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ABSTRACT

Advance-Purchase Discounts as a Price Discrimination Device

In an intertemporal setting in which individual uncertainty is resolved over time, advance-purchase discounts can serve to price discriminate between consumers with different expected valuations for the same product. Consumers with a high expected valuation purchase the product before learning their actual valuation at the offered advance-purchase discount; consumers with a low expected valuation will wait and purchase the good at the regular price only in the event where their realized valuation is high. We provide a necessary and sufficient condition under which the monopolist's optimal intertemporal selling policy features such advance-purchase discounts.

JEL Classification: D42 and L12
Keywords: advance-purchase discount, demand uncertainty, intertemporal pricing, introductory offers, monopoly pricing and price discrimination

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1 Introduction

Advance-purchase discounts (introductory offers, early-booker discounts) are frequently used in the sale of products such as holiday packages, hotel rooms, rental car hires, and, sometimes, airline tickets. In this paper, we provide a novel explanation for such a pricing policy that is based on price discrimination. The starting point of our theory is the observation that consumers are likely to face uncertainty about their valuation when the time of consumption is far ahead in the future. Consider the problem of a monopolist selling a product with a fixed consumption date (e.g., a holiday package, hotel room, rental car hire, airline ticket). If individual uncertainty over the valuation of this product is resolved over time, the monopolist can sell the product at different prior dates and charge different prices so as to price discriminate between consumers.

In our model, consumers can buy either at an early date (before individual uncertainty is resolved) or at a later date (after individual uncertainty is resolved). At the early date, consumers only know their expected valuation, which is heterogeneous across consumers. The monopolist only knows the distribution of expected valuations and ex ante commits to a price path. If the monopolist offers an advance-purchase discount (i.e., an increasing price path), consumers face a trade-off: they can either buy early at a discount or else wait and make their purchasing decision dependent on the actual realization of their valuation. Consumers with a high expected valuation will optimally purchase the good in advance while consumers with a low expected valuation will buy the good at the later date only if their actual valuation exceeds their expected valuation (and the regular price of the good). We provide a necessary and sufficient condition under which the monopolist's optimal intertemporal pricing policy involves such an advance-purchase discount.

Our paper contributes to the literature on intertemporal pricing. Previous work has identified a large class of economic environments in which, absent capacity costs, a monopolist will optimally sell all units at a uniform price over time (Riley and Zeckhauser, 1983; Wilson, 1988; Courty, 2003). Advance-purchase discounts emerge as the optimal selling policy in a number of economic environments where capacity is scarce and the aggregate level of demand is uncertain (Gale and Holmes, 1992, 1993; Dana, 1998, 1999a, 1999b, 2001). Our paper is the first to show the optimality of advance-purchase discounts in a setting in which neither scarce capacity nor aggregate demand uncertainty play any role.[1]

[1] However, in independent recent work, Möller and Watanabe (2007) consider a model with two consumer types in which the seller can ex ante commit to her capacity. They show that advance-purchase discounts can be optimal if capacity costs are zero. Our paper is also related to the analysis of refund contracts by Courty and Li (2001). In their model, there is heterogeneity not only in the mean but also in the variance of consumers' valuations. Before the individual uncertainty is resolved, consumers choose from a menu of contracts which may include fixed-price contracts and contracts with full or partial refunds. If a consumer has chosen a contract with full or partial refund, he can return the product for refund after learning his actual valuation. Courty and Li show that the optimal menu of contracts involves both full and partial refund contracts so as to screen between different types of consumers. In our model, refund contracts are not available but the firm sets prices for both periods. Translated into Courty and Li's setting, this means that partial refund contracts are unavailable.

2 The Model

Consider a monopolist who can choose to sell a product at two different dates, $t = 1$ and $t = 2$, at prices $p_1 \in \mathbb{R}_+ \cup \{\infty\}$ and $p_2 \in \mathbb{R}_+ \cup \{\infty\}$, respectively, and announces her prices prior to $t = 1$. We write $p_t = \infty$ if the monopolist does not make the product available at date t. The monopolist can produce any quantity of the good at constant marginal cost $c \geq 0$.

There is a unit mass of consumers with unit demand. A consumer is described by his type r, which denotes his expected valuation of the good. The consumer's actual valuation of the good is given by

$$v = r + \varepsilon,$$

where ε is the realization of a random variable. This random variable has zero mean and can take a positive value $\sigma > 0$ or a negative value $-\sigma$, each with probability 1/2. We assume that the realization of the random variable is independent across consumers. That is, there is no aggregate uncertainty. At $t = 1$, each consumer privately learns his own type r. At $t = 2$, each consumer privately learns the realization of his valuation v.

Consumption takes place after $t = 2$. If a consumer with valuation v purchases the good (at either date) at price p, his (ex post) surplus from consuming the good is $v - p$. If he does not purchase the good, the surplus is zero.

Consumers differ in their expected valuation r. The cumulative distribution function over r is denoted by $F : \mathbb{R}_+ \to [0, 1]$. We assume that F is continuous on $\mathbb{R}_+$ and has a density f which takes values $f(r) > 0$ for all $r \in (\underline{r}, \overline{r})$, and $f(r) = 0$ for all $r \notin [\underline{r}, \overline{r}]$ and is continuous on $[\underline{r}, \overline{r}]$. Note that we allow for $f(\underline{r})$ and $f(\overline{r})$ to be zero or strictly positive. To avoid negative realizations, we assume that $\underline{r} \geq \sigma$. The upper bound of the support may be finite or infinite: $\overline{r} \in (\underline{r}, \infty) \cup \{\infty\}$.

Throughout this paper we make the monotone hazard rate assumption with respect to F.

Assumption *The hazard rate $f(\cdot)/(1 - F(\cdot))$ is strictly increasing in r on the support of F.*

The monotone hazard rate assumption is satisfied if and only if $1 - F$ is strictly log-concave and holds for a variety of parametric distribution functions (see e.g. Bagnoli and Bergstrom, 2005). Taking limits of the inverse hazard rate, we define

$$\underline{\phi} \equiv \lim_{r \downarrow \underline{r}} \frac{1 - F(r)}{f(r)} \text{ and } \overline{\phi} \equiv \lim_{r \uparrow \overline{r}} \frac{1 - F(r)}{f(r)}.$$

Under the monotone hazard rate assumption, $\underline{\phi} \in \mathbb{R}_+ \cup \{\infty\}$ and $\overline{\phi} \in \mathbb{R}_+$ with $0 \leq \overline{\phi} < \underline{\phi}$.

3 Advance-Purchase Discounts: Analysis and Results

The strategy of the monopolist is given by the price path (p_1, p_2). Each price path is an element of one of three "categories" of selling policies. First, an advance-purchase discount (APD) policy is defined by prices $p_1 < p_2$ that induce strictly positive demand at both dates. Second, an advance-selling policy is defined by an increasing price path (p_1, p_2) with $p_1 < p_2$

that induces positive demand at $t = 1$ only. This advance-selling policy is payoff-equivalent to (p_1, ∞), that is, the monopolist sells the product at $t = 1$ only. Third, a spot-selling policy is defined by a decreasing price path, (p_1, p_2) with $p_1 \geq p_2$. A decreasing price path necessarily induces positive demand at $t = 2$ only. This spot-selling policy is payoff-equivalent to (∞, p_2), that is, the monopolist sells the product at $t = 2$ only.

In the following, we first characterize the profit-maximizing APD policy. We then compare this policy with the profit-maximizing advance-selling and spot-selling policies. Finally, we provide a necessary and sufficient condition under which the monopolist's optimal strategy involves advance-purchase discounts.

We now derive the monopolist's optimal APD policy. Suppose consumers face an increasing price path $p_1 < p_2$. If a consumer of type r purchases the good at $t = 1$, his expected surplus is $r - p_1$. Since the price at date 1 is lower than the price at date 2, a rational consumer of type r will delay his purchasing decision until $t = 2$ only if he does not intend to buy the good in the event where $\varepsilon = -\sigma$. If $r < p_2 - \sigma$, the consumer will not buy the product even when $\varepsilon = \sigma$. If instead $r \geq p_2 - \sigma$, the consumer will purchase the product at $t = 2$ if and only if $\varepsilon = \sigma$, and so his expected surplus is $1/2(r + \sigma - p_2)$. The gain in expected surplus from delaying the purchasing decision until $t = 2$ is therefore $\Lambda(r) \equiv 1/2(r + \sigma - p_2) - (r - p_1)$, which is strictly decreasing in r. Let $\widetilde{r} = \sigma + 2p_1 - p_2$ denote the consumer type who is indifferent between purchasing at $t = 1$ and delaying the purchasing decision, i.e., $\Lambda(\widetilde{r}) = 0$. If $r > \widetilde{r}$, the consumer will optimally purchase the product at $t = 1$; if $r \in [p_2 - \sigma, \widetilde{r}]$, he will purchase at $t = 2$ if and only if $\varepsilon = \sigma$; otherwise, he will not buy at all. Thus, demand at date 1 is $1 - F(\widetilde{r})$, while demand at date 2 is $[F(\widetilde{r}) - F(p_2 - \sigma)]/2$. The monopolist's profit is

$$\pi(p_1, p_2) = (p_1 - c)(1 - F(\widetilde{r})) + \frac{1}{2}(p_2 - c) \begin{cases} F(\widetilde{r}) & \text{if } p_2 \leq \underline{r} + \sigma, \\ [F(\widetilde{r}) - F(p_2 - \sigma)] & \text{otherwise.} \end{cases} \tag{1}$$

Clearly, any profit-maximizing price at date 2 satisfies $p_2 \geq \underline{r} + \sigma$.

Lemma 1 *Consider the set of prices p_1 and p_2 such that demand at both dates is positive, i.e., $p_1 < p_2 < \min\{p_1 + \sigma, 2p_1 + \sigma - \underline{r}\}$. If $\partial\pi(p_1, p_2)/\partial p_1 < 0$, then $\partial\pi(p_1', p_2)/\partial p_1 < 0$ for all $p_1' \geq p_1$. That is, π is quasi-concave in its first argument.*

Proof. Note first that

$$\frac{\partial\pi(p_1, p_2)}{\partial p_1} = 1 - F(2p_1 + \sigma - p_2) - [2(p_1 - c) - (p_2 - c)]\, f(2p_1 + \sigma - p_2).$$

This expression is negative if and only if

$$[2(p_1 - c) - (p_2 - c)] \left\{ \frac{f(2p_1 + \sigma - p_2)}{1 - F(2p_1 + \sigma - p_2)} \right\} > 1. \tag{2}$$

If $\partial\pi(p_1, p_2)/\partial p_1 < 0$, then the term in brackets is positive and strictly increasing in p_1. The term in curly brackets is positive and, from the monotone hazard rate assumption, strictly increasing in p_1. Hence, if $\partial\pi(p_1, p_2)/\partial p_1$ is negative, then it must remain negative after an infinitesimal increase in p_1. But this means that if $\partial\pi(p_1, p_2)/\partial p_1 < 0$, we cannot have that $\partial\pi(p_1', p_2)/\partial p_1 \geq 0$ for $p_1' \geq p_1$. ■

The following lemma characterizes the unique candidate of the profit-maximizing APD policy $(\widehat{p}_1, \widehat{p}_2)$.

Lemma 2 *If $c - \sigma > \underline{r} - \underline{\phi}$ the candidate for the profit-maximizing APD policy, $(\widehat{p}_1, \widehat{p}_2)$, is uniquely determined by*

$$\widehat{p}_2 = c + \frac{1 - F(\widehat{p}_2 - \sigma)}{f(\widehat{p}_2 - \sigma)}; \tag{3}$$

$$\widehat{p}_1 = c + \frac{1}{2}\left\{\frac{1 - F(\widehat{p}_2 - \sigma)}{f(\widehat{p}_2 - \sigma)} + \frac{1 - F(2\widehat{p}_1 - \widehat{p}_2 + \sigma)}{f(2\widehat{p}_1 - \widehat{p}_2 + \sigma)}\right\}. \tag{4}$$

If $c - \sigma \leq \underline{r} - \underline{\phi}$ the candidate for the profit-maximizing APD policy, $(\widehat{p}_1, \widehat{p}_2)$, is uniquely determined by

$$\widehat{p}_2 = \underline{r} + \sigma; \tag{5}$$

$$\widehat{p}_1 = \frac{\underline{r} + \sigma + c}{2} + \frac{1 - F(2\widehat{p}_1 - \underline{r})}{2f(2\widehat{p}_1 - \underline{r})}. \tag{6}$$

In both cases, if the price path of the optimal selling policy induces positive demand at each date, $(\widehat{p}_1, \widehat{p}_2)$ constitutes the optimal APD policy.

Proof. The two first-order conditions of profit maximization yield

$$2(\widehat{p}_1 - c) = \widehat{p}_2 - c + \frac{1 - F(2\widehat{p}_1 - \widehat{p}_2 + \sigma)}{f(2\widehat{p}_1 - \widehat{p}_2 + \sigma)} \tag{7}$$

and

$$\widehat{p}_2 - c = \frac{2(\widehat{p}_1 - c)f(2\widehat{p}_1 - \widehat{p}_2 + \sigma) + F(2\widehat{p}_1 - \widehat{p}_2 + \sigma) - F(\widehat{p}_2 - \sigma)}{f(\widehat{p}_2 - \sigma) + f(2\widehat{p}_1 - \widehat{p}_2 + \sigma)}. \tag{8}$$

Inserting (7) into (8), and simplifying, we obtain (3). Recall that the optimal APD policy must be such that $\widehat{p}_2 \in [\underline{r} + \sigma, \overline{r} + \sigma)$. Note that the LHS of (3) is strictly increasing in $\widehat{p}_2$, while (by the monotone hazard rate assumption) the RHS is strictly decreasing in $\widehat{p}_2$. Continuity then implies that if $\widehat{p}_2 \in [\underline{r}+\sigma, \overline{r}+\sigma)$, it is uniquely determined by equation (3). Inserting (3) into (7) yields equation (4). By definition, an APD policy involves positive demand at both dates, i.e., $\widetilde{r} = 2\widehat{p}_1 - \widehat{p}_2 + \sigma \in (\widehat{p}_2 - \sigma, \overline{r})$. The RHS of (4) is strictly increasing in $\widehat{p}_1$, while (by the monotone hazard rate assumption) the RHS is strictly decreasing in $\widehat{p}_1$. Continuity then implies that if $\widetilde{r} \in (\widehat{p}_2 - \sigma, \overline{r})$, $\widehat{p}_1$ is uniquely determined by equation (4). Note also that the monotone hazard rate assumption implies that $\widehat{p}_1 < \widehat{p}_2$. To see this, suppose otherwise that $\widehat{p}_1 \geq \widehat{p}_2$. From (3) and (4), it follows that $[1 - F(\widehat{p}_2 - \sigma)]/f(\widehat{p}_2 - \sigma) \leq [1 - F(2\widehat{p}_1 - \widehat{p}_2 + \sigma)]/f(2\widehat{p}_1 - \widehat{p}_2 + \sigma)$. From the monotone hazard rate assumption, $\widehat{p}_2 - \sigma \geq 2\widehat{p}_1 - \widehat{p}_2 + \sigma$, i.e., $\widehat{p}_2 \geq \widehat{p}_1 + \sigma$, contradicting that $\widehat{p}_1 \geq \widehat{p}_2$.

The unique solution to (3) satisfies $\widehat{p}_2 > \underline{r} + \sigma$ if and only if $c - \sigma > \underline{r} - \underline{\phi}$. Otherwise, if $c - \sigma \leq \underline{r} - \underline{\phi}$, then $\widehat{p}_2$ must be given by the corner solution $\widehat{p}_2 = \underline{r} + \sigma$. Substituting $\widehat{p}_2$ in (7) and rewriting yields (6). The RHS of (6) is strictly increasing in $\widehat{p}_1$, while (by the monotone hazard rate assumption) the RHS is and strictly decreasing in $\widehat{p}_1$ for $\widehat{p}_1 \leq (\overline{r}+\underline{r})/2$. If $\widetilde{r} \in (\underline{r}, \overline{r})$, continuity implies that $\widehat{p}_1$ is uniquely determined by (6) and $\widehat{p}_1 < \widehat{p}_2$.[2] ■

It can easily be verified that the price path $(\widehat{p}_1, \widehat{p}_2)$ induces positive demand at both dates only if $c < \overline{r} - \sigma$, i.e., only if it is efficient to sell to the highest type(s) even when $\varepsilon = -\sigma$.

We now turn to the characterization of the profit-maximizing advance-selling policy where, by definition, demand at $t = 2$ is zero. Under such an advance-selling policy, the monopolist's profit can be written as

$$\pi(p_1, \infty) = (p_1 - c)(1 - F(\widetilde{\widetilde{r}}))$$

where $\widetilde{\widetilde{r}} = p_1$ denotes the consumer type who is indifferent between purchasing at $t = 1$ and not purchasing at all. Accordingly, the monopolist solves a standard monopoly problem with demand function $1 - F(p)$. Let $p^a \equiv \arg\max_{p_1} \pi(p_1, \infty)$ denote the profit-maximizing advance-selling price. From the first-order condition,

$$p^a = c + \frac{1 - F(p^a)}{f(p^a)} \tag{9}$$

By the monotone hazard rate assumption, p^a is uniquely determined by (9) if $c > \underline{r} - \underline{\phi}$. If the reverse inequality holds, $c \leq \underline{r} - \underline{\phi}$, then the profit-maximizing advance-selling price is $p^a = \underline{r}$.

Intuitively, the monopolist will prefer APD over advance selling (i) if marginal cost is high (since, in this case, it is inefficient to sell to low types when the realization of ε is negative) and (ii) if consumers face a great degree of uncertainty about their valuation, i.e., σ is large (since, in this case, it becomes attractive for the monopolist to sell to low types only when the realization of ε is positive). The following lemma confirms this intuition.

Lemma 3 *There exists an APD policy that yields larger profits to the monopolist than the optimal advance-selling policy* (p^a, ∞) *if and only if* $c + \sigma > \underline{r} - \underline{\phi}$.

Proof. Suppose first that $c > \underline{r} - \underline{\phi}$, and so $p^a > \underline{r}$. In this case, the profit from advance selling is equal to $\pi(p^a, p^a + \sigma)$. We show that $\lim_{p_2 \uparrow p^a + \sigma}[\partial\pi(p^a, p_2)/\partial p_2] < 0$: starting from $p_1 = p^a$ and $p_2 = p^a + \sigma$ (where demand at $t = 2$ is zero), a firm can increase its profit by marginally reducing its price at date 2 and thereby making sales at date 2.

$$\begin{aligned}
& \lim_{p_2 \uparrow p^a + \sigma} \frac{\partial\pi(p^a, p_2)}{\partial p_2} \\
= \; & \frac{d}{dp_2}\left[(p^a - c)\left[1 - F(\sigma + 2p^a - p_2)\right] + \frac{1}{2}(p_2 - c)\left[F(\sigma + 2p^a - p_2) - F(p_2 - \sigma)\right]\right]\Bigg|_{p_2 = p^a + \sigma} \\
= \; & (p^a - c)f(p^a) + \frac{1}{2}(p^a + \sigma - c)\left[-f(p^a) - f(p^a)\right] \\
= \; & -\sigma f(p^a) \\
< \; & 0.
\end{aligned}$$

[2] We also note that, evaluated at $\widehat{p}_1 = \underline{r}$, the RHS of (6) is greater or equal to the LHS since $c - \sigma \leq \underline{r} - \underline{\phi}$. Note also that the RHS of (6) evaluated at $\widehat{p}_1 = \underline{r} + \sigma$ is $\{[\underline{r} + \sigma + c] + [1 - F(\underline{r} + 2\sigma)]/f(\underline{r} + 2\sigma)\}/2$, which is larger than $\underline{r} + \sigma$ if and only if $\underline{r} + \sigma > c + [1 - F(\underline{r} + 2\sigma)]/f(\underline{r} + 2\sigma)$, which, because of the monotone hazard rate assumption, is implied by $\underline{r} + \sigma > c + \underline{\phi}$.

Suppose now that $c \leq \underline{r} - \underline{\phi}$, and so $p^a = \underline{r}$. Again, the profit from advance selling is equal to $\pi(p^a, p^a + \sigma)$, but now with $p^a = \underline{r}$. We show that $\lim_{p_1 \downarrow \underline{r}}[\partial \pi(p_1, \underline{r} + \sigma)/\partial p_1] > 0$ if and only if $c + \sigma > \underline{r} - \underline{\phi}$: starting from $p_1 = \underline{r}$ and $p_2 = \underline{r} + \sigma$ (where demand at $t = 2$ is zero), by marginally increasing its price at date 1 (and thereby making sales at both dates), a firm can increase its profit if and only if $c + \sigma > \underline{r} - \underline{\phi}$. To see the "if" part, note that

$$\lim_{p_1 \downarrow \underline{r}} \frac{\partial \pi(p_1, \underline{r} + \sigma)}{\partial p_1} = 1 - [\underline{r} - c - \sigma] \, f(\underline{r}),$$

which is positive if and only if $c + \sigma > \underline{r} - \underline{\phi}$. To see the "only if" part, note that, under our assumption that $c + \sigma \leq \underline{r} - \underline{\phi}$, the candidate for the optimal APD policy is such that $p_2 = \underline{r} + \sigma$. Hence, the this policy and the optimal advance-selling policy differ only in the price at date 1. The assertion then follows from our earlier observation that π is quasi-concave in p_1 for a fixed p_2 (Lemma 1): if $\lim_{p_1 \downarrow \underline{r}}[\partial \pi(p_1, \underline{r} + \sigma)/\partial p_1] < 0$, then $\partial \pi(p_1, \underline{r} + \sigma)/\partial p_1 < 0$ for all $p_1 \geq \underline{r}$. ■

Note that if $f(\underline{r})$ is sufficiently small, there exists an APD policy that necessarily yields higher profits than advance selling.

We now turn to the characterization of the profit-maximizing spot-selling policy where, by definition, demand at $t = 1$ is zero. Let $p^s \equiv \arg\max_{p_2} \pi(\infty, p_2)$ denote the profit-maximizing spot price. Suppose first that the spot price satisfies $p^s < \overline{r} - \sigma$ (i.e., there are some consumer types that consume independently of the realization of ε). In this case, the monopolist's profit can be written as

$$\begin{aligned} \pi(\infty, p_2) &= \frac{1}{2}(p_2 - c) \left[1 - F(p_2 - \sigma)\right] + \frac{1}{2}(p_2 - c) \left[1 - F(p_2 + \sigma)\right] \\ &= (p_2 - c) \left[1 - F(p_2 + \sigma)\right] + \frac{1}{2}(p_2 - c) \left[F(p_2 + \sigma) - F(p_2 - \sigma)\right]. \end{aligned}$$

From the first-order condition, we obtain that the profit-maximizing spot price satisfies

$$p^s = c + \frac{[1 - F(p^s + \sigma)] + [1 - F(p^s - \sigma)]}{f(p^s + \sigma) + f(p^s - \sigma)}. \tag{10}$$

Note that $p^s < \overline{r} - \sigma$ if and only if $c + \sigma < \overline{r} - [1 - F(\overline{r} - 2\sigma)]/f(\overline{r} - 2\sigma)$.

Suppose now that $c + \sigma \geq \overline{r} - [1 - F(\overline{r} - 2\sigma)]/f(\overline{r} - 2\sigma)$, and so $p^s \geq \overline{r} - \sigma$, i.e., each consumer buys the good only when $\varepsilon = \sigma$ if at all. In this case, the monopolist's profit can be written as

$$\pi(\infty, p_2) = \frac{1}{2}(p_2 - c) \left[1 - F(p_2 - \sigma)\right],$$

and so the profit-maximizing spot price p^s satisfies

$$p^s = \begin{cases} c + \frac{1 - F(p^s - \sigma)}{f(p^s - \sigma)} & \text{if } c - \sigma > \underline{r} - \underline{\phi}, \\ \underline{r} + \sigma & \text{otherwise.} \end{cases} . \tag{11}$$

Note that this spot price coincides with the price at $t = 2$ under the optimal APD policy, $p^s = \widehat{p}_2$, as can be seen from equations (3) and (5).

The following lemma shows that spot selling yields higher profits than advance-purchase discounts if and only if it is socially efficient to serve even a consumer of type $\overline{r}$ only when $\varepsilon = \sigma$.

Lemma 4 *There exists an APD policy that yields larger profits to the monopolist than the optimal spot-selling policy* (∞, p^s) *if and only if* $c + \sigma < \overline{r} - \overline{\phi}$.

Proof. Suppose first that $p^s < \overline{r} - \sigma$ (which can hold only if $c + \sigma < \overline{r} - \overline{\phi}$). Since $\pi(\infty, p^s) = \pi(p^s, p^s)$, spot selling is not optimal if

$$\lim_{p_1 \uparrow p^s} \partial\pi(p_1, p^s)/\partial p_1 < 0,$$

i.e., if introducing a small advance-purchase discount, holding the date-2 price fixed at p^s, increases the monopolist's profit. We have

$$\begin{aligned} & \lim_{p_1 \uparrow p^s} \frac{\partial\pi(p_1, p^s)}{\partial p_1} \\ = \quad & [1 - F(p^s + \sigma)] + (p^s - c)\,[-2f(p^s + \sigma)] + \frac{1}{2}(p^s - c)2f(p^s + \sigma) < 0, \end{aligned}$$

which is equivalent to

$$p^s - c > \frac{1 - F(p^s + \sigma)}{f(p^s + \sigma)}.$$

Using (10), this inequality can be rewritten as

$$\frac{[1 - F(p^s + \sigma)] + [1 - F(p^s - \sigma)]}{f(p^s + \sigma) + f(p^s - \sigma)} > \frac{1 - F(p^s + \sigma)}{f(p^s + \sigma)},$$

which is equivalent to

$$\frac{1 - F(p^s - \sigma)}{f(p^s - \sigma)} > \frac{1 - F(p^s + \sigma)}{f(p^s + \sigma)}.$$

But this inequality is implied by the monotone hazard rate assumption.

Suppose second that $p^s \geq \overline{r} - \sigma$. Under the parameter restriction that $c + \sigma < \overline{r} - \overline{\phi}$ consider the solution to the optimal spot-selling policy and show that selling to the highest types close to $\overline{r}$ at date $t = 1$ at price $\widetilde{p}_1 = (\overline{r} - \sigma + p_2)/2 - \varepsilon/2$ increases the firm's profit for ε small,

$$\begin{aligned} & \frac{\partial\pi(\widetilde{p}_1, p_2)}{\partial p_1} \\ = \quad & [1 - F(\overline{r} - \varepsilon)] - 2(\widetilde{p}_1 - c)f(\overline{r} - \varepsilon) + (p_2 - c)2f(\overline{r} - \varepsilon) \\ = \quad & [1 - F(\overline{r} - \varepsilon)] + f(\overline{r} - \varepsilon)(-\overline{r} + \sigma + c) < 0 \end{aligned}$$

for ε small. Hence, rearranging and taking the limit as $\varepsilon \to 0$ we must have

$$\overline{\phi} = \lim_{r \uparrow \overline{r}} \frac{1 - F(r)}{f(r)} < \overline{r} - \sigma - c$$

which is equivalent to the above parameter restriction. Hence, it cannot be optimal for the monopolist to sell at date $t = 2$ only.

Consider now the reverse parameter restriction that $c+\sigma \geq \overline{r}-\overline{\phi}$ or, equivalently, $\overline{r}-(c+\sigma) \leq \overline{\phi}$. Consider any APD policy $(\widetilde{p}_1, \widetilde{p}_2)$, which (by definition) induces positive demand in each period. From equation (2), $\partial\pi(\widetilde{p}_1, \widetilde{p}_2)/\partial p_1 > 0$ if and only if

$$\widetilde{r} - (c + \sigma) < \frac{1 - F(\widetilde{r})}{f(\widetilde{r})}, \tag{12}$$

where $\widetilde{r} = 2\widetilde{p}_1 - \widetilde{p}_2 + \sigma \in (\underline{r}, \overline{r})$. The LHS is increasing in $\widetilde{r}$ while, by the monotone hazard rate assumption, the RHS is strictly decreasing in $\widetilde{r}$ and converges to $\overline{\phi}$ as $\widetilde{r} \to \overline{r}$. Hence the inequality is implied by $\overline{r} - (c + \sigma) \leq \overline{\phi}$, as postulated. By continuity of π, we thus have

$$\pi(\widetilde{p}_1, \widetilde{p}_2) < \pi(\widetilde{p}_2, \widetilde{p}_2) = \pi(\infty, \widetilde{p}_2) \leq \pi(\infty, p_2^s),$$

i.e., any APD policy yields strictly lower profits than the optimal spot-selling policy. ■

Using Lemmas 2, 3, and 4 we obtain our main result.

Proposition 1 *The monopolist's optimal selling policy involves advance-purchase discounts, characterized in Lemma 2, if and only if* $\underline{r} - \underline{\phi} < c + \sigma < \overline{r} - \overline{\phi}$.

Proof. By Lemma 3 there exist APD policies that yield higher profits than the profit-maximizing advance-selling policy if and only if $\underline{r} - \underline{\phi} < c + \sigma$; by Lemma 4 there exist APD policies that yield higher profits than the profit-maximizing spot-selling policy if and only if $c+\sigma < \overline{r}-\overline{\phi}$. This implies that the optimal selling policy must induce positive demand at each date. In this case the monopolist's optimal selling policy is the APD policy $(\widehat{p}_1, \widehat{p}_2)$, which is characterized in Lemma 2. ■

The proposition implies that, for advance-purchase discounts to be the monopolist's optimal selling policy, there must be sufficient heterogeneity in the expected valuation r. In the limit as $\underline{r} \to \sigma$ and $\overline{r} \to \infty$, the profit-maximizing selling policy is necessarily an APD policy. In contrast, as heterogeneity in the expected valuation disappears, $\overline{r} - \underline{r} \to 0$, advance-purchase discounts are never optimal.[3]

4 Conclusion

In this paper, we have provided a novel theory of advance-purchase discounts in which advance-purchase discounts serve as a pure price discrimination device. In contrast to existing explanations of advance-purchase discounts, our theory does not rely on scarce capacity or aggregate demand uncertainty. The key feature of our theory is that consumers face individual uncertainty over their future valuation for the good and this uncertainty is resolved over time. This allows a monopoly seller to charge different prices for the same product at different dates prior

[3]This confirms the finding by Courty (2003) who considers the case where all consumers have the same expected valuation, which corresponds to $\underline{r} = \overline{r}$.

to consumption. Consumers with a high expected valuation will purchase the product in advance at a discount while consumers with a low expected valuation will delay their purchasing decision and buy at the regular price only when their realized valuation turns out to be high. Assuming that the distribution of expected valuations has a monotone hazard rate, we obtain a necessary and sufficient condition under which the monopolist's optimal strategy involves advance-purchase discounts.

Our analysis has been motivated by the selling policies for holiday packages, hotel rooms, and rental car hires. Another group of examples are tickets for the soccer world cup and other sports or cultural events. For instance, consider the ticket sale for a particular match. Consumers are willing to pay a premium in the event that their favorite team or player makes it into this match. Thus, the organizers of the tournament can use advance-purchase discounts as a price discrimination strategy (provided secondary markets can be dried out, which may be achieved by "personalizing" tickets as in the 2006 soccer world cup). Our analysis also extends to the introduction of new experience goods where consumers face uncertainty about the product's characteristics. For our theory to be directly applicable, this uncertainty must reflect horizontal taste heterogeneity. In such a setting, consumers with a high expected valuation tend to buy early while those with a lower expected valuation wait and buy only if the actual product characteristics fit their taste.[4]

[4] An example is the practice by premium vineries to post subscription prices for wine; that is, consumers are offered to buy the vintage at a discount before it is bottled. A buyer who buys such a subscription cannot rely on taste tests and recommendations since the product does not yet exist.

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