157
Taylor rule response to output gap	r_y	Gamma (0.125,0.1)	0.342	0.272	0.421
Taylor rule inertia	ρ_s	Beta (0.75,0.1)	0.773	0.728	0.810
Steady-state growth rate	γ_z	Uniform (1,1.5)	1.004	1.003	1.005

This table reports the prior and posterior distribution of the estimated structural parameters. For the uniform distribution, the two numbers in parentheses are the lower and upper bounds. Otherwise, the two numbers are the mean and the standard deviation of the distribution.

Table 2: Prior and posterior distribution of shock parameters

		Prior distribution	Posterior distribution Median	5%	95%
(a) *Autoregressive parameters*					
Productivity growth rate	ρ_z	Beta (0.5,2)	0.131	0.071	0.198
Preferences	ρ_b	Beta (0.5,2)	0.703	0.639	0.764
Investment-specific technology	ρ_i	Beta (0.5,2)	0.597	0.517	0.674
Price markup	ρ_p	Beta (0.5,2)	0.802	0.744	0.857
Bargaining power	ρ_η	Beta (0.5,2)	0.278	0.200	0.349
Government spending	ρ_g	Beta (0.5,2)	0.990	0.984	0.995
Monetary policy	ρ_r	Beta (0.5,2)	0.212	0.133	0.299
(b) *Standard deviations*					
Productivity growth rate	σ_z	IGamma (0.15,0.15)	1.028	0.966	1.090
Preferences	σ_b	IGamma (0.15,0.15)	0.362	0.278	0.502
Investment-specific technology	σ_i	IGamma (0.15,0.15)	0.168	0.121	0.229
Price markup	σ_p	IGamma (0.15,0.15)	0.063	0.048	0.080
Bargaining power	σ_η	IGamma (0.15,0.15)	0.579	0.516	0.651
Government spending	σ_g	IGamma (0.15,0.15)	0.361	0.331	0.396
Monetary policy	σ_r	IGamma (0.15,0.15)	0.228	0.208	0.251

This table reports the prior and posterior distribution of the estimated parameters of the exogenous shock processes. The two numbers in parentheses are the mean and the standard deviation of the distribution.

Table 3: Optimized monetary policy rules in benchmark model

Rule	Coefficient			
	r_π	r_y	r_u	ρ_s
Gap rules				
Output	0.206	0.117		1.054
Unemployment	0.201		−0.136	1.084
Difference rules				
Output	0.721	1.230		1.132
Unemployment	0.136		−1.414	1.107

This table shows the optimized coefficients in the monetary policy rules (45)–(48) in the estimated model with median parameter values. The objective function is given by equation (44), with $\lambda_u = 0.833$, $\lambda_r = 0.08$.

Table 4: Performance of optimized monetary policy rules in benchmark model

Rule	Standard deviation				Loss	Unemployment equivalent
	$\overline{\pi}_t$	x_t^y	x_t^u	$\overline{r}_t$		
Optimal policy	2.19	2.20	1.55	2.61	7.33	
Gap rules						
Output	2.41	1.71	1.34	2.68	7.87	0.11
Unemployment	2.40	2.01	1.47	2.54	8.09	0.15
Difference rules						
Output	2.32	4.60	2.86	3.69	13.26	1.02
Unemployment	2.28	3.59	2.21	3.69	10.37	0.56

This table shows the standard deviations for key variables and the value of the loss function (44) in the estimated model with median parameter values under the unconstrained optimal monetary policy and the optimized monetary policy rules in Table 3. Bars denote annualized values (four times quarterly values); loss is calculated using equation (44) with $\lambda_u = 0.833$, $\lambda_r = 0.08$; the unemployment equivalent is the permanent percentage point deviation of the unemployment rate from a natural rate of 6 percent that is equivalent in terms of loss to moving from the optimal policy to the optimized rule.

Table 5: Evaluating optimized benchmark monetary policy rules with parameter uncertainty

Rule		Standard deviation				Loss	Unemployment equivalent
		$\overline{\pi}_t$	x_t^y	x_t^u	$\overline{r}_t$		
Gap rules							
Output	Mean	2.46	1.74	1.38	2.73	8.25	0.074
	5%	2.13	1.58	1.21	2.39	6.39	−0.30
	95%	2.86	1.92	1.58	3.11	10.84	0.53
Unemployment	Mean	2.46	2.04	1.49	2.60	8.44	0.068
	5%	2.14	1.83	1.34	2.26	6.57	−0.31
	95%	2.84	2.28	1.67	2.98	11.08	0.53
Difference rules							
Output	Mean	2.35	4.92	3.03	3.71	14.29	0.15
	5%	2.12	3.70	2.30	3.27	10.64	−0.42
	95%	2.59	6.44	3.96	4.17	19.90	0.90
Unemployment	Mean	2.33	3.91	2.38	3.72	11.27	0.15
	5%	2.05	2.79	1.71	3.35	8.15	−0.40
	95%	2.64	5.36	3.24	4.11	16.08	0.86

This table shows the standard deviations for key variables and the value of the loss function (44) as averages and 5th and 95th percentiles over 5,000 draws from the posterior parameter distribution under the unconstrained optimal monetary policy and the optimized monetary policy rules in Table 3. Bars denote annualized values (four times quarterly values); loss is calculated using equation (44) with $\lambda_u = 0.833$, $\lambda_r = 0.08$; the unemployment equivalent is the permanent percentage point deviation of the unemployment rate from a natural rate of 6 percent that is equivalent in terms of loss to moving from the benchmark model without parameter uncertainty to the model with uncertainty.

Table 6: Evaluating optimized benchmark monetary policy rules with parameter uncertainty and misperceived natural rates

Rule		Standard deviation				Loss	Unemployment equivalent
		$\overline{\pi}_t$	x_t^y	x_t^u	$\overline{r}_t$		
Gap rules							
Output	Mean	2.47	1.74	1.37	2.73	8.26	0.076
	5%	2.13	1.58	1.22	2.38	6.40	−0.30
	95%	2.87	1.91	1.58	3.12	10.89	0.54
Unemployment	Mean	2.46	2.04	1.49	2.60	8.45	0.069
	5%	2.14	1.83	1.35	2.26	6.56	−0.31
	95%	2.85	2.27	1.66	2.98	11.10	0.53

This table shows the standard deviations for key variables and the value of the loss function (44) as averages and 5th and 95th percentiles over 5,000 draws from the posterior parameter distribution under the unconstrained optimal monetary policy and the optimized monetary policy rules in Table 3. In each draw, the natural rates of output and unemployment are constructed with the median parameter values. Bars denote annualized values (four times quarterly values); loss is calculated using equation (44) with $\lambda_u = 0.833$, $\lambda_r = 0.08$; the unemployment equivalent is the permanent percentage point deviation of the unemployment rate from a natural rate of 6 percent that is equivalent in terms of loss to moving from the benchmark model without parameter uncertainty to the model with uncertainty.

Table 7: Evaluating optimized benchmark monetary policy rules with persistent measurement errors

Rule		Standard deviation				Loss	Unemployment equivalent
		$\overline{\pi}_t$	x_t^y	x_t^u	$\overline{r}_t$		
Gap rules, no parameter uncertainty							
Output		2.42	2.06	1.52	2.71	8.35	0.094
Unemployment		2.44	2.68	1.82	2.65	9.29	0.22
Gap rules, parameter uncertainty							
Output	Mean	2.48	2.08	1.55	2.76	8.74	0.17
	5%	2.14	1.94	1.39	2.42	6.87	−0.20
	95%	2.87	2.23	1.74	3.14	11.33	0.62
Unemployment	Mean	2.50	2.71	1.84	2.70	9.67	0.29
	5%	2.19	2.55	1.71	2.37	7.77	−0.062
	95%	2.88	2.89	2.00	3.09	12.33	0.73

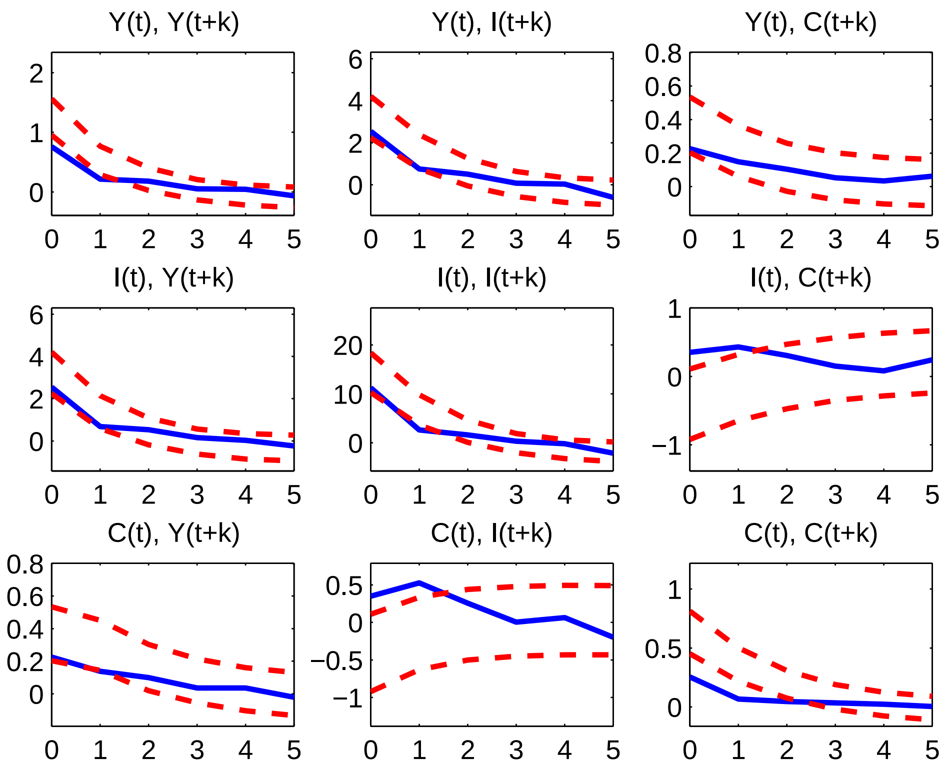
This table shows the standard deviations for key variables and the value of the loss function (44) in the model with median parameter values and averages and 5th and 95th percentiles over 5,000 draws from the posterior parameter distribution in the model with persistent measurement errors under the optimized monetary policy rules in Table 3. The central bank indicators for the output and unemployment gaps are given by $\widetilde{x}_t^j = x_t^j + \varepsilon_t^j$ with $\varepsilon_t^j = \rho_j \varepsilon_{t-1}^j + \zeta_t^j$ for $j = y, u$, where ζ_t^j is i.i.d. with mean zero and standard deviation σ_j. Bars denote annualized values (four times quarterly values); loss is calculated using equation (44) with $\lambda_u = 0.833$, $\lambda_r = 0.08$; the unemployment equivalent is the permanent percentage point deviation of the unemployment rate from a natural rate of 6 percent that is equivalent in terms of loss to moving from the benchmark model with observable gaps to the model with noisy gap indicators.

Table 8: Optimized monetary policy rules with parameter uncertainty, misperceived natural rates, and persistent measurement errors

Rule	Coefficient				Loss	Unemployment equivalent
	r_π	r_y	r_u	ρ_s		
Gap rules, parameter uncertainty						
Output	0.213	0.114		1.056	8.23	0.004
Unemployment	0.203		−0.132	1.085	8.43	0.003
Difference rules, parameter uncertainty						
Output	0.695	1.277		1.127	14.28	0.001
Unemployment	0.124		−1.516	1.100	11.27	0.000
Gap rules, parameter uncertainty and misperceived natural rates						
Output	0.213	0.113		1.058	8.24	0.004
Unemployment	0.203		−0.132	1.085	8.43	0.002
Gap rules, persistent measurement errors						
Output	0.261	0.110		0.988	8.33	0.005
Unemployment	0.417		−0.136	0.889	9.01	0.052
Gap rules, parameter uncertainty and persistent measurement errors						
Output	0.265	0.107		0.992	8.69	0.009
Unemployment	0.410		−0.133	0.898	9.36	0.055

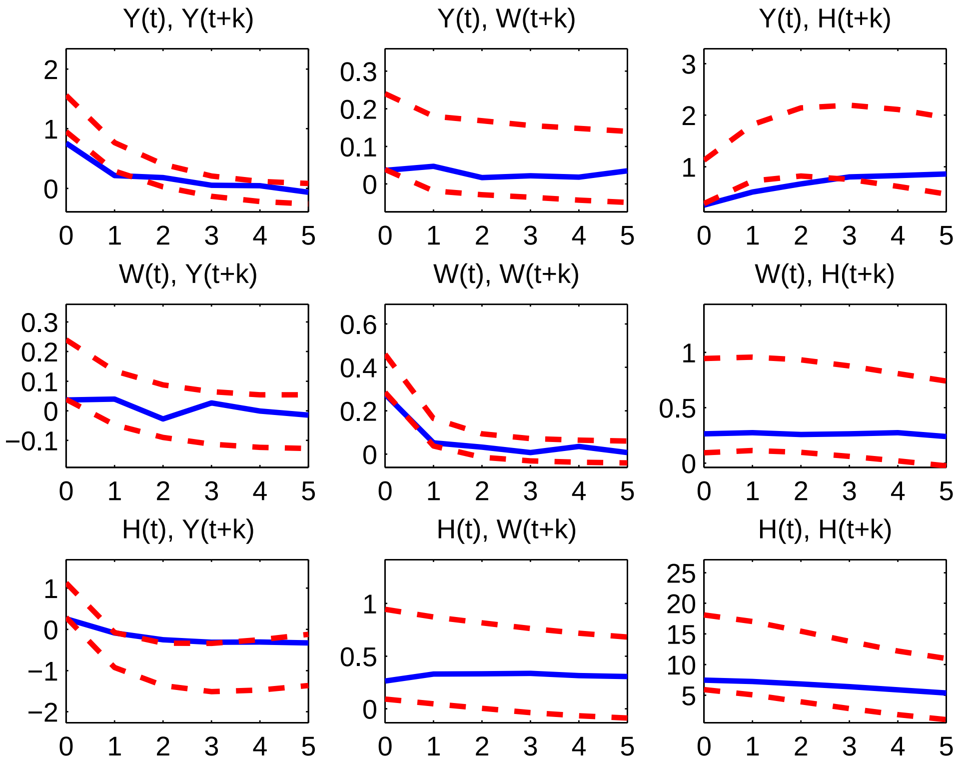
This table shows the optimized coefficients in the monetary policy rules (45)–(48) and the average loss and unemployment equivalent in the model with parameter uncertainty, misperceived natural rates, and/or persistent measurement errors, using 1,000 draws from the estimated posterior parameter distribution. The misperceived natural rates are constructed with the median parameter values; with persistent measurement errors, the central bank indicators for the gaps are given by $\widetilde{x}_t^j = x_t^j + \varepsilon_t^j$ with $\varepsilon_t^j = \rho_j \varepsilon_{t-1}^j + \zeta_t^j$ for $j = y, u$, where ζ_t^j is i.i.d. with mean zero and standard deviation σ_j. The objective function is given by equation (44), with $\lambda_u = 0.833$, $\lambda_r = 0.08$; the unemployment equivalent is the permanent percentage point deviation of the unemployment rate from a natural rate of 6 percent that is equivalent in terms of loss to moving from the the optimized rule benchmark rule in Table 3.

Figure 1: Autocovariance functions of aggregate demand variables in U.S. data and in the estimated model



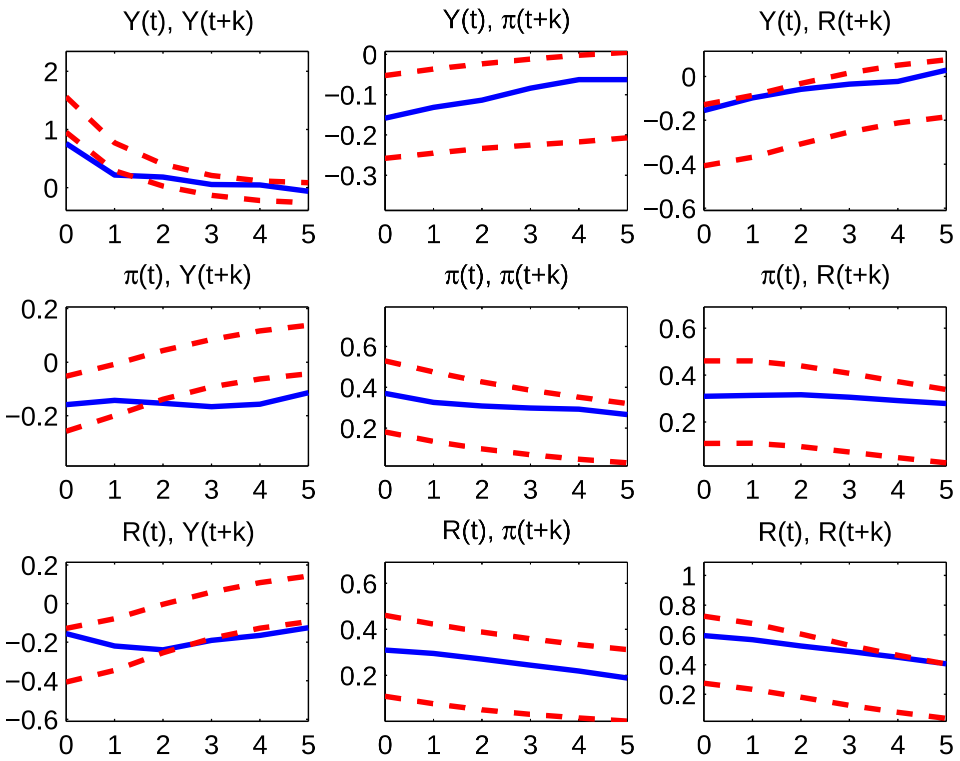
This figure shows the autocovariance functions of the growth rates of output, investment, and consumption in U.S. data (solid lines) and in the estimated model (dashed lines, representing the 5th and 95th percentiles over 500 draws from the posterior parameter distribution and 100 simulated samples of 160 observations for each draw). The panels on the diagonal show the univariate autocovariance functions of each series, while off-diagonal panels show the covariances across two series at different leads and lags.

Figure 2: Autocovariance functions of output growth and labor market variables in U.S. data and in the estimated model



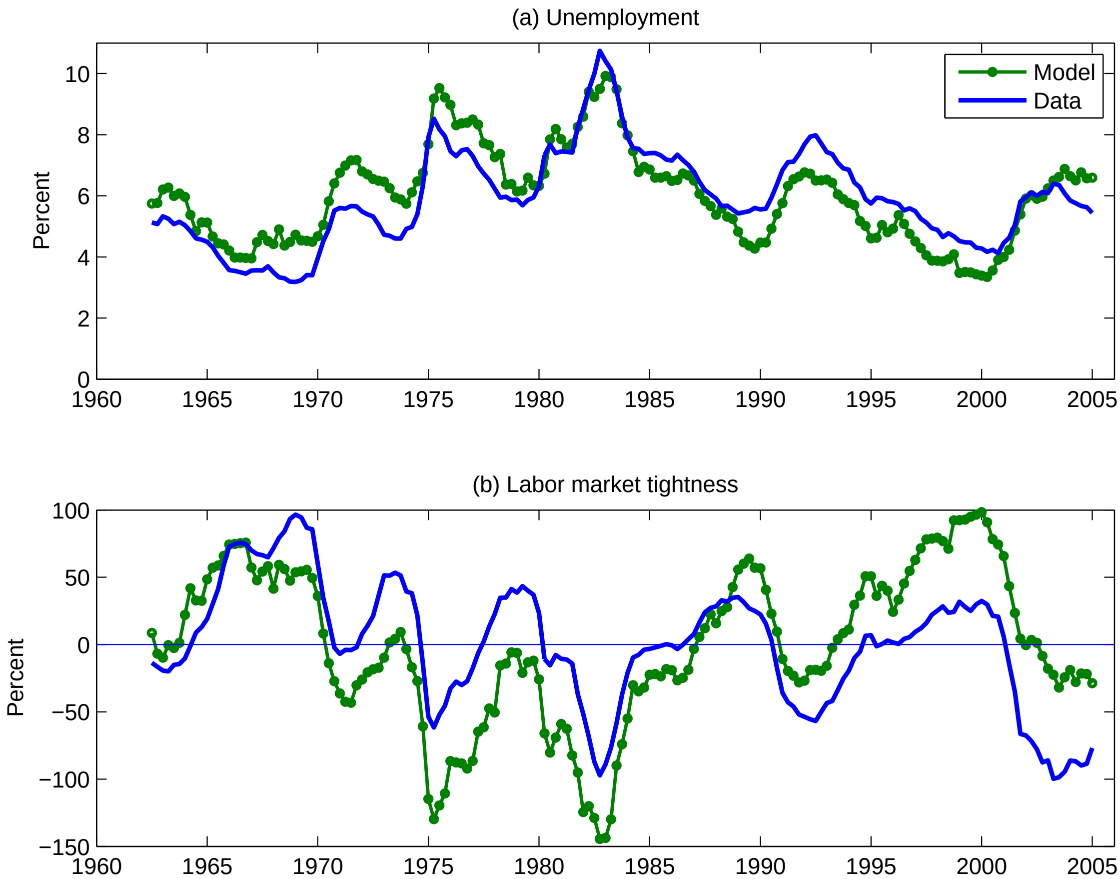
This figure shows the autocovariance functions of the growth rates of output and the real wage, and the level of employment (total hours per capita) in U.S. data (solid lines) and in the estimated model (dashed lines, representing the 5th and 95th percentiles over 500 draws from the posterior parameter distribution and 100 simulated samples of 160 observations for each draw). The panels on the diagonal show the univariate autocovariance functions of each series, while off-diagonal panels show the covariances across two series at different leads and lags.

Figure 3: Autocovariance functions of monetary policy variables in U.S. data and in the estimated model



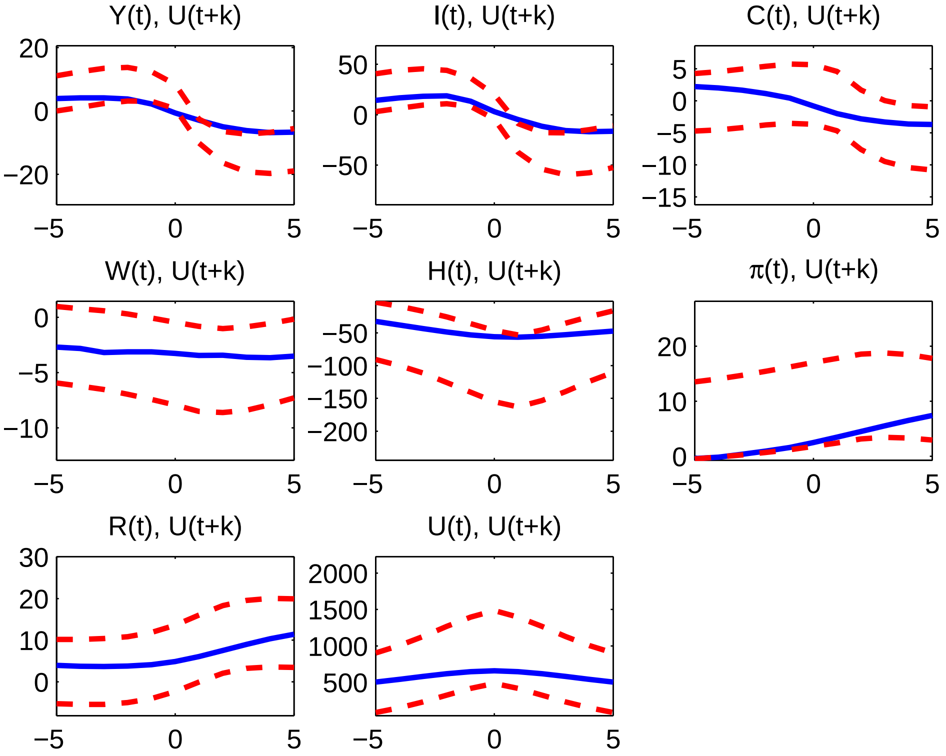
This figure shows the autocovariance functions of output growth, inflation, and the federal funds rate in U.S. data (solid lines) and in the estimated model (dashed lines, representing the 5th and 95th percentiles over 500 draws from the posterior parameter distribution and 100 simulated samples of 160 observations for each draw). The panels on the diagonal show the univariate autocovariance functions of each series, while off-diagonal panels show the covariances across two series at different leads and lags.

Figure 4: Unemployment and labor market tightness: U.S. data and model estimate, 1962Q3–2005Q1



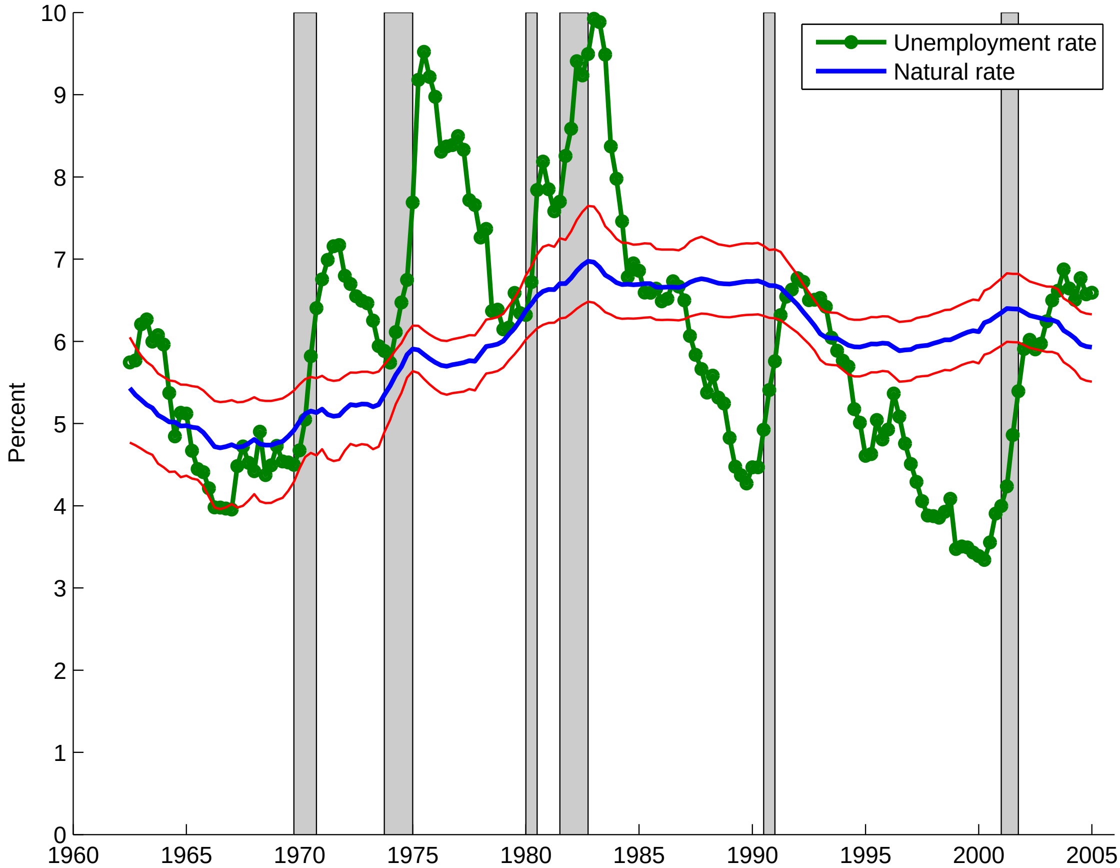
This figure shows the rate of unemployment and the degree of labor market tightness (unemployment/vacancies) in U.S. data and in the estimated model over the period from 1962Q3 to 2005Q1. The unemployment rate in the data is total unemployment/population, adjusted to obtain a sample mean of 6%; the unemployment rate from the model has been calculated assuming a steady-state rate of unemployment of 6%. The tightness in the data is measured as the percent deviation from the sample mean, while the tightness from the model is measured as the percent deviation from steady state (normalized to zero).

Figure 5: Autocovariance functions of unemployment with other variables in U.S. data and in the estimated model



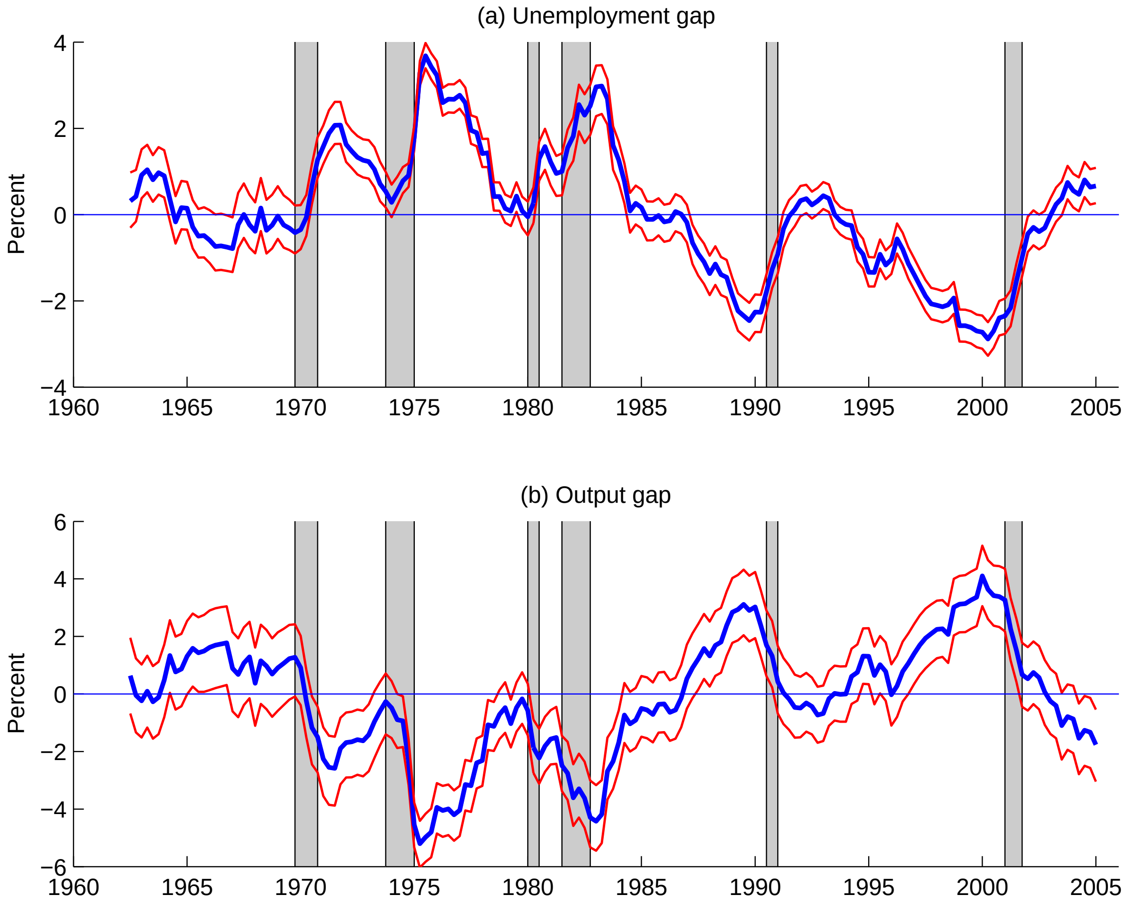
This figure shows the autocovariance functions of the rate of unemployment with the growth rates of output, investment, consumption and the real wage, and the rate of employment (total hours per capita), inflation, the federal funds rate, and with itself at different leads and lags. Solid lines represent U.S. data, dashed lines represent the estimated model (5th and 95th percentiles over 500 draws from the posterior parameter distribution with 100 simulated samples of 160 observations for each draw).

Figure 6: Estimated actual and natural rate of unemployment, 1962Q3–2005Q1



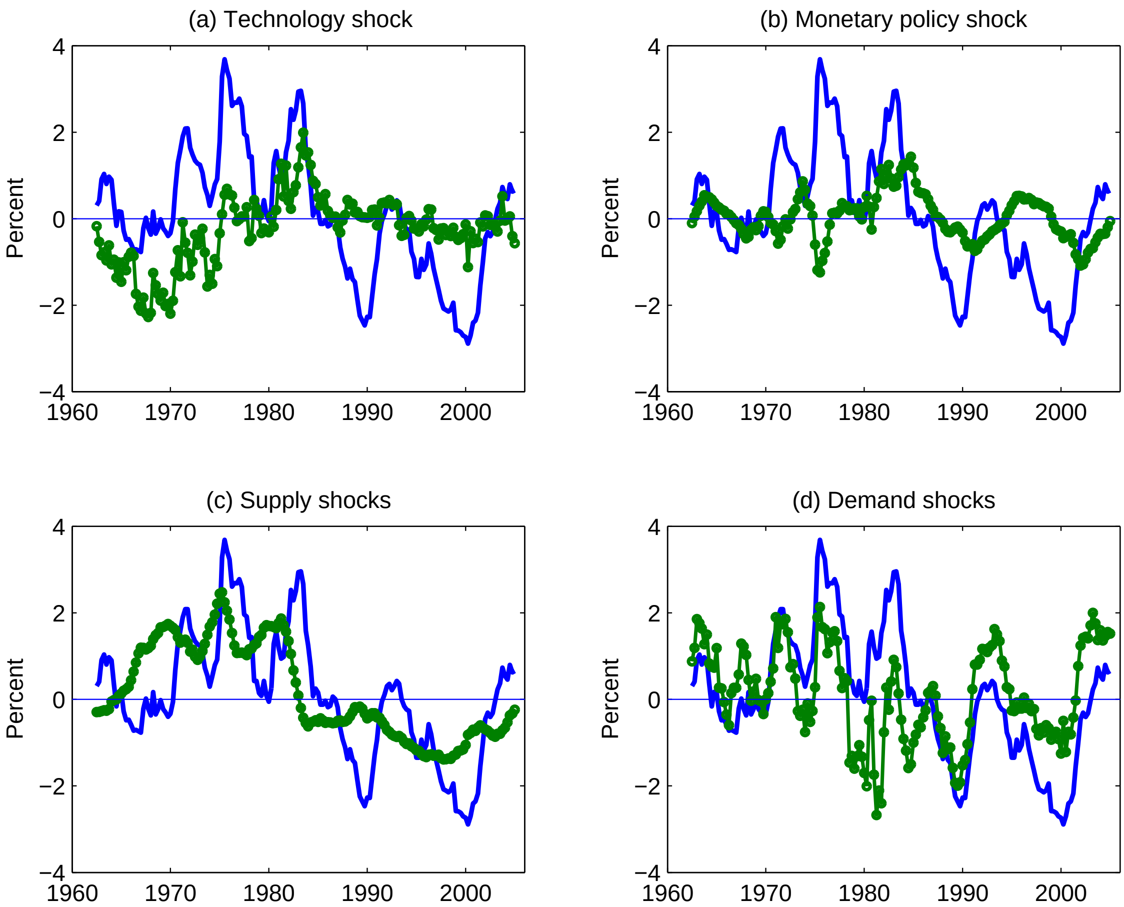
This figure shows the estimated path for the actual and natural rates of unemployment over the period from 1962Q3 to 2005Q1. The unemployment rates have been calculated assuming a steady-state rate of unemployment of 6%. For the natural rate, the thick line is the median and the thin lines are the 5th and 95th percentiles, taking into account parameter uncertainty and Kalman filter uncertainty. Shaded areas correspond to recessions dated by the NBER.

Figure 7: Estimated unemployment and output gaps, 1962Q3–2005Q1



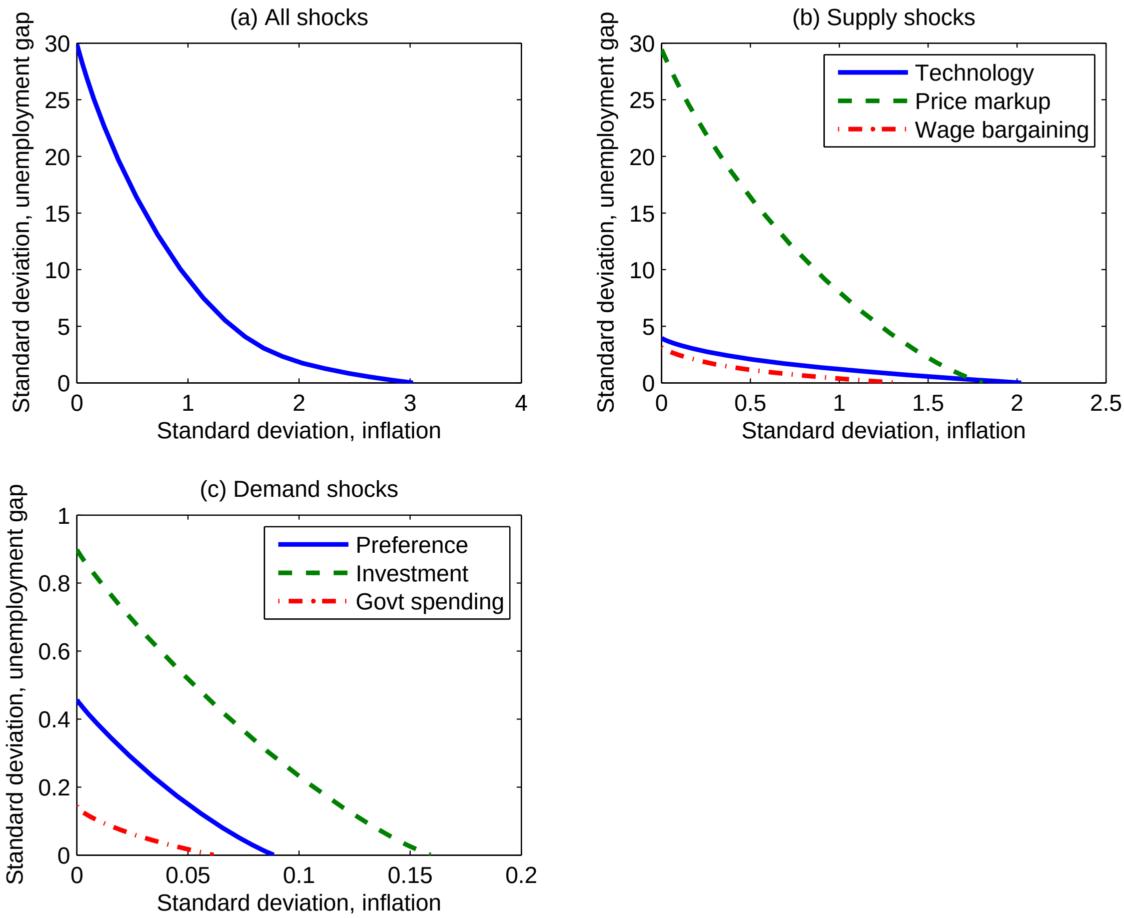
This figure shows the estimated unemployment gap (the percentage deviation of the actual rate of unemployment from the natural rate of unemployment) and output gap (the percent deviation of the actual level of output from the natural level of output) over the period from 1962Q3 to 2005Q1. The unemployment rates have been calculated assuming a steady-state rate of unemployment of 6%. The thick line is the median and the thin lines are the 5th and 95th percentiles, taking into account parameter uncertainty and Kalman filter uncertainty. Shaded areas correspond to recessions dated by the NBER.

Figure 8: Historical decomposition of estimated unemployment gap, 1962Q3–2005Q1



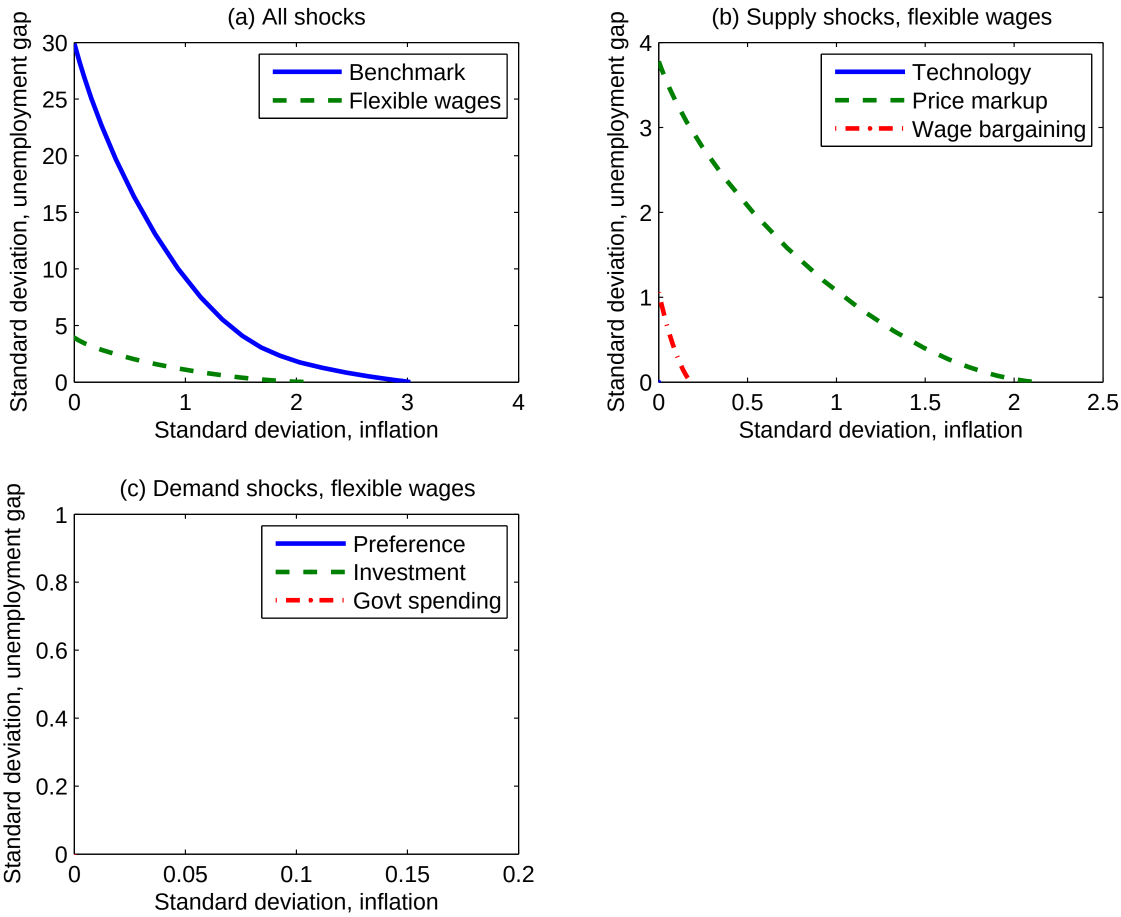
This figure shows the contribution of four sets of shocks to the estimated unemployment gap (the percentage deviation of the actual rate of unemployment from the natural rate of unemployment) over the period from 1962Q3 to 2005Q1. The solid line is the estimated unemployment gap, the dotted lines are the respective contributions of each set of shocks.

Figure 9: Efficient policy frontiers in benchmark model



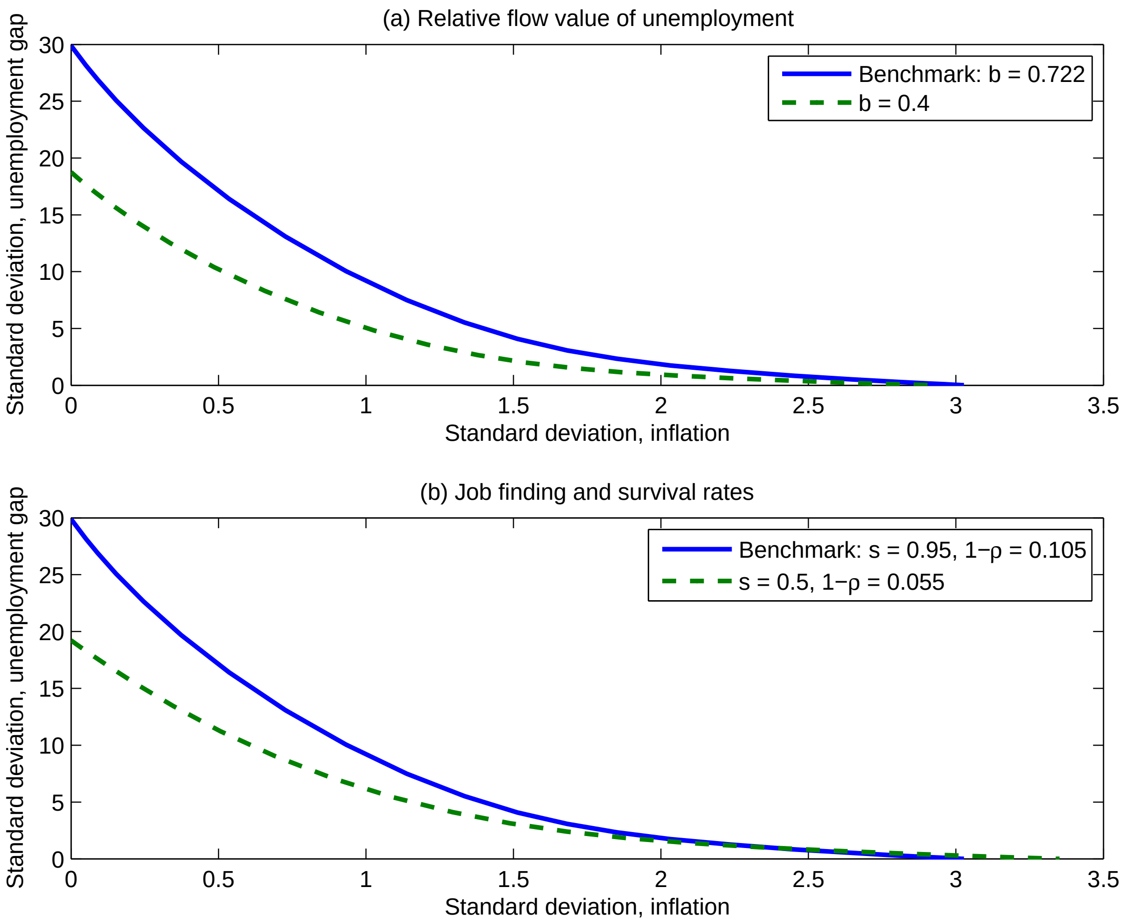
This figure shows the standard deviations of annualized inflation and the unemployment gap (the percentage deviation of unemployment from the natural rate) under the fully optimal policy in the benchmark model as the weight on inflation/unemployment stabilization varies from strict inflation targeting (upper left corner) to strict unemployment targeting (lower right corner). Panel (a) shows the frontier with all shocks, Panels (b) and (c) show the frontiers for each shock in the model.

Figure 10: Efficient policy frontiers with flexible wages



This figure shows the standard deviations of annualized inflation and the unemployment gap (the percentage deviation of unemployment from the natural rate) under the fully optimal policy in the model with flexible wages as the weight on inflation/unemployment stabilization varies from strict inflation targeting (upper left corner) to strict unemployment targeting (lower right corner). Panel (a) shows the frontier with all shocks (including also the benchmark model with sticky wages), Panels (b) and (c) show the frontiers for each shock in the model.

Figure 11: Sensitivity of efficient policy frontiers to labor market parameters



This figure shows the standard deviations of annualized inflation and the unemployment gap (the percentage deviation of unemployment from the natural rate) under the fully optimal policy for alternative parameterizations of the labor market as the weight on inflation/unemployment stabilization varies from strict inflation targeting (upper left corner) to strict unemployment targeting (lower right corner).