

DISCUSSION PAPER SERIES

No. 6696

**DEBT STABILISATION BIAS AND
THE TAYLOR PRINCIPLE: OPTIMAL
POLICY IN A NEW KEYNESIAN
MODEL WITH GOVERNMENT
DEBT AND INFLATION PERSISTENCE**

Sven Jari Stehn and David Vines

INTERNATIONAL MACROECONOMICS



Centre for Economic Policy Research

www.cepr.org

Available online at:

www.cepr.org/pubs/dps/DP6696.asp

DEBT STABILISATION BIAS AND THE TAYLOR PRINCIPLE: OPTIMAL POLICY IN A NEW KEYNESIAN MODEL WITH GOVERNMENT DEBT AND INFLATION PERSISTENCE

**Sven Jari Stehn, International Monetary Fund
David Vines, Oxford University, ANU and CEPR**

Discussion Paper No. 6696
February 2008

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

This Discussion Paper is issued under the auspices of the Centre's research programme in **INTERNATIONAL MACROECONOMICS**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as a private educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions. Institutional (core) finance for the Centre has been provided through major grants from the Economic and Social Research Council, under which an ESRC Resource Centre operates within CEPR; the Esmée Fairbairn Charitable Trust; and the Bank of England. These organizations do not give prior review to the Centre's publications, nor do they necessarily endorse the views expressed therein.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Sven Jari Stehn and David Vines

February 2008

ABSTRACT

Debt Stabilisation Bias and the Taylor Principle: Optimal Policy in a New Keynesian Model with Government Debt and Inflation Persistence*

Leith and Wren-Lewis (2007) have shown that government debt is returned to its pre-shock level in a New Keynesian model under optimal discretionary policy. This has two important implications for monetary and fiscal policy. First, in a high-debt economy, it may be optimal for discretionary monetary policy to cut the interest rate in response to a cost-push shock - thereby violating the Taylor principle - although this will not be true if inflation is significantly persistent. Second, the optimal fiscal response to such a shock is more active under discretion than commitment, whatever the degree of inflation persistence.

JEL Classification: E52, E60, E61 and E63

Keywords: fiscal policy, government debt, monetary policy and stabilisation bias

Sven Jari Stehn
Fiscal Affairs Department
International Monetary Fund
700 19th Street, N.W.
Washington
D.C. 20431
USA
Email: sstehn@imf.org

David Vines
Department of Economics
University of Oxford
Manor Road
OXFORD OX1 3UQ
UK
Email:
david.vines@economics.ox.ac.uk

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=163866

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=100459

* We would like to thank Charles Bean, Tanya Kirsanova, Campbell Leith, Warwick McKibbin, John Muellbauer and Simon Wren-Lewis, and participants at seminars at Oxford University, the Australian National University, the Massachusetts Institute of Technology, the Reserve Bank of Australia, the Australian Treasury, the Bank of England, and the International Monetary Fund. Sven Jari Stehn is grateful to the ESRC for providing financial support while this paper was written.

Submitted 08 January 2008

"Central banks are often accused of being obsessed with inflation. This is untrue. If they are obsessed with anything, it is with fiscal policy." (Mervyn King, 1995)

1 Introduction

It is conventional wisdom that well-designed monetary policy can, on its own, do a good job of stabilising an economy in the face of cost-push shocks (Svensson, 1997, Clarida et al 1999, Woodford, 2003a). Such well-designed monetary policy satisfies the Taylor principle, i.e. it raises the real interest rate in response to a rise in inflation. It is also widely believed that the role for fiscal policy in the macroeconomic stabilisation of an economy can be limited to that of ensuring that the fiscal position is solvent (Allsopp and Vines, 2005). Kirsanova and Wren-Lewis (2007), for example, show that the optimal policy under commitment has this form. Many countries have established policymaking institutions whose purpose is to ensure that macroeconomic policy is conducted in this manner. For example, in the UK, the Bank of England is given the task of achieving an inflation target, and, subject to that, of stabilising demand; it appears to follow the Taylor principle.¹ Furthermore, fiscal policy has been circumscribed by rules which, in effect, tightly constrain discretionary fiscal policy and ensure that fiscal policy is only used, gradually, so as to ensure the sustainability of public debt.

Monetary policy cannot be conducted in this way if fiscal policy fails to ensure debt sustainability (Leeper 1991, Woodford 2000).² With such 'irresponsible' fiscal policy, the optimal policy regime becomes one in which monetary policy lowers the interest rate in response to a cost-push shock to stabilise debt. That is, optimal monetary policy becomes 'passive', and violates the Taylor principle - essentially because the actions which are possible for monetary policy are tightly constrained by the need to stabilise debt.

In this paper we show that, under discretionary policy it is possible that two aspects of the conventional wisdom may be overturned, even although both monetary and fiscal policy are set *optimally*. First, we show that optimal monetary policy does not follow the Taylor principle in a high-debt economy, although this ceases to be true if inflation is sufficiently persistent. Second, we show that optimal fiscal policy is actively involved in the stabilisation of cost-push shocks. Our argument makes use of the fact that, both the control of inflation and the control of debt are subject to 'stabilisation bias'.

1.1 Stabilisation Bias

Stabilisation bias results from the inability to commit to a time-inconsistent policy path.³ The effects of stabilisation bias in the control of inflation by means of monetary policy have been widely explored in New Keynesian models (Clarida et al 1999, Woodford 2003b). Following a cost-push shock, optimal time-inconsistent commitment policy reduces inflation in the current period partly by promising tight monetary policy in the future. Without the ability to reduce current inflation by manipulating inflation expectations optimal discretionary policy leads to a suboptimally slow rate of disinflation.

¹See, for example, Nelson (2003).

²See also Sims (2005) and Benigno and Woodford (2006), who have shown that the standard inflation targeting regime becomes inappropriate when fiscal policy is exogenous.

³This paper focuses on the dynamic bias that arises when benevolent policymakers are unable to commit to a time-inconsistent policy path (Currie and Levine 1993) and abstracts from a level bias that arises if policymakers target levels of output in excess of potential (Barro and Gordon 1983).

Recently, these ideas about stabilisation bias have also been applied to the optimal control of public debt in New Keynesian models. Under optimal commitment policy, it is well established that government debt follows a random walk (Benigno and Woodford 2003, Schmitt-Grohe and Uribe 2004). Leith and Wren-Lewis (2007), however, have shown that such behaviour of debt is time inconsistent. They show that time-consistent optimal discretionary policy is required to return debt to its pre-shock level - rather than following a random walk. We will label this distortion ‘debt stabilisation bias’. Leith and Wren-Lewis (2007) further establish that the means of adjustment of debt to its initial value depends on the steady-state ratio of debt to output, because this determines the relative effectiveness of monetary and fiscal policy in controlling debt. With a low steady-state value of debt, the burden of adjustment is shared by fiscal and monetary policy. With a high steady-state level of debt, however, they show that monetary is so effective in controlling debt that it is optimal to *cut* interest rates in response to a cost-push shock. That is, the violation of the Taylor principle becomes optimal, even although both monetary and fiscal policy are set optimally. This is a striking result and does not correspond what we observe in practice.⁴

1.2 The Contribution of this Paper

Using a New Keynesian model with government debt and inflation persistence, this paper extends the above analysis in a number of ways. We start by showing that optimal commitment policy is time inconsistent in its control of *both* inflation and debt.⁵ This finding has two important implications for optimal policy under discretion. First, we show that the violation of the Taylor principle in a high-debt economy continues to be optimal even although discretionary policy is subject to both inflation and debt stabilisation bias. However, we then show that this is *not* true if inflation is sufficiently persistent. Second, we show that optimal fiscal policy is always more active in the stabilisation of cost-push shocks under discretion than commitment.

Our results imply that the conventional assignment, in which monetary policy stabilises cost-push shocks and fiscal policy merely ensures the sustainability of debt, ceases to be optimal if the authorities act under discretion. We conclude that whilst monetary policy should fulfil the Taylor principle in an economy with significant inflation persistence, fiscal policy should be actively involved in the stabilisation of cost-push shocks.

The remainder of the paper is structured as follows. Section 2 introduces the model and Section 3 solves for optimal policy. Section 4 presents simulations for optimal policy under commitment and discretion. Section 5 concludes.

2 The Model

We use a microfounded model which extends the standard closed-economy New Keynesian model of, for example, Woodford (2003a) in two ways. Firstly, following Steinsson (2003) it contains rule-of-thumb price setters which induce inflation persistence. Secondly, the setup includes fiscal policy with govern-

⁴Clarida et al (1998), for example, find that the Taylor principle has been fulfilled by major central banks over the last two decades.

⁵Leith and Wren-Lewis (2007) studied debt stabilisation bias independently of inflation stabilisation bias. They treated the distortionary income tax rate as a fiscal instrument which enters the Phillips curve directly and hence allows exact control of inflation in each period.

ment spending and public debt accumulation. The model we use in this paper is based on Kirsanova and Wren-Lewis (2007).⁶

2.1 Consumers

The economy is populated by a continuum of infinitely lived individuals, who specialise in the production of a differentiated good (indexed by z), and who spend $h(z)$ of effort in its production. They consume a basket of goods C , and derive utility from per capita government consumption G . The individual's maximisation problem is:

$$\max_{\{C_s, h_s\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t} [u(C_s) + f(G_s) - v(h_s(z))] \quad (1)$$

The price of the differentiated good z is given by $p(z)$ and the corresponding aggregate price level is given by P . Each individual chooses his optimal consumption and work effort to maximise his utility function (1) subject to the demand system and the intertemporal budget constraint:

$$P_t C_t + E_t R_{t,t+1} \bar{A}_{t+1} \leq \bar{A}_t + (1 - \tau) (\omega_t(z) h_t(z) + \Omega_t(z)) + T_t$$

where $P_t C_t = \int_0^1 p(z) c(z) dz$ is nominal consumption, \bar{A}_t are nominal financial assets of a household, Ω_t is profit and T_t is a lump sum subsidy. The nominal wage rate is given by ω_t and τ is an exogenous labour income tax rate. $R_{t,t+1}$ is the stochastic discount factor which denotes the price in period t of carrying the state-contingent asset \bar{A}_{t+1} into period $t + 1$. We can express the stochastic discount factor in terms of the riskless one period nominal interest rate i_t :

$$E_t (R_{t,t+1}) = \frac{1}{1 + i_t}$$

Individuals consume identical baskets of goods which are aggregated into a Dixit and Stiglitz (1977) consumption index. The elasticity of substitution between any pair of goods is assumed to be stochastic to allow for shocks to the mark-up of firms and is given by $\varepsilon_t > 1$ with mean ε . The consumption index is given by $C_t = \left[\int_0^1 c_t^{\frac{\varepsilon_t - 1}{\varepsilon_t}}(z) dz \right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}}$.

We assume no Ponzi schemes, that the net present value of individual's income and wealth is bounded⁷ and that the nominal interest rate is always positive. By ruling out infinite consumption, this allows us to summarise the infinite sequence of budget constraints as a single intertemporal constraint:

$$E_t \sum_{s=t}^{\infty} R_{t,s} C_s P_s \leq \bar{A}_t + E_t \sum_{s=t}^{\infty} R_{t,s} [(1 - \tau) (\omega_s(z) h_s(z) + \Omega_s(z)) + T_s]$$

⁶The models of Benigno and Woodford (2003) and Leith and Wren-Lewis (2007) are similar to our setup but differ to the extent that they treat distortionary taxes as a fiscal instrument which enters the Phillips curve and therefore allow for direct stabilisation of cost-push shocks. Whilst either taxes or government spending can be taken as the fiscal instrument to study the problem of this paper, we choose government spending which affects inflation only through its effect on aggregate demand.

⁷The requirement that the household's wealth accumulation satisfies the transversality condition is given by $\lim_{s \rightarrow \infty} E_t (R_{t,s} \bar{A}_s) = 0$.

Household optimisation determines the following dynamic evolution of consumption:

$$\beta E_t \left(\left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\sigma}} \frac{P_t}{P_{t+1}} \right) = \frac{1}{1+i_t} \quad (2)$$

where the intertemporal elasticity of substitution is defined as $\sigma = -u_C/u_{CC}C = -f_G/f_{GG}G$. Aggregate nominal assets accumulate according to:

$$\bar{A}_{t+1} = (1+i_t) (\bar{A}_t + (1-\tau) P_t Y_t - P_t C_t) \quad (3)$$

We define real assets as $A_t = \bar{A}_t/P_{t-1}$ and linearise (2) and (3) around the steady state. For each variable X_t we denote its steady-state value as X and its logarithmic deviation from this steady state as $\hat{X}_t = \ln(X_t/X)$. Linearising equation (2) leads to a standard Euler equation:

$$\hat{C}_t = E_t \hat{C}_{t+1} - \sigma (\hat{i}_t - E_t \pi_{t+1}) \quad (4)$$

where we defined the inflation rate as $\pi_t = P_t/P_{t-1} - 1$ and assume that inflation is zero in equilibrium. Linearising (3) gives:

$$\hat{A}_{t+1} = \hat{i}_t + \frac{1}{\beta} \left(\hat{A}_t - \pi_t + \frac{(1-\tau)}{A} \hat{Y}_t - \frac{\theta}{A} \hat{C}_t \right) \quad (5)$$

where $\theta = C/Y$ is the steady-state share of private consumption in output⁸ and A is the steady-state level of real assets as a share of Y .

2.2 Price Setting

Following Steinsson (2003), we model price setting as a mix of Calvo contracting and rule-of-thumb behaviour. As in Woodford (2003a), agents re-calculate their prices with fixed probability $(1-\gamma)$. If prices are re-calculated then a proportion ω of the price re-setting agents use a rule of thumb to set their price and proportion $(1-\omega)$ calculate the optimum price. With probability γ prices are not re-calculated and are assumed to rise at the average rate of inflation.

Using superscript $*$ to denote firms that re-set their price we see that the average price is a weighted average between forward (P_t^F) and backward-looking prices (P_t^B):

$$P_t^* = (P_t^F)^{1-\omega} (P_t^B)^\omega$$

Backward-looking agents set their prices P_t^B using the rule of thumb:

$$P_t^B = P_{t-1}^* \Pi_{t-1} \left(\frac{Y_{t-1}}{Y_{t-1}^n} \right)^\delta \quad (6)$$

where $\Pi_t = P_t/P_{t-1}$ and Y_t^n is the flexible-price equilibrium of output which we define later. The coefficient δ defines the relative weight of output in the rule of thumb. The forward-looking price setters solve the first order conditions for profit maximisation and obtain the optimal solution as in Rotemberg

⁸A model-consistent value of θ can be derived from the utility function in steady state, see Kirsanova and Wren-Lewis (2007).

and Woodford (1997). The rest of the prices rise at the steady-state rate of inflation Π with probability γ . We can write the price equation for the economy as a whole as:

$$P_t = \left[\gamma (\Pi P_{t-1})^{1-\varepsilon_t} + (1-\gamma)(1-\omega)(P_t^F)^{1-\varepsilon_t} + (1-\gamma)\omega(P_t^B)^{1-\varepsilon_t} \right]^{\frac{1}{1-\varepsilon_t}}$$

Steinsson (2003, equation B.2) has shown that the optimising price setters reset prices in period t according to the following approximate log-linear rule:

$$p_t^F = \theta\beta E_t p_{t+1}^F + (1-\theta\beta)(mc_t + p_t) \quad (7)$$

To determine real marginal cost (mc_t) we assume that the production function for good z is given by $y_t(z) = h_t(z)$ and that the cost of supplying a good is given by $w_t(z)h_t(z)$. Real marginal cost, which is equal to the real wage, is given by:

$$mc_t = \frac{1}{\psi}\hat{Y}_t + \frac{1}{\sigma}\hat{C}_t \quad (8)$$

where we defined $\psi = v_y/v_{yy}Y$. Steinsson (2003, equation A.3) has further shown that:

$$p_t^B = (1-\omega)p_{t-1}^F + \omega p_{t-1}^B + \pi_{t-1} + \delta y_{t-1} \quad (9)$$

And average inflation is defined as:

$$\pi_t = \frac{1-\gamma}{\gamma} \left((1-\omega)p_t^F + \omega p_t^B - p_t \right) \quad (10)$$

Manipulation of equations (7), (9) and (10) leads to the following hybrid Phillips curve (see Steinsson, 2003):

$$\pi_t = \chi^f \beta E_t \pi_{t+1} + \chi^b \pi_{t-1} + \kappa_c \hat{C}_t + \kappa_{y0} \hat{Y}_t + \kappa_{y1} \hat{Y}_{t-1} + \hat{\mu}_t \quad (11)$$

where $\hat{\mu}_t$ is a mark-up shock. The coefficients are defined as:

$$\begin{aligned} \chi^f &= \frac{\gamma}{\gamma + \omega(1-\gamma + \gamma\beta)}, \quad \chi^b = \frac{\omega}{\gamma + \omega(1-\gamma + \gamma\beta)} \\ \kappa_c &= \frac{(1-\gamma\beta)(1-\gamma)(1-\omega)\psi}{(\gamma + \omega(1-\gamma + \gamma\beta))(\psi + \varepsilon)}, \quad \kappa_{y1} = \frac{(1-\gamma)\omega}{\gamma + \omega(1-\gamma + \gamma\beta)}\delta \\ \kappa_{y0} &= \frac{(1-\gamma\beta)(1-\gamma)(1-\omega)}{(\gamma + \omega(1-\gamma + \gamma\beta))(\psi + \varepsilon)} - \frac{(1-\gamma)\gamma\beta\omega}{\gamma + \omega(1-\gamma + \gamma\beta)}\delta, \quad \delta = \frac{(1-\gamma\beta)(\psi + \sigma)}{\gamma\sigma(\psi + \varepsilon)} \end{aligned}$$

2.3 Aggregate Demand

Aggregate demand is given by the national income identity:

$$Y_t = C_t + G_t \quad (12)$$

In steady state we assume $G = (1-\theta)Y$ where θ is the share of private consumption in output.

Linearising the income identity:

$$\hat{Y}_t = \theta \hat{C}_t + (1 - \theta) \hat{G}_t$$

2.4 Fiscal Policy

The government buys goods, taxes income with a constant income tax rate τ and issues nominal debt \bar{B} . The evolution of nominal debt is given by:

$$\bar{B}_{t+1} = (1 + i_t) (\bar{B}_t + P_t G_t - \tau P_t Y_t) \quad (13)$$

Linearising the debt evolution equation:

$$\hat{B}_{t+1} = \hat{i}_t + \frac{1}{\beta} \left(\hat{B}_t - \pi_t + \frac{(1 - \theta)}{B} \hat{G}_t - \frac{\tau}{B} \hat{Y}_t \right) \quad (14)$$

where we defined the real debt stock as $B_t = \bar{B}_t / P_{t-1}$ and B as the steady-state ratio of debt to output.⁹

2.5 The System

Finally, we obtain the system of equations that describes the evolution of the out-of-equilibrium economy. We follow convention in denoting lower case letters to denote ‘gap’ variables, where the gap is the difference between actual and natural levels (that is we define $x_t = \hat{X}_t - \hat{X}_t^n$). As government debt is the only asset in the economy we have $\hat{A}_t = \hat{B}_t$. We obtain the following system:

$$c_t = E_t c_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \quad (15)$$

$$\pi_t = \chi^f \beta E_t \pi_{t+1} + \chi^b \pi_{t-1} + \kappa_c c_t + \kappa_y 0 y_t + \kappa_{y1} y_{t-1} + \mu_t \quad (16)$$

$$y_t = (1 - \theta) g_t + \theta c_t \quad (17)$$

$$b_{t+1} = i_t + \frac{1}{\beta} \left(b_t - \pi_t + \frac{(1 - \theta)}{B} g_t - \frac{\tau}{B} y_t \right) \quad (18)$$

The complete model consists of four equations. Equation (15) is a standard intertemporal Euler equation in which current consumption depends on its future expected value, because consumers smooth consumption, and negatively on the intertemporal price of consumption, the real interest rate. Secondly, (16) describes a hybrid Phillips curve in which current inflation depends on both forward- and backward-looking components due to firms that set their prices optimally and use the rule of thumb respectively. Equation (17) describes a linearised aggregate demand relationship. Finally, (18) describes public debt accumulation in which debt at the beginning of period $t + 1$ depends on existing debt, real interest payments, government spending and tax revenues through the constant income tax rate.

⁹With lump-sum taxes, any debt stock in steady state can be matched by the appropriate level such taxes. We therefore take this debt ratio as given and discuss its calibration in Section 2.7. The choice of B in turn determines the steady-state tax rate τ .

2.6 Social Welfare Function

Next we follow Kirsanova and Wren-Lewis (2007) and Steinsson (2003) in using a second-order approximation of the aggregate utility function to show that the model-consistent social welfare function can be expressed as (see Appendix A):

$$\frac{1}{2} E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[u(C_s) + f(G_s) - \int_0^1 v(h_s(z)) dz \right] = \frac{1}{2} E_t \sum_{s=t}^{\infty} \beta^{s-t} W_s$$

where the period loss function W_s given by:

$$W_s = \lambda_c c_s^2 + \lambda_g g_s^2 + \lambda_y y_s^2 + \pi_s^2 + \lambda_2 (\Delta \pi_s)^2 + \lambda_3 y_{s-1}^2 + \lambda_4 y_{s-1} \Delta \pi_s + O(3) \quad (19)$$

where $O(3)$ denotes terms of higher than second order and terms independent of policy. The coefficients are determined by the parameters of the model and are given by:

$$\lambda_c = \frac{\theta \psi (1 - \gamma \beta) (1 - \gamma)}{\sigma \varepsilon (\varepsilon + \psi) \gamma}, \quad \lambda_g = \frac{(1 - \theta) \psi (1 - \gamma \beta) (1 - \gamma)}{\sigma \varepsilon (\varepsilon + \psi) \gamma}, \quad \lambda_y = \frac{(1 - \gamma \beta) (1 - \gamma)}{\varepsilon (\varepsilon + \psi) \gamma}$$

$$\lambda_2 = \frac{\omega}{(1 - \omega) \gamma}, \quad \lambda_3 = \frac{\omega (1 - \gamma)^2 \delta^2}{(1 - \omega) \gamma}, \quad \lambda_4 = -2 \frac{\omega (1 - \gamma) \delta}{(1 - \omega) \gamma}$$

We see that the social welfare function consists of three sets of terms in a model with fiscal policy and inflation persistence. Firstly, quadratic terms in c , g and y arise because the representative consumer attempts to smooth both private and public consumption and dislikes fluctuations in hours worked. Secondly, with nominal rigidities, inflation induces price dispersion across industries, which carries a quadratic cost. Thirdly, social welfare contains terms in $\Delta \pi$ and y_{-1} in the presence of rule-of-thumb price setters who base current prices on past period's output and prices using (6), because those past values contribute to price dispersion. A larger proportion of rule-of-thumb price setters raises the weights λ_2 , λ_3 and λ_4 and they start to dominate the loss function for high ω .¹⁰

2.7 Calibration

We follow the recent literature in assuming a time period to be a quarter and set $\beta = 0.99$, $\sigma = 0.5$, $\psi = 2$, $\varepsilon = 5$ and $\gamma = 0.75$ (Rotemberg and Woodford 1997). The Calvo parameter γ implies that prices are on average set once a year. Following Kirsanova and Wren-Lewis (2007), we set the steady-state share of private consumption in output to $\theta = 0.75$.

Whilst the above calibration is standard, there is little consensus on how to calibrate the proportion of rule-of-thumb price setters ω and the steady-state ratio of debt to output B . Estimates of the persistence of inflation vary widely.¹¹ We will therefore consider both a purely forward-looking 'New Keynesian' Phillips curve ($\omega = 0$) and a 'hybrid' Phillips curve which places equal weights on forward and backward-looking components ($\omega = 0.75$). Similarly, we will consider two calibrations for government debt: firstly, a 'low-debt' economy with a debt-to-output ratio of 0.1 and, secondly, a 'high-debt'

¹⁰We notice, however, that there is no solution to the model in the polar case of $\omega = 1$: whilst the Phillips curve converges to an accelerationist form, the social welfare function is not well defined in this limit.

¹¹Gali and Gertler (1999), for example, find a predominantly forward-looking Phillips curve (with $\chi^b = 0.3$) whilst Mehra (2004) finds a predominantly backward-looking inflation process (with $\chi^b = 0.7$).

economy with a debt ratio of 0.3. These calibrations imply annual debt to output ratios of 2.5% and 7.5% respectively.

2.8 The Role of Rule-of-Thumb Price Setters

The introduction of rule-of-thumb price setters affects the relative effectiveness of monetary and fiscal policy in controlling inflation in our model. We can write the inflation rate as the sum of a forward-looking component ($\pi_t^F = \chi^f \beta E_t \pi_{t+1} + \kappa_c c_t + \kappa_{y0} y_t$), a backward-looking component ($\pi_t^B = \chi^b \pi_{t-1} + \kappa_{y1} y_{t-1}$) and the mark-up shock. For a given level of inflation expectations, forward-looking inflation depends on both consumption and the output gap because Calvo price setters base their decisions on real marginal cost, which in turn depends on consumption and output (see equation (8)). For a given level of past prices, backward-looking price setters are assumed to base their decisions on past output, and not real marginal cost. Holding expectations of future inflation and consumption constant and dropping time subscripts for simplicity, we obtain a simple expression for the relative effectiveness of monetary and fiscal policy in controlling the forward-looking and backward-looking elements of inflation:

$$\frac{\delta \pi^F}{\delta c} / \frac{\delta \pi^B}{\delta c} = \frac{\kappa_c}{\theta \kappa_{y1}} + \frac{\kappa_{y0}}{\kappa_{y1}}, \quad \frac{\delta \pi^F}{\delta g} / \frac{\delta \pi^B}{\delta g} = \frac{\kappa_{y0}}{\kappa_{y1}}$$

We see that monetary policy is relatively more effective in controlling the forward-looking component of inflation than the backward-looking component as compared to fiscal policy.¹² That is, monetary policy has a ‘comparative advantage’ in controlling forward-looking inflation and fiscal policy has a comparative advantage in controlling rule-of-thumb inflation. Intuitively this is because monetary policy has a relatively stronger effect on real marginal cost than on output, because real marginal cost depends on consumption directly and also indirectly through output.¹³ Unlike in simple backward-looking models, such as Kirsanova et al (2005), monetary and fiscal policy are therefore not perfect substitutes in the control of inflation. This finding will have implications for optimal policy.

3 Solving for Optimal Policy

The policymaker minimises the social loss by choosing the interest rate and spending subject to the evolution of the economy (15) to (18). We will outline a canonical representation that we will subsequently use to solve for optimal commitment and discretionary policy.

3.1 Canonical Form

Following Currie and Levine (1993) a linear quadratic optimisation problem for the policymaker can be written as:

$$\min_{\{U_s\}_{s=t}^{\infty}} \frac{1}{2} E_t \sum_{s=t}^{\infty} \beta^{s-t} L_s$$

¹²This holds for a hybrid Phillips curve ($0 < \omega < 1$, and hence $\kappa_c > 0$) and as long as the rule of thumb includes past output ($\delta > 0$).

¹³The forward-looking elements of the system will leave this finding unchanged as monetary policy will become more effective in both consumption and output control.

subject to the constraints

$$\begin{bmatrix} X_{1,t+1} \\ E_t X_{2,t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} U_t + \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \varepsilon_{t+1} \quad (20)$$

where $X_{1,t}$ is a $n_1 \times 1$ vector of predetermined (or ‘state’) variables, $X_{2,t}$ is a $n_2 \times 1$ vector of forward-looking (‘jump’) variables, U_t is the instrument vector with dimension k and ε_{t+1} is a white noise process. For our model we have $X_{1,t} = (\mu_t, \pi_{t-1}, y_{t-1}, b_t)'$, $X_{2,t} = (\pi_t, c_t)'$, $U_t = (i_t, g_t)'$ and $\varepsilon_{t+1} = (\eta_{t+1}, 0, 0, 0)'$, where η_t is an i.i.d process.

The quadratic loss function L_t has target variables G_t , such that $L_t = G_t' Q G_t$, where the target variables are functions of the state variables and the instruments of the system, $G_t = C Z_t$ where $Z_t = (X_{1,t}', X_{2,t}', U_t')'$. The period loss function L_t can hence be re-written as:

$$L_t = Z_t' \Omega Z_t \quad (21)$$

with $\Omega = C' Q C$. Appendix B defines the matrices A_{ii} , B_i , E_i and Ω in terms of the structural parameters of our model.

3.2 Optimal Policy under Commitment

Following Currie and Levine (1993) we can write the objective function of the policymaker under commitment as a constrained loss function:

$$H^C = \min_{\{U_s\}_{s=t}^{\infty}} \frac{1}{2} E_t \sum_{s=t}^{\infty} H_s^C \quad (22)$$

with

$$H_s^C = \frac{1}{2} \beta^{s-t} \{ L_s + \hat{\mu}'_{s+1} (A_{11} X_{1,s} + A_{12} X_{2,s} + B_1 U_s - X_{1,s+1}) + \hat{\rho}'_{s+1} (A_{21} X_{1,s} + A_{22} X_{2,s} + B_2 U_s - X_{2,s+1}) \}$$

where L_s is defined in (21), $\hat{\mu}_{t+1}$ is a n_1 -dimensional non-predetermined Lagrange multiplier (or ‘co-state’) associated with the predetermined variables $X_{1,t}$ and $\hat{\rho}_{t+1}$ is a n_2 -dimensional predetermined Lagrange multiplier associated with the non-predetermined variables $X_{2,t}$. In referring to these multipliers, it will be helpful to use the notation $\mu_s = \beta^{-s} \hat{\mu}_s$ and $\rho_s = \beta_s^{-s} \hat{\rho}_s$. The Lagrange multipliers have the usual interpretation as shadow prices of the system constraints.

The first order conditions are obtained by differentiating with respect to X_1 , X_2 , U , $\hat{\mu}$ and $\hat{\rho}$. There are $2n_1 + 2n_2 + k$ of these, i.e. equations for both predetermined and non-predetermined states, their associated co-states, and policy instruments. Appendix C presents the complete set of these conditions. This is a two-point boundary-value problem: the initial values of the predetermined variables and the Lagrange multipliers attached to the non-predetermined variables both have initial conditions, the non-predetermined variables and the co-states of predetermined variables both have terminal conditions. The initial conditions for the co-states of the non-predetermined variables are set to zero ($\rho_s = 0$). The reason for this is that these non-predetermined variables are free to jump, and policy can ensure that they move to the optimal point, which will be one at which the associated Lagrange multiplier is zero (see Currie

and Levin (1993) for further details).

Three aspects of these first order conditions are important. First, the optimality conditions for the non-predetermined variables $X_{2,s}$ highlight the problem of time inconsistency in the optimal commitment solution (see equations (40) and (41) in Appendix C).¹⁴ Once optimal policy has been found at time $t = 0$, and ρ_0 is set to zero, such optimal policy may imply a time path for ρ_s such that ρ_s is not necessarily equal to zero anymore for $s > 0$. That is, given a chance to re-optimize at $s > 0$ the policymaker will choose to set ρ_s equal to zero, reneging on the previously optimal plan. The magnitude of ρ_s therefore captures the extent of the time inconsistency problem. With two jump variables we have two sources of time inconsistency: the control of both inflation and consumption is time inconsistent, indicated by non-zero values of ρ_s^π and ρ_s^c respectively.

Second, we see from the first order condition for the debt stock (equation (39) in Appendix C) that the Lagrange multiplier for debt follows a random walk:

$$\mu_{s+1}^b = \mu_s^b \quad (23)$$

This behaviour of the Lagrange multiplier associated with debt will underpin the result that debt under optimal commitment policy follows a random walk.

Third, the first order condition for the interest rate (equation (42) in Appendix C) reveals a crucial aspect of optimal commitment policy. It describes how optimal interest rate setting at time s equates the marginal cost of raising debt through higher interest payments (μ_{s+1}^b) with the marginal cost of lower consumption through a higher intertemporal price ($-\sigma\rho_{s+1}^c$):

$$\mu_{s+1}^b = -\sigma\rho_{s+1}^c \quad (24)$$

The intuition behind this condition is as follows. Whilst in standard New Keynesian models the interest rate, and hence consumption, is determined solely to achieve the optimal rate of disinflation (Clarida et al 1999), in the present model with public debt there is a cost of interest rate movements as it affects the debt stock which has to be serviced to ensure solvency. Optimal interest rate setting therefore takes the effect on debt into account and, at the margin, balances the costs and benefits of interest rate movements. This relationship implies that along the optimal commitment path, the non-predetermined Lagrange multiplier on debt is proportional to the predetermined Lagrange multiplier of the jump variable, c_s . A non-zero value for ρ_{s+1}^c will imply a non-zero value for μ_{s+1}^b . This allows us to connect the time-inconsistent control of consumption to the time-inconsistent control of debt. Given the structure of the problem in (22), we see that negative values of ρ_s^π and ρ_s^c indicate that the social loss under optimal commitment could be reduced by raising inflation and consumption, or equivalently by raising inflation and lowering debt.

Finally, we follow Currie and Levine (1993) in finding a certainty equivalent solution to these first

¹⁴Woodford (2003b) has suggested an alternative solution approach in which the system, in all periods including the initial, follows the first order conditions for $s > 0$. This ‘timeless perspective’ policy is not time inconsistent as it involves ignoring the conditions that prevail at the regime’s inception ($s = 0$). We want to analyse the time inconsistency inherent in commitment policy and will therefore consider fully optimal commitment policy.

order conditions. The solution can be written as:

$$\begin{bmatrix} U_s \\ X_{2,s} \\ \mu_s \end{bmatrix} = \Phi \begin{bmatrix} X_{1,s} \\ \rho_s \end{bmatrix} \quad (25)$$

$$\begin{bmatrix} X_{1,s+1} \\ \rho_{s+1} \end{bmatrix} = \Psi \begin{bmatrix} X_{1,s} \\ \rho_s \end{bmatrix} + E\varepsilon_{s+1} \quad (26)$$

where Φ and Ψ are found by solving the system of first order conditions using the initial and terminal conditions, as described above. Equations (25) and (26) together with the initial conditions $X_{1,0}$ and $\rho_0 = 0$ provide a complete description of the evolution of the economy. For future reference we define the coefficients of the instrument reaction functions under optimal policy as follows:

$$\begin{aligned} i_t &= \theta_\mu^C \mu_t + \theta_\pi^C \pi_{t-1} + \theta_y^C y_{t-1} + \theta_b^C b_t + \Theta^C \rho_s \\ g_t &= \phi_\mu^C \mu_t + \phi_\pi^C \pi_{t-1} + \phi_y^C y_{t-1} + \phi_b^C b_t + \Phi^C \rho_s \end{aligned} \quad (27)$$

For example, θ_μ^C constitutes the optimal feedback of the interest rate onto the cost-push shock under optimal commitment policy. The expressions $\Theta^C \rho_s$ and $\Phi^C \rho_s$ capture the feedback of the policy instruments onto the predetermined co-states of the system.

3.3 Optimal Policy under Discretion

Optimal policy under discretion, in contrast, must be time consistent. Currie and Levine (1993) show that the first step in finding the discretionary solution is to postulate how private agents determine their expectations of non-predetermined variables. Given the linear-quadratic setup of the model, the solution method postulates that the reaction function of the public takes the following linear form:

$$X_{2,t} = -GX_{1,t} - KU_t \quad (28)$$

where the matrices G and K are unknown and will be found later. We substitute for (28) and form the Lagrangian:

$$H^D = \min_{\{U_s\}_{s=t}^\infty} \frac{1}{2} E_t \sum_{s=t}^\infty H_s^D \quad (29)$$

with

$$H_s^D = \frac{1}{2} \beta^{s-t} \{L_s + \hat{\eta}'_{s+1} ((A_{11} - GA_{12}) X_{1,s} + A_{12} X_{2,s} + (B_1 - A_{12}K) U_s - X_{1,s+1})\}$$

where L_s is defined in (21) and $\hat{\eta}_{t+1}$ is a vector of non-predetermined Lagrange multipliers associated with the predetermined variables $X_{1,t}$. To ensure time consistency, the objective function is only constrained by predetermined variables, as the policymaker takes non-predetermined ones as given, i.e. time consistency requires $\rho_s = 0$ for all s . As a result of this, the time path for non-predetermined variables is determined by (28). Currie and Levine (1993) show that the certainty-equivalent solution of

the first order conditions converges to:¹⁵

$$U_t = F X_{1,t} \tag{30}$$

$$X_{2,t} = C X_{1,t} \tag{31}$$

where F and C are found by means of a generalised Schur decomposition (Soderlind 1999). The dynamics of $X_{1,t}$ are then found by substituting these expressions into (20). Together with the initial conditions $X_{1,0}$, these expressions give a complete description of the evolution of the economy. For future reference we define the instrument feedback coefficients under optimal discretionary policy as:

$$\begin{aligned} i_t &= \theta_\mu^D \mu_t + \theta_\pi^D \pi_{t-1} + \theta_y^D y_{t-1} + \theta_b^D b_t \\ g_t &= \phi_\mu^D \mu_t + \phi_\pi^D \pi_{t-1} + \phi_y^D y_{t-1} + \phi_b^D b_t \end{aligned} \tag{32}$$

where we notice that the instruments under discretionary policy do not feed back onto any Lagrange multipliers (as there are no predetermined co-states in the system).

4 Simulating Optimal Policy

We use the algorithm of Soderlind (1999) to simulate the impulse responses of the system to a one-period cost-push shock under optimal policy. We will analyse optimal monetary and optimal fiscal policy under commitment in Section 4.1 and under discretion in Section 4.2. We will look at both a low-debt ($B = 0.1$) and high-debt economy ($B = 0.3$). For each of these debt calibrations, we will consider a New Keynesian Phillips curve ($\omega = 0$) and a hybrid Phillips curve ($\omega = 0.75$). In Section 4.3 we will extend the analysis to more general values of debt and inflation persistence.

Table 1 summarises the simulations of optimal policy for these Phillips curve specifications for the low and high-debt economies in the left and right parts of the Table respectively. Columns labelled C and D present optimal policy outcomes under commitment and discretion respectively. We report both the absolute welfare loss ('Loss') and the excess loss of the discretionary solution over the commitment solution in terms of percentage of steady-state consumption foregone (% C). We also present the maximum eigenvalue of the system of predetermined variables (ν_{Max}^C), which is indicative of the speed of adjustment of debt in our model.¹⁶ We also report the optimal feedback coefficients under commitment and discretion, which we defined in (27) and (32) respectively.¹⁷ With a unit cost-push shock, the first-period interest rate and spending movements are identical to their feedback coefficients onto the cost-push shock (for example, θ_μ^C denotes the first-period movement of the interest rate under commitment).¹⁸

¹⁵This solution fulfills the Bellman constraint and is therefore sub-game perfect.

¹⁶The maximum eigenvalue of a system describes the speed of adjustment of the system and hence that of its most persistent process.

¹⁷The coefficients under commitment have to be interpreted with care as the fully optimal rule includes feedback onto the predetermined Lagrange multipliers (see above).

¹⁸This is seen by substituting the unit shock and the initial conditions into (25) and (30) for commitment and discretion respectively.

4.1 Optimal Policy under Commitment

We will consider optimal policy for our two values of inflation persistence in Figures 1 and 2. The solid line in these Figures (labelled C) plots the dynamic responses of the model under optimal commitment policy. We will turn to discretionary policy, labelled D , in Section 4.2 below.

4.1.1 The Low-Debt Economy

Let us start by characterising optimal commitment policy in a low-debt economy with a purely forward-looking Phillips curve. Figure 1 shows that the policymaker raises the nominal interest rate to control inflation. We see from column (1) in Table 1 that nominal interest rates rise sufficiently strongly to increase the real interest rate (θ_μ^C exceeds unity). That is, the policymaker fulfils the Taylor principle to ensure inflation stability.¹⁹ This rise in real interest rates induces a fall in consumption, and hence the output gap, which reduce real marginal cost and therefore inflation. Optimal policy under commitment is highly effective in achieving such a disinflation by steering inflation expectations through committing to and delivering contractionary policy in the future. This is achieved through a gradual response to the cost-push shock in which interest rates are slowly smoothed back to zero. We further see that, in the benchmark New Keynesian model without inflation persistence, optimal fiscal policy is almost inactive in response to the cost-push shock (ϕ_μ^C is negative but small at -0.11). The fiscal authority therefore leaves the stabilisation of the cost-push shock almost entirely to monetary policy. This is because movements in the fiscal instrument, in contrast to the monetary instrument, are costly as they induce a suboptimal quantity of public spending. This policy mix for a New Keynesian Phillips curve underpins the conventional view that monetary policy performs almost the entire stabilisation of cost-push shocks. Fiscal policy with a New Keynesian Phillips curve, as we are about to see, simply ensures the sustainability of debt.

Figure 1 further shows that debt accumulates strongly through persistently higher interest rates and the corresponding fall in income tax revenues. Following the shock, debt remains permanently higher: debt under optimal commitment policy follows a random walk, as in Benigno and Woodford (2003) and Schmitt-Grohe and Uribe (2004).²⁰ Column (1) in Table 1 confirms this with a maximum eigenvalue of unity. This random walk result is underpinned by the random walk of the debt Lagrange multiplier in equation (23). (We will discuss this in more detail below). The intuitive reason for the random walk of debt is as follows. We saw above that the cost-push shock induces contractionary monetary policy which in turn creates debt. When determining to what extent to reduce such debt, the policymaker will weigh benefits against costs. The benefits of reducing debt are that permanently higher debt leads to permanently higher interest payments, which will require a permanently lower level of government spending as the government needs to be solvent at given rates of tax. Lower government spending is costly both because the level of public spending appears in the welfare function directly and also because lower government spending leads to permanently higher consumption.²¹ The costs of reducing debt are that doing so would be inflationary: this is both because higher inflation helps to reduce real interest

¹⁹Given the setup of the model, the condition $\theta_\mu^i > 1$ amounts to the Taylor principle in the first period. However, there is no simple expression available for the Taylor principle for subsequent periods.

²⁰Notice that with flexible prices, debt would not follow a random walk but optimally be inflated away as inflation would be costless (Schmitt-Grohe and Uribe 2004).

²¹It is a standard result in baseline New Keynesian models that government spending crowds out private consumption (Gali et al 2007).

payments directly and because any optimal mix of lower interest rates and spending will, on balance, raise inflation.²² As the benefits from permanently reducing government debt are discounted, there will only be finite gains. This means that the commitment solution will be a point such that, at the margin, these gains are balanced with the costs of debt reduction which hence involves a permanent increase in the ratio of debt to output. Consequently, in the face of random cost-push shocks, it is optimal to allow debt to become a random walk. Such a permanent increase in debt requires permanently lower government spending, so that the new, permanently higher, level of debt can be serviced. Table 1 shows that debt sustainability is ensured through permanently lower spending via small fiscal feedback on debt (ϕ_b^C is negative).²³ This implies that in response to the shock not only public debt, but also government spending, and hence consumption and output, will converge to a new steady state.²⁴

Inflation Persistence. The introduction of inflation persistence through rule-of-thumb price setters has important consequences for this optimal policy mix. With a hybrid Phillips curve Figure 2 shows that, as inflation becomes harder to control through expectations of future policies, monetary policy has to raise interest rates significantly more in the first period and induce a larger fall in consumption. With persistent inflation, we also see in column (3) of Table 1 that monetary policy furthermore feeds back onto past inflation and output in a stabilising manner along the disinflation path (θ_π^C and θ_y^C are positive). The combination of positive monetary responses to the cost-push shock and the resulting output and inflation dynamics ensures the stability of inflation.

Fiscal policy now plays an active role with inflation persistence: government spending falls in response to the cost-push shock. The explanation follows from our ‘comparative advantage’ discussion in Section 2.8. During a disinflation, the Calvo component of inflation falls strongly due to low current marginal cost and expectations of low future marginal cost (as we saw in Figure 1). The rule-of-thumb component of inflation, in contrast, falls less rapidly as it depends on past output and prices. This difference in price adjustment speed contributes to price dispersion in the economy and is costly. We showed above that fiscal policy has a ‘comparative advantage’ in controlling the rule-of-thumb component of inflation whilst monetary policy is relatively more effective in affecting the Calvo part of inflation. Fiscal policy therefore becomes helpful in raising the speed of disinflation of rule-of-thumb price setters towards that of the Calvo price setters through cuts in government spending. The gains in terms of better inflation control outweigh the costs of moving the fiscal instrument and column (3) in Table 1 shows that spending optimally falls on impact of the shock (ϕ_μ^C is now significantly negative at -4.48).²⁵ Despite this fall in spending, the strong rise in interest rates leads to a more rapid accumulation of public debt than without inflation persistence. For the same reasons as with the New Keynesian Phillips curve, debt remains permanently higher and follows a random walk.

²²Reducing debt is necessarily inflationary in this setup because it will be done, to some extent, by lowering interest rates. That is both because inflation helps to reduce debt directly and because reducing debt only by lowering government spending would be costly, because the level of government expenditure features in the utility function.

²³As in Kirsanova and Wren-Lewis (2007) the optimal fiscal feedback on debt is negative and small.

²⁴The linearisation of the system remains valid provided that this shift in steady state is small (see also Benigno and Woodford, 2003, and Schmitt-Grohe and Uribe 2004).

²⁵We further notice that inflation falls below zero and then rises back to zero. For the same reasons as just discussed, it becomes optimal for fiscal policy to raise spending to align the rule-of-thumb price setters with the Calvo price setters when inflation is negative.

Time Inconsistency. We have already mentioned in Section 3.2 that optimal commitment policy is time inconsistent in its control of *both* inflation *and* debt. The fourth row in Figures 1 and 2 plots the evolution of the predetermined Lagrange multipliers ρ_s^π and ρ_s^c for the optimal policy scenarios discussed above. We see that both Lagrange multipliers are different from zero during the disinflation process for each level of inflation persistence.

Firstly, as in Currie and Levine (1993) and Woodford (2003b), we see that the control of inflation along the optimal policy path is time inconsistent. We observe that ρ_s^π is negative for $s > 0$, which indicates that the social loss could be reduced by setting a higher inflation rate in periods after the initial than was optimal at time $s = 0$, where we had $\rho_0^\pi = 0$. It is therefore optimal for the policymaker to announce at time $s = 0$ a rapid disinflation through higher interest rates in that and future periods. Expectations of future tight policy help to reduce current inflation without a large fall in current period consumption through the forward-looking part of the Phillips curve. However, at $s > 0$, once inflation has fallen substantially, it becomes optimal for the policymaker not to implement tight policy to lower inflation as this would depress demand. This incentive to renege induces less contractionary policy at time $s > 0$ than announced at time $s = 0$. As inflation converges back to zero, the incentive to renege disappears gradually.

Secondly, as in Leith and Wren-Lewis (2007), the control of debt under optimal commitment policy is time inconsistent. Figures 1 and 2 show that ρ_s^c is different from zero for $s > 0$, which from (24) implies a non-zero value of μ_s^b . A negative value of ρ_s^c is equivalent to a positive value of μ_s^b and hence indicates the incentive to reduce debt under optimal commitment policy. This incentive to cut debt does not vanish over time because μ_s^b and ρ_s^c follow a random walk. The intuition for this result is as follows. In any period, there is a benefit from reducing debt through cutting government expenditure and/or interest rates so as to cut debt service costs. We have explained above that doing so entails a cost because it will be inflationary. The key insight is that, whilst the gain of cutting debt is constant over time, the cost of reducing debt in the first period is smaller than in subsequent periods. This is because, in the first period, the effect on inflation will be confined to that and subsequent periods; there will be no effects on inflation in previous periods (since they do not exist). But in all subsequent periods any attempt to change policy so as to reduce debt which was expected would, because it was expected in the periods before the period in which it occurred, lead to an increase in inflation not only in the period in which it happened (and in subsequent periods) but also in periods before it was implemented. It would thus be more costly to cut debt in future periods, as compared with cutting debt in the first period. But this means that a policymaker who re-optimises every period would face an incentive to unexpectedly lower debt in every period in the future because he had not been expected to do such lowering of debt. The random walk in debt under optimal commitment is therefore time inconsistent.²⁶ This discussion suggests that the random walk of μ_s^b in (23) serves as a sufficient condition for the random walk of debt under commitment: if optimal policy is described by a permanent incentive to cut debt after starting from an initial equilibrium position, then the debt stock must be permanently different from its initial level.

Following Currie and Levine (1993) we quantify the incentive to renege on the optimal inflation and debt paths in terms of the gain in social welfare (in % of steady-state consumption gained). For the

²⁶This discussion implies that policy under fully optimal commitment policy will induce slightly higher inflation and lower debt in the first period as compared with ‘timeless’ commitment policy, see Leith and Wren-Lewis (2007).

New Keynesian Phillips curve, the last row of Figure 1 plots the period-to-period incentive to renege on the optimal inflation and consumption path respectively. We see that the incentive to deviate from the optimal path of inflation is largest in the first period for which expectations have already been set and then fall over time as inflation returns to zero. The incentive to renege on the optimal consumption path, and therefore on the debt path, follows a random walk. This is because, as discussed above, the gain from reducing debt is constant over time as it stems from steady-state gains resulting from higher government spending and lower consumption. Figure 1 further suggests that the welfare gains from renegeing on the optimal inflation path are significantly larger than those on debt. Given the importance of inflation in the social welfare function, the smaller welfare consequences of renegeing on debt control is not surprising.²⁷ Figure 2 shows that the incentive to renege on the optimal inflation path disappears more slowly for economies with higher inflation persistence. We observe that the incentive to renege on the inflation path becomes larger in magnitude because, with higher ω , the weights on inflation related terms in the social welfare function increase strongly (see Section 2.8).

4.1.2 The High-Debt Economy

Let us now turn to optimal commitment policy for an economy with high steady-state debt in Figures 3 and 4. We see from these impulse responses, and from the feedback coefficients in columns (5) and (7) of Table 1, that optimal commitment policy is similar in low and high-debt economies. For the hybrid Phillips curve, for example, Figure 4 shows that, as before, the interest rate rises and spending falls in response to the cost-push shock. Monetary policy fulfils the Taylor principle and debt remains a random walk. The only important change, as one might expect, occurs with respect to debt control. We see from both of the Figures that less debt is accumulated in economies with higher steady-state debt.²⁸ This is because, the interest payments for an additional percentage of debt are larger and therefore even lower government spending is needed in steady-state to service them. Table 1 shows that the tighter control of debt is reflected in stronger fiscal debt feedback coefficients on debt.

As one might expect, the steady-state ratio of debt strengthens the time inconsistency problem with respect to debt control. Taking the hybrid Phillips curve as an example, the comparison of the bottom rows of Figures 2 and 4 shows that the welfare incentive of renegeing on the announced optimal path of debt is larger in the high-debt economy. This is because the gains from cutting debt - in terms of higher government spending - are higher with more debt as the cost-push shock leads to a larger shift in steady state. The incentive to renege on the inflation path, in contrast, remains roughly unchanged.²⁹

4.2 Optimal Policy under Discretion

We will next characterise optimal discretionary policy which, as discussed above, has to be time consistent. As both the control of inflation and debt are time inconsistent under optimal commitment policy, it follows that a time-consistency constraint will impose two distortions onto optimal discretionary policy.

²⁷A smaller incentive to renege under commitment does not mean, however, that the requirement to conduct time-consistent control of debt will impose a smaller distortion onto the system. We will return to this issue below.

²⁸That is, we observe a smaller percentage deviation of debt from its steady-state level.

²⁹We also see this by observing that B does not feature in (40) in Appendix C, which is the source of the time inconsistency problem in inflation control.

4.2.1 The Low-Debt Economy

The dashed lines in Figures 1 and 2 plot optimal discretionary policy in a low-debt economy. Starting again with optimal policy with a New Keynesian Phillips curve we see in Figure 1 that inflation is controlled much less effectively under discretion than commitment. The inability to control inflation tightly by steering inflation expectations under discretion results in the classic inflation stabilisation bias of Currie and Levine (1993) and Woodford (2003b). Unable to promise high interest rates in the future, the policymaker raises interest rates very strongly in the first period. This first period hike in interest rates induces a large recession but then interest rates are much more quickly returned to zero than under commitment which leads to slower inflation control.

We further see from Figure 1 that debt does not follow a random walk under optimal discretionary policy but returns to its initial value. This result was first discovered by Leith and Wren-Lewis (2007). This ‘debt stabilisation bias’ is a direct consequence of the incentive to cut debt that we found under commitment. Under discretion the policymaker cannot commit to ‘not cutting debt in the next period’. The only time-consistent solution is one in which there is no incentive, at any stage, to reduce debt through unexpected changes in government spending or interest rates. As inflation, the interest rate, and spending fall back to towards their steady-state values, the only time-consistent solution is one in which debt returns to its pre-shock level (i.e. equals its steady-state value).³⁰ Otherwise, as described above, there would always be an incentive to carry out an unexpected reduction in debt. Debt under optimal discretionary policy does therefore not follow a random walk. For a New Keynesian Phillips curve, column (2) in Table 1 shows that the maximum eigenvalue for the simulated system is considerably below unity (at 0.73).

Next we consider how the adjustment of debt takes place. The key difference between the control of inflation and debt is that inflation is a forward-looking process, whilst debt is an entirely backward-looking process. This implies that inflation in the first period may be reduced through expectations of future contractionary policy. Debt, in contrast, can only be reduced in the first period through lower interest rates and/or lower spending *in that period*. As discussed above, the policymaker will choose to do the bulk of the debt adjustment in the first period when the inflationary costs of doing so are smallest. Leith and Wren-Lewis (2007) show that whether to lower the interest rate or spending, or both, to do this adjustment in the first period depends critically on the steady-state ratio of debt to output. This is because the steady-state value of debt determines the relative effectiveness of monetary and fiscal policy in affecting the debt stock. For the low-debt economy, we see from Figure 1 that the adjustment of debt in the first period is done mostly through lower spending.³¹ Column (2) in Table 1 shows that spending under discretionary policy falls much more strongly in response to the cost-push shock than under commitment (ϕ_{μ}^D is significantly larger in absolute value than ϕ_{μ}^C). Strongly negative fiscal and monetary feedback onto the debt stock subsequently ensures a fast convergence of debt back to its initial level. The debt stabilisation bias therefore necessitates a much more active role for fiscal policy under discretion than commitment for a New Keynesian Phillips curve.

³⁰An argument by *reductio ad absurdum* makes this point clear. Any candidate for a discretionary outcome with debt above its pre-shock level would be vulnerable, at any point after this supposed equilibrium had been reached, to re-optimisation by the policymaker to reduce debt, taking inflation expectations as given. But this vulnerability would cause the candidate equilibrium to unravel, by backwards induction.

³¹In fact, the behaviour of interest rates also helps to accumulate less debt than under commitment. Whilst interest rates rise more strongly under discretion they are returned much more quickly to zero than under commitment. This lower cumulative effect of interest rates helps to limit the accumulation of debt.

The inability to commit to a time-inconsistent policy plan entails significant costs in a model with government debt. Column (2) in Table 1 shows that the combination of the inflation and debt stabilisation bias induces a loss that is equivalent to a 0.09% fall in steady-state consumption. This excess loss of discretionary policy is about twice as large as that in a standard New Keynesian model without government debt, which is subject to inflation stabilisation bias alone (see Woodford 2003a, Steinsson 2003).

Inflation Persistence. The introduction of inflation persistence again has important consequences for the optimal policy mix under discretion. We see in Figure 2 that monetary policy raises interest rates more strongly and that fiscal policy cuts spending more actively. This is because with rule-of-thumb price setters fiscal policy needs to assist monetary policy *both* in its control of both inflation - as it did under commitment - *and* debt.

Furthermore, with inflation persistence, the dynamics of inflation and debt under discretion become more similar to that of commitment. We see that the disinflation under discretion is less slow compared to commitment than it was in a purely forward-looking regime. This is because the link from expected inflation to current inflation weakens and time-inconsistent promises about future policy become less effective in controlling inflation. With low debt the costs of stabilisation bias associated with the control of debt becomes smaller with persistence, which means that debt can be allowed to behave nearly as under commitment. Table 1 confirms that the maximum eigenvalue rises from 0.73 to 0.98 as we introduce inflation persistence. Nevertheless, we see that the welfare cost of stabilisation bias rises to 0.28% of steady-state consumption. These significantly higher welfare costs are driven by the fact that the weight on price dispersion in the social welfare function rises strongly with inflation persistence.

4.2.2 The High-Debt Economy

We saw for the low-debt economy that the adjustment of debt to its pre-shock level was shared between monetary and fiscal policy. As the steady-state ratio of debt to output in the economy rises, monetary policy becomes more powerful in controlling the debt stock relative to fiscal policy, because its leverage over interest payments rises. For a New Keynesian Phillips curve in a high-debt economy we see in Figure 3 that it turns out to be optimal for the interest rate to *fall* in the first period (we see in column (6) of Table 1 that θ_μ^D is strongly negative). As found in Leith and Wren-Lewis (2007), monetary policy is forced to lower interest rates in the first period - and hence violate the Taylor Principle³² - because debt has to be returned to its initial level to ensure time consistency. Lower interest rates also serve to fuel inflation and hence additionally reduce debt through lower real interest payments. The optimal behaviour of monetary policy is therefore ‘passive’ as in Leeper (1991), even although fiscal policy is set *optimally*.

Notice that we have shown that interest rates optimally fall in the first period, even although the control of inflation is subject to inflation stabilisation bias. Under discretion, the effect of promises about the effects of future monetary policy is weakened because these promises cannot be time inconsistent. Nevertheless the effects of debt stabilisation bias are so severe that it remains optimal to cut interest rates, despite the weak link of expected future monetary policy to current inflation in a regime

³²Davig and Leeper (2007) have generalised the concept of the Taylor principle and suggest that it can be interpreted as a long-term requirement between interest rates and inflation. Here we refer to the first-period, or short-term, relationship.

of discretionary policy. This finding is interesting because the effects of inflation stabilisation bias and debt stabilisation bias point in opposite directions as to the initial movement of monetary policy. We saw above that inflation stabilisation bias would, if operating on its own, cause interest rates to be raised strongly initially, compared with optimal commitment policy. And we have noted that debt stabilisation bias, operating on its own, pulls interest rates down initially, when the level of debt is high. The results here show that this debt stabilisation bias effect dominates, when the initial level of debt is high and inflation is purely forward looking. This interaction of the inflation and debt stabilisation biases makes the inability to commit particularly costly for a high-debt economy, as we will see below.

Once interest rates have fallen in the first period to reduce debt, they rise strongly in the second period. Even under discretion rational agents anticipate in the first period that interest rates will have to rise in subsequent periods to control inflation. Expectations of future contractionary policy ensure the stability of inflation in the first period, despite the cut in interest rates.³³ In other words, optimal policy incurs the ‘damage’ necessary for debt control in the first period and postpones the control of inflation to subsequent periods.

Given that monetary policy is constrained by having to cut debt, it is not surprising that fiscal policy reduces spending very aggressively in the first period to assist monetary policy in the control of inflation.³⁴ In fact, we see in Figure 3 that on impact of the shock, interest rates and spending are cut so strongly that debt actually falls below its steady-state value and then returns to its initial level from below. This is, as we discussed above, because interest rates have to rise in future periods to ensure inflation stability. The only way optimal policy can deliver contractionary policy in future periods, but still fulfil the time consistency requirement, is by cutting debt below its pre-shock value in the first period. Debt is then rather quickly returned to its initial level which is reflected in a small maximum eigenvalue (equal to 0.22, see column (6) of Table 1).

Inflation Persistence. As we introduce inflation persistence it becomes more and more difficult to control inflation despite cutting interest rates in the first period. Figure 4 shows that optimal monetary policy *raises* interest rates in response to the cost-push shock and hence returns to fulfilling the Taylor principle in the first period (this is confirmed by $\theta_\mu^C > 1$ in column (8) of Table 1). Cutting interest rates ceases to be optimal, because the effect of expected future tight monetary policy on current inflation is not strong enough. We conclude that the debt stabilisation bias ceases to force optimal monetary policy into violating the Taylor principle in the first period with a hybrid Phillips curve.³⁵

We further see from Figure 4 that the system implications of stabilisation bias are more severe in the high-debt economy. In contrast to the low-debt case, where the dynamics of the system under discretion approached that under commitment with inflation persistence, we see from Figure 4 that this is not true with a high-debt economy. Even although inflation stabilisation bias weakens as the degree of inflation persistence rises, the costs under discretion of allowing debt to behave in a manner similar to commitment remain high. As a result, even with inflation persistence, debt continues to converge

³³That is, the policymaker under discretion can still make promises about future policy but these promises have to be time consistent.

³⁴Notice, however, that this cut in spending is not necessary to control inflation in the first period. We have checked that interest rates under optimal discretionary policy continue to be cut initially even if government spending is unable to fall because fiscal policy is constrained to a simple feedback on debt.

³⁵This conclusion is consistent with the description of optimal policy in a fully backward-looking system, in which interest rates rise and spending is broadly inactive in response to an inflation shock (see Kirsanova et al 2005).

much faster to its pre-shock level than in a low-debt economy. Column (8) in Table 1 confirms that the maximum eigenvalue of a high-debt economy remains considerably below unity at 0.53. As a result the excess loss of discretionary policy over commitment is significantly larger in the high-debt economy with inflation persistence and equivalent to a 0.75% fall in steady-state consumption.

4.3 Robustness

Having analysed optimal policy for selected calibrations of inflation persistence and steady-state debt, let us now turn to a summary of optimal policy outcomes for a wider range of calibrations. Doing so will allow us to identify the conditions under which falling interest rates in the first period cease to be optimal.

Figure 5 plots the optimal feedback coefficient on the cost-push shock under optimal commitment and discretionary policy respectively (which we recall is equivalent to the first-period instrument movement). We plot the optimal monetary (θ_μ^i) and fiscal feedback coefficients (ϕ_μ^i) against different proportions of rule-of-thumb price setters (ω) for different steady-state debt levels in the economy. The top row of Figure 5 confirms that under optimal commitment policy the social planner always raises the interest rate and cuts spending in the first period (that is $\theta_\mu^C > 0, \phi_\mu^C < 0$ for all ω and B). As the degree of inflation persistence rises, we see that the social planner becomes more active in using both monetary and fiscal policy to stabilise the cost-push shock.

Turning to the bottom two panels in Figure 5 we see the implications of stabilisation bias for optimal monetary and fiscal policy behaviour. The right-hand panel shows that fiscal policy is more active in the stabilisation of the economy under discretion than commitment and that this degree of activism rises with the steady-state ratio of debt to output. The left-hand panel shows that for low values of steady-state debt ($B \leq 0.2$) optimal monetary policy under discretion raises the interest rate in response to the cost-push shock and fulfils the Taylor principle for all levels of inflation persistence. For economies with $B > 0.2$, in contrast, we see how monetary policy begins to stabilise debt through cutting the interest rate in response to the cost-push shock. However, Figure 5 shows that this is *only* true if the Phillips curve is predominantly forward-looking and it is possible to control inflation in the first period by expectations of future tight policy. For $B = 0.25$, for example, the perverse monetary response vanishes at about $\omega = 0.1$. For higher levels of steady-state debt this threshold is larger (e.g. for $B = 0.3$ the point at which monetary policy returns to fulfilling the Taylor principle in the first period rises to about $\omega = 0.5$). This is because the higher the steady-state value of debt, the stronger is the debt stabilisation bias and the more powerful monetary policy is in cutting debt and hence the more optimal policy trades off slower control of inflation for lowering debt in the first period. However, we also observe that the violation of the Taylor principle in the first period remains optimal at all levels of inflation persistence for very high debt calibrations ($B \geq 0.5$). In those cases monetary policy is so powerful in affecting debt that it remains optimal to lower interest rates in the first period, even for $\omega = 0.99$.³⁶

Figure 6 reports simulation results on the strength of the stabilisation bias both by reporting the speed of adjustment of the system under discretion and the welfare consequences of the inability to commit.

³⁶We would expect the violation of the Taylor principle to disappear for an entirely backward-looking inflation process, as cutting the interest rate would lead to an explosive inflation process. However, as the micro-founded social loss function (19) is not defined in this limit, we cannot compute optimal policy. Kirsanova et al (2005), for a non-microfounded model, show that monetary policy fulfils the Taylor principle with an accelerationist Phillips curve, regardless of the level of steady-state debt.

The left hand panel summarises the severity of the stabilisation bias with respect to debt control by plotting the maximum eigenvalue under discretion (ν_{\max}^D). Under commitment, we recall that the maximum eigenvalue equals one. For low values of steady-state debt the largest eigenvalue under discretion rises towards unity as the system becomes increasingly backward looking because inflation stabilisation bias imposes less tight debt control onto the policymaker.³⁷ The eigenvalues fall for higher steady-state debt as the debt stabilisation bias becomes more severe. For very high-debt economies ($B \geq 0.5$) we see that the maximum eigenvalue remains significantly below unity even as ω approaches unity. This underpins our earlier finding that the stabilisation bias continues to impose tight debt control and the violation of the Taylor principle onto optimal discretionary policy.

The right hand panel of Figure 6 evaluates the welfare consequences of these policies by plotting the excess loss of discretionary policy over commitment policy (in % steady-state consumption foregone). We see that this excess loss is large and higher for bigger values of steady-state debt. As the persistence of the system rises, the excess loss of discretionary policy over commitment policy increases as the cost of delivering less tight inflation control rises as the welfare function places more weight on price dispersion. As the persistence of the Phillips curve rises past the hybrid specification, however, the excess loss of discretionary policy over commitment policy starts to fall for economies with small debt ratios because with fewer forward-looking agents the stabilisation bias becomes less severe. For very high-debt economies, which always violate the Taylor principle, we observe that the inability to commit becomes more and more costly.

4.3.1 Discussion

We have shown that the violation of the Taylor principle in a high-debt economy ceases to be optimal with high degrees of inflation persistence. Figures 5 and 6, however, suggest that optimal discretionary policy may violate the Taylor principle for reasonable debt calibrations. The quantitative results depend on the model setup in two important ways. Firstly, the thresholds depend critically on how we define debt. Public debt in this model has a one-period maturity, which means that the entire debt stock is rolled over each period which gives monetary policy large leverage over interest payments. In practice, the fraction of debt which is refinanced every period is considerably lower.³⁸ Denoting debt in nominal terms, in contrast, would raise the effect of inflation on debt and increase the severity of debt stabilisation bias (Leith and Wren-Lewis 2007).

Secondly, the threshold of inflation persistence at which it ceases to be optimal to violate the Taylor principle falls with the intertemporal elasticity of substitution, because changes in interest rates have stronger effects on consumption. In response to a cost-push shock monetary policy can dis-inflate by accumulating less debt which renders the debt stabilisation bias less severe.

³⁷However, even as ω approaches unity, we see that the dynamics of debt does not converge all the way to a random walk.

³⁸For example, whilst Euroland has a debt-to-GDP ratio of about 60%, the amount of debt re-financed per year is much lower at around 11% of GDP (ECB 2007).

5 Conclusion

Leith and Wren-Lewis (2007) have shown that public debt under optimal discretionary policy does not follow a random walk but has to be returned to its pre-shock level to ensure time consistency. This finding has two important implications for optimal monetary and optimal fiscal policy under discretion. Firstly, as Leith and Wren-Lewis (2007) show, optimal monetary policy in a high-debt economy cuts the interest rate in response to a cost-push shock - and therefore violates the Taylor principle. This is a striking and unintuitive result. We have shown that this is not true with significant degrees of inflation persistence. Secondly, because debt does not follow a random walk under discretionary policy, we have shown that optimal fiscal policy is more active in the stabilisation of cost-push shocks under discretion than commitment. These results imply that the conventional assignment, in which monetary policy stabilises cost-push shocks and fiscal policy merely ensures the sustainability of debt, ceases to be optimal if the authorities act under discretion. We have shown that whilst monetary policy should fulfil the Taylor principle in an economy with significant inflation persistence, fiscal policy should be actively involved in the stabilisation of cost-push shocks.

These findings further suggest that the gains from commitment are much larger in an economy with public debt, especially if it is high, than the traditional monetary-policy analysis identified. In low-debt countries with effective institutions, such as Britain, monetary policy is unlikely to be tightly constrained by public debt. In very high-debt countries with weak commitment mechanisms, in contrast, the central bank might well be hindered in its control of inflation. These considerations help to explain recent empirical findings which show that the level of public debt has impeded the ability of central banks to control inflation in some high-debt developing countries (Giavazzi 2003, Baig et al 2006, Mitra 2007). Institutions which promote the commitment of both monetary policy and fiscal policy are therefore highly desirable.

A The Social Welfare Function

The derivations in this Section are based on Kirsanova and Wren-Lewis (2007). The social welfare function can be written as:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} W_s = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[u(C_s) + f(G_s) - \int_0^1 v(h_s(z)) dz \right]$$

Following Woodford (2003a), we linearise the intra-temporal utility W_s around its equilibrium using $\hat{X}_t = \ln(X_t/X)$:

$$\begin{aligned} W_s = & C u_C(C) \left(\hat{C}_s + \frac{1}{2} \left(1 - \frac{1}{\sigma} \right) \hat{C}_s^2 \right) + G f_G(G) \left(\hat{G}_s + \frac{1}{2} \left(1 - \frac{1}{\sigma} \right) \hat{G}_s^2 \right) \\ & - Y v_y(Y) \left(\hat{Y}_s + \frac{1}{2} \left(1 + \frac{1}{\psi} \right) \hat{Y}_s^2 + \frac{1}{2} \left(\frac{1}{\psi} + \frac{1}{\varepsilon} \right) \text{var}_z \hat{y}_s(z) \right) + O(3) \end{aligned} \quad (33)$$

where $O(3)$ denotes terms of higher than second order and terms independent of policy. Further, we linearise the aggregate demand equation (12):

$$\hat{C}_s = \frac{1}{\theta} \left(\hat{Y}_s - (1 - \theta) \hat{G}_s - \theta \frac{1}{2} \hat{C}_s^2 - \frac{1}{2} (1 - \theta) \hat{G}_s^2 + \frac{1}{2} \hat{Y}_s^2 \right) + O(3)$$

Substituting this expression into (33) we obtain:

$$\begin{aligned} W_s = & \theta u_C \left[\left(1 - \frac{v_y}{u_C} \right) \hat{Y}_s - (1 - \theta) \left(1 - \frac{f_G}{u_C} \right) \hat{G}_s - \frac{\theta}{2\sigma} \hat{C}_s^2 - \frac{(1 - \theta)}{2} \left(1 + \frac{f_G}{u_C} \frac{(1 - \sigma)}{\sigma} \right) \hat{G}_s^2 \right. \\ & \left. - \frac{1}{2} \left(\frac{v_y}{u_C} \frac{1 + \psi}{\psi} - 1 \right) \hat{Y}_s^2 - \frac{1}{2} \frac{v_y}{u_C} \frac{\varepsilon + \psi}{\varepsilon \psi} \text{var}_z \hat{y}_s(z) \right] + O(3) \end{aligned}$$

The next step is to eliminate the linear terms in output and government spending. We can always choose a steady-state such that $\theta = 1 - \frac{G}{Y}$ such that $\frac{f_G}{u_C} = \frac{v_y}{u_C}$. The government is assumed to eliminate both the distortions resulting from monopolistic competition and the distortions resulting from income taxation with a lump sum of $\mu_w = \frac{\mu}{1 - \tau}$ in steady state. We set $\frac{f_G}{u_C} = \frac{v_y}{u_C} = 1$ and the welfare function does not contain any linear terms. We can then re-write it in 'gap' form in deviations from its natural levels (denoting $x_t = \hat{X}_t - \hat{X}_t^n$):

$$W_s = -\theta u_C \left[\frac{\theta}{2\sigma} c_s^2 + \frac{(1 - \theta)}{2\sigma} g_s^2 + \frac{1}{2\psi} y_s^2 + \frac{\varepsilon + \psi}{2\varepsilon\psi} \text{var}_z \hat{y}_s(z) \right] + O(3) \quad (34)$$

Finally, Steinsson (2003) has shown that:³⁹

$$\text{var}_z \hat{y}_s(z) = \frac{\varepsilon^2}{(1 - \gamma\beta)(1 - \omega)} \left[\frac{\gamma(1 - \omega)}{(1 - \gamma)} \pi_s^2 + \frac{\omega}{(1 - \gamma)} (\Delta\pi_s)^2 + (1 - \gamma) \omega \delta^2 y_{s-1}^2 - 2\omega \delta y_{s-1} \Delta\pi_s \right] \quad (35)$$

Substituting (35) into (34) and normalising, so there is a unit coefficient on inflation variability, we obtain (19) in the main text.

³⁹Notice that we make use of the erratum to Steinsson (2003).

B Canonical Form

For our model we have:

$$A_{11} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\beta} \end{pmatrix}, A_{12} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & \theta \\ -\frac{1}{\beta} & -\frac{\tau\theta}{\beta B} \end{pmatrix},$$

$$A_{21} = \begin{pmatrix} -\frac{1}{\chi^f \beta} & -\frac{\chi^b}{\chi^f \beta} & -\frac{\kappa_{y1}}{\chi^f \beta} & 0 \\ \frac{\sigma}{\chi^f \beta} & \frac{\sigma \chi^b}{\chi^f \beta} & \frac{\sigma \kappa_{y1}}{\chi^f \beta} & 0 \end{pmatrix}, A_{22} = \begin{pmatrix} \frac{1}{\chi^f \beta} & \frac{(\kappa_c - \theta \kappa_{y1})}{\chi^f \beta} \\ -\frac{\sigma}{\chi^f \beta} & \frac{(\chi^f \beta + \sigma \kappa_c + \sigma \theta \kappa_{y1})}{\chi^f \beta} \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 - \theta \\ 1 & \frac{(1-\tau)(1-\theta)}{\beta B} \end{pmatrix}, B_2 = \begin{pmatrix} 0 & -\frac{\kappa_{y0}(1-\theta)}{\chi^f \beta} \\ \sigma & \frac{\sigma \kappa_{y0}(1-\theta)}{\chi^f \beta} \end{pmatrix}, E_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ and } E_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

The weight matrix is given by:

$$\Omega = \begin{pmatrix} \Omega_{11} & \Omega_{12} & 0 \\ \Omega_{21} & \Omega_{22} & \Omega_{23} \\ 0 & \Omega_{32} & \Omega_{33} \end{pmatrix} \text{ with } \Omega_{11} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & -\frac{1}{2}\lambda_4 & 0 \\ 0 & -\frac{1}{2}\lambda_4 & \lambda_3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\Omega_{12} = \Omega'_{21} = \begin{pmatrix} 0 & 0 \\ -\lambda_2 & 0 \\ \frac{1}{2}\lambda_4 & 0 \\ 0 & 0 \end{pmatrix}, \Omega_{22} = \begin{pmatrix} 1 + \lambda_2 & 0 \\ 0 & \lambda_c + \theta^2 \lambda_y \end{pmatrix},$$

$$\Omega_{23} = \Omega'_{32} = \begin{pmatrix} 0 & 0 \\ 0 & \theta(1-\theta)\lambda_y \end{pmatrix} \text{ and } \Omega_{33} = \begin{pmatrix} 0 & 0 \\ 0 & \lambda_g + (1-\theta)^2 \lambda_y \end{pmatrix}.$$

C Solving for Optimal Commitment Policy

In this Section we provide the first order conditions for optimal policy under commitment. For notational simplicity we will define $\mu_s = \beta^{-s} \hat{\mu}_s$ and $\rho_s = \beta^{-s} \hat{\rho}_s$ and drop the rational expectations operator E_s (that is, denote $E_s X_{s+1} = X_{s+1}$). The first block of optimality conditions are for the state variables in vector $X_{1,s}$:

$$\frac{\partial H^C}{\partial \mu_s} = -\frac{1}{\chi^f} \rho_{s+1}^\pi + \frac{\sigma}{\chi^f} \rho_{s+1}^c - \mu_s^\mu = 0 \quad (36)$$

$$\frac{\partial H^C}{\partial \pi_{s-1}} = \lambda_2 \pi_{s-1} - \frac{1}{2} \lambda_4 y_{s-1} - \lambda_2 \pi_s - \frac{\chi^b}{\chi^f} \rho_{s+1}^\pi + \frac{\sigma \chi^b}{\chi^f} \rho_{s+1}^c - \mu_s^\pi = 0 \quad (37)$$

$$\frac{\partial H^C}{\partial y_{s-1}} = -\frac{1}{2}\lambda_4\pi_{s-1} + \lambda_3y_{s-1} + \frac{1}{2}\lambda_4\pi_s - \frac{\kappa_{y1}}{\chi^f}\rho_{s+1}^\pi + \frac{\sigma\kappa_{y1}}{\chi^f}\rho_{s+1}^c - \mu_s^y = 0 \quad (38)$$

$$\frac{\partial H^C}{\partial b_s} = \mu_{s+1}^b - \mu_s^b = 0 \quad (39)$$

The second block of optimality conditions are for the jump variables in vector $X_{2,s}$:

$$\frac{\partial H^C}{\partial \pi_s} = -\lambda_2\pi_{s-1} + \frac{1}{2}\lambda_4y_{s-1} + (\lambda_\pi + \lambda_2)\pi_s + \mu_{s+1}^b + \sigma\rho_{s+1}^c - \rho_s^\pi = 0 \quad (40)$$

$$\begin{aligned} \frac{\partial H^C}{\partial c_s} &= (\theta^2\lambda_y + \lambda_c)c_s + 2\theta\lambda_y(1-\theta)g_s + \beta(1-\theta)\mu_{s+1}^y \\ &+ \frac{(1-\tau)(1-\theta)}{B}\mu_{s+1}^b + \frac{\sigma\kappa_{y0}(1-\theta)}{\chi^f}\rho_{s+1}^c - \frac{\kappa_{y0}(1-\theta)}{\chi^f}\rho_{s+1}^\pi - \rho_s^c = 0 \end{aligned} \quad (41)$$

The third block for the instruments in vector U_s is:

$$\frac{\partial H^C}{\partial i_s} = \mu_{s+1}^b + \sigma\rho_{s+1}^c = 0 \quad (42)$$

$$\begin{aligned} \frac{\partial H^C}{\partial g_s} &= 2\theta\lambda_y(1-\theta)c_s + \left((1-\theta)^2\lambda_y + \lambda_g\right)g_s + (1-\theta)\mu_{s+1}^y \\ &+ \left(\frac{(1-\tau)(1-\theta)}{\beta B}\right)\mu_{s+1}^b - \frac{\kappa_{y0}(1-\theta)}{\chi^f\beta}\rho_{s+1}^\pi + \left(\frac{\sigma\kappa_{y0}(1-\theta)}{\chi^f\beta}\right)\rho_{s+1}^c = 0 \end{aligned} \quad (43)$$

The final block of first order conditions is the evolution of the system (20) which, for brevity, we do not replicate here.

References

- Allsopp, Christopher and David Vines. (2005). "The Macroeconomic Role of Fiscal Policy." *Oxford Review of Economic Policy* 21, 485-508.
- Baig, Taimur, Manmohan Kumar, Garima Vasishta, and Edda Zoli. (2006). "Fiscal and Monetary Nexus in Emerging Market Economies: How Does Debt Matter?" IMF Working Paper, 06/184.
- Barro, Robert and David Gordon. (1983). "Rules, Discretion and Reputation in a Model of Monetary Policy." *Journal of Monetary Economics* 12, 101-121.
- Benigno, Pierpaolo and Michael Woodford. (2003). "Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach." In *NBER Macroeconomics Annual 2003*, edited by Mark Gertler and Kenneth Rogoff. Cambridge (US): MIT Press.
- Benigno, Pierpaolo and Michael Woodford. (2006). "Inflation targeting under alternative fiscal regimes." NBER Working Paper, 12158.
- Clarida, Richard, Jordi Gali, and Mark Gertler. (1998). "Monetary policy rules in practice: some international evidence." *European Economic Review*, 42, 1033-67.
- Clarida, Richard., Jordi Gali, and Mark Gertler. (1999). "The science of monetary policy: a New Keynesian perspective." *Journal of Economic Literature* 37, 1661-1707.
- Currie, David and Paul Levine. (1993). *Rules, Reputation and Macroeconomic Policy Coordination*. Cambridge (UK): Cambridge University Press.

- Davig, Troy and Eric Leeper (2007). "Generalizing the Taylor principle." *American Economic Review* 97, 607-635.
- Dixit, Avinash and Joseph Stiglitz. (1977). "Monopolistic competition and optimum product diversity." *American Economic Review* 67, 297-308.
- ECB (2007). *Monthly Bulletin* October 2007. European Central Bank.
- Gali, Jordi and Mark Gertler. (1999). "Inflation dynamics: a structural econometric analysis." *Journal of Monetary Economics* 44, 195-222.
- Gali, Jordi, David López-Salido, and Javier Vallés. (2007). "Understanding the Effects of Government Spending on Consumption." *Journal of the European Economic Association* 5, 227-270.
- Giavazzi, Francesco. (2003). "Inflation Targeting and the Fiscal Policy Regime: The Experience in Brazil." *Bank of England Quarterly Bulletin*, Autumn 2003.
- King, Mervyn. (1995). "Commentary: Monetary Policy Implications of Greater Fiscal Discipline." *Federal Reserve Bank of Kansas City Proceedings*, 71-183.
- Kirsanova, Tatiana, Jari Stehn, and David Vines. (2005). "The Interactions of Monetary and Fiscal Policy." *Oxford Review of Economic Policy* 21, 532-564.
- Kirsanova, Tatiana and Simon Wren-Lewis. (2007). "Optimal fiscal feedback in an economy with nominal rigidities." *Department of Economics Series Working Paper No. 306*, University of Oxford.
- Leeper, Eric. (1991). "Equilibria under 'active' and 'passive' monetary and fiscal policies." *Journal of Monetary Economics* 27, 129-47.
- Leith, Campbell and Simon Wren-Lewis. (2007). "Fiscal sustainability in a New Keynesian model." *Department of Economics Series Working Paper, No. 310*, University of Oxford.
- Mehra, Yash. (2004). "The output gap, expected future inflation and inflation dynamics: another look." *Topics in Macroeconomics* 4, 1194-1213.
- Mitra, Srobona. (2007). "Is the Quantity of Government Debt a Constraint for Monetary Policy?" *IMF Working Paper, No. 07/62*.
- Nelson, Edward. (2003). "UK Monetary Policy 1972-97: A Guide Using Taylor Rules." In *Central Banking, Monetary Theory and Practice: Essays in Honour of Charles Goodhart, Volume One*, edited by Paul Mizen. Cheltenham, UK: Edward Elgar.
- Rotemberg, Julio and Michael Woodford. (1997). "An optimization-based econometric framework for the evaluation of monetary policy." In *NBER Macroeconomics Annual Vol. 12*, edited by Ben Bernanke and Julio Rotemberg. Cambridge (US): MIT Press.
- Schmitt-Grohe, Stephanie and Martin Uribe. (2004). "Optimal Simple And Implementable Monetary and Fiscal Rules." *Journal of Monetary Economics*, forthcoming.

- Sims, Chris. (2005). "Limits to Inflation Targeting." In *The Inflation Targeting Debate*, edited by Ben Bernanke and Michael Woodford. NBER.
- Soderlind, Paul. (1999). "Solution and estimation of RE macromodels with optimal policy." *European Economic Review* 43, 813-823.
- Steinsson, Jon. (2003). "Optimal Monetary Policy in an Economy with Inflation Persistence." *Journal of Monetary Economics* 50, 1425-1456.
- Svensson, Lars. (1997). "Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets." *European Economic Review* 41, 1111-1146.
- Woodford, Michael. (2000). "Fiscal Requirements for Price Stability." *Journal of Money, Credit and Banking* 33, 669-728.
- Woodford, Michael. (2003a). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.
- Woodford, Michael. (2003b). "Optimal interest rate smoothing." *Review of Economic Studies* 70, 861-886.

	Low Debt ($B = 0.1$)				High Debt ($B = 0.3$)			
	$\omega = 0$		$\omega = 0.75$		$\omega = 0$		$\omega = 0.75$	
	C	D	C	D	C	D	C	D
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Loss	0.18	0.28	4.03	4.31	0.19	0.28	4.06	4.82
$\%C$	-	0.09	-	0.28	-	0.10	-	0.75
ν_{Max}^i	1.00	0.73	1.00	0.98	1.00	0.22	1.00	0.53
	Monetary Feedback Coefficients				Monetary Feedback Coefficients			
θ_{μ}^i	2.99	6.27	13.95	24.63	2.75	-2.39	13.20	4.90
θ_{π}^i	-	-	3.68	4.26	-	-	3.52	1.78
θ_y^i	-	-	0.23	0.26	-	-	0.22	0.11
θ_b^i	-0.003	-0.012	-0.003	-0.001	-0.027	-0.394	-0.022	-0.211
	Fiscal Feedback Coefficients				Fiscal Feedback Coefficients			
ϕ_{μ}^i	-0.11	-4.10	-4.48	-5.73	-0.20	-4.86	-4.89	-19.65
ϕ_{π}^i	-	-	-3.47	-3.62	-	-	-3.57	-6.35
ϕ_y^i	-	-	-0.21	-0.22	-	-	-0.22	-0.39
ϕ_b^i	-0.005	-0.107	-0.005	-0.011	-0.017	-0.342	-0.019	-0.311

Table 1: Summary of Optimal Policy under Commitment (C) and Discretion (D)

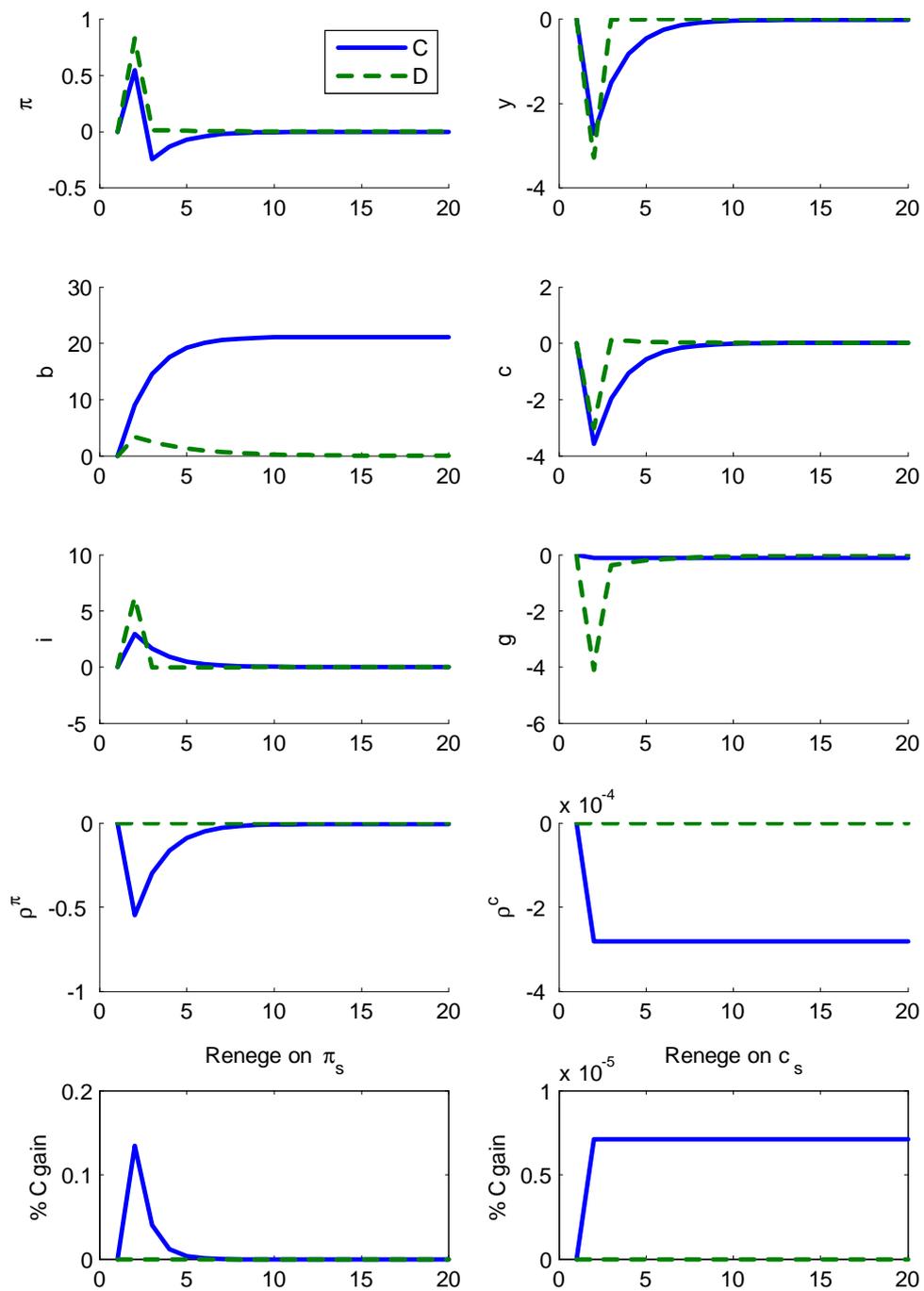


Figure 1: Optimal Policy with a New Keynesian Phillips curve ($\omega = 0$) in a low-debt economy ($B = 0.1$).

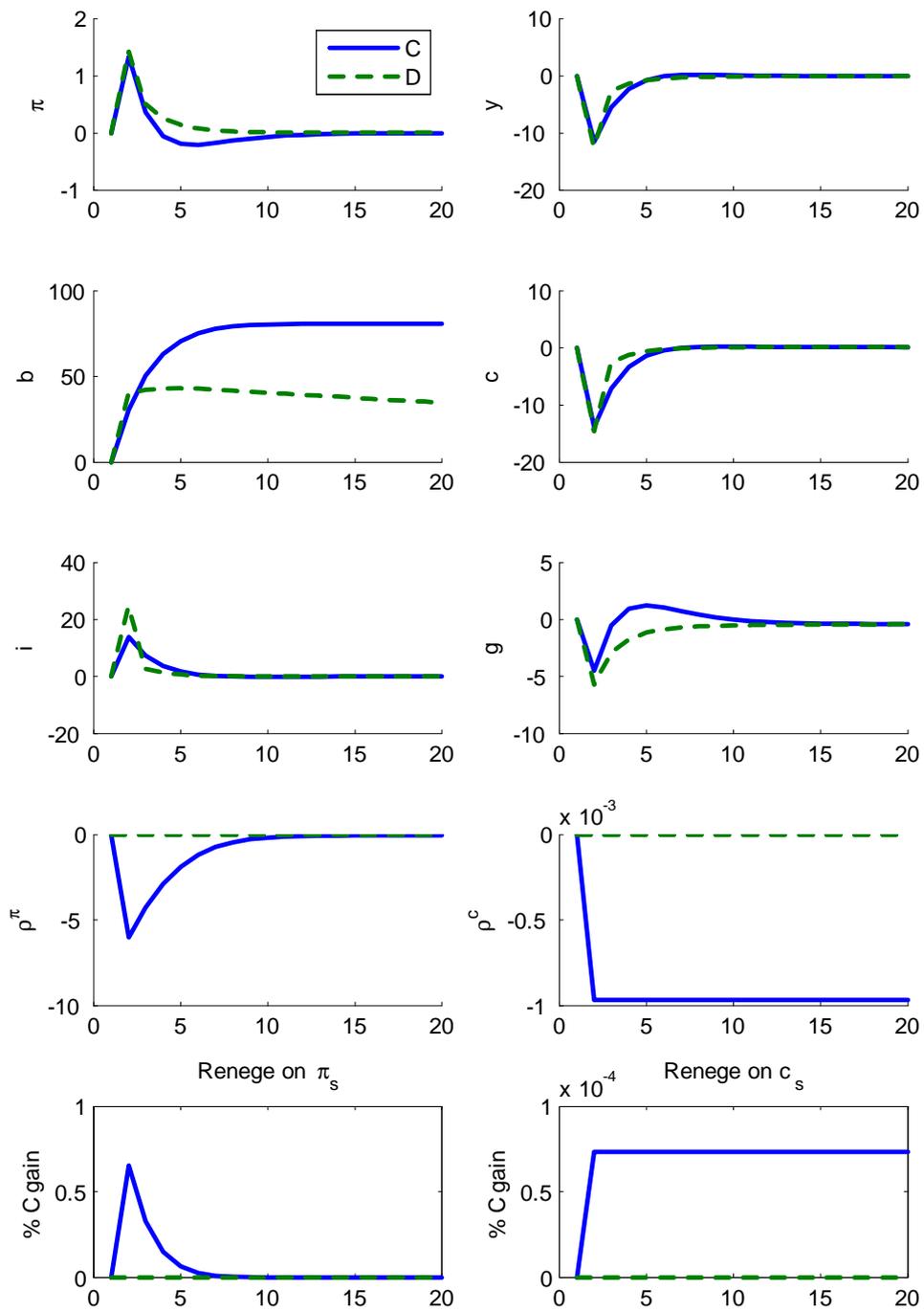


Figure 2: Optimal Policy with a hybrid Phillips curve ($\omega = 0.75$) in a low-debt economy ($B = 0.1$).

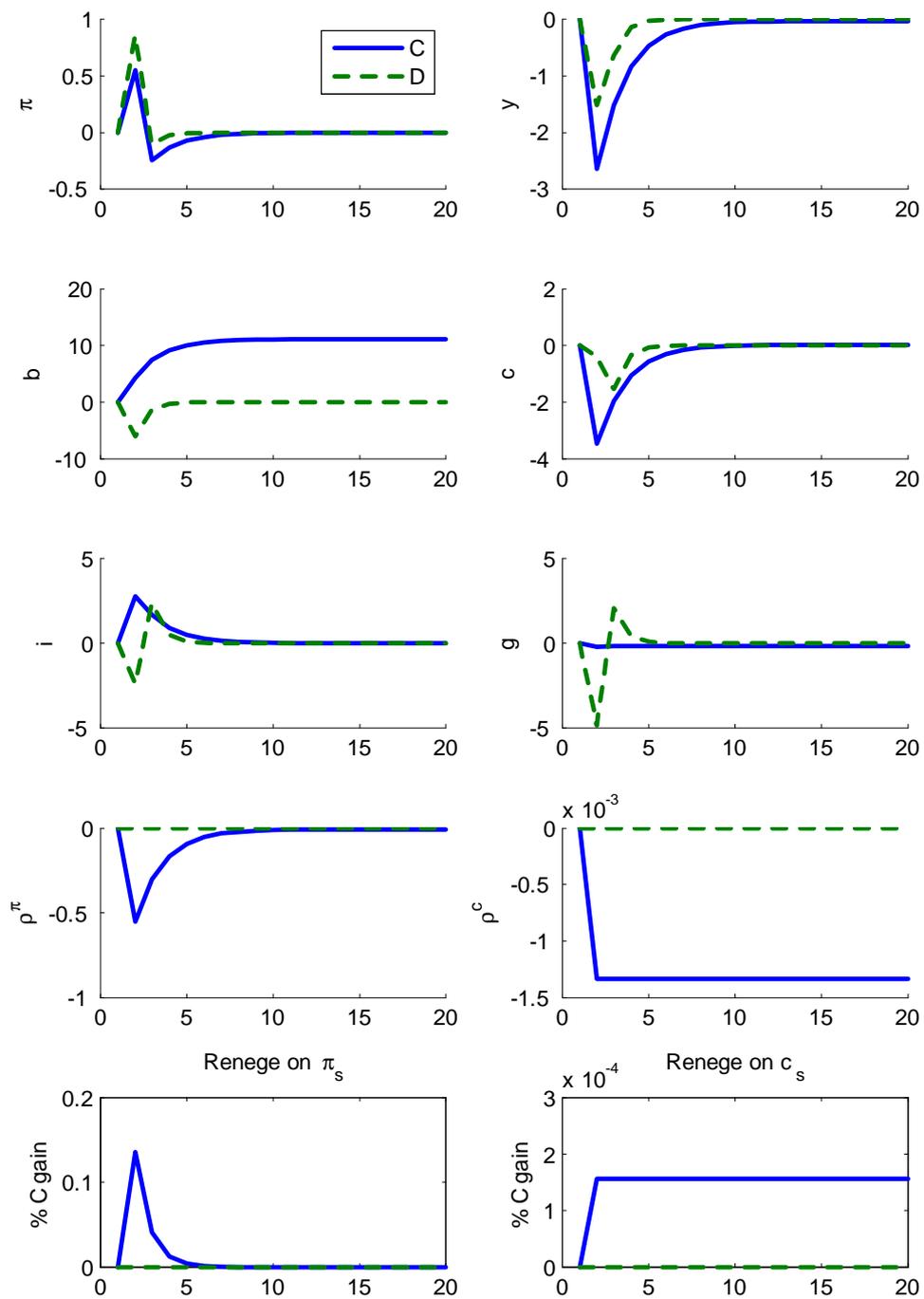


Figure 3: Optimal Policy with a New Keynesian Phillips curve ($\omega = 0$) in a high-debt economy ($B = 0.3$).

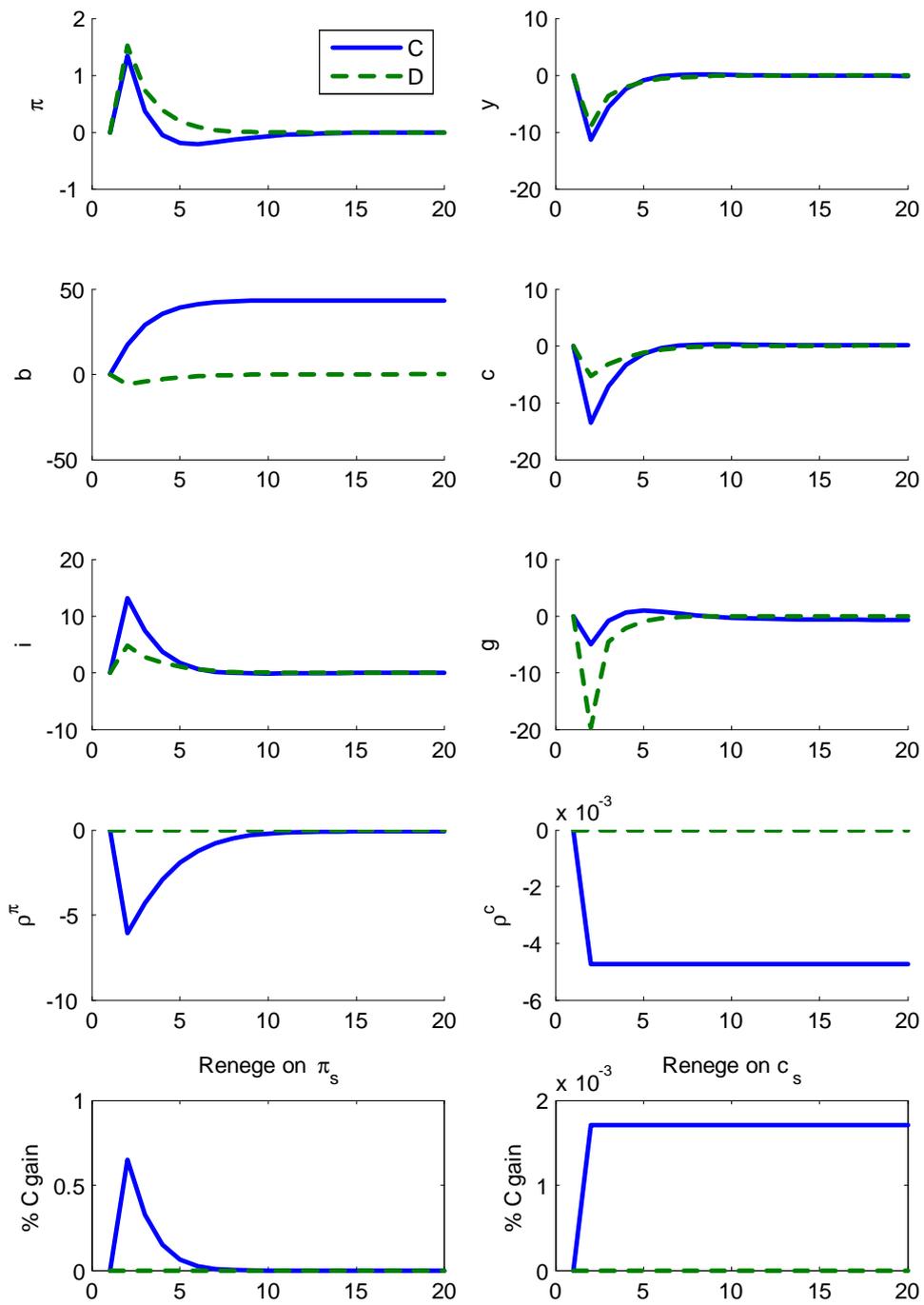


Figure 4: Optimal Policy with a hybrid Phillips curve ($\omega = 0.75$) in a high-debt economy ($B = 0.3$).

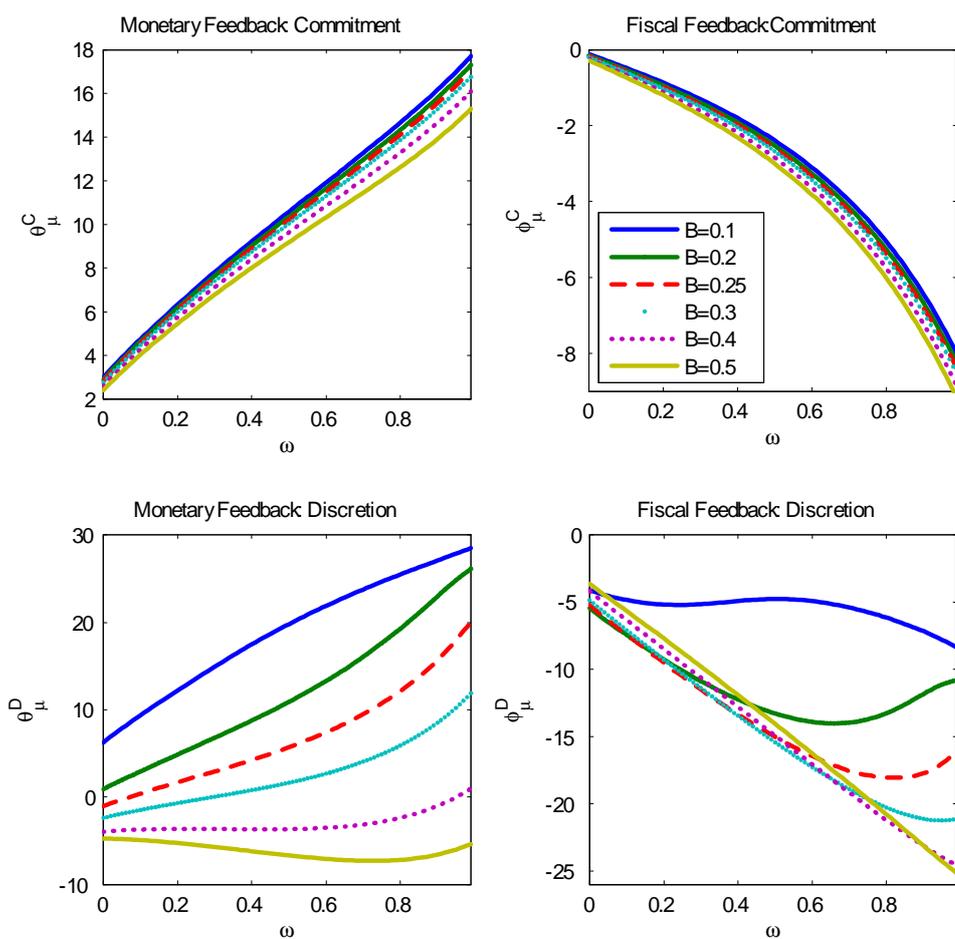


Figure 5: Monetary (θ_μ^i) and fiscal (ϕ_μ^i) feedback coefficients on the cost-push shock under optimal commitment ($i = C$) and discretion ($i = D$) policy.

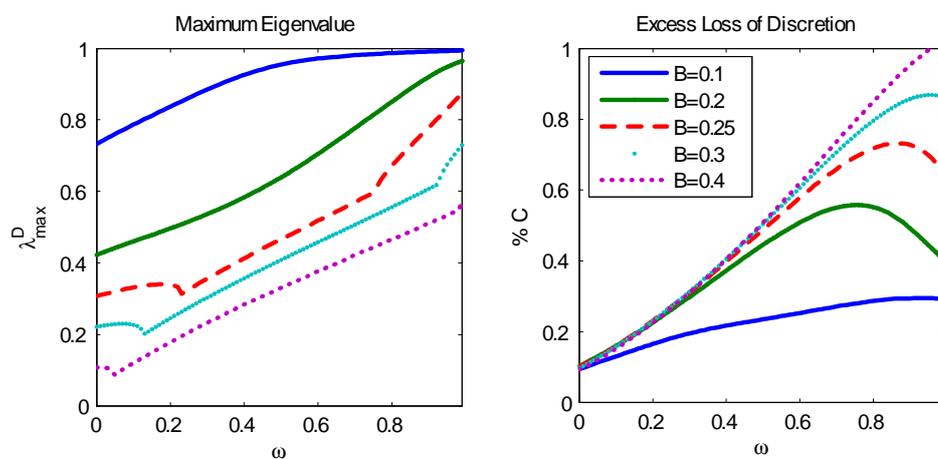


Figure 6: The excess loss of discretion over commitment (in % of steady-state consumption foregone) and the maximum eigenvalue (λ_{\max}^D) under discretion.